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A Survey of Fluid Flow and Heat Transfer in Rotating Ducts

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Introduction

This Paper is intended as an introduction to the study of fluid flow and heat transfer in rotating ducts, and to serve as a basis for discussion of this field of work. The interest in this subject results mainly from the need to determine heat transfer between the surfaces of ducts or passages in the components of rotating machines and a confined or flowing fluid. There are numerous situations in the engineering field where this type of problem occurs. Fig. 1 shows a number of cases of rotating machine components and illustrates the wide range of duct shapes and attitudes of rotation which may be encountered in practice. The fluids in these convection problems may be liquid or gas, and in some cases heat transfer may be accompanied by phase change as in the case of internal cooling of a turbine blade by coolant evaporation. A wide range of liquids has been studied because of the importance of this problem in the chemical industry. Gases other than air are of interest here also. In the cooling of rotating electrical machines for example, hydrogen may be employed on account of its low density (which minimises windage losses) and its high thermal conductivity.

The main difference between the problem of heat transfer in a rotating duct and that of the stationary duct is in the existence of a secondary flow superimposed on the through flow (should there be one). This secondary flow arises partly (if not entirely) as a result of buoyant motion. The buoyancy effects are due to centrifugal and coriolis forces in the presence of a density gradient. As an illustration of these effects, the flow of a fluid through a revolving duct as shown in Fig. 2(a) might be considered. In this case, a secondary flow may exist even when the duct is stationary because of its shape. With rotation, however, the cooler and more dense fluid near the centre will tend to move radially outwards under centrifugal loading with the result that the secondary flow pattern shown will occur. There will be some coriolis force effect but this is less important (see Fig. 2(b)). With practical centrifugal accelerations up to the order of 70,000g significant buoyancy effects can be produced in the presence of heat transfer.

Complete/

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* Replaces A.R.C.30 525

Complete generalisation of the problem is clearly difficult on account of the numerous geometries, attitudes or rotation of revolution and types of fluid which are involved. However, some classification of the types of system which are of immediate interest here is possible, and an attempt at this has been made in Fig. 3 by classifying three particular modes of rotation. These are:

Case I. Rotation about the duct axis.

(e.g. a bored transmission shaft)

Case II. Rotation about an axis perpendicular to the duct axis.

(e.g. a coolant passage in a turbine blade)

and Case III. Revolution about an axis parallel to the duct axis.

(e.g. an axial coolant duct in a turbo alternator).

Within these three categories, there are many variations of the problem as reference to Fig. 3 will show. For a particular attitude there is a wide variety of duct geometries, and with the rotating flow the entry condition and tube length are very important. In Case III, for example, the restricted entry case is likely to be a different problem from the free entry case unless the tube is exceptionally long when presumably the separate entry effects will be reduced to a minimum.

It is not surprising then that a brief survey of the earlier literature indicates that many investigations have been directed at the solution of one particular situation with the result that their data and correlations are restricted in application. This will become more evident in the next Section where it has been found necessary to deal with the earlier work under the three headings pertaining to the attitudes of rotation.

At this point some mention might be made of the basic equations of flow and heat transfer in rotating ducts. In Appendix 1 an indication is given of the modifications which are introduced into these equations when the system rotates. Only in the most idealised geometries and flow conditions are the fundamental equations amenable to approximate solution so that they best serve to indicate the dimensionless variables which are relevant in problems of this kind. Extensive experimental investigation is required to obtain the appropriate functional relationships and some difficulty is obtained in varying the separate parameters independently. The relevant dimensionless parameters are referred to in Appendix 1.

Previous Work

(a) Case I. The simplest and perhaps most common rotating configuration is that in which the duct axis and the axis of rotation are coincident. The case of a steady flow through a duct of circular cross-section has important technical applications and will be considered first. At the outset, however, it is necessary to make a preliminary examination of the nature of the flow in such a situation with and without heat transfer. (Many investigators have found an understanding of the flow mechanism to be of special value in the explanation of the heat transfer behaviour in the present field of study). Perhaps the main feature here is that of flow stability. Many years ago, Lord Rayleigh showed that when an inviscid fluid moves in a curved path, the flow tends to be stable provided the angular momentum increases with radius. (In a flow where the angular momentum decreases radially, as for example in the case of an annulus where the inner boundary rotates, the flow is unstable and can lead to Taylor-Görtler vortices). This stabilization effect in axially rotating pipe flow has been convincingly demonstrated by White [1]*. At high rotational speeds, reductions in pressure loss of up to 40% were obtained at values of Re corresponding to the turbulent flow of water in a stationary pipe. White [2] also extended the Rayleigh criterion for stability by an approximate method to show that stability is dependent on compressibility effects and temperature gradient. On the basis of his analysis he concluded that heating would produce instability and so increase heat transfer (cooling produces the reverse effect) and that compressible fluids are more stable than incompressible fluids. He had made an earlier experimental study [3] of heat transfer to air and water in an axially rotating pipe. In the case of heating, there was a considerable increase of heat transfer in both the case of air and water but the heat transfer to air tended to decrease again at high speeds. Cooling of the air produced the reverse effect. The tests were made with and without vanes in the entry. In general, White's data were in agreement with his criteria for stability.

An almost identical test to that made by White has been carried out more recently by Cannon and Kays [4]. These workers employed air as the working fluid in their heat transfer investigation, and water for visual flow observations. It was found that the major effect of tube rotation was on the transition from laminar to turbulent flow otherwise the heat transfer coefficient was unaffected. Like White, they recognised the stabilization effects and suggested the possibility of retransition or laminarization of an initially turbulent flow. There appears to be some disagreement between the results of White for air and those of Cannon and Kays. These differences are possibly due to differences of geometry, particularly in regard to the length of the heated section (See Table 1). It would have been interesting to have had results at larger values of Re in White's equipment: a more critical examination of the two sets of data would then have been possible. Perhaps the most interesting feature of Cannon and Kays' work is that of the entry effect. Changing the entry conditions had the effect of changing the transition delay. The thermal effects, i.e. heating and cooling, did not greatly alter the transition delay pattern.

Some of the work of Kuo et al [5] on heat transfer in filled and partly filled rotating pipes and annuli is relevant here, although the range of geometries and flow variables are somewhat specialised and pertain to particular applications in the industry. Their data for the case of a full tube (water being heated) indicates increases in average heat transfer far greater than those observed by White [3].

