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THE INFLUENCE OF ELEVATOR MOVEMENT ON THE NORMAL ACCELERATIONS EXPERIENCED BY A TRANSPORT AIRCRAFT IN MODERATE TURBULENCE

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SUMMARY

Some excerpts from a record of data obtained in an encounter with moderate turbulence during routine operations by a transport aircraft are analysed in order to assess the contribution to the cg normal accelerations from the elevator movement produced by the automatic control system. It is found that this contribution is quite small and does not have a consistent effect on the amplitudes of the accelerations.

The influence of an autostabiliser on the response in turbulent conditions is discussed for an idealised situation. From the analytical and numerical results obtained it is seen that the effectiveness of an autostabiliser will usually be strongly frequency-dependent. However, it is suggested that for the case considered here an autostabiliser with a pitch-rate law could give a useful reduction in the cg normal accelerations experienced in turbulence: the most suitable value of the gearing appears to be that which produces a value of about 0·7 for the relative damping of the aircraft's short-period longitudinal mode.

* Replaces R.A.E. Technical Report 69008 - A.R.C. 31270

CONTENTS

1 INTRODUCTION

When an aircraft encounters moderate to severe atmospheric turbulence there are large and rapid changes in its normal acceleration. Associated with these changes is a considerable amount of activity in elevator movement as the autopilot, or the human pilot, reacts to the response of the aircraft. Such elevator movements contribute to the accelerations experienced by the aircraft and the question of the magnitude and nature of this contribution therefore arises. The problem can be solved if the complete histories of elevator angle and normal acceleration are known and a reliable mathematical model of the aircraft is established. This state of affairs exists all too rarely since most detailed records of input and output parameters result from experiments of short duration in which encounters with turbulence are unlikely to occur.

Since October 1962 the Civil Aircraft Airworthiness Data Recording Programme (CAADRP)[1] has furnished data, recorded during routine airline operations, on a variety of parameters relevant to airworthiness, including elevator angle and normal acceleration. The histories of these parameters have been recorded as continuous analogue traces on photographic paper. Unfortunately, the need to record data from a large number of flying hours and the unpracticality of frequent changes of the cassettes of photographic paper have dictated a choice of paper speed for cruising flight that is too low to allow the full analogue records of rapidly changing parameters to be recovered and only peak values can be determined. However, during both the take-off and climb-out and the descent and landing phases of flight a higher paper speed is used and this permits the complete histories to be read with fair accuracy. Due to a fault in a switch, the whole of one cassette in a large turbojet transport aircraft was run at the higher speed of about 40 mm/min as against 10 mm/min. During the comparatively short time (about 25 hr) for which the recorder operated with this fault, the aircraft happened to encounter in cruising flight a patch of turbulence of moderate intensity and long duration, such as is met by this aircraft, on average, only once in about 500 to 1000 hours. It was decided to take advantage of this fortunate coincidence by using the data acquired to examine the influence which elevator movements, produced by the automatic control system, had on the normal accelerations experienced.

The general subject of the possibilities which exist for modifying the response of an aircraft to turbulence by employing autostabilisation was also considered, albeit within a restricted framework.

Three excerpts from the CAADRP record are selected for analysis. The normal accelerations due to the recorded elevator movements are computed by the method described in section 3 and their contribution to the total normal accelerations is determined; these results are presented in section 4. The discussion then turns to an assessment of the influence of an idealised autostabiliser on an aircraft's response to harmonic vertical gusts.

This topic is first considered in general terms (section 5.1); next the analysis employed for the case chosen here of a pitch-rate autostabiliser law is developed (section 5.2) and the results obtained are presented and discussed (section 5.3).

2 THE COMPONENTS OF THE RESPONSE DURING FLIGHT THROUGH TURBULENCE

If the aircraft behaves as a linear system then the response to any combination of disturbances is the sum of the responses due to each disturbance alone. From a symmetrical initial flight condition where turbulence* has negligible influence the subsequent response, measured relative to sustained level flight, may be expressed as

(total response) = (response due to turbulence)

+ (response due to non-zero initial values of w and q)

+ (response due to difference between η and η_t),

where it is assumed that for sustained level flight the z component of the velocity (w) is zero (i.e. aerodynamic-body axes[2,3] are employed) and that the elevator angle (η) has the value η_t.

By calculating the sum of the second and third terms on the right-hand side of the above expression and subtracting this from the left-hand side the contribution of turbulence alone to the disturbance from level flight may be found.

The elevator angle history during flight through turbulence has not only a component associated with the disturbance due to the turbulence but also a component associated with any required changes in the overall flight path.

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* Within this Report the word 'turbulence' is used to mean any departure of the atmosphere from rest, on such a scale that the motion of an aircraft is affected.

Strictly, because these two components cannot be separated, it is not correct to speak of the third term on the right-hand side as 'the contribution of elevator movement to the response in turbulence' since part of this, together with the preceding term, should be regarded as the response in a manoeuvre. In moderate to severe turbulence the changes in normal acceleration and elevator angle associated with the turbulence are considerably larger and more rapid than those associated with any superimposed manoeuvre; the latter may therefore be disregarded for the present purpose of assessing, in largely qualitative terms, the influence of elevator movements on the normal accelerations experienced.

The extent to which the elevator movements exert a favourable or unfavourable influence on the various structural loads experienced or the comfort of the crew and passengers is not considered: it could be determined only from a much more detailed analysis than is carried out here, backed up by the recording of a number of additional parameters.

3 METHOD FOR CALCULATING THE RESPONSE TO ELEVATOR MOVEMENT

The response of the aircraft to elevator movement is assumed to be governed by the equations of short-period longitudinal motion for a rigid aircraft (which may also be employed when, as here, aeroelastic effects are considered only on a quasi-static basis), viz

$$\hat{z}_e + \{(1 + \hat{z}_{\dot{w}})\hat{D} + \hat{z}_w\}\hat{w} + \{\hat{z}_{\dot{q}}\hat{D} + \hat{z}_q - 1\}\hat{q} + \hat{z}_\eta \eta = 0 \qquad (1)$$

and

$$\hat{m}_e + \{\hat{m}_{\dot{w}}\hat{D} + \hat{m}_w\}\hat{w} + \{(1 + \hat{m}_{\dot{q}})\hat{D} + \hat{m}_q\}\hat{q} + \hat{m}_\eta \eta = 0 \qquad (2)$$

The notation is that of Refs.2 and 3; note in particular that the various coefficients in the aerodynamic force and moment expressions are dynamic-normalised concise quantities and not the 'stability coefficients' of Bryant and Gates.

The incidence (here to be identified with $\hat{w}$) and elevator deflection may be measured from any convenient origins. In the present work the origin for η was determined by the CAADRP instrumentation. For each case the value, η_t, of η for sustained level flight could be found from a neighbouring portion of the flight record. The origin for $\hat{w}$ was taken to correspond to this condition.

The aero-normalised derivatives[2,3] were supplied by the manufacturers of the aircraft and are appropriate in each case to the recorded values of air-speed, altitude, mass and cg position. (The last was deduced from the recorded tailplane angle.) All the derivatives incorporate allowances, on a quasi-static basis, for the effects of aeroelasticity: the derivatives $Z_{\dot{q}}$ and $M_{\dot{q}}$ arise entirely from aeroelasticity.

In accordance with the above choices of the origins for $\hat{w}$ and η, $\hat{z}_e$ and $\hat{m}_e$ were derived from

$$\hat{z}_e = - \hat{z}_\eta \eta_t \qquad (3)$$

and

$$\hat{m}_e = - \hat{m}_\eta \eta_t \quad . \qquad (4)$$

At any instant when the contribution of turbulence to the total aircraft response was negligible the values of $\hat{w}$ and $\hat{q}$ could be found, under certain simplifying assumptions, from the recorded variations in η and n (cg normal acceleration) in conjunction with equations (1) to (4). The equations of motion were solved by a marching method, using the recorded elevator angle as the input $\eta(t)$.

In order to gain an idea of the validity of the mathematical model thus defined it was decided to calculate the responses to the measured elevator movements in symmetric manoeuvres in calm air and compare these with the measured responses. From the cruise portions of the CAADRP record two periods which seemed to cover such manoeuvres were identified: these were designated cases 1 and 2. The flight conditions for these are given in Table 1 below (section 4).

The results of computing the aircraft's response in the two manoeuvres are shown in the dashed-line graphs of Figs.1 and 2. It will be seen that in both cases the agreement with the recorded response (solid-line graph) is generally good. It is as well to point out that, in the present context, a calculated normal acceleration is that of the actual cg whereas a measured normal acceleration is that experienced by an accelerometer attached to a part of the aircraft's structure which is close to the cg for some datum mass distribution. However, since attention is being concentrated on fairly low-frequency (below about 1 Hz) variations, the two quantities may justifiably be compared directly.

As a partial check on the accuracy of the estimated cg positions, which would be affected by, for example, a change in datum for the tailplane angle, the responses in the two manoeuvres were also calculated for arbitrarily chosen cg positions of 0·3 smc and 0·4 smc. From the results obtained it was seen that, although in either case some improvement in the agreement of certain features could be obtained by suitably repositioning the cg, there was no evidence that the overall agreement would be improved by any systematic repositioning. The originally deduced cg positions for the manoeuvre and the turbulence cases were therefore taken to be correct.

All in all, the results give a high degree of confidence in the mathematical model.

4 RESULTS OF RESPONSE CALCULATIONS FOR ENCOUNTERS WITH TURBULENCE

The flying covered by the CAADRP record included one fairly long period of flight through a region containing moderate turbulence and three excerpts from this were chosen for investigation. These were designated cases 3 to 5. As is indicated in Table 1 below, the flight conditions for these cases were not the same as for case 1 or case 2; however, they do not differ sufficiently for there to be any reason to doubt that the mathematical model is as valid here as in the manoeuvre cases.

Table 1

Flight conditions for manoeuvres and turbulence encounters

	Case 1 (manoeuvre)	Case 2 (manoeuvre)	Cases 3-5 (turbulence encounters)
Airspeed (V) m/s	234·1	237·0	242·0
Altitude km	11·13	9·51	10·21
Mach number	0·793	0·786	0·809
Mass (m) kg	100400	97100	132400
cg position % standard mean chord (smc)	37·0	36·5	34·3

The recorded histories of elevator angle (η) and cg normal acceleration (n) are given by the solid-line graphs in Figs.3 to 5.

Cases 3 and 4 start at times when the effects of turbulence are negligible. It was therefore possible to find the initial values of $\hat{w}$ and $\hat{q}$ and to calculate the response due to these initial conditions and the subsequent elevator movements. The results of these calculations are shown by the dashed-line graphs in Figs.3 and 4. The dotted-line graphs were produced by differencing the corresponding values on the other two graphs and referring this difference to a datum of 1 g. They represent the response due to turbulence alone of an aircraft with a fixed elevator which is initially in level flight.

At the start of case 5 there is already considerable response due to turbulence and therefore the 'turbulence alone' component cannot be deduced by the above technique. It was decided in this case to subtract from the total response only the contribution due to elevator movement. This contribution and the result of subtracting it from the total response are shown by the dashed- and the dotted-line graphs, respectively, in Fig.5. The neglect of the influence of non-zero initial values of w and q will have a noticeable effect only during the first 1 sec of the response.

In all three cases the contributions from elevator movement to the largest increments (from the 1 g level) in cg normal acceleration are mostly small. Increments of lower magnitude are neither consistently increased nor consistently decreased by elevator movement. It is clear that the general character of the history of cg normal acceleration in this period of flight through turbulence has not been significantly influenced by the actions of the autostabiliser.

