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# The Difference between the Spinning of Model and Full-scale Aircraft

By  
G. E. PRINGLE, PH.D.

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# The Difference between the Spinning of Model and Full-scale Aircraft

By

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),  
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1. *Summary.*—1.1. *Purpose.*—It was required to review the technique of model spinning tests with the object of improving the reliability of model standards as applied to full scale.

1.2. *Range.*—The empirical basis of present model standards was examined critically. To obtain quantitative information, the difference between model and full-scale recoveries was analysed on a statistical basis. Possible causes of excessive scatter were investigated by further model tests.

1.3. *Conclusions.*—Some possible causes of error in assessing full-scale behaviour by existing standards have been eliminated and the difference from model behaviour is presented numerically as a single parameter subject to statistical variation, *i.e.* the scale effect in units of yawing moment, in the wider sense defined in section 4. In special cases the admission of a more complex variation may be advisable; especially if the model is unduly sensitive to applied rolling moments, the corresponding constituent of the overall scale effect is separated from the rest and is allowed for independently in arriving at a safe standard. In such cases, model results are being interpreted cautiously until more definite full-scale evidence leads to an eventual revision of standards.

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\* R.A.E. Reports Nos. B.A. 1693 received 26th September, 1941, and Aero. 1820 received 17th July, 1943.

2. *List of Symbols.*—In this report the following symbols all refer to the model :—

- $S$  gross wing area  
 $s$  semi-span  
 $A, B, C$  principal moments of inertia  
 $a = \frac{B - C}{\rho S s^3}$ ,  $b = \frac{C - A}{\rho S s^3}$ ,  $c = \frac{A - B}{\rho S s^3}$   
 $\rho$  air density at equivalent altitude of spin  
 $\mu$  relative density =  $\frac{\text{mass of model}}{\rho S s}$   
 $V$  rate of descent  
 $\Omega$  rate of rotation  
 $\lambda$  spin parameter =  $\frac{\Omega s}{V}$   
 $v$  sideslip velocity  
 $\alpha$  incidence of wing chord  
 $\beta$  sideslip angle (positive for inward sideslip)  
 $\theta_y$  tilt angle ; inclination of wing span to horizontal (positive if outer wing tip is up)  
 $\chi$  spiral pitch  
 $l$  rolling moment coefficient =  $\frac{\text{rolling moment in body axes}}{\rho V^2 S s}$   
 $m$  pitching moment coefficient  
 $n$  yawing moment coefficient =  $\frac{\text{yawing moment in body axes}}{\rho V^2 S s}$   
 $N$  yawing moment coefficient measured in steady spin, just sufficient to prevent recovery on moving controls for recovery  
 $X$  mean difference between model and full-scale aircraft expressed as a yawing moment coefficient  
 $Y$  probable error of this difference  
 $Z$  individual difference for a given type of aircraft  
 $Y_A, Y_B$  yawing moment coefficients equivalent to an increase of 15 per cent. in  $A$  and  $B$   
 $\theta$  correction to threshold value  $N$  for random errors in model inertias  
 $C_L$  lift coefficient  
 $\overline{l}_i$  rolling moment coefficient due to inertia of body axes  
 $\overline{l}'_p$  rolling moment coefficient due to rotation  
 $\overline{l}'_{pv}$  rolling moment coefficient due to rotation and sideslip  
 $l_v$  sideslip derivative of rolling moment =  $\frac{\partial}{\partial \beta} \overline{l}'_{pv}$   
 $n_v$  sideslip derivative of yawing moment =  $\frac{\partial}{\partial \beta} \overline{n}'_{pv}$   
 $\lambda_v, \nu_v$  corresponding derivatives augmented by inertia terms  
 $j = -\lambda_v / \nu_v$



- (ii) Failure to achieve exact similarity of loading. This takes the form of error in equivalent altitude, and error in weight and moments of inertia. Some error in altitude is to be expected, and a further departure from similarity results from the fact that the model is held at constant altitude, whereas the full-scale aircraft is necessarily changing during the spin. Errors in moments of inertia are regarded as materially affecting the precision of the results, and are further discussed in § 7.
- (iii) Difference between left- and right-handed spins (*see* § 8).
- (iv) Accelerations of the tunnel due to unsteadiness, or intentional accelerations required to keep the model in the test section.
- (v) Control movements are not exactly represented. Aerodynamic balance is not attempted on the models, and there is no restriction of the automatic movement of controls either by appreciable hinge moments or by the discretion of a pilot. The optimum use of controls may remain undiscovered in either model or full scale.

Factors (ii) and (v) are already partly eliminated because higher tunnel speeds are available than heretofore, enabling the rate of descent at sufficient altitude to be balanced against tunnel speed, and on some models improved mechanisms allow the controls to be moved separately as they are in full scale. These factors cannot be considered on all models retrospectively, but it is intended that in future the standard should be applied to the best use of controls for recovery. Misuse of controls is a contingency that can best be allowed for by ensuring a margin of safety with normal use, except that if tunnel work indicates any specially dangerous condition, a warning may be issued.

5. *Theoretical form of the Failure Curve.*—Experience of the model spinning standard has not hitherto been given satisfactory numerical expression. What has usually been attempted is to estimate by comparison with spinning trials or accidents what yawing moment must be applied to make a particular model fail, especially in cases where the aeroplane fails to recover. The largest value so found has been taken as a basis for the standard to be reached in future, but it is our purpose to find a better empirical basis if possible.

The model usually spins more steeply and recovers from the spin more easily than the aeroplane. The application of a pro-spin yawing moment makes the model recover more slowly, the measured time of recovery progressively increasing according to some curve like that of Fig. 1, and there is a yawing moment,  $N$  say, such that all larger values will wholly prevent recovery. This yawing moment is called the threshold value for the particular model, loading and sense of spin. In what follows  $N$  usually refers to a mean for left- and right-handed spins.

The main problem is to make the best use of measurements of  $N$  in deciding the probability of failure to recover from full-scale spins. The applied yawing moment brings the model behaviour into better agreement with full scale by making the steady spin flatter and faster. These changes generally diminish the initial effectiveness of the rudder, and the extra pro-spin moment is a handicap against which the remaining rudder power must work in stopping rotation. We may therefore visualise a full-scale curve of recovery time in Fig. 1 displaced horizontally but qualitatively similar in shape to the curve for the model. It is assumed here that the model times are multiplied by the square root of the linear scale ratio, in order to represent the times on a common scale.

The magnitude of yawing moment required to equalise the model and full-scale times of recovery is a convenient measure of the difference between the spins and will be denoted by  $Z$ . This cannot be measured directly unless the aircraft is on the borderline, that is, recovers from spins in response to correct control movements but only in an abnormally long time. The practical importance of such cases is therefore considerable. In cases of non-recovery,  $Z$  must be redefined and can be thought of as that applied moment which will bring the steady spins into agreement. In either case it can be understood as the distance along the yawing moment axis in Fig. 1 between the curves for the model, and if it could be obtained, for the aeroplane. The only point actually resulting from full-scale observation is  $P$ . Similar curves could be obtained with, for example,

the moment of inertia  $B$  as the independent variable, and this has been done in particular cases by the use of the ballast tank ; but yawing moment happens to be the most convenient variable to manipulate for models although there is no corresponding technique for aeroplanes.

We now suppose that  $Z$  is the linear resultant of a fairly large number of independent small constituents, such as those enumerated in § 4, and that its values are distributed among various types of aircraft according to the normal error law. Then, in addition to its direct determination in some few cases,  $Z$  can be thought of as having a mean value  $X$  and a probable error  $Y$  (Fig. 2).

The probability that  $Z$  exceeds  $N$  is the same as that the aircraft will fail to recover if the model threshold is  $N$ , neglecting for this purpose the mere lack of sufficient height to recover in defining the probability. It is represented as the shaded area to the right of  $N$  in Fig. 2, and is plotted against  $N$  in Fig. 3. This is the curve of the error integral, values of which are given in standard mathematical tables.

