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Note on the Flow Near the Tail of a Two-dimensional Aerofoil Moving at a Free-stream Mach Number close to Unity

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30th June, 1954

Summary

This note discusses the flow near the trailing edge of a two-dimensional aerofoil moving at a free-stream Mach number close to unity, for cases where the effects of viscosity are small. A qualitative argument, which is supported by experimental evidence, suggests that the local Mach number downstream of the trailing-edge shock waves is approximately independent of free-stream Mach number, aerofoil geometry, and incidence. It follows from this result that there is a unique relationship between the flow deflection angle at the trailing edge and the local Mach number just upstream of the trailing-edge shock waves. This relationship is determined by using results obtained during wind-tunnel experiments on aerofoils of the R.A.E. series, and may sometimes be used to give rapid estimates of the local Mach number at the trailing edge of an aerofoil in terms of the trailing-edge angle, incidence, and control angle. When the Mach number immediately ahead of the trailing-edge shock has been determined, the local Mach numbers over the surface ahead of the trailing edge can be estimated by using simple-wave theory. The characteristics of straight-sided controls are considered as an example.

1. Introduction

Several approximate methods for estimating the pressure distribution round an aerofoil in inviscid flow at high subsonic and transonic speeds are based on some assumptions as to the location of the sonic point, or of the conditions at some other point on the forward part of the aerofoil. Although it is not suggested that these approximate methods are in error, it seems worth while to point out that an extremely simple method, based on a semi-empirical conclusion about the flow at the trailing edge, can sometimes be used in making rapid estimates of the flow over the tail of a two-dimensional aerofoil when the free-stream Mach number is close to unity. It is assumed throughout that the effects of the boundary layers are small, and in particular that flow separation is absent.

2. Qualitative Discussion of the Flow Near the Trailing Edge

It is well known that the shock waves on an aerofoil moving at fixed incidence and high subsonic speeds move rearwards as the free-stream Mach number M_0 is raised, and reach the trailing edge at some free-stream Mach number less than unity. With further increase of M_0, the shock waves become inclined, and wind-tunnel observations show that a second, near-normal, shock usually appears between the trailing-edge shocks as shown in the photographs reproduced in Fig. 1. This second shock moves downstream as the Mach number is raised and finally disappears at some free-stream Mach number close to unity;

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the trailing-edge shocks alone are then present and, of course, persist when the free-stream Mach number is raised above unity.

It is found that, for free-stream Mach numbers close to unity, the local Mach number distribution at the surface of the aerofoil is insensitive to changes of free-stream Mach number. Thus, the local Mach number ahead of the trailing-edge shock is sensibly constant, and for this reason it is possible to explain the observations described above by the following argument which will, in the first place, be confined to a symmetrical aerofoil at zero incidence.

Consider first an aerofoil moving at a free-stream Mach number M_{01} at which the shock waves have reached the trailing edge, but are approximately normal. Then Fig. 2(a) shows diagrammatically the variation of Mach number on the axis* near and downstream of the trailing edge. The local Mach number $M_{T.E.}$ at the surface ahead of the trailing edge is greater than unity, and drops to a value M_{D1} less than unity through the trailing-edge shock. The Mach number then varies gradually along the wake until the free-stream Mach number M_{01} is reached at infinity downstream. Now suppose that the free-stream Mach number is raised to a value M_{02} (still less than unity); the corresponding Mach number M_{D2} cannot then be reached through a normal shock because the upstream Mach number $M_{T.E.}$ remains unchanged. Apparently, this difficulty is overcome by the formation of an inclined shock at the trailing edge through which the Mach number drops to M'_D (greater than unity); the Mach number then varies along the wake in the manner sketched in Fig. 2(a) until a value M'_{D2} is reached where a normal shock gives a downstream Mach number lying on the curve M_{D2} M_{02}. Since the angle of incidence is zero, the oblique shock formed at the trailing edge must be such that the flow deflection angle through it is approximately equal to the semi trailing-edge angle. Because the upstream Mach number $M_{T.E.}$ is constant, the downstream Mach number M'_D is thus independent of free-stream Mach number; some support for this conclusion is provided by the fact that in Fig. 1 the inclination** of the trailing-edge shock waves ahead of the normal shock does not change appreciably with change of free-stream Mach number. If, therefore, (Fig. 2(a)) the free-stream Mach number is again raised, to M_{03}, the value of M'_D does not change, and the normal shock wave moves downstream to a position M'_{D3} and becomes weaker. This process continues (M_{04} and M'_{D4}) until eventually, for a free-stream Mach number close to unity (M_{05}), the normal shock disappears.

