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Diffraction of Blast Waves for the Oblique Case

By

R.S Srivastava

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- By -
R. S. Srivastava

SUMMARY

The problem of diffraction of an oblique shock wave has been considered in this paper. The investigations are devoted to the cases when the relative outflow behind the reflected shock before diffraction is subsonic and sonic. The distribution of pressure has been obtained for finite and infinite shock strengths for both these cases.

Introduction

The problem of diffraction of a plane straight shock wave past a small bend in a plane wall was solved by Lighthill[3]. The problem considered in this paper deals with the case of diffraction of an oblique shock wave. For studying the case of diffraction of an oblique shock wave, knowledge of the theory of regular reflection from a rigid wall is necessary. In work on shock reflection usually three critical angles of incidence are introduced[5]:

(1) α_s (sonic angle) is such that for angles of incidence $\alpha_o < \alpha_s$ one gets supersonic relative outflow behind the reflected shock and, for $\alpha_o \geqslant \alpha_s$ subsonic and sonic flows are obtained respectively.

(2) α_e is the theoretical extreme angle beyond which regular reflection is not possible.

(3) α_o', somewhat greater than α_e, is the limiting angle of incidence beyond which regular reflection is not observed experimentally.

In the present problem the physical constants defining the problem will be U the velocity of the point of intersection of incident and reflected shocks, p_o, ρ_o the pressure and density of the still air, and δ the angle of the bend. The angle of the bend is assumed to be small and so also are the variations of velocity and pressure. For the oblique shock diffraction problem one has to consider two regions, one region being the region between the incident and reflected shock and the other being the region behind the reflected shock. In an earlier paper[6] it has been

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* Replaces A.R.C.28 604

established that the region between the incident and reflected shock remains undisturbed for all incident shock strengths after the shock configuration has crossed the corner. In Ref. 5 the work of Ref. 6 has been reviewed and it has been argued there that for $\alpha_o < \alpha_s$, Mach reflection would take place after the shock configuration has crossed the corner. A referee of the present paper has pointed out that the conclusion about Mach reflection in Ref. 5 is incorrect as in this case also one would get a region of non-uniform flow enclosed by arc of the unit circle, the wall and reflected shock, even though the point of intersection of the incident and reflected shock is outside the unit circle. The case of diffraction for $\alpha_o < \alpha_s$ therefore remains to be investigated; Dr. Ter-Minasyants of Moscow University Computing Centre, in a private communication, states that he has done this.

The cases treated in the present paper refers to subsonic and sonic relative outflows, i.e., one has to be in the range $\alpha_s \leq \alpha_o \leq \alpha_e$. It is necessary to discuss the experimental and theoretical results in this range in order to make a proper choice of data for carrying out the numerical work. Bleakney and Taub[1] have stated that the theory and experiment are confused between sonic angle curve and α_o' curve but there is a good deal of evidence which shows that the theory and experiment are in good agreement (e.g., in the prediction of angle of reflection) for angle of incidence up to the theoretical extreme angle curve for all incident shock strengths[2,4]. The discrepancy between theory and experiment exists beyond $\alpha_o = \alpha_e$; in fact, between it and another curve (experimental curve $\alpha_o = \alpha_o'$ for the onset of Mach reflection) which is slightly above the theoretical extreme angle curve; in this region regular reflection appears to continue to take place. However, the numerical computation carried out does not refer to this troublesome range but to the range where theory and experiment agree well.

In the first instance the mathematical solution has been obtained for both subsonic and sonic cases. The paper has been divided into three parts. Part I and Part II deal with the theoretical solution for subsonic and sonic cases respectively. In Part III pressure distribution along the wall has been obtained for infinite and finite shock strengths for both subsonic and sonic cases. The angle of the bend has been taken to be 0·1 radian.

Part I

Mathematical Formulation

The shock relations across the incident and reflected shock (Fig. 1) before diffraction are given by equations (1) and (2) of Ref. 5.

After the shock configuration has crossed the corner, let the velocity, pressure, density and entropy at any point be $\vec{q}_2'$, p_2', ρ_2' and S_2'. Choose (X,Y) axes with origin at the corner and X-axis along the original wall produced. By the application of Lighthill's linearisation and by the help of the transformation

$$\left.\begin{aligned} \frac{X - q_2 t}{a_2 t} = x, \quad \frac{Y}{a_2 t} = y, \quad \frac{\vec{q}_2'}{q_2} = \{(1+u),\ v\} \\ \frac{p_2' - p_2}{a_2 q_2 \rho_2} \end{aligned}\right\} \quad \ldots(1)$$

the/

the equation of continuity and the equations of motion behind the diffracted reflected shock give the following equations:

$$x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \qquad \ldots(2)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\partial p}{\partial x} \qquad \ldots(3)$$

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = \frac{\partial p}{\partial y}. \qquad \ldots(4)$$

In the new axes the origin is at a point on the original wall produced. The straight part of the reflected shock lies along a fixed line $x = k - y \cot \alpha_2$ where $k = (U - q_2)/a_2$. The corner is at the point $(-M_2, 0)$ where $M_2 = q_2/a_2$. Immediately behind the reflected diffracted shock the conditions at a point will be given by the right-hand sides of equation (2) of Ref. 5 if U^* is replaced therein by the shock velocity normal to itself and $\vec{q}_1$ denotes the total velocity in the region between the incident and reflected shock.

Now since the whole field suffers a uniform expansion in time about the corner, the velocity on each point of the shock is $(X/t, Y/t)$ in the (X,Y) system of co-ordinates. Hence the velocity of the shock normal to itself is $\vec{h}$ where $t\vec{h}$ is the vector perpendicular drawn from the corner to the tangent to the shock at that point. In terms of $\vec{h}$, the boundary conditions at the shock are

$$\vec{q}_2' = \vec{q}_1 + \frac{5}{6}(\vec{h} - \vec{q}_1')\left\{1 - \frac{a_1^2}{(|\vec{h}| - |\vec{q}_1'|)^2}\right\} \qquad \ldots(5)$$

$$p_2' = \frac{5}{6}\rho_1\left\{(|\vec{h}| - |\vec{q}_1'|)^2 - \frac{a_1^2}{7}\right\} \qquad \ldots(6)$$

where $\vec{q}_1' = q_1 \sin(\theta' + \varepsilon) \sin(\alpha_2 + \varepsilon),\ q_1 \sin(\theta' + \varepsilon) \cos(\alpha_2 + \varepsilon)$, ε being small.

