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A PREDICTION METHOD FOR PRESSURE

DISTRIBUTIONS ON COMPRESSION SURFACES

OF CONICAL BODIES AT SUPERSONIC SPEEDS

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1974

PRICE 50p NET





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* CP No.1295 March 1973

A PREDICTION METHOD FOR PRESSURE DISTRIBUTIONS ON COMPRESSION SURFACES OF CONICAL BODIES AT SUPERSONIC SPEEDS

Ъy

M. J. Larcombe

SUMMARY

An approximation to the flow on the compression surfaces of delta wings and conical bodies is made by a new simple method. Surface pressure distributions are obtained by an interpolation procedure between circularly conical flow and two-dimensional flow. The technique is applicable to a wide variety of body geometries and flow conditions including cases with either attached or detached shock waves at the leading edges.

* Replaces RAE Technical Memorandum Aero 1457 - ARC 34849

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1 INTRODUCTION

Many methods for calculating the flow over delta wings and conical bodies have been proposed but few are capable of producing accurate quick estimates of, for example, the surface pressure distribution either because of the complexity of the calculation or because of the limited range of conditions for which the methods are applicable. Frequently the well-established Newtonian or tangentcone theories have been used in situations where their accuracy is seriously limited. Many modifications to these theories have been made to improve their range of applicability and accuracy in view of their simplicity.

The introduction of thin shock layer theory by Messiter¹ and its development by Squire^{2,3} has provided an alternative which gives a relatively straightforward method of calculating pressure distributions over a variety of bodies. Although the method uses a hypersonic approximation it has been shown also to have an adequate accuracy at lower supersonic Mach numbers. However, for certain types of body at low incidences the accuracy diminishes rapidly, and in some cases a solution cannot be found.

More recently Woods⁴ and Roe⁵ have further developed the thin shock layer theory to study the problem of flows with attached shock waves at the leading edges.

The method presented here relies on established experimental data to determine the general characteristics of the flow field and hence determine its affinity to other known flow fields, in contrast to the more usual practice of proposing a completed mathematical model of the flow field. The method is based on a technique that was originally developed for correlating and predicting the pressures along the ridge lines of conical bodies⁶.

2 DEVELOPMENT OF THE METHOD

For a body with triangular or diamond cross-section the pressures over the compression surface, particularly near the ridge line, could not be predicted with any accuracy using existing simple methods such as tangent-cone or Newtonian theory. The experimental data obtained^{7,11} at a Mach number of 4.0 which are presented in Fig.1 show that for such bodies the included angle of the ridge $(2\zeta_0)$ is a dominant parameter, the pressure at the ridge line increasing with the value of ζ_0 . Only local conditions at the ridge line are required in order to obtain the value of C_p . Following Ref.6, it is demonstrated in Fig.2 that the separate experimental results for all ridge angles in Fig.1 can be correlated with sufficient accuracy by the parameter $\delta_r (\sin \zeta_0)^{\frac{1}{2}}$, where δ_r is the inclination of the ridge line to the free-stream direction.

The data in Figs.1 and 2 have been restricted to bodies which have aspect ratios less than 4/3 because the effect of increasing aspect ratio above this limit is to increase the pressure at the ridge line for a fixed ridge angle and ridge inclination. This effect is only noticeable on the flatter wings $(2\zeta_0 \rightarrow 180^\circ)$ which have aspect ratios greater than unity along with large ridge inclinations so that the flow becomes locally more twodimensional in character. Therefore, for the majority of applications, the pressure coefficient at the ridge line can be correlated with sufficient accuracy using the equation

$$C_{p} = f\left(\delta_{r} \left(\sin \zeta_{0}\right)^{\frac{1}{2}}\right) . \tag{1}$$

A prediction of the pressure coefficient at the ridge line can be achieved by assuming that the flow in the neighbourhood of the ridge line is similar to that over an equivalent circular cone at zero incidence where the cone semi-vertex angle is given by

 $\sigma = \delta_r (\sin \zeta_0)^{\frac{1}{2}} .$

Pressure coefficients calculated using this principle are also plotted in Fig.2 for a Mach number of 4.0. The effects of free stream Mach number on the pressure coefficient, although small for circularly conical flow, are automatically included. A comparison with experimental data⁶ demonstrates that the computed values of the pressure coefficient at the ridge line follow the same variation with Mach number as the experimental results.