A number of studies of the annulus geometry have been reported [5,6,7,8,9]. Details of a selected number of these are also recorded in Table 1, where it will be seen that the range of conditions etc. makes comparison difficult. Perhaps

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* See Tables 1 and 2 for a summary of Previous Work

the work of Bjorklund and Kays[8] is of special importance here because it is related to problems in connection with the thermal behaviour of enclosed shafting, multiple concentric drives (see Fig. 1) and turbine rotors etc. The fluid in this study was air confined between concentric rotating cylinders with heat transfer radially outwards. Nusselt numbers in the rotating case were related to the Nusselt number by conduction (Appendix 3) alone through the Taylor number. For example, with the outer cylinder stationary which is very common the following correlation applied:-

$$\frac{Nu}{Nu_{co}} = 0{\cdot}175\ Ta^{1/2} \qquad \left(\begin{array}{l} 90 < Ta < 2000 \\ \dfrac{d_o - d_i}{d_i} = 0{\cdot}05 \rightarrow 0{\cdot}25 \end{array}\right) \qquad \cdots (1)$$

This implies that for a rotational Reynolds number of 64,000, the heat transfer coefficient is 8 times that in conduction. This correlation at least affords a method of determining heat transfer in a single idealised geometry which is very relevant in rotating machinery design. In this study, there was no axial flow, but in a contribution to the paper by Bjorklund and Kays, Kreith[10] makes the point that even with a low axial velocity phenomena similar to those in the enclosed system occur. It would seem therefore, that the work of Bjorklund and Kays is most useful in long concentric shaft problems. The use of the Taylor number in the correlation of heat transfer coefficient, Nu

i.e. $$Nu = f\ (Ta,\ \text{geometry}) \qquad \cdots (2)$$

is consistent with the general correlations which have been discussed in Appendix 2 and must not be considered as a special equation. It will be a useful exercise to test this correlation with other data as they occur. Inclusion of the Prandtl number will be necessary, however, when fluids other than air are tested, in accordance with the requirements of equation (A.1).

Case II

This Section is concerned with heat transfer to fluids which are either contained or in steady flow through passages whose centre-lines are perpendicular to the axis of rotation. The internal convection cooling of turbine rotor blades falls within this category as reference to Fig. 1 will show. An excellent review of work in the turbine application has been given by Ellerbrock and Livingood[18] and many references are included. Understandably, air is perhaps the most common working medium in this connection on account of its availability, particularly in aircraft engine applications. Liquid coolant, on the other hand, may be more useful in marine and stationary power plants. Cohen and Bayley[19], have made a survey of possible methods of liquid cooling of gas turbine blades and deal, in particular, with the thermosyphon system which will be referred to later. The coolant passage or duct geometry in the blading problem is dictated to a large extent by design and manufacturing techniques so that a wide range of pos ible shapes is encountered.

While the effects of buoyant forces must be recognised in the general problem, it would appear from an examination of papers on this subject that in some circumstances this is unnecessary and the use of standard forced convection correlations based on equivalent diameter may be resorted to. For gas flows,

e.g. air/

e.g. air cooling, apart from modifications to allow for length and temperature effects, the familiar formulae for the stationary duct situation have been employed. Horlock[20] gives

$$\overline{Nu} = 0{\cdot}034\, Pr^{0\cdot4} \left(\frac{L}{de}\right)^{-0\cdot1} \overline{Re}^{\,0\cdot8} \left(\frac{\overline{T}_c}{\overline{T}_w}\right)^{0\cdot55} \qquad \dots (3)$$

which is the Humble, Lowdermilk and Desmon correlation. This equation may be simplified to

$$\overline{Nu} = 0{\cdot}02\, \overline{Re}^{\,0\cdot8} \left(\frac{\overline{T}_c}{\overline{T}_w}\right)^{0\cdot55} \quad \left\{ \begin{array}{c} Pr = 0{\cdot}7 \\ 25 < \dfrac{L}{de} < 100 \end{array} \right\} \qquad \dots (4)$$

The nature of the geometry of blade coolant passages for maximum heat transfer is considered by Ainley[21] who introduces a shape parameter for comparison of the relative merits of various duct configurations. Further evidence of this procedure of using forced convection correlations for stationary pipes is to be found in the paper by Fray and Barnes[22] where a formula similar to that given in equation (4) has been used.

It would seem, therefore, that in the case of a gas in flow through a radial rotating tube or passage, there is negligible centrifugal and coriolis force effect. It will be interesting to have the results of a theoretical analysis in this case to substantiate the foregoing procedure of calculation. (We shall see later that theoretically predicted increases in heat transfer in another rotating geometry are not large for the case of air).

In the case of liquid flow, we must anticipate large buoyancy effects when the duct is revolved. An illustration of such an effect is presented in Appendix 4 which shows a calculation for the heat transfer coefficient to a contained fluid following the idea of Schmidt[23]. It can be seen that the thermal resistance on the coolant side is substantially reduced by the natural convection effect which is enhanced by the rotation of the duct. Provided the flow does not choke, the calculation procedure which uses results relevant to the case of a plane surface in an infinite medium is adequate. (Alternative designs are available to alleviate choking in the passage.) Even so, the outside heat transfer coefficient is controlling in this case and it would appear that better accuracy in determining the value of the inside coefficient is unnecessary. Further modifications to this very simple thermosyphon effect are reported by Cohen and Bayley[19]. With these modifications, the range of possible fluids is increased markedly and phase change can occur in some cases.

Finally, some points might be made in connection with the combined forced and free convection mode of heat transfer. With steady flow of a liquid through a radial rotating passage under large centrifugal loading, significant buoyancy effects can occur and give rise to flow reversal phenomena. This effect has been demonstrated recently by Rao[24] in his experiments on combined forced and free convection in a stationary vertical tube. It would seem, however, that the well established forced convection correlations suffice in the calculation of the heat transfer coefficient in the rotating case[18]. There is then an important question in this field which deserves attention and further enquiry must be made to obtain reliable methods for prediction. Basically, the problem is one of forced convection in the entry region of a duct in the presence of large axial body forces and transverse temperature gradients. It will be necessary to consider both radially inward and outward net flow to cater for the different designs of coolant passages.

Case III

Finally, we consider those problems where the axis of rotation is parallel to the axis of the duct. Again the duct of circular cross-section deserves special consideration.