Let the equation of the shock in the new co-ordinates be $x = k - y \cot \alpha_2 + f(y)$ where $f(y)$ could be regarded as small as the angle of bend is small. In Fig. 2 $\vec{ON}$ is $t\vec{h}$ and is denoted by

$$\left(X - Y\frac{dX}{dY}\right)\sin^2\psi, \qquad \left(X - Y\frac{dX}{dY}\right)\sin\psi\cos\psi$$

where $\psi = \alpha_2 + \varepsilon$. Therefore

$t\vec{h}$/

$$t\vec{h} \approx \left\{ \left(X - Y\frac{dX}{dY}\right) \sin\alpha_2 \cos\alpha_2 (\tan\alpha_2 + 2\varepsilon), \right.$$

$$\left. \left(X - Y\frac{dX}{dY}\right)(\sin\alpha_2 \cos\alpha_2 + \varepsilon\cos 2\alpha_2)\right\} .$$

Hence

$$\vec{h} \approx \{U + a_2 f(y) - a_2 yf'(y) + U \sin 2\alpha_2 f'(y)\}\sin^2\alpha_2 ,$$

$$\{(U + a_2 f(y) - a_2 yf'(y))\sin\alpha_2 \cos\alpha_2 + U \sin^2\alpha_2 \cos 2\alpha_2 f'(y)\}.$$

As $f(y)$ is small, terms containing $f(y)f'(y)$, $y\{f'(y)\}^2$ have been neglected. Now since the tangential velocities are equal, equation (2) of Ref. 5 gives

$$\vec{q}_2 - \vec{q}_1 = \frac{5}{6}(\vec{U}^* - \vec{q}_1)\left\{1 - \frac{a_1^2}{(U^* - \bar{q}_1)^2}\right\} \qquad \ldots(7)$$

where

$$\left.\begin{aligned} (\vec{U}^* - \vec{q}_1) &= (V\sin\alpha_2, V\cos\alpha_2) \\ V &= (U\sin\alpha_2 - q_1 \sin\theta') \end{aligned}\right\} . \qquad \ldots(8)$$

Now from equations (5), (6) and (7) one obtains after simplification

$$u = AF + Bf'(y) \qquad \ldots(9)$$

$$v = A_1 F + B_1 f'(y) \qquad \ldots(10)$$

$$p = A_2 F + B_2 f'(y) \qquad \ldots(11)$$

where $F = a_2 f(y) - a_2 yf'(y) - q_2 \cos\alpha_2 f'(y)\sin\alpha_2$ and A, A_1, A_2, B, B_1 and B_2 are constants. At the shock boundary therefore we obtain

$$\frac{\partial p}{\partial y} = \frac{B_2 - A_2 G}{B - AG} \cdot \frac{\partial u}{\partial y} = \frac{B_2 - A_2 G}{B_1 - A_1 G} \cdot \frac{\partial v}{\partial y} \qquad \ldots(12)$$

where $G = (a_2 y + q_2 \cos\alpha_2 \sin\alpha_2)$. Now equations (2), (3) and (4) have to be solved under the following boundary conditions:

on $y = 0$ $\quad v = -\delta \quad x > -M_2$

$\quad v = 0 \quad x < -M_2$.

On the shock boundary $x = k - y\cot\alpha_2$, u, v, p are related by equations (12). On the remaining boundary between the disturbed flow and uniform flow $u = v = p = 0$.

Elimination/

Elimination of u and v

By eliminating u and v from equations (2), (3) and (4) we get a single second order partial differential equation in p. The equation is

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}. \qquad ...(13)$$

This equation is hyperbolic for $x^2 + y^2 > 1$ and elliptic for $x^2 + y^2 < 1$; its characteristics are all tangents to the unit circle $x^2 + y^2 = 1$. It is, therefore, reasonable to assume that the region of disturbance will be enclosed by an arc of the unit circle, and by the reflected shock. As in Lighthill's paper we obtain

(a) $M_2 < 1$

(i) On the wall $y = 0$, $\frac{\partial p}{\partial y} = 0$ except at the corner. ...(14)

At the corner

$$\underset{y\to 0}{\mathrm{Lt}} \int_{-M_2 - C}^{-M_2 + C} \frac{\partial p}{\partial y}\, dx = M_2 \delta. \qquad ...(15)$$

(ii) On the circle $x^2 + y^2 = 1$

$$p = 0, \qquad y > 0, \qquad x < k - y \cot \alpha_2. \qquad ...(16)$$

(b) $M_2 > 1$

(i) On the wall $\frac{\partial p}{\partial y} = 0$. ...(17)

(ii) On the unit circle $x^2 + y^2 = 1$

$$p = -M_2\delta(M_2^2 - 1)^{-\frac{1}{2}}, \qquad x < -\frac{1}{M_2}$$

$$p = 0 \qquad , \qquad x > -\frac{1}{M_2}. \qquad ...(18)$$

On the shock boundary $x = k - y \cot \alpha_2$, p satisfies the equation

$$(k - y\cot\alpha_2)\left\{(k - y\cot\alpha_2)\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right\}$$

$$= \frac{\partial p}{\partial x} - y\,\frac{B - AG}{B_2 - A_2G}\,\frac{\partial p}{\partial y} + (k - y\cot\alpha_2)\,\frac{B_1 - A_1G}{B_2 - A_2G}\,\frac{\partial p}{\partial y}. \qquad ...(19)$$

Now,/

Now, since $v = -\delta$ at $(k,0)$

$$\int \frac{\partial v}{\partial y}\,dy = \int_{\Gamma} \frac{B_1 - A_1G}{B_2 - A_2G}\,dp = \delta \qquad \ldots(20)$$

where Γ denotes the diffracted portion of the shock starting from the wall.

We have, therefore, to obtain a value of p which satisfies the boundary conditions (14), (15), (16), (19) and (20) in the case $M_2 < 1$. In the case $M_2 > 1$ (17), (18), (19) and (20) hold good on the boundaries.

<u>Busemann's Transformation</u>

Under the transformation $x = r\cos\theta$, $y = r\sin\theta$ where

$$\rho = \frac{[1 - (1 - r^2)^{\frac{1}{2}}]}{r}$$

equation (13) becomes Laplace's equation

$$\frac{\partial^2 p}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial p}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2 p}{\partial\theta^2} = 0$$

in (ρ,θ) as polar co-ordinates.

Now the circle $r = 1$ becomes the circle $\rho = 1$. Also we have $r = \dfrac{2\rho}{(1 + \rho^2)}$ so that the line $x = k - y\cot\alpha_2$ becomes an arc of the circle

$$\frac{2\rho\sin(\theta + \alpha_2)}{1 + \rho^2} = k\sin\alpha_2.$$

Let the initial line be rotated through an angle $(\frac{\pi}{2} - \alpha_2)$. (Fig. 3.) The circle $\dfrac{2\rho\sin(\theta + \alpha_2)}{(1 + \rho^2)} = k\sin\alpha_2$ is transformed into the circle $2\rho\cos\Theta = k\sin\alpha_2(1 + \rho^2)$ which cuts the circle $\rho = 1$ orthogonally at $\cos\Theta = k\sin\alpha_2$.

Following Lighthill we now have

$$\frac{\partial p/\partial x'}{\partial p/\partial y'} = \frac{K^2}{(1 - K^2)}\tan\Theta + \frac{(1 - K^2\sec^2\Theta)^{\frac{1}{2}}}{(1 - K^2)}\frac{\partial p/\partial n}{\partial p/\partial s} \qquad \ldots(21)$$

where x', y' refer to new axes, Θ is measured from the new initial line, $K = k\sin\alpha_2$ and dn and ds are elements normal and tangential to the circular arc $2\rho\cos\Theta = k\sin\alpha_2(1 + \rho^2)$ respectively towards its centre and away from the initial line $\Theta = 0$.

Now/

Now

$$\frac{\partial p/\partial x'}{\partial p/\partial y'} = \frac{1 + \dfrac{\partial p/\partial x}{\partial p/\partial y}\tan\alpha_2}{\tan\alpha_2 - \dfrac{\partial p/\partial x}{\partial p/\partial y}} .$$

Substituting the value of $\dfrac{(\partial p/\partial x)}{(\partial p/\partial y)}$ from (19) and using the fact that $y = K[\cos\alpha_2 + \sin\alpha_2 \tan\Theta]$ we obtain

$$\frac{(\partial p/\partial x')}{(\partial p/\partial y')} = \frac{C_1 + D_1\tan\Theta + E_1\tan^2\Theta + F_1\tan^3\Theta}{C_1' + D_1'\tan\Theta + E_1'\tan^2\Theta + F_1'\tan^3\Theta} \qquad \ldots(22)$$

where C_1, D_1, E_1, F_1, C_1', D_1', E_1' and F_1' are known constants.