The pressure distributions over complete compression surfaces of conical bodies can be approximated by representing a generator in the surface of the body by an equivalent circular cone at zero incidence. The semi-vertex angle of the equivalent cone (σ) can be found by application of equation (1) and is given by

$$\sigma = \delta (\sin \zeta)^{\frac{1}{2}}$$
 (2)

where δ is the angle between a generator in the surface and the free-stream direction and ζ is the local inclination of the surface with respect to the free-stream plane passing through the generator. The angles δ and ζ are

shown diagrammatically in Fig.3 and a derivation of the equations for computing these angles is given in the Appendix.

The method outlined above is principally suitable for bodies with detached shock waves that can be considered as a perturbation from circularly conical flow. For example, calculated pressure distributions over a conical body with a cross-section consisting of an equilateral triangle are compared with experiment¹¹ in Fig.4. These show particularly good agreement with experimental values and also indicate the deficiencies of tangent-cone and Newtonian theories which would predict a constant pressure across the span. The success is related to the fact that even when the body cross-section is far-removed from circular, the shape of the enveloping shock wave represents only a small perturbation from a circular cross-section. The shape of the shock wave obtained experimentally¹² for the same model and flow conditions as for the surface pressure data is shown in Fig.5.

The perturbations from circularly conical flow produced by caret wings, even at the off-design condition with detached shock waves at the leading edges, are too large for the above method to be of sufficient accuracy. A perturbation from two-dimensional flow should provide a more realistic flow pattern and hence an improved accuracy. Also, as mentioned previously, the effect of increasing aspect ratio for flatter wings is to increase the pressure near the centre-line. This pressure approaches the value predicted from two-dimensional oblique shock wave theory when applied in the axial direction so that an improvement could be expected for high aspect ratio flat wings.

Thus, there are a number of applications for which perturbations from two-dimensional theory would be a better approximation. To deal with these situations the present method has been developed to allow for perturbations from both circularly conical flow and two-dimensional flow by an interpolation between these two exact flow processes.

The cross-section shape of the body, its aspect ratio and also its incidence, determine the proximity of the flow to either two-dimensional or circularly conical. Therefore an effective cross-section area A of the body and flow (Fig.6a) which takes into account the above parameters is proposed as the variable for interpolation. The cross-section defining the area for the limiting case of circularly conical flow A_c is the sector of the circle passing through the leading edges with centre on the free-stream axis (Fig.6b),

thus representing a section of a circular cone at zero incidence. At all incidences the limiting flow is considered to be equivalent to that around a cone of semi-angle equal to the angle γ between the leading edge and the free-stream direction. The area A_0 for the limit of two-dimensional flow is obtained from an effective cross-section of the body on the assumption that a plane oblique shock wave exists between the leading edges (Fig.6c) and can be produced by a particular caret wing at the design condition.

The pressure coefficient can then be approximated by simple linear interpolation as

$$C_{p} = C_{p} + \frac{(A_{c} - A)}{(A_{c} - A_{0})} (C_{p_{0}} - C_{p_{c}})$$
(3)

where C_{p_c} is the pressure on a circular cone of semi-angle σ given by equation (2) and, for bodies with flat facets, is variable across the span. C_{p_0} is the pressure on a two-dimensional wedge whose inclination is equal to the inclination of the ridge line δ_r .

