C.P. No. 996

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MINISTRY OF TECHNOLOGY

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CURRENT PAPERS

Correlation of Voltage-Current Characteristics of Wall-Stabilised, Free-Burning and Cross-Flow Arcs

by

W. T. Lord

LONDON. HER MAJESTY'S STATIONERY OFFICE

1968

PRICE 5s 6d NET

U.D.C. No. 621.365.2 : 537.311 : 536.2 : 538.6 : 621.3.015.1

C.P. No.996*
April 1967

CORRELATION OF VOLTAGE-CURRENT CHARACTERISTICS OF WALL-STABILISED, FREE-BURNING AND CROSS-FLOW ARCS

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SUMMARY

A comparison is made between non-dimensional forms of the voltage-current characteristics of wall-stabilised, free-burning and cross-flow arcs. Effects of representative length-scale, external heat transfer, the relation between electric conductivity and heat-flux potential, and radiation are displayed.

*Replaces R.A.E. Technical Report 67087 - A.R.C. 29326

CONTENTS

1 INTRODUCTION

This paper attempts to show how the voltage-current characteristics of the wall-stabilised arc, the free-burning arc and the arc in a transverse magnetic field can be correlated by using non-dimensional parameters. It also indicates how a voltage-current characteristic is built up from different components which depend on the current range, the type of arc (governed by the outermost boundary conditions), the properties of the gas and whether radiation plays a part or not. The gas considered specifically is nitrogen, and it is assumed that the properties of air are sufficiently similar to the properties of nitrogen that they may be regarded as the same for present purposes. The spirit of the paper follows that of Suits and Poritsky[1], and its theme is developed by using results from a larger collection of notes on arc theory[2].

By the voltage-current characteristic of a uniform arc column is meant the plot of electric field (or voltage gradient) E against current I (Fig.1). At first sight E-I plots look much the same whatever type of arc they refer to, and whatever the gas. The main qualitative distinction is whether there is a portion of rising characteristic or not. It will appear that this resemblance between all characteristics is in some ways genuine and in some ways misleading. It is usual in practice to consider the graph of E against I, and if this is made on log-log paper and the plot is turned through 45° in a counter clockwise direction the result is a plot which shows EI against I/E; that is, a plot of the power gradient against the inverse of the resistance gradient (commonly and loosely called the conductance). Theory generally gives the relation between EI and I/E first, and henceforth in this paper the phrase voltage-current characteristic implies such a relation. One aim of the paper is to express voltage-current characteristics in non-dimensional form; since there is not yet a standard notation for the non-dimensional parameters involved, it is necessary to introduce some rather unfamiliar symbols.

Use is made of the heat-flux potential φ, which is the integral of the thermal conductivity κ with respect to the temperature T and is a function of temperature if thermal conductivity depends only on temperature:

$$\varphi = \int_0^T \kappa \, dT \quad . \qquad (1)$$

One of the properties of a gas which has an important bearing on the behaviour of an arc is the relation between the electric conductivity σ and the heat-flux potential (Fig. 2). The concept of a definite arc boundary, the periphery, is introduced by stipulating that the electric conductivity is identically zero for values of the heat-flux potential below the peripheral value $\hat{\varphi}$. There exists the corresponding peripheral value of temperature $\hat{T}$, which may be derived from $\hat{\varphi}$ when the φ-T relationship is known. However, $\hat{\varphi}$ can be derived more directly than $\hat{T}$ since $\hat{\varphi}$ is regarded as a parameter to be determined from an analysis of experimental arc characteristics, and it is associated with how the σ-φ curve starts at the periphery. The initial curve is taken to be the straight line

$$\sigma = \alpha(\varphi-\hat{\varphi}) \quad . \tag{2}$$

The parameter α, which is the gradient at the periphery of the electric conductivity with respect to the heat-flux potential, is also regarded as an empirical constant. The periphery and the linear σ-φ law may be thought of as a means of bridging the regime of thermal non-equilibrium in order that thermal equilibrium theory may be started in a definite manner. For arcs of low power gradient, which implies that the temperature inside the arc does not rise much above the peripheral temperature, the linear σ-φ law is sufficient, but for arcs of higher power gradient an extended and more realistic σ-φ law must be taken up to the appropriate value of heat-flux potential. Mathematically, it will be assumed that the σ-φ law is then

$$\frac{\sigma}{\bar{\sigma}} = f_\sigma\left(\frac{\varphi-\hat{\varphi}}{\bar{\varphi}}\right) \quad , \tag{3}$$

where $\bar{\varphi}$ and $\bar{\sigma}$ are reference values needed to express the σ-φ law in non-dimensional form; when $(\varphi-\hat{\varphi})/\bar{\varphi}$ is small, $f_\sigma = (\varphi-\hat{\varphi})/\bar{\varphi}$ and hence $\bar{\sigma}/\bar{\varphi} = \alpha$.

In addition, for arcs of higher power gradient it may be necessary to take into account the loss of energy from the arc by radiation. Here, radiation is treated by considering only the bulk radiation loss per unit volume, denoted by ψ. This has a stronger dependence on pressure than the electric conductivity and for the sake of simplicity the pressure effect is separated from the temperature effect by writing

$$\psi = \left(\frac{p}{\bar{p}}\right)^m \psi' \quad , \tag{4}$$

where $\bar{p}$ is a reference pressure used to preserve the same dimensions for ψ' as for ψ no matter what the value of m; m = 5/4 is appropriate in a moderate range of φ. ψ' is treated as a function of heat-flux potential only, as in Fig.3. The radiated power density is very small for temperatures not much higher than the peripheral temperature and to illustrate this point ψ' in such a temperature range is taken to be

$$\psi' = \beta(\varphi-\hat{\varphi})^2 \quad , \tag{5}$$

where β is a parameter to be determined empirically. It follows that radiation may be neglected for arcs with low power gradient because low power gradient implies low temperature inside an arc. The more general form of ψ'-φ law is taken to be

$$\frac{\psi'}{\bar{\psi}} = f_{\psi}\left(\frac{\varphi-\hat{\varphi}}{\bar{\varphi}}\right) \quad , \tag{6}$$

where $\bar{\psi}$ is a reference value and when $(\varphi-\hat{\varphi})/\bar{\varphi}$ is small $f_{\psi} = [(\varphi-\hat{\varphi})/\bar{\varphi}]^2$ so $\bar{\psi}/\bar{\varphi}^2 = \beta$.

This paper considers three kinds of arc: the wall-stabilised arc (Fig.4), the free-burning arc (Fig.5) and the arc in a cross flow and a transverse magnetic field (Fig.6). (In principle the free-burning arc should be regarded as being in a suitable transverse magnetic field so that its centre line is straight.) The prescribed dimensional parameters are shown in the figures. The arc in a magnetic field is considered to be held at rest against an imposed flow by the applied field and henceforth this arc will be referred to as a cross-flow arc because it will be the velocity of the imposed flow which is taken as an important parameter and not the magnitude of the applied magnetic field. This enables the discussion to be confined to voltage-current characteristics, but the cross-flow arc not only has an electric characteristic but a magnetic characteristic also. Indeed, for the cross-flow arc the question of the relation of the applied field to the imposed velocity and the current is of very great importance.

These three kinds of arc are dealt with in the following way. First, they are all treated in the case of low power gradient, and the idea of a representative length-scale for a particular kind of arc is introduced; this length-scale plays a very important part in the construction of similarity parameters. Then arcs of higher power gradient are considered, it being shown how the voltage-current characteristic is influenced in turn by the different ways in which heat is transferred from the arc to the environment, by the σ-φ law, and by radiation.