From (21) and (22)

$$\frac{(\partial p/\partial n)}{(\partial p/\partial s)} = f(\tan\Theta) = f\left(\tan\left(\theta + \alpha_2 - \frac{\pi}{2}\right)\right)$$

where θ in the second expression is measured from the original position of the initial line. The function f is known. Putting $\xi = \rho\cos\theta$, $\eta = \rho\sin\theta$ the condition holding at the corner is

$$\underset{\eta\to 0}{\mathrm{Lt}} \int \frac{\partial p}{\partial \eta}\, d\xi = \frac{M_2\,\delta}{(1 - M_2^2)^{\frac{1}{2}}} . \qquad \ldots(23)$$

The other boundary conditions are unaltered.

<u>Conformal Transformation</u>

Now p is given as a harmonic function satisfying certain boundary conditions in a triangle ABC with AB and BC circular arcs and AC a straight segment as shown in Fig. 4.

To solve this problem conformal transformation is necessary. The transformation introduced is

$$Z = (K + iK')\left\{i - \frac{2K'}{\zeta - (K + iK')}\right\}$$

where $K^2 + K'^2 = 1$ and $\zeta = \rho e^{i\Theta}$.

On/

On the shock boundary $(\rho^2 + 1)K = 2\rho \cos \theta$ we get

$$Z = \frac{\sqrt{\cos^2\theta - K^2}}{[K' \cos \theta - K \sin \theta]} \qquad \ldots(24)$$

which is purely real and increases from

$$\frac{[\sin^2\alpha_2 - K^2]^{\frac{1}{2}}}{[K' \sin \alpha_2 + K \cos \alpha_2]} \quad \text{to} \quad +\infty.$$

Solving (24) we get

$$\tan \theta = \frac{K'}{K} \frac{(Z^2 - 1)}{(Z^2 + 1)}. \qquad \ldots(25)$$

On the unit circle $X = 0$ and Y varies from

$$\frac{K + \sin \alpha_2}{1 + K \sin \alpha_2 - K' \cos \alpha_2} \quad \text{to} \quad +\infty.$$

The wall gets transformed into the circle

$$X^2 + \left(Y - \frac{\cos \alpha_2}{(K' \sin \alpha_2 + K \cos \alpha_2)}\right)^2 = \frac{K'^2}{[K' \sin \alpha_2 + K \cos \alpha_2]^2}.$$

The region enclosed inside the triangle in the ζ-plane goes into the shaded region of Z-plane (Fig. 5).

The transformation

$$z_1 = \frac{1}{2}\left\{\left(\frac{bZ + 1}{bZ - 1}\right)^{\frac{\pi}{\lambda}} + \left(\frac{bZ + 1}{bZ - 1}\right)^{-\frac{\pi}{\lambda}}\right\} \qquad \ldots(26)$$

where

$$b = \left(\frac{K' \sin \alpha_2 + K \cos \alpha_2}{K' \sin \alpha_2 - K \cos \alpha_2}\right)^{\frac{1}{2}}$$

and

$$\lambda = \cot^{-1}\left\{\frac{\cos \alpha_2}{(\sin^2\alpha_2 - K^2)^{\frac{1}{2}}}\right\}$$

converts the shaded region in Z-plane into the lower half z_1-plane. The shock boundary corresponds to the real z_1-axis with $z_1 > 1$. The wall

becomes/

becomes a part of the real axis with $z_1 < -1$. The unit circle becomes the part of the real axis with $-1 < z_1 < 1$ (Fig. 6). With this transformation the shock boundary conditions transforms to

$$\frac{C_1 + D_1 \tan\theta + E_1 \tan^2\theta + F_1 \tan^3\theta}{C'_1 + D'_1 \tan\theta + E'_1 \tan^2\theta + F'_1 \tan^3\theta} - \frac{K^2}{(1 - K^2)} \tan\theta$$

$$= -\frac{[1 - K^2 \sec^2\theta]^{\frac{1}{2}}}{(1 - K^2)} \cdot \frac{(\partial p/\partial y_1)}{(\partial p/\partial x_1)} \qquad \ldots(27)$$

for $x_1 > 1$, $y_1 = 0$.

Here $\tan\theta = \frac{K'}{K}\frac{(Z^2 - 1)}{(Z^2 + 1)}$ where Z is replaced by $z_1 = x_1$ from (26). The wall boundary condition is that $\frac{\partial p}{\partial y_1} = 0$ when $x_1 < -1$, $y_1 = 0$. The discontinuity condition (23) now becomes

$$-\underset{-y_1 \to 0}{\mathrm{Lt}} \int \frac{\partial p}{\partial y_1} dx_1 = \frac{M_2 \delta}{(1 - M_2^2)^{\frac{1}{2}}} \qquad \ldots(28)$$

and holds at the point

$$z_1 = x_0 = -\cosh\left\{\frac{\pi}{\lambda} \tanh^{-1} \frac{(1 - M_2^2)^{\frac{1}{2}}\sqrt{(\sin^2\alpha_2 - K^2)}}{(M_2 K + \sin\alpha_2)}\right\} < -1 \quad \ldots(29)$$

corresponding to the point

$$\left\{-\frac{\{1 - (1 - M_2^2)^{\frac{1}{2}}\}}{M_2} \sin\alpha_2 + i\,\frac{\{1 - (1 - M_2^2)^{\frac{1}{2}}\}}{M_2} \cos\alpha_2\right\}$$

in the ζ-plane.

The condition on the third boundary can be written $\frac{\partial p}{\partial x_1} = 0$ when $-1 < x_1 < 1$. But when $M_2 > 1$ this must be supplemented with the condition

$$\underset{-y_1 \to 0}{\mathrm{Lt}} \int \frac{\partial p}{\partial x_1} dx_1 = \frac{M_2 \delta}{(M_2^2 - 1)^{\frac{1}{2}}} \qquad \ldots(30)$$

which holds at the point

$$z_1 = x_0 = \cos\left(\frac{2\theta\pi}{\lambda}\right) > -1 \qquad \ldots(31)$$

where/

where

$$\lambda = \tan^{-1}\frac{(\sin^2\alpha_2 - K^2)^{\frac{1}{2}}}{\cos\alpha_2} \quad \text{and} \quad \theta = \tan^{-1}\frac{[1 + M_2(K\sin\alpha_2 - K'\cos\alpha_2)]}{b[(M_2\sin\alpha_2 + K) - K'(M_2^2 - 1)^{\frac{1}{2}}]}$$

corresponding to the point

$$\left\{(-\sin\alpha_2 + i\cos\alpha_2)\left[\frac{1}{M_2} - \frac{i\sqrt{(M_2^2 - 1)}}{M_2}\right]\right\}.$$

Solution

Now we shall find out a function which satisfies all the boundary conditions. The solution is effected by the introduction of a function

$$\omega(z_1) = \frac{\partial p}{\partial x_1} + i\frac{\partial p}{\partial y_1}$$

which is regular throughout the lower half-plane since p is harmonic. In terms of ω the discontinuity condition (28) and (30) can be written

$$\left.\begin{aligned} \omega &\sim -\frac{\dfrac{M_2\delta}{\pi(1 - M_2^2)^{\frac{1}{2}}}}{(z_1 - x_0)}, \quad M_2 < 1 \\ \omega &\sim -\frac{\dfrac{iM_2\delta}{\pi(M_2^2 - 1)^{\frac{1}{2}}}}{(z_1 - x_0)}, \quad M_2 > 1 \end{aligned}\right\} . \qquad \ldots(32)$$

Equation (20) becomes

$$\delta = \int_{\infty}^{1}\frac{B_1 - A_1G}{B_2 - A_2G}\frac{\partial p}{\partial x_1}dx_1 \qquad \ldots(33)$$

where $y = K(\cos\alpha_2 + \sin\alpha_2\tan\Theta)$ from the section on Busemann's transformation and $\tan\Theta$ is replaced by its value in terms of z_1 by the help of (25) and (26).

