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An Application of Fast Frequency-Sweep Excitation  
to the Measurement of Sub-Critical Response  
of a Low Speed Wind Tunnel Model

by

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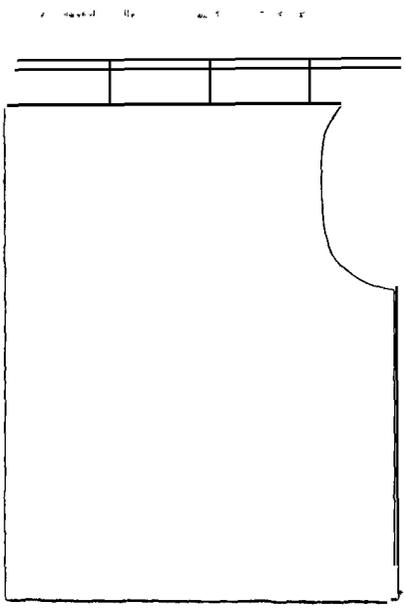
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AN APPLICATION OF FAST FREQUENCY-SWEEP EXCITATION TO THE MEASUREMENT OF  
SUB-CRITICAL RESPONSE OF A LOW-SPEED WIND TUNNEL MODEL

by

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D..R. Gaukroger

SUMMARY

Transient excitation has been applied to a low-speed flutter model in order to determine the sub-critical response characteristics. The transient input was a fast frequency-sweep and the resulting response and force input records were processed using digital Fourier analysis techniques. The results are compared with those obtained from sustained sinusoidal excitation. Comparisons are given for the frequency and damping characteristics of the first four modes of the model. Frequencies are in reasonable agreement between the two sets of results but the high degree of scatter in the values of damping ratio makes detailed comparison difficult.

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## 1 INTRODUCTION

Some years ago, a series of wind tunnel tests was made on a swept-wing flutter model to determine its sub-critical response characteristics'. The tests were made by exciting the model with sustained sinusoidal forces, and measuring the vector responses. The response measurements **were** displayed as vector plots, and, from these, the modal frequencies and damping ratios were derived by graphical analysis. At the time the tests were made, there was a stirring of interest in the application to flutter testing, of new techniques, particularly those which offered the possibility of reducing the duration of tests, or of improving the accuracy of test results which **were** contaminated with extraneous noise. The adoption of random process analysis techniques, coupled with transient excitation, appeared to offer improvements over existing techniques in both these respects. In order to obtain test data which could be used to assess the potential of new techniques, the swept-wing model of Ref.1 was subjected to a further series of tests in which the model was excited by an oscillatory force which swept rapidly through a range of frequency (the 'fast frequency-sweep'). Both the force input and the model response signals were recorded on magnetic tape at each aerodynamic test condition that was investigated.

Unfortunately, lack of adequate analysis equipment prevented the making of a full analysis of the records. At that time, the analysis could only be made on an analogue basis, and processes such as **auto-** and cross-correlation of signals were found to make unacceptable demands on manpower with the large quantities of data that were involved. The tape records were therefore stored, to await the provision of improved analysis facilities. These facilities are now available in the form of a digital Fourier analyser, and the records have been analysed.

The analysis that has now been made differs from what was originally intended. This is hardly surprising in view of the intervening developments in the application of digital techniques to dynamic testing and analysis. In fact, the analysis has been made with two main aims in mind. The first of these was to provide sub-critical characteristics for the model that could be compared with the test results of Ref.1. The second was to contribute to the RAE research programme on methods of making flutter tests of short **duration**<sup>2</sup>.

In this Report, two methods of analysis have been applied. The first method, which is described in some detail, makes use of the time-histories of

both the frequency-sweep excitation and of the response, and enables the modal frequencies and damping ratios to be determined when, in addition to the frequency-sweep, there is some degree of random background excitation. The second method, described in detail in Ref.2, and outlined in this Report, makes use only of the time-history of the response and is based on the assumption that the spectra of the frequency-sweep and of any background excitation is sensibly flat within the bandwidth of each mode.

The results from these methods of analysis are compared with those of the earlier tests<sup>1</sup>, and show reasonable agreement, bearing in mind the degree of scatter that is evident in all the test results.

## 2 METHOD OF RECORD ANALYSIS

### 2.1 Requirements

The basic requirement of the analysis process is that the end-product should be accurate values of the natural frequencies and damping ratios of all the modes which might significantly affect the flutter characteristics of the model.

Several methods of analysis are considered in Ref.2 for handling typical response data from wind tunnel or flight flutter tests. The method which is likely to give optimum results depends on the test conditions and, in particular, on the level of background excitation to which the system under test is subjected. The investigation described in Ref.2 concluded that over a wide range of random background excitation levels, the preferred technique of test is to apply external excitation in the form of a fast frequency sweep, and to analyse the response via the autocorrelation function of the response. A knowledge of the input sweep is not needed, although the spectrum of the input must be sensibly flat over the bandwidth of each modal resonance.

Where the level of response to background excitation is relatively low compared to the level produced by the frequency sweep, (or by other forms of transient excitation), satisfactory results may be obtained by dividing the Fourier transform of the response by that of the excitation in order to derive a transfer function of the system. (This technique was only indirectly demonstrated in Ref.2 by using an impulse to excite the system, thus obtaining a transfer function without the need for Fourier transformation and subsequent division of response by the excitation.) In the present analysis, it was judged that, despite the significant level of background excitation, it would

be of interest to apply a modified form of the technique and the test records have therefore been analysed by initially dividing the Fourier transform of the response to the frequency-sweep excitation by the Fourier transform of the excitation.

White<sup>4</sup>, and others, have shown that in the presence of extraneous noise, the derivation of the transfer function in this way can lead to significant errors in the values of the modal characteristics. The errors are characterised on the vector plot (i.e. on the Argand presentation of the transfer function) by a random scatter-ing of the plotted points about their true positions. In order to avoid this difficulty, a smoothing process has been applied in obtaining the transfer function. The resulting vector plots are virtually free of scatter, and are readily analysed by graphical methods. This method of analysis, which is fully described in the next section, is complementary to the methods of Ref.2, in that it is an alternative which can be used in fast flutter testing when the waveform of the transient input is known.

In order to compare analysis techniques, the test records have also been analysed by the method recommended in Ref.2 using the autocorrelation function of the response, To avoid confusion, this method (which does not directly make use of the time-history of the excitation) is subsequently referred to in this Report on the 'indirect' analysis, and the method involving the Fourier transforms of both excitation and response is referred to as the 'direct' analysis.

## 2.2 Details of 'direct' analysis

Consider a system subjected at point P to a force  $f_P(t)$  in the finite time interval  $0 < t < T$ . The system is also subjected to a random force which may be assumed to act spatially over the system and may be represented by a generalised force  $n(t)$  which, for simplicity of analysis, will be assumed to be applied only during the period of application of  $f_P(t)$ . Such a system is an approximate representation of a wind tunnel model which is subject to both deliberate excitation and excitation from unsteadiness in the tunnel flow.

Let the response at Q due to a unit impulse at P be  $h_{PQ}(t)$ , and the response at Q due to a unit impulse applied to the system in the same spatial loading pattern on the system as the random input, be  $h_{NQ}(t)$ . Then, using the convolution integral, and superimposing the responses to  $f_P(t)$  and  $n(t)$ , the total response at Q is,

$$g_Q(t) = \int_{-\infty}^{+\infty} h_{PQ}(\tau) f_P(t - \tau) d\tau + \int_{-\infty}^{+\infty} h_{NQ}(\tau) n(t - \tau) d\tau . \quad (1)$$

Taking the Fourier transforms of both sides of equation (1) gives:-

$$G_Q(w) = H_{PQ}(w)F_P(w) + H_{NQ}(w)N(w) \quad (2)$$

where  $G_Q(w)$ ,  $H_{PQ}(w)$ ,  $F_P(w)$ ,  $H_{NQ}(w)$  and  $N(w)$  are the Fourier transforms of  $g_Q(t)$ ,  $h_{PQ}(t)$ ,  $f,(t)$ ,  $h_{NQ}(t)$  and  $n(t)$  respectively. Equation (2) may be rewritten:-

$$\frac{G_Q(w)}{F_P(w)} = H_{PQ}(w) + \left( H_{NQ}(w) \frac{N(w)}{F_P(w)} \right) = R(w) \quad , \quad \text{say} \quad . \quad (3)$$

In the wind tunnel tests of this Report,  $g_Q(t)$  and  $f,(t)$  are measured, so that  $R(w)$  may be determined, and the problem is to obtain  $H_{PQ}(w)$  when  $N(w)$  is unknown. ( $H_{NQ}(w)$  is also unknown, but it contains the same modal characteristics as  $H_{PQ}(w)$  and it would be possible to replace it by  $H_{PQ}(w)$  by letting  $H_{NQ}(w)N(w) = H_{PQ}(w)M(w)$ . However, this is unnecessary, as will be shown.)