6. *Empirical Determination of the Failure Curve.*—The relevant data for correlating model and full-scale tests are collected in Table 1, in which  $N$  is the threshold measured for the model with normal inertias.  $Y_B$  is the measured yawing moment equivalent to an increase of 15 per cent. in  $B$ . Previous practice has been to use  $N - \frac{2}{3}Y_B$  as the parameter both for establishing the standard and for predicting full-scale behaviour. The probability of failure can be estimated by sorting each group of models having similar values of the parameter. The histogram of Fig. 4 shows this done, with borderline cases counting  $\frac{1}{2}$ . A mean curve has then been drawn, of the theoretical form described in § 5. Fig. 5 shows a histogram resulting from the revised method of correcting  $N$ ; the separation of passes from failures is hardly better but a rather safer standard is set. It may also be concluded tentatively that inertia errors are not the predominant cause of "scatter".

7. *Inertia Errors.*—The most important constituents of  $Z$  due to loading errors are those for deviations of all-up weight, pitching moment of inertia  $B$ , and rolling moment of inertia  $A$ . It may be assumed that comparisons with full scale are made with the model weight correct, apart from altitude error. With  $A$  and  $B$  it is otherwise because the full-scale values are not measured directly and a statistical error results. Usually this possibility has been allowed for by supposing  $B$  increased by 10 per cent. on the model. However convenient this may be, it is liable to lead to an eventual lowering of the standard which will defeat its purpose; it is more rational to compare model and aeroplane at the same loading or as near as possible. Systematic errors in the inertias can be minimised by a simple procedure. The mass of each item is entered in a table in which the individual moments of inertia are calculated. Systematic errors in the resulting radii of gyration are unlikely, so that the moments of inertia will also be free from systematic error if the total weight is all accounted for in the inertia sheet. This is checked by agreement with the known weight. If the weight proves incorrect, the calculated moments of inertia can be scaled proportionally. The final errors in moments of inertia may then be assumed to follow a normal error law.

The problem now is, to make use of the fact that, although  $A$  and  $B$  themselves have unknown errors, the effect of a given error in either can be measured on the model. In Appendix II it is argued that such effects can be regarded chiefly as influencing the precision of the error integral curve. The model normally loaded corresponds to a certain full-scale loading, from which the actual  $A$  and  $B$  in spinning trials differ positively or negatively. Therefore in going from the scaled-up model loading to the actual loading we must pass to a flatter failure curve, so situated that for a given  $N$  the probability of full-scale failure is brought nearer to 50 per cent. The probability 50 per cent. throws the least light on full-scale behaviour so that if the loading condition is a dominating variable, only a correspondingly large measured value of  $N$  would enable any confidence to be placed in the prediction of full-scale recovery.

This procedure based on Appendix II is adopted in Table 1 and Fig. 5, in which it is seen that the Moth Minor and Bristol 133 are over the borderline, leaving only the Typhoon and Wellesley erroneously passed (but see § 9.2).

TABLE 1

Comparison of Model and Full-scale Spins

See Fig. 5 and 6.

Fig. 4.

No.	Type	$\frac{B}{A}$	$\frac{C}{A}$	N	$\frac{N}{(\delta l = \delta n)}$	$N - X$	$Y_A$	$Y_B$	$1 - \frac{Y}{\sqrt{Y^2 + Y_A^2 + Y_B^2}}$	$\theta$	$N - \theta$	$N - \frac{2}{3} Y_B$	Full Scale
1	Supermarine F7/30	1.36	2.28	35		22.5	≈ 3	≈ 5	0.55	12.4	22.6	31.7	P*
2	Night Hawk	1.01	1.91	19.5		7	1.5	7.5	0.63	4.4	15.1	14.5	B
3	Magister (i)	1.50	2.34	13.5		1	≈ 1.5	≈ 6	0.61	0.6	12.9	9.5	F
4	Magister (ii)	1.61	2.43	22		9.5	≈ 1.5	≈ 6	0.61	5.8	16.2	18.0	P
5	Hurricane	1.37	2.24	14.5		2	≈ 1	≈ 4.5	0.47	0.9	13.6	11.5	B
6	Spitfire	1.29	2.10	15		2.5	0	3	0.29	0.7	14.3	13.0	P
7	Defiant	2.12	2.99	16		3.5	0	1	0.05	0.2	15.8	15.3	P
7	Defiant	2.12	2.99		29	16.5	0	1	0.05	0.9	28.0		P
8	Australian Trainer	1.24	2.07	27		14.5	≈ 1	7	0.61	8.8	18.2	22.3	P
9	Miles M18 (i)	1.47	2.33	24		11.5	5	13.5	0.79	9.1	14.9	15	B
10	Miles M18 (ii)	1.47	2.33	31		18.5	5	13.5	0.79	14.6	16.4	22	P
11	Miles M20 (i)	0.98	1.81	6		-6.5	3	3	0.43	-2.8	8.8	4	F
12	Miles M20 (ii)	0.98	1.81	8		-4.5	3	3	0.43	-1.9	9.9	6	B
13	Miles M20 (iii)	0.98	1.81	22		+9.5	3	3	0.43	+4.1	17.9	20	P
14	Moth Minor	1.20	2.08	24.5		12	2.5	7.5	0.65	7.8	16.7	18.5	B
14	Moth Minor	1.20	2.08		23	10.5	2.5	7.5	0.65	6.8	16.2		B
15	Bristol 133	0.75	1.55	25		12.5	1	9	0.68	8.5	16.5	19	B
16	Typhoon	0.67	1.57	22.5		10	3	1.5	0.33	3.3	19.2	21.5	B
16	Typhoon	0.67	1.57		16.5	4	3	1.5	0.33	1.3	15.2		B
17	Wellesley	0.52	1.37	53		40.5	5	12	0.56	22.5	30.5	45	B
17	Wellesley	0.52	1.37		44	31.5	5	12	0.56	17.5	26.5		B
18	Percival Trainer	0.67	1.54	40		27.5	2	15	0.80	22.0	18.0	30	P

\* P = Pass B = Borderline F = Fail

8. *Other Individual Errors.*—8.1. *Difference between Right and Left Spins.*—The difference between the two directions of full-scale spinning is probably due to aerodynamic and gyroscopic effects of the propeller, to manufacturing tolerances leading to minor asymmetries, and to design asymmetries if there are any. Model differences are not correlated to any very marked degree with the sense of rotation of the idling propeller and are dominated by tolerance of manufacture. Thus it is reasonable to use the mean model result and, in comparing with the worse full-scale case, to make a calculated allowance. In Table 1 the comparison is between the mean model result and the worse full-scale result, so that the allowance is already present in the resulting failure curve. The magnitude of the full-scale effect on modern aircraft is discussed elsewhere.<sup>3</sup>

8.2. *Rolling Moment Effects.*—It has been suggested that the model in a given condition will generally spin steeper and, at a given incidence with more outward sideslip than the corresponding full-scale aeroplane. In that wing tip vanes flatten the spin, the scale difference of attitude may be thought of as due to Reynolds Number effect on yawing moment. If so, a constant  $Z$  would be a physical representation of aerodynamic scale effect, and we should then have to reckon with the statistical variability of the true Reynolds Number effect according to aeroplane design.

Error in sideslip requires rather more detailed consideration. This may be partly, indeed largely, due to scale effect on rolling moment, since  $l_v$  is usually larger numerically than  $n_v$  in the spin. In fact it is probable that Reynolds Number effect has a fairly large rolling component in body axes, since not only are tangential forces on stalled thin aerofoils fairly small but it is known that  $\bar{l}\bar{p}$  tends to be the major component, taking the aircraft as a whole.

Evidence has therefore been sought that models are unduly sensitive to rolling moment errors.