We know that $\log\omega(z_1)$ is such a function that the value of its imaginary part is known on the real axis of the z_1-plane. In such a case an extension of Poisson's integral formula gives the value of $\log\omega(z_1)$ as

log/

$$\log \frac{\omega(z_1)}{C_1} = -\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\left[\tan^{-1}\dfrac{(\partial p/\partial y_1)}{(\partial p/\partial x_1)}\right]_{x_1=t}}{t - z_1}\,dt \qquad \ldots(34)$$

where C_1 is a real constant and $\left[\tan^{-1}\dfrac{(\partial p/\partial y_1)}{(\partial p/\partial x_1)}\right]_{x_1=t}$ means that x_1 in $\left[\tan^{-1}\dfrac{(\partial p/\partial y_1)}{(\partial p/\partial x_1)}\right]$ has been replaced by t.

Now when the above integral is evaluated for points on the real axis due to the discontinuity in the cases $M_2 > 1$ or $M_2 < 1$ at the point $z_1 = x_0$ we get two other constants C_2 and D. Finally in the expression of $\omega(z_1)$ we will get two constants C and D, C being determined by the condition (32) and D is determined from the condition (33).

Part II

Sonic case $\left(\dfrac{U - q_2}{a_2} = 1\right)$

In this case the boundaries in the Z-plane are different than in the case $\dfrac{(U-q_2)}{a_2} < 1$. Here in the Z-plane the shock boundary runs from 0 to ∞ as the point C' of Z-plane (Fig. 5) shifts to the origin, the wall becomes the semi-circle of radius $\dfrac{1}{2\sin\alpha_2}$ with centre at $\left(0, \dfrac{1}{2\sin\alpha_2}\right)$ and the unit circle runs from $\dfrac{1}{\sin\alpha_2}$ to ∞ on the imaginary axis. The boundaries are shown in Fig. 7.

Now a fresh transformation is to be introduced for transforming the boundaries from the Z-plane to z_1-plane.

The transformation introduced is

$$z_1 = \cosh\left(\frac{\pi}{Z}\frac{1}{\sin\alpha_2}\right). \qquad \ldots(35)$$

This transformation transforms the shaded region in the Z-plane into lower half plane in the z_1-plane. The shock boundary corresponds to the real z_1-axis with $z_1 > 1$. The wall becomes the part of the real axis with $z_1 < -1$. The unit circle becomes the part of the real axis with $-1 < z_1 < 1$.

The/

The function $\omega(z_1)$ defined before is the same for this case also. Equations (27), (28), (30), (32), (33) and (34) hold good here also. However the equations corresponding to (29) and (31) are

$$z_1 = x_0 = -\cosh\left(\pi \cot \alpha_2 \sqrt{\frac{1 - M_2}{1 + M_2}}\right) < -1, \quad M_2 < 1 \qquad \ldots(36)$$

and

$$z_1 = x_0 = -\cos\left(\pi \cot \alpha_2 \sqrt{\frac{M_2 - 1}{M_2 + 1}}\right) > -1, \quad M_2 > 1 \qquad \ldots(37)$$

respectively.

Part III

Numerical Solution

Subsonic case $\left(\frac{U - q_2}{a_2} < 1\right)$

The calculations have been carried out for two shock strengths. The table given below gives the choice of the data

p_0/p_1	α_0	α_2	$\frac{U - q_2}{a_2}$	M_2
0	39•97°	32•97°	0•94699	1•48137
0•5	42° 27'	51° 6'	0•95765	0•67255

Now for determining the function $\omega(z_1)$, the integral on the left hand side of (34) could be broken into three integrals ranging from $-\infty$ to -1, -1 to $+1$ and $+1$ to $+\infty$. Then applying the boundary condition on $\omega(z_1)$ and simplifying we would obtain

$$\log \omega(z_1) = \log \frac{C\delta[D(z_1 - x_0) - 1]}{(z_1 - x_0)\sqrt{(z_1^2 - 1)}} - \frac{z_1}{\pi}\int_0^1 \frac{\left[\tan^{-1}\frac{(\partial p/\partial y_1)}{(\partial p/\partial x_1)}\right]_{x_1 = t = \frac{1}{x}}}{(1 - xz_1)}\, dx. \qquad \ldots(38)$$

For/

For $z_1 > 1$ we write

$$\int_0^1 \frac{\left[\tan^{-1}\frac{(\partial p/\partial y_1)}{(\partial p/\partial x_1)}\right]_{x_1=t=\frac{1}{x}}}{(1 - xz_1)}\,dx$$

$$= \beta\int_0^1 \frac{dx}{(1 - xz_1)} + \int_0^1 \frac{\left[\tan^{-1}\frac{(\partial p/\partial y_1)}{(\partial p/\partial x_1)}\right]_{x_1=t=\frac{1}{x}} - \beta}{(1 - xz_1)}\,dx$$

where $\beta = \left[\tan^{-1}\frac{(\partial p/\partial y_1)}{(\partial p/\partial x_1)}\right]_{x_1=t=\frac{1}{x}=z_1}$.

Having done that we find after approximate numerical evaluation that for $M_2 = 1{\cdot}48137$

$$\omega(z_1) = \exp\left[-\frac{z_1}{12\pi}\left\{(-1{\cdot}51698 - \beta) + \frac{4(-0{\cdot}05779 - \beta)}{(1 - 0{\cdot}25z_1)} + \frac{2(0{\cdot}05767 - \beta)}{(1 - 0{\cdot}5z_1)}\right.\right.$$

$$\left.\left. + \frac{4(0{\cdot}18751 - \beta)}{(1 - 0{\cdot}75z_1)} + \frac{(\frac{\pi}{2} - \beta)}{(1 - z_1)}\right\}\right](z_1 - 1)^{\frac{\beta}{\pi}} \cdot e^{i\beta} \cdot \frac{C\delta[D(z_1 - x_0) - 1]}{(z_1 - x_0)\sqrt{(z_1^2 - 1)}},$$

$$z_1 > 1 \qquad \ldots(39)$$

$$\omega(z_1) = \exp\left[-\frac{z_1}{12\pi}\left\{(-1{\cdot}51698) + \frac{4(-0{\cdot}05779)}{(1 - 0{\cdot}25z_1)} + \frac{2(0{\cdot}05767)}{(1 - 0{\cdot}5z_1)}\right.\right.$$

$$\left.\left. + \frac{4(0{\cdot}18751)}{(1 - 0{\cdot}75z_1)} + \frac{\pi/2}{(1 - z_1)}\right\}\right] \cdot \frac{C\delta[D(z_1 - x_0) - 1]}{(z_1 - x_0)\sqrt{(z_1^2 - 1)}}, \; z_1 < 1. \qquad \ldots(40)$$

The function $\omega(z_1)$ satisfies all the boundary conditions. The argument of the function $\omega(z_1)$ for $z_1 > 1$ is β which one should get for the shock boundary condition to be satisfied. $\omega(z_1)$ is purely imaginary for $-1 < z_1 < 1$ and is purely real for $z_1 < -1$. The constants C and D are known from conditions (32) and (33).

