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Vibration Levels Experienced in Take-Off on a Large Flexible Aircraft

by

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VIBRATION LEVELS EXPERIENCED IN TAKE-OFF ON A LARGE FLEXIBLE AIRCRAFT

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SUMMARY

On the basis of some American theoretical predictions and flight experience it was suggested qualitatively by Zbrozek that there was a possibility of unacceptably large responses at the cockpit of Concorde during take-off. The work described in this paper was done in order to provide experimental evidence aganist which a general theoretical model of take-off response behaviour could be checked.

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*Replaces RAE Technical Report 68193 - ARC 32 600

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1 INTRODUCTION

It was shown in Ref.1 that the ratio of cockpit to cg rms acceleration for an American supersonic transport design, travelling at constant velocity of 100 ft/sec on the Langley 12 runway, was 2.84. The corresponding figure for the Boeing 707 was 1.92. Operational results indicated that this latter ratio was of the correct order and gave confidence in the theoretical method used. An analysis in an earlier paper[2] gave an almost identical ratio to the value for the supersonic transport above for a slender delta operating in turbulent air. The cockpit acceleration in the runway case, that led to the above ratio for the supersonic transport, was 0.54 g rms. Although the significance of this level was hard to assess, the fact that accelerations due to turbulence with 0.3 g rms were classified as 'heavy' or 'very heavy' led to the suggestion that the level on Concorde should be assessed.

A digital computer programme was written by A. J. Sobey of Structures Department, R.A.E. to investigate deterministically the response of Concorde in the take-off run. The input to this programme was the profile of the Langley 12 runway. Responses predicted by this calculation reached a maximum of the order of ±2 g during the critical rotation phase. Independent calculations made by Sud Aviation using the San Francisco 28R profile yielded levels of the same order. A drawback with both these analyses was that they had to use as input runway surfaces that were considered poor from an operational point of view and indeed are probably no longer used operationally for take-off and landing.

In a calculation of this nature certain assumptions had to be made regarding aircraft and undercarriage characteristics which could materially alter the predicted responses. These assumptions could be checked only against experimentally derived responses. Due to the fact that Concorde had not reached the stage at which trials could be made, it was decided to generalise the programme so that it could

(1) be used for analysis of other existing aircraft, and hence provide a check on the assumptions made;

and (2) provide a practical research tool that would be of value to aircraft designers generally.

The generalised programme has now been written and is described in Ref.3. The first application will be to a VC 10 aircraft that made special take-offs and landings from the main runway at A. & A.E.E. The runway profile was surveyed for this purpose and the surface is considered to be more representative of a commercially acceptable surface (in terms of surface irregularities) than either of the American runways mentioned earlier.

2 TESTS MADE WITH VC 10 (XV 105)

Six landings and take-offs were made using this aircraft from the main runway at A. & A.E.E., Boscombe Down under conditions of negligible turbulence. For the purposes of this paper only the take-offs are of interest and further consideration is accordingly limited to these.

2.1 Take-off schedule

The first three take-offs were made under nominally identical conditions. The pilot began the ground roll as near as possible at the same base position on the runway on each of the three occasions. Pilot technique on each of these runs was similar in that the aircraft was rotated at approximately 130 kt. The fourth take-off was started at the same base position on the runway but rotation was delayed until a speed of 165 kt had been achieved. On the fifth take-off the starter position was 1000 ft down the runway from the base, rotation being at 130 kt. For all these tests the weight was sensibly constant, ranging between 228 000 and 236 000 lb, and the same pilot operated the aircraft.

On the final take-off run the weight was increased to 260 000 lb, by increasing fuel weight, and the ground roll was begun 2000 ft down the runway from the base position. The rotation speed was normal, i.e. 130 kt, and the aircraft was flown by another pilot.

12 passengers were carried on all flights in the fuselage near the centre of gravity.

2.2 The instrumentation

The aircraft was instrumented to measure the following responses.

(1) Cockpit vertical acceleration. The accelerometer was positioned slightly aft of the first pilot station.

(2) Cockpit lateral acceleration (measured at the same station).

(3) cg vertical acceleration.

(4) Rear fuselage vertical acceleration

(5) Rear fuselage lateral acceleration

The accelerometers were positioned in the rear freight hold just forward of the engine station.

The accelerometers used in the test were supplied by Structures Department, Type 15L; the 70% response level was at 10 Hz and the 10% response level at 21 Hz. Outputs from the accelerometers were taken to an F and E 50-channel photographic recorder which produced negligible attenuation up to 50 Hz.

The aircraft position on the runway during take-off was determined from analysis of the kinetheodolite record obtained during the tracking of each take-off. A secondary source of information on aircraft position was a pair of magnets mounted diametrically opposite each other on the rear outer wheel of the port main undercarriage assembly. As the wheel rotated, these magnets passed a coil mounted on the main bogie and a pulse was generated that was taken to the photographic recorder. The pulse generation method proved to be very satisfactory but aircraft position was not extracted from the information. The effective rolling radius of the wheel could not be established with sufficient accuracy to enable a reliable estimate of distance travelled to be made.

2.3 The runway

The parts of the runway used on the third, fourth, fifth and sixth take-offs are indicated on Fig.1 which illustrates the profile. The illustration indicates the macroscopic structure of the profile. Some indication of the finer grain structure may be gained from the results plotted in Fig.2 which are spectral density estimates of the profile at particular wavelengths. For comparison purposes, the corresponding estimates for the Langley 12 runway are included.

The long wavelength component in the Boscombe profile is so large that spectral density estimates based on the raw data are inaccurate as shown by the plot in Fig.2. However, frequencies below approximately $\frac{3}{4}$ cycle/sec are not significant for the undercarriage designer and at the speeds we are considering, this means that wavelengths significantly above 250 ft are not of importance. If these longer wavelengths that cause the non-stationarity are filtered out, then useful estimates of spectral density may be obtained. The raw Boscombe profile elevation data, available in five hole punched paper tape form, has been treated in this manner using a moving average digital filter. This filter cuts off above 250 ft, the half power point being at 370 ft and one tenth power point at 650 ft. Data points at 2 ft intervals have been modified as follows

$$Y'_I = Y_I - \sum_{J=-K}^{K} B_J Y_{I+J} \quad .$$

Details of the weighting function B_J for $K = 100$ are given in Ref.4 and reproduced in Table 4. Spectral density estimates have been calculated from this modified data firstly by calculating the autocorrelation function (130 lags) and using Bartlett's smoothed periodogram[5], a particular spectral window. The estimates were formed for frequencies up to 0.25 cycle/ft at intervals of 0.0066 cycle/ft. Spectral estimates for Langley 12, Fig.2, have been made using the same technique (40 lags). It is interesting that the estimates made in this way are very similar to those made for presentation in Ref.6 using a somewhat different technique. Data points in the latter case were pre-whitened (the transformation being $Y'_I = Y_I - Y_{I-1}$ which is equivalent to modification by a frequency response function), the autocorrelation function calculated from the modified data, 'initial' spectral density estimates formed from the autocorrelation functions, final 'smoothed' spectral estimates formed from the weighted averages of the 'initial' estimates (the 'Hanning' spectral window) and finally these estimates were post darkened by an inverse frequency transformation.

When the Boscombe and Langley runways are compared on this spectral density basis, the former is seen to be the smoother over the frequency range considered apart from, possibly, the higher values of frequency.

The presentation of the Boscombe runway in spectral form is completed in Figs.3 and 4 where estimates are made of those parts of the runway over which individual take-off runs were made. These estimates were derived from 40 lag autocorrelation functions (the take-off runs were similar in length to the Langley profile) and are presented at intervals of 0.0066 cycle/ft at frequencies up to 0.25 cycle/ft. The estimates for individual runs are slightly higher than for the runway as a whole.

2.4 Results

These are restricted for the purpose of this paper to the cockpit vertical and lateral accelerations and, because aircraft response data is customarily described in this manner, by cg acceleration. The acceleration data was read at 25 msec intervals. Visual inspection of response records for the first three take-offs showed them to be determistically similar and the analysis was confined to that of the responses on the third take-off.