Following the reasoning given elsewhere, large speeds of revolution give rise to a secondary flow due to centrifugal and coriolis buoyancy forces. This secondary flow effect will be additional to that accruing from entry swirl which is inherent in any real case. However, the results of the study of the so-called fully developed case where entry swirl is absent is not without interest and this problem has been studied analytically [11,13]. The most recent theoretical work by Nakayama [13] for turbulent flow uses the boundary layer concept and a core region where the flow is assumed to be irrotational. The analysis is complex. The Nusselt number ratio (Nu/Nu_s) is presented as a function of a new parameter (Γ) which is a measure of the significance of the inertia forces compared with the body forces. A disintegrated form of the complex expression for Γ facilitates quick determination of (Nu/Nu_s). It suffices here to indicate the order of magnitude of (Nu/Nu_s). It is shown in the paper, that for the following data:-

$$\bar{u} = 20 \text{ m/s}$$
$$d = 10 \text{ mm}$$
$$H = 50 \text{ cm}$$
$$\Omega = 523 \text{ rad/s } (5000 \text{ rpm})$$
$$\text{air temp.} = 40°\text{C}$$
$$\text{axial temp. gradient} = 20°\text{C/m}$$

the increase in the Nusselt number over that in the stationary tube is about 30 per cent. The coriolis force exerts very little influence and so the flow pattern is similar to that in Fig. 2(b), being very nearly symmetrical about a diameter which is coincident with the radius arm of revolution. The effect of secondary flow increases with increase in angular velocity and decreases with increase in through velocity.

Two experimental studies of more practical geometries in this category warrant some mention. Humphreys[12] has considered the case of a short revolving pipe through which air was introduced via a radial rotating pipe and a bend. Significant increases in mean heat transfer coefficient were detected and these were attributed mainly to entry swirl. Centrifugal buoyancy effects were detectable but it was found impossible to isolate these from the entry swirl effect. Later a transient system was employed with a view to overcoming this difficulty. Three fluids, water, glycol and engine oil were used and the transient convection heat transfer was compared with transient heat conduction. On this basis large percentage increases in Nu were found. Details of this transient study are to be published in due course [16]. A similar study on heat transfer in rotor cooling ducts has been carried out by Le Feuvre[15], the main difference here being the inlet condition. As in Humphreys steady flow case, air was used as the working fluid. The speed of rotation was more

realistic/

realistic ranging up to 1500 rpm. Again large increases in heat transfer were measured and the results show some agreement with those of reference 12.

As might be expected, the two latter experimental investigations indicate an enhancement of heat transfer greater than that predicted by Nakayama[13] on account of the swirling nature of the flow at entry to the test sections. These three separate works provide useful information for the assessment of heat transfer in the design stage. A brief summary of the range of variables etc. is given in Table 2, where the differences of geometry are more readily apparent.

Discussion and Conclusions

A brief survey of some investigations which were concerned with flow and heat transfer in rotating ducts has been made. It has been found necessary to consider the different attitudes of rotation of the ducts separately, as each has its own particular problems with regard to flow geometry and flow phenomena. However, some general remarks and conclusions can be made as follows:-

1. When a duct is rotated or revolved, heating or cooling of the fluid which is contained in or flowing through it, can give rise to significant buoyant force effects.

2. The centrifugal buoyant force is predominant in most of the cases which are encountered in practice.

3. The direction of heat transfer can be important as this can influence flow stability.

4. The nature of the fluid with regard to its compressibility can be important.

5. Film heat transfer coefficients can be increased many times (when compared with those in the stationary duct case).

6. The enhancement due to the secondary flow effects which result from buoyant motion (and possibly entry swirl) is reduced with increase in the axial or through flow velocity.

7. General correlation is impossible even within the separate categories of attitude of rotation because there has been insufficient control of the geometry and the boundary conditions of the problem.

8. Methods of prediction are available for the special cases which have been studied.

It is interesting to compare the effects of rotation or revolution of a duct with the effects of swirl in a stationary duct. As might be expected, similar results are obtained because the characteristics of the flow in both situations are generally similar if not in detail. In this connection, the study of Yeh[25] and contributions to that work by Kreith[26] and Wislicenus[27] are worthy of note.

In a decaying swirl flow in an annular passage, Yeh[25] made measurements of the velocities and turbulence intensities and studied the problem analytically. Sketches of the flow characteristics for this type of flow

are/

are shown in Fig. 6: the changes in flow pattern along the length are indicated. Yeh found that the wall stress on the convex wall decreased, but on the concave wall the stress remained constant or increased slightly. The outer portion of the outer boundary layer crept rapidly into the mainstream, Fig. 6(b), and this was attributed to the intense turbulence near the outer wall, Figs. 6 (c) and (d). Earlier, Kreith[26] had made measurements on convection heat transfer to water and butyl alcohol. He found that the heat transfer coefficient is substantially larger when the fluid is heated from a concave surface than when it is heated from a convex surface. Because heat transfer increases with increased turbulence and surface shear, the results of Kreith[26] are compatible with those of Yeh[25]. Kreith[26] also observed that the heat transfer on the convex surface was smaller than that on the flat surface. For geometries encountered in practice, the ratio of the Nusselt number (concave surface) to the Nusselt number (convex surface) is between 1·25 and 1·6 at the same Reynolds number and for Prandtl numbers greater than unity. In particular, for air the ratio was 1·62 at $Re = 5 \times 10^4$. In a later study[28], (see Ref. 26), swirling motion in water and air flow was produced by spiral wires and strips and the heat transfer in the non-decaying flow was as much as four times that in the straight pipe flow. An extremely interesting comment made by Kreith was that the increase was larger when the fluid was heated than when it was cooled. He explained this as being due to instability which has been explained earlier and also concluded that centrifugal forces affect the turbulent mixing motion (and consequently heat transfer) appreciably. Wislicenus[27] drew attention to the Rayleigh-Prandtl criterion regarding the stability of a fluid moving in a curved path. As explained previously (under "Previous Work"), if the angular momentum increases radially outward the flow is stable. This is because an element of fluid maintains its momentum and would have a larger peripheral velocity than its environment when displaced radially inward, and a lower peripheral velocity when displaced radially outward. Forces are then generated tending to return the fluid element to its original position. On this basis, Wislecenus[27] suggested that the flow near the inner wall of Yeh's annulus was stable whereas near the outer wall it was unstable. Yeh[25] recognised that the simple stability criterion was pertinent to his problem of swirling flow in the stationary duct and pointed out that it essentially agreed with his consideration of the turbulent energy equations.

Further evidence of the similarity of the effects of rotation of the duct and rotation of the fluid is to be found in the paper by Gambill and Bundy[29], (see Ref. 30). These authors compiled and examined the data of experiments conducted with twisted tapes on the cooling and heating of water, ethylene glycol, and air. For the non-decaying swirl flow they reached the following conclusions:-

1. The <u>swirl</u> flow Nusselt number was greater than that for <u>axial</u> flow. This was attributed to

    (a) larger local velocities

    (b) increased turbulence level

    and (c) centrifugal buoyancy forces.

2. The Nusselt number was greater with water than with air at the same flow conditions. The compressibility of the air allows the density at larger radii to be increased by the pressure gradient. In a heated flow, the density increases radially inward, but the magnitude of this will be reduced by the radially outward positive pressure gradient due to the centrifugal force field.