9. *Experiments with Applied Rolling Couples.*—9.1. *Measurements on the Wellesley and other Models.*—The Wellesley model seemed a suitable model for this enquiry as full-scale behaviour has caused serious doubt of its ability to recover by normal use of the controls (Appendix III). The original model tests by Alston and Cohen 1933<sup>4</sup> and subsequent repetitions all point to good recovery with the largest margin of safety so far recorded.

With the model loaded to represent the aeroplane in the condition which gave trouble, a peculiarity of the spin is the large outward tilt. This must certainly make the centrifugal rolling and yawing couples more significant than they usually are for monoplanes. In fact there seemed to be a strong case for further investigation of the lateral behaviour. The actual investigation took the form of an analysis of the spin with rolling moments applied, as in Fig. 6, by means of auxiliary vanes placed in the same plane as the wing. The first attempt failed because with pro-spin rolling moments the spin did not appear steady enough for photography, and also because of experimental difficulties in determining the rolling moment when  $\lambda$  is large, as in this model it is. Tests were therefore made with anti-spin rolling moments. These showed a marked effect even allowing for some interference between the two vanes placed, as these now were, close together on a wire attached to the inner wing-tip. In the steady spin, the results of Figs. 7 and 8 apply to the effect on incidence, whereas those of Figs. 9 and 10 apply to the effect on sideslip.

Referring to equation (2) we now find that the value

$$j = \delta_1 n / \delta l = 0.4$$

is in reasonable accord with the experimental results over a range of incidences. There is no reason to expect  $j$  to be constant and this value may be in error, as a result of mutual interference of the vanes, by perhaps 25 per cent.

A continuation of these tests included an investigation of recovery from the spin with rolling moments applied. These did not show an effect of the same magnitude as in the steady spin, thus contradicting the hypothesis of § 4.1, probably because (a) the rolling moment does not vary during recovery in the same manner as the yawing moment, and (b) probably  $j$  is in any case not constant during recovery.



Eventually recovery tests were made with pro-spin rolling couples applied to several different models. These results certainly seemed to show that the effect is not consistent but is correlated with the sign of  $(A - B)$  as it would be if the "equivalence" formula were true. It is noted that if  $\lambda$  is sufficiently large, equation (2) leads to the simple formula

$$j = \frac{A - B}{C - B} \simeq 1 - B/A$$

The measured ratio  $j$  on the other hand appears to be numerically less than this, as shown in Table 2, probably because in fact the  $l_v$  term is not negligible in comparison with  $a\lambda^2 \sin \alpha$ .

TABLE 2  
*Equivalent Yawing and Rolling Moments*

Type	$j$ (measured for recovery)	$j$ (calculated for $l_v = -0.2$ )	$(1 - B/A)$ in model test
Defiant .. ..	-0.32	-0.3	-1.68
Bristol 133 .. ..	0	0	-0.02
Bristol 133 .. ..	+0.14	+0.3	+0.49
Moth Minor .. ..	variable	0.05	0.11
Wellesley .. ..	0.25	0.13	0.37
Typhoon .. ..	0.19	0.07	0.19

It also seemed that the value of  $Z$  was correlated with  $A - B$  in the sense that when  $A > B$  the model tends, on the simpler interpretation, to an over-optimistic conclusion about the recovery of the aeroplane and vice versa, especially in the case of Defiant, Spitfire, Bristol 133, Wellesley and Typhoon, whereas for the Moth Minor the correlation was negative. A critical examination of these cases, however, makes the list much less impressive, for in three of them the over-optimism could easily be due to statistical causes and experimental error, whereas the other two are complicated by doubts of the elevator operation to recover.

The results in Table 2 are presented graphically in Fig. 11. In ignorance of the true relation between  $\delta l$  and  $\delta n$  we might naturally search for some line with which the intersections of these graphs would give an improved separation into passes and failures. However, the number of authentic cases is insufficient. It is noted that in the present routine tests, the applied moments are in a fixed ratio given by  $\delta l \simeq -\delta n \tan 40^\circ$ , and if there is a valid equivalence ratio  $j$  we expect the corresponding deviation on the scale of yawing moments to be given by

$$(\delta n + j\delta l) - (\delta n - j \tan 40^\circ \delta n) = j (\delta l + \delta n \tan 40^\circ).$$

Column 6 of Table 1 is based on an assumption of equal values for  $\delta l$  and  $\delta n$ , but further progress requires an experimental investigation of the scale effect on autorotational moments, as well as more data for statistical analysis.

9.2. *The Wellesley and Typhoon Spinning Tests.*—The Wellesley was placed on the borderline in Table 1 on the evidence of a flight report (Appendix III) which appeared to receive some corroboration from a later fatal accident. Against the full-scale evidence is that it is not the result of systematic spinning trials, but of an accident to a particular aircraft that appeared, on the showing of the same report, to behave exceptionally. At the stall, most production Wellesleys did not tend to spin at all whereas this one did so. With this reservation the evidence points however quite clearly to unsatisfactory behaviour in the spin.

The aerodynamic criteria for the Wellesley are low ;  $b = 0.24$ , damping coefficient =  $0.007$ , unshielded rudder volume coefficient =  $0.004$ . There is an inadequate number of other cases with so low a value of  $b$  to arrive at any fair comparison ; it is doubtful whether the graphs of Ref. 5 are applicable here.

No reasonable interpretation of § 9.1 seems likely to bring the model over the borderline. The question now is whether model tests throw any further light at all on this puzzling discrepancy. Two salient facts are :—(a) The Wellesley model is sensitive to loading of the fuselage such as ballasting of the tail to bring the C.G. to its aftmost position, or accidental rearward movement of a large mass. However, model tests indicate that such a weight would have to be of the order of 1,000 lb. to make the spin dangerous. A comparable error in estimating  $B$  is hardly a possibility. (b) The misuse of controls to recover is important. Use of the rudder to recover is not essential ; use of the elevator is quite essential. Because of the former, we can ignore any suggestion that the rudder failed to work. In the case of elevator we ought to consider whether the pilot could be mistaken in thinking he had applied it, *e.g.* by failure of the control circuit or excessive stretch.

Similar considerations apply with greater force to the Typhoon. In this case (Appendix IV) the pilot's definite impression was of excessive stick forces ; this evidence is to be taken in conjunction with the observation that on the model Typhoon as on the Wellesley model the stick movement is a very important factor in recovery.

The following threshold values are observed with the model loaded to represent approximately the condition of the machine on the occasions in question when difficulty was experienced :—

TABLE 3  
*Recovery as Affected by Elevator Movement*

Model	Controls for recovery		Threshold $N$
	Rudder	Elevator	
Wellesley .. .. .	Reversed	Down	47
		Central	20
Half up		12.5	
Up		0	
	Fixed	Down	>36
Typhoon .. .. .	Reversed	Down	19
		Central	14
		Up	11
	Central	Down	13

It may be correct to explain the Typhoon experience on these lines but the Wellesley should perhaps be left as an open question as there was no independent evidence of control troubles.

10. *Conclusions.*—(a) The basis of the spinning model vane technique is empirical and must remain so until more is known of scale effects in the spin.

(b) The difference between models and full-scale aircraft can be broadly represented as a single parameter subject to statistical variation on a scale of yawing moments.

(c) Analysis of results of full-scale spins can eventually lead to a knowledge of the variation of this difference as well as its mean value and any exceptional values. Inertia errors can be regarded as chiefly diminishing the precision of predicting full-scale behaviour.

(d) The Reynolds number effect can probably be represented principally by a rolling moment and a yawing moment. The moments applied in model tests are of this nature but the rolling component is in all probability of the wrong sign. In principle this can lead to an unduly large "scatter" on the yawing moment scale. A full investigation of the factors affecting the magnitude of the scale effect by direct experiment is very desirable.

(e) In practice the measured effect of applied rolling couples is usually not large, but it is correlated as theory would indicate with the value of  $A - B$  in sign and magnitude.

