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Some Curves for use in Calculations of the Performance of Conical Centrebody Intakes at Supersonic Speeds and at Full Mass Flow

By

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ROYAL AIRCRAFT ESTABLISHMENT

Some curves for use in calculations of the performance of conical centrebody intakes at supersonic speeds and at full mass flow

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SUMMARY

This note presents curves of a number of parameters which frequently occur in calculations of the performance of conical centrebody intakes at supersonic speeds and full mass flow. These parameters follow directly from the theoretical supersonic flow past cones. Semi-cone angles of 15°, 20°, 22.5°, 25°, and 30° are considered, and the range of free stream Mach number is approximately 1.3 to 3.0.

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1 Introduction and Description of Curves

This note presents curves of a number of parameters which frequently occur in calculations of the performance of conical centrebody intakes at supersonic speeds. The curves apply only to the full-mass-flow condition for a given geometry and Mach number, and contain nothing that is new in principle; they follow directly from the Taylor-Maccoll solutions for the supersonic flow past cones which have been tabulated in ref. 1.

The following parameters are plotted.

(i) The inclination of the cone shock wave θ_w, plotted as a function of Mach number (M_∞ = 1.3 to 3.0) for a range of cone angles (θ_c = 15, 20, 22.5, 25, 30 degrees).

(ii) The 'pre-entry' drag coefficient*, $C_{D_{pre\,o}}$, which is due to pressures acting on the boundary of the entering stream tube between the cone shock and the inlet section. (It is assumed that the second shock is attached to the cowl lip.) $C_{D_{pre\,o}}$ is thus defined by

$$C_{D_{pre\,o}} = \frac{1}{A_1} \int_{R_w}^{R_\ell} C_p \quad 2\pi R \quad dR \tag{1}$$

(It should be noted that the reference area here is A_1, and not the maximum cross-section area of the intake.)

(iii) The full mass flow ratio A_∞/A_1.

(iv) The angle of the flow (with respect to the intake axis), the Mach number, and the pressure coefficient at points in the conical flow field. These are denoted by λ, M, C_p respectively.

λ and M are included because their values immediately ahead of the cowl lip must be known if the lip is to be designed for an attached shock. If C_p is also known, the initial pressure on the cowl may be determined exactly by means of oblique shock or Prandtl-Meyer expansion tables.

2 Details of the Calculations

In the polar coordinates (r, θ) of Fig. 1 the equation of a streamline is

$$\frac{1}{r}\,\frac{dr}{d\theta} = \frac{u}{v}$$

i.e.

$$\log r = \int_{\theta_1}^{\theta} \frac{u}{v}\, d\theta + \log k, \tag{2}$$

---

* This quantity must be included in the drag if the conventional definition is used for the thrust of a turbojet or ramjet, because this definition of thrust considers the change in momentum between the exit of the engine and the undisturbed stream ahead of the intake.

where θ_1 is some fixed reference value such that $\theta_c < \theta_1 \leqslant \theta \leqslant \theta_w$, and k is a constant to be determined. Putting $r = r_w$ when $\theta = \theta_w$ we obtain

$$\log \frac{r}{r_w} = \int_{\theta}^{\theta_w} \frac{u}{-v}\, d\theta ,$$

and from $r = R/\sin\theta$, $r_w = R_\infty/\sin\theta_w$, where R is the radius from the intake axis we have

$$\frac{R}{R_\infty} = \frac{\sin\theta}{\sin\theta_w} \cdot \exp\left[\int_{\theta}^{\theta_w} \frac{u}{-v}\, d\theta\right] . \tag{3}$$

Values of R/R_∞ were calculated by evaluating the integral in (3) numerically using the values of u and v tabulated in ref. 1 and increments of θ of two degrees.

The pressure at the centre of each increment of streamline was then calculated from

$$\frac{p}{p_{tw}} = \left(\frac{2a^2}{\gamma - 1}\right)^{\frac{\gamma}{\gamma - 1}} \tag{4}$$

where a is the ratio of the local velocity of sound to the limiting velocity (or the velocity 'into a vacuum'), and is tabulated in ref. 1.

The local Mach number and flow inclination were calculated from

$$M^2 = \frac{1}{a^2} - \frac{2}{\gamma - 1} , \tag{5}$$

and

$$\lambda = \theta + \tan^{-1}\frac{v}{u} . \tag{6}$$

The ratios

$$\frac{p_{tw}}{p_\infty} = \frac{p_{tw}}{p_{t_\infty}} \Big/ \frac{p_\infty}{p_{t_\infty}} \quad \text{and} \quad \frac{p_\infty}{q_\infty} \tag{7}$$

were calculated from the tables of ref. 2; p_{tw}/p_{t_∞} was found by taking the component of Mach number normal to the shock and using normal shock wave tables.

$C_{D_{pre\ o}}$ was calculated from the relation

$$C_{D_{pre\ o}} = \frac{p_\infty}{q_\infty} \left(\frac{R}{R_1}\right)^2 \int_{R_c}^{R_1} \left[\frac{p}{p_{tw}} \frac{p_{tw}}{p_\infty} - 1 \right] d\left[\frac{R^2}{R_\infty{}^2}\right] \qquad (8)$$

the integral being again evaluated numerically with increments of θ of two degrees

3 The Effect of Different values of γ

The value of γ in ref 1 is 1 405, this value was therefore used throughout the work, with the exception of the ratio p_{tw}/p_∞, which for convenience was calculated from tables with $\gamma = 1\ 400$

When the work had been completed data became available* which made possible an approximate check on the effect of different values of γ

The comparisons below were made for fixed values of M_∞, θ_c and θ They can only be regarded as approximate estimates because they are the result of considerable interpolation, and of numerical integration in the case of $C_{D_{pre\ o}}$ and A_∞/A_1 However they do show that, as might be expected, the effect of using different values of γ is not a serious one

TABLE I

Maximum differences
(Values with γ = 1 405 throughout) - (values of this note)

θ_c	M_∞	Differences	
		C_p	$C_{D_{pre\ o}}$
15	1 34	0 002	0 0006
	2 59	-0 001	-0 0001
30	1 52	0 003	0 0016
	3 17	-0 002	-0 0009

---

* The author is indebted to Dr C W Jones of the University of Liverpool for providing data on a number of cone flows with γ = 1 400

TABLE II

Maximum differences:
(Values with $\gamma = 1.405$ throughout) - (values with $\gamma = 1.400$ throughout)

θ_c	M_∞	Differences					
		θ_w	λ	M	C_p	$C_{D_{pre\ o}}$	$\frac{A_\infty}{A_1}$
15	1.34	0.02	0.1	0.006	0.004	0.001	0.002
	2.59	0.01	-0.1	0.003	0.002	0.000	0.004
30	1.52	0.23	0.2	-0.02	0.009	0.009	-0.005
	3.17	0.04	-0.1	-0.01	0.005	0.003	-0.002

NOTATION

(See also Fig. 1)

a	ratio of the velocity of sound to the limiting velocity
A	total cross-section area
C_p	pressure coefficient $(p - p_\infty)/\frac{1}{2}\rho_\infty V_\infty^2$
M	Mach number
p	static pressure
p_t	total head pressure
q	dynamic pressure $\frac{1}{2}\rho V^2$
r	radial coordinate (from the vertex of the cone)
R	radius (from the axis of the intake)
u,v	components of velocity in the directions of increasing r and θ
γ	ratio of the specific heats of air
θ	inclination with respect to the intake axis of a ray through the vertex of the cone
λ	inclination of a streamline with respect to the intake axis
$(\ )_\infty$	in the free stream
$(\ )_w$	immediately behind the cone shock
$(\ )_1$	at the inlet section
$(\ )_\ell$	immediately ahead of the cowl lip
$(\ )_c$	on the cone

LIST OF REFERENCES

No.	Author	Title, etc.
1	Z. Kopal	Tables of supersonic flow around cones MIT Tech. Report 1, 1947
2	–	Notes and tables for use in the analysis of supersonic flow NACA Tech. Note No. 1428, 1947

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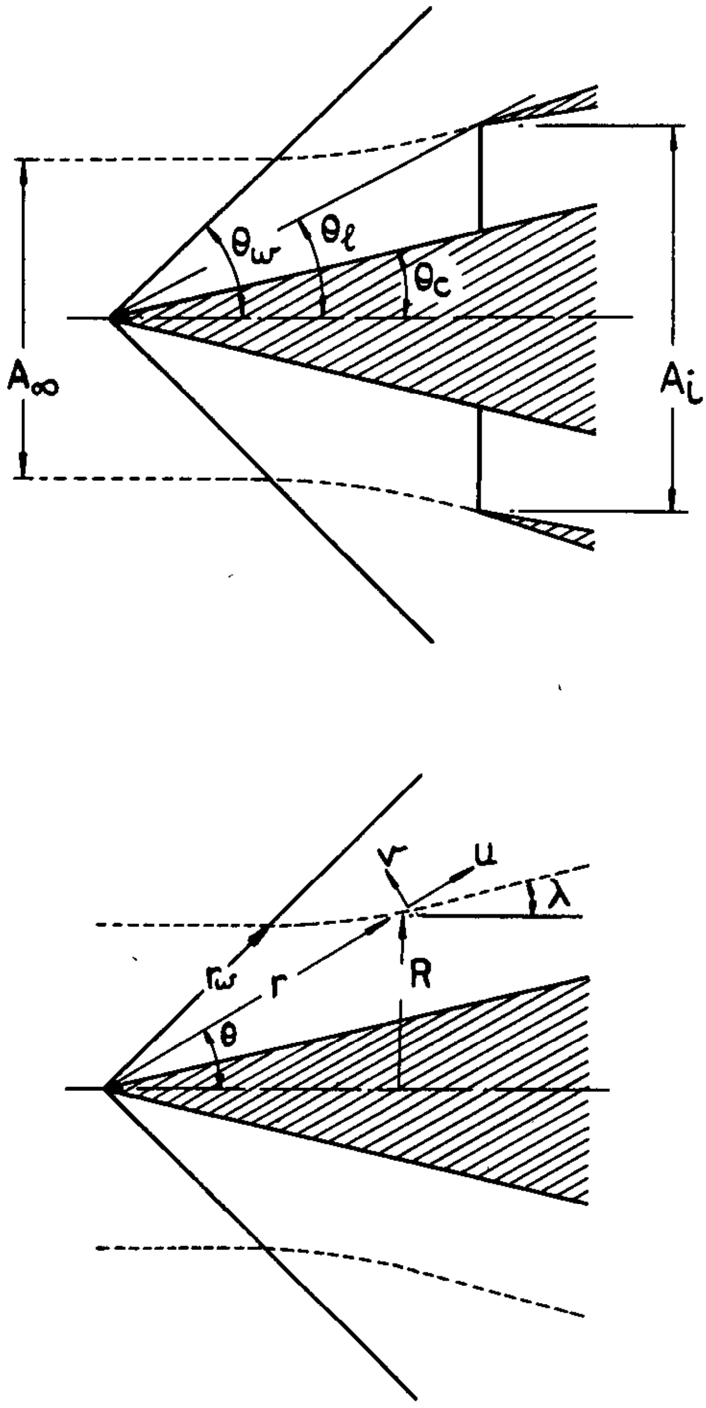


FIG.1. NOTATION.

FIG.2.