The expression for $\omega(z_1)$ for $M_2 = 0{\cdot}67255$ is

$\omega(z_1)$/

$$\omega(z_1) = \exp\left[-\frac{z_1}{12\pi}\left\{(-0{\cdot}83807-\beta)+\frac{4(-0{\cdot}00712-\beta)}{(1-0{\cdot}25z_1)}+\frac{2(-0{\cdot}40698-\beta)}{(1-0{\cdot}5z_1)}\right.\right.$$

$$\left.\left.+\frac{4(-0{\cdot}78184-\beta)}{(1-0{\cdot}75z_1)}+\frac{(\frac{\pi}{2}-\beta)}{(1-z_1)}\right\}\right]\cdot(z_1-1)^{\frac{\beta}{\pi}}e^{i\beta}\,\frac{C\delta[D(z_1-x_0)-1]}{(z_1-x_0)\sqrt{(z_1^2-1)}},$$

$$z_1 > 1 \qquad \ldots(41)$$

$$\omega(z_1) = \exp\left[-\frac{z_1}{12\pi}\left\{-0{\cdot}83807+\frac{4(-0{\cdot}00712)}{(1-0{\cdot}25z_1)}+\frac{2(-0{\cdot}40698)}{(1-0{\cdot}5z_1)}\right.\right.$$

$$\left.\left.+\frac{4(-0{\cdot}78184)}{(1-0{\cdot}75z_1)}+\frac{\pi/2}{(1-z_1)}\right\}\right]\cdot\frac{C\delta[D(z_1-x_0)-1]}{(z_1-x_0)\sqrt{(z_1^2-1)}}, \qquad z_1 < 1. \qquad \ldots(42)$$

Pressure Distribution along the Wall

At a point $(x, 0)$ of the wall $(-1 < x < \frac{U-q_2}{a_2})$ the x_1 co-ordinate is

$$x_1 = -\cosh\left[\frac{\pi}{\lambda}\tanh^{-1}\frac{(1-x^2)}{(1-kx)}\sqrt{1-k^2}\right]$$

which satisfies $x_1 < -1$.

The pressure derivative in this region is obtained from (40) for $M_2 = 1{\cdot}48137$ and from (42) for $M_2 = 0{\cdot}67255$. After integrating the pressure derivative for different points of the wall, the pressure distribution along the wall has been obtained. In Figs. 8 and 9 the value of $\frac{(p_2' - p_2)}{\delta(p_2 - p_1)} = \frac{a_2 q_2 \rho_2}{(p_2 - p_1)}(-\frac{p}{\delta})$ has been plotted for different points of the wall. The disturbed region has also been shown. In the case $M_2 = 1{\cdot}48137$ the pressure maintains a constant value from the corner to the point of intersection of unit circle and wall as it is given by Prandtl-Meyer expansion theory. From the point of intersection of wall and unit circle to the point of intersection of shock and wall we find that there is a monotonic decrease in the value of $(p_2' - p_2)/\delta(p_2 - p_1)$ (Fig. 8). In the case $M_2 = 0{\cdot}67255$ the value of $(p_2' - p_2)/\delta(p_2 - p_1)$ which is zero at the boundary increase to infinity at the corner. From infinity it again decreases and finally rises (Fig. 9).

Sonic case $\left(\frac{(U - q_2)}{a_2} = 1\right)$

In this case also the numerical work has been carried out for two shock strengths. The table given below gives the choice of data

p_0/p_1 /

p_0/p_1	α_0	α_2	$\frac{U - q_2}{a_2}$	M_2
0	39•91°	31° 14'	1•00009	1•44938
0•5	42° 11'	48° 52'	1•00002	0•64616

The function $\omega(z_1)$ is determined from the equation (38). We find here for $M_2 = 1{\cdot}44938$

$$\omega(z_1) = \exp\left[-\frac{z_1}{12\pi}\left\{\left(-\frac{\pi}{2}-\beta\right)+\frac{4(-0{\cdot}09153-\beta)}{(1-0{\cdot}25z_1)}+\frac{2(-0{\cdot}31151-\beta)}{(1-0{\cdot}5z_1)}\right.\right.$$
$$\left.\left.+\frac{4(-0{\cdot}58272-\beta)}{(1-0{\cdot}75z_1)}+\frac{\left(\frac{\pi}{2}-\beta\right)}{(1-z_1)}\right\}\right](z_1-1)^{\frac{\beta}{\pi}}\cdot e^{i\beta}\,\frac{C\delta[D(z_1-x_0)-1]}{(z_1-x_0)\sqrt{(z_1^2-1)}},$$
$$z_1 > 1. \qquad \ldots(43)$$

$$\omega(z_1) = \exp\left[-\frac{z_1}{12\pi}\left\{-\frac{\pi}{2}+\frac{4(-0{\cdot}09153)}{(1-0{\cdot}25z_1)}+\frac{2(-0{\cdot}31151)}{(1-0{\cdot}5z_1)}\right.\right.$$
$$\left.\left.+\frac{4(-0{\cdot}58272)}{(1-0{\cdot}75z_1)}+\frac{\pi/2}{(1-z_1)}\right\}\right]\cdot\frac{C\delta[D(z_1-x_0)-1]}{(z_1-x_0)\sqrt{(z_1^2-1)}}, \qquad z_1 < 1.$$
$$\ldots(44)$$

Similarly for $M_2 = 0{\cdot}64616$

$$\omega(z_1) = \exp\left[-\frac{z_1}{12\pi}\left\{\left(-\frac{\pi}{2}-\beta\right)+\frac{4(-0{\cdot}04179-\beta)}{(1-0{\cdot}25z_1)}+\frac{2(-0{\cdot}43690-\beta)}{(1-0{\cdot}5z_1)}\right.\right.$$
$$\left.\left.+\frac{4(-0{\cdot}81216-\beta}{(1-0{\cdot}75z_1)}+\frac{\left(\frac{\pi}{2}-\beta\right)}{(1-z_1)}\right\}\right]\cdot(z_1-1)^{\frac{\beta}{\pi}}\cdot e^{i\beta}\cdot\frac{C\delta[D(z_1-x_0)-1]}{(z_1-x_0)\sqrt{(z_1^2-1)}},$$
$$z_1 > 1 \qquad \ldots(45)$$

$$\omega(z_1) = \exp\left[-\frac{z_1}{12\pi}\left\{-\frac{\pi}{2}+\frac{4(-0{\cdot}04179)}{(1-0{\cdot}25z_1)}+\frac{2(-0{\cdot}43690)}{(1-0{\cdot}5z_1)}\right.\right.$$
$$\left.\left.+\frac{4(-0{\cdot}81216)}{(1-0{\cdot}75z_1)}+\frac{\pi/2}{(1-z_1)}\right\}\right]\cdot\frac{C\delta[D(z_1-x_0)-1]}{(z_1-x_0)\sqrt{(z_1^2-1)}}, \qquad z_1 < 1.$$
$$\ldots(46)$$

At/

Pressure Distribution along the Wall

At a point $(x, 0)$ of the wall $\left(-1 < x < \frac{U - q_2}{a_2}\right)$ the x_1 co-ordinate is $x_1 = -\cosh\left(\pi \cot \alpha_2 \sqrt{\frac{1 + x}{1 - x}}\right)$ which satisfies $x_1 < -1$.