The length of the normal shock is fixed by its position downstream of the trailing edge, and by the inclination of the trailing-edge shocks; the distance over which it extends laterally must, however, also be consistent with the fact that further from the axis the upstream Mach number and flow

deflection/

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* There is, of course, some variation of Mach number with distance from the axis (i.e., perpendicular to the chord), but this should be small in the vicinity of the trailing edge at free-stream Mach numbers close to unity for many practical sections, and especially for sections which have straight surfaces at the rear such as the R.A.E. series[1] of sections.

** The inclination agrees with that calculated for a plane shock wave of deflection angle equal to the semi trailing-edge angle and with the measured Mach number upstream. The calculated value of the Mach number downstream is about 1.08 which is consistent with the curves of Fig. 3.

deflection angle are such that the required subsonic flow can be achieved downstream of a single shock wave (of inclination different from that of the tail shock closer to the aerofoil, see Fig. 1).

So far, the discussion has been confined to a symmetrical aerofoil at zero incidence, but for an aerofoil producing lift it is again found that the local surface Mach numbers are insensitive to free-stream Mach number, and a similar argument can be used. Referring to Fig. 2(b), let us suppose that the upper-surface shock wave has reached the trailing edge at a free-stream Mach number M_{01}, and is then normal. The Mach number just downstream of this shock is M_{D1}, and this must be approximately equal* to the Mach number at the trailing edge of the lower surface since the static pressure must be the same at this point on both surfaces. The lower-surface shock occurs at a distance ahead of the trailing edge such that the pressure rise through the shock, plus the pressure rise between the shock and the trailing edge, gives the Mach number M_{D1} there. With further increase of free-stream Mach number the upper-surface shock becomes inclined (the downwash changing), and the Mach number downstream gradually rises. The lower-surface shock moves back towards the trailing edge (compare curves for M_{01} and M_{02} in Fig. 2 (b)), and reaches the trailing edge at a free-stream Mach number M_{03} when the downstream Mach number M_{D3} is approximately equal to that downstream of the shock on the upper surface. When the flow behind the trailing-edge shocks becomes supersonic, it might be expected that the downwash would fall to zero (experiment suggests that this is approximately true). The discussion for higher values of the free-stream Mach number (M_{04} to M_{07}) is thus the same as for the case of zero incidence except that the trailing-edge shocks ($M_{T.E.U.}$ M_D' and $M_{T.E.L.}$ M_D') are not of equal strength.

3. Comparison with Experiment

The discussion given above is highly speculative, and is advanced solely in an attempt to explain the part played by the near-normal shock wave. The only conclusion which is required for the purposes of the present paper is, however, that there is some reason for supposing that the Mach number M_D' immediately downstream of the trailing-edge shocks will be approximately independent of free-stream Mach number. Further, if the variation of Mach number along the wake is independent of incidence and trailing-edge angle, the value of M_D' will also be independent of these angles. If these conclusions are valid, the Mach number should be constant downstream of the oblique shock wave which gives a flow deflection**, δ, equal to that at the trailing edge and has the observed local Mach number $M_{T.E.}$ upstream. Values of $M_{T.E.}$ calculated from the equations for plane oblique shock waves are plotted on Fig. 3 against the flow deflection angle for several values of the downstream Mach number, and in Fig. 4 a similar plot is made showing the observed variation of $M_{T.E.}$ with δ for several aerofoil sections tested in wind tunnels under conditions where boundary layer separation did not occur ahead of the shock waves when they had moved back to the trailing edge. It is seen that the experimental observations for the

R.A.E./

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* For the purposes of the present qualitative discussion it is permissible to neglect differences between the changes of total head through the shocks on the two surfaces. Equality of static pressure thus implies equality of Mach number.