If, in equation (3),  $N(w)$  is zero (i.e. there is no force input  $n(t)$  due to flow unsteadiness) then

$$\frac{G_Q(w)}{F_P(w)} = H_{PQ}(w) \quad (4)$$

and the transfer function  $H_{PQ}(w)$  will, in the complex plane, take the form of the well-known vector response plot from which modal frequencies and **dampings** may be determined. If  $N(w)$  is not zero, then the evaluation of  $G_Q(w)/F_P(w)$  will result in a vector plot in which the vector at each point (that is to say, at each frequency) is the sum of the vectors of  $H_{PQ}(w)$  and  $\left( H_{NQ}(w)(N(w)/F_P(w)) \right)$ . Now  $N(w)$  has random variation of phase with frequency, and the vectors of  $\left( H_{NQ}(w)(N(w)/F_P(w)) \right)$  therefore appear on the vector plot as scatter of the plotted points; the vector lengths of  $\left( H_{NQ}(w)(N(w)/F_P(w)) \right)$  are proportional to the amplitudes of  $N(w)$  as well as being functions of  $H_{NQ}(w)/F_P(w)$ . In other words, the amount of scatter of the points on the vector plot is proportional to the noise-to-signal ratio of the exciting forces.

Now, taking the inverse Fourier transforms of both sides of equation (3),

$$r(t) = h_{PQ}(t) + h_{NQ}(t) * \bar{n}(t) \quad (5)$$

where  $r(t)$  is the inverse Fourier transform of  $R(w)$ ,  $\bar{n}(t)$  is the inverse Fourier transform of  $N(w)/F_P(w)$ , and  $*$  denotes convolution. If it is assumed

that the spectra of both  $n(t)$  and  $f_p(t)$  are smooth, then  $N(w)/F_p(w)$  will have the random characteristics of  $N(w)$ , and  $\bar{n}(t)$  will be a continuous function with time having the random characteristics of  $n(t)$ . It can now be seen that  $r(t)$  is composed of the sum of the impulse response  $h_{pQ}(t)$ , which decays to a small value after a finite time, and the response of the system to a continuous input  $\bar{n}(t)$ , which produces a continuous random response at a (statistically) constant level.

When  $r(t)$  is obtained in practice, the response of the system is measured from a time datum coinciding with the application of the transient force  $f_p(t)$ ; the measurement is continued for a time that is much greater than that required for the transient response to fall to the level of the continuous response. Thus  $r(t)$  is a finite record consisting of an initial transient response superimposed on a continuous response which extends to the end of the record.

A weighting function may now be applied to the record of  $r(t)$ , and it is convenient to make this an exponential function because subsequent correction is facilitated. The record is therefore multiplied by a function of the form  $Ae^{-\lambda t}$ . The effect of this is to modify the transient response, and, more important, to reduce the level of continuous response (following the decay of the transient) by a large factor. If the process is examined on the basis of energy levels, it can be seen that the application of exponential weighting reduces the energy in the transient, but the energy in the continuous response is reduced by a much larger factor. This is because the continuous response extends over the whole length of the record, whereas the transient response is significant only in the initial portion where the weighting function has less effect. Thus the weighting function improves the signal-to-noise ratio in the new function  $r'(t) = r(t)Ae^{-\lambda t}$ . Equation (5) may now be written

$$r'(t) = Ae^{-\lambda t}h_{pQ}(t) + Ae^{-\lambda t}(h_{nQ}(t) * \bar{n}(t)) . \quad (6)$$

The final step is to take the Fourier transform of both sides of equation (6) to obtain:-

$$R'(w) = H'_{pQ}(w) + S(w) \quad (7)$$

where  $R'(w)$ ,  $H'_{pQ}(w)$  and  $S(w)$  are respectively Fourier transforms of  $Ae^{-\lambda t}r(t)$ ,  $Ae^{-\lambda t}h_{pQ}(t)$  and  $Ae^{-\lambda t}(h_{nQ}(t) * \bar{n}(t))$ .  $R'(w)$  is composed of the sum of weighted transfer function  $H'_{pQ}(w)$  and the function  $S(w)$  which is now

small compared to  $H'_{PQ}(w)$  . The scatter in the plot is therefore less than in the original plot of  $R(w)$  .

It can be shown that the effect of an exponential weighting term on  $H_{PQ}(w)$  is to increase the decay rate of all the modal components. The values of modal frequency and decay rate can thus be obtained from analysis of the vector plot of  $R'(w)$  , followed by a correction to the decay rates, by subtracting the decay rate of the exponential weighting.

### 2.3 Summary of 'direct' analysis process

For each test, records of the sweep-frequency excitation  $f_p(t)$  at P , and of the response  $g_Q(t)$  at Q were taken. The analysis carried out on the Fourier analyser then consisted of the following steps:-

- (1) Digitise  $f_p(t)$  and  $g_Q(t)$  , and obtain their Fourier transforms  $F_P(w)$  and  $G_Q(w)$  respectively.
- (2) Calculate  $R(w)$  from  $R(w) = (G_Q(w))/(F_P(w))$  .
- (3) Obtain the inverse Fourier transform  $r(t)$  of  $R(w)$  .
- (4) Apply an exponential weighting function  $Ae^{-\lambda t}$  to  $r(t)$  giving
 
$$r'(t) = r(t)Ae^{-\lambda t}$$
- (5) Obtain the Fourier transform  $R'(w)$  of  $r'(t)$  .
- (6) Display  $R'(w)$  as a vector plot, and analyse it to derive the frequencies and decay rates of the modal components.
- (7) Correct the values of decay rates by subtracting the decay rate of the exponential weighting function.

### 2.4 Summary of 'indirect' analysis

The 'indirect' method of analysis was identical to that used in Ref.2, and consisted of the following stages:-

- (1) Form the autocorrelogram of the response.
- (2) Apply an exponential weighting function to the autocorrelogram.
- (3) Obtain the Fourier transform of half the weighted autocorrelogram (associated with positive lag values).
- (4) Analyse the resultant vector plot to derive frequency and decay rates.
- (5) Correct the decay rates for exponential weighting.

### 3 WIND TUNNEL MODEL AND TESTS

Full details of the model are given in **Ref.1**, and only an outline description is given here.

The model was a half-span, straight-tapered wing with a leading-edge sweepback of  $28^{\circ}47'$ . The **planform** is shown in **Fig.1**. The basic structure was an aluminium alloy spar which carried eighteen segments projecting fore-and-aft. The spar and segments were covered with Flexalkyd plastic foam, which was foamed in a mould of the external wing shape.

The model was mounted vertically in the RAE 5ft wind tunnel with the tip uppermost. The root was attached to a substantial base plate, and a reflector plate was fitted at the inboard end of the model.

A small electromagnetic exciter was connected to the model in the position shown in **Fig.1**. Close to this position, a displacement transducer was mounted, and a set of semi-conductor strain gauges was bonded to the spar at the root.

The electromagnetic exciter was driven by a current amplifier having a high impedance output, and the amplifier input was taken from a sweep-frequency oscillator. The force applied to the model by the exciter was proportional to the current in the exciter coil, which was proportional to the voltage across the feedback resistor in the amplifier. This voltage was used as the force input signal. The force input signals and the response signals of either the displacement or strain gauge transducer were recorded on magnetic tape using a two-channel FM recorder.

Tests were made on the model at five values of airspeed 30.5, 36.6, 39.6, 42.6 and 46.1 m/s and also at zero airspeed. At each airspeed condition, two frequency sweeps of force input were made. For each frequency sweep the force input and the response of one of the transducers was measured, the other transducer signal being recorded with a subsequent sweep. The sweep frequency was from 0 Hz to 37.5 Hz, and the rate of change of frequency with time was constant. The sweep duration was 35 s.