It may be asked whether or not there is any scope for obtaining a useful reduction in the response to turbulence by employing an autostabiliser. A partial answer to this question is provided by the following sections.

5 THE EFFECT OF AN AUTOSTABILISER ON THE RESPONSE TO VERTICAL GUSTS

In recent years great strides have been taken in the design and implementation of autostabilisation systems for controlling various features of the dynamic motions of flexible aircraft. An idea of the scope of these developments can be obtained from the relevant papers in Ref.4. The purpose of the following discussion of the influence of an autostabilisation system on the response to gusts is not to analyse the effect of any particular system over a wide range of flight conditions, let alone to determine the best types of system to employ. Rather, it is limited to setting out the principles which govern the performance of idealised autostabilisers in turbulence of a particular form and illustrating these by assessing the scope for reducing the normal-acceleration response by means of a pitch-rate autostabiliser. It is hoped that by this approach it will be possible to gain some insight into the basic problem of modifying the response to gusts by employing autostabilisation.

In the following sections all quantities are defined relative to their level-flight values.

5.1 General considerations

It is assumed that the aircraft is flying through a region in which the vertical velocity of the air, the 'gust velocity', varies sinusoidally with the distance along the flight path. That is, if s denotes the flight-path distance, the gust velocity w_W (positive downwards) is given by

$$w_W = \bar{w}_W \sin \Omega s \quad * , \tag{5}$$

where $\Omega = 2\pi/L$ and L is the wavelength of the gusts.
If the speed of flight is constant the variation of the gust velocity at a typical station along the aircraft is given by

$$w_W = \bar{w}_W \sin \omega t \quad , \tag{6}$$

where

$$\omega = V\Omega = 2\pi V/L . \tag{7}$$

The forced-oscillation response in a quantity δ, which is in general a linear combination of w, q and their derivatives and integrals, may be expressed as

$$\frac{\delta_G}{\bar{w}_W} = \bar{\delta}_G \sin (\omega t - \phi_{\delta_G}) , \tag{8}$$

or, considering the response to the complex input $\bar{w}_W \exp(i\,\omega t)$,

$$\frac{\delta_G}{\bar{w}_W \exp (i\,\omega t)} = \bar{\delta}_G \exp (- i\,\phi_{\delta_G}) . \tag{9}$$

---

* All amplitudes, i.e. quantities with a superscribed bar, are positive.

ϕ_{δ_G} is the phase angle, which is defined to lie in the interval $(-\pi, \pi]$, and is positive for the output lagging the input.

Similarly, let the forced-oscillation response in δ to a sinusoidal elevator motion of frequency ω, i.e. $\eta = \bar{\eta} \sin \omega t$, be expressed as

$$\frac{\delta_E}{\bar{\eta}} = \bar{\delta}_E \sin (\omega t - \phi_{\delta_E}) , \tag{10}$$

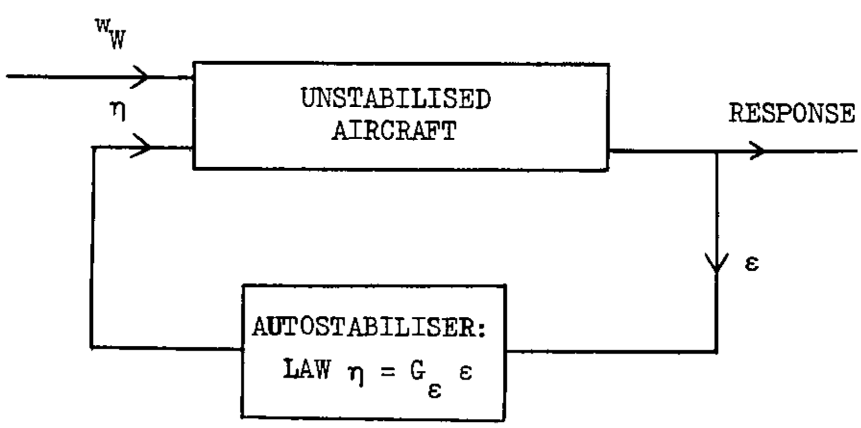
or, c.f. (9)

$$\frac{\delta_E}{\bar{\eta} \exp (i \omega t)} = \bar{\delta}_E \exp (- i \phi_{\delta_E}) . \tag{11}$$

It is now assumed that the elevator is moved according to the law

$$\eta = G_\varepsilon \varepsilon , \tag{12}$$

where ε is a linear combination of w, q and their derivatives and integrals. This represents the action of an autostabiliser, idealised by neglecting all lags in the system. The stabilised aircraft can be represented by the block diagram below.



This differs from most systems with feedback in that the feedback term (η) cannot simply be added to the input (w_W) since these two terms produce response of the aircraft in essentially different ways.

From the above block diagram it may be seen that the response of the stabilised aircraft to gusts is expressible as (response of stabilised aircraft to gusts) = (response of unstabilised aircraft to gusts)

+ (response of unstabilised aircraft to induced elevator movement).

Now the induced elevator movement is, by the autostabiliser law (12) above, proportional to a quantity ε in the response of the stabilised aircraft. Then, if quantities relating to the unstabilised aircraft are distinguished from the corresponding ones for the stabilised aircraft by the addition of the suffix o to their symbols, the response of the stabilised aircraft is given by

$$\frac{\delta_G}{\bar{w}_W \exp(i\,\omega t)} = \bar{\delta}_G \exp(-i\,\phi_{\delta_G})$$

$$= \bar{\delta}_{Go} \exp(-i\,\phi_{\delta_{Go}}) + G_\varepsilon\,\bar{\varepsilon}_G \exp(-i\,\phi_{\varepsilon_G})\,\bar{\delta}_{Eo} \exp(-i\,\phi_{\delta_{Eo}}) \qquad \ldots (13)$$

$$= \exp(-i\,\phi_{\delta_{Go}})\,[\bar{\delta}_{Go} + G_\varepsilon\,\bar{\varepsilon}_G\,\bar{\delta}_{Eo} \exp\{-i(\phi_{\delta_{Eo}} + \phi_{\varepsilon_G} - \phi_{\delta_{Go}})\}]\,. \qquad \ldots (14)$$

Write

$$\phi_{\delta_{Eo}} + \phi_{\varepsilon_G} - \phi_{\delta_{Go}} = e' \qquad (15)$$

and define

$$e'' = e' - \pi\,\mathrm{sign}\,(e') < e', \pi > , \qquad (16a)$$

$$e^* = e'' - \pi\,\mathrm{sign}\,(e'') < e'', \pi/2 > , \qquad (16b)$$

and

$$\sigma = \exp\{-i(e' - e^*)\} , \qquad (16c)$$

where $< a,b > \equiv$ integer part of $|a/b|$.

Then

$$\frac{\delta_G}{\bar{w}_W \exp(i\,\omega t)} = \exp(-i\,\phi_{\delta_{Go}})\,[\bar{\delta}_{Go} + \sigma\,G_\varepsilon\,\bar{\varepsilon}_G\,\bar{\delta}_{Eo} \exp(-i\,e^*)]\,. \qquad \ldots (17)$$

e^* is termed the 'phase error' and is in the interval $[-\pi/2, \pi/2]$ while σ takes the values +1 and -1.

Only when the phase error is zero can the autostabiliser produce the same proportional change in δ throughout a cycle, and in general it is likely that attempts to achieve a satisfactory response in δ by employing an autostabiliser will be successful only if the phase error is fairly small.

From (17)

$$\bar{\delta}_G^2 \;=\; \bar{\delta}_{Go}^2 + 2\sigma G_\varepsilon\, \bar{\varepsilon}_G\, \bar{\delta}_{Go}\, \bar{\delta}_{Eo} \cos e^* + (G_\varepsilon\, \bar{\varepsilon}_G\, \bar{\delta}_{Eo})^2 \,. \qquad (18)$$

Hence if G_ε is positive (and not too large) the amplitude of the response in δ is reduced if σ is negative, and vice versa.

Forasmuch as the expressions derived above contain on their right-hand sides symbols relating to the response of the stabilised aircraft, it could be argued that they have little practical value since they do not permit one to draw conclusions about the effect of an autostabiliser before performing a full analysis. However, as is shown below, it is possible through them to predict the effects of small changes in the gearing from values for which a full analysis is available. Also, they are useful in aiding one's interpretation of the results from such an analysis and they will be employed in this role when the results presented in section 5.3, which were obtained by the method described in the next section, are discussed.

Suppose that the gearing is small enough for $\bar{\varepsilon}_G$ and ϕ_{ε_G} to differ little from $\bar{\varepsilon}_{Go}$ and $\phi_{\varepsilon_{Go}}$ respectively. Then from (18)

$$\bar{\delta}_G^2 - \bar{\delta}_{Go}^2 \;\simeq\; 2\sigma G_\varepsilon\, \bar{\varepsilon}_{Go}\, \bar{\delta}_{Go}\, \bar{\delta}_{Eo} \cos e_o^* + (G_\varepsilon\, \bar{\varepsilon}_{Go}\, \bar{\delta}_{Go})^2 \,. \qquad (19)$$

If G_ε is sufficiently small the second term on the right-hand side may be ignored and the left-hand side approximated by $2\bar{\delta}_{Go}\,(\bar{\delta}_G - \bar{\delta}_{Go})$.

Then

$$\bar{\delta}_G - \bar{\delta}_{Go} \;\simeq\; \sigma G_\varepsilon\, \bar{\varepsilon}_{Go}\, \bar{\delta}_{Eo} \cos e_o^* \,, \qquad (20)$$

or

$$\frac{\bar{\delta}_G - \bar{\delta}_{Go}}{\bar{\delta}_{Go}\, G_\varepsilon} \;\simeq\; \frac{\sigma\, \bar{\varepsilon}_{Go}\, \bar{\delta}_{Eo} \cos e_o^*}{\bar{\delta}_{Go}} \;=\; P_o, \quad \text{say}. \qquad (21)$$

Now the aircraft with any particular amount of autostabilisation may be considered in place of the unstabilised aircraft as constituting the 'datum' system. More generally, then, if the gearing is changed by a small amount ΔG_ε,

$$\frac{\Delta\bar{\delta}_{G\ell}}{\bar{\delta}_{G\ell}\ \Delta G_\varepsilon} \simeq \frac{\sigma\ \bar{\varepsilon}_{G\ell}\ \bar{\delta}_{E\ell}\ \cos e^*_\ell}{\bar{\delta}_{G\ell}} = P, \quad \text{say}, \tag{22}$$

where the suffix ℓ denotes that the amplitudes and phase angles are 'local' quantities, i.e. appropriate to the aircraft with an autostabiliser gearing of G_ε. It may be noted that a previously unconsidered response, that of the stabilised aircraft to harmonic elevator movement, has been introduced.

P, which is termed the 'pay-off function', is a measure of the effectiveness of a change in gearing of the autostabiliser in modifying the response in δ. The magnitude of P for a single combination of frequency and gearing is not very significant but the variation of P with frequency and/or gearing is, as will be seen in section 5.3, a useful pointer in predicting the benefits of a change in gearing.

Since

$$\bar{\delta}^2_G = \bar{\delta}^2_{G\ell} + 2\sigma\ \Delta G_\varepsilon\ \bar{\varepsilon}_{G\ell}\ \bar{\delta}_{G\ell}\ \bar{\delta}_{E\ell}\ \cos e^*_\ell + 0\ (\Delta G^2_\varepsilon) ,$$

P = 0 corresponds to a turning point of $\bar{\delta}_G$. $\bar{\varepsilon}_{G\ell}$ will not usually be zero for practical frequencies and therefore the turning points of $\bar{\delta}_G$ occur when $|e^*_\ell| = \pi/2$.