The presentation of results in a suitable form for comparison with theory has been made in three ways:

(i) rms response levels are determined for each second of the take-off run and plotted against time. (Figs.5, 6, 7 and 8.) (This method has been found useful for presenting non-stationary vibration data derived from spacecraft launching[7] in which each sample record has a common underlying time varying deterministic characteristic. It seemed likely that this was true of the runway response data too and accordingly this method of presentation was adopted.) Lateral responses presented in Figs.5, 6 and 7 will be in error due to difficulty in defining a datum for this type of motion and particularly so in the case of low vibration levels. The response presented in Fig.8 for this motion is accurate because of a fortuitous choice of datum.

(ii) Spectral density estimates of the response amplitude distribution over the complete length of the take-off run have been prepared. The results are presented in Figs.9 to 12 as plots of spectral density against frequency and are described in section 2.4.2. The same technique as that described in section 2.3 was employed in making these spectral density estimates; in this instance as there were no very long wavelength components the initial filtering stage was omitted. Estimates were formed from autocorrelation functions employing eighty lags for frequencies up to 20 Hz at intervals of 0.34 Hz. In the figures the curves have been arbitrarily terminated at 13 Hz, frequencies above this were outside the range of interest for the investigation and had been heavily attenuated during the recording process.

(iii) Peak exceedances of specified positive response levels throughout the take-off have been counted. Positive response is upwards for cockpit vertical and cg vertical and to port laterally. Tables of exceedances at various levels have been prepared for take-offs 3, 4, 5 and 6 and are presented in Tables 1 to 3 for the three acceleration traces selected for study.

2.4.1 rms response levels

The development of rms response throughout the third take-off is shown in Fig.5. Cockpit vertical response rises to its first peak 0.117 g in the fifteenth second and then continues on a rising trend to an ultimate peak of 0.133 g in the twenty-third second, i.e. immediately prior to the rotation of the aircraft. This general behaviour is mirrored on a lower level by the aircraft response at the cg. The peak response at the cg occurs slightly earlier than that for the cockpit vertical response.

In the fourth take-off the development of individual responses (Fig.6) follows the same pattern as the third although the general level is a little higher. Initial peaks on the cockpit vertical and cg accelerations occur somewhat later than during the third take-off. The final peak on cockpit response is approximately double that experienced in the previous take-off. This result must be due to a combination of higher aircraft speed prior to rotation and movement over a different part of the runway during the latter part of the take-off run.

The cockpit vertical response development during the fifth take-off (Fig.7) is similar to that during the third take-off, apart from the somewhat lower level immediately prior to rotation. A similar comment applies to cg vertical response. The fact that the responses are in essence the same indicates that the runway input does not differ significantly except possibly at the extreme end of the portion appropriate to the fifth take-off.

The character of the cockpit vertical response during the sixth take-off (Fig.8) for the heavier aircraft differs from that recorded in the other take-offs; after an initial rise the level remains sensibly constant throughout the remainder of the run apart from a single peak after 18 seconds. The levels recorded at the cg are typical of those in all take-offs, peaks being recorded late in the take-off. Cockpit lateral acceleration is at a low level throughout the take-off showing no tendency to build up with speed.

2.4.2 Response spectra

The measured response spectra for the third take-off are shown in Fig.9. It can be seen that in the frequency range of physiological significance, 1-8 Hz, the spectral density estimates for vertical acceleration at the cockpit are of the order 100 times the lateral; on this basis the ratio of response amplitudes in this frequency range will be about 10:1. Peaks in the vertical response at the cockpit occur at approximately 3.5, 4.5, 6, 9 and 12 Hz, whereas the dominant frequency for lateral response occurs at 12 Hz at reduced power. Response frequency peaks for the cg acceleration are at 1.7, 3.4, 4.4 and 12 Hz.

Fig.10 shows spectra for the fourth take-off that commenced at the base position on the runway but in which the aircraft was not rotated until a speed of 165 kt had been achieved, thus an additional part of the runway was used in excess of that required for the third take-off. The three response spectra are essentially similar to those of Fig.9, with peaks occurring at the same frequencies but the levels for pilot vertical and cg acceleration are somewhat greater, particularly at the low frequencies. The cockpit lateral response levels are virtually the same in the two cases apart from a slight peak at 2.5 Hz occurring on this take-off.

Spectral densities for three responses of interest on the fifth take-off are plotted in Fig.11. The estimated magnitudes are very similar to those appropriate to the third take-off over much of the frequency range and the dominant frequencies are essentially similar. It will be remembered that a similar take-off technique was applied in the two cases, the principal difference was that the fifth commenced approximately 1000 ft down the runway. The slight differences between Figs.9 and 11 may be attributed to variations in aircraft speed between the two take-off runs and the different runway surface. The differences between these spectra for cockpit vertical and cg responses and the corresponding ones for the fourth take-off are most marked at the low frequency end of the range below 1.5 Hz.

Spectral responses for the sixth take-off are plotted in Fig.12. The aircraft weight during this take-off was approximately 30000 lb greater than that appropriate to the other three take-off cases and the ground roll commenced 2000 ft down the runway from the base position. The dominant response frequencies are not changed substantially from the other cases. Response levels for cockpit vertical and cg responses are a little lower than those experienced in the third take-off below 2.5 Hz but above this frequency are virtually the same. Below 7 Hz the cockpit lateral response is greater than that experienced in the third take-off. Comparison with results for the fourth take-off shows that the levels for cockpit vertical and cg response recorded during the sixth take-off were lower throughout the complete frequency range 0-10 Hz whilst the opposite was the case for cockpit lateral response.

The frequency components at 3.5, 4.5, 6 and 12 Hz show up strongly and it is accordingly felt that modes at these frequencies should certainly be included in any theoretical analysis of the problem. If further tests are required to investigate physiological effects it is suggested that the experiment should be designed to filter out the higher of these frequencies.

2.4.3 Peak responses

Table 1 indicates that from the point of view of the vertical response at the cockpit, the fourth take-off was the most severe by a considerable margin and that the sixth gave the smoothest response. Levels in take-offs 3 and 5 were intermediate between these values with the third being slightly the poorer ride.

Table 2 shows that lateral accelerations recorded were all very small. The highest levels were obtained on the sixth take-off.

The cg vertical accelerations experienced in the third and fourth take-offs were of roughly the same severity. The values were greater than those for the fifth and sixth take-offs which were of about equal severity. At lower response levels the fourth take-off is more severe than the third but at levels in excess of 0.1 g the third take-off response levels are the greater.

3 CONCLUSIONS

The results of a series of six closely monitored take-offs by a VC 10 operating from the main runway at Boscombe Down are included in this paper. Response levels obtained in the tests were low and this is probably a result of a well matched aircraft undercarriage combination, the good runway surface, and negligible turbulence. Three methods of presentation of the results are adopted.

Table 1

PEAK EXCEEDANCES OF STATED POSITIVE RESPONSE LEVELS - COCKPIT VERTICAL

Peak level g absolute	3rd take-off	4th take-off	5th take-off	6th take-off
	Expt.	Expt.	Expt.	Expt.
1.02	113	142	95	130
1.04	88	108	71	96
1.06	70	78	54	61
1.08	43	60	36	37
1.10	29	46	27	21
1.12	18	30	18	10
1.14	15	24	12	4
1.16	12	18	8	2
1.18	4	14	5	2
1.20	0	11	3	1
1.22	-	8	1	1
1.24	-	6	-	1
1.26	-	6	-	1
1.28	-	5	-	-
1.30	-	5	-	-
1.32	-	5	-	-
1.34	-	4	-	-
1.36	-	3	-	-
1.38	-	3	-	-
1.40	-	2	-	-
1.42	-	1	-	-
1.44	-	1	-	-
1.46	-	1	-	-
1.48	-	-	-	-

Table 2

PEAK EXCEEDANCES OF STATED RESPONSE LEVELS TO PORT - COCKPIT LATERAL

Peak level g absolute	3rd take-off	4th take-off	5th take-off	6th take-off
	Expt.	Expt.	Expt.	Expt.
0.02	30	30	30	57
0.04	2	3	2	12
0.06	0	0	0	0