2 ARCS OF LOW POWER GRADIENT

2.1 Wall-stabilised arc

2.1.1 Internal solution

First consider the wall-stabilised arc in a circular tube. The theory of the inside of a circular static arc with a linear σ-φ law is now standard. The distribution of heat-flux potential inside the arc is a Bessel function and any information required about the inside of the arc can be obtained exactly. In particular, the radius $\hat{r}$ of the arc periphery and the total amount of heat being conducted across the periphery per unit length per unit time can be identified and related to E and I. The results needed here are the expressions for the internal energy balance and Ohm's law, which may be written respectively

$$EI = Q \quad , \tag{7}$$

$$I/E = \left(\frac{2}{2.405}\right)^2 a \frac{Q}{4\pi} \pi \hat{r}^2 \quad , \tag{8}$$

where Q is the amount of heat being conducted across the periphery per unit length per unit time.

2.1.2 External heat transfer

Outside the arc the solution for the heat-flux potential is a logarithm and in particular it is easy to show that Q is related to the radius of the tube r_∞ and the radius of the arc $\hat{r}$ by

$$\frac{Q}{4\pi(\hat{\varphi}-\varphi_\infty)} = \frac{1}{\log(r_\infty^2/\hat{r}^2)} \quad , \tag{9}$$

where $(\hat{\varphi}-\varphi_\infty)$ is the difference of heat-flux potential between the arc periphery and the tube. This result may be written in the form

$$S = \exp\left(-\frac{1}{N}\right) \quad , \tag{10}$$

where the non-dimensional parameters S and N are defined by

$$S = \left(\frac{\hat{r}}{r_\infty}\right)^2 \quad , \tag{11}$$

$$N = \frac{Q}{4\pi(\hat{\varphi}-\varphi_{\infty})} \quad . \tag{12}$$

N is a Nusselt number for the arc and this definition is used here for all types of arc, with φ_{∞} denoting the ambient heat-flux potential in all cases. S is a non-dimensional form of the arc radius and this definition changes with change of the type of arc. To facilitate such a change (11) is now written as

$$S = \left(\frac{\hat{r}}{\tilde{r}}\right)^2 \quad , \tag{13}$$

where

$$\tilde{r} = r_{\infty} \quad , \tag{14}$$

and this latter relation is interpreted by stating that the representative length of a wall-stabilised arc is the radius of the stabilising tube r_{∞}.

2.1.3 Voltage-current characteristic

By combining the above relation for the external heat transfer with those for the internal energy balance and Ohm's law it is possible to derive the voltage-current characteristic for a wall-stabilised arc of low power gradient. To obtain the characteristic in non-dimensional form the symbols J and K, denoting the non-dimensional forms of the power gradient EI and the inverse of the resistance gradient I/E, are introduced. J and K are given by

$$J = \frac{EI}{\tilde{Q}} \quad , \tag{15}$$

$$K = \frac{I/E}{\tilde{\sigma}\,\pi\tilde{r}^2} \quad , \tag{16}$$

where the representative quantities $\tilde{Q}$ and $\tilde{\sigma}$ are given by

$$\tilde{Q} = 4\pi\,(\hat{\varphi}-\varphi_{\infty}) \quad , \tag{17}$$

$$\tilde{\sigma} = \left(\frac{2}{2.405}\right)^2 \alpha(\hat{\varphi}-\varphi_{\infty}) \quad . \tag{18}$$

It will be noted later that $\tilde{Q}$ and $\tilde{\sigma}$ may be deduced from experimental measurements, but for the present purposes they will be eliminated from consideration by noting that if the gas composition is kept fixed and the ambient temperature of the tube is kept constant then both $\tilde{Q}$ and $\tilde{\sigma}$ are constant. Hence, under these conditions

$$J \propto EI \quad , \tag{19}$$

$$K \propto \frac{I/E}{\tilde{r}^2} \quad . \tag{20}$$

From (7) and (8), and (11) and (12) and (15) to (18), the non-dimensional forms of the internal energy balance and Ohm's law are found to be

$$J = N \quad , \tag{21}$$

$$K = NS \quad . \tag{22}$$

Hence, by using (10) and eliminating N, the non-dimensional form of the voltage-current characteristic is obtained as

$$K = J \exp\left(-\frac{1}{J}\right) \quad . \tag{23}$$

The non-dimensional parameters S, N, J, K are listed again in Table 1, and the forms of $\tilde{r}$, S and K given in Table 2; the derivation of the voltage-current characteristic is set out in Table 3.

2.2 Free-burning arc

The free-burning arc of low power gradient may be regarded, by means of a hypothesis[2], as equivalent, as far as the heat transfer and the electric characteristic are concerned, to a static arc in a tube with radius given by

$$\tilde{r} = \frac{2.72}{Pr_2^{\frac{1}{3}}} \left[\frac{\eta_2^2 \, T_2}{8 \, \rho_2^2 \, (\hat{T}-T_\infty) \, g}\right]^{\frac{1}{3}} \tag{24}$$

where the suffix 2 denotes a mean value outside the arc, Pr being the Prandtl number, η the viscosity and ρ the density; g is the acceleration due to gravity. The factor 2.72 is an appropriate normalising factor; it is obtained numerically[2] and its resemblance to e may or may not be significant.

Hence the previous analysis also applies for the free-burning arc of low power gradient provided $\tilde{r}$ is given by equation (24). The representative length scale for a free-burning arc therefore depends on the properties of the gas and the values of the ambient temperature and pressure, and on the acceleration due to gravity. If the gas composition is regarded as fixed and the ambient temperature as constant then $\rho_2 \propto p_\infty$, where p_∞ is the ambient pressure, and so

$$\tilde{r} \propto \frac{1}{p_\infty^{\frac{2}{3}} g^{\frac{1}{3}}} . \tag{25}$$

Therefore, the representative length-scale of a free-burning arc decreases with increase of ambient pressure and with increase of the acceleration due to gravity. These relations for $\tilde{r}$ are included in Table 2.

2.3 Cross-flow arc

The cross-flow arc of low power gradient may also be regarded as equivalent, as far as the heat transfer and the electric characteristic are concerned, to a static arc in a tube of appropriate radius. This is not the result of a hypothesis but of a detailed theory of such an arc[3]. The appropriate radius of the equivalent tube and hence the appropriate representative length-scale for a cross-flow arc is

$$\tilde{r} = \frac{4\,\eta_2}{e^{\gamma}\, Pr_2\, \rho_2\, U} , \tag{26}$$

where γ is Euler's constant ($e^{\gamma} = 1.781$), and U is the velocity of the imposed flow. The analysis for the wall-stabilised arc now applies to the cross-flow arc of low power gradient provided $\tilde{r}$ is given by equation (26). If the gas is fixed and the ambient temperature constant,

$$\tilde{r} \propto \frac{1}{p_\infty U} , \tag{27}$$

which shows that the representative length of a cross-flow arc decreases with increase of ambient pressure and also with increase of flow velocity. These relations for $\tilde{r}$ are included in Table 2.