3./

3. The Nusselt number ratios were smaller with cooling than with heating, because the already cooled dense fluid tends to remain near the wall. The ratio in cooling, however, was still greater than unity.

4. For given conditions, large heat fluxes produced great heat transfer coefficients.

It is clear from a comparison of the conclusions made by Gambill and Bundy[29] and those listed earlier in this Section, that there is close similarity between the effects produced by swirl in a stationary pipe and the effects produced by rotation of the pipe itself.

As for the rotating pipe case, however, numerous variables are involved and it is not possible to obtain a "general" correlation and use will still have to be made of correlations pertaining to particular cases.

List/

List of Symbols

$A\left(=\dfrac{H\Omega^2}{g},\ \dfrac{d\,\Omega^2}{g}\right)$	acceleration ratio
d(de)	diameter (equivalent dia.) of duct
f, f_1	functions
g	acceleration due to gravity
H	radius arm
h	convective heat transfer coefficient
k	thermal conductivity
L	length of duct
Q	heat transfer
T, ΔT	temperature, temperature difference
u, v, w	absolute velocity components
u', v', w'	relative velocity components
x, y, z	cartesian co-ordinates
$\bar{u}$	mean velocity
β	coefficient of volumetric expansion
Ω	rotational speed
Γ	correlation parameter (see text)
$Re\left(=\dfrac{\bar{u}d}{\nu},\ \dfrac{\bar{u}de}{\nu}\right)$	Reynolds number
$Ro\left(=\dfrac{\bar{u}}{d\Omega},\ \dfrac{\bar{u}}{H\Omega}\right)$	Rossby number
$Re_r\left(=\dfrac{\Omega d^2}{\nu},\dfrac{\Omega de^2}{\nu}\right)$	rotational Reynolds number
Pr	Prandtl number
$Nu\left(=\dfrac{hd}{k},\ \dfrac{hde}{k},\dfrac{hL}{k}\right)$	Nusselt number

Nu_{co}/

Nu_{co} Nusselt number in conduction (see Appendix 3)

$Ta \left(= \frac{d_i\Omega_i}{4}\left(\frac{d_o-d_i}{\nu}\right)\sqrt{\frac{d_o-d_i}{d_i}} \right.$ Taylor number

$Gr \left(= \frac{\rho^2 \Delta T d^3 \beta \Omega^2 d}{\mu^2} \right.$ rotational Grashof number

$\left. \frac{\rho^2 \Delta T d^3 \Omega^2 H \beta}{\mu^2} \right)$

$Gr \left(= \frac{\rho^2 \Delta T d^3 \beta g}{\mu^2} \right.$ gravitational Grashof number

$\left. \frac{\rho^2 \Delta T L^3 \beta g}{\mu^2} \right)$

(Bar refers to the average value)

<u>Suffixes</u>

i	inner
o	outer
c	coolant
w	wall
E	entry length
s	stationary case

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Appendix 1

Basic Equations and Theoretical Considerations

In the study of fluid flow and heat transfer in rotating ducts, it is convenient to employ the basic equations of mass conservation, momentum and energy in terms of a rotating co-ordinate system. This has already been done in Refs. 11, 12 and 13 for example, where the polar co-ordinate system was used. The modification to the general expressions for the velocity and acceleration in cartesian co-ordinates are given in Fig. 4 [17]. In Fig. 5, which refers to the case of steady rotation about the z-axis, it will be seen more clearly that in addition to the convective terms there are coriolis and centripetal components of acceleration. It should be noted that the velocity components u', v', w' are those relative to the rotating co-ordinate system. Particular attention should be paid to the signs of the coriolis components: the directions of the components must be such as to rotate the corresponding velocity components in the same sense as Ω_z. Accordingly, the components $(2v'\Omega_z)$ and $(2u'\Omega_z)$ take (-ve) and (+ve) respectively. The two centripetal components on the other hand are both (-ve) being directed towards the corresponding axes as shown. The forces in the equations of motion take the same form as those in the stationary frame analysis being written in terms of x, y, z and u', v', w' The three momentum equations in laminar flow can then be formed.

The energy equation is identical to that for the stationary frame, apart from the use of u', v', w' in place of the absolute components of velocity as is the case in the mass continuity equation.

The fundamental equations may be used either in a theoretical analysis of fluid flow and heat transfer or to derive those dimensionless parameters which are relevant to the problem. The former procedure is complex in the extreme [11,13], even with a simplified form of the equations so in the present report the results of the second simpler procedure only will be referred to.

It is a relatively easy matter to use the basic equations (which pertain to laminar flow) to show that

$$Nu = f_1 (Re, Pr, Gr_r, Ro) \quad \cdots (A.1)$$

for similar geometries, and hydrodynamic and thermal boundary conditions. In addition to the mean flow Reynolds number and Prandtl number, there appears a rotational Grashof number and the Rossby number which respectively measures the significance of the centrifugal and coriolis buoyancy forces. The rotational Grashof number may be written more conveniently as the product of the gravitational Grashof number and an acceleration ratio, following Humphreys [12]. In this way, the effects of rotation may be more easily measured.

Equation (A.1) is complete and completely general and can be adopted in various forms for the rotating systems which are being considered here. At first sight the absence of a rotational Reynolds number in equation (A.1) might cause some confusion: this parameter however can be obtained by the division of the mean flow Reynolds number by the Rossby number and is not therefore an additional variable. An alternative form of equation (A.1) is then

Nu/

$$Nu = f_2\,(Re, Pr, Re_r, Gr_r) \qquad \cdots \text{(A.2)}$$

or,

$$Nu = f_3\,(Re, Pr, Re_r, Gr, A) \qquad \cdots \text{(A.3)}$$

which can readily be derived using other methods of dimensional analysis.

In the case of a duct rotating about its centre-line for example, the pertinent parameters will be

(i) $Nu = hd/k$

(ii) $Re = \bar{u}d/\nu$

(iii) $Re_r = \Omega_z^2 d/\nu$

(iv) $Gr_r = \dfrac{\Delta T d^3 \beta(\Omega_z^2 d)}{\nu^2}$

(v) $Gr = \dfrac{\Delta T d^3 \beta g}{\nu^2}$

(vi) $Pr = c\mu/k$

(vii) $A = \Omega_z^2\, d/g$

together with other groups needed to specify geometry[14,15]

Appendix 2

Relationship between the Rotating Reynolds number Re_r and the Taylor number Ta

In Appendix 1, mention has been made of a rotational Reynolds number $\Omega_z^2 d/\nu$ and its derivation from the mean flow Reynolds number $\bar{u}d/\nu$ and the Rossby number $\bar{u}/d\Omega_z$.