### 4 TEST RESULTS

#### 4.1 'Direct' analysis

The records were analysed, following the process described in sections 2.2 and 2.3. The analysis was carried out on a Hewlett-Packard Digital Fourier Analyser. The analyser is equipped with two-channel input, so that the force and response signals for each record were replayed simultaneously to the analyser.

To illustrate the various stages in the analysis and some of the points that were mentioned in section 2.2, Figs.2 to 5 have been prepared from photographs of the display oscilloscope in the analyser.

Fig.2a shows the strain-gauge response and the force input signals at zero airspeed. It will be seen that the peak force level was constant throughout the frequency sweep. (The 'loops' in the force signal result from coincidence of multiples of sweep frequency with sampling rate.) The resonance frequencies of the model were known to be 1.73, 7.05, 14.17, 16.92 Hz for the first four modes, and there were two close frequencies between 30 and 33 Hz. The response signal in Fig.2a clearly shows the modal peak at 7.05 Hz, and a compound peak for the 14.17 and 16.92Hz modes as well as the modes near 30 Hz. The response in the fundamental mode at 1.73 Hz is not obvious as a separate peak, although there is clearly a significant level of response. The reason for this is that the force input sweeps through the bandwidth of a low-frequency mode very rapidly, and the mode does not have time to build up to a large response amplitude. (A mode having the same value of damping ratio, but twice the natural frequency, would experience excitation in its bandwidth for twice the time and four times the number of cycles.)

Fig.2b shows the strain gauge response at an airspeed of 36.6 m/s. The separate modal peaks are almost lost in the broad-band response to tunnel unsteadiness although some increase in the general level can be seen at approximately 17 and 30 Hz. When the ratio of the Fourier transforms of the signals in Fig.2b is computed the vector plot shown in Fig.3a is obtained. Analysis of this plot to derive modal frequencies and decay rates is clearly out of the question. However, the application of an exponential weighting function, following the procedure of section 2.2 leads to the vector plot of Fig.3b which presents no analysis difficulties.

Fig.4a shows the force input and displacement transducer response at an airspeed of 45.7 m/s and Fig.4b is the corresponding vector plot before weighting. When the Fourier transform of the plot is computed, the resulting function  $r(t)$  of section 2.2) is as shown in Fig.5a. It will be seen that the initial transient decay is followed by a steady mean level of random response. The shape of the exponential weighting function is also shown on Fig.5a and after the waveform has been multiplied by this weighting function, the inverse Fourier transform of the product  $R'(w)$  is the vector plot shown in Fig.5b. Comparison of Fig.5b and Fig.4b demonstrates the improvement resulting from the weighting process.

All the vector plots were analysed, using the RAE digital computer program<sup>5</sup>, to derive the natural frequencies and damping ratios of the first four modes of the model. The results are given in Table 1. It will be seen that the values obtained from the strain gauge and displacement transducers sometimes differ in frequency and often differ significantly in damping ratio. Omissions in Table 1 denote that the computer program failed to 'find' a mode from the vector plot. There is an obvious error in the value of frequency of mode 2 at 42.6 m/s and this value has been ignored, subsequently. The results can be more readily appreciated in graphical form, and Figs.6 and 7 show the values of frequency and damping ratio of Table 1 plotted against airspeed. Included in these figures are the results of calculations made for comparison with the tests of Ref.1. The calculations have been factored because the flutter speed and the natural frequencies of the modes at zero airspeed, at the time the fast-frequency sweep tests were made, were slightly lower than the corresponding values at the time of the tests of Ref.1. (The changes were almost certainly due to stiffness changes in the Flexalloyd foam covering.) The measured frequency values from Ref.1 have not been plotted on Fig.6 in order to avoid overcrowding the figure. However these frequency values were in good agreement with the calculations, and the 'calculation' curves on Fig.6 can be taken to represent the measured frequencies of the earlier tests. The individual test results from Ref.1 have been shown on the damping ratio graphs of Fig.7.

Examination of Fig.6 shows that the scatter in the modal frequency values is much greater for modes 1 and 2 than for 3 and 4. It is not altogether surprising that the values for mode 1 show a high degree of scatter, because the damping in that mode rises rapidly with airspeed and analysis of the vector plot is difficult. The scatter in mode 2 cannot be explained on this basis, and compares unfavourably with the results of Ref.1, all of which were close to the calculated values. The mean levels of frequency in the first three modes are somewhat higher than the calculated values; this again contrasts unfavourably with the tests of Ref.1.

Fig.7 shows that the values of damping ratio obtained from the frequency-sweep records are, in general, lower than the calculated values, but, with one exception, are in agreement with the values of Ref.1. However, in modes 1 and 2, the scatter is such that the results from both sets of tests delineate bands of damping ratio values rather than mean lines, and a detailed comparison of the results is impossible. The exception mentioned above occurs in the results for mode 3, where the tests of Ref.1 indicated a considerable peak in the damping

ratio curve at an airspeed of 45 m/s, whereas the frequency-sweep tests give results that are in good agreement with calculation, and show no evidence of the peak. It should be noted, however, that the existence of the peak was supported by several measurements at small increments of airspeed, but the frequency-sweep tests were not as comprehensive. The authors of Ref.1 suggested that the lack of agreement between calculation and experiment in the region of the peak might be due to inadequate structural or aerodynamic modelling in the calculations. The evidence from the results of the frequency-sweep tests, although by no means incontrovertible, does suggest that the peak may have been due to an unsuspected experimental error.

#### 4.2 'Indirect' analysis

The records of response to the frequency-sweep excitation were also analysed by the indirect method (see section 2.4). In order to preserve uniformity in the analysis processes, the same records, digitisation sampling rates, and weighting functions were used as for the 'direct' analyses.

The values of modal frequencies and damping ratios obtained from the 'indirect' analysis are given in Table 2. The frequency values have been plotted on Fig.6, and the damping ratios on Fig.8. (The values of damping ratio from the 'direct' analysis and the calculated values from Ref.1 are also shown for comparison.)

Comparison of Tables 1 and 2 shows that 'indirect' analysis resulted in a larger number of cases in which the computer program failed to 'find' a mode from the vector plot. Moreover, one or two of the results are seriously in error. (For example, values of 20.11 and 11.41 for the percentages of critical damping in the fundamental mode in still air are obviously nonsensical, and the frequency and damping ratio on mode 3 at 42.6 m/s are suspect.) On the whole, the values of modal frequency (Fig.6) are subject to the same level of scatter as for the 'direct' analysis, but the values of damping ratio (Fig.8) are, if anything, rather more scattered.

#### 5 DISCUSSION

It was stated in section 1 that the first aim of the frequency-sweep tests was to provide a set of sub-critical response characteristics for the model that could be compared with the test results of Ref.1. This aim has been satisfied, but although the comparison can be made, it cannot be said to do more than indicate that the quality of the results from the frequency-sweep tests is no worse than that from the tests with a sustained sinusoidal force. Both test

programmes have yielded results that leave much to be desired. For example, without the calculated curves in Fig.7, and without knowing from test that the observed flutter speed agreed almost exactly with the calculated value (indicated on the curve for mode 2 in Fig.7) it would be almost impossible to predict the flutter speed from extrapolation of the sub-critical values of damping ratio. It may be concluded that in the conditions in which the tests were made (notably with considerable model excitation due to unsteadiness in the tunnel flow) fast-frequency-sweep excitation and the analysis presented in this Report gave results that were broadly compatible with those of Ref.1, though falling far short of the accuracies required for flutter prediction. It is, in fact, a matter of continuing concern that so many measurements of modal damping ratio at sub-critical speeds, wherever and however they are made, exhibit a degree of scatter that seriously undermines their value as a means of flutter prediction. In order to investigate the possibility that a different method of flutter prediction from the measured values of frequency and damping ratio might be more rewarding, the Zimmerman flutter criterion<sup>6</sup> was applied to the results of Ref.1 and to the 'direct' analysis results of this Report. This criterion which assumes a binary flutter system takes the form of a flutter margin  $F$ , the value of which depends on the modal frequencies and damping ratios of the two modes which combine to produce a flutter condition. At the critical flutter speed,  $F = 0$ . The method of prediction is to plot values of  $F$  against an appropriate aerodynamic variable and to extrapolate the resulting curve to  $F = 0$ , so obtaining the critical aerodynamic condition. The advantage of plotting values of  $F$  rather than values of modal damping ratio is that  $F$  is likely to vary smoothly up to the flutter condition (even in cases where modal damping may vary rapidly).