The value of $(A_c - A)/(A_c - A_0)$ should vary between zero and unity and determines the affinity of the body with either a circular cone or a caret wing at the design condition. For a given body cross-section the value increases towards the two-dimensional limit (unity) with increasing aspect ratio. This trend towards a more two-dimensional flow is in accordance with the experimental results of Ref.7. Also, as incidence is increased the value decreases such that the flow tends towards circularly conical. These effects are illustrated in Fig.7 in which values of $(A_c - A)/(A_c - A_0)$ are plotted for two types of body cross-section and two aspect ratios.

The free-stream Mach number has a significant influence on the value of $\begin{pmatrix} A_c - A \end{pmatrix} / \begin{pmatrix} A_c - A_0 \end{pmatrix}$ owing to variations of A_0 . However, when the angle between the free-stream direction and the plane containing the leading edges is less than the Mach angle a value for A_0 cannot be found because, with reference to Fig.6c, a shock wave cannot lie between the leading edges at angles below the Mach angle. It is therefore assumed for these conditions that the value of A_0 is zero and correspondingly there is no effect of Mach number on the value of $\begin{pmatrix} A_c - A \end{pmatrix} / \begin{pmatrix} A_c - A \\ c \end{pmatrix}$ for angles less than the Mach angle corresponding to the highest Mach number considered. This effect is illustrated in Fig.7 where, for a fixed body geometry, the values collapse onto a single curve independent of Mach number for incidences less than the Mach angle whereas for incidences in excess of this value $\begin{pmatrix} A_c - A \\ C \end{pmatrix} / \begin{pmatrix} A_c - A \\ C \end{pmatrix}$ increases with Mach number.

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It is also possible, depending on the body cross-section, for the value of A to be either less than A_0 or greater than A_c . Rather than use extrapolation under these circumstances the following assumptions are made. If $A > A_c$ then A is put equal to A_c , and if $A < A_0$ then A is put equal to A_c . These assumptions have the effect of restricting the approximation to the flow process within the two-dimensional and circularly conical limits.

It should be noted that the value of C_{PO} in equation (3) generally is not the same as the pressure coefficient that would be obtained on the assumption of an oblique shock-wave lying between the leading edges. The latter condition is used solely for determining a body cross-section area for the twodimensional limit. However, when dealing with a caret wing at the design condition the two values of C_p are identical. Similar reasoning also applies when considering the value of C_{PC} . The basic values of C_{PC} and C_{PO} are shown in Fig.8 together with calculated surface pressure distributions for a flat delta wing.

Further calculations of surface pressure distributions using the method outlined above are presented in Ref.8 for four delta wings. These wings were proposed by Roe⁹ in order to provide a basis for comparing various prediction methods.

3 REFINEMENTS TO THE GENERAL METHOD

The existing method uses a two-dimensional flow as one of the limits of an interpolation procedure principally to improve the solution for flows with detached shock waves. However the method is not expected to be particularly accurate for large aspect ratio delta wings with the shock waves attached to the leading edges. A more accurate method by Pike¹⁰, which is only capable of dealing with attached shock configurations, predicts a region of constant pressure near the leading edge which is obtained from oblique shock wave theory. For such conditions however, the present method does give an indication of the occurrence of an attached shock condition, since the distributions of C_{P_C} and C_{P_0} across the span are found to intersect. An example is shown in Fig.9 for a wing of aspect ratio 3.36 at a Mach number of 4.0. From a comparison with the results of Pike it appears that an improvement to the approximation of the pressure distribution can be made by taking only the twodimensional solution, C_{P_0} , outboard of the point of intersection of the pressure distributions. This implies the need for a spanwise variation of the interpolation variable whereas so far it has been assumed constant. It should also be possible to allow for spanwise variations for more complex body shapes provided sufficient experimental data becomes available to confirm the validity of the technique.

