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NOTES ON THE INTERNAL DRAG, LIFT AND PITCHING MOMENT OF A DUCTED BODY

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SUMMARY

This paper considers the problem of measuring the internal forces on a ducted body when it is assumed that the post exit flow effects can be neglected.

In Ref.1 a standard internal drag force has been defined but, as pointed out, the expression given only applies to a particular duct exit configuration. The treatment followed here is similar but extends the work to cover general planar exits and gives the corresponding expressions for lift and pitching moment.

* Replaces RAE Technical Memorandum Aero 1292 - ARC 32988

CONTENTS

		Page
1	INTRODUCTION	3
2	DERIVATION OF FORCES AND MOMENTS ASSOCIATED WITH INTERNAL FLOW	3
	2.1 Duct with any planar exit configuration	3
	2.2 Ducts with particular exit configurations	7
3	COMMENTS ON THE APPLICATION OF THE ANALYSIS	8
4	CONCLUSIONS	8
Acknowledgement		8
Symbols		9
References		10
Illustrations		Figures 1-5
Detachable abstract cards		-

1 INTRODUCTION

The thrust and drag associated with the flow through and about a ducted body have been considered in detail by a Definitions Panel of the Aeronautical Research Council in Ref.1. The resultant thrust on the ducted body has been divided into 'internal thrust' and 'external drag', associated with internal flow and external flow respectively. The internal flow is defined in Ref.1 as the flow of fluid that passes through the body including the flow in the pre-entry and post-exit streamtubes.

The internal thrust (caused by the fluid that passes through the duct) was further subdivided into pre-entry, intrinsic and post-exit thrusts. If the latter be neglected, the resultant thrust is given by the sum of the pre-entry and intrinsic thrusts. This is defined in Ref.1 as the net standard thrust and is the thrust normally specified in engine brochures. Also defined is the standard internal drag which is 'the negative of the net standard thrust'. The derivation of standard internal drag considers only the case in which the exit plane is normal to the freestream direction.

A treatment of momentum and pressure forces acting at a single reference plane in the internal flow is considered in Ref.2; this includes the case where the flow is not normal to the plane. A momentum parameter function, which depends on the local Mach number in the internal flow, is introduced to simplify the expression for the force in the local flow direction. Some discussion of net thrust is also included.

This paper studies the forces and moments associated with the internal flow for a duct with any planar exit configuration. The case considered in Ref.1 and also another associated with a particular duct exit configuration of practical importance are illustrated.

2 DERIVATION OF FORCES AND MOMENTS ASSOCIATED WITH INTERNAL FLOW

2.1 Duct with any planar exit configuration

Three typical duct exit configurations associated with internal flows are illustrated in Fig.1.

Fig.1a shows a general exit configuration in which the exit plane is not normal to the exit flow direction, and the exit flow direction is not parallel to freestream.

Considering the internal flow from freestream far ahead of the duct to the duct exit plane, vector diagrams may be used to derive the relevant forces, as shown in Fig.2.

Fig.2a shows the forces exerted on the internal flow at the control surfaces at the ends of the internal stream tube. Vector F_{a_1} represents the resultant force given by the vector equation:-

$$\vec{F}_{a_1} = \overrightarrow{p_e A_e} + \overrightarrow{p_\infty A_\infty} \quad . \tag{1}$$

Fig.2b shows the momentum vectors at the freestream and duct exit stations; vector F_1 represents the resultant force exerted on the internal flow which produces the change in momentum and is given by:-

$$\vec{F}_1 = \overrightarrow{m_e V_e} - \overrightarrow{m_\infty V_\infty} \tag{2}$$

For simplicity and convenience, the internal and external flows are treated separately. To this end, the surface of the pre-entry streamtube is replaced by a membrane which cannot sustain a pressure difference. This enables the introduction of a force which is exerted on the internal flow at the internal surface of the pre-entry streamtube, this force being balanced by an equal and opposite force on the external flow. The internal flow is now considered quite independently of the external flow. A force F_{b_1} is introduced which is the sum of the force exerted on the internal flow at the internal surface of the pre-entry streamtube and the force exerted on the internal flow by the duct.

The force F_1 in equation (2), which produces the momentum change, is the resultant of all forces acting on the internal flow, i.e. F_{a_1} and F_{b_1}. The relationship between F_{a_1}, F_{b_1} and F_1 is shown in Fig.2c and the vector equation is:-

$$\vec{F}_1 = \vec{F}_{a_1} + \vec{F}_{b_1} \tag{3}$$

Of these forces, both F_1 and F_{a_1} may be readily measured.

Fig.2d is the superimposition of Figs.2a, 2b and 2c and shows the relationship between all the forces acting on the internal flow and the resulting momentum vectors in the internal flow.