(f) In cases where the ratio of equivalence of yawing and rolling moments is such as to indicate that the routine method of testing may be in error, attempts have been made to explain full-scale behaviour in terms of a revised method of testing models. This is thrown into doubt by the full-scale evidence and by alternative explanations of crucial cases. The only immediate practical outcome is to maintain a watch for exceptional sensitivity to applied rolling couples, and to use caution in applying model results. Eventually a revised standard may be possible.

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## APPENDIX I

*Equivalent Rolling and Yawing Couples*

According to the simple theory of spinning, the pitching equilibrium of the model in a given condition is determined by only two variables, the incidence  $\alpha$  and spin parameter  $\lambda (= \Omega s/V)$ , so that in steady spins  $\lambda$  is a function of  $\alpha$ . On the whole it is found that this function is not markedly changed by the application of rolling couples to the model.

The effect of the rolling couple at constant  $\alpha$  and  $\lambda$  may be found from the equation of rolling moments

$$a\lambda^2 \sin \alpha \cdot \theta_y + \overline{l} \overline{p} + \beta l_v = 0,$$

where  $\theta_y$  is the inclination of the outer wing above horizontal, and  $\beta$  is the sideslip angle, so that

$$\beta = \theta_y - \chi,$$

where  $\chi$ , the spiral pitch,  $= \frac{C_L}{2\mu\lambda}$ .

Hence 
$$\theta_y = \frac{l_v C_L / 2\mu\lambda - \overline{l} \overline{p}}{a\lambda^2 \sin \alpha + l_v}.$$

In this expression the coefficients are functions of  $\alpha$  and  $\lambda$ , so that if these are fixed and  $\overline{l} \overline{p}$  is increased by the application of an extra rolling moment  $\delta l$ , there will be a new equilibrium with  $\theta_y$  changed by an amount  $\delta\theta_y$ , where

$$\begin{aligned} \delta\theta_y = \delta\beta &= \frac{-\delta l}{a\lambda^2 \sin \alpha + l_v} \\ &= -\delta l / \lambda_v, \end{aligned}$$

where  $\lambda_v$  takes the place of the ordinary sideslip derivative  $l_v$  and includes the inertia term.

Such a change in  $\theta_y$  and  $\beta$  will produce an unbalanced yawing moment  $\delta_1 n$  due to (i) change in  $\overline{n}'v$  resulting from the change in  $\beta$ , and (ii) a change in the inertia yawing moment:—

$$\begin{aligned} \delta_1 n &= n_v \delta\beta + c\lambda^2 \cos \alpha \cdot \delta\theta_y \\ &= (n_v + c\lambda^2 \cos \alpha) \delta\beta \\ &= r_v \delta\beta \end{aligned}$$

in which  $r_v$  is the total derivative of directional stability and replaces the ordinary aerodynamic term  $n_v$ .

Hence 
$$\delta_1 n = -\delta l r_v / \lambda_v$$

and the ratio  $j$  of equivalent yawing and rolling moments, is given by

$$j = -r_v / \lambda_v.$$

The validity of the initial assumption that  $\alpha$  and  $\lambda$  are unchanged by  $\delta l$  may be examined by taking  $\alpha$ ,  $\beta$  and  $\lambda$  as independent variables, and considering the effect of applied moment coefficients  $\delta l$ ,  $\delta m$  and  $\delta n$ ; these satisfy linear equations derived from the equations of equilibrium of the three moments

$$\left. \begin{aligned} 0 &= \delta l + \frac{\partial l}{\partial \alpha} \delta \alpha + \frac{\partial l}{\partial \beta} \delta \beta + \frac{\partial l}{\partial \lambda} \delta \lambda \\ 0 &= \delta m + \frac{\partial m}{\partial \alpha} \delta \alpha + \frac{\partial m}{\partial \beta} \delta \beta + \frac{\partial m}{\partial \lambda} \delta \lambda \\ 0 &= \delta n + \frac{\partial n}{\partial \alpha} \delta \alpha + \frac{\partial n}{\partial \beta} \delta \beta + \frac{\partial n}{\partial \lambda} \delta \lambda \end{aligned} \right\}$$

to the first order in  $\delta \alpha$ ,  $\delta \beta$  and  $\delta \lambda$ , where  $\partial l / \partial \alpha$  etc. are derivatives involving both aerodynamic and centrifugal terms.

If we now change to  $\delta l$ ,  $\delta m$  and  $\delta n$  as independent variables we find

$$\frac{\partial \alpha}{\partial l} = -\frac{1}{J} \frac{\partial (m, n)}{\partial (\beta, \lambda)},$$

where

$$J = \frac{\partial (l, m, n)}{\partial (\alpha, \beta, \lambda)},$$

so that

$$\frac{\partial \alpha}{\partial l} = -\frac{1}{J} \left( \frac{\partial m}{\partial \beta} \frac{\partial n}{\partial \lambda} - \frac{\partial m}{\partial \lambda} \frac{\partial n}{\partial \beta} \right),$$

$$\frac{\partial \alpha}{\partial n} = -\frac{1}{J} \left( \frac{\partial l}{\partial \beta} \frac{\partial m}{\partial \lambda} - \frac{\partial l}{\partial \lambda} \frac{\partial m}{\partial \beta} \right).$$

Hence if  $\partial m / \partial \beta = 0$  or is negligible by comparison with  $\partial m / \partial \lambda$ ,

$$\frac{\partial \alpha / \partial \alpha}{\partial l / \partial n} = -\frac{\partial n / \partial l}{\partial \beta / \partial \beta} = j,$$

and  $j$  then gives the ratio of equivalence of  $\delta n$  and  $\delta l$  both as regards changing  $\beta$  (when  $\alpha$  and  $\lambda$  are constant) and changing  $\alpha$  (when first  $\delta l = 0$ , then  $\delta n = 0$ ). A similar argument shows that

$$\frac{\partial \lambda / \partial \lambda}{\partial l / \partial n} = j,$$

but it is untrue that

$$\frac{\partial \beta / \partial \beta}{\partial l / \partial n} = j.$$

This equivalence therefore, has reference only to features of the spin which are independent of the tilt angle. Neglecting minor effects, it is probable true that incidence and spin parameter are the important variables and that they influence recovery more than does the initial tilt angle.

## APPENDIX II

### *Allowance for Random Errors of Inertia*

The function shown graphically in Fig. 2 is the normal probability density

$$\phi(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}$$

$$x = 0.477 \left( \frac{Z - X}{Y} \right),$$

and  $Y$  is the probable error of  $Z$  relative to its mean value  $X$ .  $Y$  may be defined as the probable error in absence of errors of inertia. If such errors are present the curve of Fig. 2 will be flatter through a change of scale given by replacing  $Y$  by  $\sqrt{Y^2 + Y_A^2 + Y_B^2}$ . In this expression we are assuming that  $A$  and  $B$  have independent probable errors of 15 per cent, and that  $Y_A$  and  $Y_B$  are the measured equivalent yawing moments.

The function given in Fig. 3 is defined as  $\frac{1}{2}\{1 - \Phi(x)\}$ ,

where 
$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

This function equals  $\frac{3}{4}$  or  $\frac{1}{4}$ , if  $N$  differs from  $X$  by the relevant probable error.

The chance of full-scale failure is now given by the flatter of the two curves in Fig. 3, in which  $N$  is the measured threshold. In order to reduce all aircraft to a common basis, *i.e.* the steeper curve with probable error  $Y$ , we have to decrease  $N$  by a quantity  $\theta$  (Fig. 3). For corresponding points the argument of  $\Phi$  must be the same and so

$$\frac{N - X - \theta}{Y} = \frac{N - X}{\sqrt{Y^2 + Y_A^2 + Y_B^2}}$$

therefore 
$$\theta = (N - X) \left\{ 1 - \frac{Y}{\sqrt{Y^2 + Y_A^2 + Y_B^2}} \right\}.$$

Since  $\theta$  cannot be calculated without first knowing  $X$  and  $Y$ , the determination for Table 1 was done by successive approximation, using a preliminary estimate to give the first set of values of  $\theta$ .