3 ARCS OF HIGHER POWER GRADIENT

3.1 Effect of type of arc

It is now assumed that, even when the power gradient is increased, convection remains negligible inside all three types of arc and that the external heat transfer relation is that given by a solid circular cylinder. Both of these assumptions become less justified for the free-burning arc and the forced-convection arc as the power gradient increases. They are made simply to allow the influence of the external heat transfer relation on the voltage-current characteristic to be illustrated, the correct external heat transfer relations for the free-burning arc and the forced-convection arc not being known at present. On the assumption that they are the same as for a solid circular cylinder, and by retaining the linear σ-φ law, it is possible to write

$$S = g(N) = \begin{cases} \exp\left(-\frac{1}{N}\right) , & \text{wall-stabilised arc;} \\ g_{fb}(N) , & \text{free-burning arc;} \\ g_{cf}(N) , & \text{cross-flow arc;} \end{cases} \tag{28}$$

g_{fb} and g_{cf} are shown in Fig.7, where they are seen in relation to the result for the wall-stabilised arc. This figure gives a comparison between the effectiveness of heat transfer by conduction to a tube, by free convection and by forced convection, when normalised to be equally effective at low Nusselt number. The range of Nusselt number over which they are all equivalent and the range over which free and forced convection are equivalent are shown. It then follows from (21), (22) and (28), as displayed in Table 3, that the voltage-current characteristics corresponding to the external heat transfer relations above are

$$K = J\, g(J) , \tag{29}$$

with g(J) given by Fig.7. These characteristics are given in Fig.8, which shows that there is likely to be a large difference between the voltage-current characteristic for arcs in convection on the one hand and the wall-stabilised arc on the other, even when the characteristics are non-dimensionalised by the use of the appropriate representative length-scales so that they are the same for arcs of low power gradient. It will become apparent later which portions of the K scale are currently covered in practical cases.

3.2 Effect of relation between electric conductivity and heat-flux potential

If the assumption that the σ-φ law is linear is relaxed and the more realistic law given by equation (3) is used, then by using a very simple but powerful approximate method[2] of solution for the internal energy equation, it may be shown that the resulting non-dimensional form of Ohm's law is

$$K = \left[\frac{f_{\sigma}(\lambda N)}{\lambda}\right] S \quad , \tag{30}$$

where λ is given by

$$\lambda = \frac{(\hat{\varphi}-\varphi_{\infty})}{\bar{\varphi}} \tag{31}$$

and for fixed gas and constant ambient temperature λ is constant. Hence, by using equation (30) in conjunction with (21) and (28), as displayed in Table 3, it follows that the voltage-current characteristic is:

$$K = \left[\frac{f_{\sigma}(\lambda J)}{\lambda}\right] g(J) \quad . \tag{32}$$

This relation shows how the σ-φ law influences directly the voltage-current characteristic. If the influence were calculated exactly it would be much more complicated than that shown here, but equation (32), when compared with (29), shows that there is a tendency for the effects of gas properties to be separated from the effects of the type of arc. The modified characteristic for a wall-stabilised arc is shown in Fig.9. The characteristics of the other arcs would be similarly modified.

3.3 Effect of radiation

The main effect of radiation is to alter the internal energy balance from the simple relation (7) to

$$EI = Q + R \tag{33}$$

where R is the net power per unit length lost by radiation. There is reason[2] to suppose that for low power gradients R may be conveniently represented by the form

$$R = c\left(\frac{p_{\infty}}{\bar{p}}\right)^{m} \beta\left(\frac{Q}{4\pi}\right)^{2} \pi\hat{r}^{2} \quad , \tag{34}$$

where c is a constant, unknown in the absence of an exact solution of the internal energy equation in these circumstances. Equation (34) may be written as

$$R = \tilde{Q} Z N^2 S \quad , \tag{35}$$

where Z is a radiation parameter, involving only independent parameters, which is of the form

$$Z = c \left(\frac{p_\infty}{\bar{p}}\right)^m \frac{\beta(\hat{\varphi}-\varphi_\infty)^2 \pi \tilde{r}^2}{\tilde{Q}} \quad . \tag{36}$$

Hence, for fixed gas and constant ambient temperature

$$Z \propto p_\infty^m \tilde{r}^2 \quad , \tag{37}$$

and from equations (14), (25) and (27) it follows that

$$Z \propto \begin{cases} p_\infty^m r_\infty^2 \quad , & \text{wall-stabilised arc;} \\ \dfrac{p_\infty^{m-4/3}}{g^{2/3}} \quad , & \text{free-burning arc;} \\ \dfrac{p_\infty^{m-2}}{U^2} \quad , & \text{cross-flow arc.} \end{cases} \tag{38}$$

These forms of Z are included in Table 2, where m is taken to be 5/4 for completeness. When higher power gradients are considered and the same method of approximation used for ψ as for σ, it follows[2] that

$$R = \tilde{Q} Z \left[\frac{f_\psi(\lambda N)}{\lambda^2}\right] S \quad . \tag{39}$$

Hence the modified form of the internal energy balance is

$$J = N + Z \left[\frac{f_\psi(\lambda N)}{\lambda^2}\right] S \quad . \tag{40}$$

The voltage-current characteristic now follows, as in Table 3, from equations (28), (30) and (40). In this case it is not possible to obtain an explicit relation between K and J, but, if S is eliminated, J and K are given parametrically in terms of N. The resulting characteristics are shown schematically for a fixed ambient pressure in Fig.10. Since N represents, non-dimensionally, the amount of heat actually being conducted across the periphery of the arc it follows that for a given amount of such heat the radiation loss increases with increase of pressure for the wall-stabilised arc, changes only slightly with change of pressure for the free-burning arc, and decreases with increase of pressure for the cross-flow arc. Also, radiation loss increases with increase of tube radius for the wall-stabilised arc and decreases with increase of acceleration due to gravity for the free-burning arc and with increase of flow velocity for the cross-flow arc. The fact that the radiation loss from a wall-stabilised arc depends on the square of the radius of the tube provides a powerful means of deducing the radiative properties of the gas from the measurements of arc characteristics in tubes of different radii.

4 COMPARISON WITH EXPERIMENTS

Only the treatment of the wall-stabilised arc is near the truth for arcs of high power gradient and many as-yet-uncalculated influences exist in the free-burning and cross-flow arcs. This point is well-illustrated by comparing the theoretical characteristics with experimental measurements. To present experimental characteristics in non-dimensional form it is necessary to use numerical values for the constants $\hat{\varphi}$ and α and for the mean values Pr_2, η_2/ρ_2 and T_2 for given ambient temperature and pressure. For nitrogen or air at one atmosphere pressure and at room temperature these quantities have been derived[4] and they lead to the characteristics shown in Figs.9, 11 and 12.

The experimental characteristic given by Maecker[5] for the wall-stabilised arc, Fig.9, is indistinguishable from the theoretical characteristic because the gas properties used in the theoretical calculation were obtained[2] from an analysis of the experimental characteristic! The properties are therefore open to objection on that score, but theoretically-calculated properties are in general agreement, and are not necessarily more reliable. The empirically-derived properties are ideal for use in calculations of the characteristics of other types of arcs.

The experimental characteristic given by King[5] for the free-burning arc, Fig.11, agrees with the theoretical characteristic for low power gradients because the appropriate mean kinematic viscosity outside the arc was deduced from it[4]. The departure of the experimental characteristic from the theoretical characteristic (with radiation neglected) at higher power gradients shows a mixture of influences, particularly those of external heat transfer and radiation. Indeed, if the external heat transfer relation were known accurately (and the internal solution of the arc also) it would be possible to use King's characteristic to deduce the radiation power density of the gas as a function of the heat-flux potential. Since this information is not yet available, such radiation data is better deduced from experiments on wall-stabilised arcs in tubes of various radii.

The experimental characteristic obtained by Adams[7] and reported by Broadbent[8], for the cross-flow arc, Fig.12, does not go down to low power gradients. However, it seems significant that the non-dimensional parameters collapse the data, and that the collapsed curve appears to be well in line with the theoretical curve for low power gradients. Adams' curve should be largely free from radiation since the velocities involved were high (of the order of one or two hundred metres per second) and hence there is a large discrepancy between the experimental and theoretical curves. This is not in the least surprising, and major contributory reasons could be the possibilities of errors in the numerical factors used to non-dimensionalise the experimental results and in the interpretation of the experimental results themselves, and the inadequate theoretical model of a solid circular cylinder for high power gradients. There is no conflict intended between the belief that a cross-flow arc is not a solid body and the important recent observation by Roman and Myers[9] that it appears to behave like one. More experimental results at low and intermediate power gradients will be needed before the present correlation can be improved and extended, and its relation to the results of Roman and Myers examined more closely.