The forces that are of significance are those associated with pressures measured relative to the undisturbed static pressure, p_∞. The integration of p_∞ around the complete closed boundary of the internal flow must give zero force; that is, the pressure forces included in F_{b_1} and the pressure forces associated with the ends of the streamtube can all be referred to the pressure p_∞. Fig.3 illustrates the development of the vector diagram in these terms. Fig.3d shows the relationship between the forces involved, where the resultant force F remains unchanged from that shown in the previous case, Fig.2d, i.e.

$$F = F_1 . \tag{4}$$

Drag

The component of the force F_b acting on the internal flow in the direction X, (directions indicated in Fig.3d) is represented by $\overrightarrow{GE}$.

The required force is that exerted by the internal flow on the internal surfaces of the pre-entry streamtube and the duct, resolved in the direction X; this is $-\overrightarrow{GE}$.

Let $D = -\overrightarrow{GE}$ then:-

$$D = (m_\infty V_\infty - m_e V_e \cos\theta) - [(p_e - p_\infty)A_e \cos\phi] . \tag{5}$$

Now

$$m_\infty = \rho_\infty V_\infty A_\infty \tag{6}$$

and from Fig.1a

$$m_e = \rho_e V_e A_e \cos\psi \tag{7}$$

then the equation (5) becomes:-

$$D = \rho_\infty V_\infty^2 A_\infty - [(p_e - p_\infty)\cos\phi + \rho_e V_e^2 \cos\psi \cos\theta] A_e . \tag{8}$$

<u>D is the standard internal drag.</u>

Lift

The component of the force F_b acting on the internal flow in the direction Z is represented by $-\overrightarrow{OG}$.

The required force is that exerted by the internal flow on the internal surfaces of the pre-entry streamtube and the duct resolved in the direction Z; this is $\vec{OG}$.

Let $L = \vec{OG}$ then:-

$$L = m_e V_e \sin\theta + (p_e - p_\infty) A_e \sin\phi \tag{9}$$

which can also be expressed in the form:-

$$L = [(p_e - p_\infty)\sin\phi + \rho_e V_e^2 \sin\theta \cos\psi] A_e \quad . \tag{10}$$

<u>L is the standard internal lift.</u>

Pitching moment

The resultant force acting on the internal flow is F_b (see Fig.3d). The required force is that exerted by the internal flow on the internal surfaces of the pre-entry streamtube and the duct; this is $-F_b$ where:-

$$(\overrightarrow{-F_b}) = -(\overrightarrow{m_e V_e} - \overrightarrow{m_\infty V_\infty}) + [\overrightarrow{(p_e - p_\infty) A_e}] \quad . \tag{11}$$

The standard internal pitching moment is the moment of $-F_b$ about a reference point.

Let M be the moment of force $(-F_b)$ about an arbitrary reference point. Then:

$$\vec{M} = -\left\{(\overrightarrow{m_e V_e} \wedge \vec{d_2}) - (\overrightarrow{m_\infty V_\infty} \wedge \vec{d_1})\right\} + \left\{[\overrightarrow{(p_e - p_\infty) A_e}] \wedge \vec{d_3}\right\} \tag{12}$$

where $\vec{d_1}$, $\vec{d_2}$ and $\vec{d_3}$ are position vectors of points on the lines of action of the various components of F_b relative to the arbitrary reference point.

A simple example is given in Fig.4, where for convenience the position vectors are the normals from the reference point to the lines of action of the forces. Taking clockwise moments as positive, the standard internal pitching moment becomes:

$$M = m_e V_e d_2 - m_\infty V_\infty d_1 + [(p_e - p_\infty) A_e] d_3 \tag{13}$$

that is

$$M = (\rho_e V_e^2 \cos\psi A_e) d_2 - (\rho_\infty V_\infty^2 A_\infty) d_1 + [(p_e - p_\infty) A_e] d_3 \quad . \tag{14}$$

The precise location of the pre-entry streamtube in the freestream far ahead of the body (i.e. d_1 in Fig.5 and equation (14)) is often not known and therefore it is not always possible to define an absolute value of standard internal pitching moment. If, in a model test, the pre-entry flow is correctly represented then the position of the pre-entry streamtube relative to the ducted body is the same for both the model, where the intrinsic and post-exit flows are not correctly represented, and the full-scale configuration. Hence the pitching moment contribution from the freestream pre-entry momentum flux is the same in both cases, and the problem is reduced to comparing pitching moments due to pressures and momentum flux at the duct exit planes.

2.2 Ducts with particular exit configurations

Figs.1b and 1c show duct exit configurations of practical significance. These are, in fact, special cases of the general treatment in which:-

Case 1 $\phi = \theta, \quad \psi = 0^o$ Fig.1b

Case 2 $\phi = 0^o, \quad \psi = \theta$ Fig.1c .