It is noted that the allowance for  $Y_A$  is always adverse if the model has  $N > X$ . In the previous routine, no allowance was made unless  $Y_A$  was negative.

### APPENDIX III

#### *Accident to Wellesley K.7737, 5th July, 1937*

While carrying out a routine production flight test on this machine, and after having done normal adjustments to the rigging and chassis, the machine was taken up to rated altitude and speeds carried out at 8,500, 13,000, 13,400 and 12,600 feet.

After the speeds had been carried out steep turns were carried out in both directions and general handling.

During the general handling the machine was completely stalled at a height between 9 and 10 thousand feet as a lateral stability test. This test has been carried out on all the 19 Production Wellesleys which I have tested, and the normal behaviour is that one or other wing drops and the machine commences a slow spiral of anything up to one turn from which recovery is practically instantaneous.

On this occasion the machine commenced a spiral to the right and controls were set for recovery before a turn was completed. Opposite rudder was applied and the stick held slightly aft of the central position. Instead of recovering from the spiral the nose lifted and the machine commenced a gentle spin to the right at a fairly flat attitude.

Opposite rudder alone was held on for about two turns and then the stick held hard forward also. The machine continued spinning for about five turns and then full engine was applied in bursts and the stick rocked violently fore and aft.

This method was continued for some time—but by this time the machine was down to between four and five thousand feet and I did not consider there was time to attempt lowering the flaps owing to the slow action of these. On the first attempt to leave the machine I felt a tendency to go out forwards on the right-hand side and owing to the danger of hitting the propeller I got back in the cockpit and switched off the engine.

I finally left the machine at what I estimated to be between three and four thousand feet, over the left-hand trailing edge.

(Signed) J. K. QUILL.

6th July, 1937.

#### APPENDIX IV

The following is a copy of a flight report by one of the contractors' test pilots.

FR/L. 680

##### *Test Flight Report*

Aircraft—Typhoon R.7692

Subject—Spinning, C.G. normal\* and extended aft.†

Date—28th July, 1943.

Duration ---

As a result of Seth Smith's report that he had experienced difficulty in recovering from a 2-turn right-hand spin (*see* Flight Report No. FR/L.679) further spinning was carried out by myself at the same loading.

On the first flight, six spins to the right were made, three 2 turns, then increasing to 2½, 3 and 4 before starting to recover, but in no case was any trouble experienced. It must, however, be pointed out that the aircraft did not stabilise itself on any one of the spins before starting to recover, including the 4 turn.

The spins were particularly violent, the aircraft pitching and yawing alarmingly. On the first two spins entry was made at between 120 and 130 A.S.I. but on the subsequent spins the aircraft was stalled in at approximately 90 A.S.I. as that was the condition of entry when Seth Smith experienced trouble.

Recovery was effected in every case in between 1½ and 2½ turns (including the 4-turn spin) by applying full opposite rudder and after about ½ turn easing the stick forward.

The rudder was light and ineffective for the first movement but very heavy for the last few degrees and considerable pressure had to be used to hold it hard on.

On the next flight the aircraft was spun from between 25,000 and 25,500 ft. On this flight three spins to the right were made and one to the left. The first three were to the right, recovery on the first two being made after 2 turns and on the third after 3 turns. Again no difficulty was experienced. Entry was not quite so severe and the aircraft stabilised itself quicker in the spin than before, being quite stable after the third turn during the last spin. Recovery in each case was again good in between 1¾ and 2½ turns depending how soon the stick was eased forward after applying opposite rudder.

Recovery from the spin to the left was similar, the same technique being employed.

\* 6.8 in. forward of datum, wheels up.

† 5.3 in. forward of datum, wheels up.

On the third flight the aircraft was spun from 28,000 ft., one  $2\frac{1}{2}$ -turn spin to the left being carried out and two  $2\frac{1}{2}$ -turn spins to the right.

On this occasion recovery from a left-hand spin was not quite so quick. After applying full opposite rudder the stick was eased forward whereupon the spin speeded up and there was considerable yaw inwards. The nose, however, progressively went down and recovery was effected in about 3 to  $3\frac{1}{2}$  turns at about 23,000 ft. It was also noticed that the elevators were very much heavier.

Recovery from the right-hand spin was just as good as before, in fact, if anything, it was slightly quicker. In every case the aircraft was out in level flight by 24,000 ft.

Afterwards, various methods of recovery were investigated and it would appear that to ensure the quickest recovery the following points are important :—

- (1) Before applying opposite rudder the stick should be pulled hard back and held there.
- (2) The rudder is heavy for the last few degrees and care should be taken to ensure that full opposite rudder is applied and held on.
- (3) There should be an appreciable pause after applying opposite rudder before moving the stick forward.
- (4) It is not either necessary or advisable to push the stick hard against the dashboard. It should be firmly and progressively eased forward to approximately central and then held there.
- (5) At least 180 to 200 A.S.I. should be attained before attempting to pull out of the resultant dive, otherwise the aircraft may be stalled and tend to spin in the opposite direction.

The aircraft was then loaded to the extended aft limit. All-up weight 10,500 lb. C.G. 5.3 in. forward of datum, wheels up.

Only one spin to the left was made, starting from 20,000 ft. Entry was perfectly normal, the aircraft pitching violently and yawing from side to side. After  $2\frac{1}{2}$  turns full opposite rudder was applied, when the aircraft suddenly became very tail heavy and the stick came hard back.

Both hands were used to try to centralise the stick but it was impossible to move the stick more than an inch or two forward and gave the impression that the control circuit had jammed. The rudder was therefore held hard on and with both hands a pitching movement was built up until finally the nose was pitched down and the aircraft recovered. There was a considerable yawing oscillation during the period of pitching the aircraft out, and when the nose finally went down the spin speeded up rapidly and the aircraft yawed inwards. Recovery was effected between 13,000 and 14,000 ft.

It is impossible to be very concise as to exactly what happened but my impression was that, had I allowed the stick to remain back, a very flat spin would have then developed. It must be appreciated that the aircraft was still pitching violently when opposite rudder was first applied, so it was never possible to get an idea of its relative angle.

(Signed) P. G. LUCAS.

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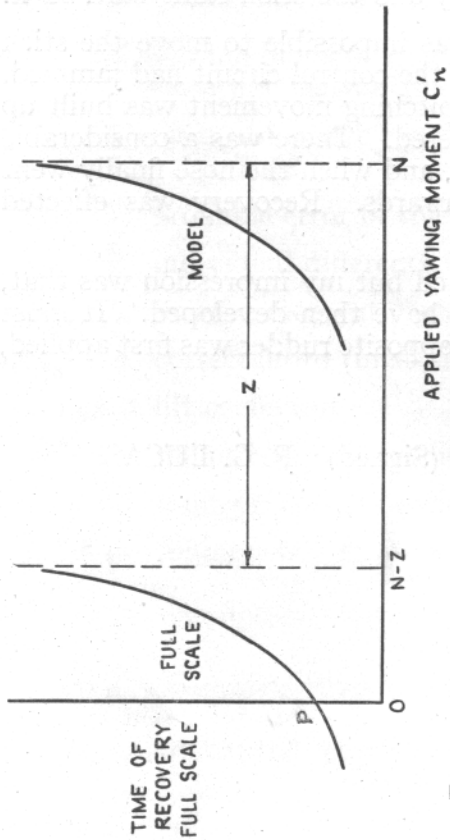
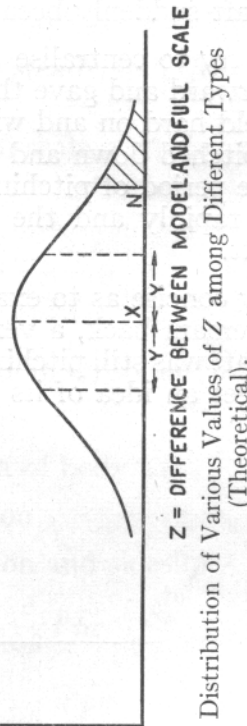


FIG. 1. Relative Position of Model and Full-scale Recovery Curves (Theoretical).