5 CONCLUSION

It has been shown that while any limited experiment of an arc may well yield a voltage-current characteristic of typical and undistinctive form, such a characteristic is composed of many ingredients which to a useful extent can be classified and correlated for several types of arc by using non-dimensional parameters. The present study is of course confined to supposedly-uniform arc columns, and its relation to presentations[10] which include the effects of column length involves a study of the complexities of non-uniform arc columns[2] and is not yet fully understood.

Table 1

NON-DIMENSIONAL PARAMETERS

$$S = \left(\frac{\hat{r}}{\tilde{r}}\right)^2$$

$$N = \frac{Q}{\tilde{Q}}$$

$$J = \frac{EI}{\tilde{Q}}$$

$$K = \frac{I/E}{\tilde{\sigma}\,\pi\tilde{r}^2}$$

$$Z = \frac{\tilde{\psi}\,\pi\tilde{r}^2}{\tilde{Q}}$$

where

$$\tilde{Q} = 4\pi(\hat{\varphi}-\varphi_\infty)$$

$$\tilde{\sigma} = \left(\frac{2}{2.405}\right)^2 \alpha(\hat{\varphi}-\varphi_\infty)$$

$$\tilde{\psi} = c\left(\frac{p_\infty}{\bar{p}}\right)^m \beta(\hat{\varphi}-\varphi_\infty)^2$$

and $\tilde{r}$ is given in Table 2.

Table 2

REPRESENTATIVE LENGTH-SCALES

	$\tilde{r}$	For fixed gas and constant ambient temperature*			
		$\tilde{r}$	S	K	Z $(m = 5/4)$
wall-stabilised	r_∞	r_∞	$\left(\frac{\hat{r}}{r_\infty}\right)^2$	$\frac{I}{E\, r_\infty^2}$	$p_\infty^{5/4}\, r_\infty^2$
free-burning	$\frac{2.72}{Pr_2^{\frac{1}{3}}}\left[\frac{\eta_2^2\, T_2}{8\, \rho_2^2\, (\hat{T}-T_\infty)\, g}\right]^{\frac{1}{3}}$	$\frac{1}{p_\infty^{\frac{2}{3}}\, g^{\frac{1}{3}}}$	$(p_\infty^{\frac{2}{3}}\, g^{\frac{1}{3}}\, \hat{r})^2$	$\frac{p_\infty^{4/3}\, g^{\frac{2}{3}}\, I}{E}$	$\frac{1}{p_\infty^{1/12}\, g^{2/3}}$
cross-flow	$\frac{4\, \eta_2}{e^\gamma\, Pr_2\, \rho_2\, U}$	$\frac{1}{p_\infty\, U}$	$(p_\infty\, U\, \hat{r})^2$	$\frac{p_\infty^2\, U^2\, I}{E}$	$\frac{1}{p_\infty^{\frac{3}{4}}\, U^2}$

* $\tilde{r}$, S, K and Z are proportional to, but not necessarily equal to, the expressions given in this part of the table; constant factors depending on the gas and the ambient temperature are involved in general.

Table 3

DERIVATION OF VOLTAGE-CURRENT CHARACTERISTICS

		External heat transfer relation	Ohm's law	Internal energy balance	Voltage-current characteristic
low power gradient		$S = e^{-\frac{1}{N}}$	$K = NS$	$J = N$	$K = J\, e^{-\frac{1}{J}}$
higher power gradients	type of arc	$S = g(N)$	"	"	$K = J\, g(J)$
	relation between conductivity and heat flux potential	"	$K = \left[\frac{f_\sigma(\lambda N)}{\lambda}\right] S$	"	$K = \left[\frac{f_\sigma(\lambda J)}{\lambda}\right] g(J)$
	radiation	"	"	$J = N + Z\left[\frac{f_\psi(\lambda N)}{\lambda^2}\right] S$	no explicit expression

SYMBOLS

E	electric field (voltage gradient) of uniform column
I	current
J	non-dimensional form of EI (Table 1)
K	non-dimensional form of I/E (Table 1)
N	non-dimensional form of Q (Table 1)
Pr	Prandtl number $c_p\ \eta/\kappa$
Q	heat conducted across arc periphery per unit length per unit time
R	heat loss by radiation from arc per unit length per unit time
S	non-dimensional form of arc radius (Table 1)
T	temperature
U	imposed velocity in cross-flow arc
Z	non-dimensional radiation parameter (Table 1)
c	constant involved in expression of radiation loss
c_p	specific heat at constant pressure
e	exponential
f_σ	function expressing electric conductivity in terms of φ
f_ψ	function expressing radiated power density in terms of φ
g	acceleration due to gravity
$g(N)$	general function expressing external heat transfer relation
$g_{fb}(N)$	form of $g(N)$ for free-burning arc
$g_{cf}(N)$	form of $g(N)$ for cross-flow arc
m	index in expression of radiation; taken to be 5/4 for illustration (Table 2)
p	pressure
$\hat{r}$	radius of arc periphery
r_∞	radius of tube in wall-stabilised arc
α	constant in initial form of σ–φ law: $\sigma = \alpha(\varphi-\hat{\varphi})$
β	constant in initial form of ψ'–φ law: $\psi' = \beta(\varphi-\hat{\varphi})^2$
γ	Euler's constant: $e^\gamma = 1.781$
η	viscosity
κ	thermal conductivity
λ	constant involved in non-dimensional characteristics: $\lambda = (\hat{\varphi}-\varphi_\infty)/\overline{\varphi}$
ρ	density
σ	electric conductivity

SYMBOLS (Contd)

φ	heat flux potential: $\varphi = \int_0^T \kappa \, dt$
ψ	radiated power density: $\psi = (p/\bar{p})^m \, \psi'$
ψ'	temperature-dependent part of radiated power density

superscripts:

$\sim$	representative values, "characteristic" of gas, ambient conditions and type of arc
$-$	reference values in description of gas properties
$\wedge$	peripheral values

subscripts:

∞	ambient values
2	mean values between ambient and peripheral values

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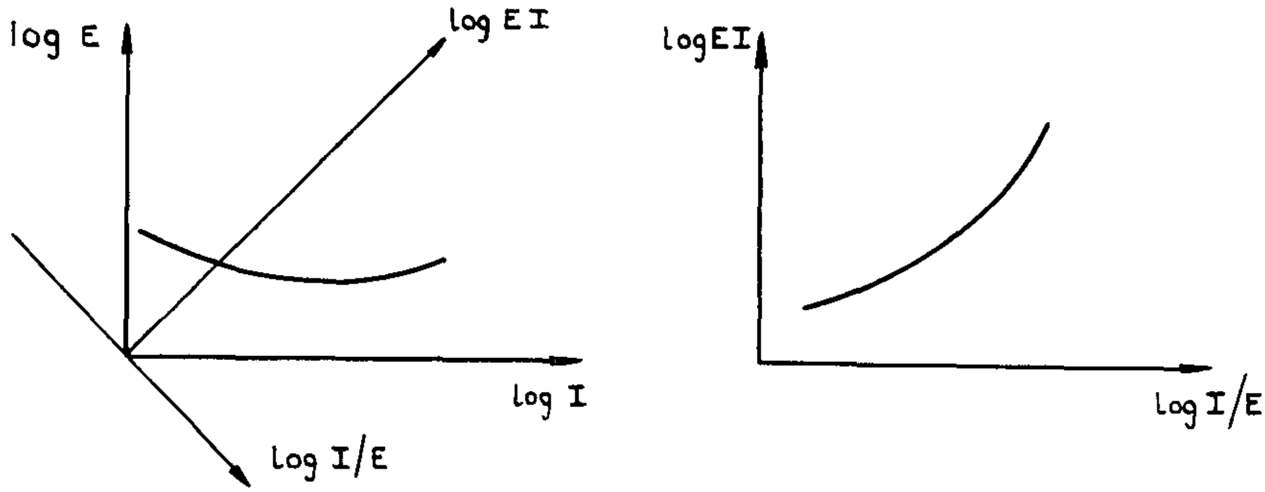