Table 3

PEAK EXCEEDANCES OF STATED POSITIVE RESPONSE LEVELS - cg VERTICAL

Peak level g absolute	3rd take-off	4th take-off	5th take-off	6th take-off
	Expt.	Expt.	Expt.	Expt.
1.02	68	90	67	79
1.04	37	47	23	29
1.06	28	30	7	11
1.08	17	17	2	2
1.10	14	7	0	0
1.12	9	3	0	0
1.14	5	0	0	0
1.16	2	0	0	0
1.18	0	0	0	0

WEIGHTS FOR THE MIN-MAX MOVING AVERAGE FILTER, K = 100

Weights for negative k's are obtained by $b_{-k} = b_k$

k = 0	0 008540	51	0 005002
1	0 008538	52	0 004891
2	0 008536	53	0 004782
3	0 008526	54	0 004672
4	0 008514	55	0.004562
5	0 008500	56	0 004451
6	0 008483	57	0 004343
7	0 008460	58	0 004235
8	0.008435	59	0 004125
9	0 008408	60	0 004016
10	0.008378	61	0.003910
11	0.008343	62	0 003804
12	0.008304	63	0 003697
13	0 008265	64	0 003591
14	0.008222	65	0 003487
15	0 008175	66	0 003384
16	0 008125	67	0 003280
17	0.008073	68	0.003177
18	0.008019	69	0 003077
19	0 007959	70	0.002977
20	0 007897	71	0 002877
21	0 007834	72	0 002777
22	0.007769	73	0 002681
23	0.007699	74	0.002585
24	0 007626	75	0 002489
25	0 007553	76	0 002393
26	0 007477	77	0 002302
27	0 007397	78	0 002212
28	0 007314	79	0 002122
29	0 007232	80	0 002034
30	0 007147	81	0 001949
31	0.007058	82	0 001868
32	0 006967	83	0.001786
33	0 006876	84	0 001707
34	0 006783	85	0 001632
35	0 006687	86	0 001560
36	0 006589	87	0.001489
37	0.006491	88	0.001420
38	0 006392	89	0 001355
39	0 006290	90	0 001291
40	0 006186	91	0 001225
41	0 006083	92	0 001159
42	0 005979	93	0.001094
43	0 005872	94	0 001029
44	0 005764	95	0 000955
45	0 005658	96	0 000894
46	0 005551	97	0.000879
47	0.005441	98	0 000865
48	0 005331	99	0 001380
49	0 005222	100	0 002720
50	0 005113		

Table 4

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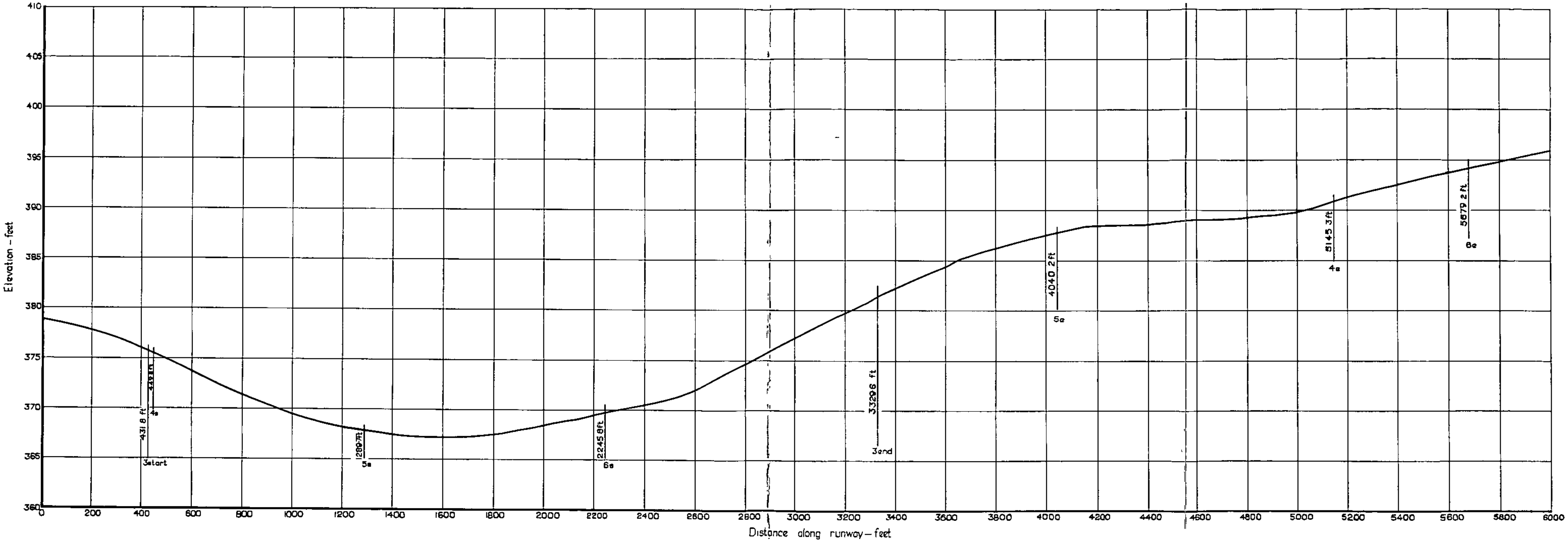