FREQUENCY OF OCCURRENCE OF Z AMONG ALL MONOPLANES



Z = DIFFERENCE BETWEEN MODEL AND FULL SCALE (Theoretical).  
 FIG. 2. Distribution of Various Values of Z among Different Types (Theoretical).

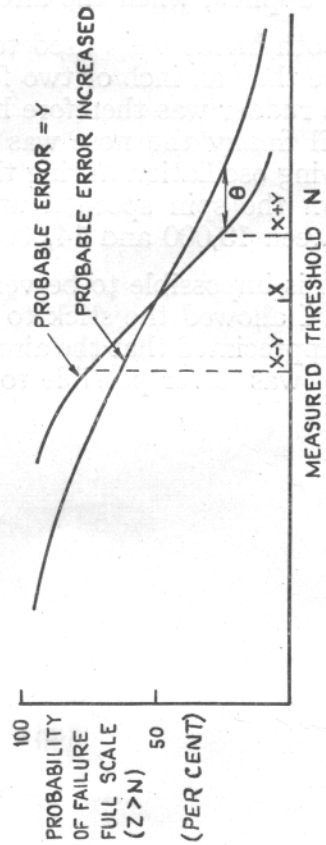


FIG. 3. Theoretical Shape of the Failure Curve.

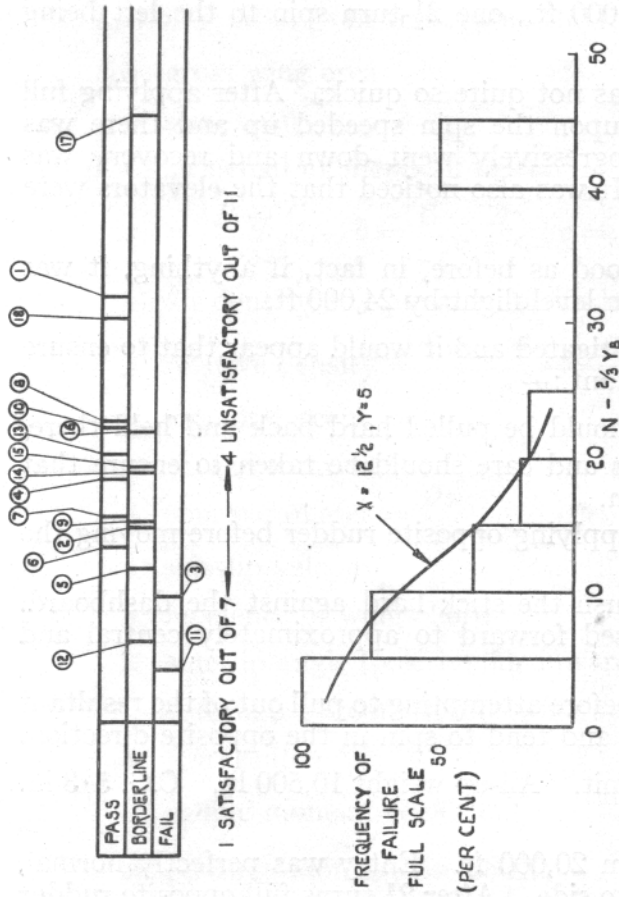


FIG. 4. Present Method of Allowing for Inertia Errors.

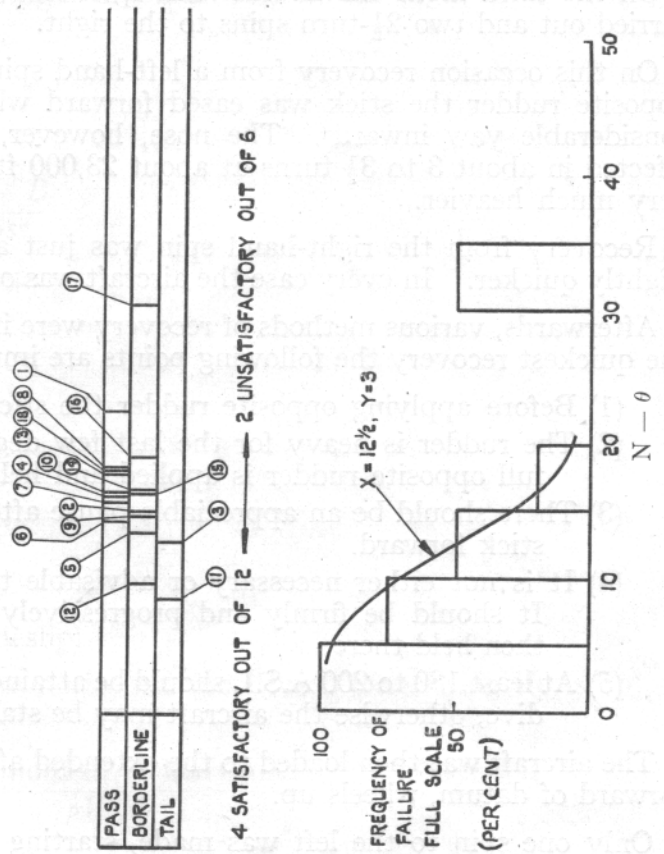


FIG. 5. Alternative Method of Allowing for Inertia Errors.

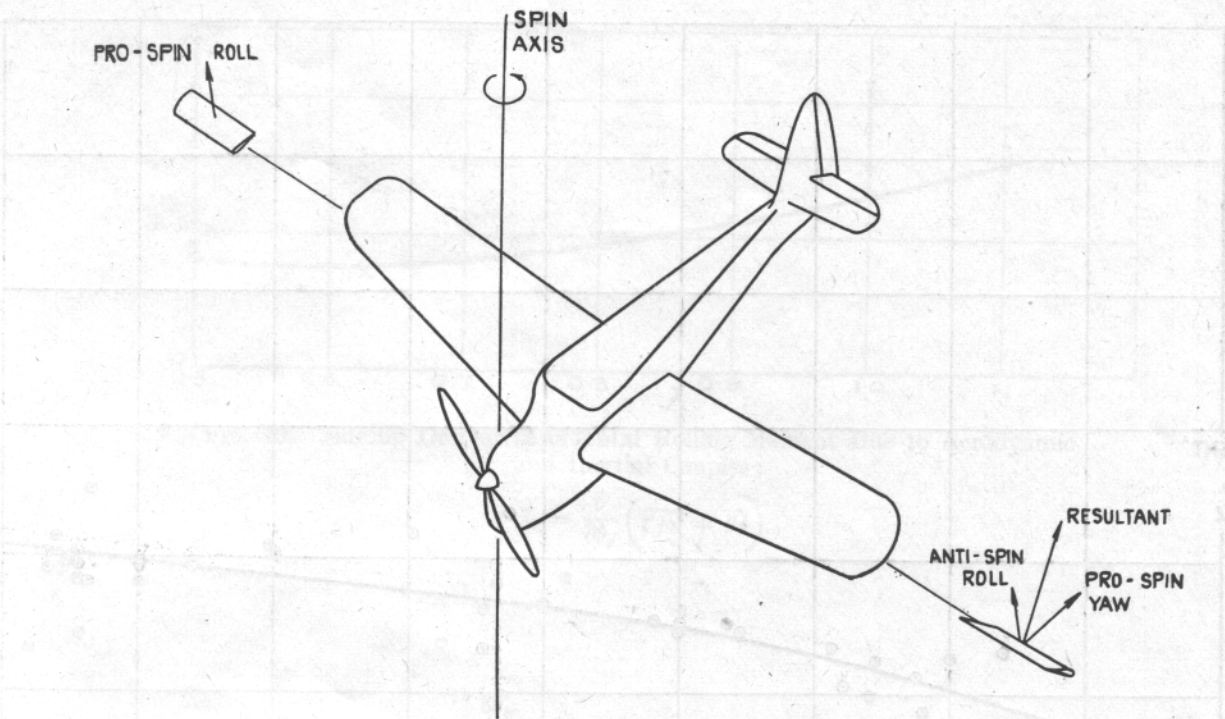


FIG. 6. Moments Applied to Spinning Model by Auxiliary Vanes.