5.2 Method for calculating the response to vertical gusts

Consider an aircraft flying at constant speed V through a region where the vertical velocity of the air w_W varies along the flight path. Now if a length L which is characteristic of the spatial variation of w_W is large in comparison with the tail arm of the aircraft it may be assumed that the vertical velocity of the air is constant along the length of the aircraft. If, furthermore, the characteristic temporal frequency of the variation of w_W, $2\pi\ V/L$, is small it may be assumed that quasi-steady aerodynamics, as used in the analysis of the aircraft's behaviour in still air, are appropriate to this case. Then the equations of motion become

$$\{(1 + \hat{z}_{\dot{w}})\ \hat{D} + \hat{z}_w\}\ \hat{w}_K + \{\hat{z}_{\dot{q}}\ \hat{D} + \hat{z}_q\}\ \hat{q} + \hat{z}_\eta\ \eta - \{\hat{z}_{\dot{w}}\ \hat{D} + \hat{z}_w\}\ \hat{w}_W = 0 \tag{23}$$

and

$$\{\hat{m}_{\dot{w}}\ \hat{D} + \hat{m}_w\}\ \hat{w}_K + \{(1 + \hat{m}_{\dot{q}})\ \hat{D} + \hat{m}_q\}\ \hat{q} + \hat{m}_\eta\ \eta - \{\hat{m}_{\dot{w}}\ \hat{D} + \hat{m}_w\}\ \hat{w}_W = 0 , \tag{24}$$

where the suffix K denotes that w_K is measured relative to the earth.

If the aircraft is fitted with a simple pitch-rate autostabiliser, for which the control law is

$$\eta = G_q\ q , \tag{25}$$

then the equations of motion for flight through vertical gusts finally become

$$\{(1 + \hat{z}_{\dot{w}})\,\hat{D} + \hat{z}_w\}\,\hat{w}_K + \{\hat{z}_{\dot{q}}\,\hat{D} + \hat{z}_q + \hat{G}_q\,\hat{z}_\eta\}\,\hat{q} + \hat{z}_\eta\,\eta - \{\hat{z}_{\dot{w}}\,\hat{D} + \hat{z}_w\}\,\hat{w}_W = 0 \quad (26)$$

and

$$\{\hat{m}_{\dot{w}}\,\hat{D} + \hat{m}_w\}\,\hat{w}_K + \{(1 + \hat{m}_{\dot{q}})\,\hat{D} + \hat{m}_q + \hat{G}_q\,\hat{m}_\eta\}\,\hat{q} + \hat{m}_\eta\,\eta - \{\hat{m}_{\dot{w}}\,\hat{D} + \hat{m}_w\}\,\hat{w}_W = 0, \quad (27)$$

where

$$\hat{G}_q = \frac{G_q}{\tau}\;.^{\dagger} \quad (28)$$

From the above equations the acceleration derivatives attached to $\hat{w}_W$ were omitted since it was convenient to employ equations of the same form to describe either the response to elevator movement or the response to vertical gusts. This further approximation, which can introduce only quite small errors at the frequencies considered, was thought to be justified in the present exploratory investigation.

The amplitude and phase angle of the forced-oscillation normal-acceleration response to a harmonic variation of either η alone or w_W alone were obtained from the particular-integral part of the analytical solution of equations (26) and (27).

5.3 Results of calculations of response to harmonic vertical gusts

The aerodynamic data used in these calculations were the same as for the turbulence encounters (cases 3, 4 and 5). The use of these data does not mean that the results obtained are necessarily valid for the actual aircraft since, although its autostabiliser law contains what is nominally a pitch-rate term, the assumed simple pitch-rate law, free from lags, is not followed. Also, the results obtained do not by themselves indicate whether or not a pitch-rate autostabiliser law would be desirable for this aircraft since any such law has to be chosen to give the stabilised aircraft good flying characteristics in a variety of situations.

The variation of the natural period, frequency and relative damping of the aircraft's short-period longitudinal mode with variations in the gearing of the autostabiliser are shown in Fig.6. For gearings up to about 1·5 the natural period is not greatly altered: for higher values the period increases rapidly and becomes infinite (corresponding to critical damping) for a gearing of about 2·2. Since large increases in the natural period are unacceptable from a general handling standpoint it may be deduced that the practical range of gearings extends to about 1·5 at most.

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† The terms $\hat{z}_\eta\,\eta$ and $\hat{m}_\eta\,\eta$ have been retained to allow for an elevator input in addition to that produced by the autostabiliser.

The amplitudes of the cg normal-acceleration response to harmonic vertical gusts of 1 m/sec amplitude, for gearings of 0, 0·25, 0·5, 1·0 and 2·0, are shown in Fig.7. For the unstabilised aircraft ($G_q = 0$) the graph of amplitude versus frequency has a pronounced peak at about 0·3 Hz, at which the amplitude exceeds the infinite-frequency value, to which all the graphs tend asymptotically, by 54%. (Since the analysis employed is invalid for high frequencies this infinite-frequency value has no physical meaning but merely serves as a convenient datum.) The overshoot is reduced markedly for quite low gearings - a gearing of 0·5 reduces it to 23% - and it is clear that little is gained by increasing the gearing above about 1·0.

At low frequencies $\bar{n}_G$ is increased by increasing G_q: these low-frequency results are not very meaningful, however, firstly because the phugoid mode has been neglected, and secondly because a pitch-attitude term would probably be incorporated in the autostabiliser law to deal with long-period disturbances. The autostabiliser is almost ineffectual at frequencies above about 0·75 Hz.

It is of interest that the greatest amplitude of gust response, for a given gust amplitude, occurs at a higher frequency than the natural frequency of the short-period mode and that the difference between these frequencies increases as the autostabiliser gearing is increased. The recorded histories of n in cases 3, 4 and 5 support the first of these results since the dominant period of oscillation appears to be in the range 2 to 3 sec. Also, they seem to indicate that the contribution of elevator movement to normal acceleration dropped as the frequency of the response increased (compare, for instance, Figs.3 and 5).

Corresponding results for the normal-acceleration response at a point 23 m ahead of the cg, roughly the position of the pilot, are presented in Fig.8. (Because of the closeness of the graphs only those for $G_q = 0$, 0·5 and 1·0 are shown.) The autostabiliser slightly increases the response at this position except in a narrow band of frequencies surrounding the frequency for peak response. Raising the gearing above about 0·5 is of little benefit.

From Fig.6 it is seen that for $G_q = 1·0$ the relative damping is just below 0·7, which is often regarded as being near to the optimum value from the general handling standpoint. It would seem, then, that a pitch-rate autostabiliser with a gearing of 1·0 could improve the longitudinal short-period handling of the aircraft and also produce a useful, though hardly dramatic, decrease in the cg normal-acceleration response to turbulence in the frequency band where this response is highest.

The above results for the cg normal-acceleration response may now be considered in the light of the analysis of section 5.1. For the moment attention is concentrated on the performance of an autostabiliser with a very low gearing. Here the phase error e_o^* and the pay-off function P_o, as defined in section 5.1, are given by

$$e_o' = \phi_{n_{Eo}} + \phi_{q_{Go}} - \phi_{n_{Go}} , \tag{29}$$

together with equations (16), and

$$P_o = \frac{\sigma \, \bar{q}_{Go} \, \bar{n}_{Eo} \cos e_o^*}{\bar{n}_{Go}} . \tag{30}$$

The three component pairs of amplitude and associated phase angle are shown in Figs.9 to 11. The resultant values of e_o^* and P_o are shown in Fig.12. The graph of P_o is very peaky, indicating that, as was seen from Fig.7, the autostabiliser is most effective only over a small range of frequencies. (The low-frequency results which indicate large positive values of P_o are omitted for convenience - as noted earlier, they are of little significance.) Examination of the various factors in P_o shows that a rapid change of cos e_o^* is the major cause of the steep slope of P_o at low frequencies. At the higher frequencies the phase error becomes quite large but the main reason for the sharp decrease of P_o is a decrease in the product $\bar{q}_{Go} \, \bar{n}_{Eo}/\bar{n}_{Go}$. The contributory effects may be seen to be first the rapid decrease, with increasing frequency, of $\bar{q}_{Go}$ compared with that of $\bar{n}_{Go}$ (compare Figs.10 and 11) and second the relatively poor performance of the elevator in producing normal acceleration at the higher frequencies (see Fig.9). With the aid of Fig.12 it may be deduced that for any frequency above 0·145 Hz the minimum amplitude of the response occurs for some positive value of G_q: for lower frequencies, a positive value of G_q increases the amplitude.

Since the phase angles for pitching velocity and pitching acceleration differ by an odd multiple of $\pi/2$ the graph of e_o^* indicates that an autostabiliser with a pitch-acceleration law would be largely ineffective, for low gearings anyway, since the phase error would be large over the most important range of frequencies. It can be shown that such an autostabiliser with a higher gearing would produce the unacceptable result of a large increase in the natural period accompanied by only a slight increase in the relative damping. Therefore the possibility of deriving much benefit from including a pitch-acceleration term in the autostabiliser law can be dismissed.

Attention is now turned to the question of the effects of further increases in G_q by considering the graphs of the pay-off function for gearings of 0·25, 0·5, 1·0 and 2·0 which are presented in Fig.13. At the frequencies where the graph of P_o indicated that sizeable benefits could be obtained by employing autostabilisation, these graphs show that as G_q is increased a given (small) increment in G_q produces a progressively smaller reduction (expressed as a proportion of the 'local' value) in the magnitude of the response. No parameter can be singled out as playing the dominant role in causing this decline of the pay-off.

Fig.13 indicates that a consideration of P may be useful when trying to fix the gearing of an autostabiliser. For instance, if analyses had been carried out for gearings of 0, 0·25 and 0·5 it would already be obvious from a consideration of the graphs of P for these cases that further increases of gearing would perhaps not be worthwhile. Used together with the values of the relative damping for these gearings, extrapolation from which is quite reliable, as can be seen from Fig.6, these graphs could have led to the conclusion that a suitable value to choose for the gearing would be between 0·5 and 1·0.

Finally, it may be remarked that since the pay-off function depends on three amplitudes and three phase angles, all of which are frequency-dependent, it would indeed be fortunate if the response quantity ε could be chosen such as to make the pay-off sensibly independent of frequency; therefore, worth-while benefits will often be obtained over only a narrow band of frequencies.

6 CONCLUDING DISCUSSION

The normal accelerations experienced by modern transport aircraft in cruising flight through turbulence contain a component due to the elevator movements produced by the automatic control system and it is often asked what is the magnitude and nature of this component. Usually one cannot answer this question since the data which would enable one to do so are either not recorded at all or not recorded in sufficient detail. However, data which permitted the calculation of the elevator-induced normal accelerations during a period of flight through moderate turbulence became available, through fortuitous circumstances, from the Civil Aircraft Airworthiness Data Recording Programme. In this Report the results of calculations for three excerpts from this period of flight are presented. The validity of the mathematical model used was checked using data from the same source.

It is found that the influence of the elevator was quite small and was such that it neither consistently increased nor consistently decreased the amplitudes of the excursions in cg normal acceleration.

In order to explore the reasons for the above findings and to assess in more general terms the scope for modifying the response of an aircraft by employing autostabilisation, the influence of an idealised autostabiliser on the response to harmonic vertical gusts was considered. From the analysis and results presented it is seen that an autostabiliser may well be of practical benefit over only a small range of frequencies. However, for the particular aircraft and flight conditions considered here a pitch-rate autostabiliser is effective in reducing the amplitude of the cg normal-acceleration response at the frequencies where it is largest. It appears that a useful reduction in amplitude could be obtained by using such an autostabiliser with a gearing of 1 deg/(deg/sec); for this gearing the relative damping of the aircraft's short-period longitudinal mode is 0·7.