The pressure derivative is obtained from (44) for $M_2 = 1{\cdot}44938$ and from (46) for $M_2 = 0{\cdot}64616$. In Figs. 10 and 11 the value of $\frac{(p_2' - p_2)}{\delta(p_2 - p_1)} = \frac{a_2 q_2 \rho_2}{(p_2 - p_1)}\left(-\frac{p}{\delta}\right)$ has been plotted for different points of the wall. The disturbed region has also been shown. In the case $M_2 = 1{\cdot}44938$ the value of $(p_2' - p_2)/\delta(p_2 - p_1)$ after maintaining a constant value from the corner to the point of intersection of unit circle and wall decreases monotomically to the point of intersection of shock and wall (Fig. 10). In the case $M = 0{\cdot}64616$ the value of $(p_2' - p_2)/\delta(p_2 - p_1)$ increases from zero at the boundary to infinity at the corner. From infinity it again decreases and finally rises (Fig. 11).

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FIG. 1.

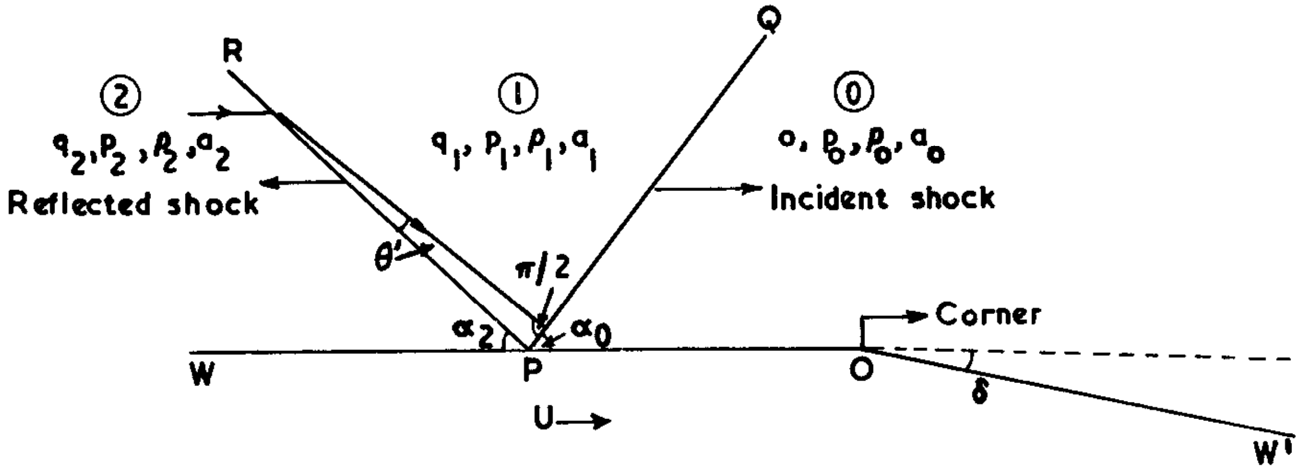


FIG. 2.

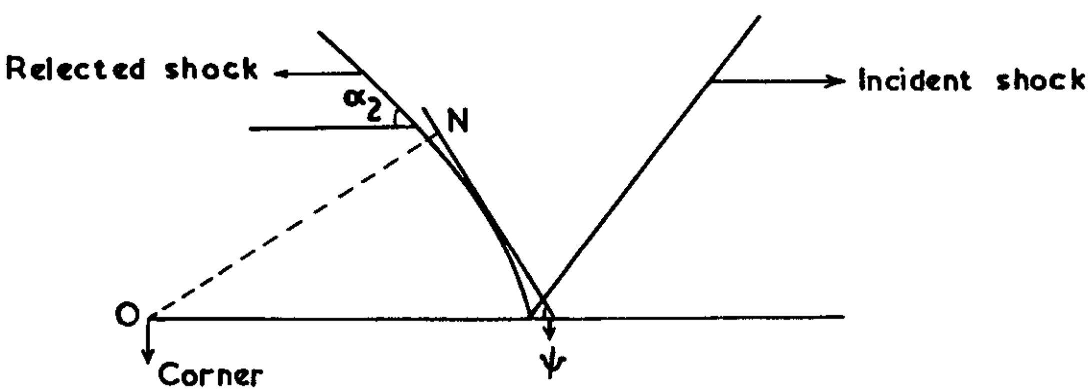
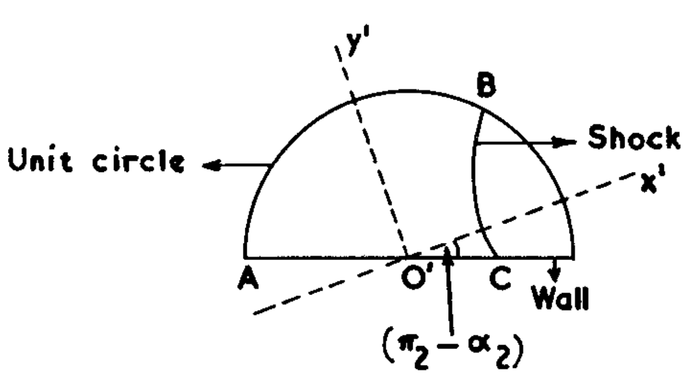


FIG. 3



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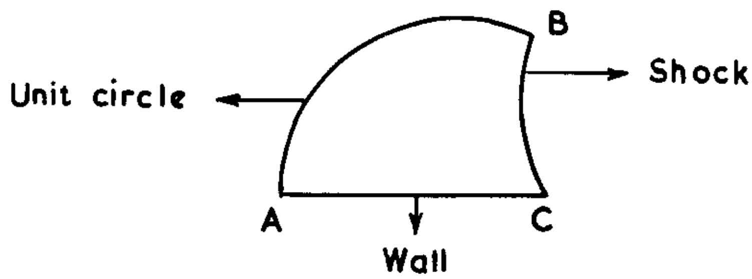
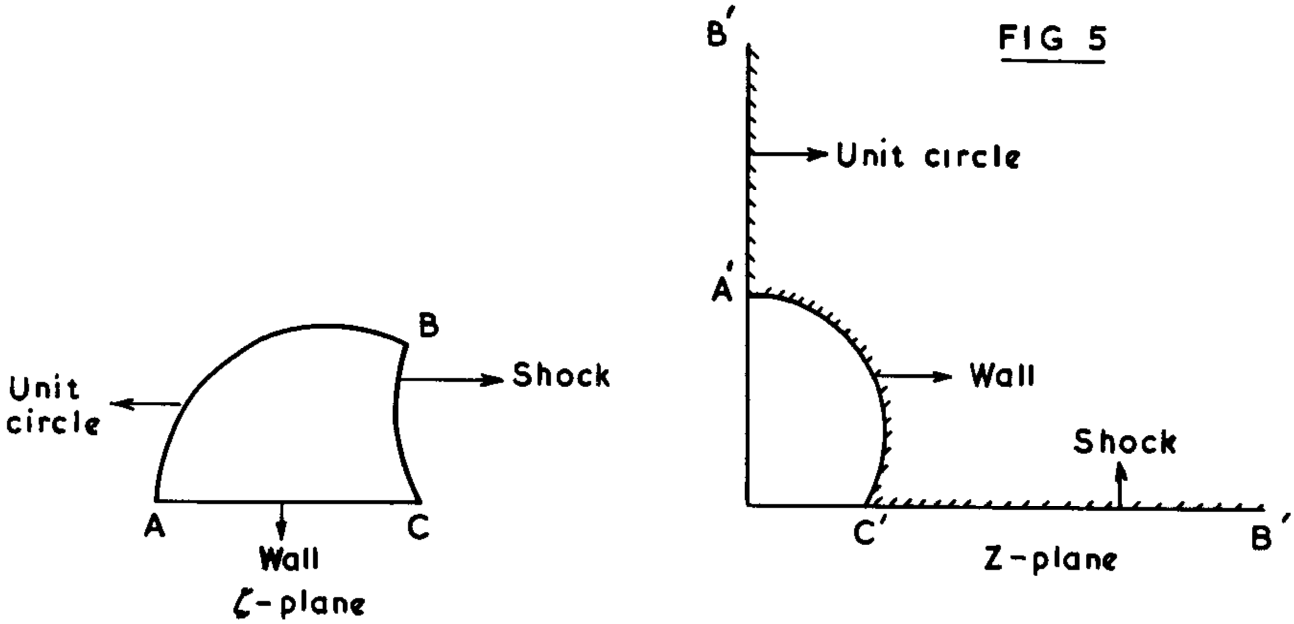