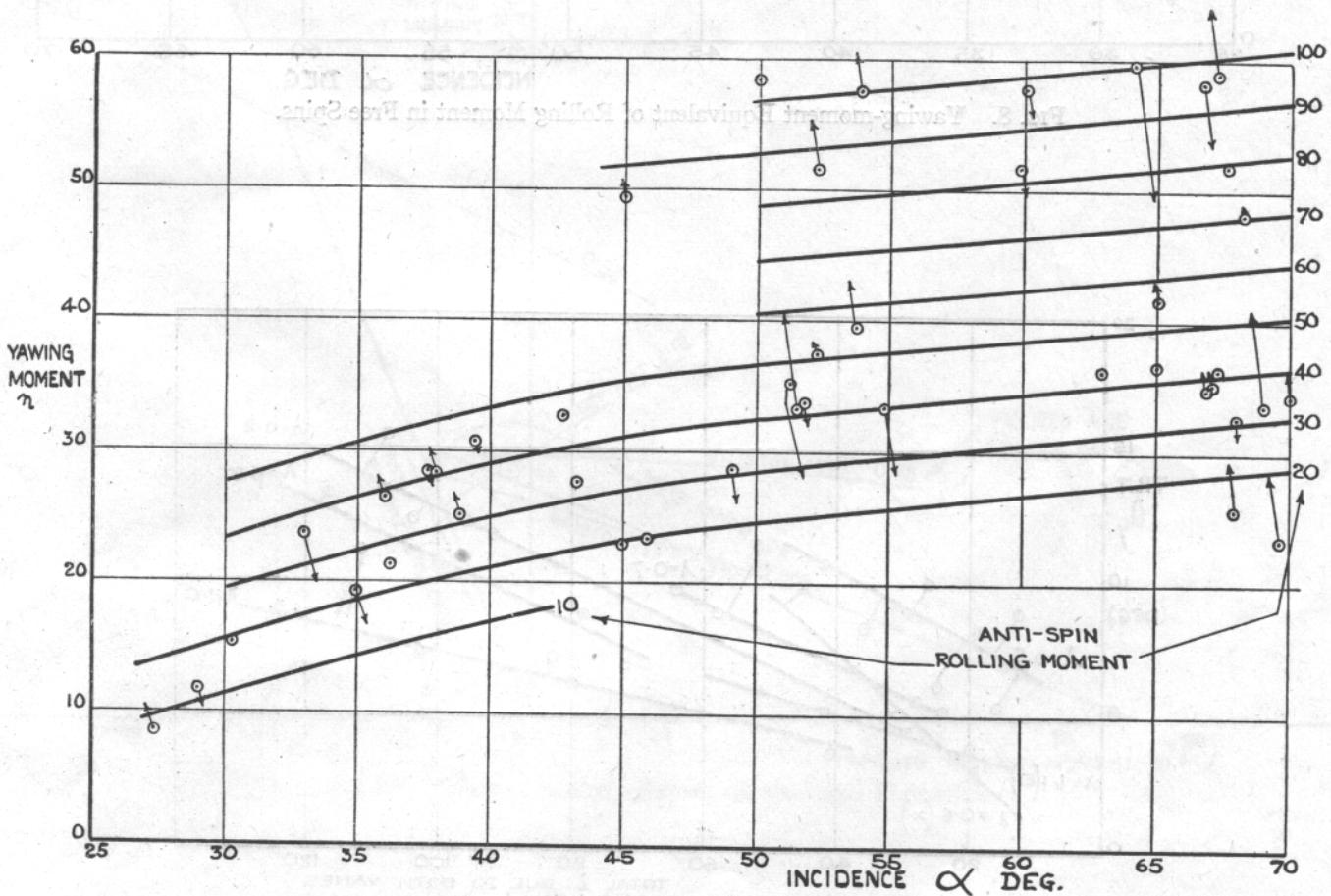


FIG. 7. Effect of Applied Rolling Moment on Steady Free Spins.

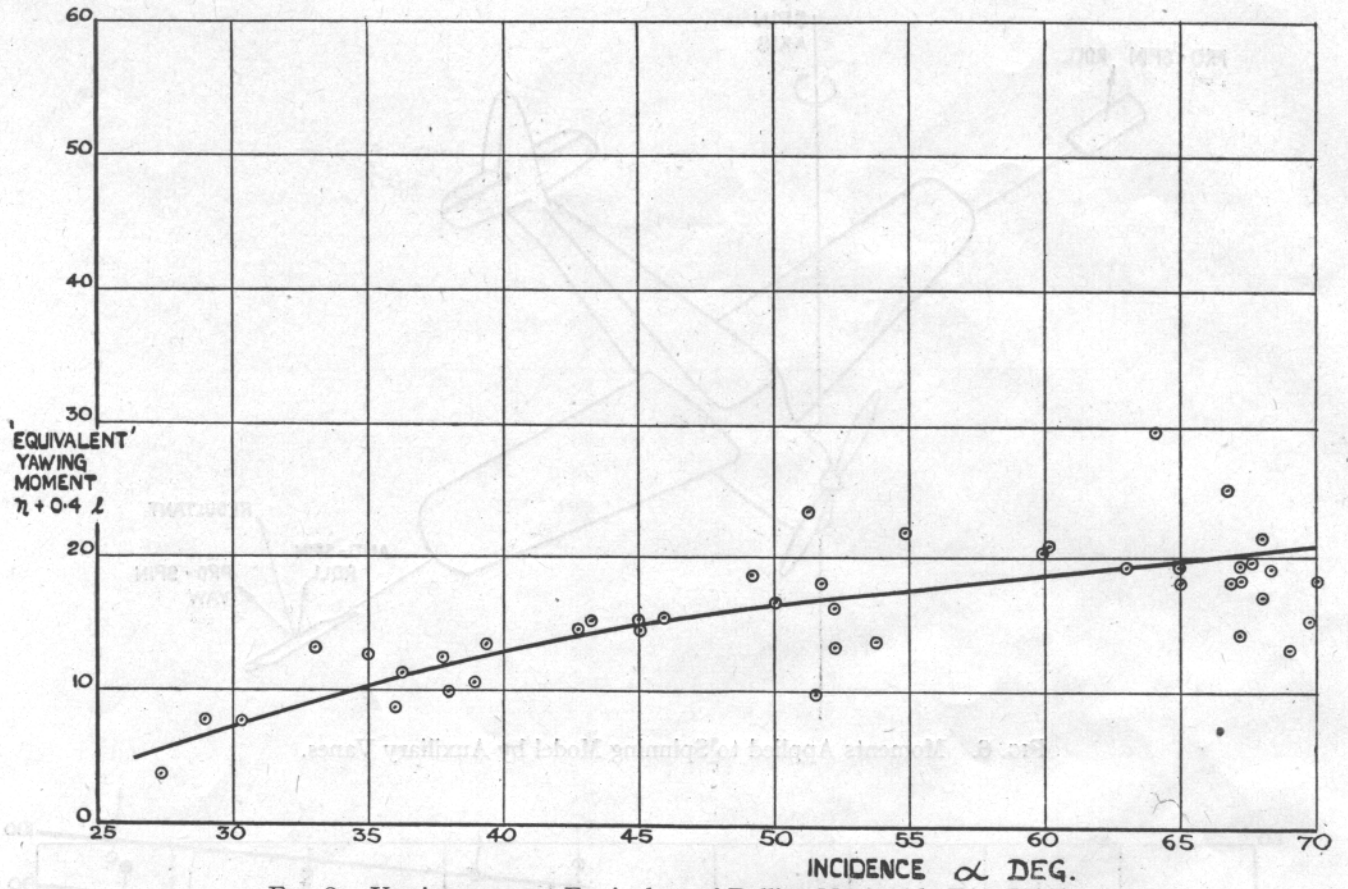


FIG. 8. Yawing-moment Equivalent of Rolling Moment in Free Spins.