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SYMBOLS

(i) General notation

$D \equiv \frac{d}{dt}$	differential operator
G_q	gearing of pitch-rate autostabiliser
G_ε	gearing of autostabiliser (equation (12))
L	wavelength of gusts: m
P	pay-off function
V	speed of flight: m/sec
e^*	phase error: rad
m	mass of aircraft: kg
n	cg normal acceleration: g units
q	rate of pitch: rad/sec
w	aircraft velocity relative to the air in the z direction: m/sec
w_K	aircraft velocity relative to the earth in the z direction: m/sec
w_W	velocity of the air relative to the earth in the z direction: m/sec
δ	typical response quantity
ε	typical response quantity, fed into autostabiliser
η	elevator angle: rad
η_t	elevator angle for sustained level flight: rad
σ	parameter defining sense of effect of autostabiliser (see section 5.1)
ϕ_δ	phase angle of response in δ: rad
ω	frequency of harmonic variation: rad/sec

Suffices

o	for the unstabilised aircraft
E	due to elevator movement
G	due to gusts
ℓ	evaluated locally

Dressings

$\bar{}$	amplitude of harmonic variation
$\hat{}$	dynamic-normalised quantity - see below

(ii) Definitions of dynamic-normalised quantities

For a full explanation of the dynamic-normalised system the reader should consult Refs.2 and 3. Some of the basic quantities used in the system are given below; the symbols used in this Report are then defined. The definitions make use of the identities between reference speeds, lengths, etc. which existed in the present numerical work and are not necessarily generally valid definitions of dynamic-normalised quantities. It is assumed that it is unnecessary to reproduce here the definitions of a certain number of standard symbols

$$C_z = \frac{Z}{\frac{1}{2}\rho V^2 S} \qquad C_m = \frac{M}{\frac{1}{2}\rho V^2 S\ell}$$

I_y aircraft moment of inertia in pitch: kg m^2

$$i_y = \frac{I_y}{m\ell^2}$$

$$\hat{D} \equiv \tau \frac{d}{dt}$$

$$\hat{G}_q = \frac{G_q}{\tau}$$

$$\hat{q} = \tau q$$

$$\hat{w} = \frac{w}{V}$$

$$\hat{z}_e = - C_{z_e} \qquad \hat{m}_e = - \frac{\mu}{i_y} C_{m_e}$$

$$\hat{z}_w = - \frac{\partial C_z}{\partial \hat{w}} \qquad \hat{m}_w = - \frac{\mu}{i_y} \frac{\partial C_m}{\partial \hat{w}}$$

$$\hat{z}_{\dot{w}} = - \frac{\partial C_z}{\partial (\hat{D}\hat{w})} \qquad \hat{m}_{\dot{w}} = - \frac{\mu}{i_y} \frac{\partial C_m}{\partial (\hat{D}\hat{w})}$$

$$\hat{z}_q = - \frac{\partial C_z}{\partial \hat{q}} \qquad \hat{m}_q = - \frac{\mu}{i_y} \frac{\partial C_m}{\partial \hat{q}}$$

SYMBOLS (Contd.)

$$\hat{z}_{\dot{q}} = -\frac{\partial C_z}{\partial(\hat{D}\hat{q})} \qquad \hat{m}_{\dot{q}} = -\frac{\mu}{i_y}\frac{\partial C_m}{\partial(\hat{D}\hat{q})}$$

$$\hat{z}_{\eta} = -\frac{\partial C_z}{\partial \eta} \qquad \hat{m}_{\eta} = -\frac{\mu}{i_y}\frac{\partial C_m}{\partial \eta}$$

$$\mu = \frac{m}{\frac{1}{2}\rho\, S\ell}$$

$$\tau = \frac{\mu\ell}{V}$$

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4	Various authors	Stability and control. AGARD Conference Proceedings No.17 (1966)