In rotating annular flows, a further parameter called the Taylor number is of interest. The Taylor number, Ta, is defined as

$$Ta = \frac{d_1 \Omega_z}{4} , \frac{d_0 - d_1}{\nu} . \sqrt{\frac{d_0 - d_1}{d_1}} \qquad \ldots \text{(A.4)}$$

and is employed in connection with stability criteria for the flow and for correlations in such cases. For example, with very narrow annuli, the outer boundaries of which are stationary, the critical value of Ta for transition from a laminar to a vortex type flow is about 40-50.

The relationship between Ta and Re_r is simply,

$$4\,Ta = Re_r . \left(\frac{d_0 - d_1}{d_1} \right)^{3/2} \qquad \ldots \text{(A.5)}$$

where $(d_0 - d_1)$ is the annulus gap and d_1 is the inner diameter of the annulus.

Appendix 3

Nusselt Number for Pure Conduction in an Internally Heated Annulus

The radial heat flow through an infinitely long thick cylinder of internal and external diameters d_1 and d_o respectively, is given by

$$Q = \left[\frac{2\pi k.(T_1 - T_o)}{\ln d_o/d_1} \right] \qquad \dots \text{(A.6)}$$

Q is the heat transfer by conduction per unit time per unit length of cylinder the thermal conductivity of which is k. T_1 and T_o are the inner and outer boundary temperatures. Defining a heat transfer coefficient as,

$$h = Q/\pi(T_1 - T_o)\, d_1 \qquad \dots \text{(A.7)}$$

When the Nusselt number based on the cylinder thickness is given by:

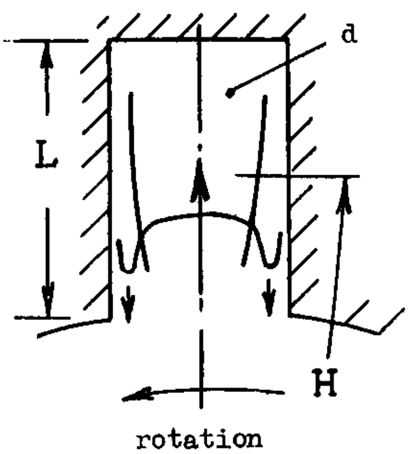
$$Nu_{co} = \left[\frac{d_o - d_1}{d_1 \ln(d_o/d_1)} \right] \qquad \dots \text{(A.8)}$$

It will be noted that for a given internal diameter (Nu_{co}) is a function of the cylinder thickness only. The introduction of (Nu_{co}) then serves as a useful common basis with which to compare heat transfer coefficients in convection in concentric cylinder geometries with different clearance ratios.

Appendix 4

Free Convection caused by Centrifugal Forces

Example. Water cooled gas turbine blade.



data:

$d = 0{\cdot}00635$ m

$L = 0{\cdot}0635$ m

$\Omega = 15{,}000$ rpm

$H = 0{\cdot}136$ m

$Pr = 1{\cdot}45$

$\Delta T \simeq 80°C$ [wall to fluid]

Calculation

$$H\Omega^2 = 224{,}000 \text{ m/sec}^2$$

with $\beta = 0{\cdot}18 \times 10^{-3}$

$$Gr_r = \frac{(H\Omega^2)\,\rho\Delta T L^3}{\nu^2} = 7{\cdot}6 \times 10^{13}$$

From McAdams,

$$\overline{Nu} = 0{\cdot}13(Gr_r)^{1/3}(Pr)^{1/3}$$

$$[10^{12} > Gr\,Pr > 10^9]$$

$$\overline{Nu} = 8600$$

or, $$\bar{h} = 92{,}000 \frac{J}{m^2 \text{ sec degC}}$$

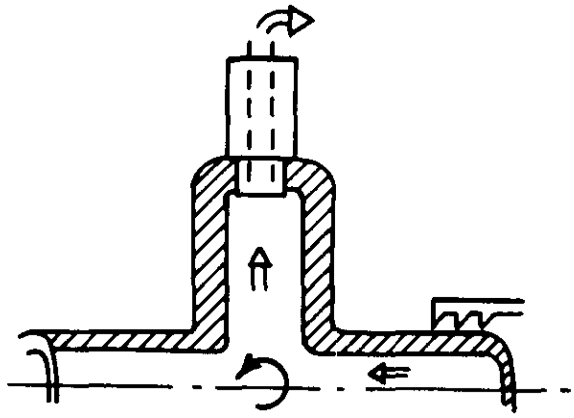
Notes:

(1) Because external heat transfer coefficient is at the order 1/10 of this value, temperature of blade can be maintained within acceptable limit.

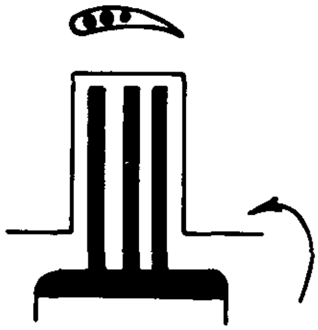
(2) Coriolis forces do not affect heat transfer appreciably.

(e) Flow can choke.

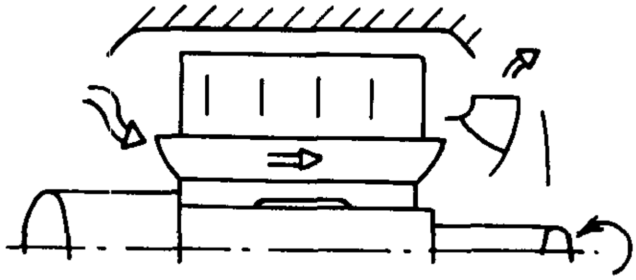
BMG



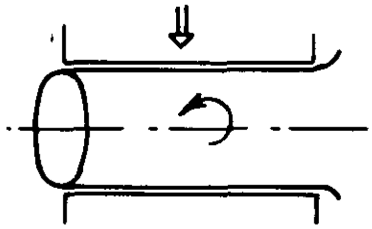
(1) Turbo jet engine with air cooled turbine blades



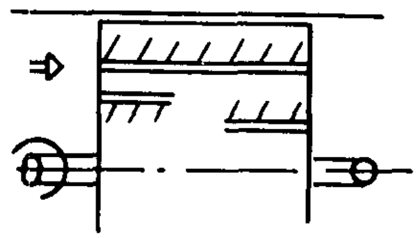
(2) Water cooled turbine blade.



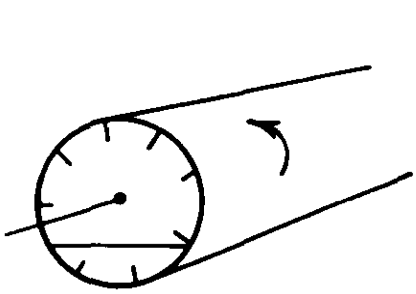
(3) Combined radial and axial ventilation of small turbo-alternator.