Fig. 1 Typical shapes of voltage-current characteristics

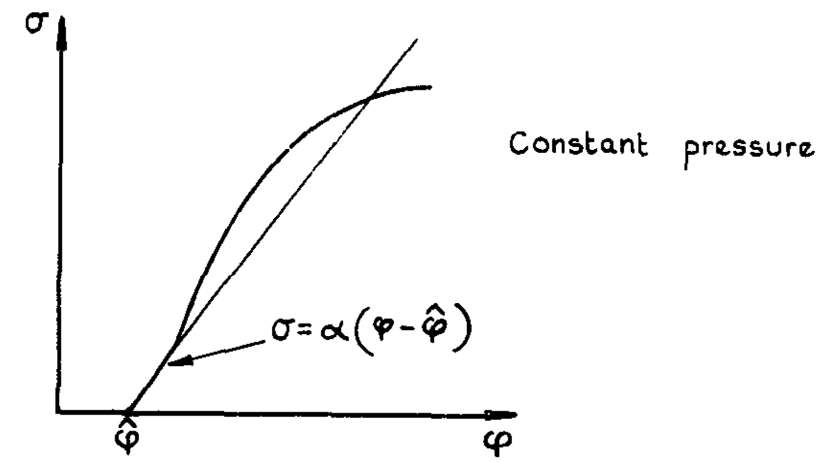

Fig. 2 Variation of electric conductivity (schematic)

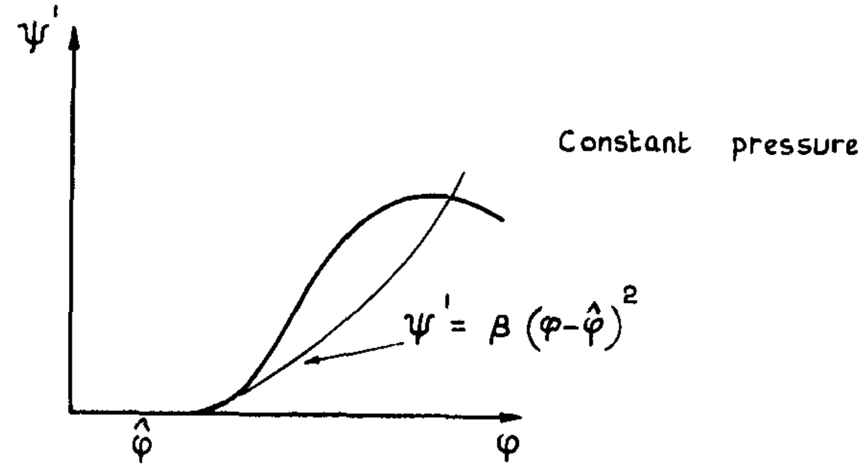

Fig. 3 Variation of radiated power density (schematic)