Fig. 1 VC-10 tests at Boscombe Down – runway profile

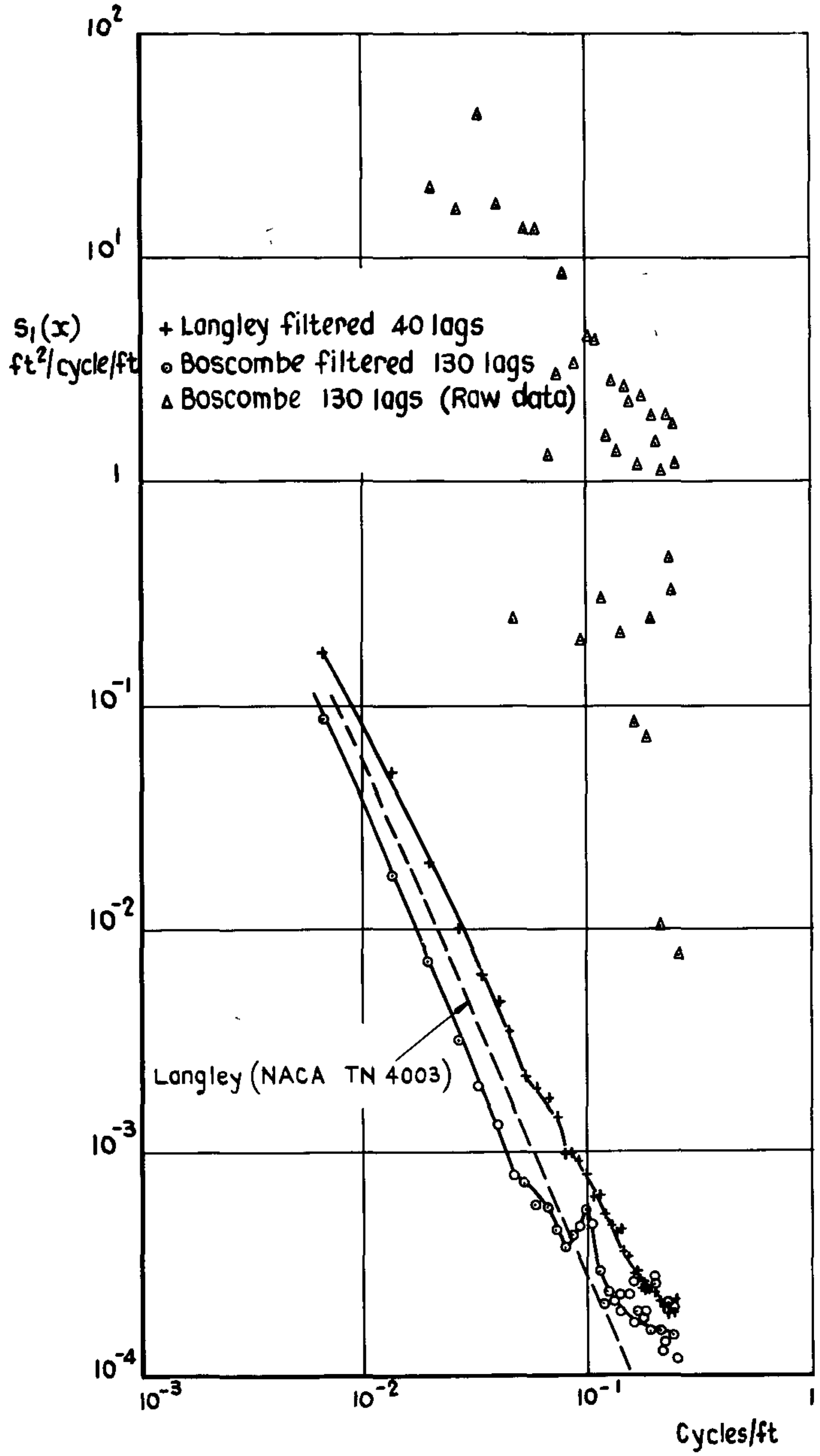

Fig. 2 Spectral density estimates of the Boscombe Down & Langley profiles

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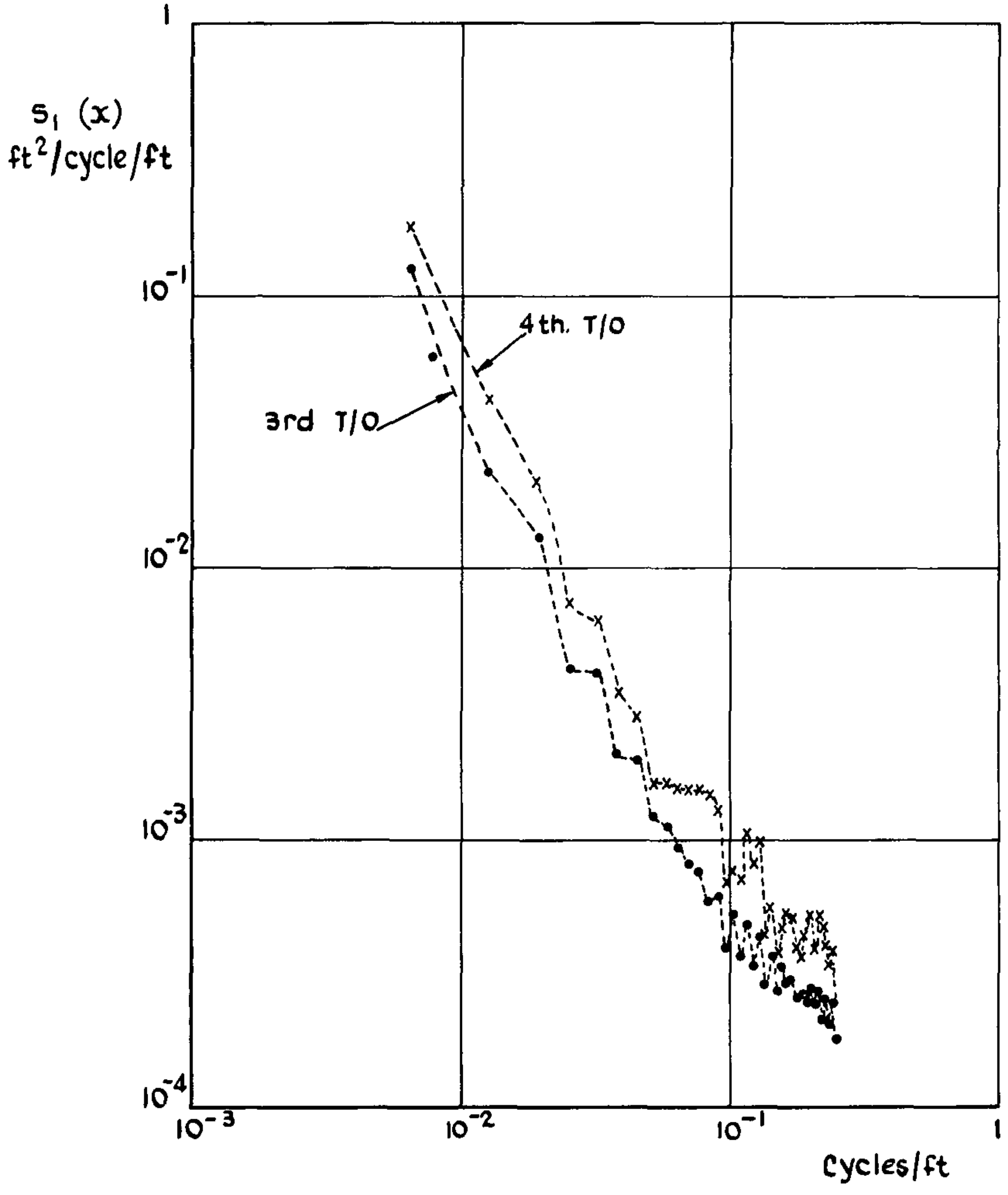

Fig. 3 Spectral density estimates of parts of the Boscombe runway from which take offs were effected

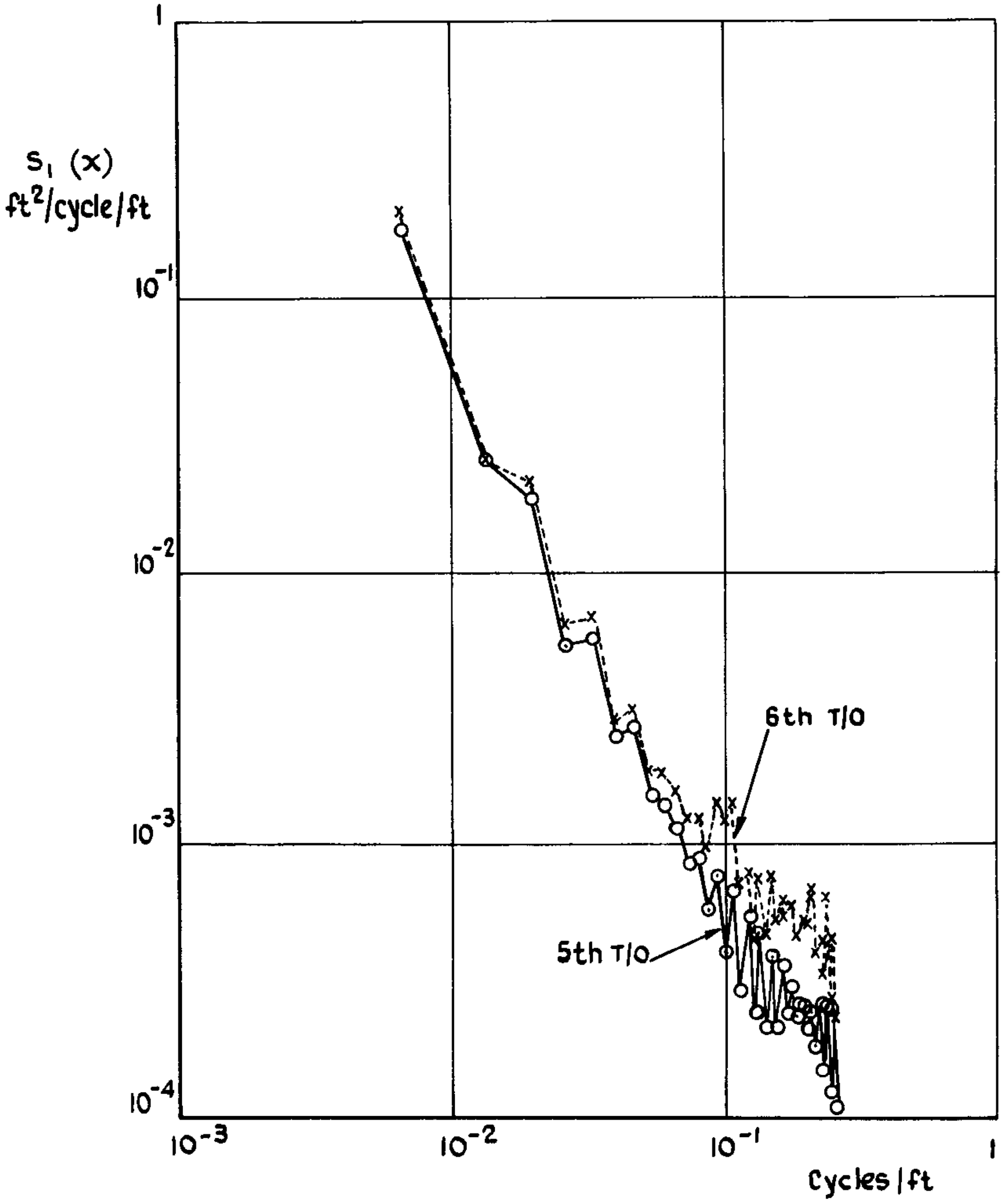

Fig 4 Spectral density estimates of parts of the Boscombe runway from which take offs were effected