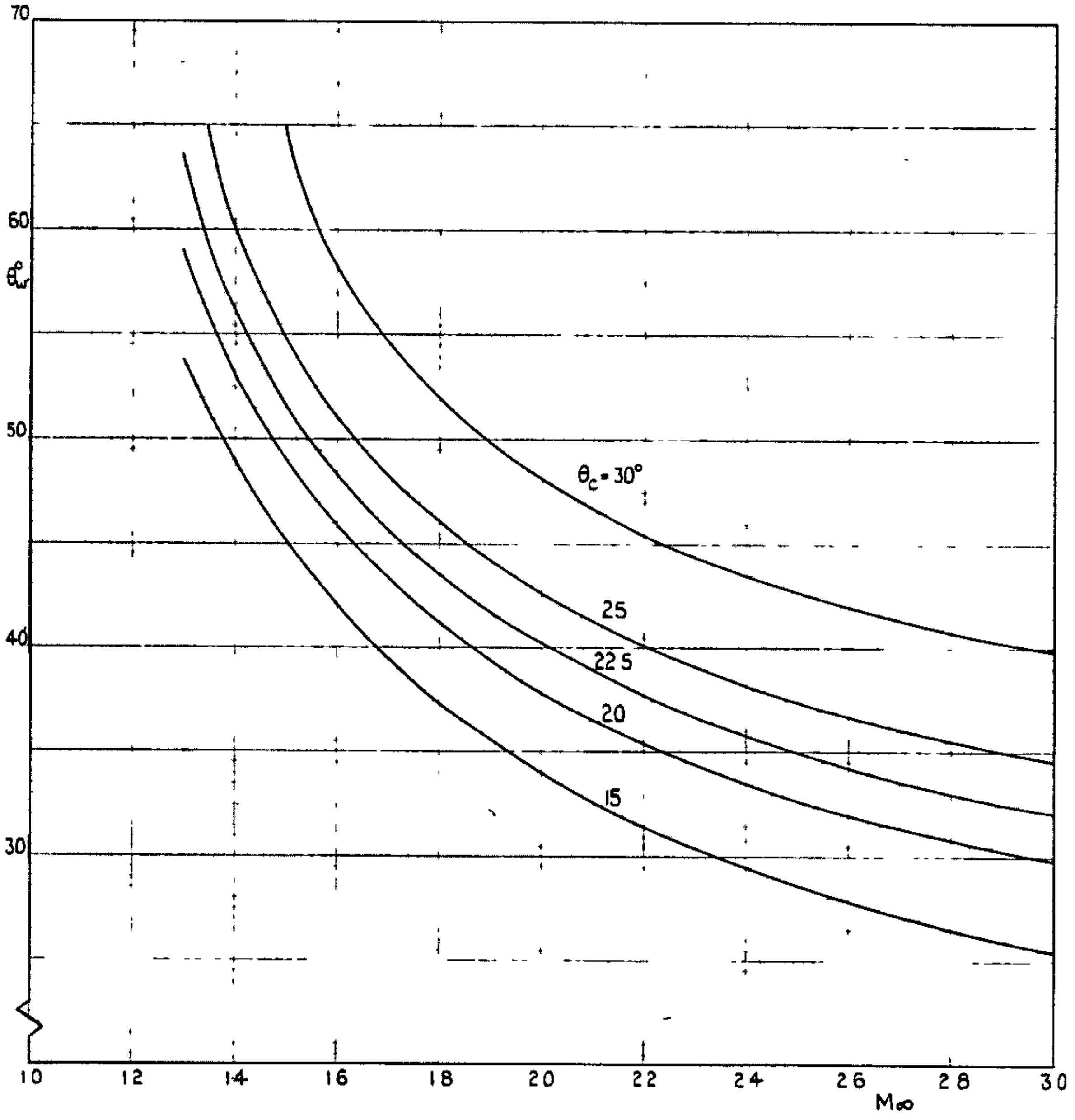
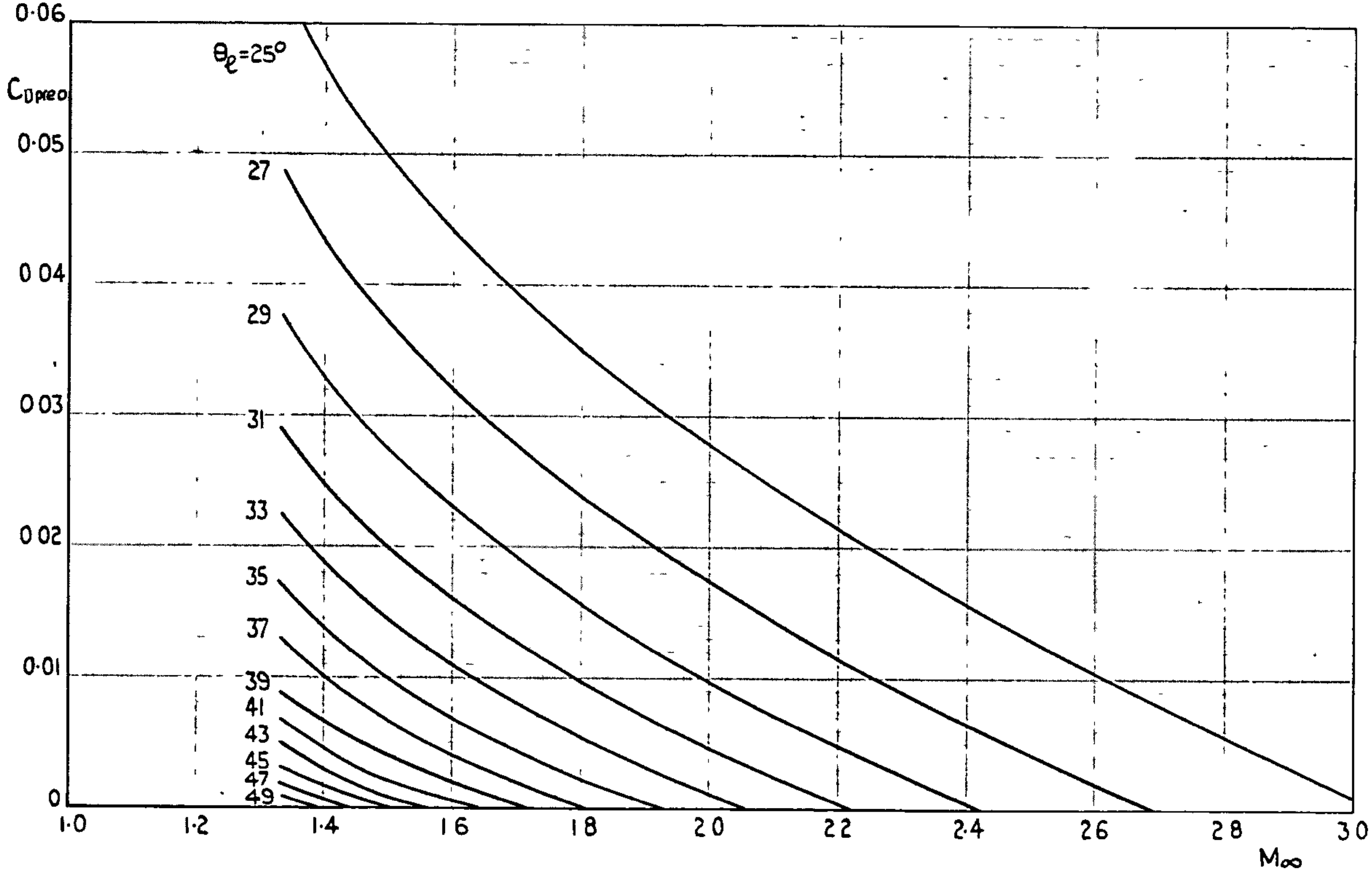


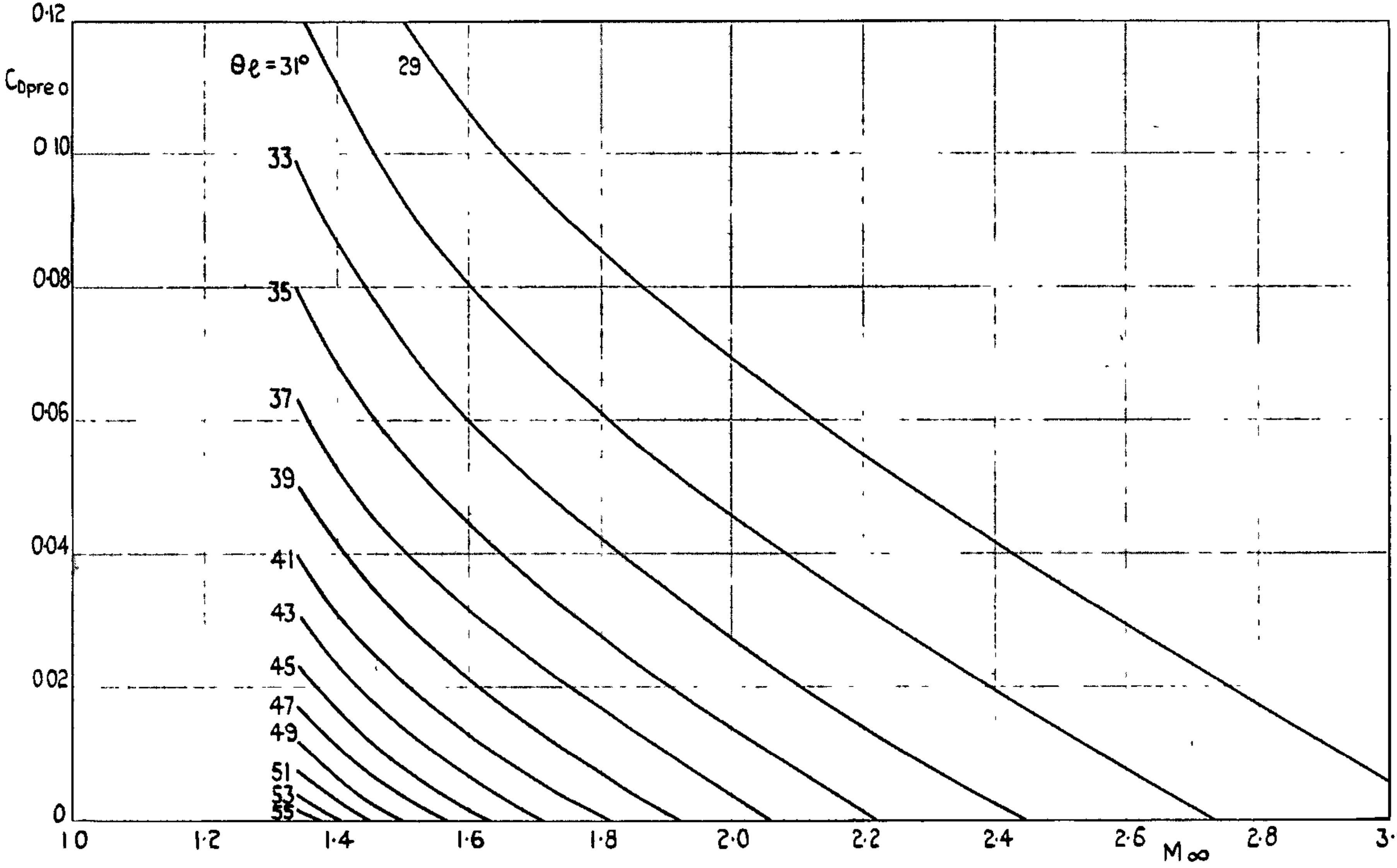
FIG.2. CURVES OF θ_w.



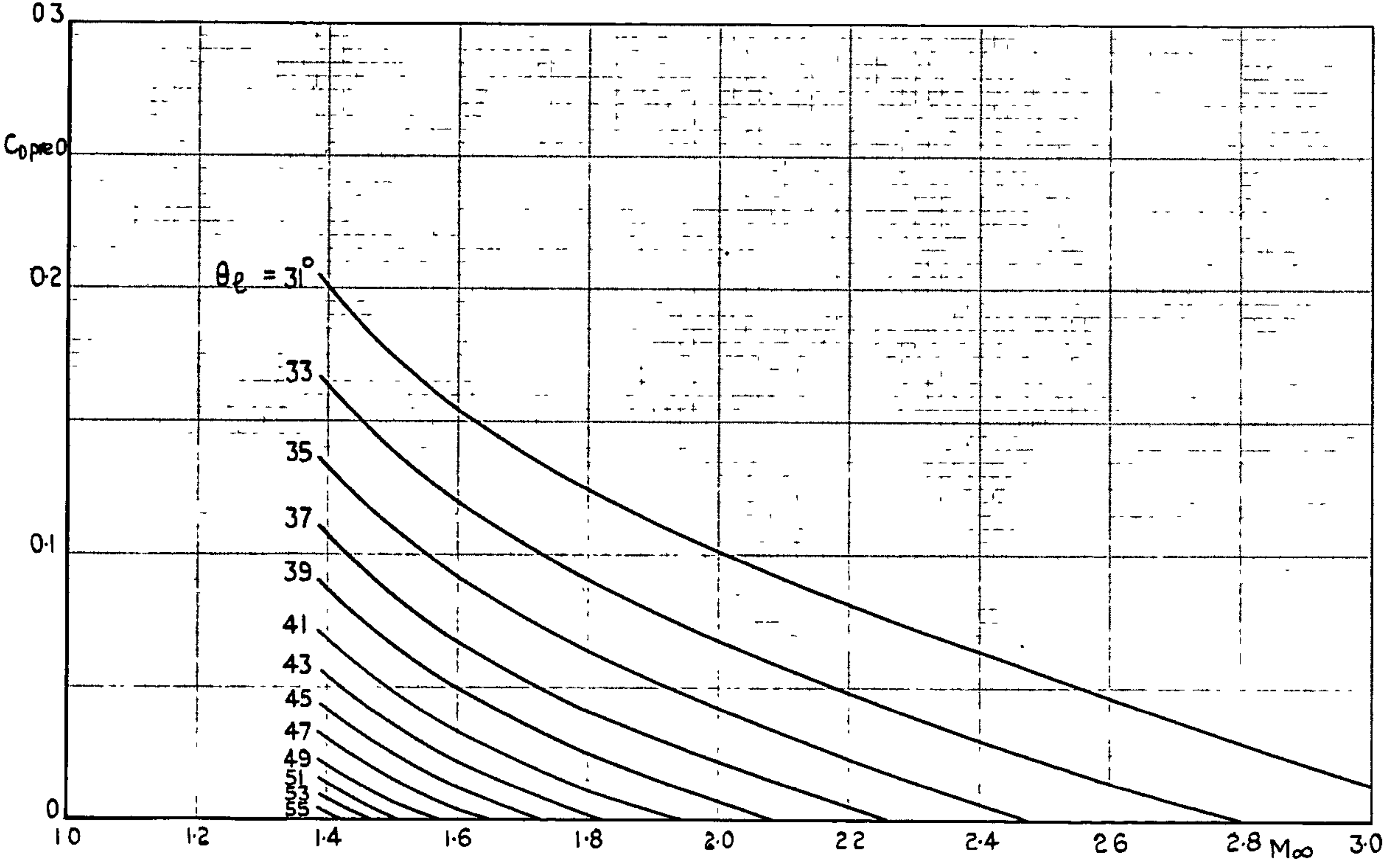
CURVES OF $C_{D\,pre\,o}$ FOR $\theta_c=15^\circ$

FIG. 3a.

FIG. 3b.



CURVES OF C_D pre o FOR $\theta_c = 20°$.



CURVES OF C_D pre o FOR $\theta=22.5°$

FIG. 3c.