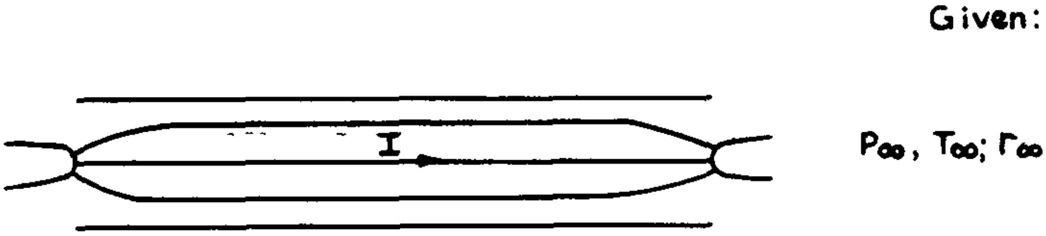

Fig. 4 Wall-stabilised arc

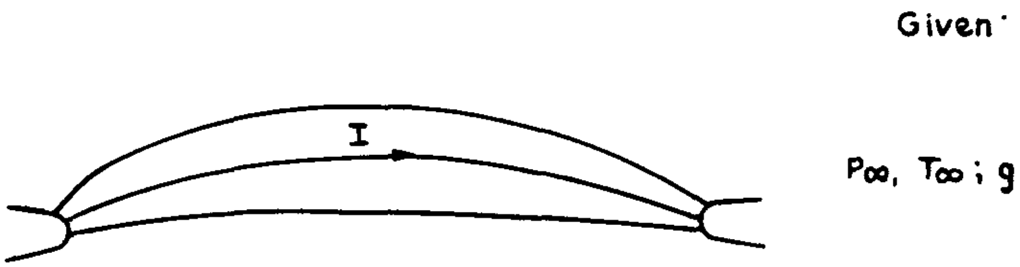

Fig. 5 Free-burning arc

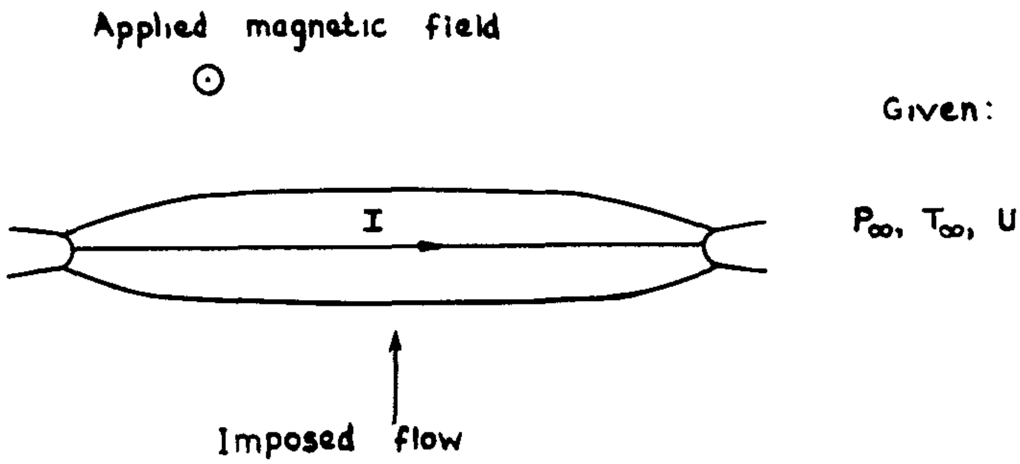

Fig. 6 Cross-flow arc

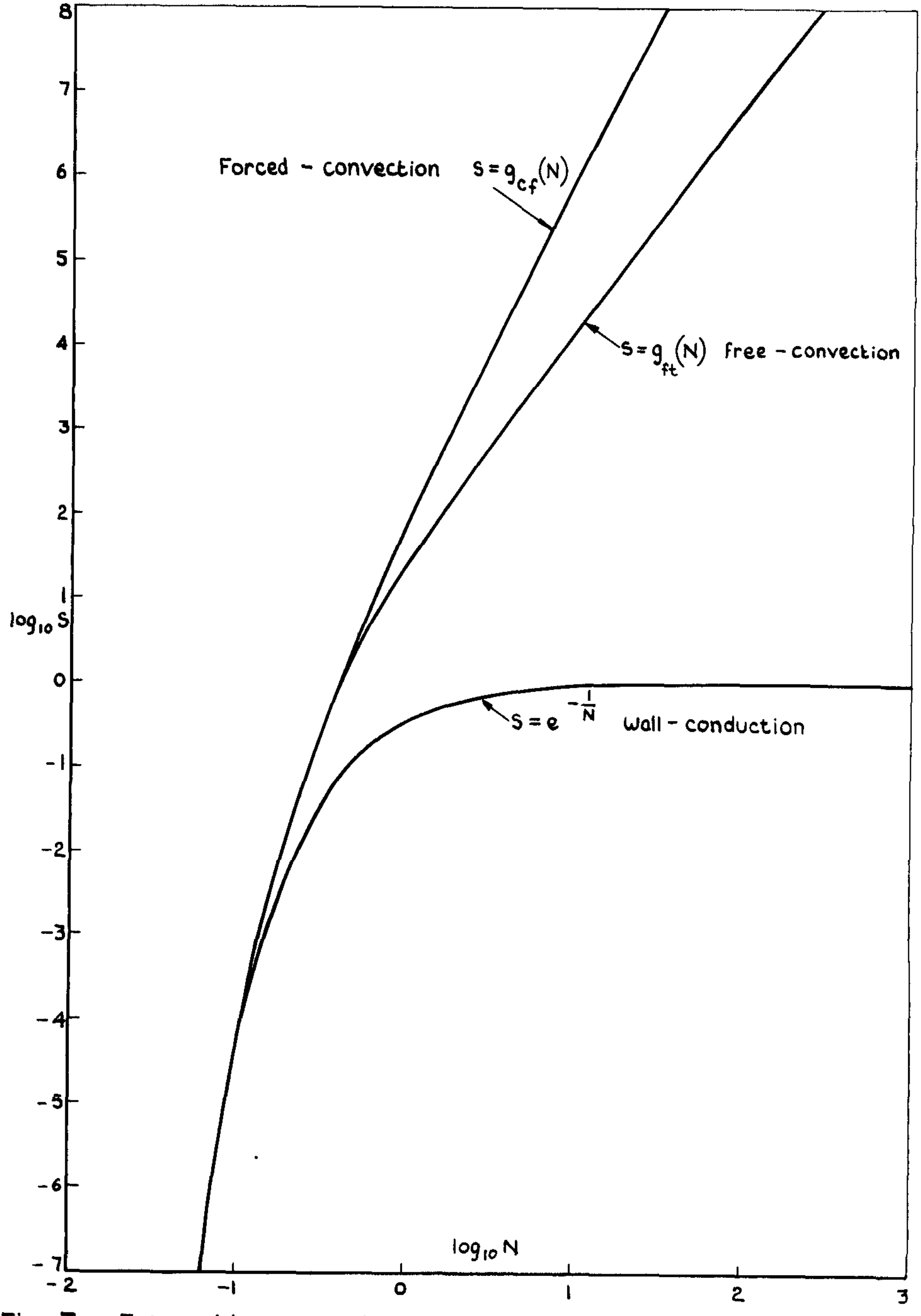

Fig. 7 External heat transfer relations for solid circular cylinders