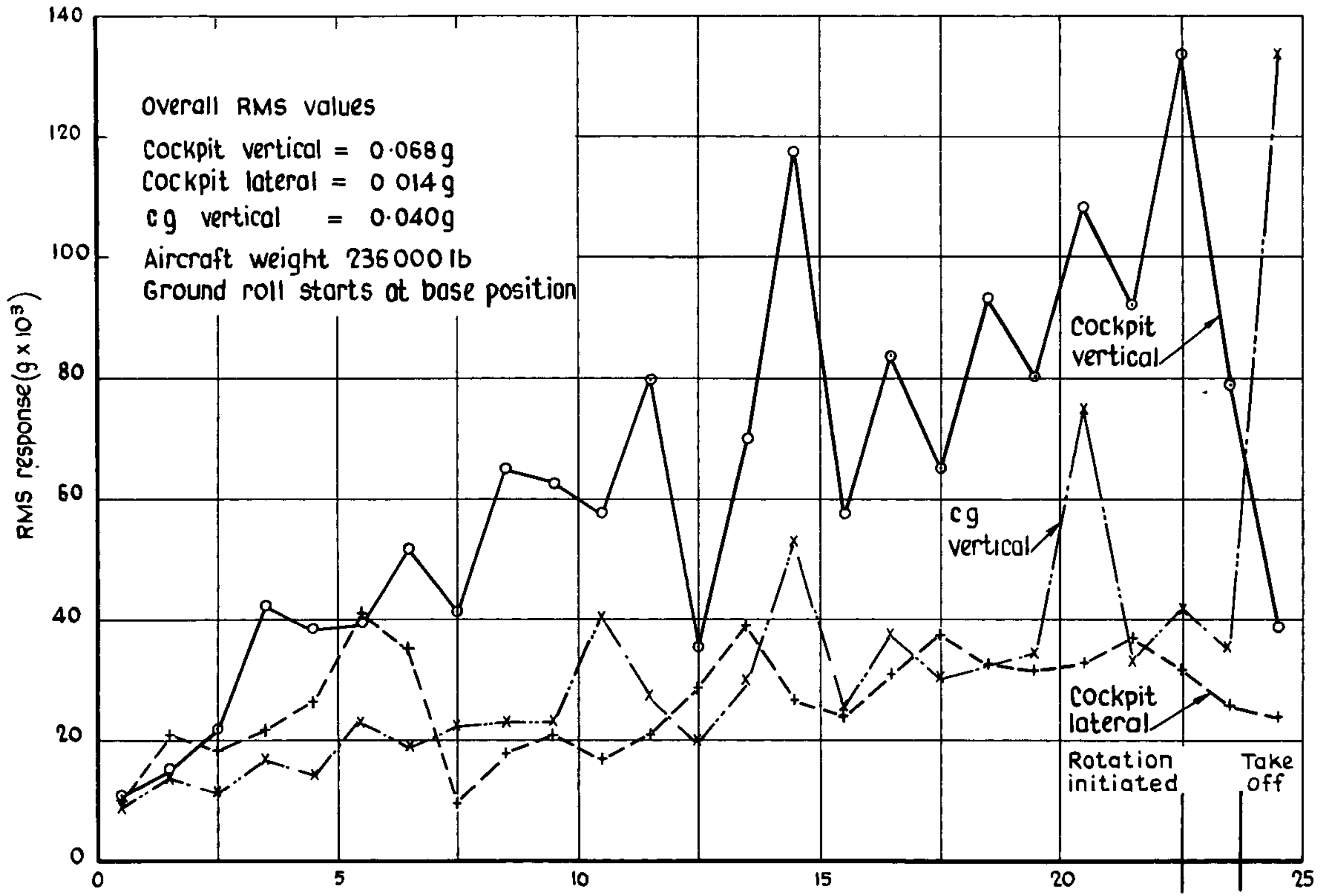

Fig. 5 The RMS value of response developed during each second of take off. 3rd take off

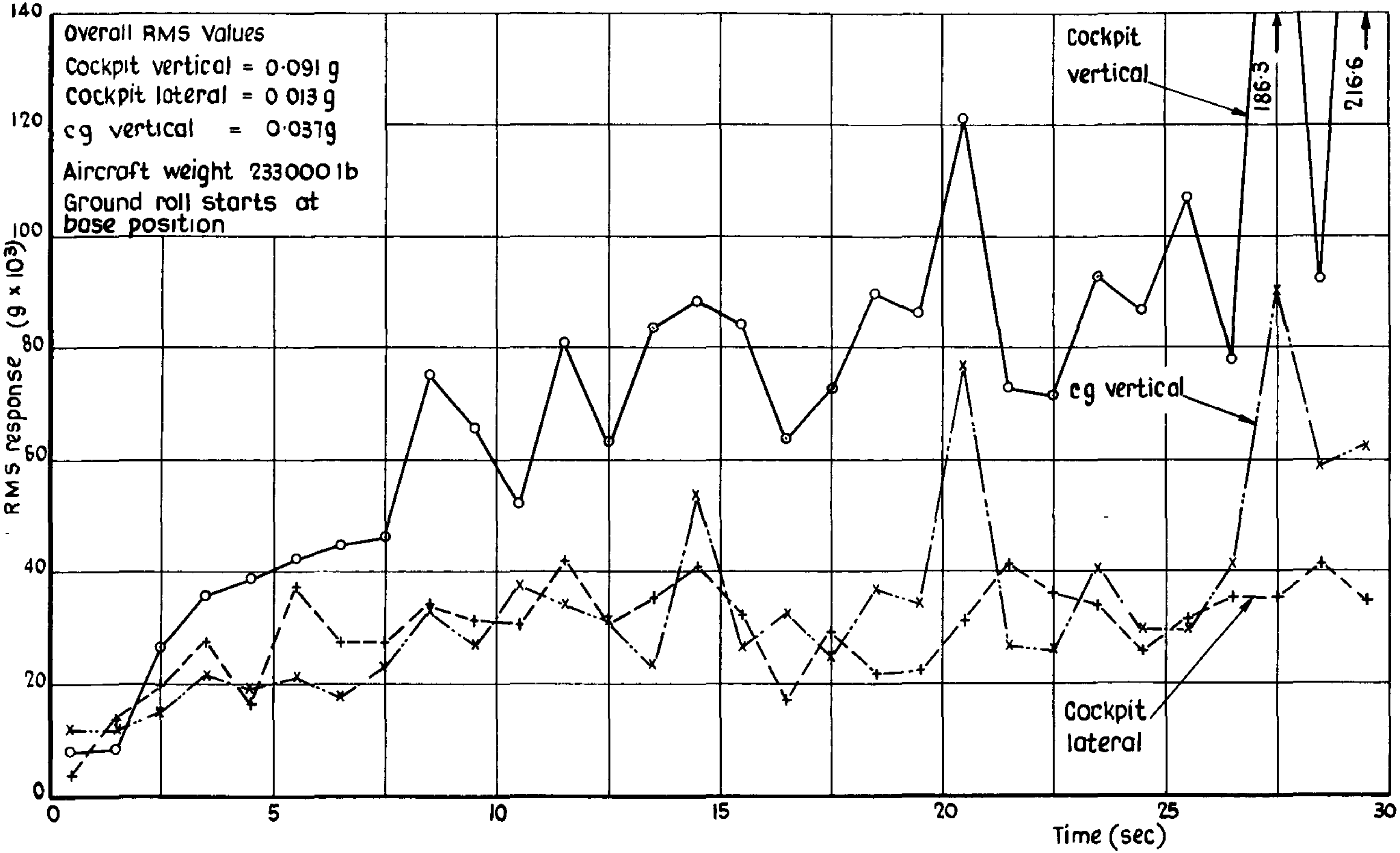

Fig. 6 The RMS values of response developed during each second of take off. 4th take off