A two-dimensional solution based on conditions along the centre-line produces a shock wave appropriate to an infinite unswept wedge (Fig. 10b) and does not provide a realistic shock-wave shape near the leading edges. However, a two-dimensional swept wedge solution near the leading edges, with a consequently higher value of C_{p} , appears a more appropriate solution, giving an attached shock wave at the leading edges as shown in Fig. 10a. The various shock-wave shapes are illustrated in cross-section in Fig.11 for the same delta wing as in Fig.9. Although a realistic shock-wave shape is obtained near the leading edges it is not possible to match the conical and oblique shock solutions with such a simple approach. Similar difficulties in matching solutions and shock wave shapes are also encountered in the more complex solutions^{3,4,5} to the problem of delta wings with attached shock waves. It is clear that a significant improvement to the method, and to the results of Ref.8, can be made by adopting the oblique shock solution appropriate to a swept wedge without any loss of generality. In Fig.12 a comparison with the results of Squire, Pike and Roe is made showing the improvement that can be achieved.

4 CONCLUSIONS

A new simple approach to the problem of determining pressure distributions on delta wings and conical bodies is proposed. The flow existing over the compression surface of the body is approximated by an interpolation procedure between circularly conical flow and two-dimensional flow. The method is particularly suitable for bodies with detached shock waves but is capable of being applied to a wide range of conical bodies over a large range of Mach number and incidence. Further improvements allow flows with attached shock waves to be calculated with reasonable accuracy.

Appendix

DERIVATION OF EQUATIONS FOR COMPUTING THE ANGLES $~\delta~~$ And $~\zeta~~$

The angles are shown in Fig.3 together with the axis system and nomenclature for the coordinates of various points in the free-stream direction and on a simplified body consisting of flat facets. In order to allow for variation of body geometry and attitude the coordinates of these points are used as data.

In this case the axis system OXYZ is fixed in the base of the body which has unit chord such that the vertex (point 1) has coordinates (1, 0, 0) and the free-stream vector also passes through this point. A facet of the model is then completely defined by points 5 and 6 in the Y-Z plane of the base. Point 2 is variable across the facet and the coordinates of point 3 define the free-stream direction and hence the attitude of the body.

The following table shows the fixed and variable data.

Point	x	Y	Z
1	1	0	0
2	0	y(z ₅)	$z_{6} - \bar{y}(z_{6} - z_{5})$
3	0	0	tan α
5	0	Coordinates of leading edge in Y-Z plane	
6	0	Coordinate	s of lower ridge in Y-Z plane

Coordinates of points along axis directions

The value of $\overline{y} = y/b$ can vary from zero to unity across the span for this simplified model.

The angle δ between the free-stream direction, specified by the line joining points 1 and 3, and the generator in the surface specified by the line joining points 1 and 2, is given by

$$\cos \delta = \ell \ell_f + m m_f + n n_f \tag{4}$$

where l_f , m_f , n_f are the direction cosines of the free-stream and l, m, n are the direction cosines of the generator. The direction cosines can be obtained from the coordinates as:-

and substituted into equation (4) to obtain δ .

The angle ζ is the angle between the plane of the wing, containing points 1, 2, 5 and 6, and the free-stream plane passing through the generator in the surface, containing points 1, 2 and 3.

If the equation of the plane of the wing is

$$A_{1}x + B_{1}y + C_{1}z + D_{1} = 0$$
 (7)

and the equation of the free-stream plane is

$$A_2 x + B_2 y + C_2 z + D_2 = 0$$
 (8)

then the angle ζ between them is given by

Appendix

$$\cos \zeta = \frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)\left(A_2^2 + B_2^2 + C_2^2\right)}}$$
(9)