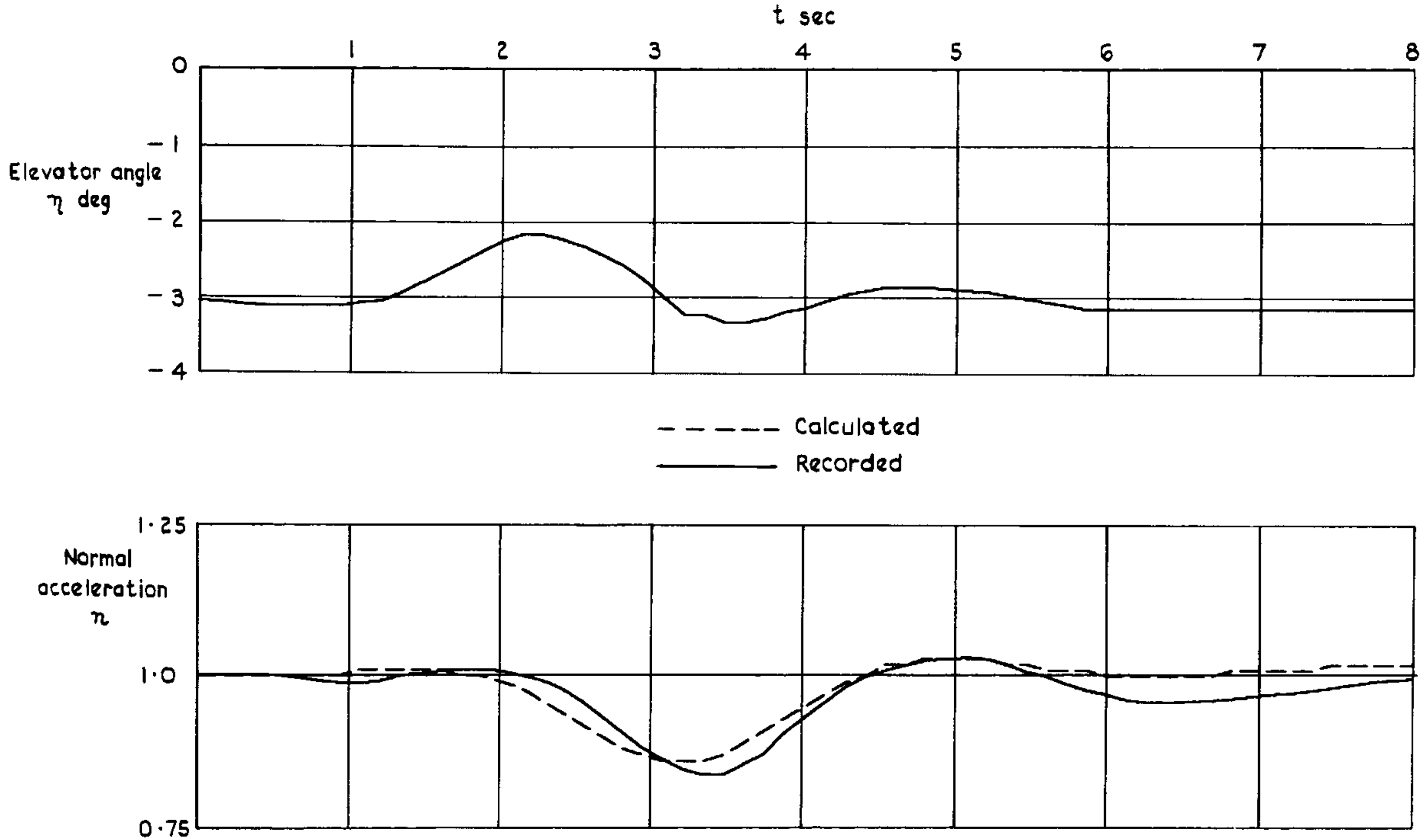


Fig.1 Comparison of calculated and recorded responses in manoeuvres · Case 1

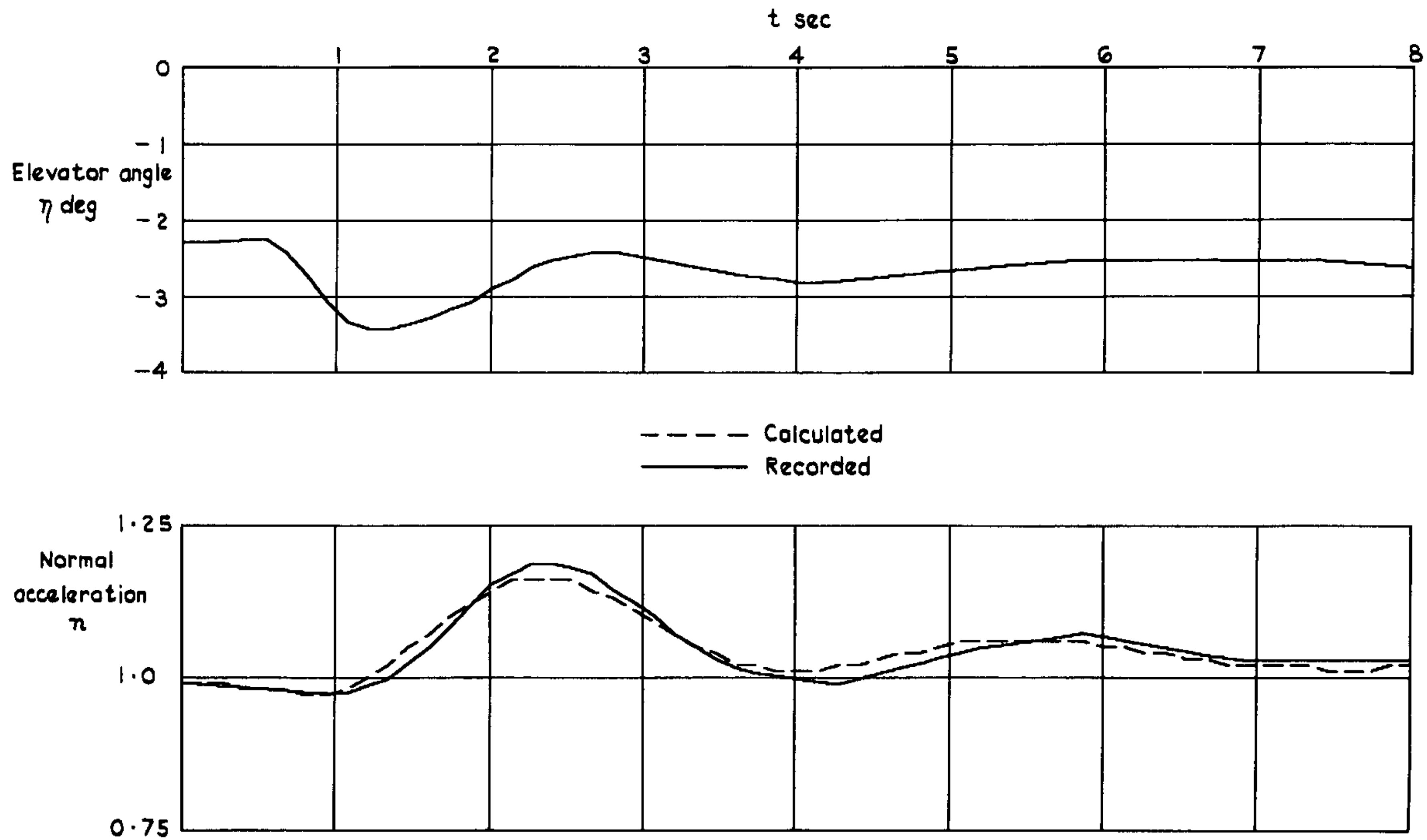


Fig. 2 Comparison of calculated and recorded responses in manoeuvres : Case 2

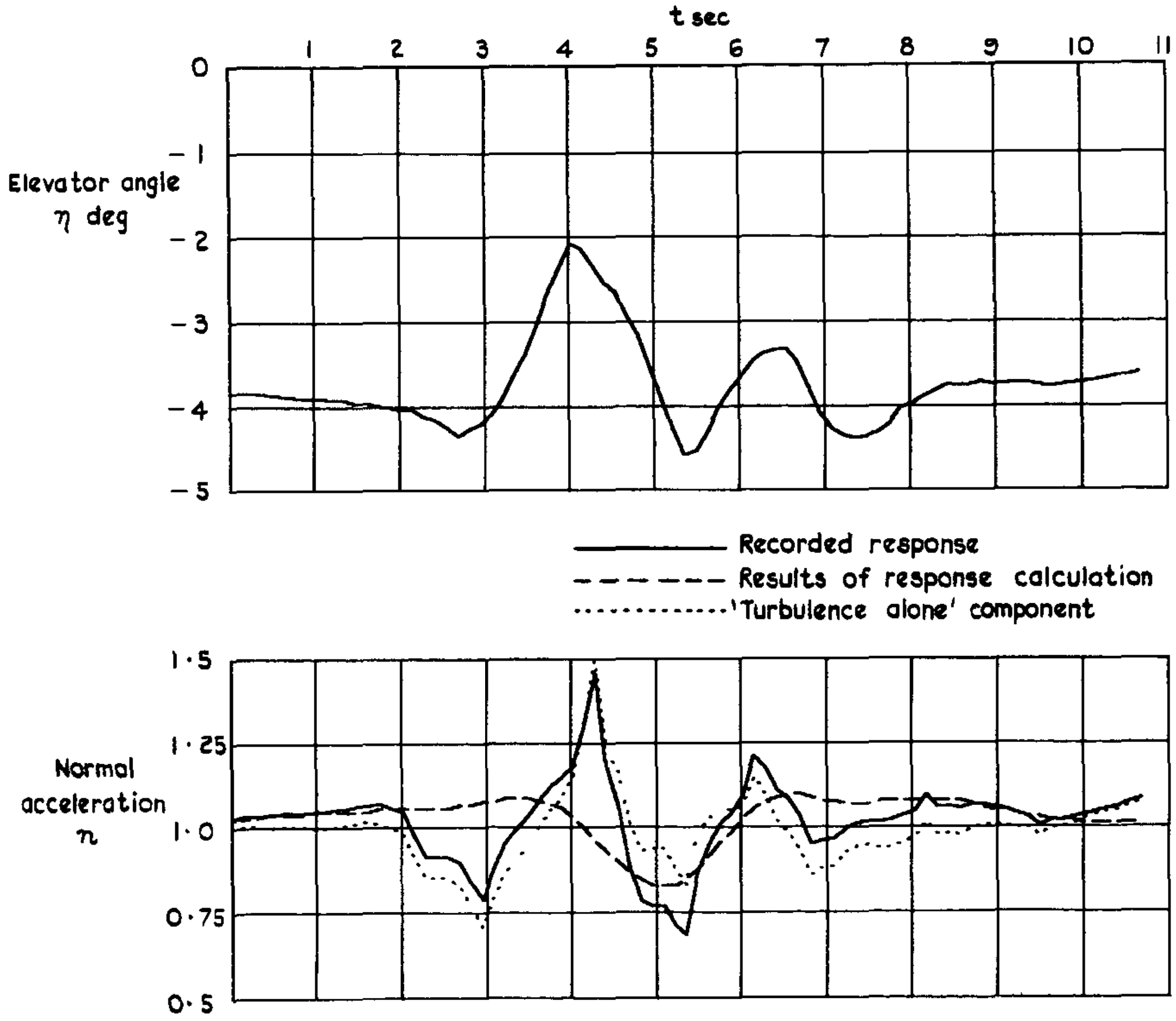


Fig.3 Components of the response in turbulence : Case 3

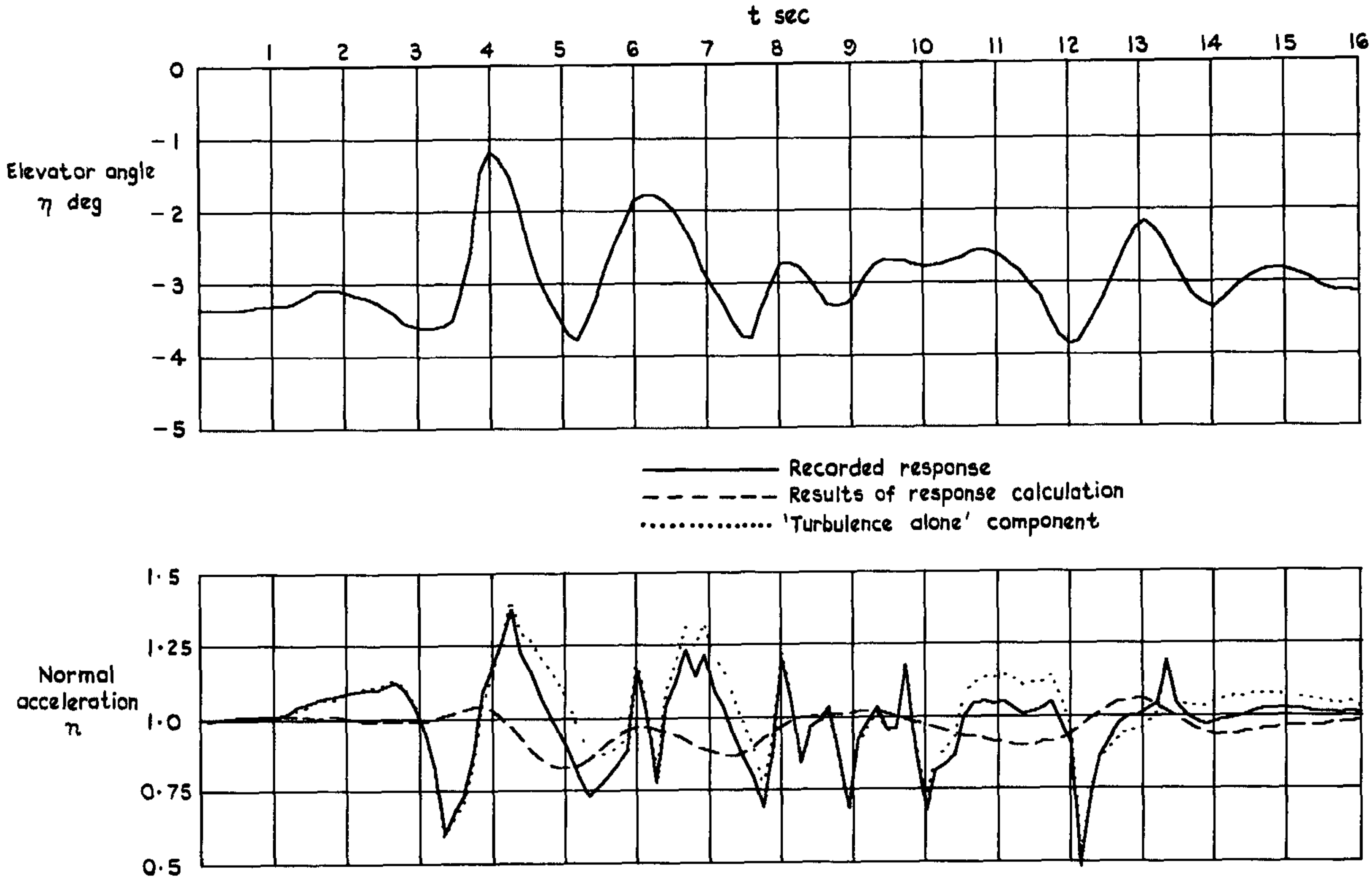


Fig. 4 Components of the response in turbulence : Case 4

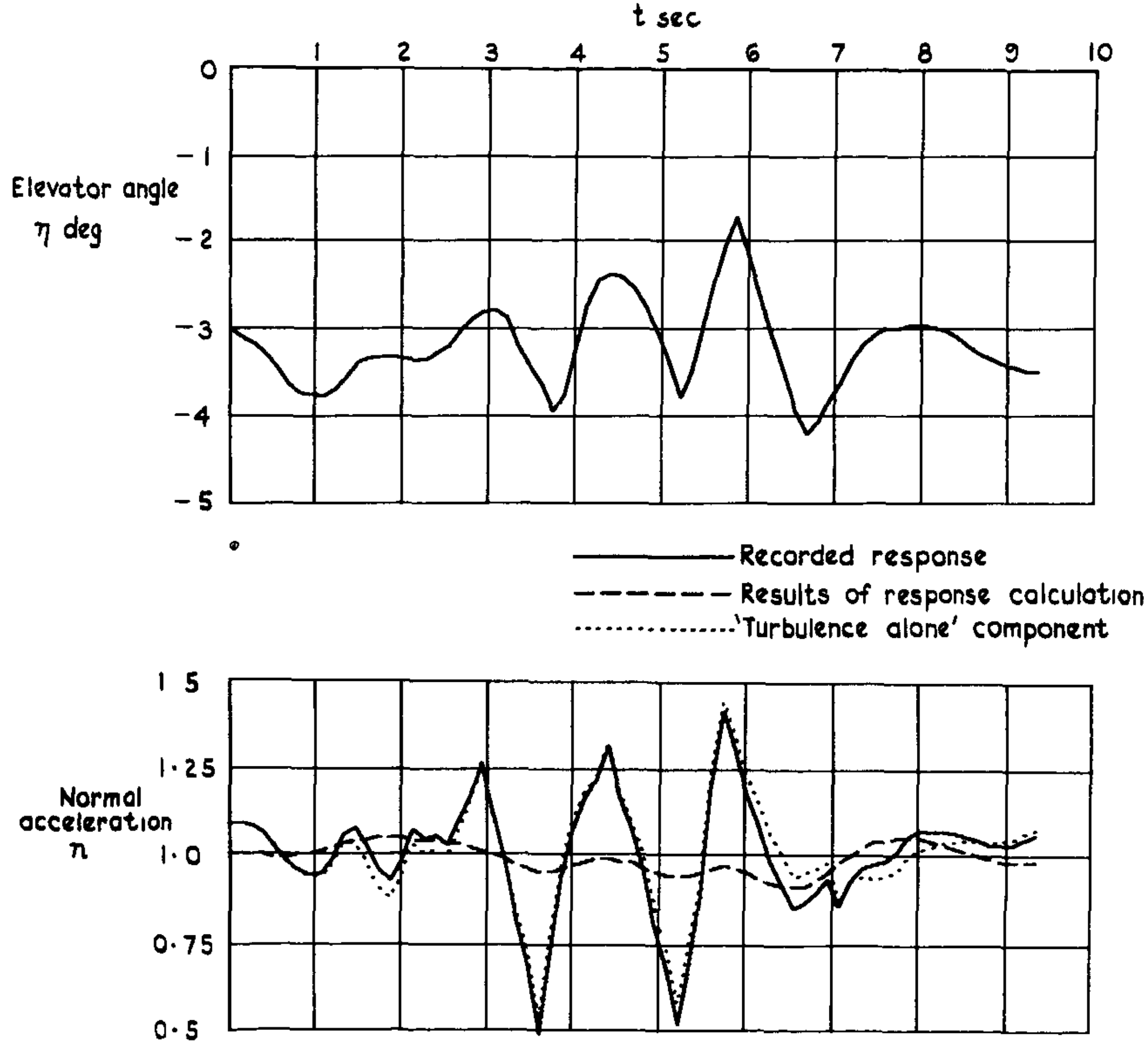
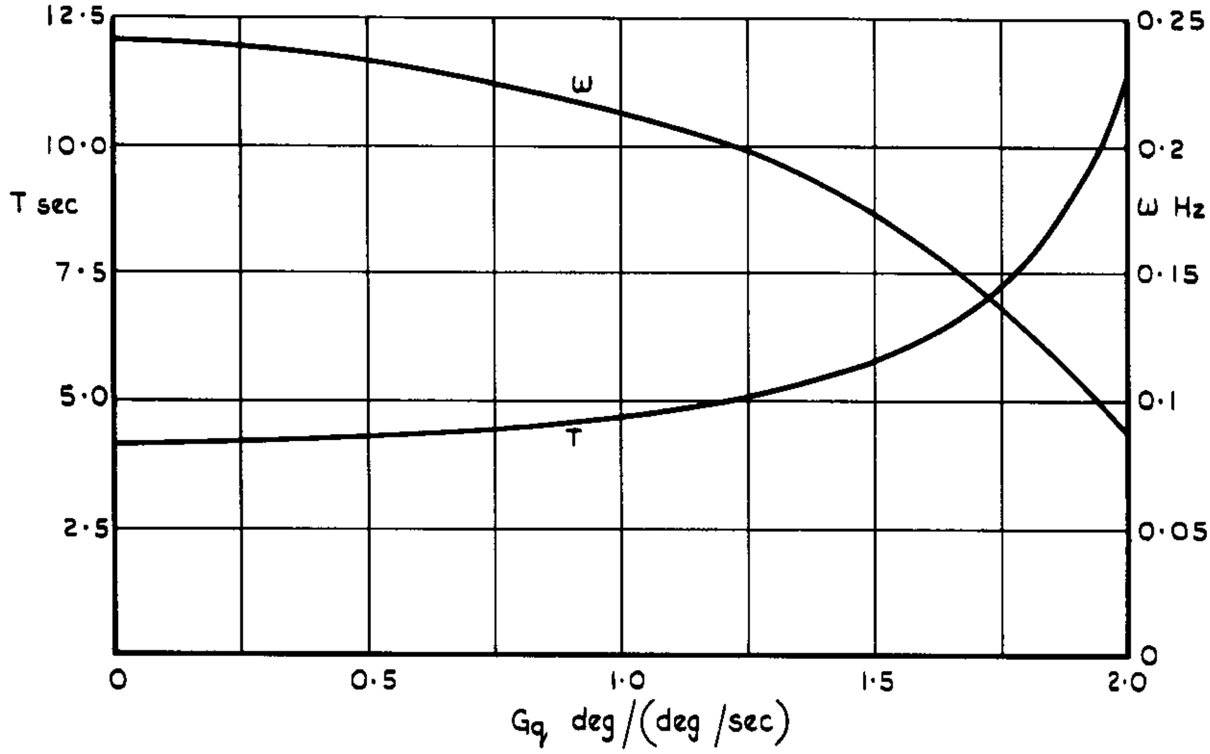