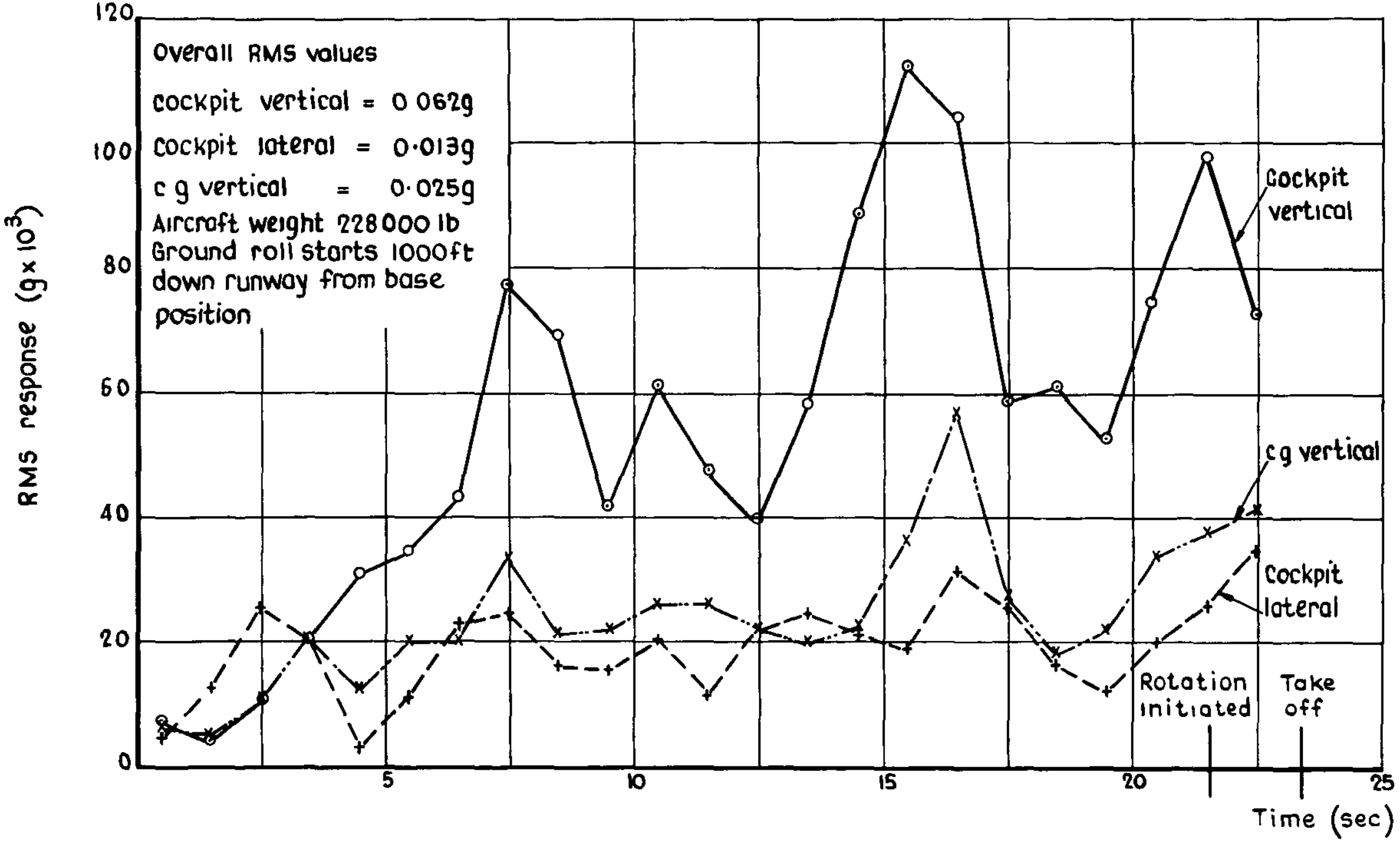

Fig. 7 The RMS values of response developed during each second of take off. 5th take off

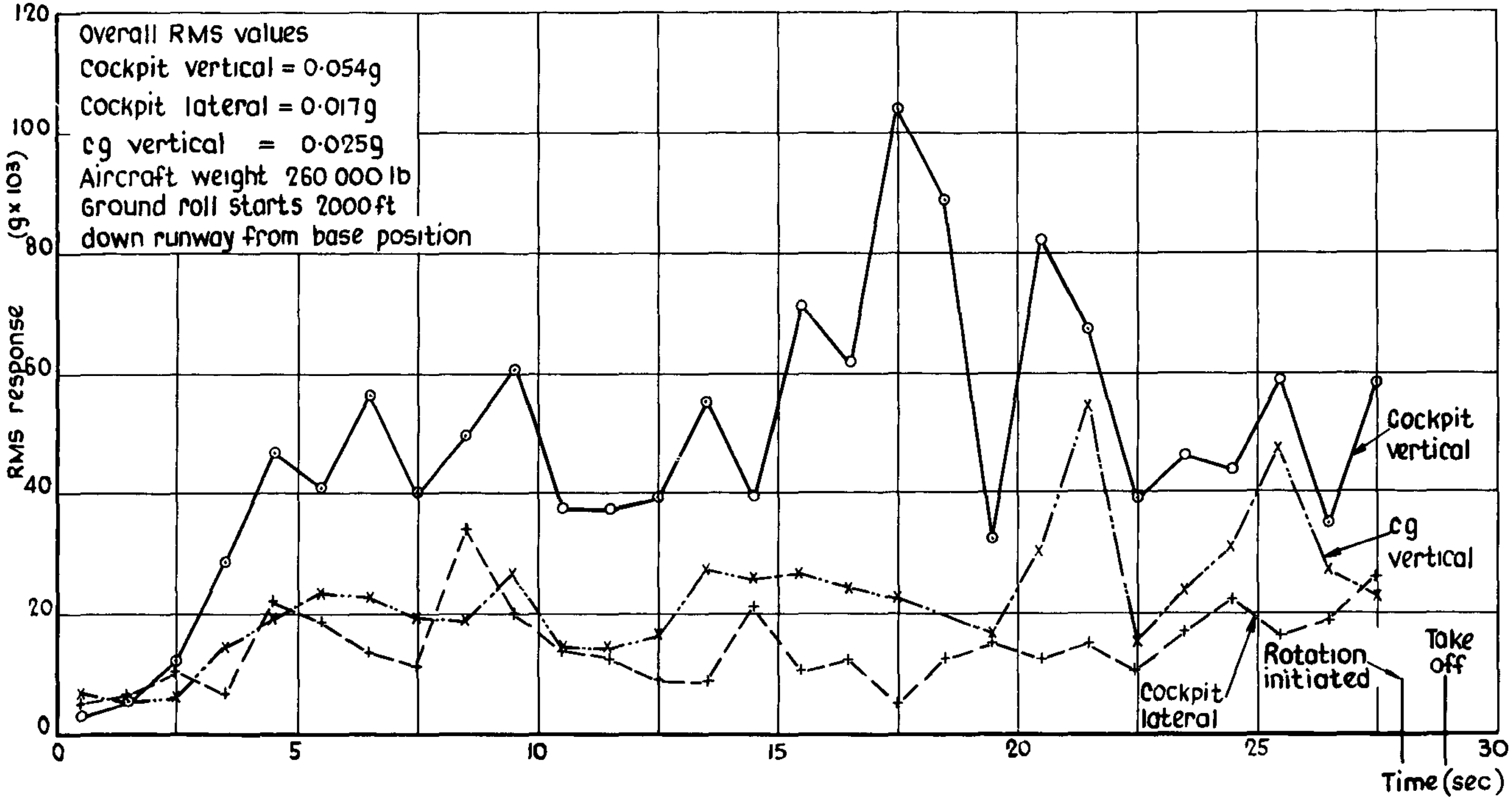

Fig. 8 The RMS values of response developed during each second of take off. 6th take off