FIG 5



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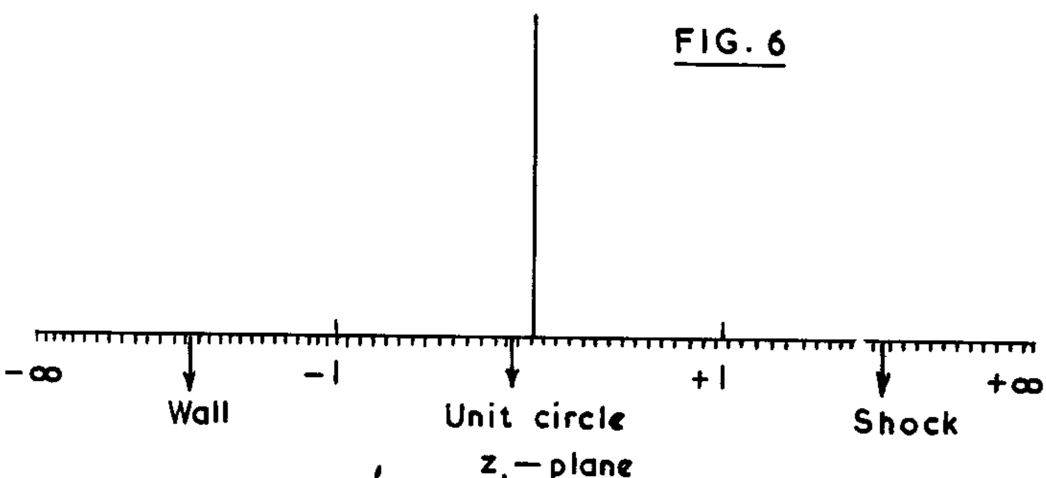

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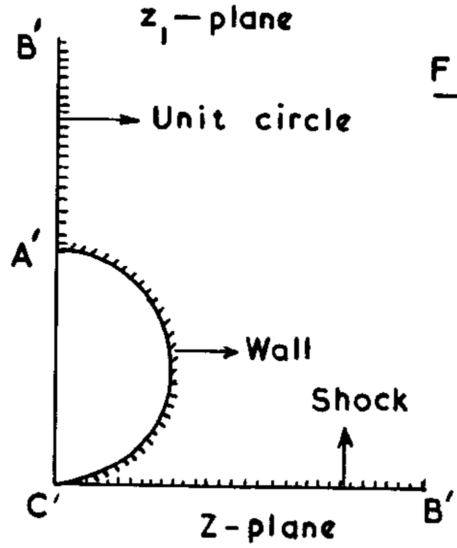


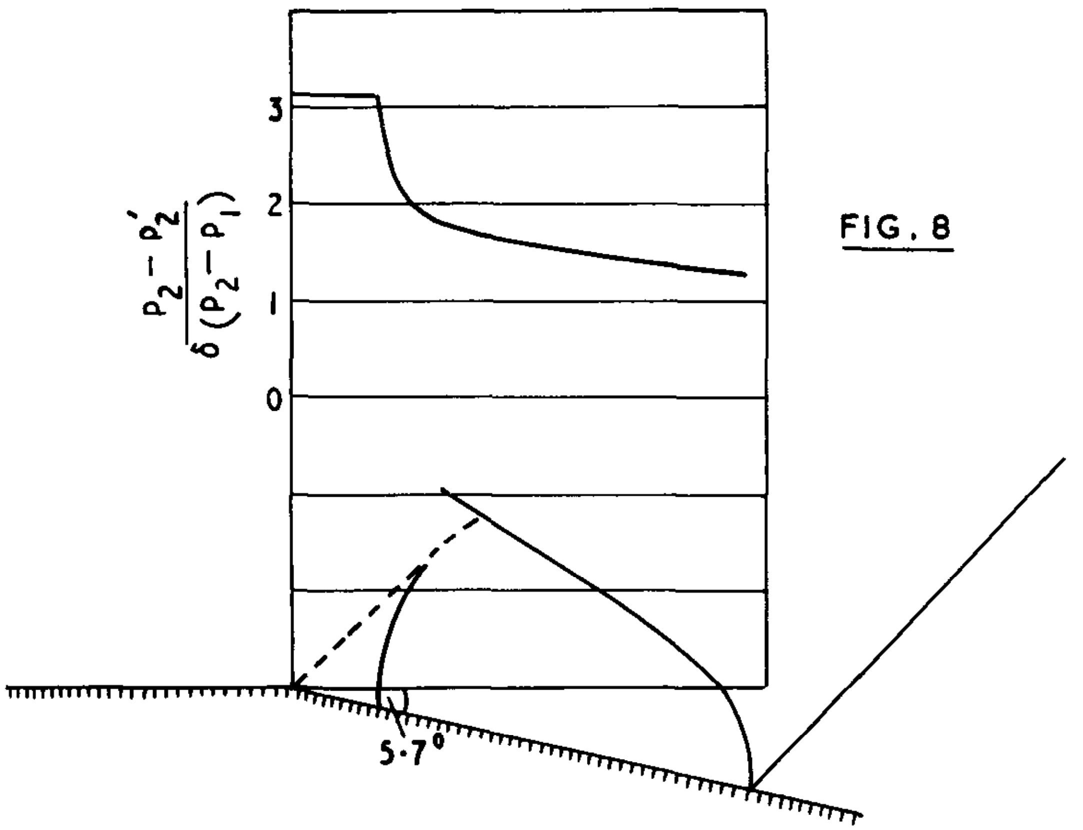
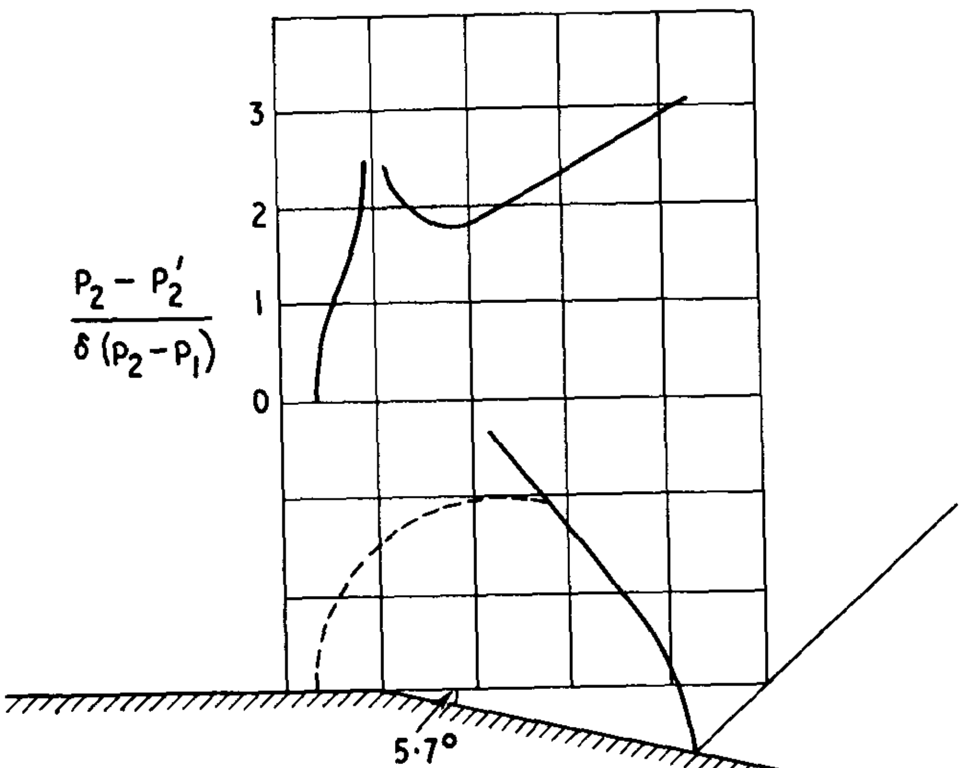