Values of the Zimmerman flutter margin  $F$  were calculated assuming that the model could be condensed to a binary system of modes 2 and 3. The results are shown in Fig.9. It will be seen that the flutter margin for the calculated results follows a curve which changes direction quite rapidly at airspeeds close to the flutter speed. This is an unusual characteristic for the Zimmerman criterion and probably indicates that the assumption of a binary system in this particular case is not justified. The values of  $F$  for the test results of Ref.1 shows slightly less scatter than those for the 'direct' analysis results, but both are in reasonable agreement. In the absence of calculated results, extrapolation of both curves would lead to overestimates of the flutter speed. Comparing Fig.9 with Fig.7, it is clear that with the degree of scatter that occurred in the measured values of damping ratio the application of the

Zimmerman criterion made it somewhat easier to predict a flutter speed than by depending only on the trend in damping values. In both cases, however, the prediction was somewhat unreliable.

The second aim of the tests was to contribute to the RAE research programme on methods of making flutter tests of short duration<sup>2</sup>. There is no doubt that the frequency-sweep techniques represent significant savings in wind tunnel test time compared with the sinusoidal test technique, and the tests described here emphasise the point. At each test condition the excitation was applied for 35 s and the records were obtained in little more than a minute. Comparable test times for the sinusoidal tests are not given in Ref.1, but it is known that, on average, more than a minute was required to obtain a single point on a vector plot, and this could even be extended when long averaging times were needed to cope with response fluctuations due to tunnel unsteadiness.

The 'direct' analysis of this Report is complementary to the analyses described in Ref.2, where the methods explored made no direct use of the time-histories of the excitation forces. The results given here demonstrate that where both the excitation and response signals were available, the difficulties associated with background noise in the response record were overcome by the weighting procedure of section 2.2, and the frequency response function was obtained from the ratio of the Fourier transforms of response and excitation. This, therefore, constitutes an alternative approach to the analyses of responses due to frequency-sweep excitation. Obviously, further research is needed to identify the background noise conditions under which one method is preferable to the other.

## 6 CONCLUSIONS

An analysis method has been developed to enable modal frequencies and damping ratios of a system to be obtained from excitation and response records in conditions where the system is also excited by unknown random forces. Basically the method consists of finding the transfer function of the system from the ratio of the Fourier transforms of the response and the known excitation, and then following a weighting process to reduce the effect of the random excitation.

The method has been applied to records of the response of a low speed flutter model to frequency-sweep excitation. Comparison of the results of the analyses with those of earlier tests, made with discrete frequency sinusoidal excitation of the model, shows that both tests gave values of modal frequency

and damping ratios that were broadly similar. However, the scatter in all the results was too large to permit detailed comparisons of the test methods to be made with confidence.

The analysis is an alternative to that used in Ref.2 where the response to frequency sweep excitation was processed via the autocorrelation function without using the excitation signal. Both analyses allow unwanted random excitation to occur simultaneously with the applied excitation. It has not been established which analysis is to be preferred in typical flutter test situations, and this will be the subject of further investigation.

Table 1

MODAL FREQUENCIES AND DAMPING RATIOS ('DIRECT' ANALYSIS)

Airspeed m/s	Transducer	Mode 1		Mode 2		Mode 3		Mode 4	
		$f_1$ (Hz)	$\zeta_1$ (% crit.)	$f_2$ (Hz)	$\zeta_2$ (% crit.)	$f_3$ (Hz)	$\zeta_3$ (% crit.)	$f_4$ (Hz)	$\zeta_4$ (% crit.)
0	Strain gauge	1.73	-1.99	7.05	2.00	14.17	2.84	16.92	2.17
	Displacement	1.75	0.15	7.05	2.01	14.13	3.51	16.92	2.06
30.5	Strain gauge	2.04	16.06	7.13	6.15	12.61	6.58	16.90	3.45
	Displacement	2.03	24.73	7.24	7.97	12.42	7.56	16.90	3.24
36.6	Strain gauge	2.65	39.6	7.00	8.92	11.65	12.41	16.96	3.71
	Displacement	2.91	44.2	7.32	3.39	11.79	8.12	16.93	3.64
39.6	Strain gauge	3.11	29.6	6.76	6.95	10.88	10.38	16.90	3.54
	Displacement	3.87	50.1	7.57	11.68	11.05	9.49	16.86	3.72
42.6	Strain gauge	-	-	7.87	8.82	-	-	16.86	3.15
	Displacement	2.11	18.8	4.09	7.37	10.3	7.52	16.79	4.55
45.7	Strain gauge	3.38	40.3	6.72	4.97	9.48	4.73	16.85	4.03
	Displacement	2.76	48.1	7.23	8.09	9.72	7.08	16.90	4.59

Table 2

MODAL FREQUENCIES AND DAMPING RATIOS ('INDIRECT' ANALYSIS)

Airspeed m/s	Transducer	Mode 1		Mode 2		Mode 3		Mode 4	
		$f_1$ (Hz)	$\zeta_1$ (% crit.)	$f_2$ (Hz)	$\zeta_2$ (% crit.)	$f_3$ (Hz)	$\zeta_3$ (% crit.)	$f_4$ (Hz)	$\zeta_4$ (% crit.)
0	Strain gauge	1.94	20.11	7.05	1.75	14.08	9.88	16.89	1.56
	Displacement	1.87	11.41	7.07	1.49	-	-	16.89	1.77
30.5	Strain gauge	-	-	7.58	9.77	-	-	16.94	3.48
	Displacement	2.78	34.64	7.49	5.56	12.81	4.91	16.79	2.29
36.6	Strain gauge	2.95	43.91	7.38	8.13	-	-	16.94	3.45
	Displacement	3.02	54.94	7.01	5.70	11.91	7.11	16.71	2.41
39.6	Strain gauge	3.35	52.27	7.09	8.07	-	-	16.82	2.84
	Displacement	3.10	29.75	6.99	0.85	11.21	9.85	16.59	2.28
42.6	Strain gauge	2.98	40.37	6.91	5.19	11.96	29.36	16.85	2.61
	Displacement	-	-	-	-	-	-	-	-
45.7	Strain gauge	-	-	6.74	4.93	9.78	7.26	16.59	4.61
	Displacement	-	-	-	-	-	-	-	-

REFERENCES

<u>No.</u>	<u>Author</u>	<u>Title, etc.</u>
1	C.A.K. Irwin P.R. Guyett	The subcritical response and flutter of a swept-wing model. ARC R & M 3497 (1965)
2	K.W. Newman C.W. Skingle D.R. Gaukroger	Development of rapid-testing techniques for flutter tests. ARC CP No.1274 (1973)
3	K.H. Heron D.R. Gaukroger C.W. Skingle	Derivation of the equations of motion of a dynamic system from vector response data. Unpublished MOD(PE) material
4	R.G. White	Evaluation of the dynamic characteristics of structures by transient testing. J. Sound and Vibration, 15(2), pp.147-161 (1971)
5	C.W. Skingle K.H. Heron D.R. Gaukroger	Numerical analysis of plots of vector response loci. J. Sound and Vibration, 9(3), pp.341-353 (1973)
6	N.H. Zimmerman J.T. Weissenberger	Prediction of flutter onset speed based on flight testing at sub-critical speeds. AIAA/AFFTC/NASA-FRC Conference on Testing of Manned Flight Systems, Edwards AFB, USA (1963)