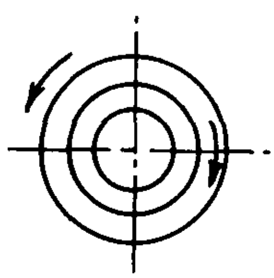
(4) Bearing.



(5) Regenerative heat exchanger matrix

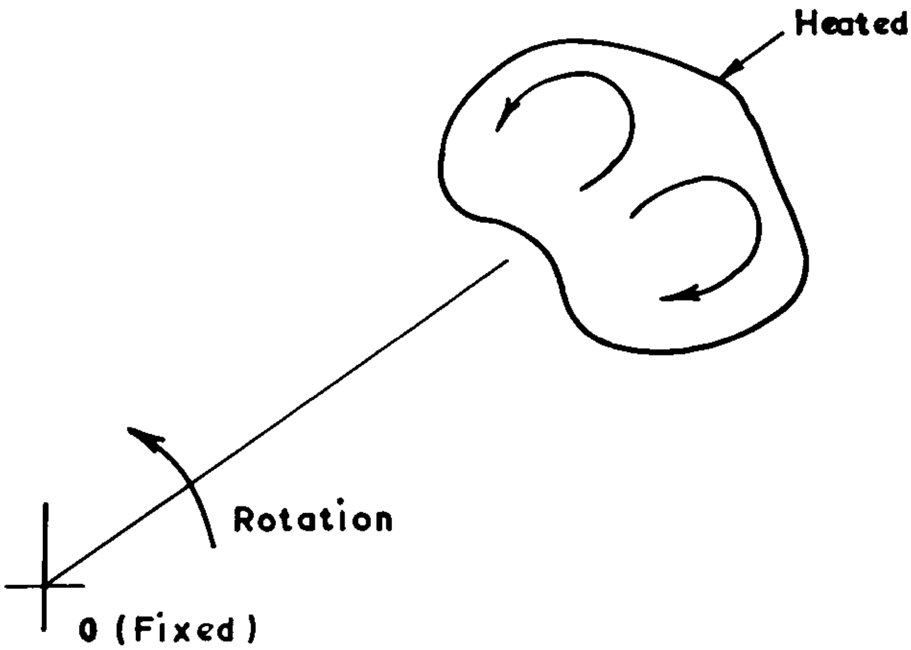


(6) Calender roll.

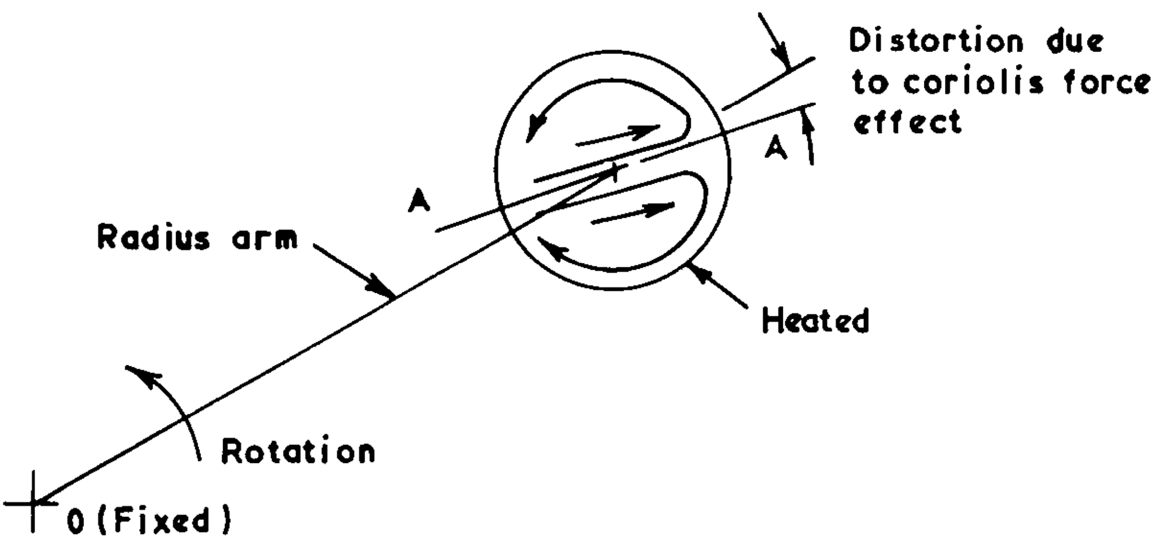


(7) Multiple concentric drive shafting

FIG.1 Practical cases of rotating ducts.



(a) The secondary flow effect due to centrifugal and coriolis forces in a heated revolving duct.



(b) The secondary flow pattern in a heated revolving pipe. When coriolis force effects are negligibly small, the axis of symmetry is coincident with the radius arm.