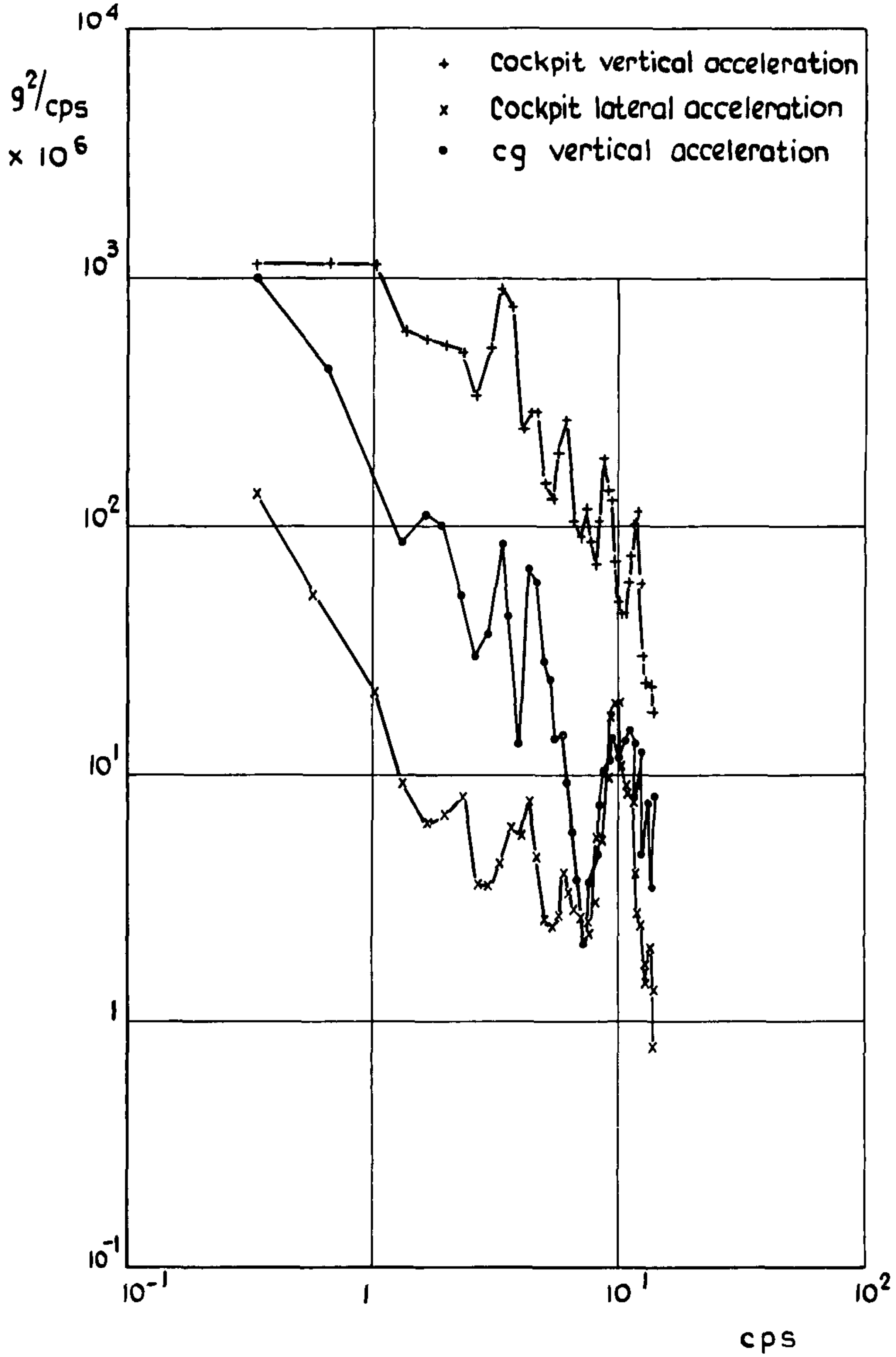

Fig. 9 Spectra of measured responses, 3rd take off

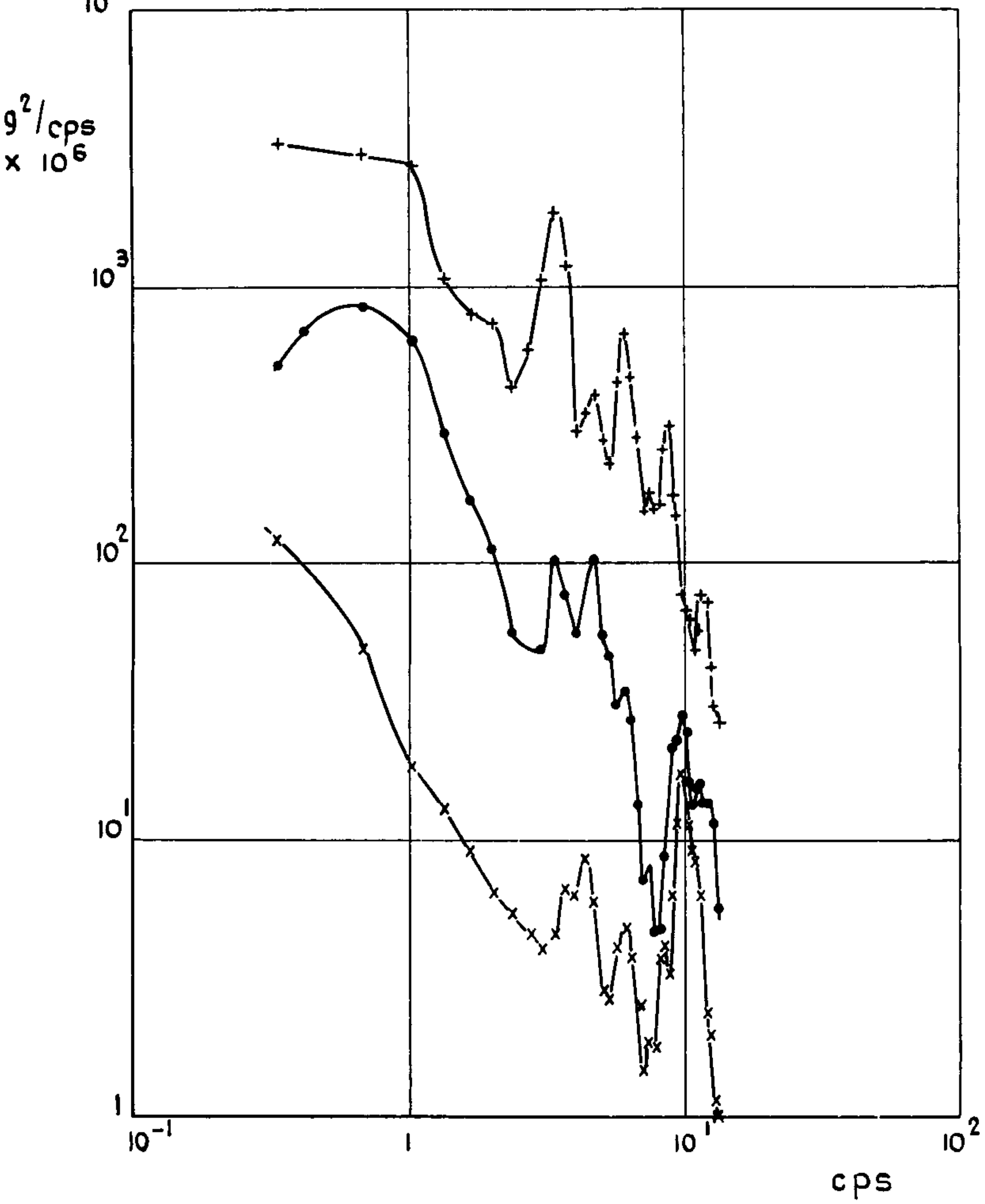

Fig. 10 Spectra of measured responses, 4th take off

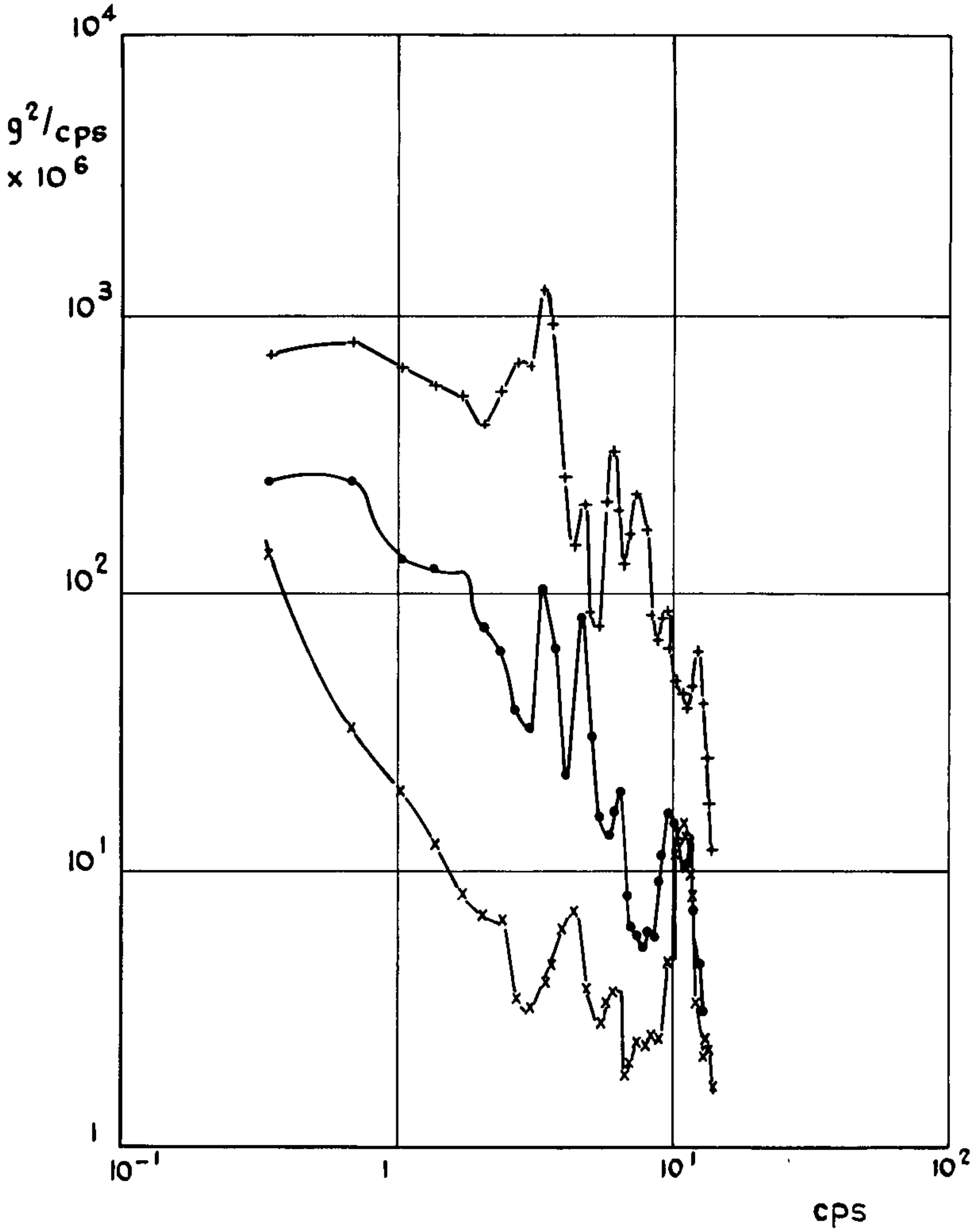