For these cases the general equations for D, L and M reduce to:-

Case 1

$$D = \rho_\infty V_\infty^2 A_\infty - [(p_e - p_\infty)\cos\theta + \rho_e V_e^2 \cos\theta] A_e \tag{15}$$

$$L = [(p_e - p_\infty)\sin\theta + \rho_e V_e^2 \sin\theta] A_e \tag{16}$$

$$M = -(\rho_\infty V_\infty^2 A_\infty) d_1 + (\rho_e V_e^2 A_e) d_2 + [(p_e - p_\infty) A_e] d_3 \ . \tag{17}$$

Case 2

$$D = \rho_\infty V_\infty^2 A_\infty - [(p_e - p_\infty) + \rho_e V_e^2 \cos^2\theta] A_e \ . \tag{18}$$

The above is the negative of the equation derived in Ref.1 for the net standard thrust.

$$L = (\rho_e V_e^2 \sin\theta\cos\theta) A_e \tag{19}$$

$$M = -(\rho_\infty V_\infty^2 A_\infty) d_1 + (\rho_e V_e^2 \cos\theta A_e) d_2 + [(p_e - p_\infty) A_e] d_3 \ . \tag{20}$$

3 COMMENTS ON THE APPLICATION OF THE ANALYSIS

Expressions for standard internal lift, drag and pitching moment have been developed in the previous sections and ducted models with simple inlet shapes in Figs.1 and 4 have been used for illustration. It should be noted, however, that these equations are perfectly general, and can be applied to any ducted model, such as, for example, the configuration shown in Fig.5, for which standard internal forces are required. This configuration incorporates a side intake with some forebody boundary layer removal and includes some intake compression geometry.

Analysis of test data on such a configuration involves a definition of the external forces which is consistent with the measured standard internal forces. This situation is considered in more detail in Ref.3.

Throughout the analysis in the earlier sections, uniform internal flow at the duct exit had been assumed. It must be emphasised, however, that this treatment can be applied to the usual practical case with non-uniform internal flow at the duct exit. This application involves integrating the relevant properties (based on pressure, velocity and flow direction) of each stream-tube in the exit flow to obtain the pressure force and momentum flux at this station. This problem is considered in Ref.4.

4 CONCLUSIONS

This paper considers the problem of measuring the internal forces on a ducted body when it is assumed that the post exit flow effects can be neglected.

In Ref.1 a standard internal drag has been defined but, as pointed out, the expression given only applies to a particular duct exit configuration. The treatment followed here is similar but extends the work to cover general planar exits and gives the corresponding expressions for lift and pitching moment.

Acknowledgment

The authors would like to thank Dr. I. McGregor for the suggestion of the vector method of analysis.

SYMBOLS

A	area
D	drag force
d, d_1, d_2, d_3	distances associated with pitching moment
F	force
L	lift force
M	pitching moment
m	mass flow ($AV\rho$)
p	static pressure
V	velocity
X	positive direction of drag
Z	positive direction of lift
θ, ϕ, ψ	angles associated with duct exit
ρ	density

Suffixes

a, b	define particular forces
e	duct exit station
∞	free stream station far ahead of the duct
1	denoting absolute pressures used in analysis

REFERENCES

No.	Author(s)	Title, etc.
1	-	Report of the Definitions Panel on the definitions of the thrust of a jet engine and of the internal drag of a ducted body. ARC CP 190 (1954)
2	C.L. Bore J.W. Fozard P.H. Dove	A table of compressible airflow functions of Mach number. Hawker Aircraft POR 114, September 1959. (R. Ae. S. Data sheet S.00.03.25 is derived from this reference)
3	J.W. Britton	*Some notes on the analysis of the forces on ducted models* with side intakes. RAE Technical Memorandum Aero 1366 (1971)
4	J.W. Britton	Measurement of the internal drag of air breathing installations on slender wing-body combinations at supersonic speeds. ARC CP 914 (1965)