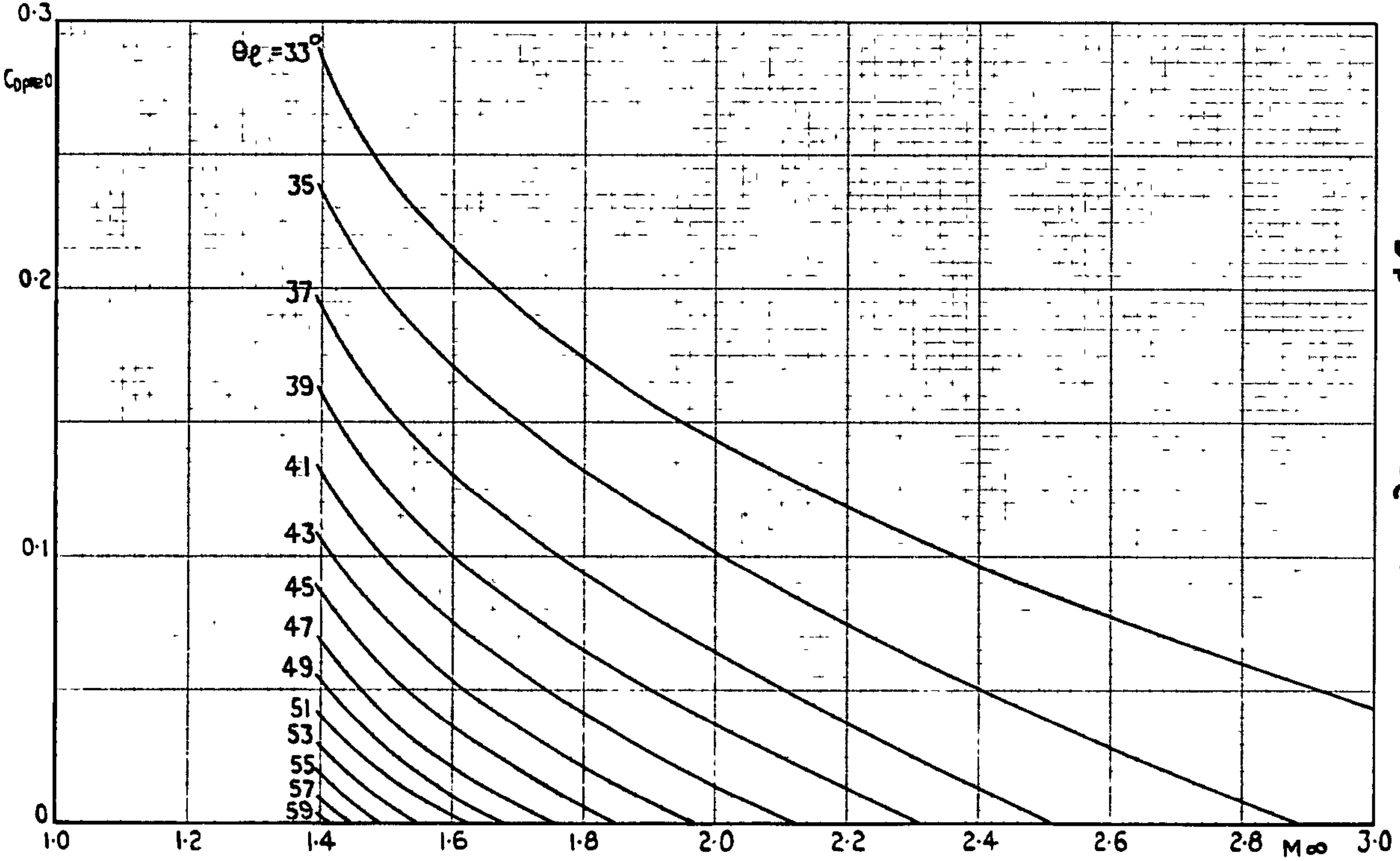
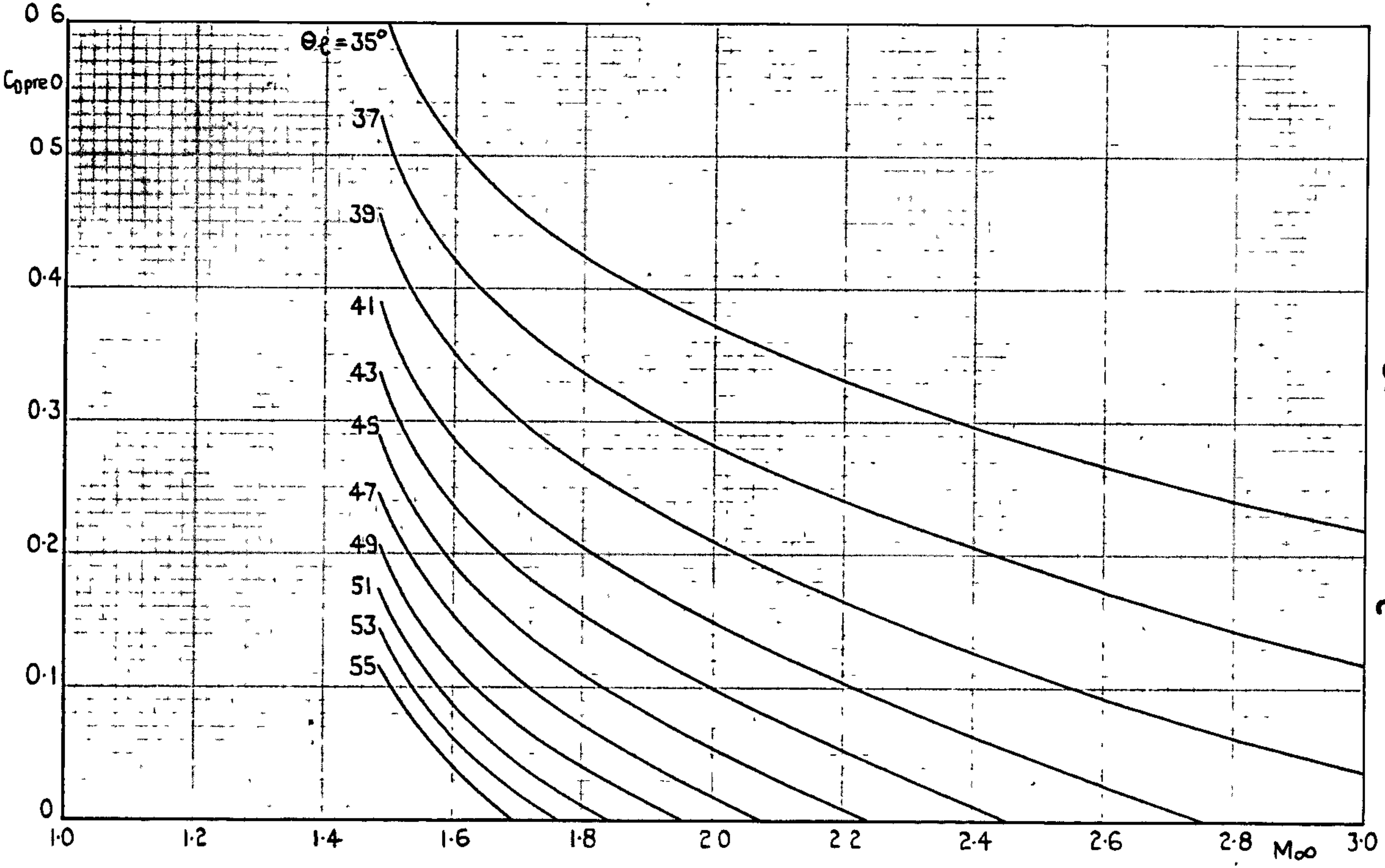


FIG. 3d.

CURVES OF C_{D} pre o FOR $\theta_c = 25^\circ$



CURVES OF $C_{D\,pre\,0}$ FOR $\theta_c = 30°$.

FIG. 3e.

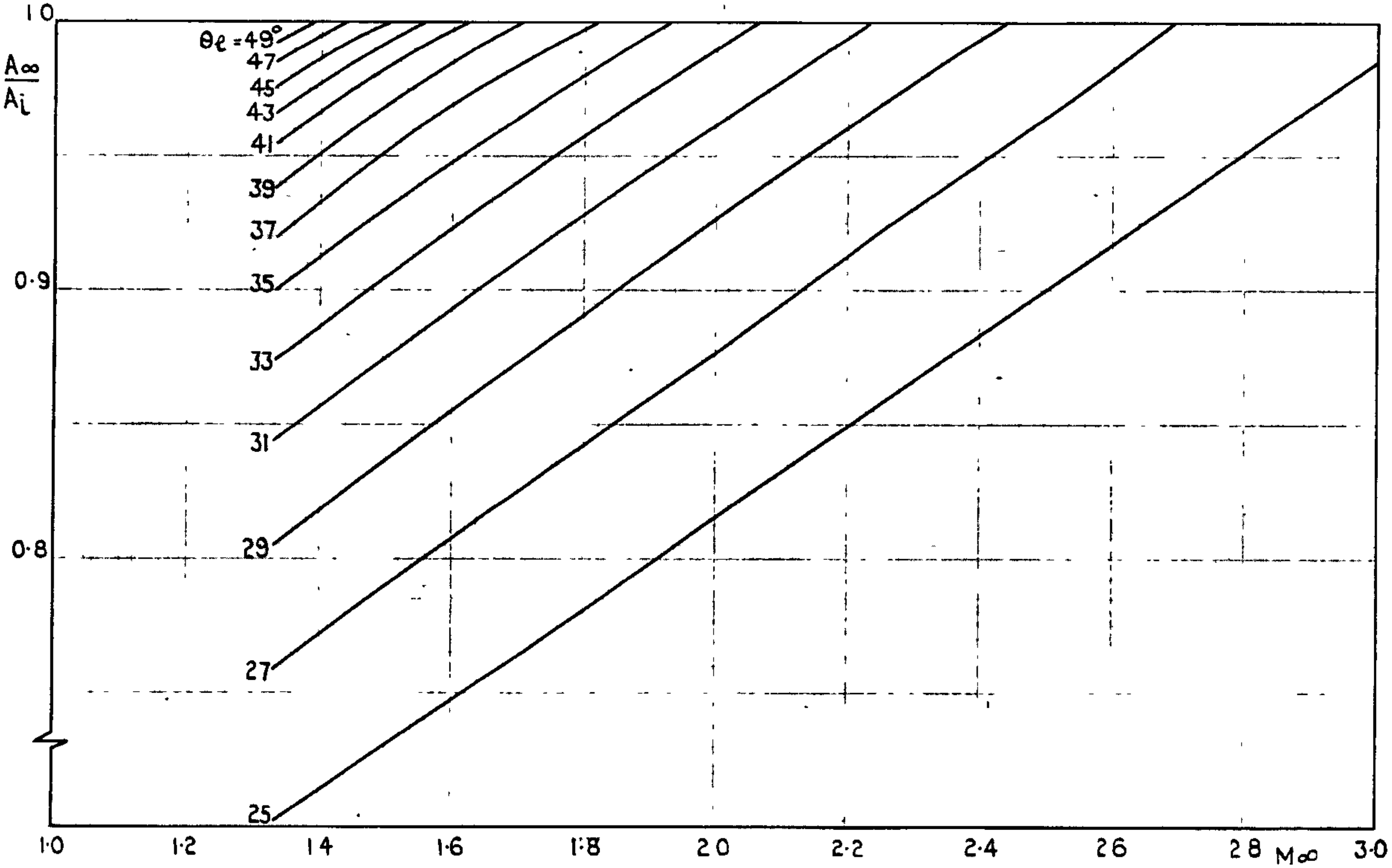
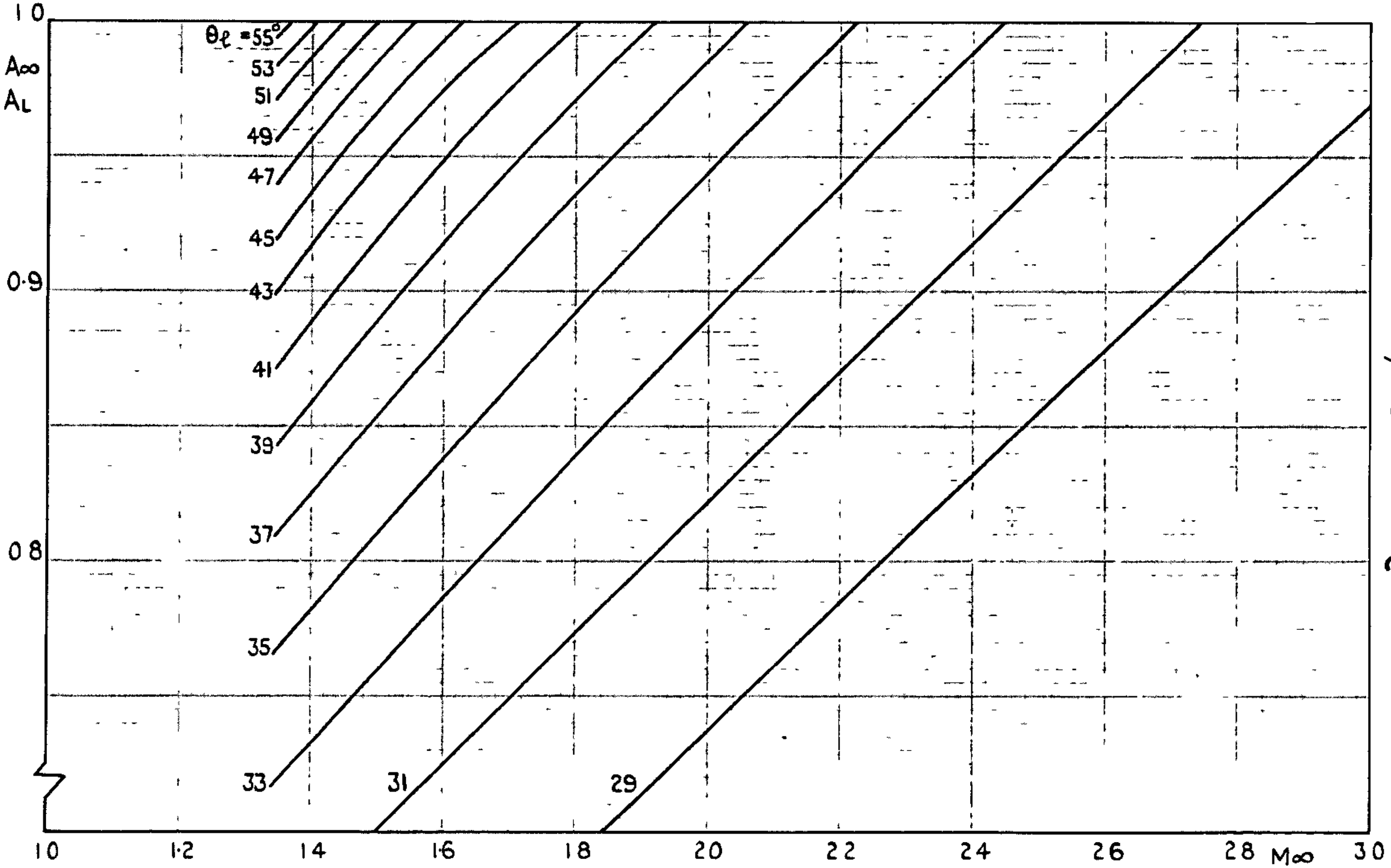


FIG. 4a.