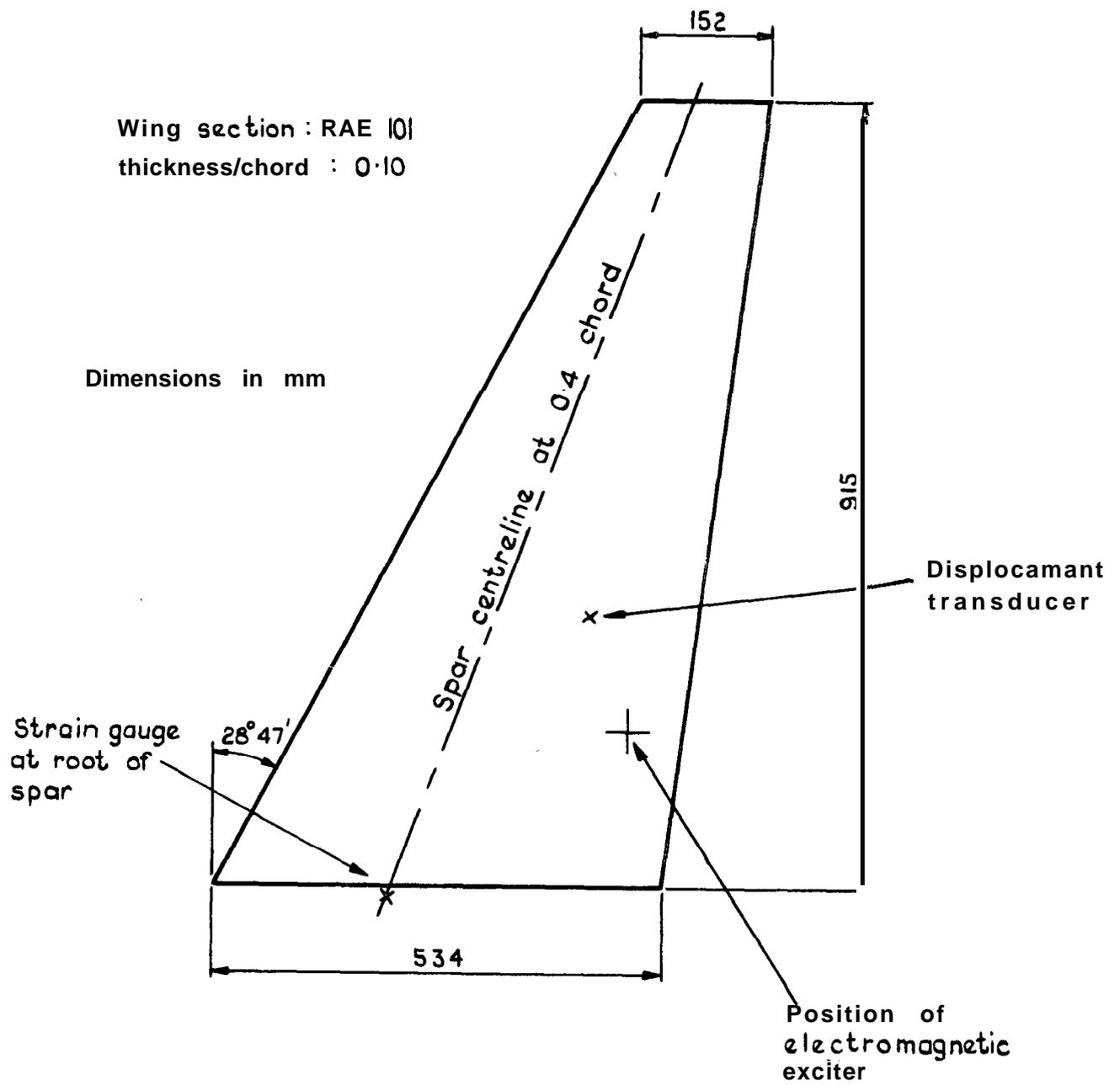
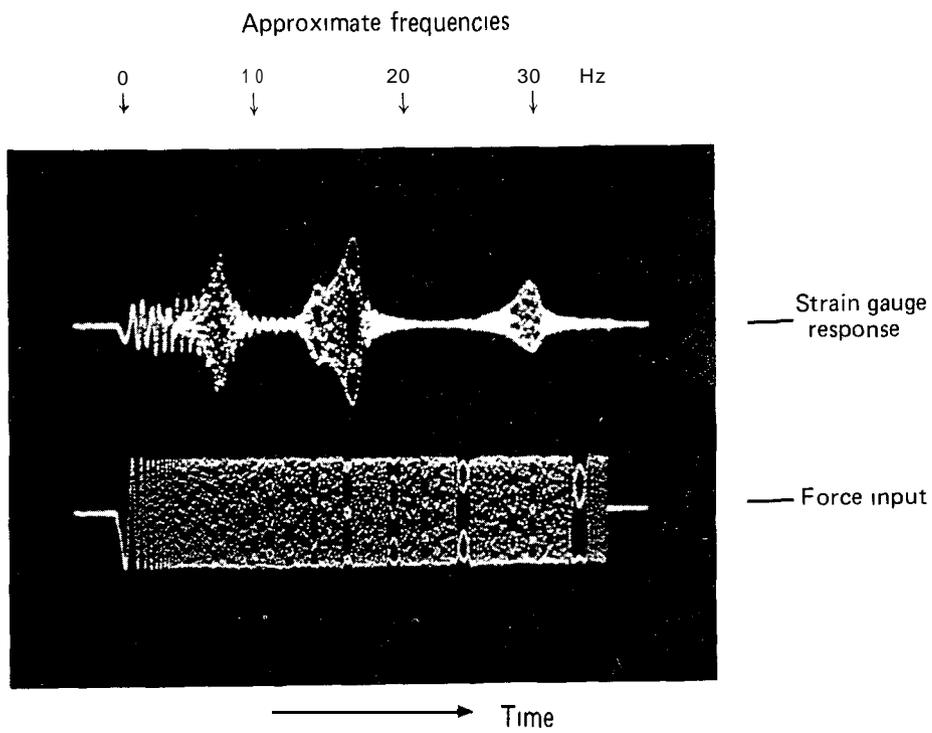
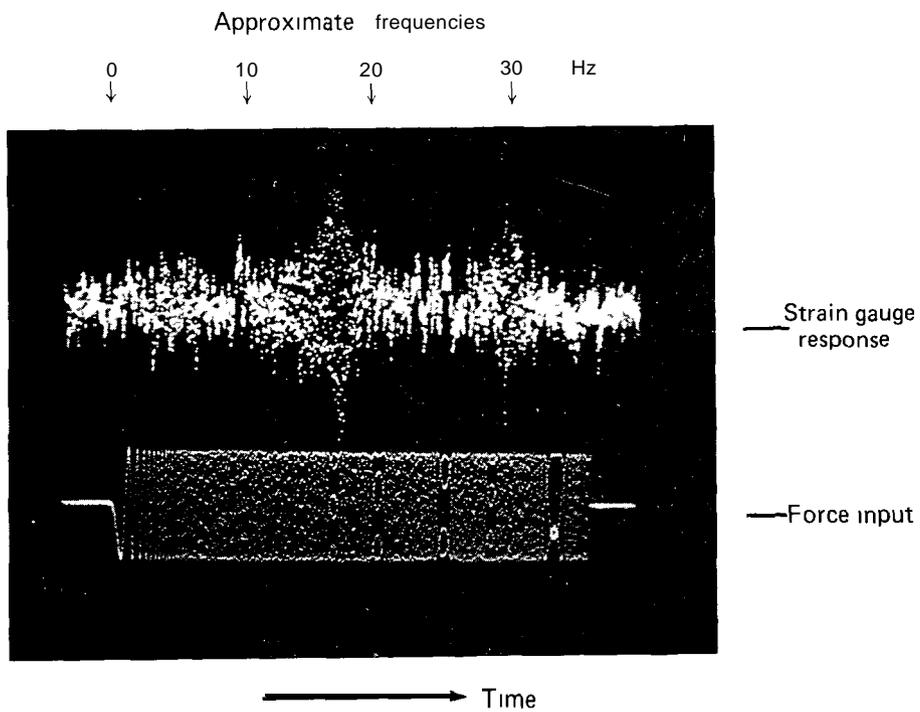