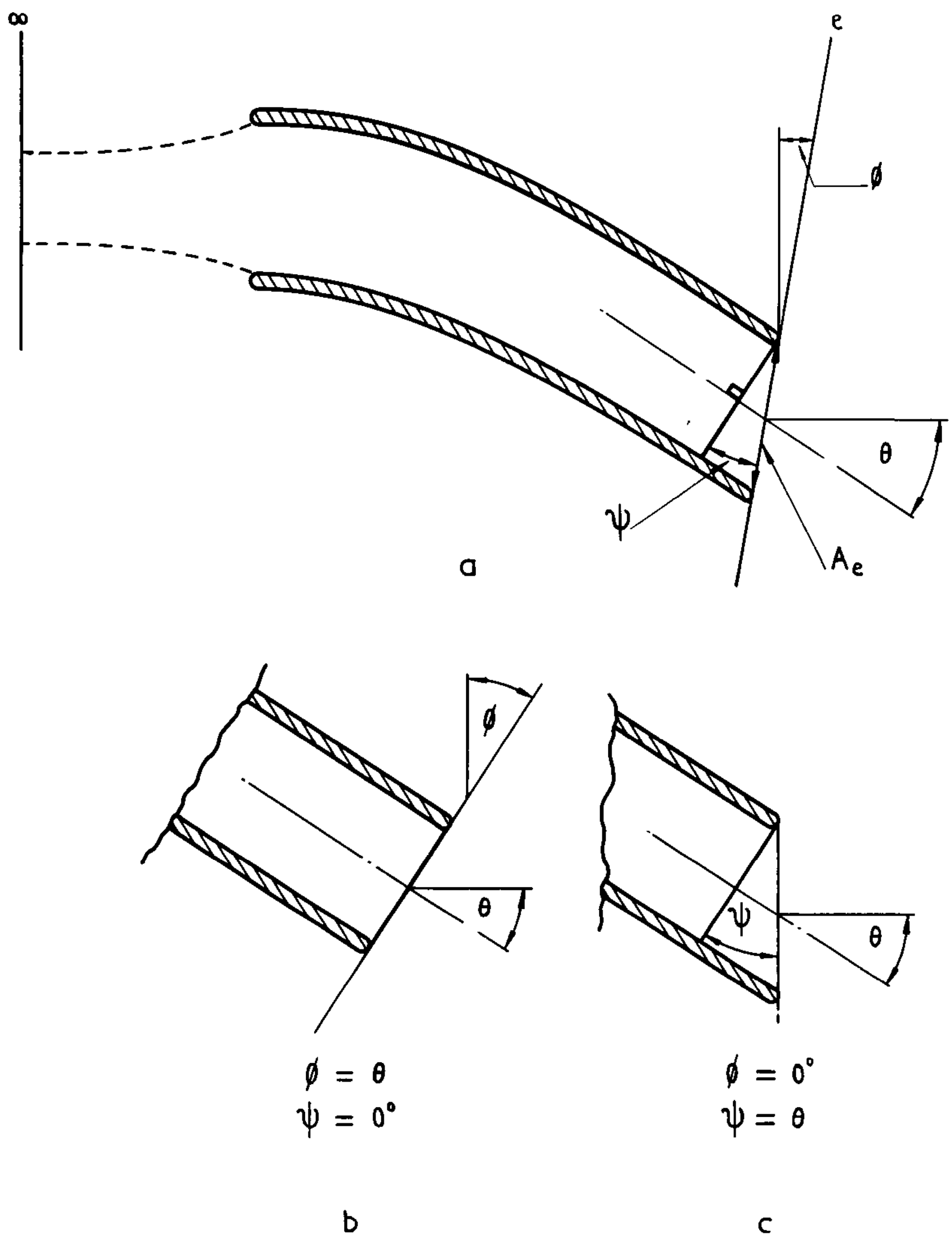


Fig1a-c Typical duct exit configurations

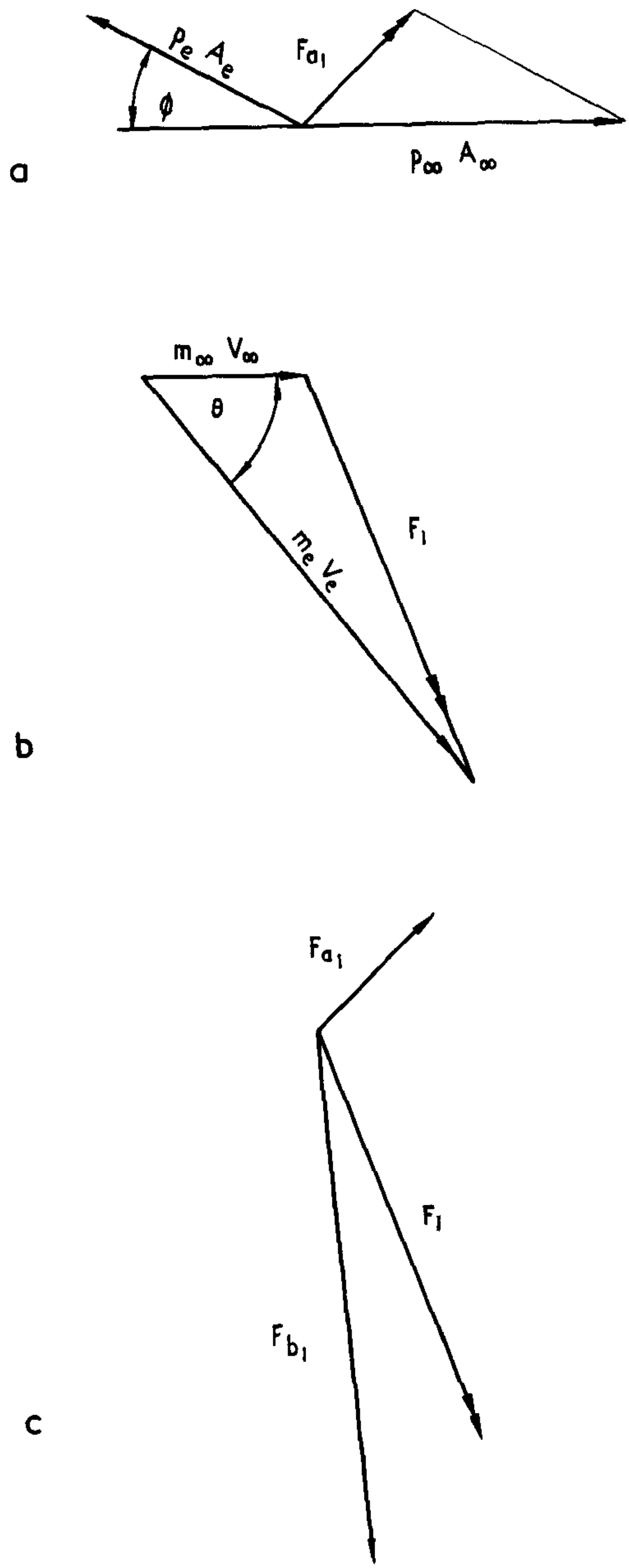


Fig 2a–c Development of vector diagram