CURVES OF A_∞/A_i FOR $\theta_c = 15°$

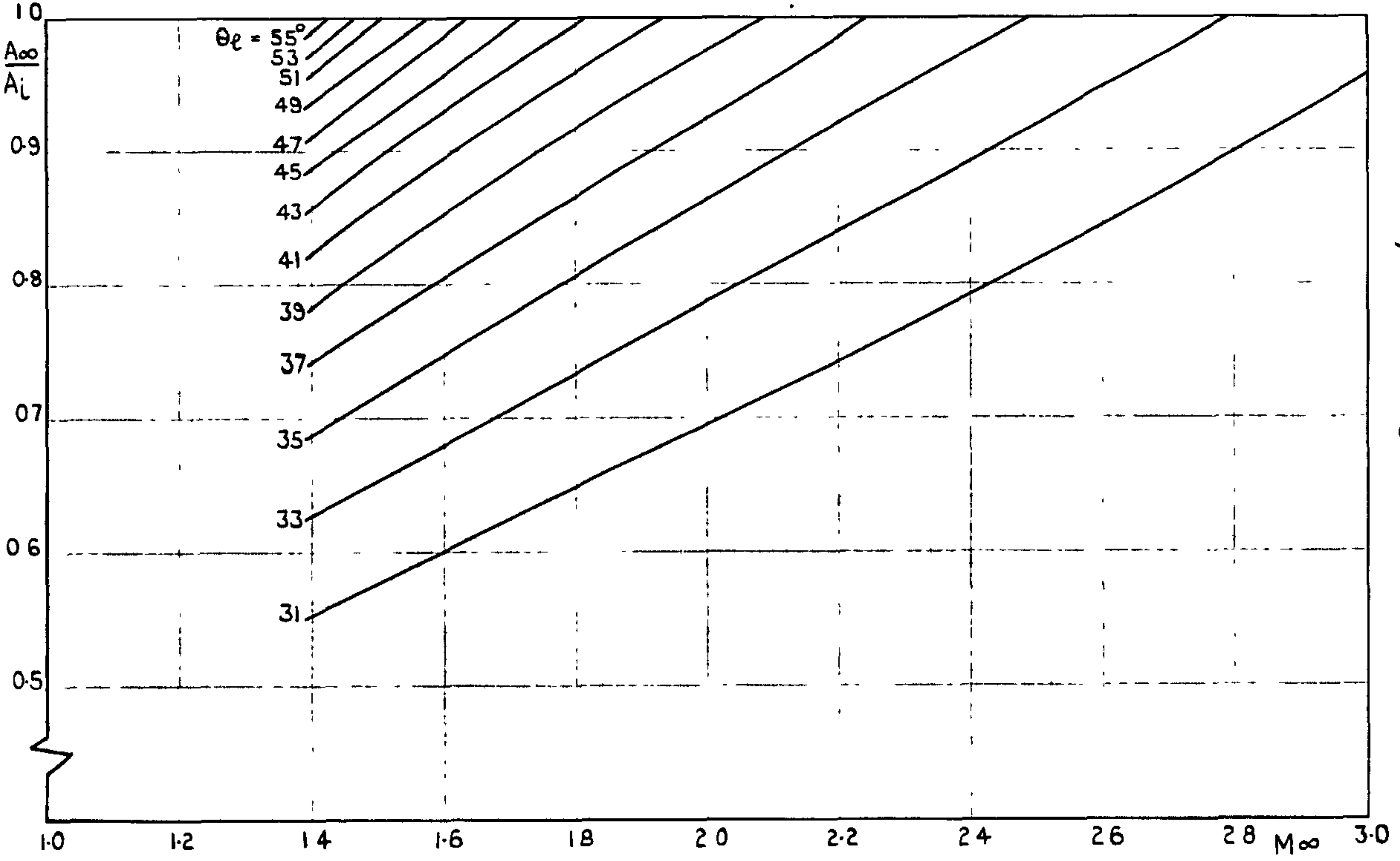


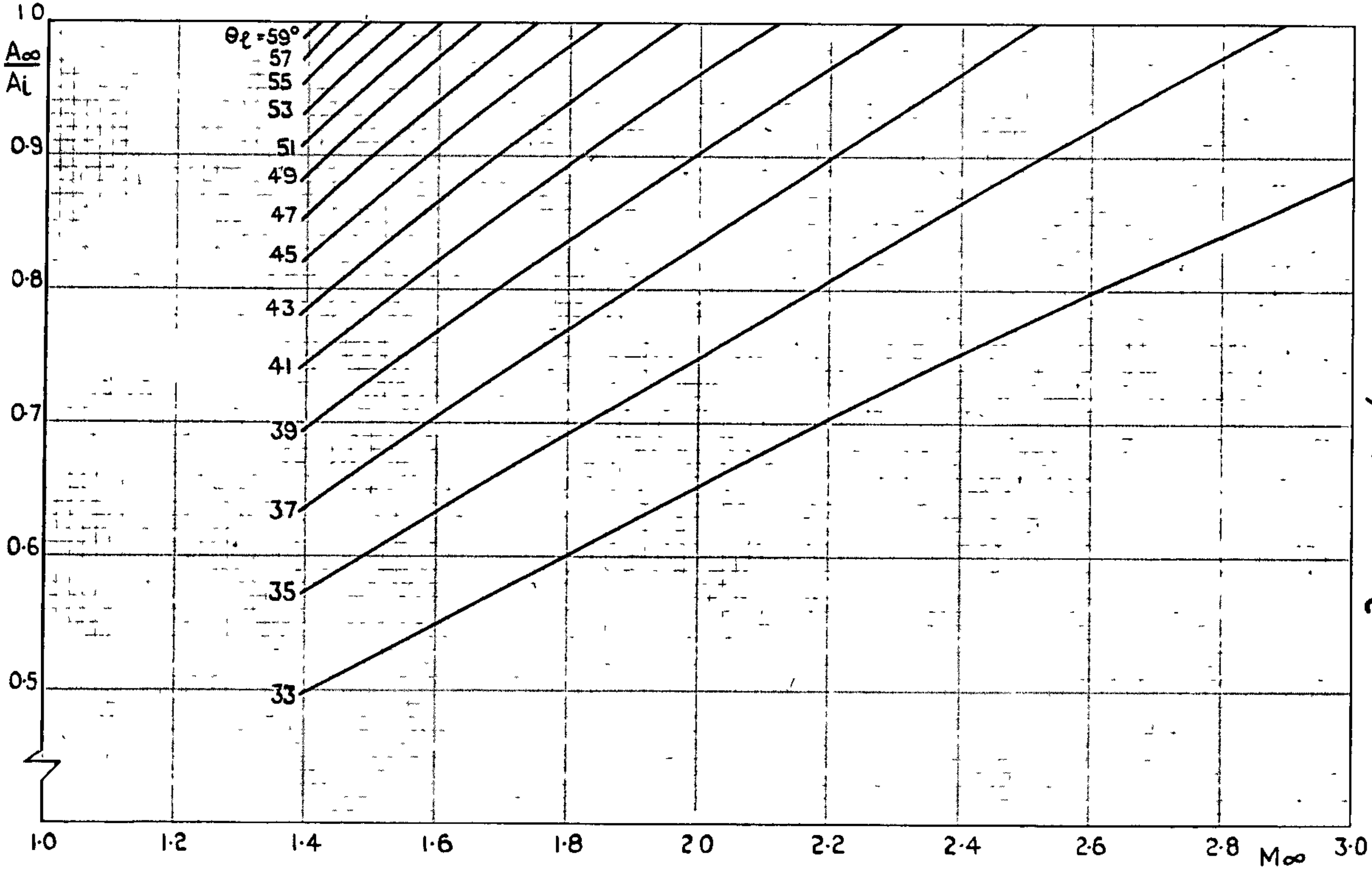
CURVES OF A_∞/A_i FOR $\theta_c = 20°$

FIG. 4b.

FIG. 4c.

CURVES OF A_∞/A_i FOR $\theta_c = 22.5°$



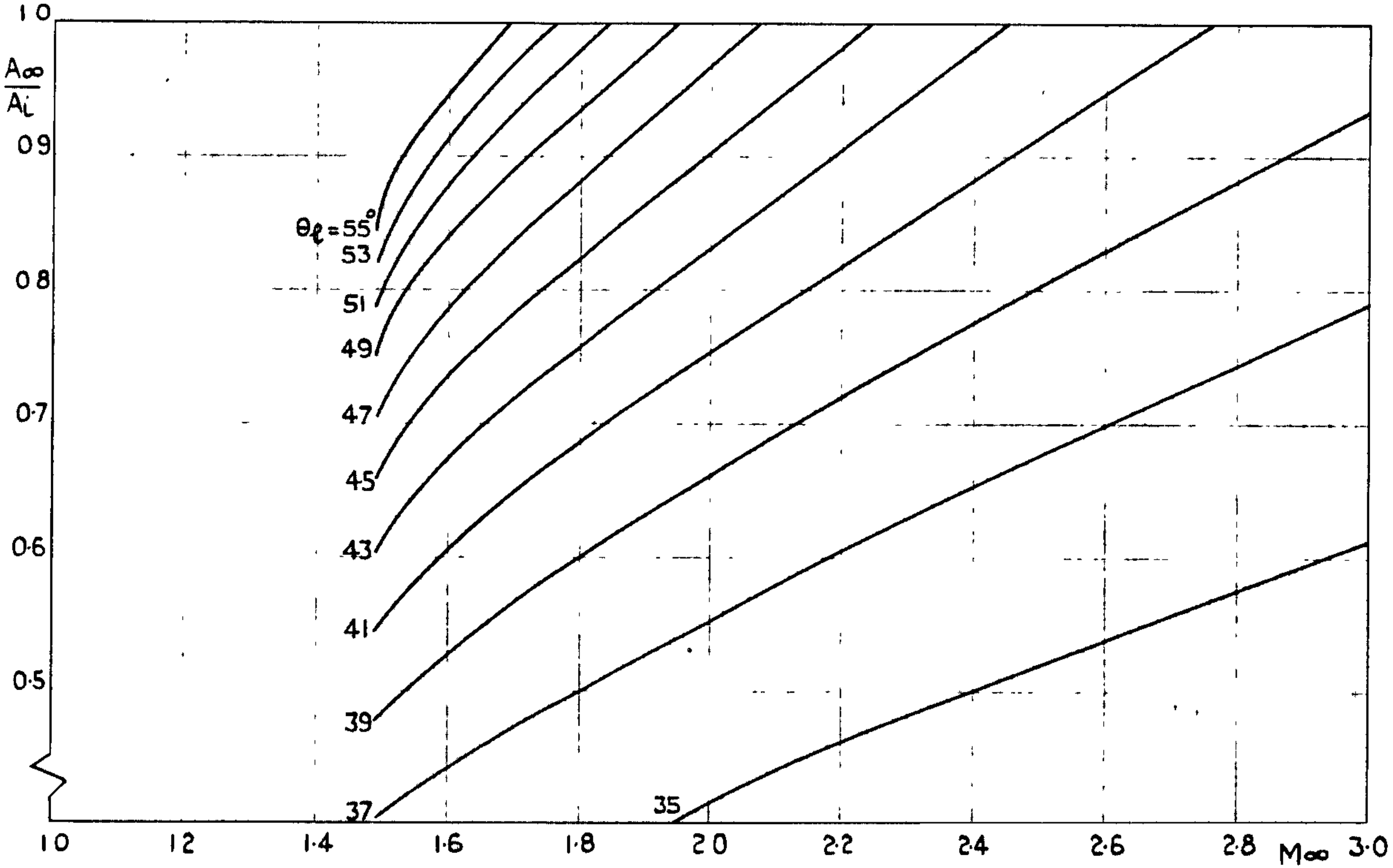


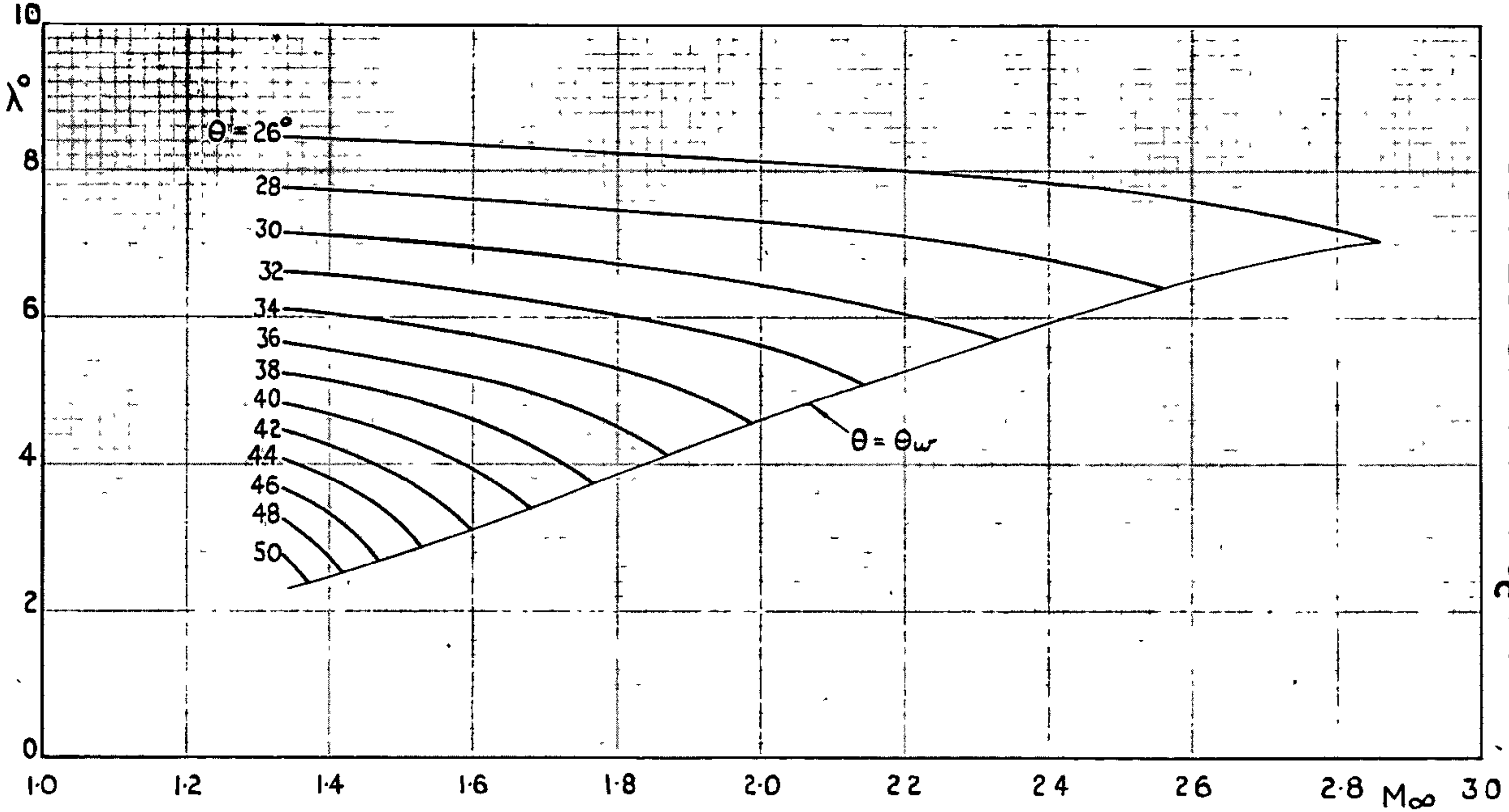
CURVES OF A_∞/A_i FOR $\theta_c = 25°$

FIG. 4d.