Fig. 1 Details of model wing and instrument positions

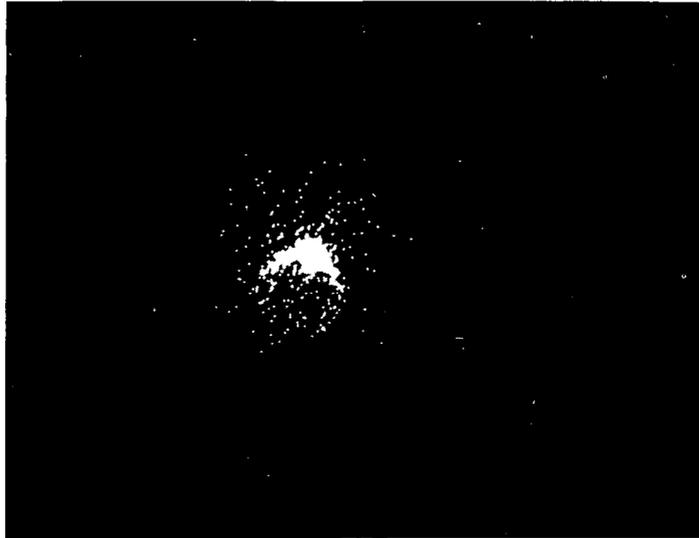


(a) Zero airspeed

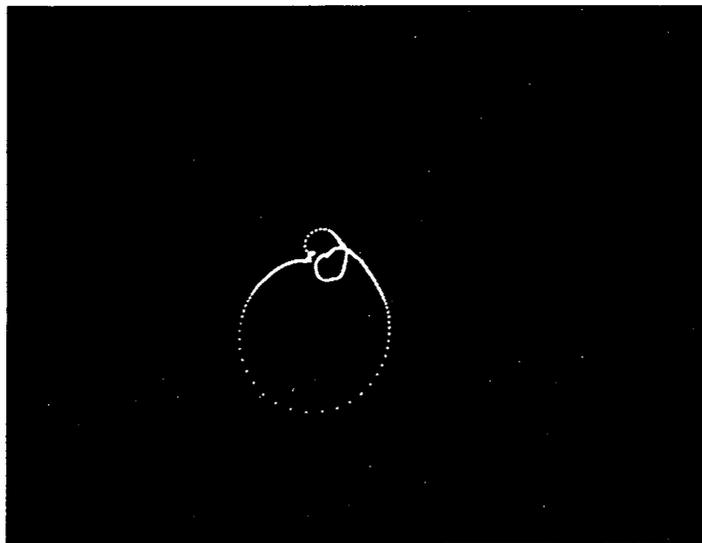


(b) Airspeed  $\approx$  36.6 m/s

**Fig.2** Response and excitation signals after digitisation

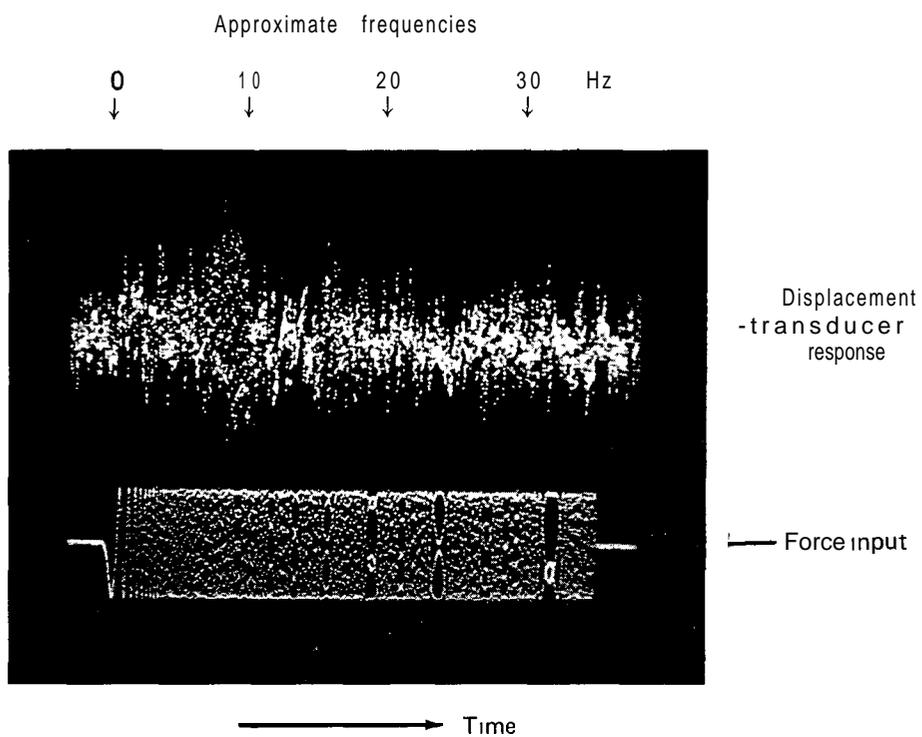


(a) Before weighting

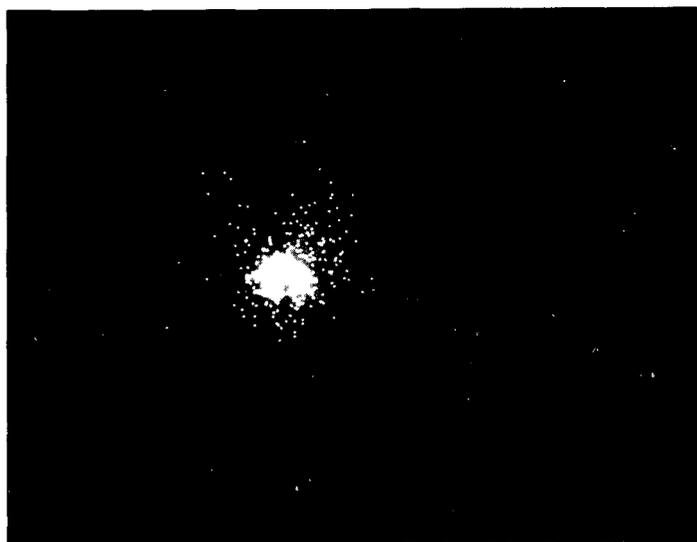


(b) After exponential weighting

**Fig.3** Vector plots before and after weighting.  
Airspeed = 36.6 m/s

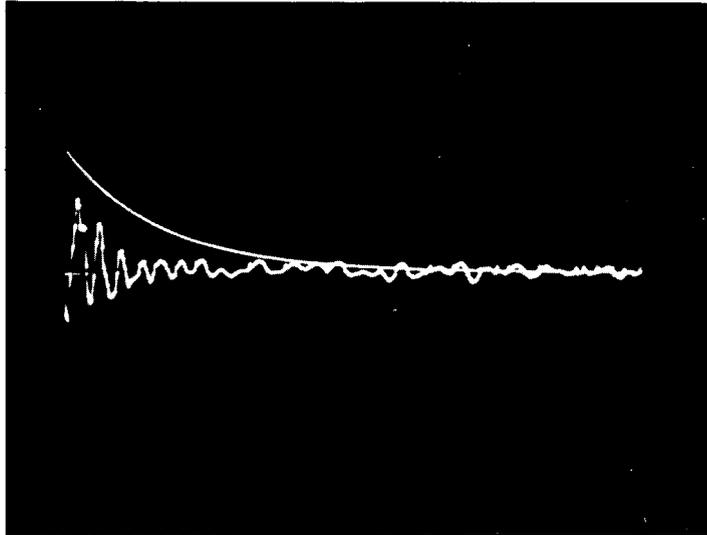


(a) Response and excitation



(b) Vector plot unweighted

**Fig.4** Response and excitation signals, and unweighted vector plot.  
Airspeed = 45.7 m/s



(a) Fourier transform of unweighted vector plot and exponential weighting function

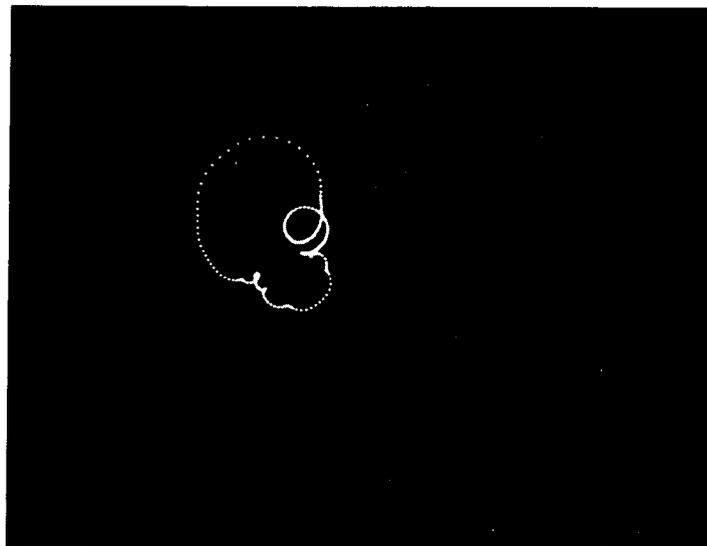


Fig.5 Fourier transform of unweighted vector plot, and vector plot after weighting. Airspeed = 45.7 m/s