** The flow deflection angle δ is taken as

$$\delta = \tau/2 + \alpha + \eta \text{ on the upper surface,}$$

$$\text{and } \delta = \tau/2 - \alpha - \eta \text{ on the lower surface}$$

where τ = trailing-edge angle; α = incidence; η = control angle. Positive values of δ only are considered (i.e., values leading to the formation of compression waves).

R.A.E. aerofoils correlate very well on this basis even though the free-stream Mach numbers of the tests covered a range extending from about 0.90 to 0.97. The curve drawn through the experimental points of Fig. 4 is reproduced in Fig. 3, and appears to correspond to a Mach number between 1.06 and 1.10 behind the trailing-edge shocks.

4. Some Practical Uses of the Correlation between the Local Mach Number and Flow-deflection Angle at the Trailing Edge

The correlation shown in Fig. 4 suggests that the conclusions of the discussion given in Section 2 are approximately correct, and also gives an extremely simple method for estimating the local Mach number at the surface just ahead of the trailing edge in terms of the geometry of the aerofoil and its incidence. When the trailing-edge Mach number has been determined by the use of Fig. 4, the distribution of Mach number over the surface ahead of the trailing edge may be estimated with reasonable accuracy by the use of simple-wave theory. This method will, of course, become progressively less accurate as the sonic point is approached, but should be satisfactory for estimating the pressures over the rear of the aerofoil.

Two examples are shown in Fig. 5. The first is for a 6% thick RAE 104 section at 2 degrees incidence, and the second for a 10% thick RAE 102 section at zero incidence but with a 25% chord plain flap deflected through 2 degrees. The agreement with experiment is reasonably good over the rear 40% of the chord except close to the trailing edge where the measured pressures rise rapidly*. This is caused by thickening of the boundary layers ahead of the trailing edge shocks, and is exaggerated in the present experimental results because the boundary layer is relatively thick due to the low Reynolds number (about 1.9×10^6) and to the fact that transition to turbulent flow was fixed artificially.

Since it gives a rapid method for obtaining the pressures over the rear of an aerofoil, the procedure outlined above may have applications to the estimation of control-surface characteristics at free-stream Mach numbers close to unity. The calculation is particularly simple if, as is frequently the case, the controls have straight sides since the pressures on the upper surface and lower surface are then constant. As an example, values of the hinge-moment coefficient are plotted in Fig. 6(a), and Fig. 6(b) shows the values of a_2 ($= \partial C_L/\partial \eta$) and b_2 ($= \partial C_H/\partial \eta$).

A further application of the method is to predict whether shock-induced boundary-layer separation will occur at transonic speeds. It has been shown[2] that a turbulent boundary layer will separate at a near-normal shock wave formed on the surface of an aerofoil moving at high Mach number when the local Mach number ahead of the shock exceeds a value of about 1.2. For given incidence and for an aerofoil without concavities ahead of the trailing edge or a deflected flap**, the value of $M_{T.E.}$ (as obtained from Fig. 4) will be greater than the local Mach number ahead of the shock at any stage during its rearward movement towards the trailing edge. Thus under these conditions if the value of $M_{T.E.}$ is less than 1.2, it may be anticipated that separation will be absent on the surface under consideration throughout the transonic range.

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* In preparing Fig. 4, this pressure rise was ignored, when present in the experimental results, and the values of $M_{T.E.}$ which are plotted are based on an extrapolation to the trailing edge of the gradually varying pressures slightly upstream.

** If concavities or a deflected flap are present, the maximum local Mach number can be estimated from $M_{T.E.}$ by the use of simple-wave theory.