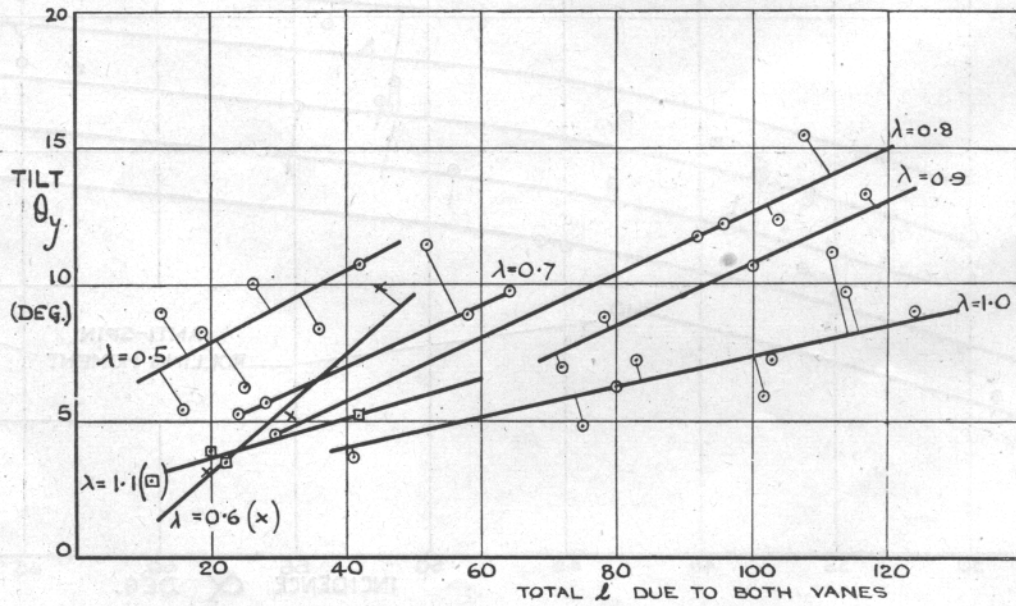


FIG. 9. Effect of Applied Rolling Moment on Tilt Angle  $\theta_y$ .

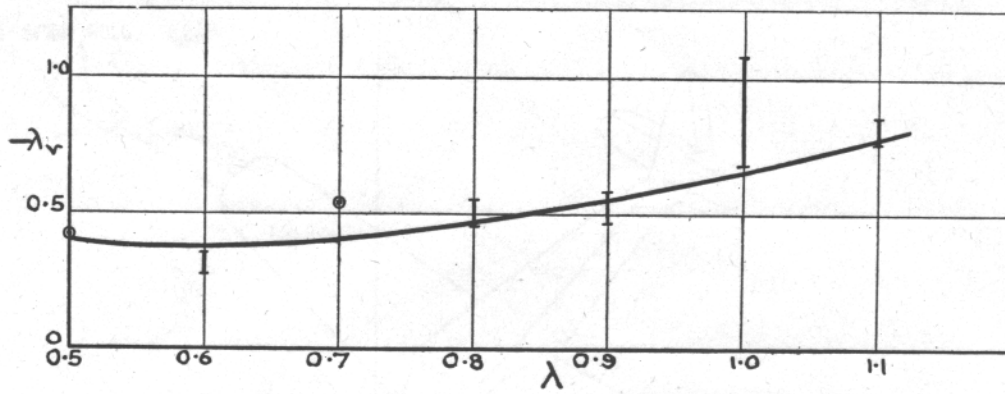


FIG. 10. Sideslip Derivative of Total Rolling Moment Due to Aerodynamic and Inertial Couples;

$$\lambda_r = \frac{\partial}{\partial \theta_r} (\bar{v}_{pv} + \bar{v}_i).$$

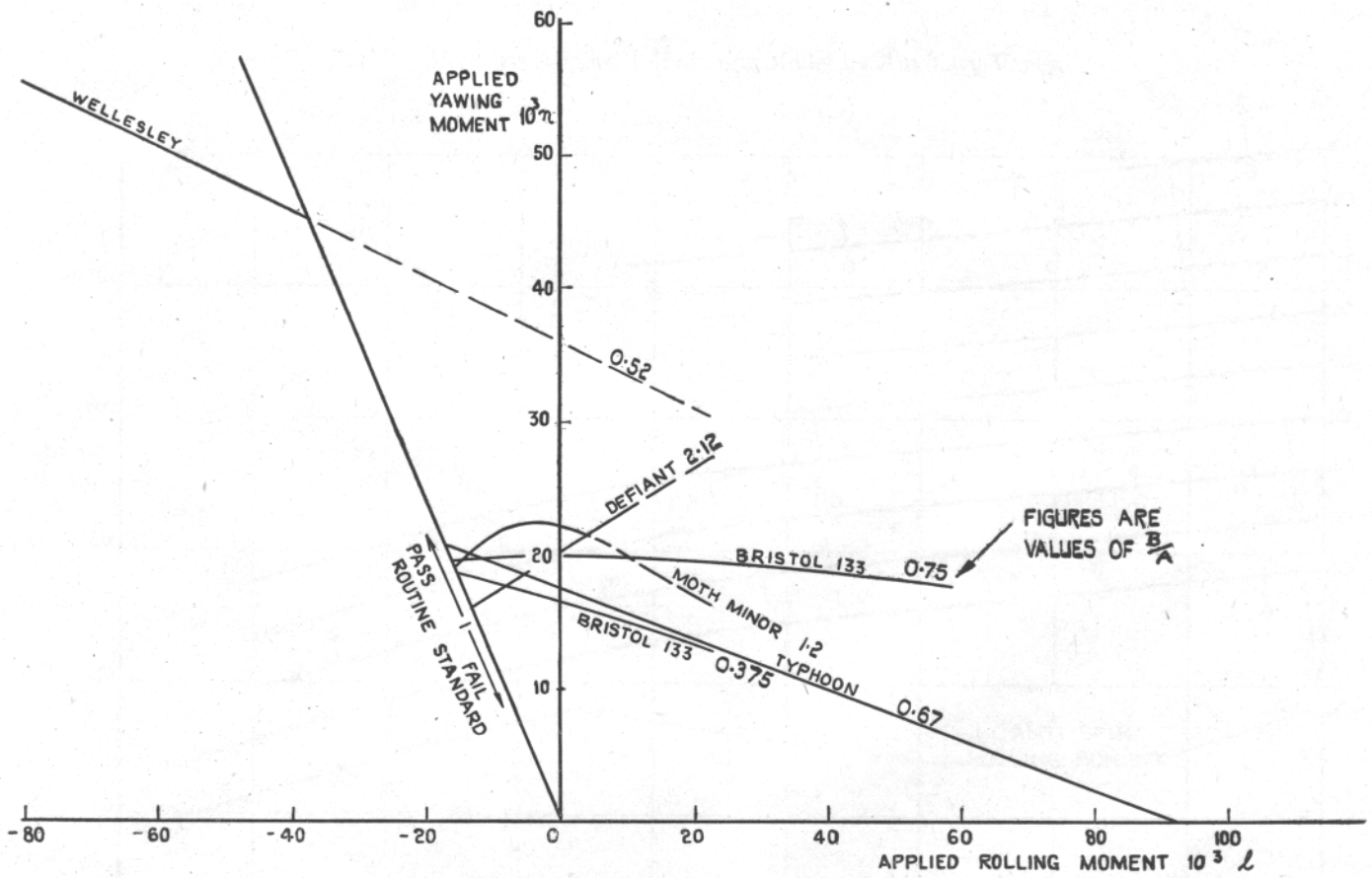


FIG. 11. Effect of Rolling Moment on Measured Threshold.

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