FIG. 2 Secondary flow patterns in heated revolving ducts

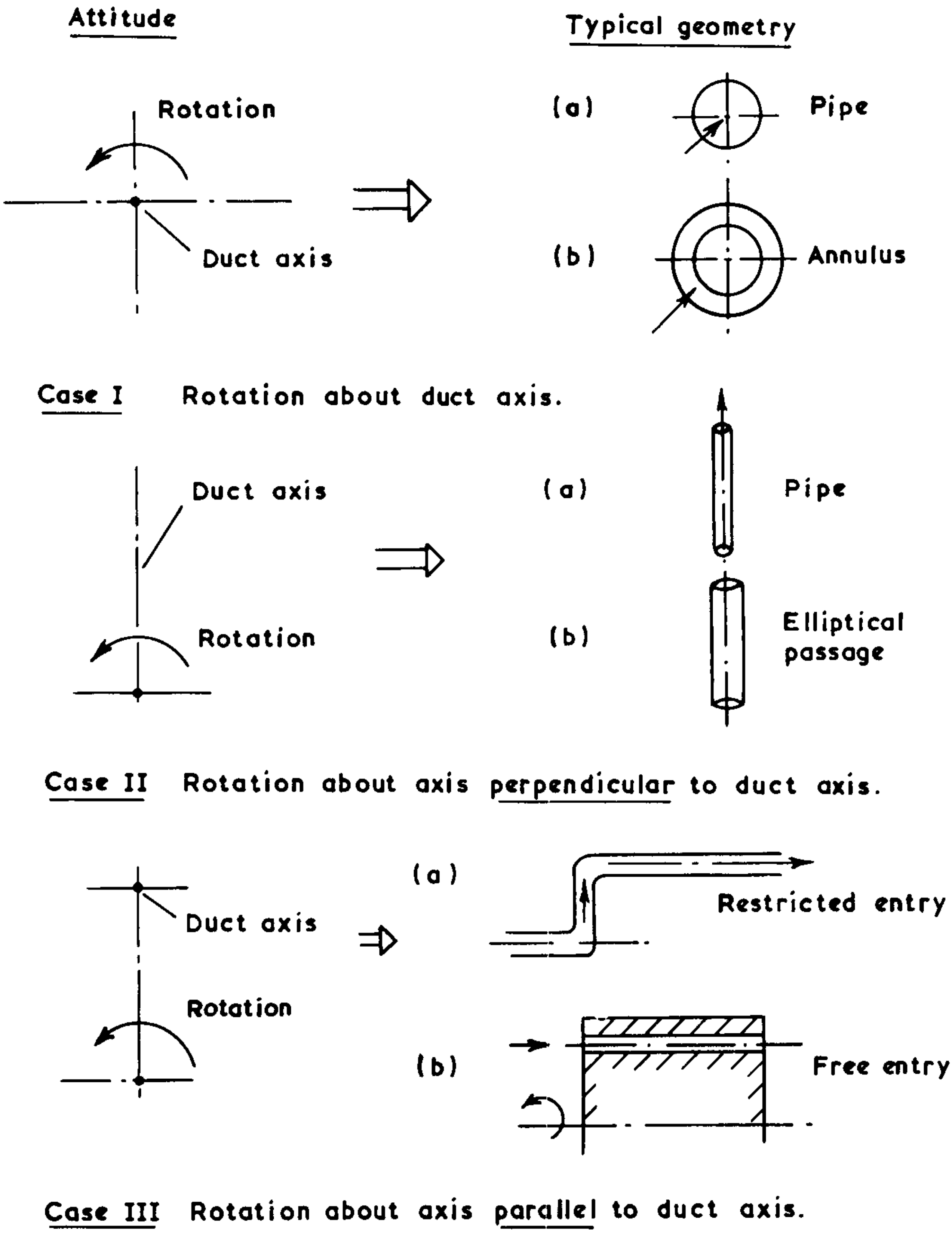
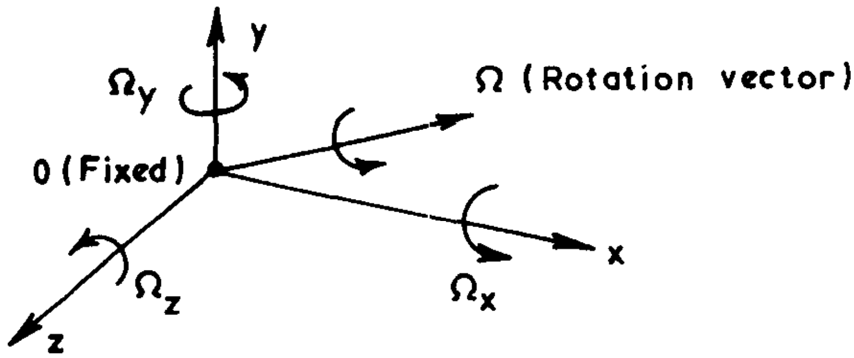


FIG. 3 Attitudes of duct rotation or revolution and duct geometries



Velocity terms

$$u = u' - y\Omega_z + z\Omega_y$$
$$v = v' - z\Omega_x + x\Omega_z$$
$$w = w' - x\Omega_y + y\Omega_x$$

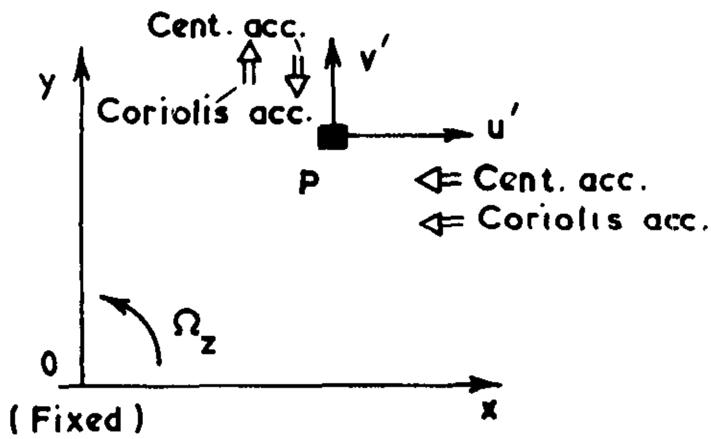
Acceleration terms

$$\frac{\partial u}{\partial t} + u'\frac{\partial u}{\partial x} + v'\frac{\partial u}{\partial y} + w'\frac{\partial u}{\partial z} - v\Omega_z + w\Omega_y$$

$$\frac{\partial v}{\partial t} + u'\frac{\partial v}{\partial x} + v'\frac{\partial v}{\partial y} + w'\frac{\partial v}{\partial z} - w\Omega_x + u\Omega_z$$

$$\frac{\partial w}{\partial t} + u'\frac{\partial w}{\partial x} + v'\frac{\partial w}{\partial y} + w'\frac{\partial w}{\partial z} - u\Omega_y + v\Omega_x$$

FIG.4 Rotating co-ordinate system.



Velocity terms

$$u = u' - y\Omega_z$$
$$v = v' + x\Omega_z$$
$$w = w'$$

Acceleration terms (Steady flow)

$$u'\frac{\partial u'}{\partial x} + v'\frac{\partial u'}{\partial y} + w'\frac{\partial u'}{\partial z} - 2v'\Omega_z - x\Omega_z^2$$

$$u'\frac{\partial v'}{\partial x} + v'\frac{\partial v'}{\partial y} + w'\frac{\partial v'}{\partial z} + 2u'\Omega_z - y\Omega_z^2$$

$$u'\frac{\partial w'}{\partial x} + v'\frac{\partial w'}{\partial y} + w'\frac{\partial w'}{\partial z}$$

Centripetal terms

Coriolis terms

FIG.5 Rotating co-ordinate system (rotation about z axis)