5. Concluding Remarks

It will be clear to the reader that the discussion of Section 2 is speculative, and that its only justification is that it appears to be supported to some extent by experiment. It should be remembered, however, that the experimental results used here were obtained on a particular family of sections at an approximately constant Reynolds number. The effects of Reynolds number are probably not large provided that the boundary layer is turbulent over most of the surface, and some support for this suggestion is provided by the agreement in Fig. 4 of the aerofoil results with results obtained on bumps attached to a wind tunnel wall on which there was a thick turbulent boundary layer. Until further data are available, however, care should be taken in applying Fig. 4, for example, to sections which differ widely from the R.A.E. series. For example, it might be anticipated that double-wedge sections, for which the position of the sonic point is fixed by the shoulder, would give results which differed from those for round-nosed sections. To investigate this point three results for symmetrical double wedges are plotted in Fig. 4. The experimental points were measured with laminar boundary layers, and for the calculated point[3] it was assumed that the flow was inviscid (this accounts for the major part of the discrepancy between the calculated and measured points shown for $\delta = 5.7$ degrees). It is seen that the measured points lie slightly above the curve drawn through the points for the R.A.E. sections, but further data are needed before it can be decided whether this is generally true.

Finally, it should be stressed that the correlation shown in Fig. 4 can be expected to apply only when boundary-layer separation is absent. For turbulent boundary layers this restricts the value of δ in Fig. 4 to about 12 degrees, this being the value of the deflection angle for separation at an oblique shock wave in purely supersonic flow[4].

References

No.	Author(s)	Title, etc.
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2	D. W. Holder, H. H. Pearcey and G. E. Gadd, N.P.L.	The interaction between shock waves and boundary layers
	J. E. Seddon, R.A.E.	With a note on The effects of the interaction on the performance of supersonic intakes. C.P. 180. 2nd February, 1954.
3	G. Guderley and H. Yoshihara	The flow over a wedge profile at Mach number 1. J.Ae.Soc., Vol. 17, p.723, 1950.
4	G. E. Gadd, D. W. Holder and J. D. Regan	An experimental investigation of the interaction between shock waves and boundary layers. A.R.C. 16,680 25th March, 1954.