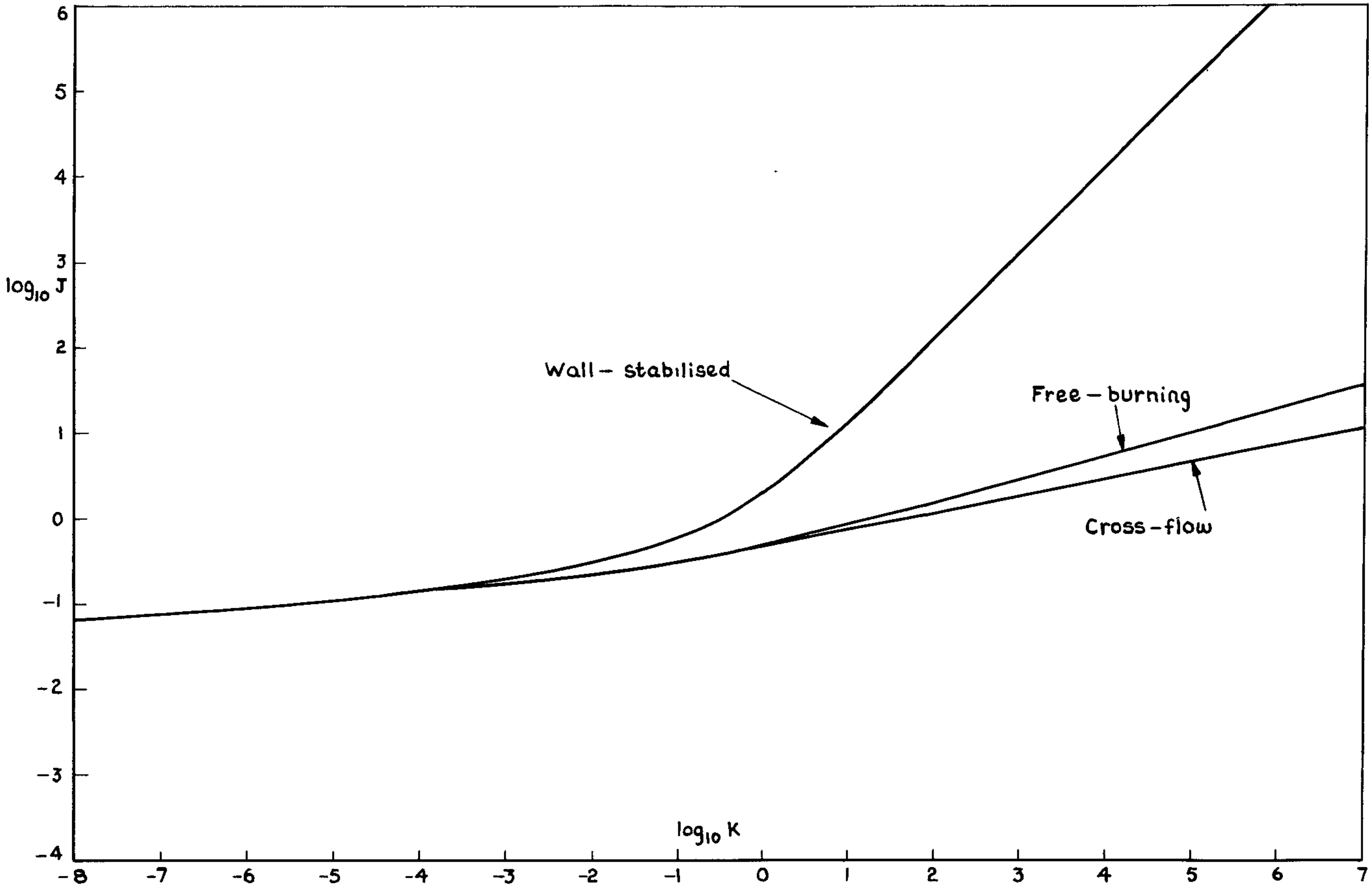

Fig 8 Effect of type of arc on voltage-current characteristic

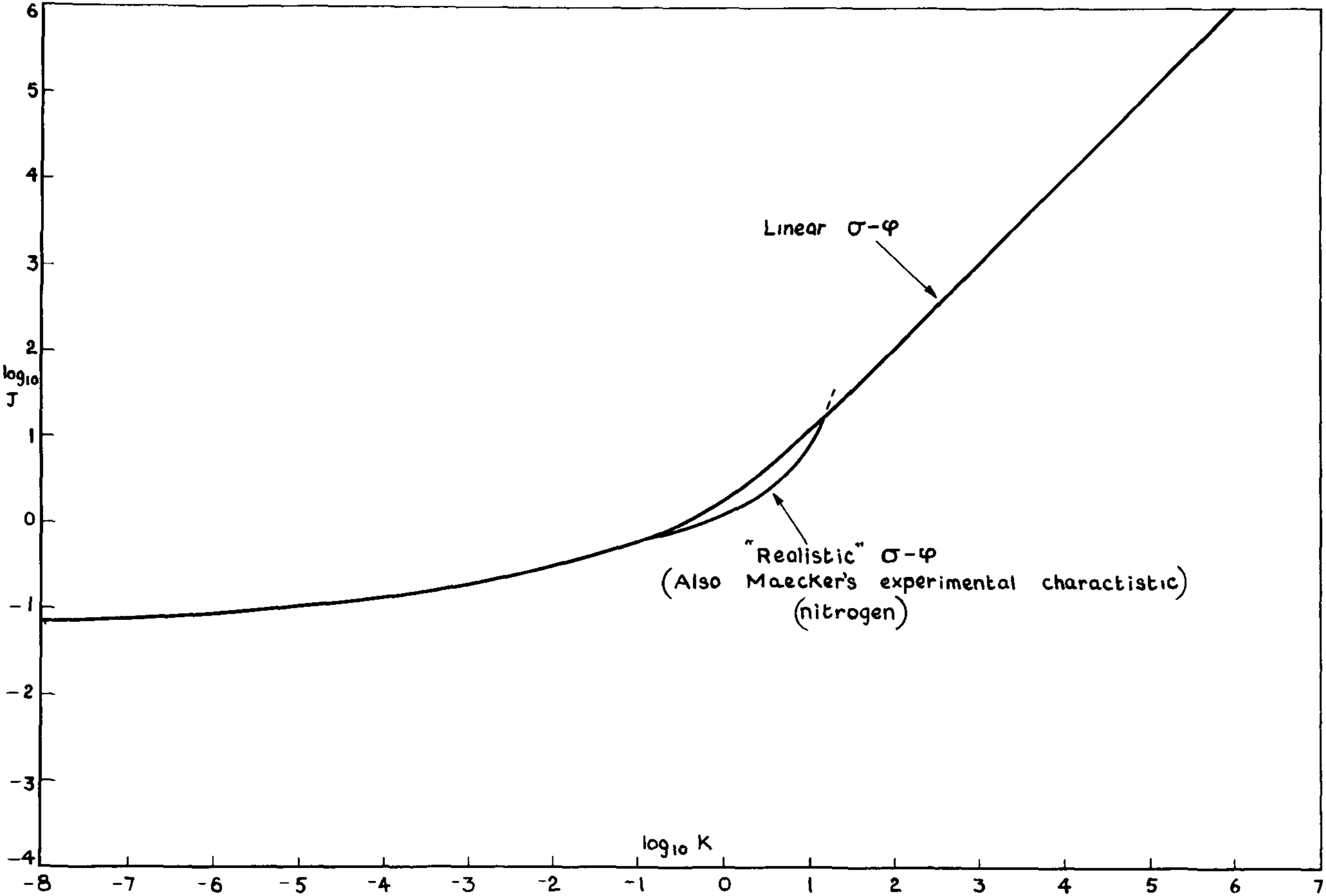

Fig 9 Effect of σ-φ relation on voltage-current characteristic of wall-stabilised arc

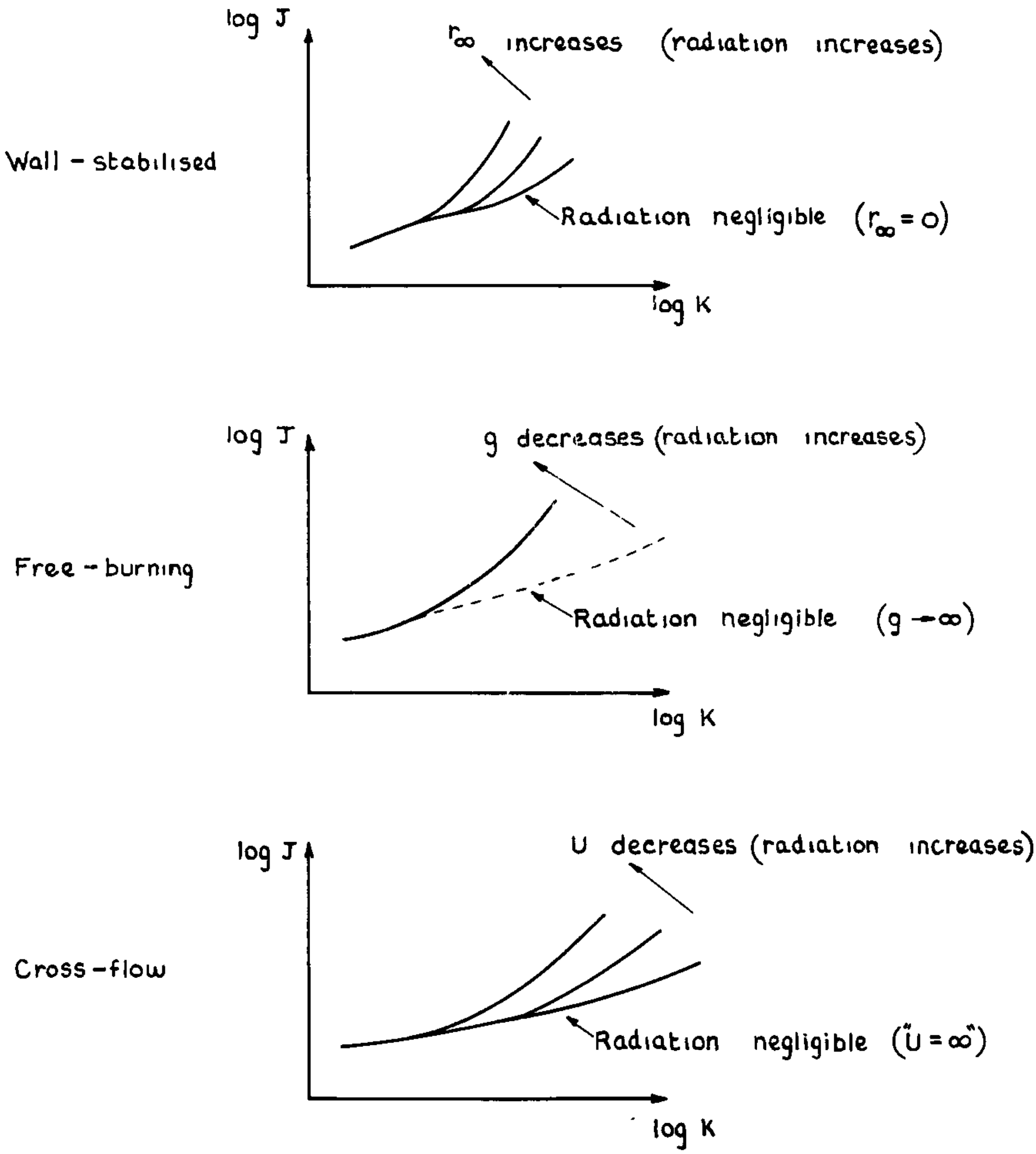

Fig. 10 Effects of radiation (schematic)