FIG. 4e.

CURVES OF A_∞/A_i FOR $\theta_c=30°$



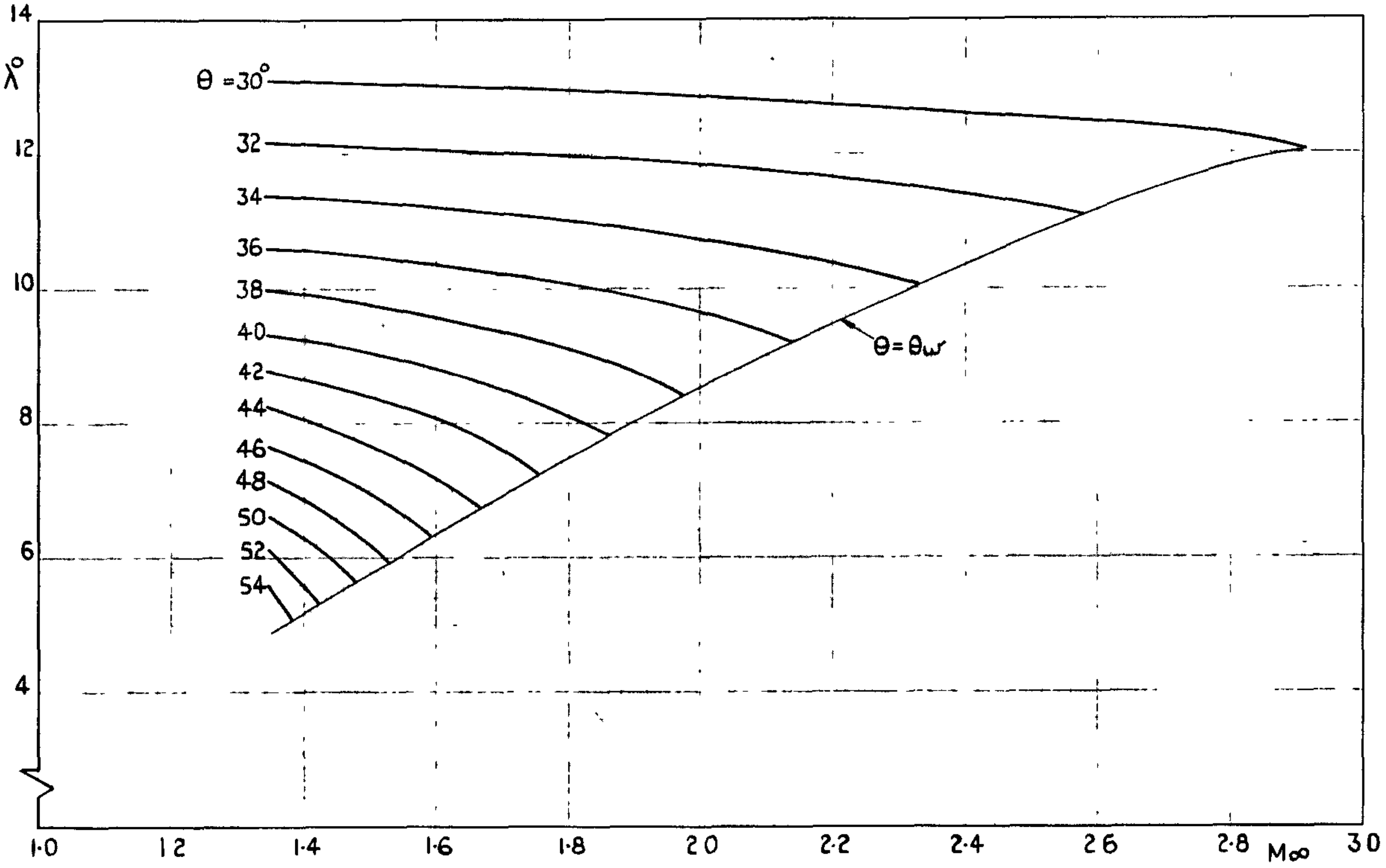


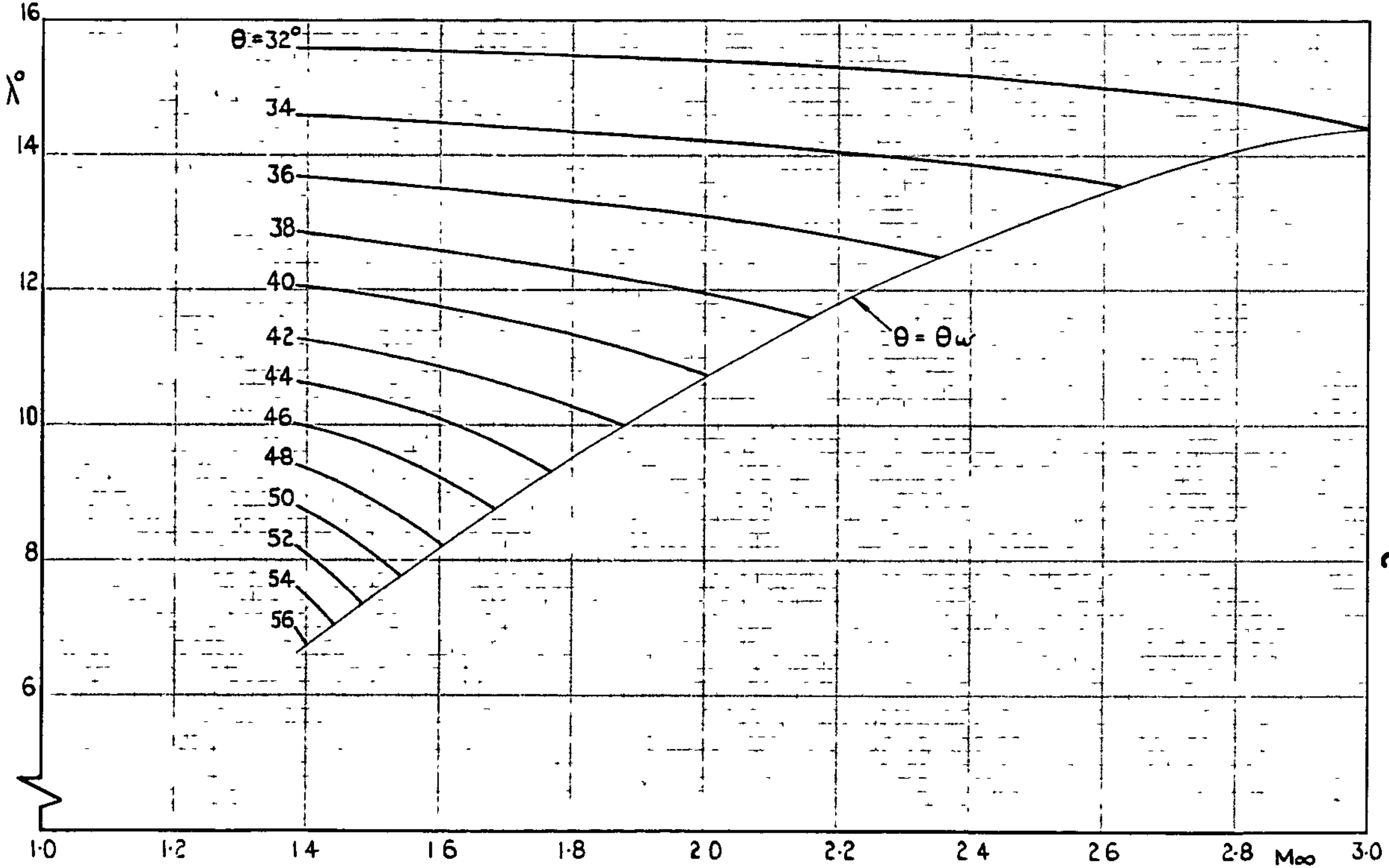
CURVES OF λ FOR $\theta_c = 15^\circ$.

FIG. 5a.

FIG.5b.