appropriate to Fig.1a —absolute pressures

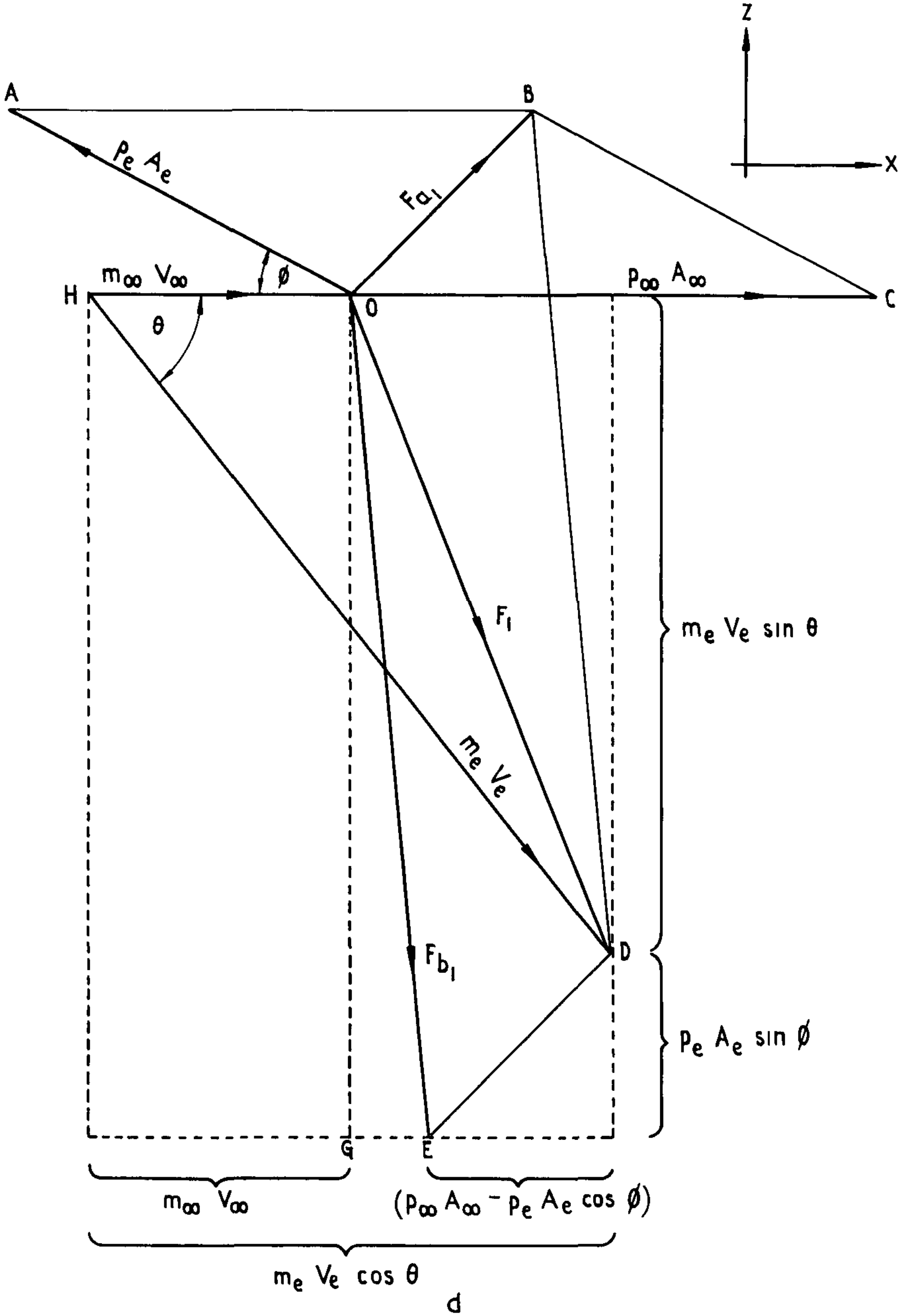


Fig 2 contd Development of vector diagram appropriate to Fig 1a — absolute pressures