The coefficients in equation (7) in terms of the coordinates of points 1, 5 and 6, are

$$A_{1} = \begin{vmatrix} y_{1} & z_{1} & 1 \\ y_{5} & z_{5} & 1 \\ y_{6} & z_{6} & 1 \end{vmatrix} - B_{1} = \begin{vmatrix} x_{1} & z_{1} & 1 \\ x_{5} & z_{5} & 1 \\ x_{6} & z_{6} & 1 \end{vmatrix} - C_{1} = \begin{vmatrix} x_{1} & y_{1} & 1 \\ x_{5} & y_{5} & 1 \\ x_{6} & y_{6} & 1 \end{vmatrix}$$
(10)

and the coefficients in equation (8) in terms of the coordinates of points 1, 2 and 3 are

$$A_{2} = \begin{vmatrix} y_{1} & z_{1} & 1 \\ y_{2} & z_{2} & 1 \\ y_{3} & z_{3} & 1 \end{vmatrix} - B_{2} = \begin{vmatrix} x_{1} & z_{1} & 1 \\ x_{2} & z_{2} & 1 \\ x_{3} & z_{3} & 1 \end{vmatrix} - C_{2} = \begin{vmatrix} x_{1} & y_{1} & 1 \\ x_{2} & y_{2} & 1 \\ x_{3} & y_{3} & 1 \end{vmatrix} .$$
(11)

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The value of ζ can then be calculated after evaluation of the determinants and substitution into equation (9).

The angles for more complex body shapes consisting of curved crosssections can be computed using a similar technique if the body is defined by separate tangent planes with the generator in the surface fixed at each point of tangency. In this situation it is convenient to define each tangent plane in terms of its intercepts with the coordinate axes as for the simple body shown in Fig.3.

The effects of yaw can also be incorporated by inserting the value - tan β for the y coordinate of point 3, where β is the angle of sideslip and the tangent definition for both incidence and sideslip have been adopted.

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SYMBOLS

A	effective cross-section area of body, see Fig.6a
A c	cross-section area of equivalent body for circularly conical flow, see Fig.6b
A ₀	cross-section area of equivalent body for two-dimensional flow, see Fig.6c
с _р	pressure coefficient
°p _c	pressure coefficient on a circular cone of semi-angle σ
с _{р0}	pressure coefficient on a two-dimensional wedge inclined at angle $\begin{pmatrix} \delta \\ r \end{pmatrix}$
γ	angle between the leading edge and the free-stream direction defining the semi-angle of an equivalent circular cone, Fig.6b
δ	angle between a generator in the surface of the body and the free-stream direction
δr	angle between the ridge line of a body and the free-stream direction
ζ	local inclination of the surface with respect to the free-stream plane passing through the generator, Fig.3
ζ0	inclination of the surface with respect to the free-stream plane passing through the ridge line, which is equivalent to half the included angle of the ridge, Fig.1

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Fig.l Pressure coefficient at ridge line on low aspect-ratio conical bodies at $M_{\infty} = 4$



Fig. 2 Correlation of pressure coefficient at ridge line of conical bodies at $M_{co} = 4$





Fig. 4 Comparison of experimental pressure distributions with the predicted values for the lower surface of a triangular section delta wing. AR = 1.05



 $M_{\infty} = 4.0$, incidence 8.8° , aspect ratio 1.05

Fig.5 Cross-section of shock wave in a plane perpendicular to the free-stream direction



Fig.6a-c Cross-section areas of flow and body for interpolation

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Fig. 7 Variation of interpolation parameter with incidence, Mach number, body shape and aspect ratio



Fig.8 Pressure distributions on a thin delta wing of aspect ratio 1.07 at M=10



Fig. 9 Basic solutions for a flat delta wing of aspect ratio 3.36 at M=4.0 and α =15°





- C Shock-wave due to swept wedge based on conditions at leading edge
- b Shock-wave due to unswept wedge based on conditions along centre-line

Fig. 10 a & b Shock wave shapes on a thin delta wing

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Fig.II Cross-section of shock wave shapes on a thin delta wing of aspect ratio 3.36 at M = 4.0 and $\alpha = 15^{\circ}$



delta wing of aspect ratio 3.36 at M = 4.0 and $\alpha = 15^{\circ}$

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ISBN 011 470879 7