CURVES OF λ FOR $\theta_c = 20°$



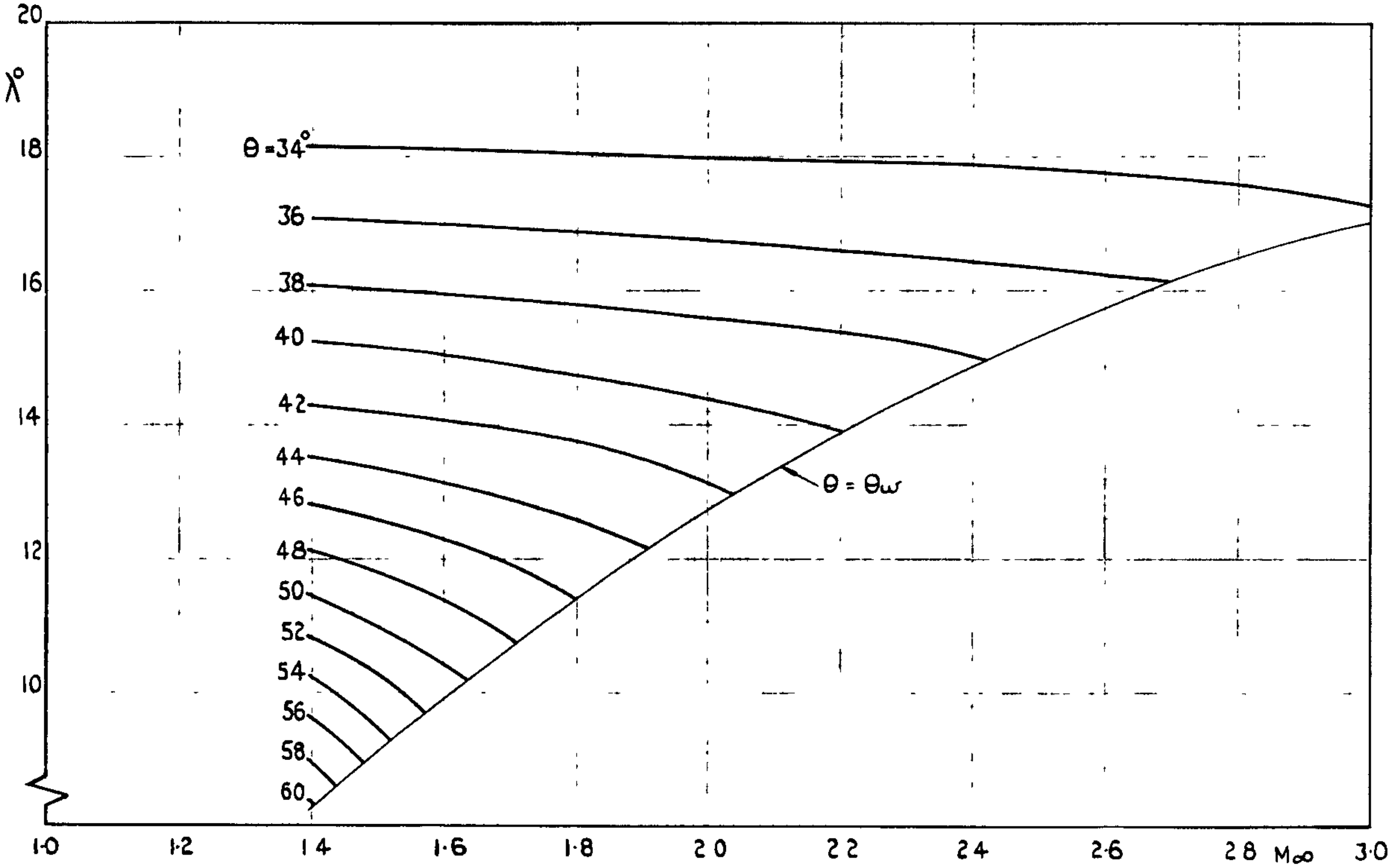


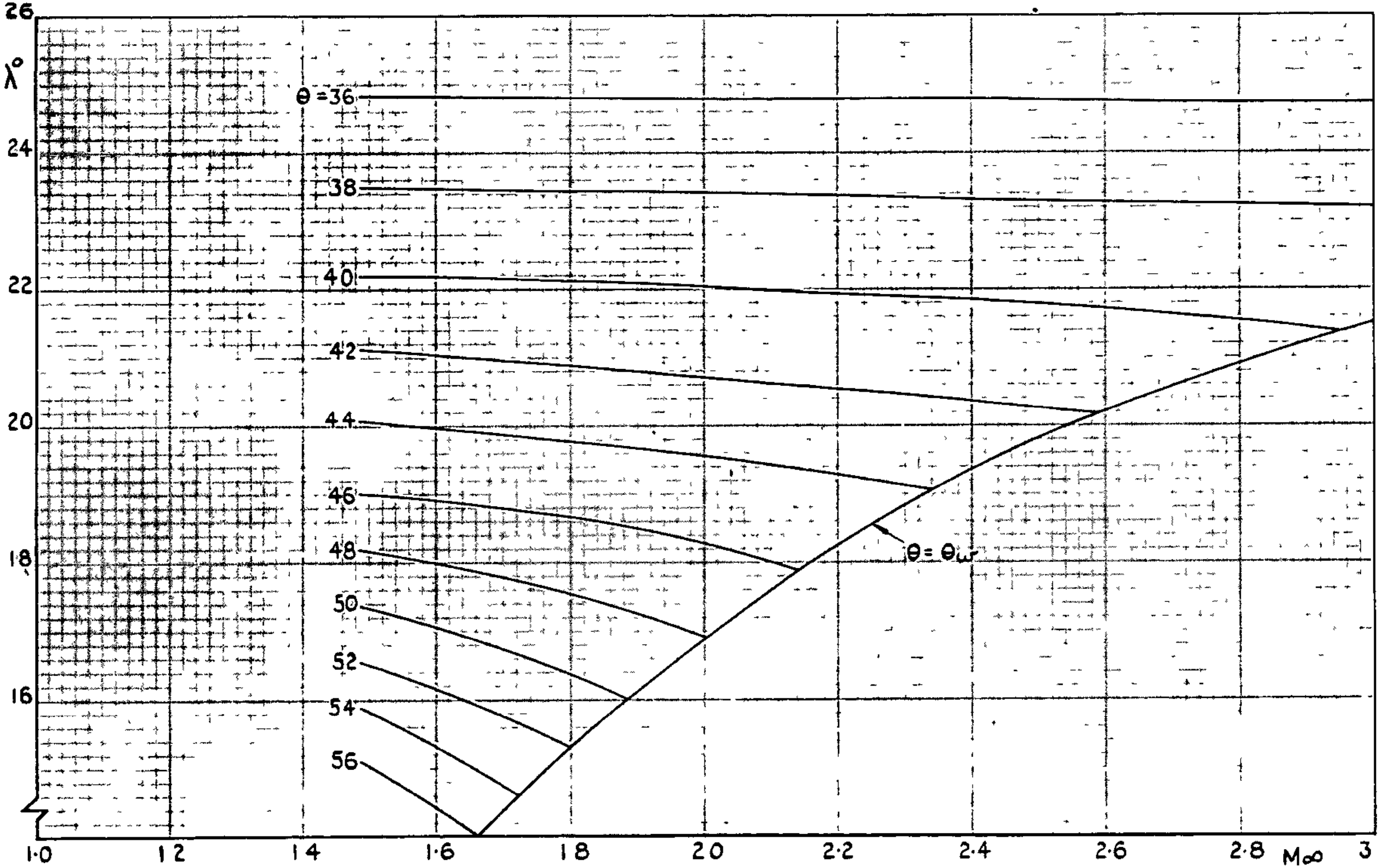
CURVES OF λ FOR θ_c=22·5°

FIG. 5c.

FIG. 5d.

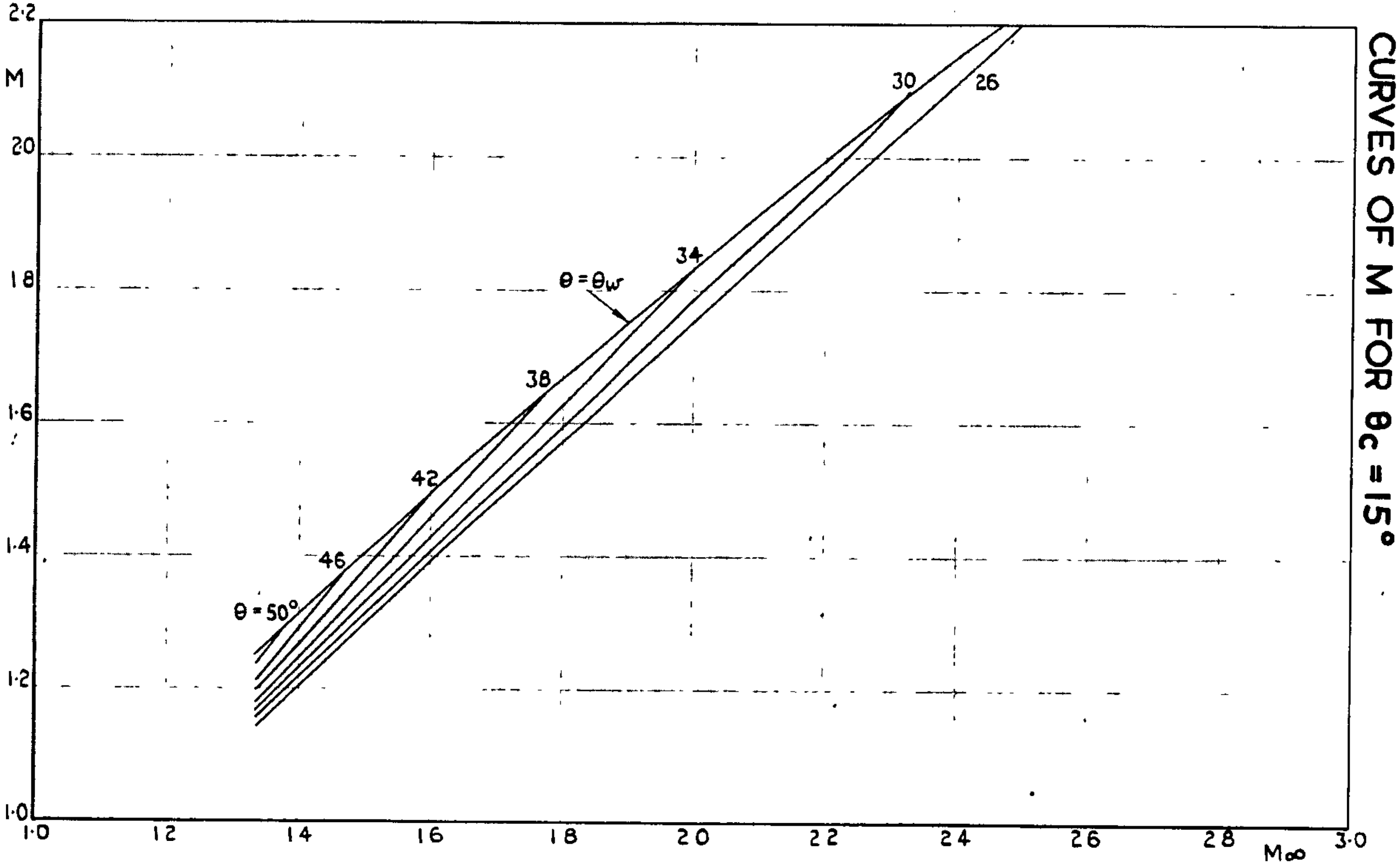
CURVES OF λ FOR $\theta_c = 25°$





CURVES OF λ FOR $\theta_c = 30°$

FIG. 5e.



CURVES OF M FOR θc = 15°

FIG. 6a.

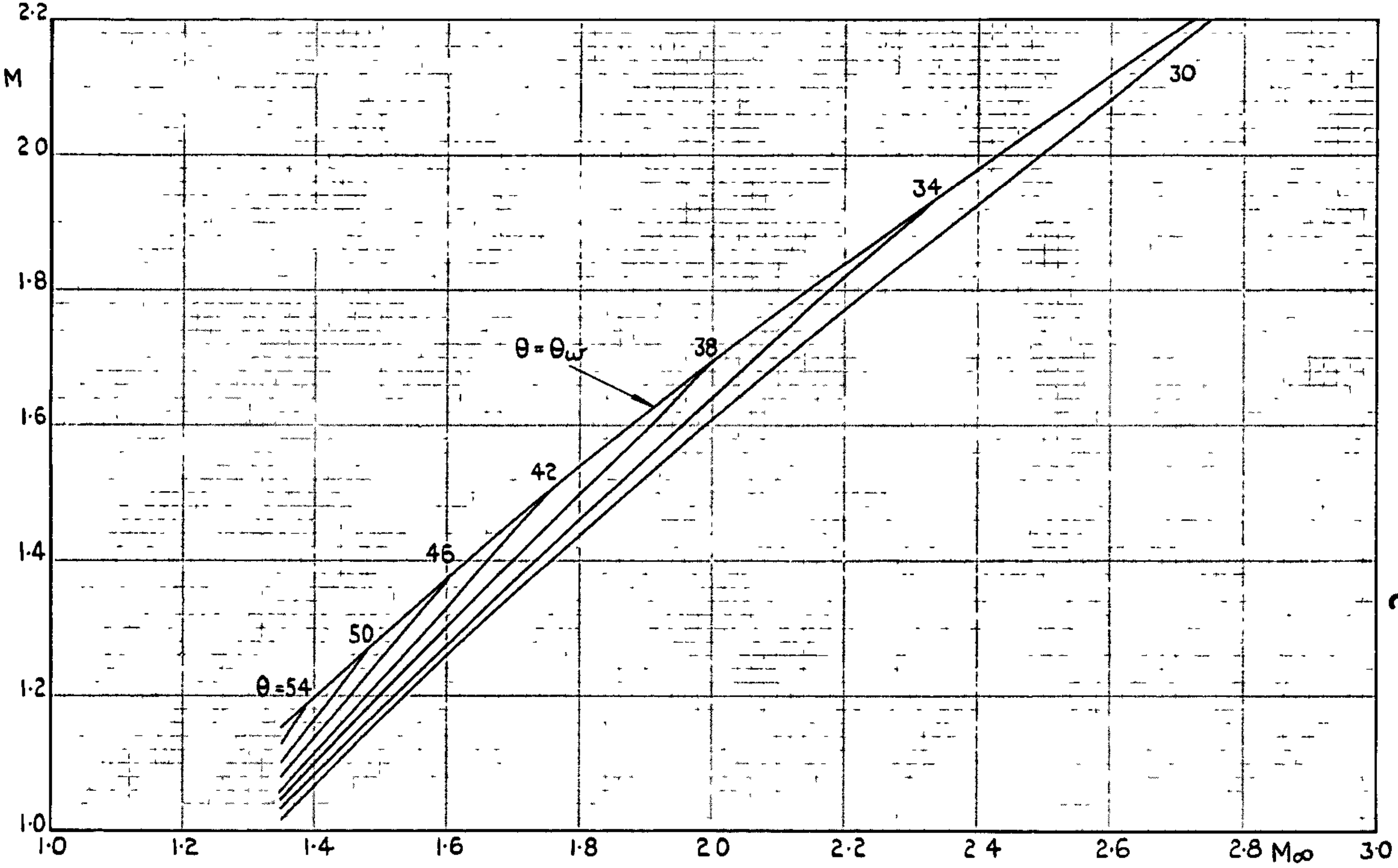


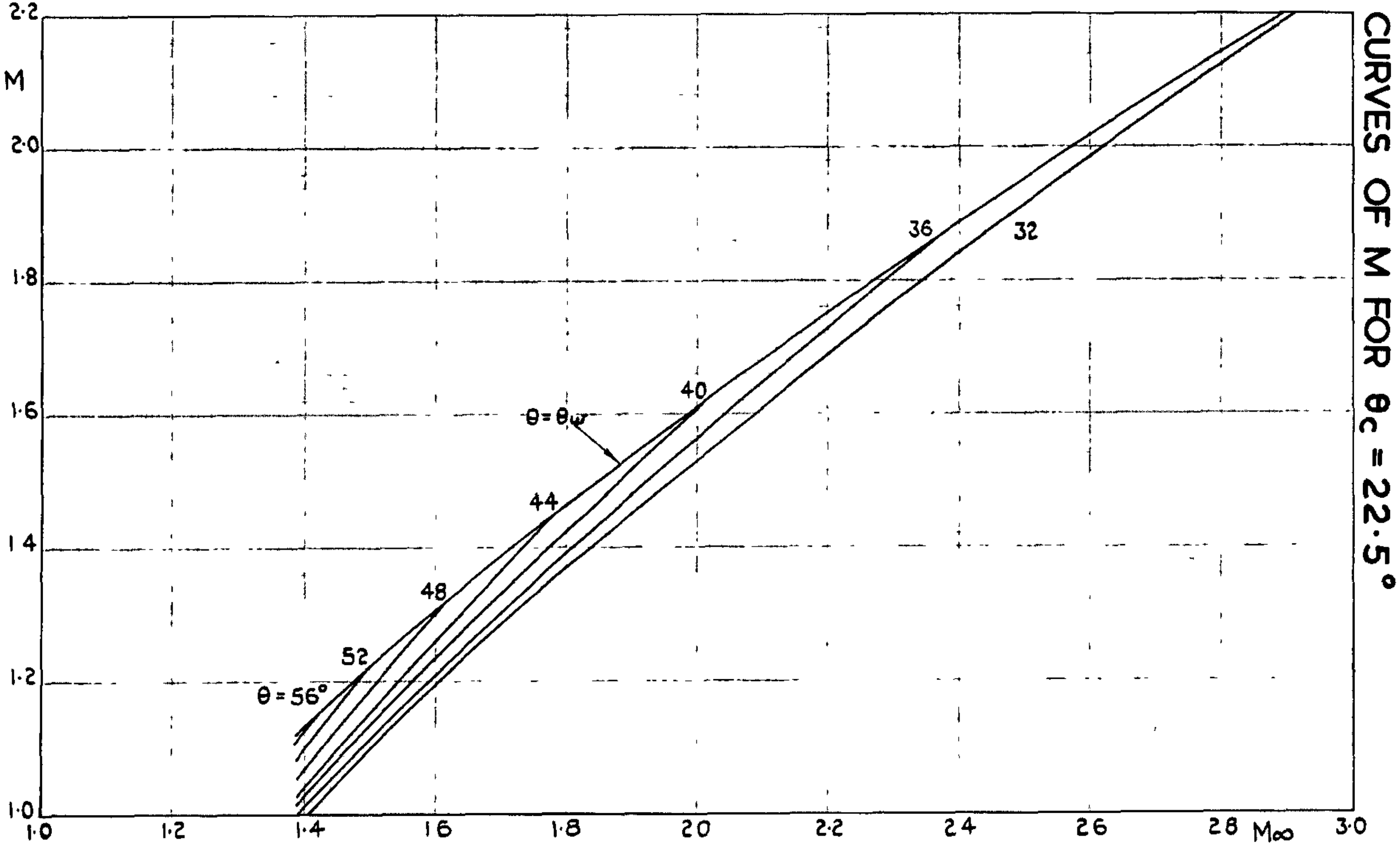
FIG. 6b.