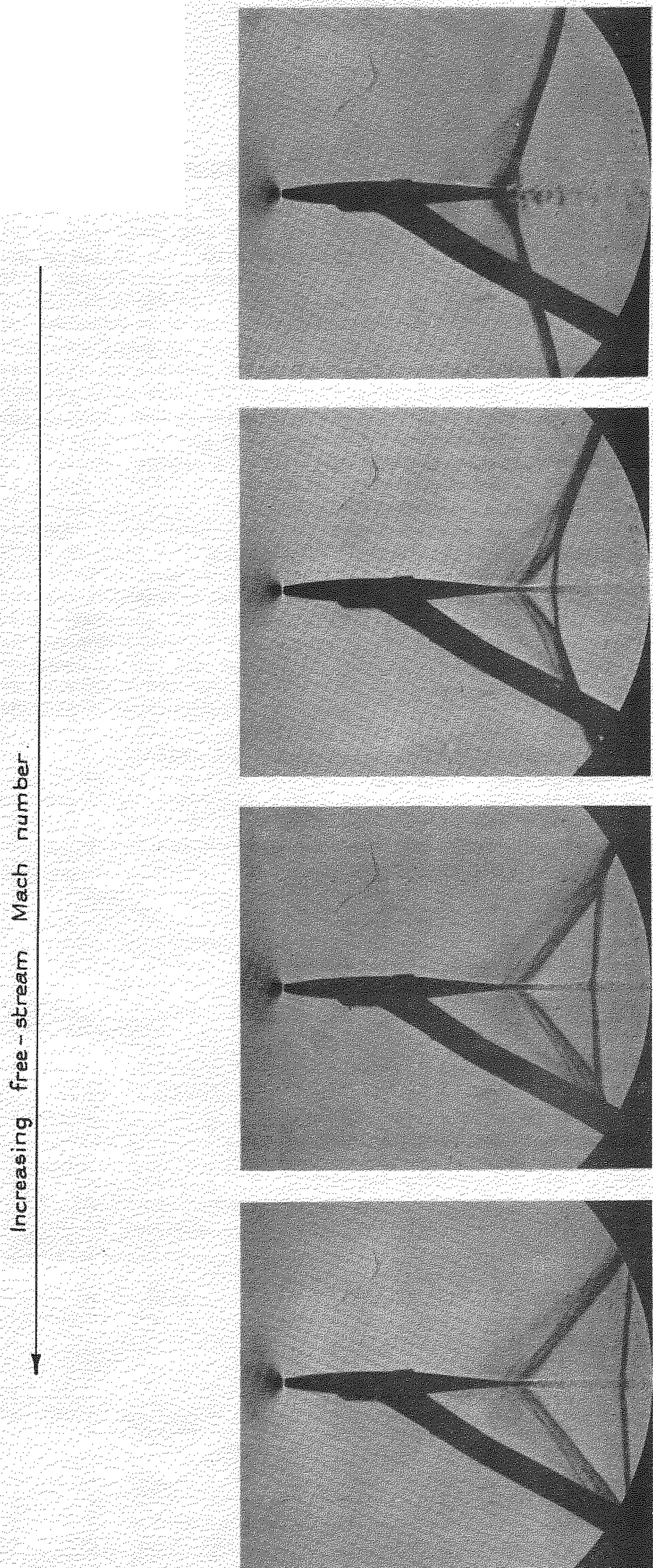
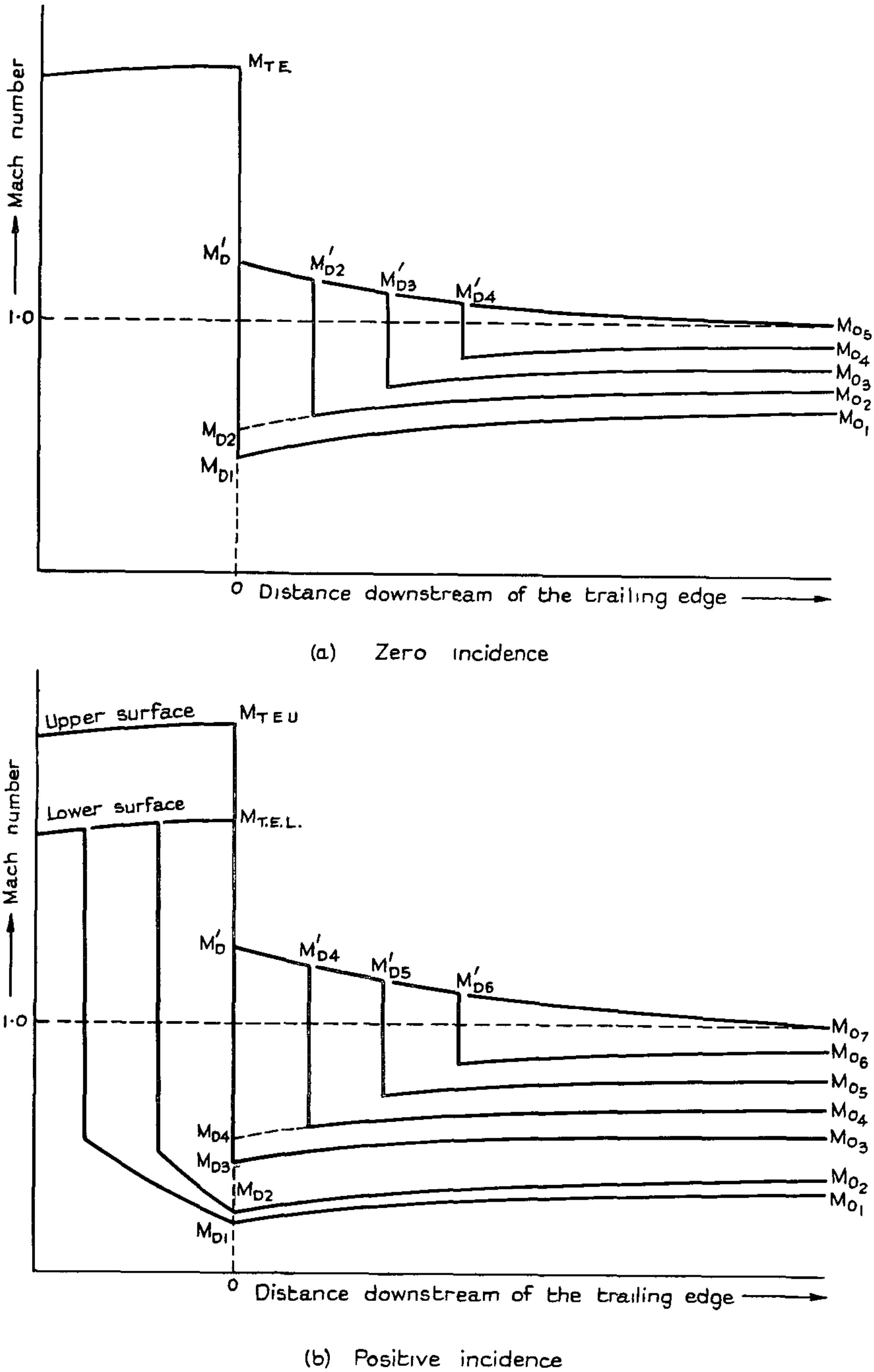