Fig. 5 Components of the response in turbulence : Case 5



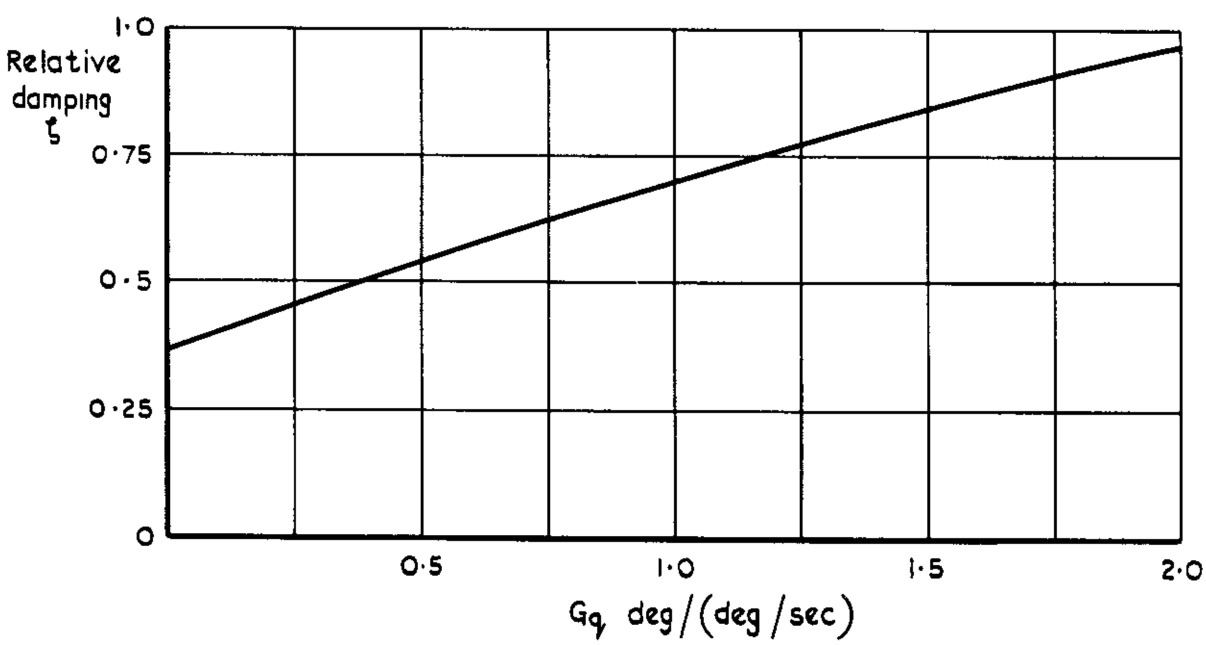


Fig. 6 Variation of natural period, frequency and damping with gearing of pitch-rate autostabiliser

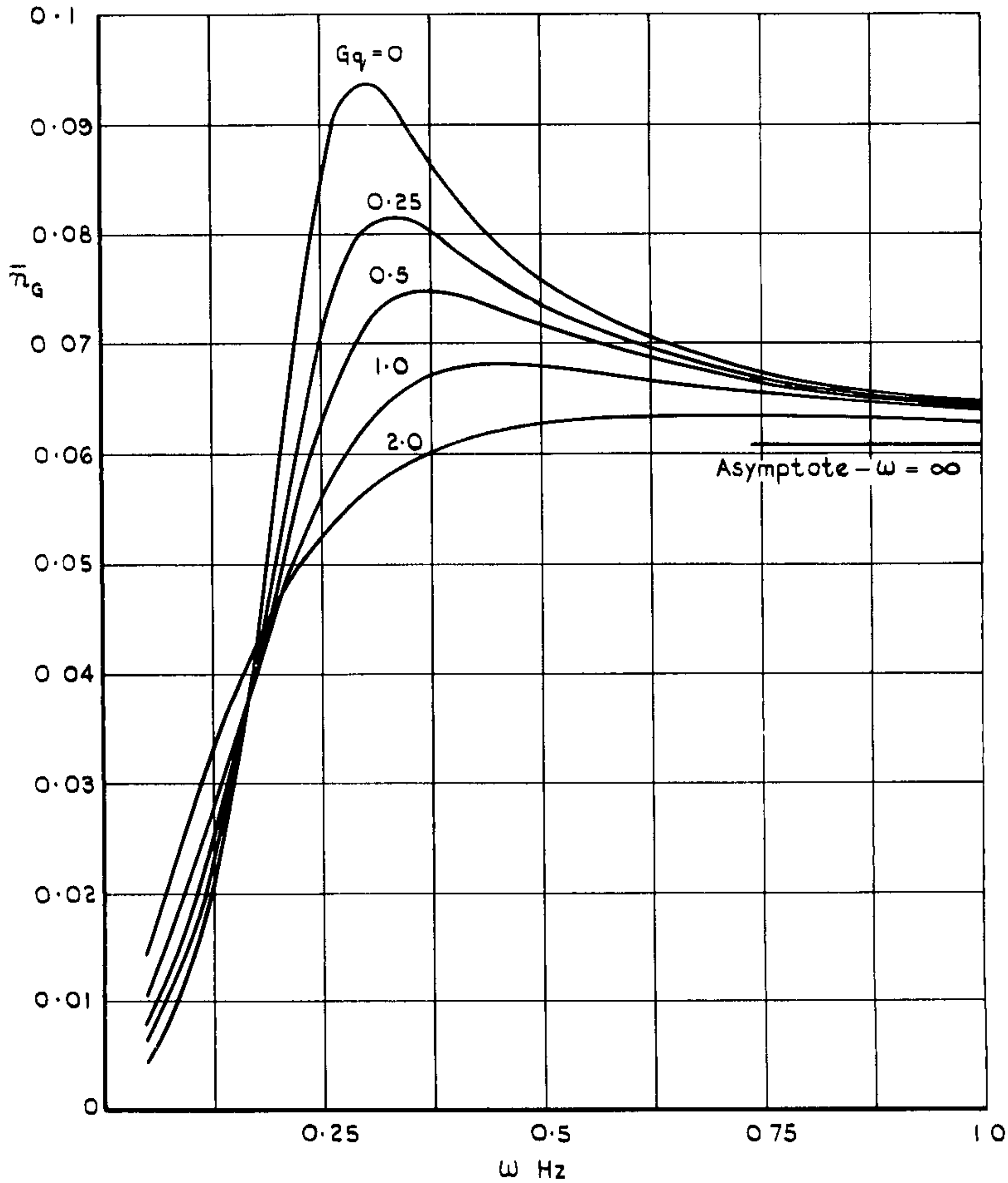


Fig 7 Amplitude of cg normal-acceleration response to 1 m/sec amplitude harmonic vertical gusts