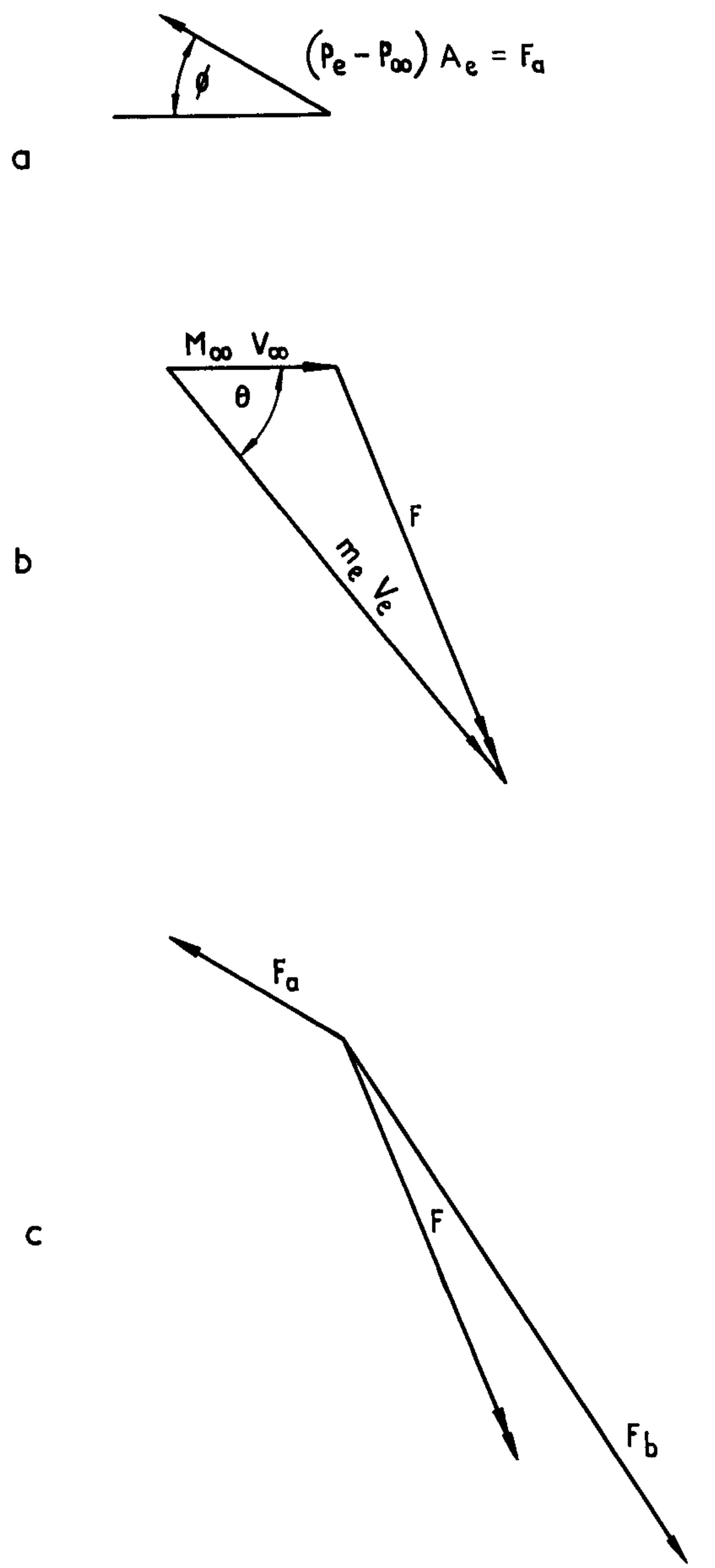