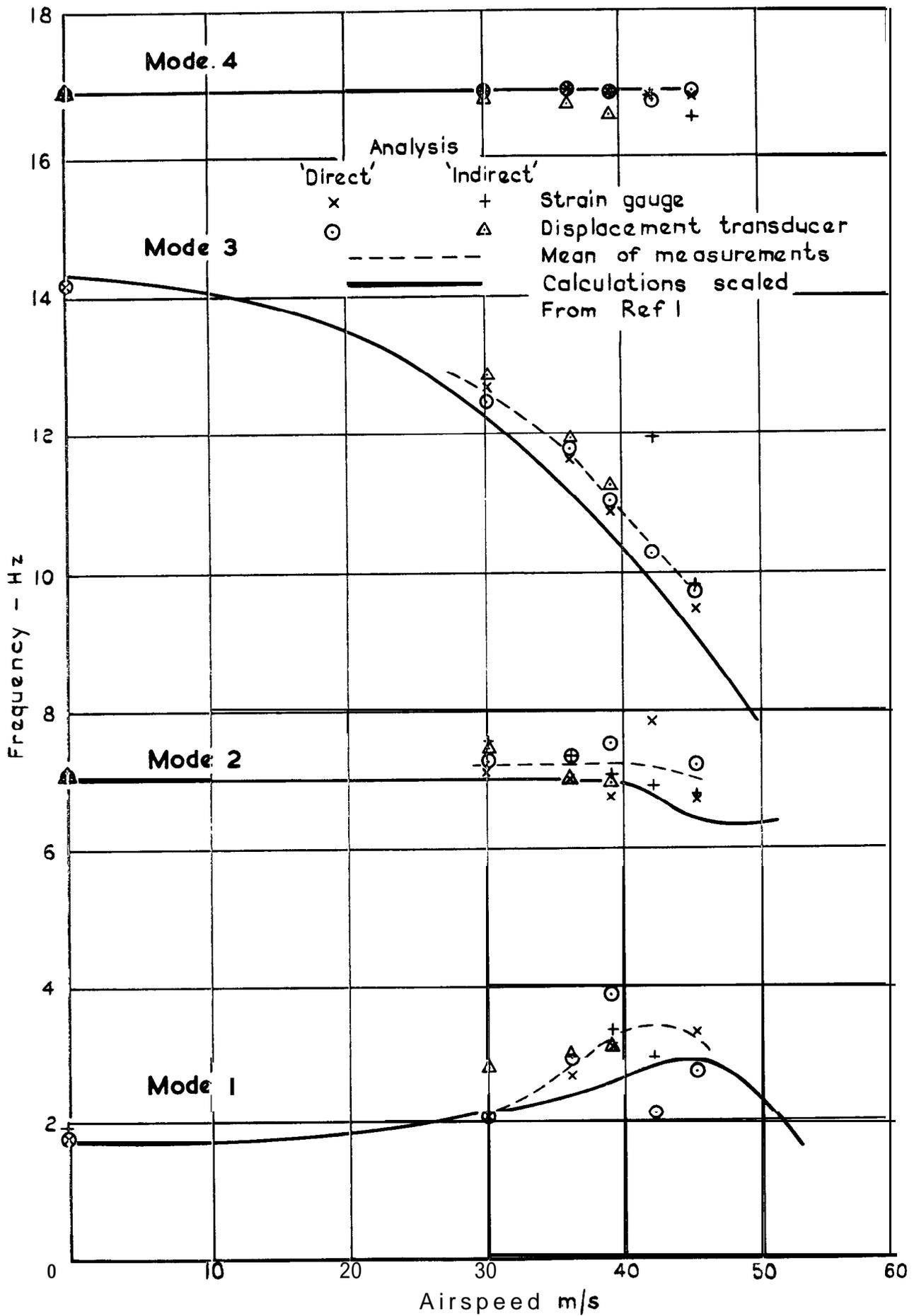


Fig. 6 Variation of modal frequency with airspeed

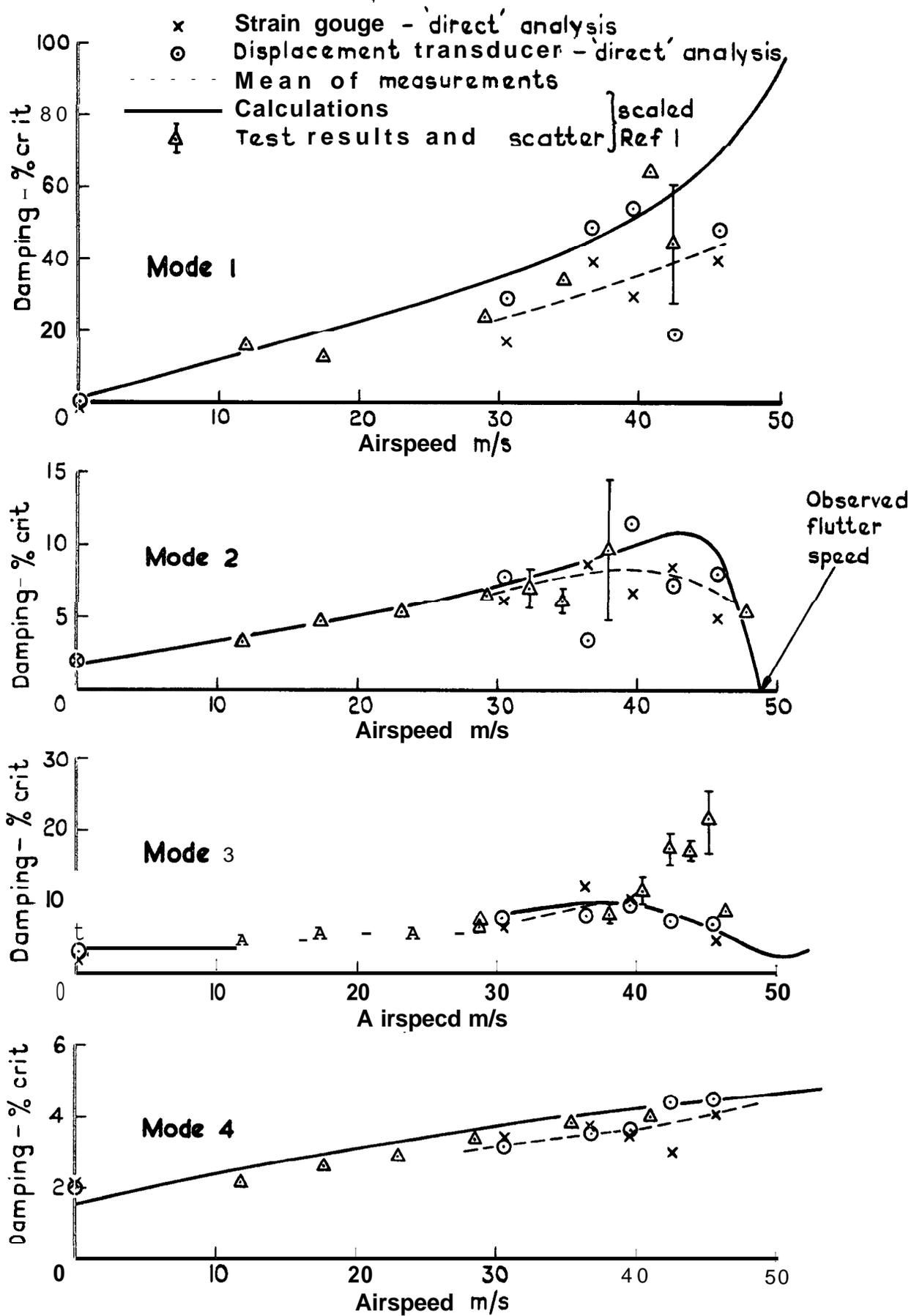
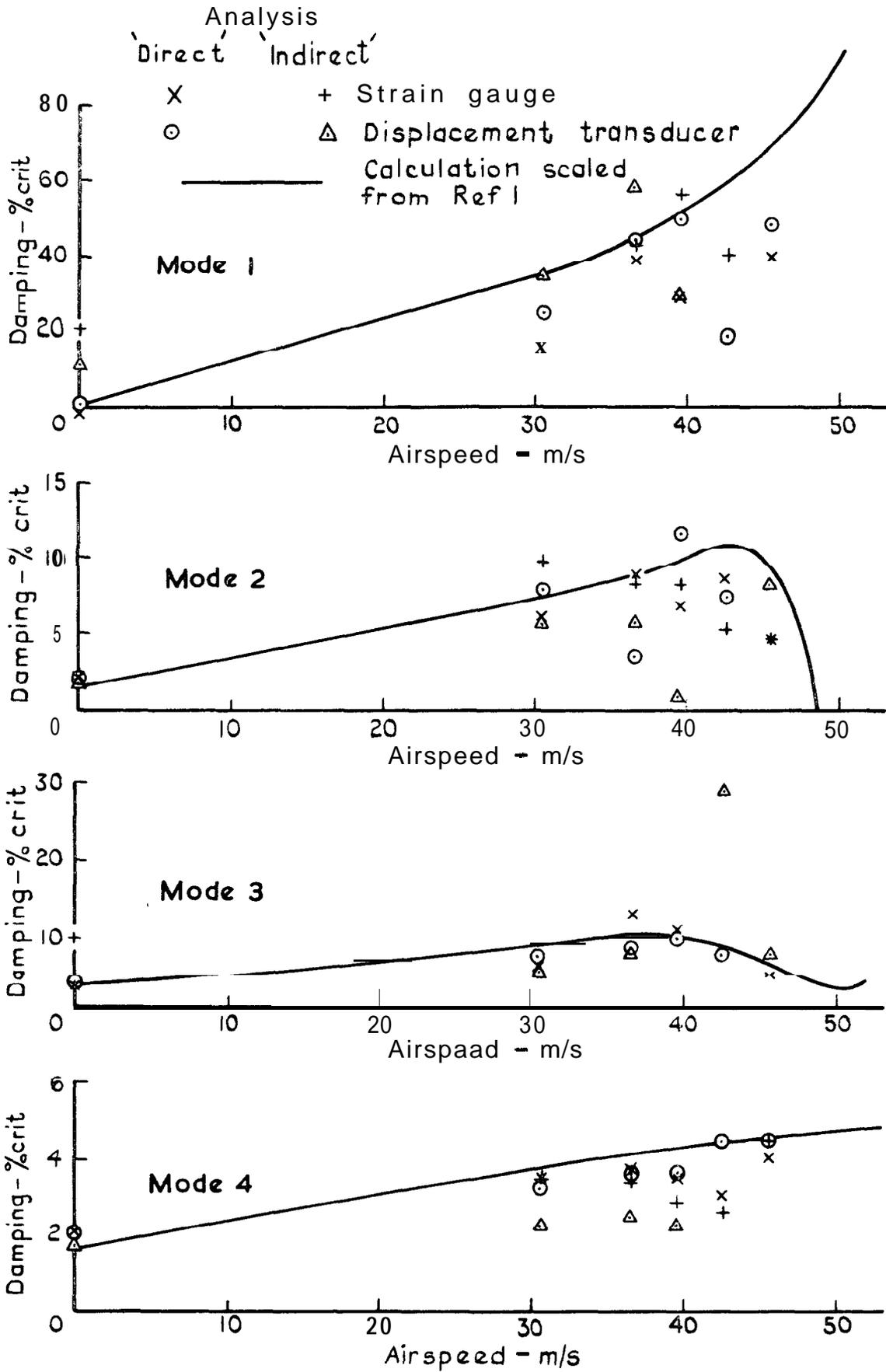