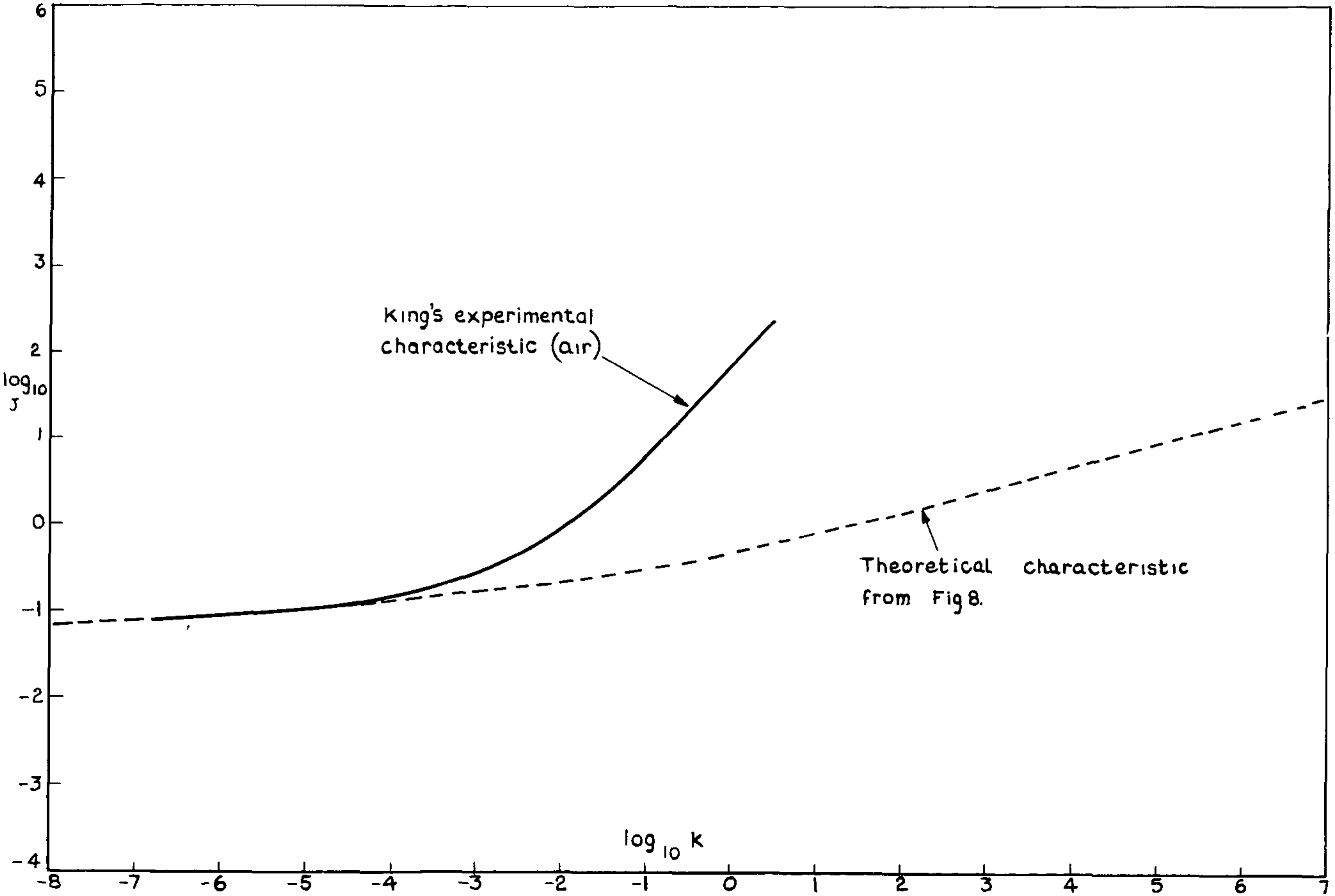

Fig. 11 Experimental voltage-current characteristic of free-burning arc

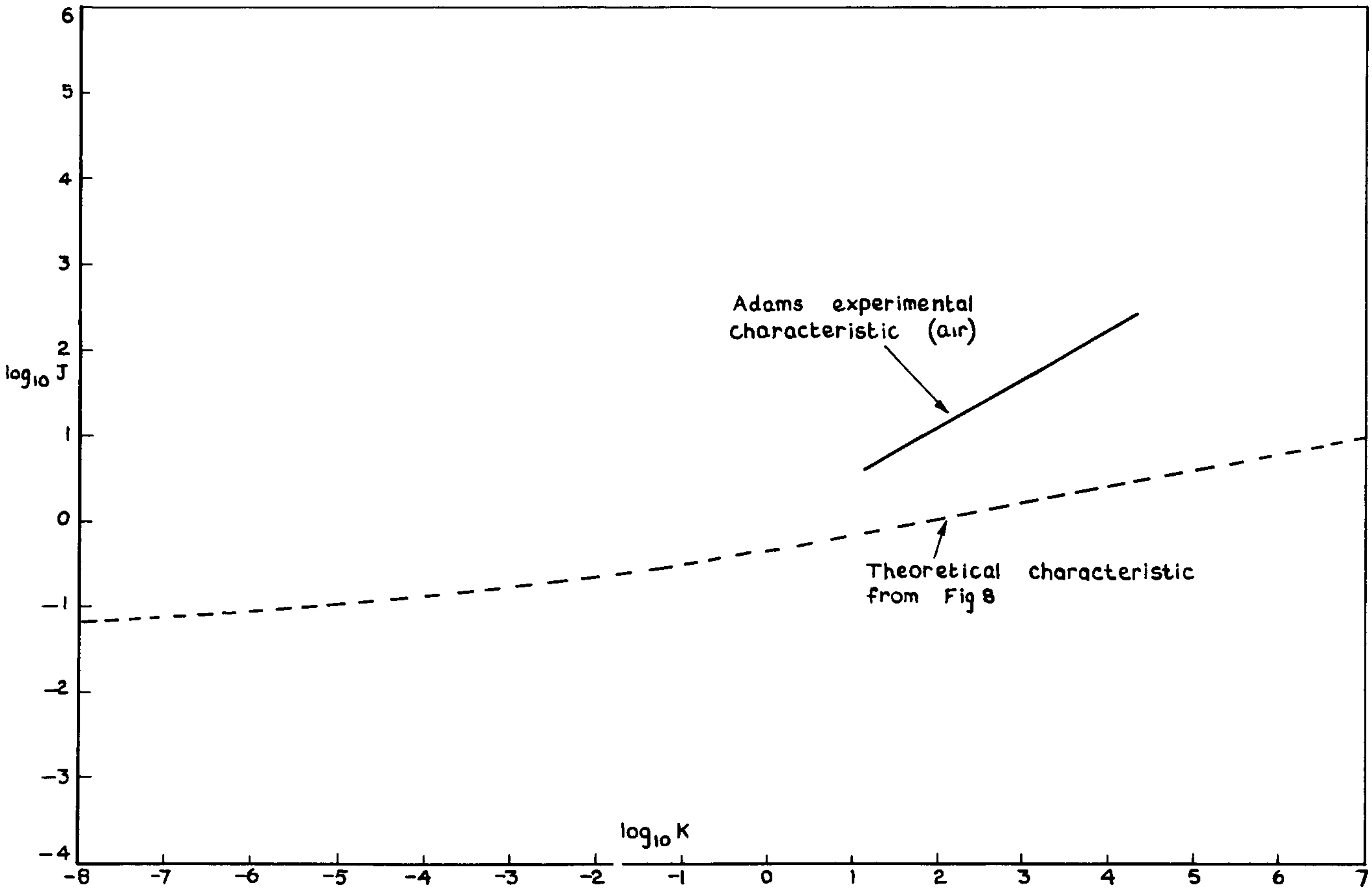

Fig. 12 Experimental voltage-current characteristic of cross-flow arc

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