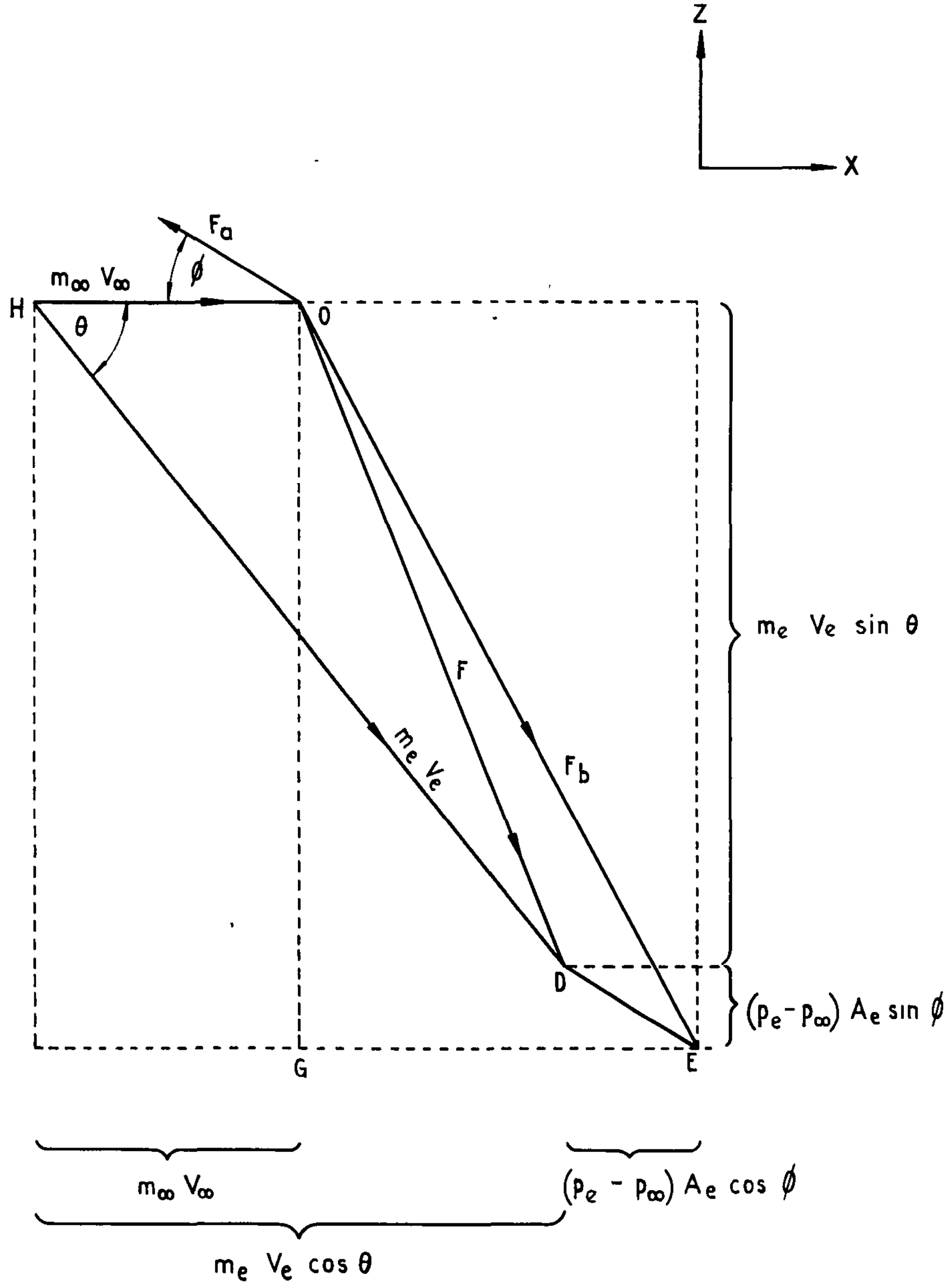