FIG. 8

Wall pressure distribution and shape of disturbed region ($\delta = 0{\cdot}1$ radian, $p_1/p_0 = \infty$, $\alpha_0 = 39{\cdot}97°$, $\alpha_2 = 32\ 97$)

FIG. 9



Wall pressure distribution and shape of disturbed region

(δ = 0·1 radian, $\frac{p_1}{p_0}$ = 2, $\alpha_o = 42° 27'$, α_2 = 51° 6')

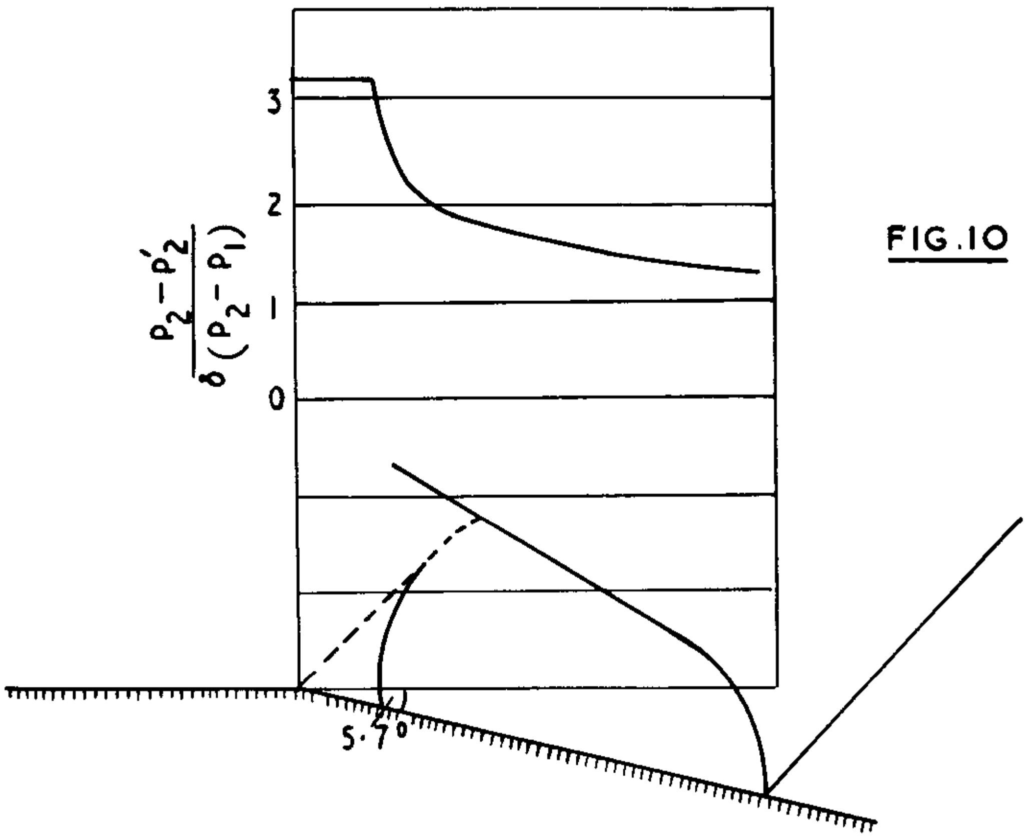
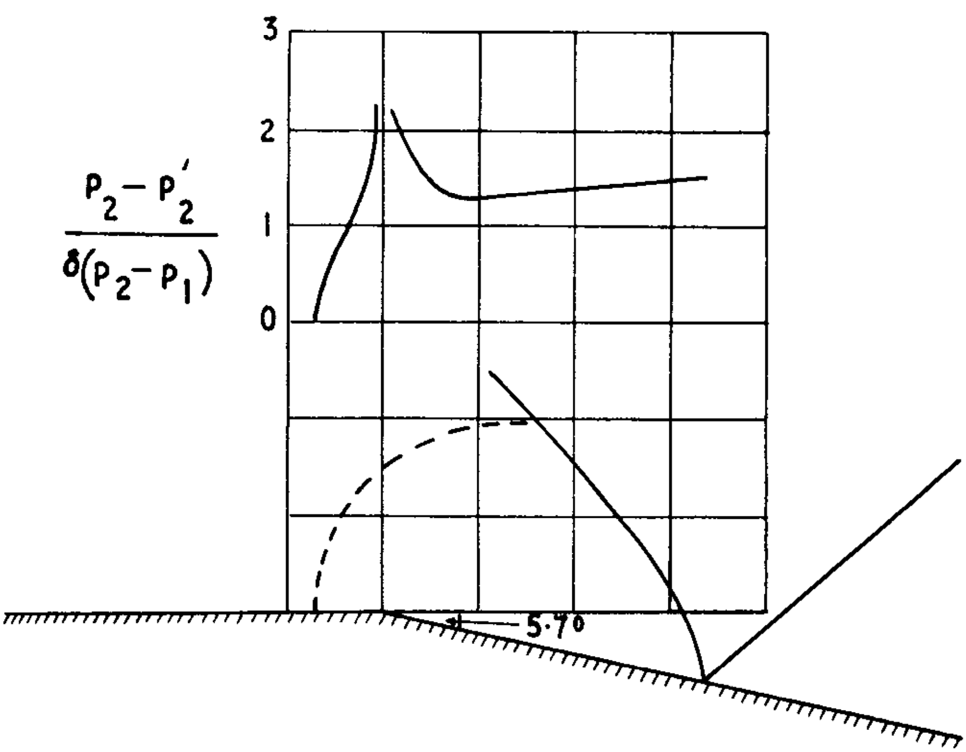


FIG. 10

Wall pressure distribution and shape of disturbed region ($\delta = 0 \cdot 1$ radian, $p_1 / p_0 = \infty$, $\alpha_0 = 39 \cdot 91^\circ$, $\alpha_2 = 31^\circ 14'$)

FIG. 11



Wall pressure distribution and shape of disturbed region ($\delta = 0 \cdot 1$ radian, $\frac{p_1}{p_0} = 2$, $\alpha_0 = 42^\circ 11'$, $\alpha_2 = 48^\circ 52'$)

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December, 1966
R. S. Srivastava

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