Fig. 7 Variation of damping ratio with airspeed from reference 1 and from 'direct' analysis



**Variation of damping ratio with airspeed for  
 'direct' and indirect analyses**

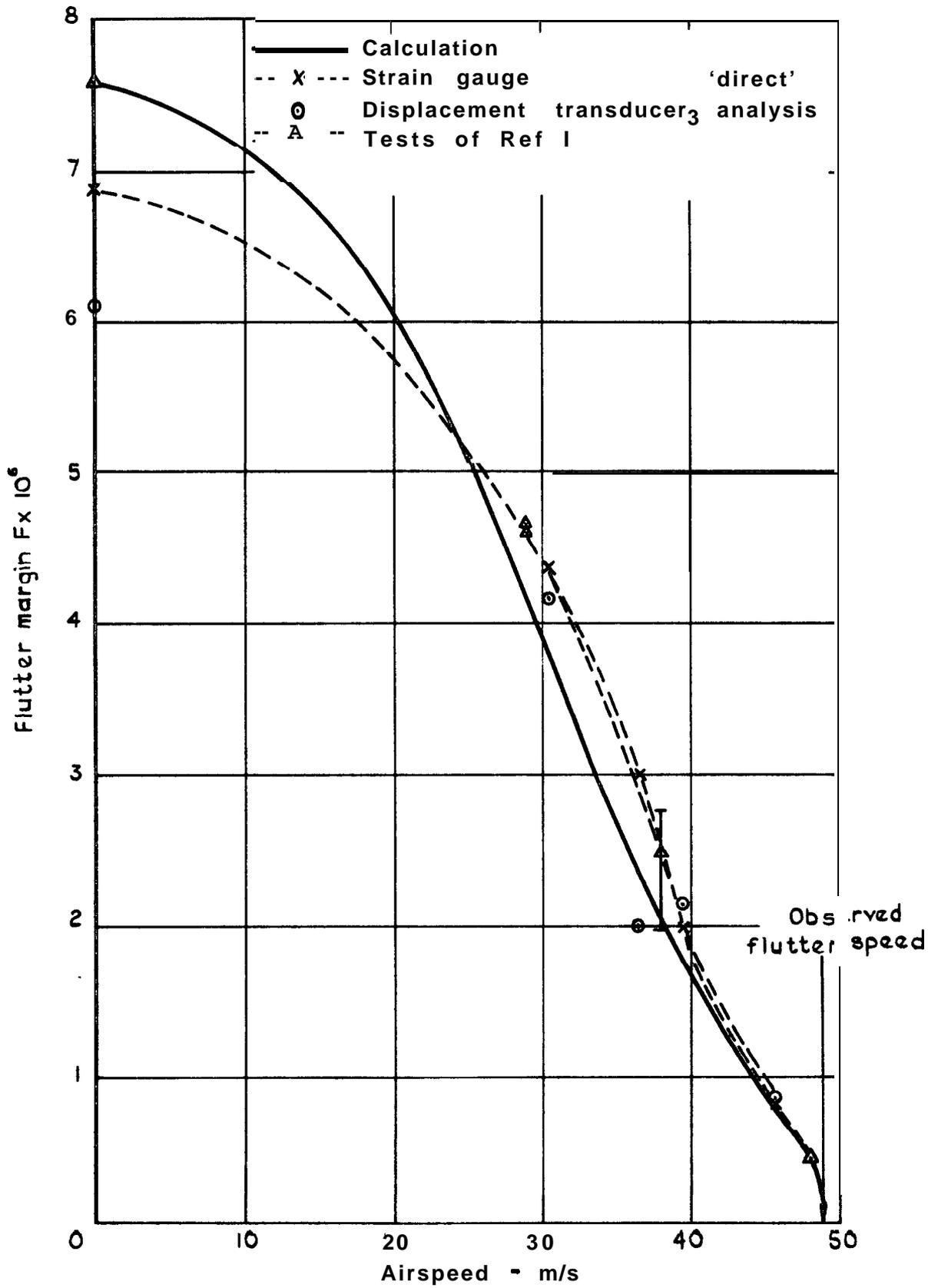


Fig. 9 Values of Zimmermon flutter margin for 'direct' analysis and results of Ref I



ARC CP No.1356  
September 1974

533.6.072 :  
533.6.013.422 :  
620.178.53

Skingle, C. W.  
Gaukroger, D. R.

AN APPLICATION OF FAST FREQUENCY-SWEEP EXCITATION TO  
THE MEASURE OF SUBCRITICAL RESPONSE OF A LOW-SPEED  
WIND TUNNEL MODEL

Transient excitation has been applied to a low-speed flutter model in order to determine the sub-critical response characteristics. The transient input was a fast frequency-sweep and the resulting response and force input records were processed using digital Fourier analysis techniques. The results are compared with those obtained from sustained sinusoidal excitation. Comparisons are given for the frequency and damping characteristics of the first four modes of the model. Frequencies are in reasonable agreement between the two sets of results but the high degree of scatter in the values of damping ratio makes detailed comparison difficult.

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