Fig.3a-c Development of vector diagram appropriate to Fig.1a pressures measured relative to undisturbed static pressure (p_∞)



d

Fig 3 contd Development of vector diagram appropriate to Fig 1a pressures measured relative to undisturbed static pressure (p_∞)

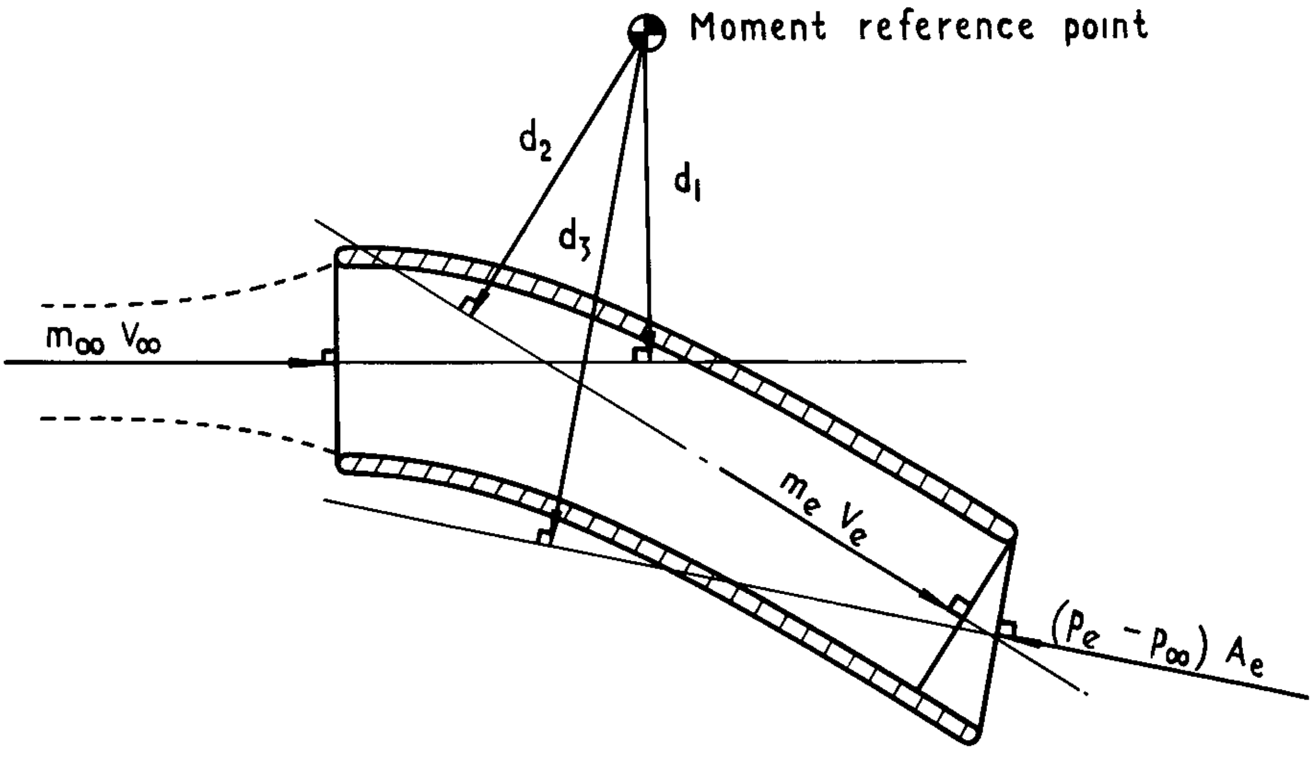


Fig 4 Definition of the moment arms associated with pitching moment

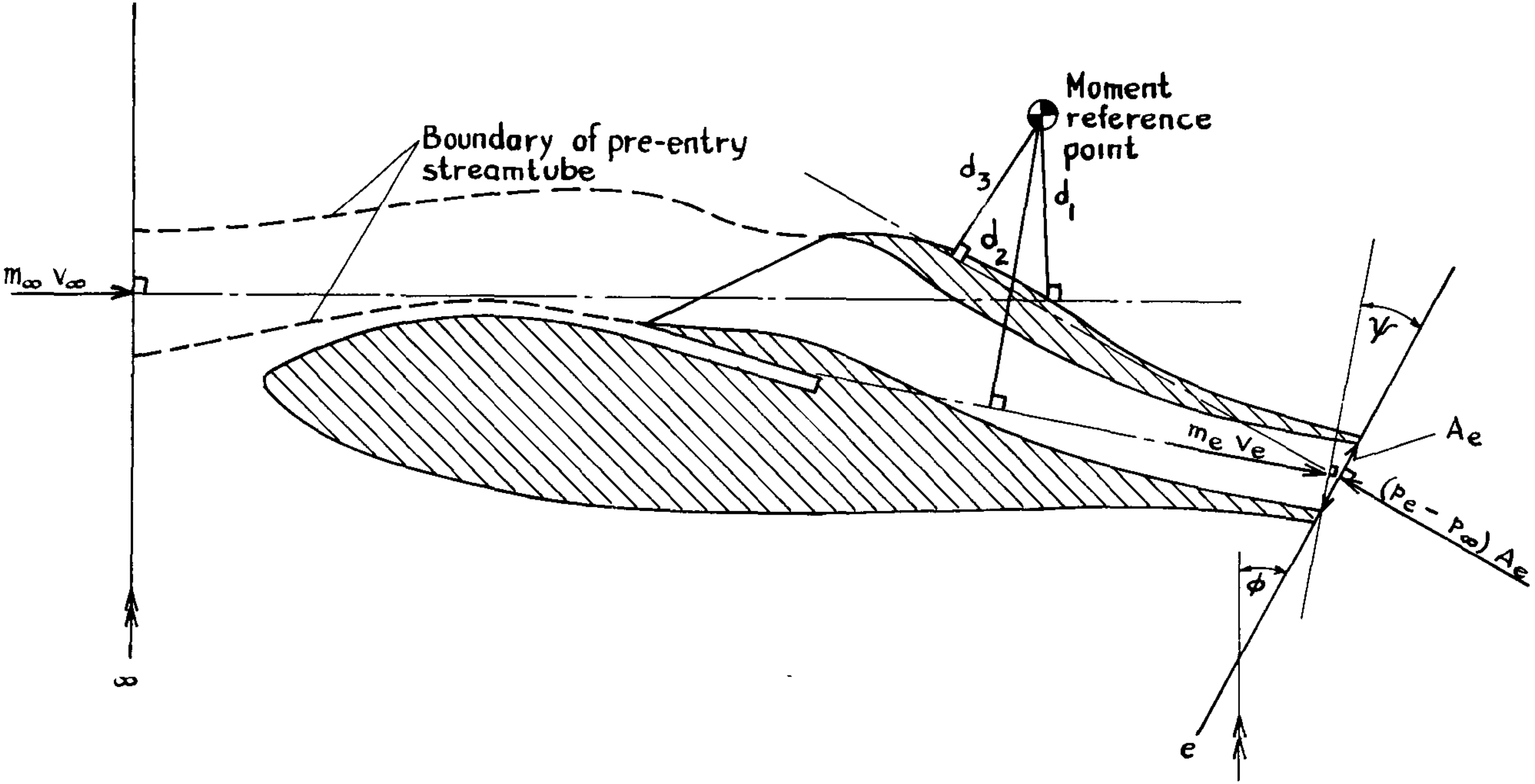


Fig 5 More general ducted configuration

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