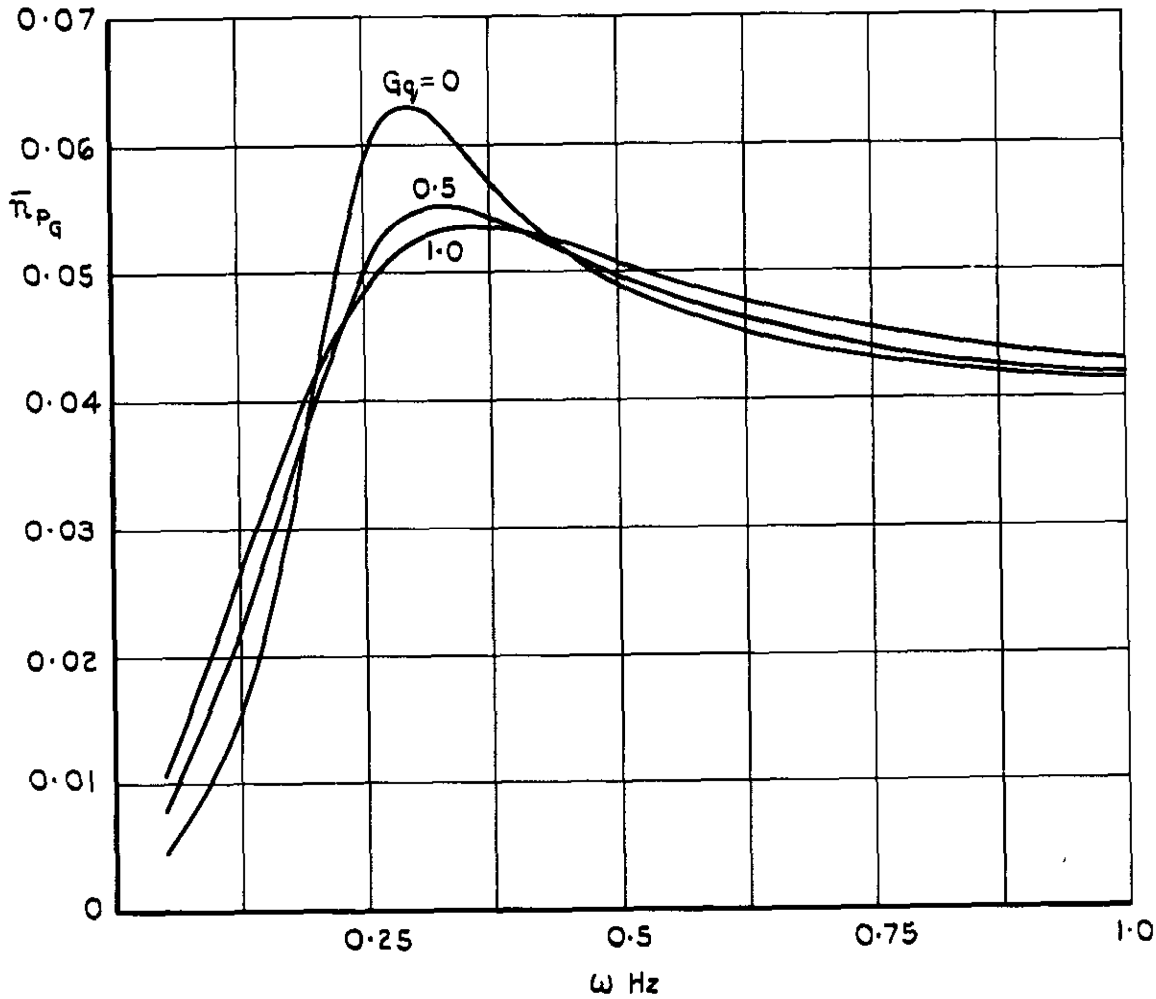


Fig. 8 Amplitude of normal acceleration response 23 m ahead of cg to 1 m/sec amplitude harmonic vertical gusts

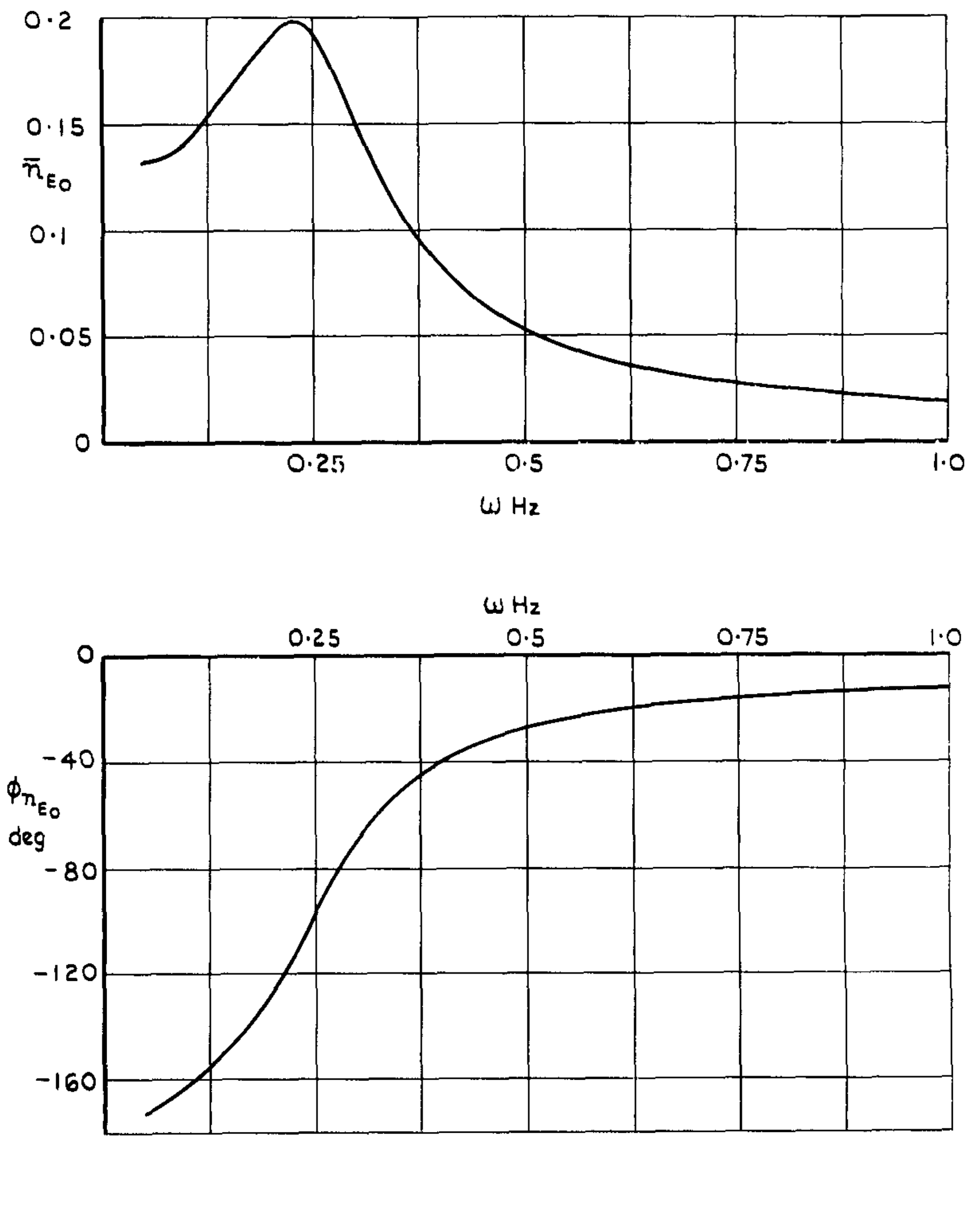
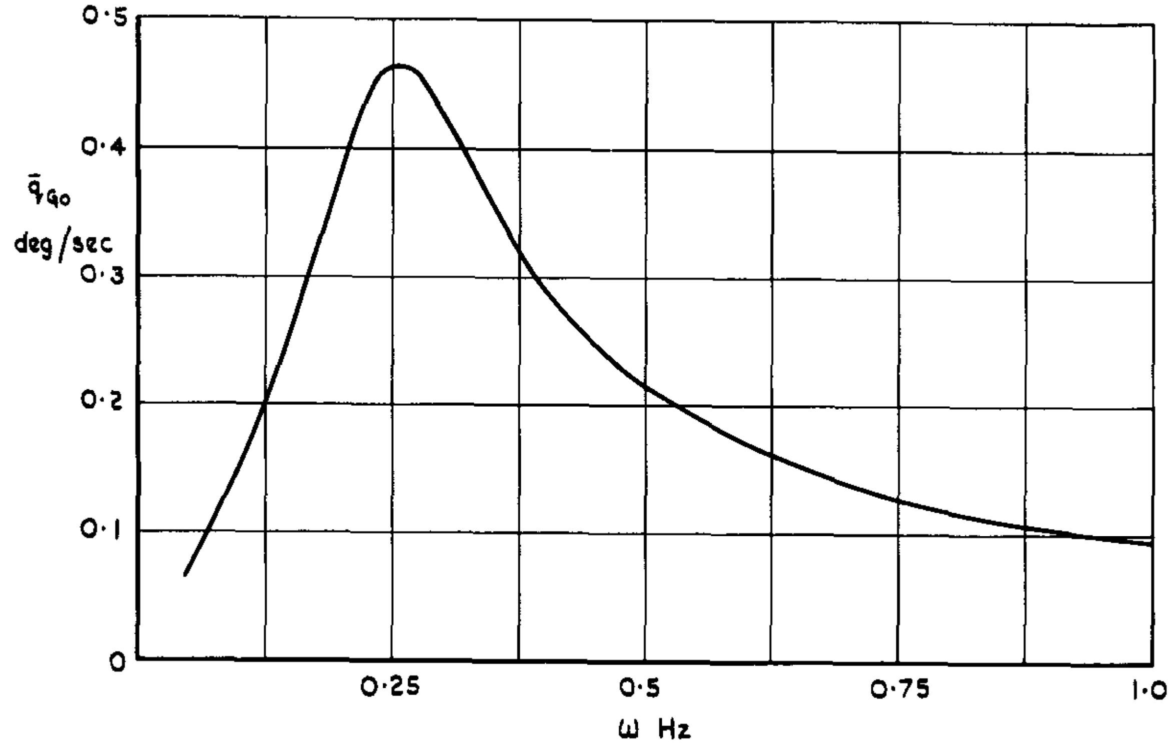


Fig.9 Amplitude and phase angle of cg normal-acceleration response to 1 deg amplitude harmonic elevator movement $- G_q = 0$



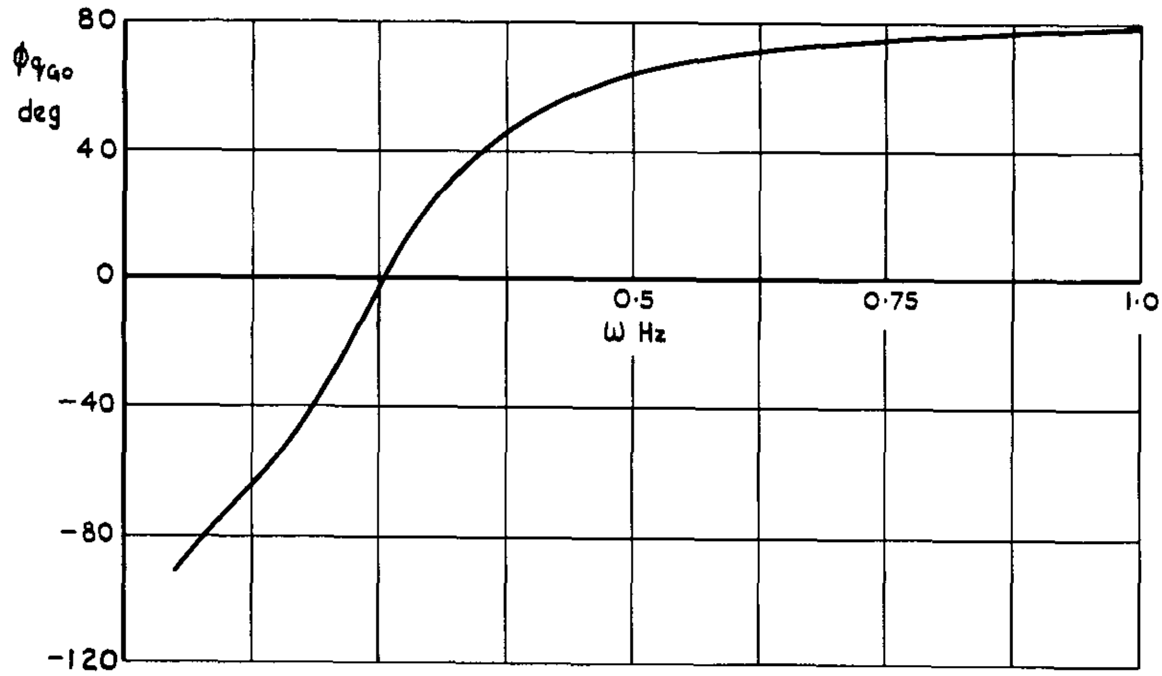


Fig. 10 Amplitude and phase angle of pitch-rate response to 1m/sec amplitude harmonic vertical gusts $- G_q = 0$