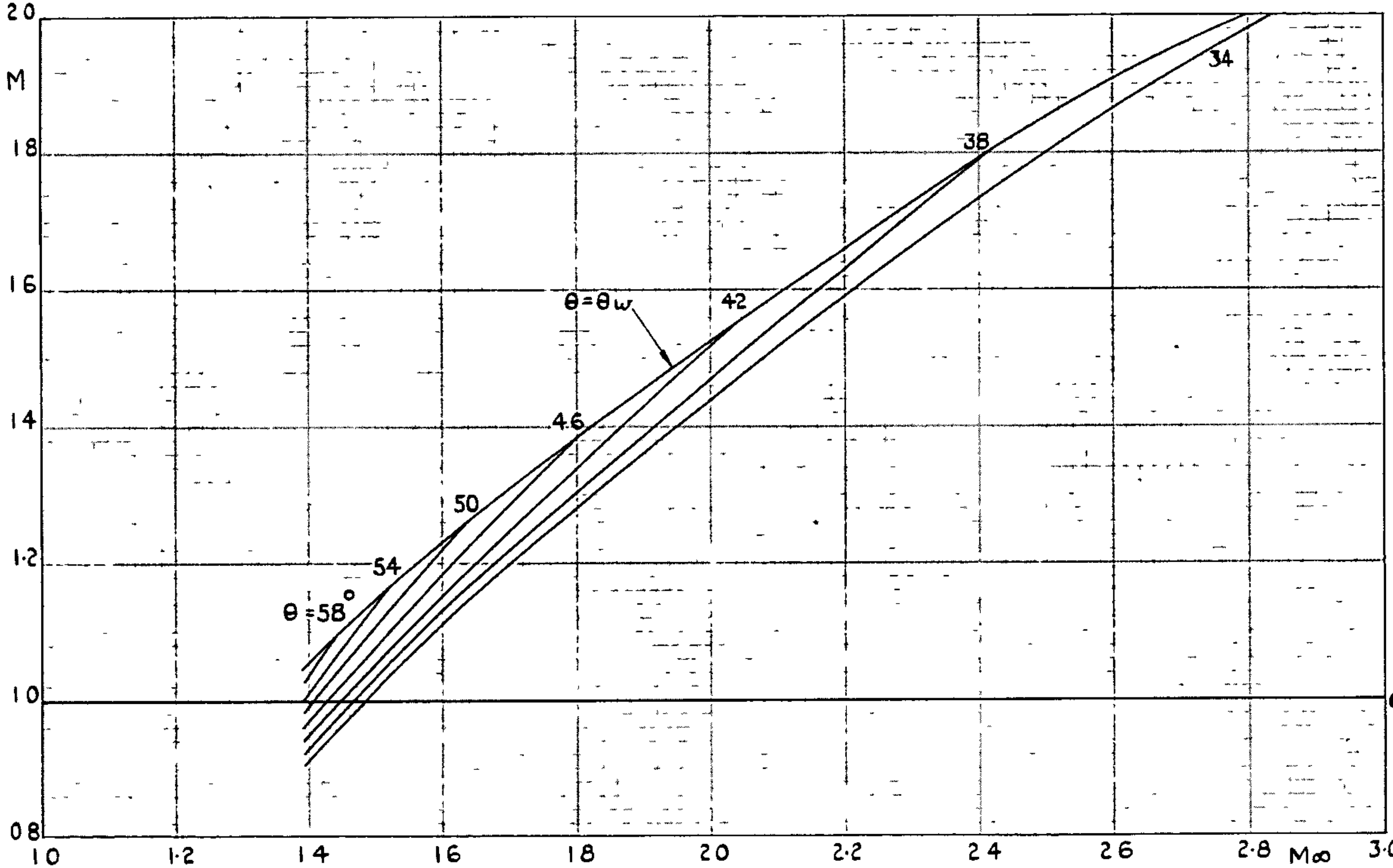
CURVES OF M FOR $\theta_c = 20°$

FIG. 6b.

FIG. 6c.

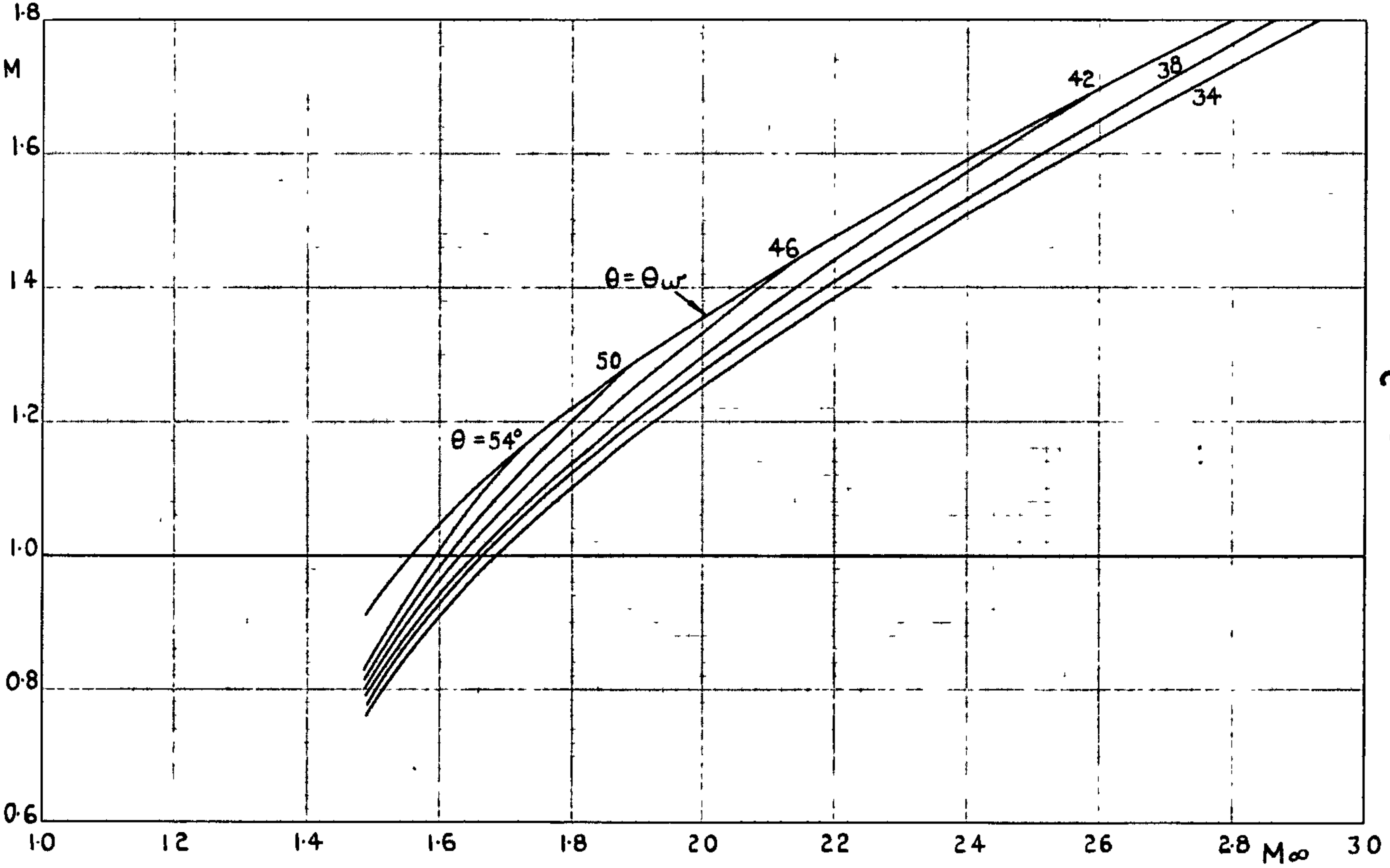
CURVES OF M FOR $\theta_c = 22.5°$





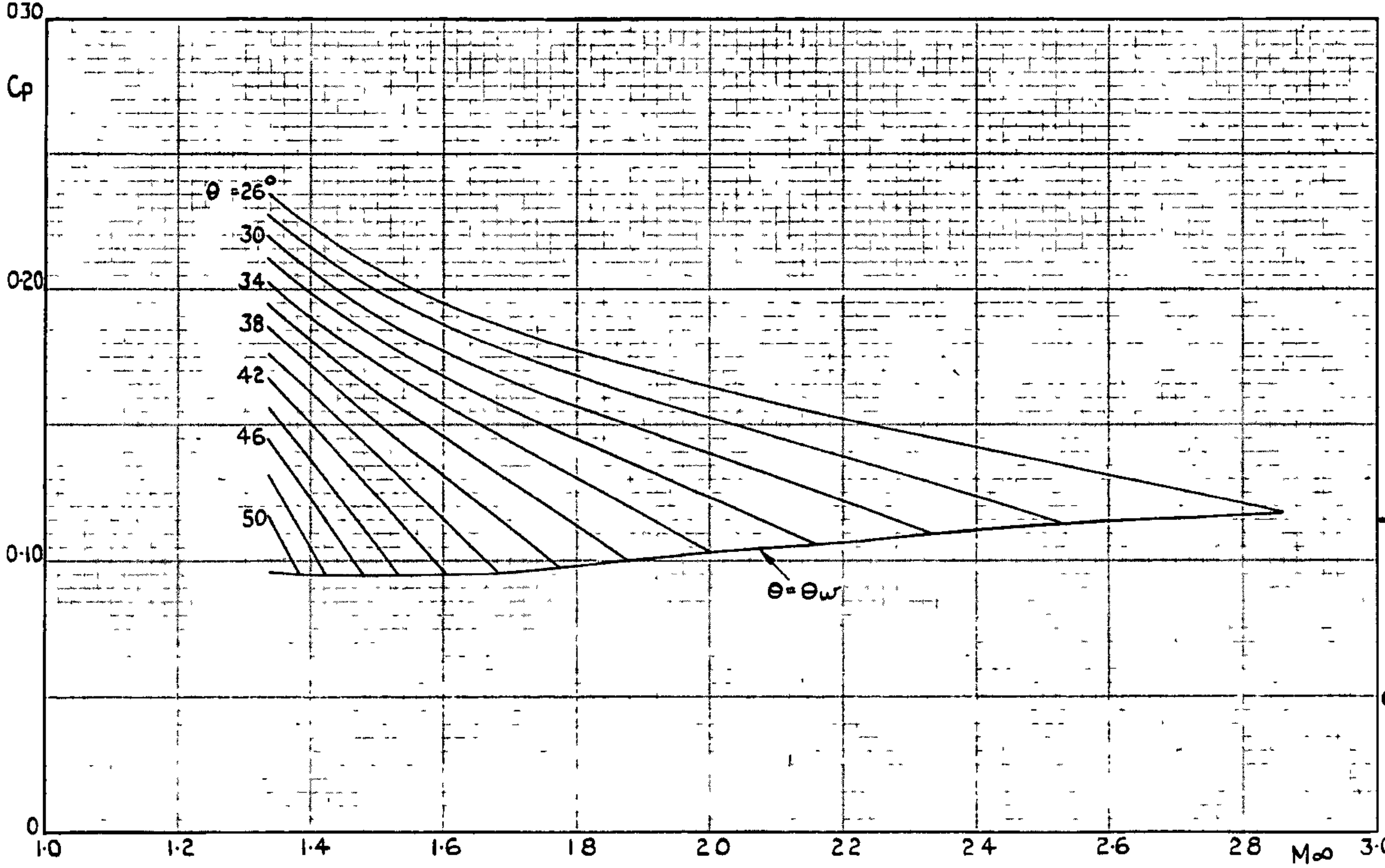
CURVES OF M FOR θ_c = 25°

FIG. 6d.



CURVES OF M FOR $\theta_c = 30°$

FIG. 6e.



CURVES OF C_p FOR $\theta_c = 15°$

FIG. 7a.