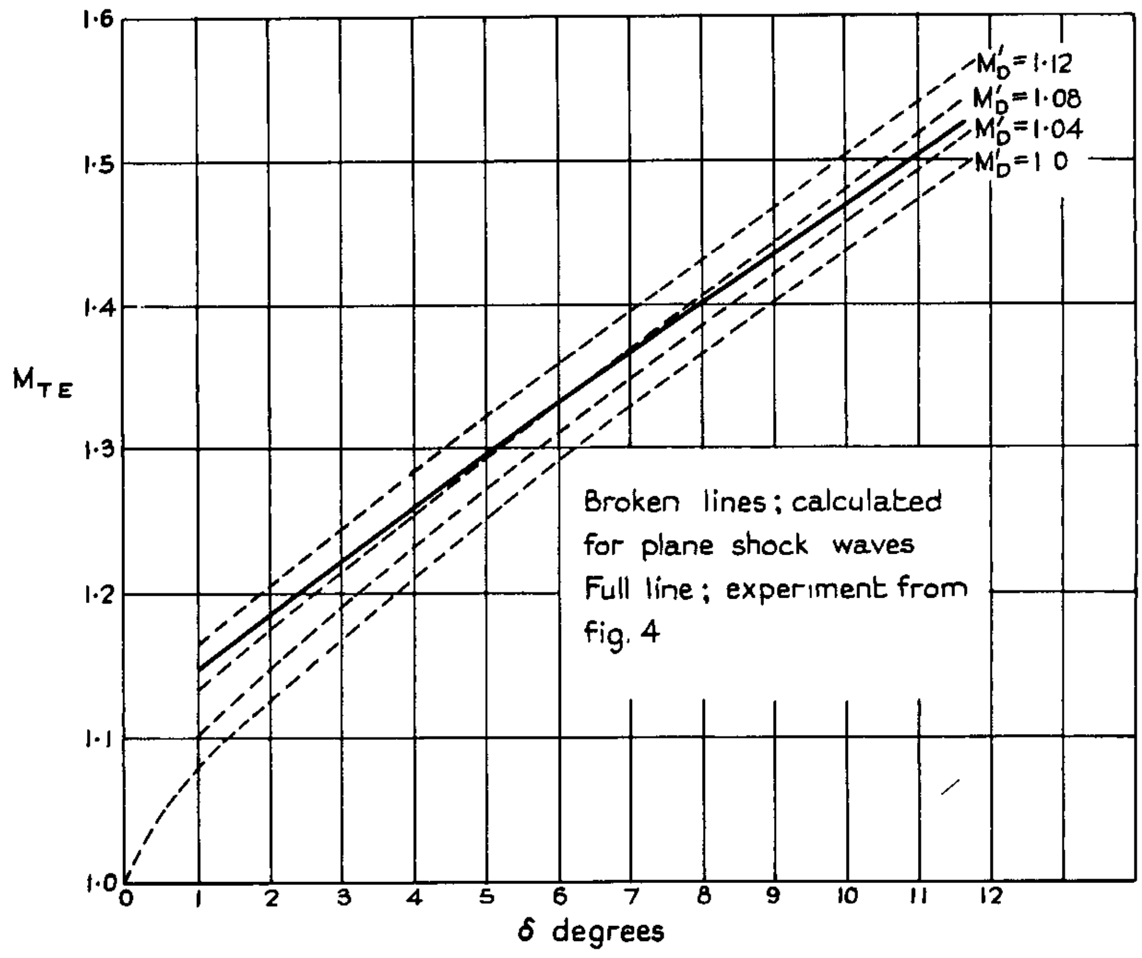
FIG. 1. Flow photographs for a 10% thick RAE 104 section at zero incidence.