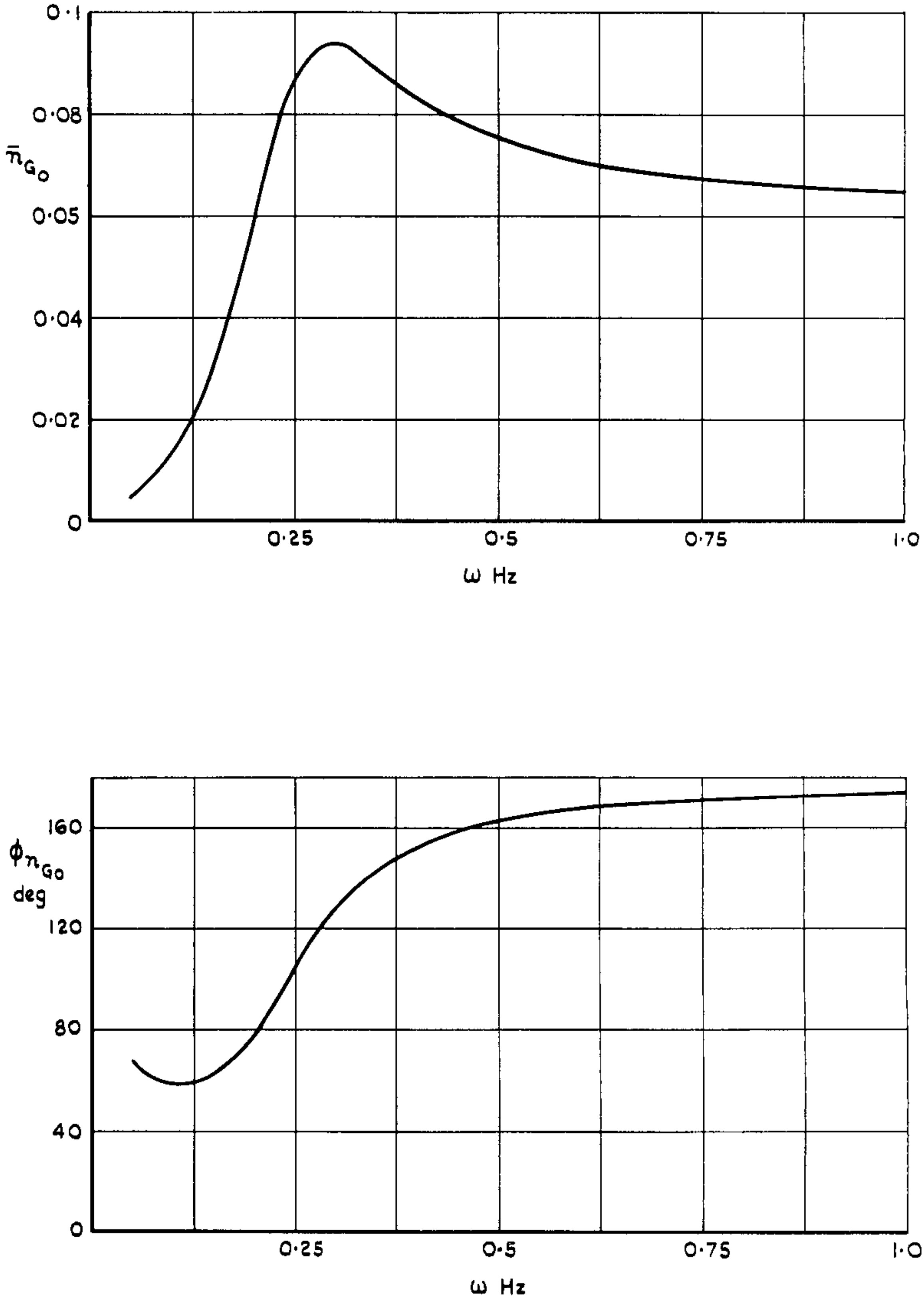


Fig. 11 Amplitude and phase angle of cg normal-acceleration response to 1m/sec amplitude harmonic vertical gusts – $G_q = 0$

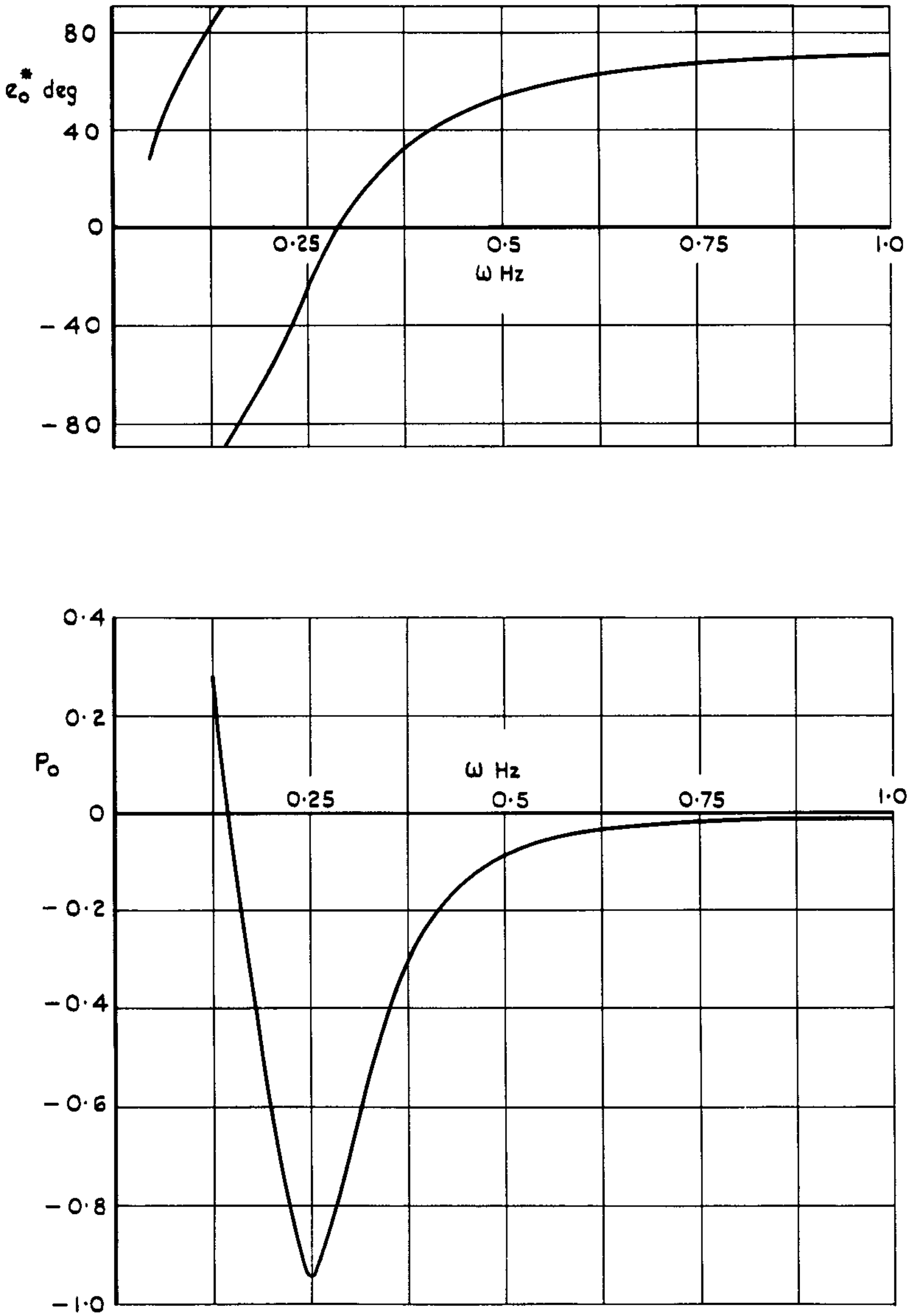


Fig.12 Phase error and pay-off function for $G_q = 0$

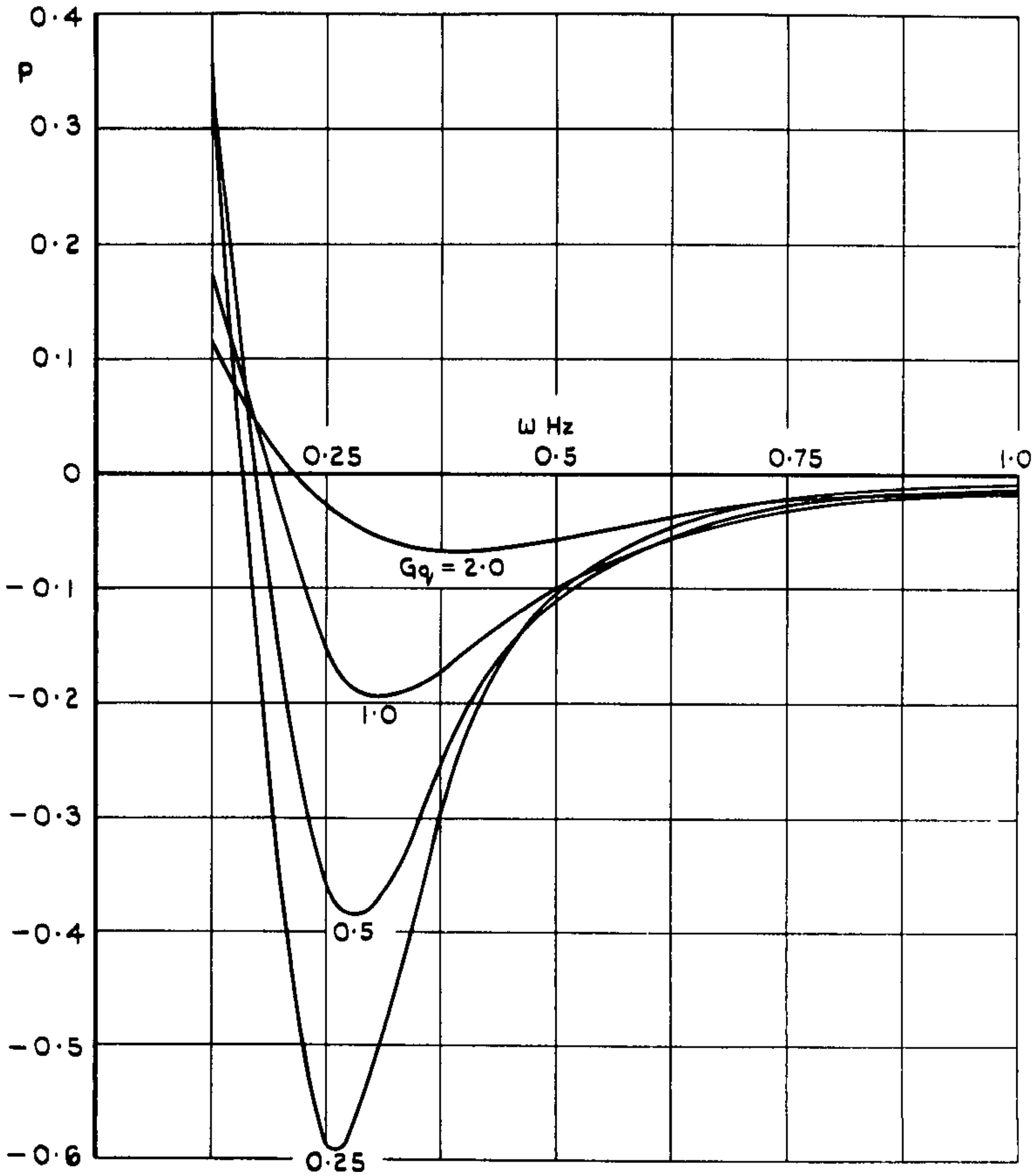


Fig.13 Pay-off function for $G_q = 0·25, 0·5, 1·0$ & $2·0$

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