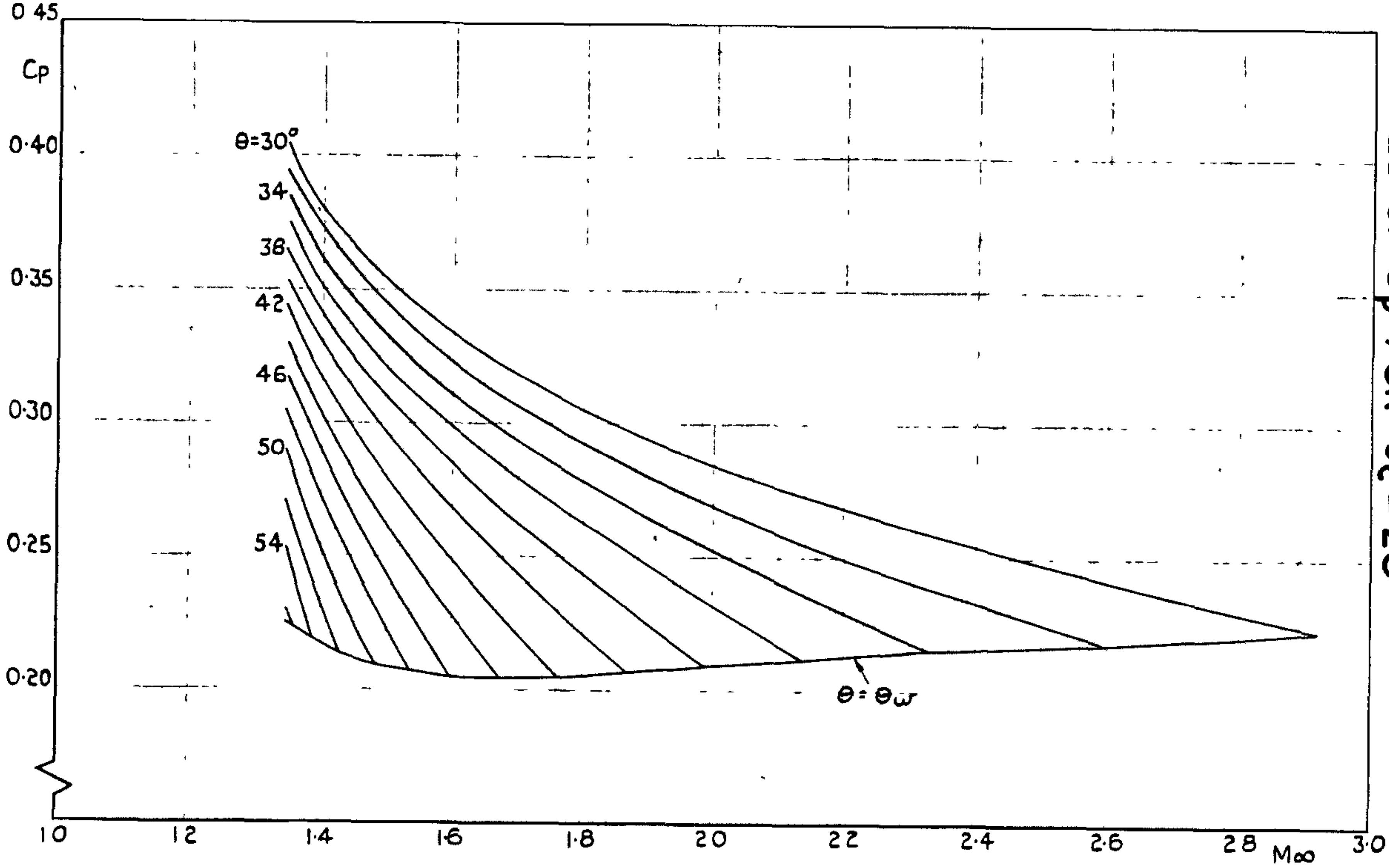
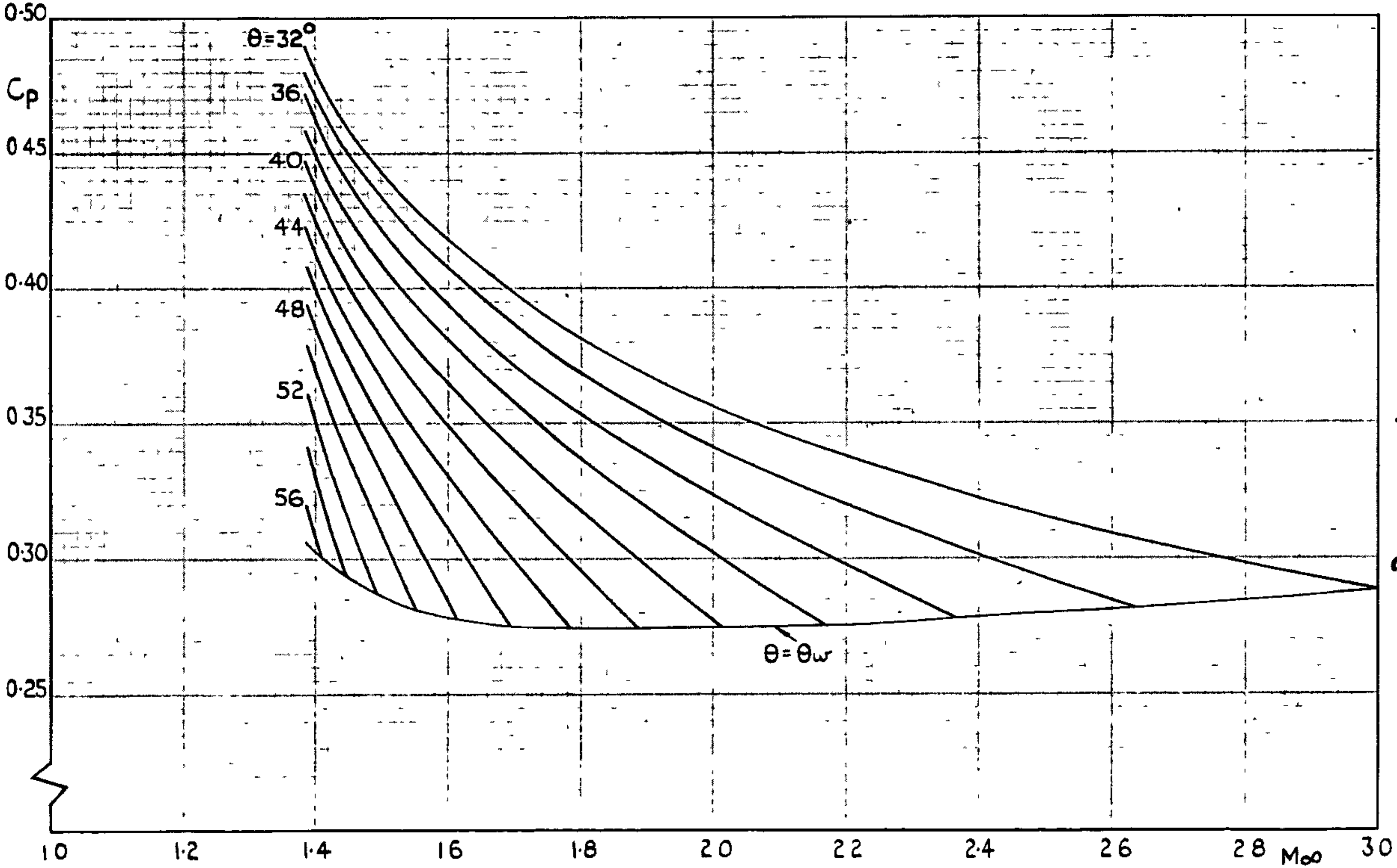


FIG. 7b.

CURVES OF C_p FOR $\theta_c = 20°$

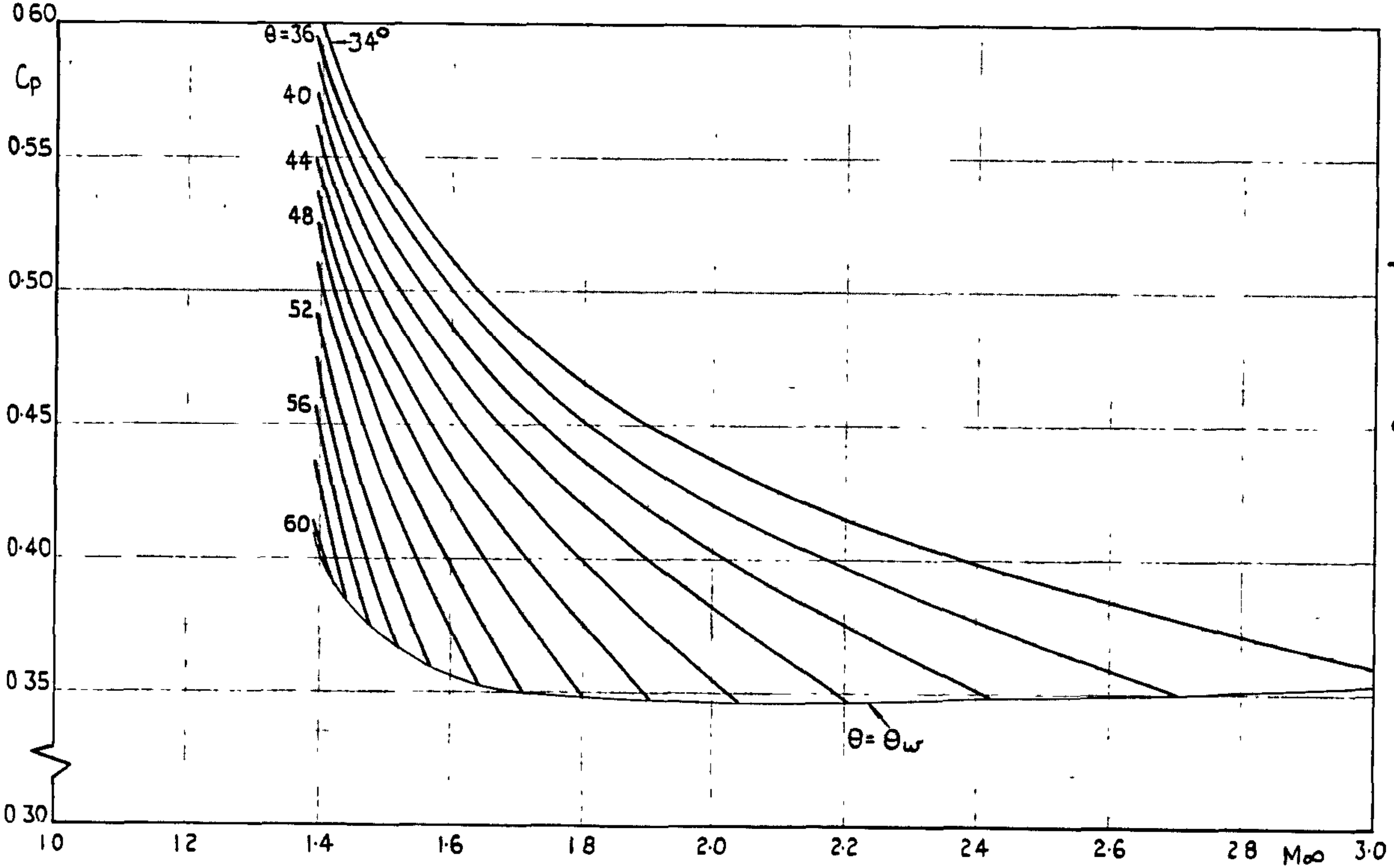


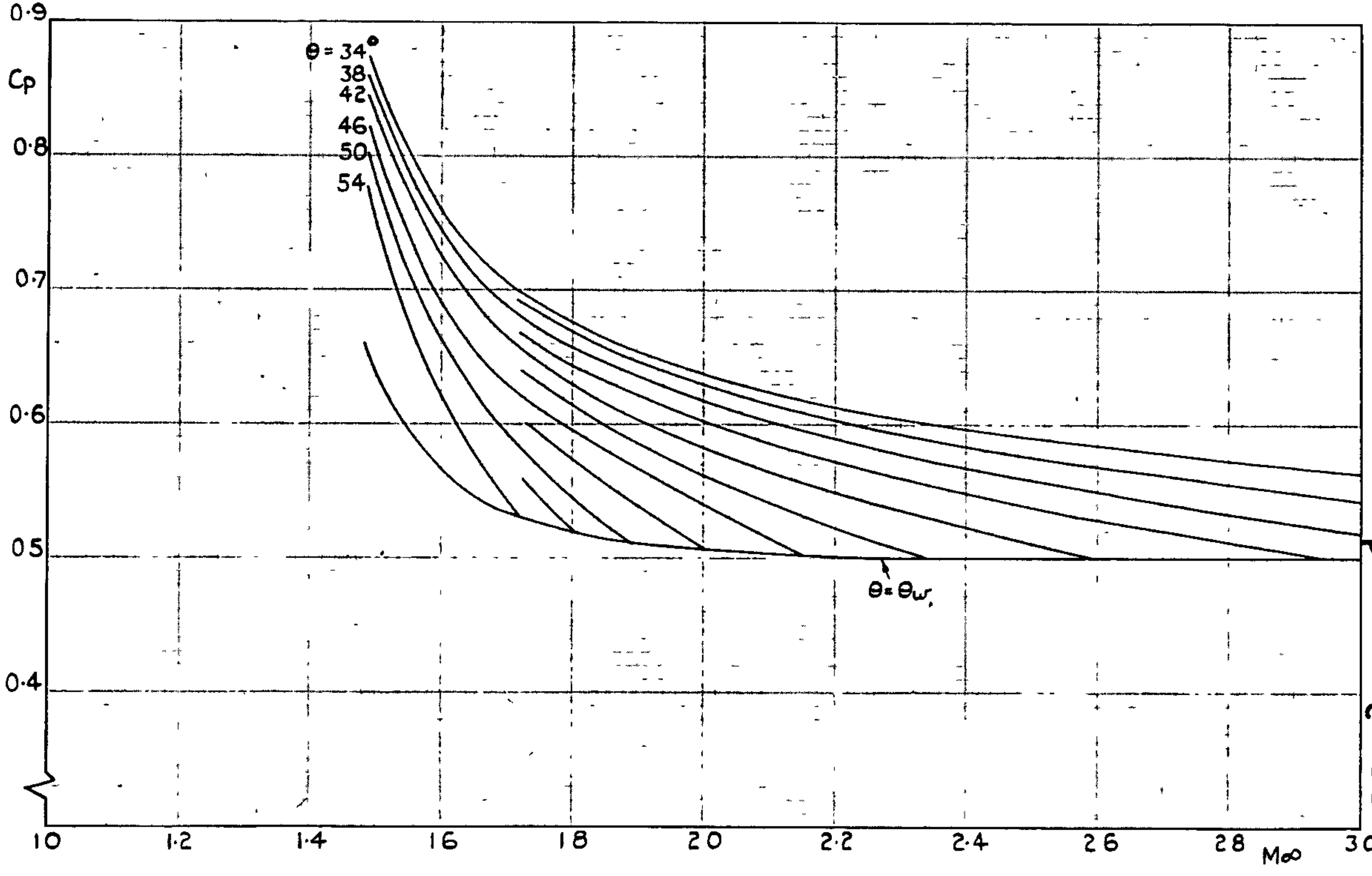
CURVES OF C_p FOR $\theta_c = 22{\cdot}5^\circ$

FIG. 7c.

FIG.7d.

CURVES OF C_p FOR $\theta_c = 25°$





CURVES OF C_p FOR $\theta_c = 30°$

FIG. 7e.

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