Fig. 11 Spectra of measured responses, 5th take off

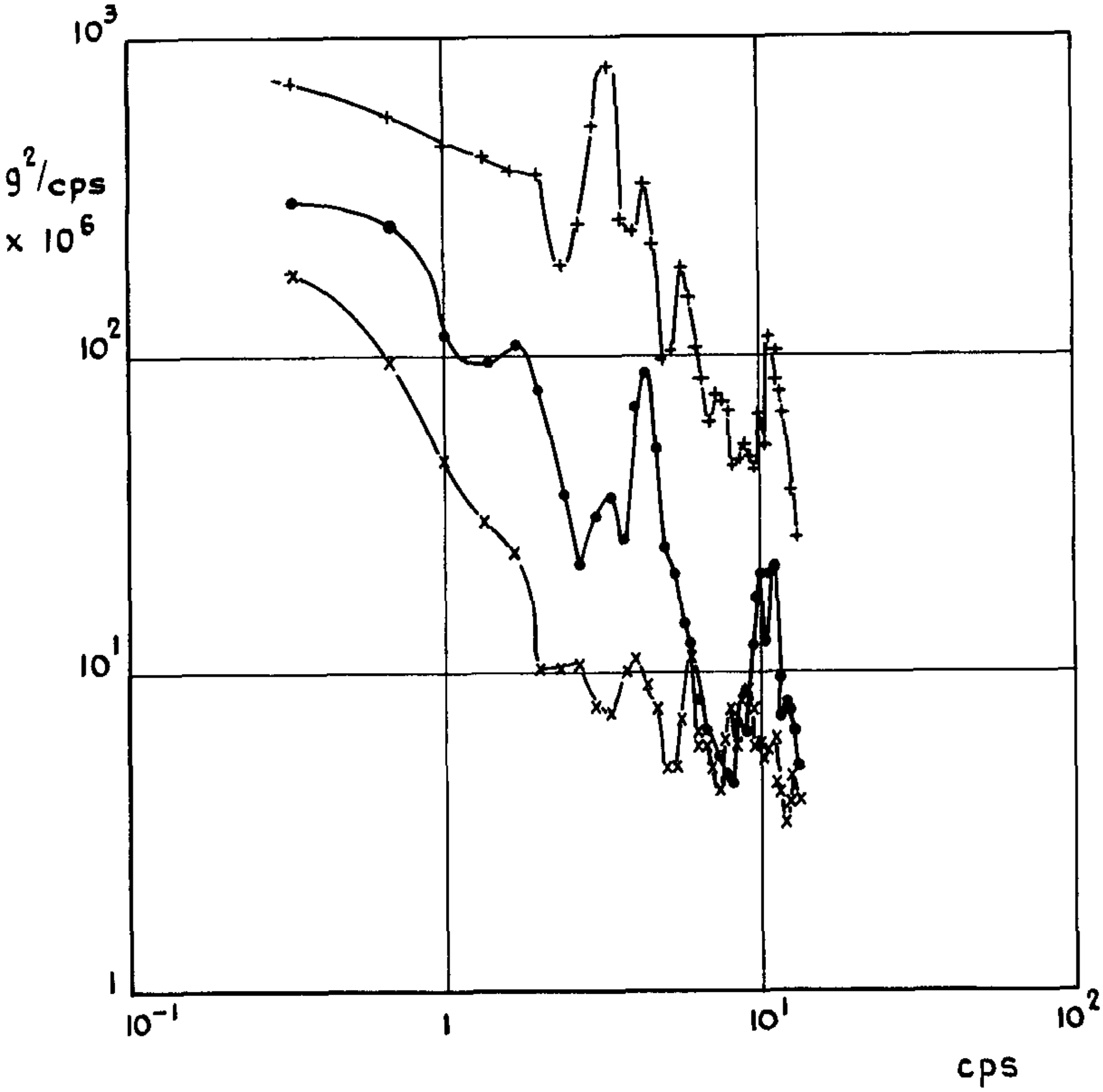

Fig. 12 Spectra of measured responses, 6th take off

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