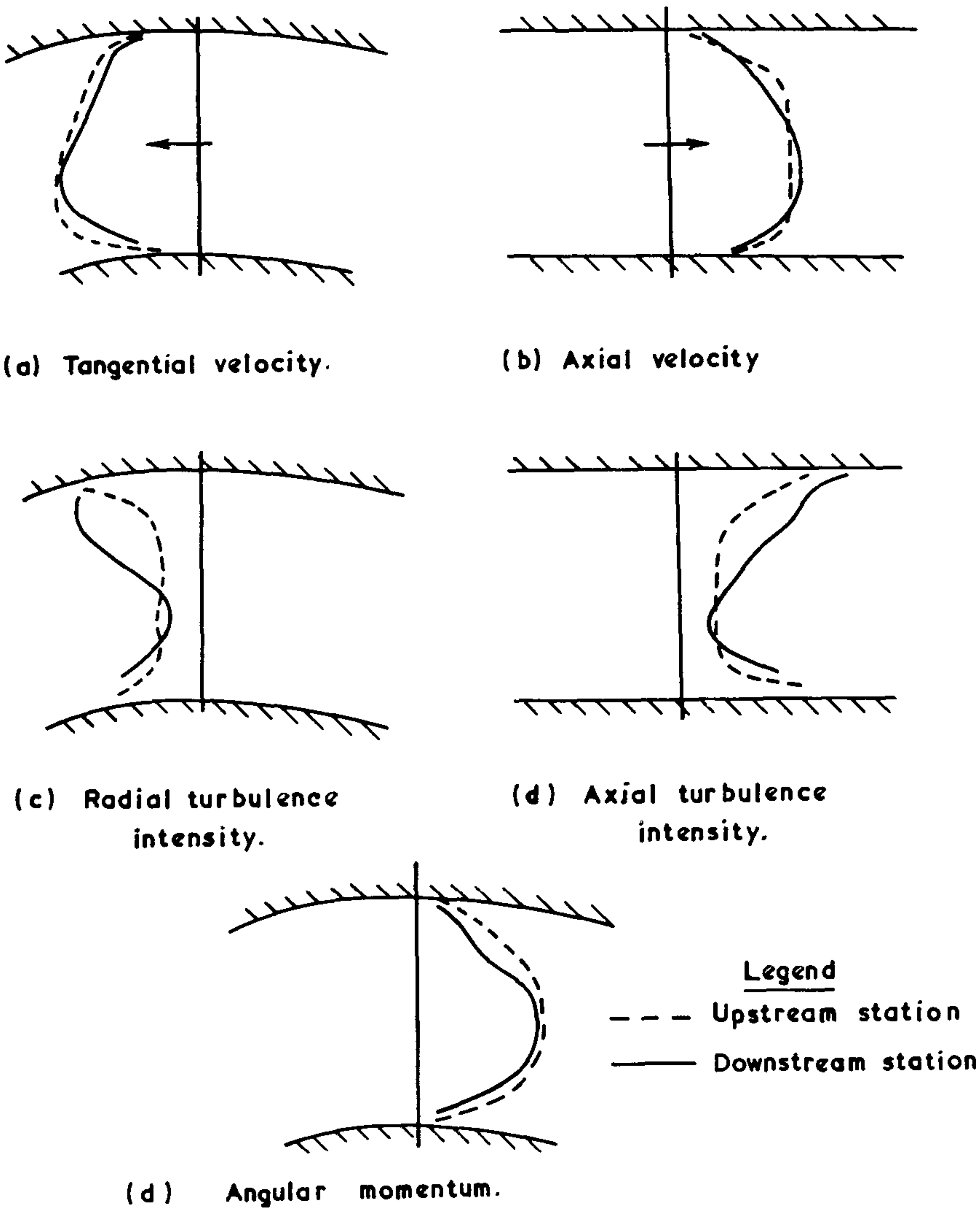


FIG.6 Distributions of velocity, turbulence intensity and angular momentum in decaying swirl flow in an annulus (after Yeh (23))

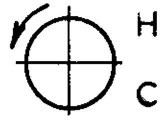
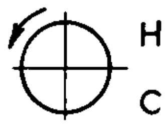
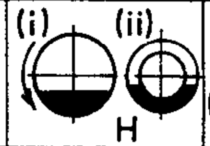
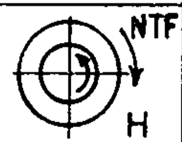
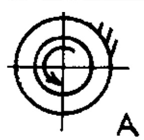
Reference	Geometry	$\frac{L}{d}\left(\frac{L}{d}\right)_e$	Ω rpm	Re	Re_r	Fluid	Comments
White, (1) (2) (3)		30 (10)	→ 5,000	→ 15,000	—	Air, Water	Heating causes large inc. in heat transfer (3-4 times). Cooling reduces heat transfer. Pressure drop in ad. flow reduced.
Cannon & Kays, (4)		57 (45)	→ 4,000	→ 60,000	→ 15,000	Air	No distinction between heating & cooling. Rotation delays transition (laminarization). Entry effects not important.
Kuo et al, (5)		(i) 8 (0) (ii) 64 (0)	→ 300	→ 950	→ 200,000	Water	Very large increases in heat transfer (8 times).
Pattenden, (6)		50 (out) 69 (inner)	→ 3,000	—	—	See comments	Hot water in inner annulus, Cold water in outer annulus. Inner film coefficient increased (10 times).
Coney, (7)		36 (inner) 13 1/2 (out)	→ 6,000	20,000 lb/h	—	See comments	Water in inner annulus, Steam in outer annulus. Overall ht.trans.coeff increased (1·7 times)
Bjorklund & Kays, (8)		Very large	Various combinations (co and contra)		—	Air	Heat transfer compared with heat transfer in conduction. For outer cylinder stationary $Nu/Nu_c = 0\,175\,Ta^{1/2}$.
Yamada, (9)		45 → 348 (0)	90 → 5,000	25,000	—	Water	No heat transfer. Increase in resistance after critical Re_r. Critical Ta varies with axial flow rate.

TABLE I Summary of previous studies (Case I)

Legend
- H Heating
- C Cooling
- /// Stationary
- A Adiabatic
- NTF No through flow

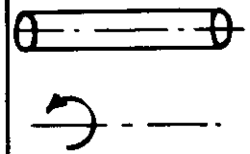
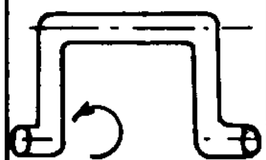
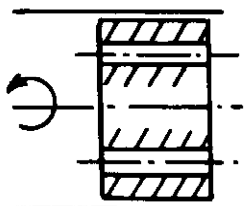
Reference	Geometry	L/d	Ω rpm	Re	Fluid	Comments
Nakayama, (13)		∞	Any	Any	Any	Theoretical (Turbulent flow) Constant axial temp. gradient Prediction
(i) Humphreys, (12) (ii) Humphreys, Morris & Barrow, (14)		12 & 48	$\rightarrow$ 500	$5.10^3 \rightarrow 2.10^4$	Air Water Glycol Oil	Experimental $\Rightarrow$ Steady flow. A $\rightarrow$ 85 No correlation in steady flow case. $\Rightarrow$ No flow, Transient heating
Le Feuvre, (15)		10·67	$\rightarrow$ 1,500	$5.10^3 \rightarrow 4.10^4$	Air	Experimental A $\rightarrow$ 191 Correlation. (Various duct geometries)

TABLE II Summary of previous studies (Case III)

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