FIG. 2



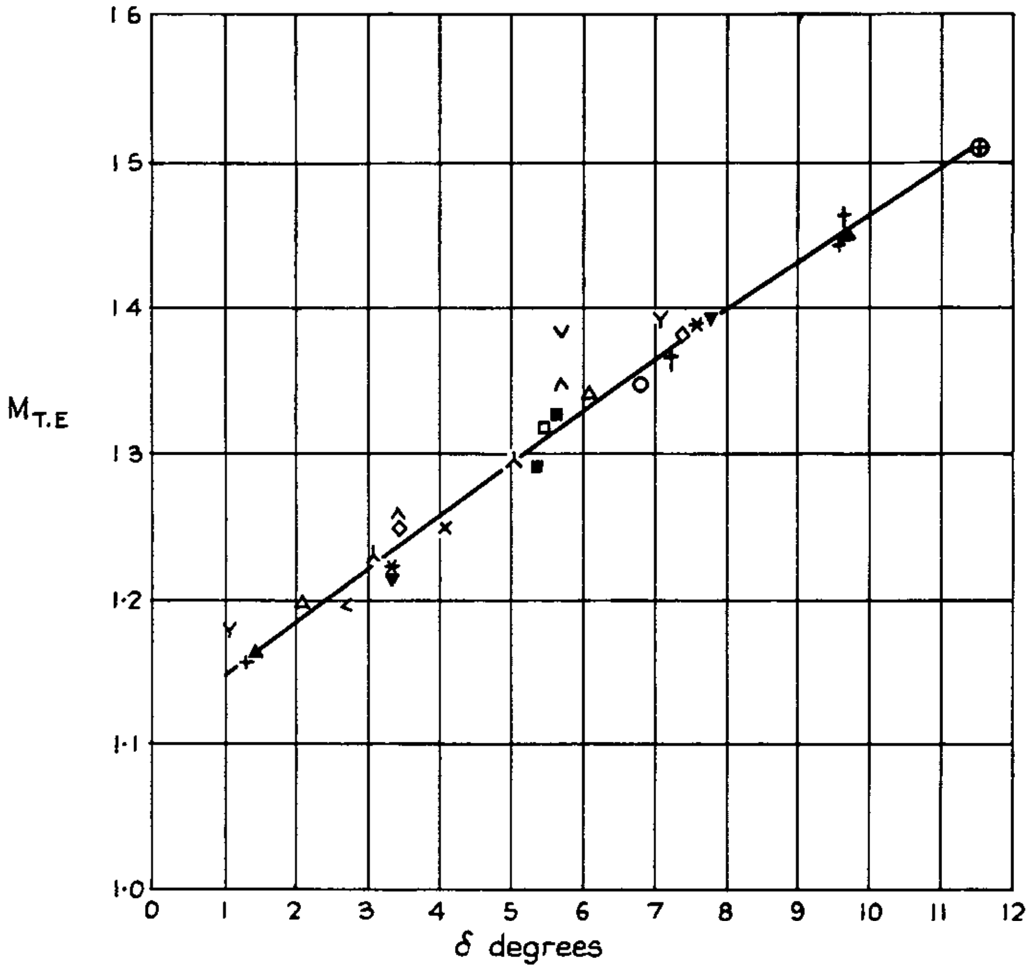
Sketches of the Mach number distribution near the trailing edge of a symmetrical aerofoil.

FIG. 3.



Calculated variation of $M_{T.E}$ with δ for various values of M'_D and comparison with the experimental curve of fig. 4.

<u>FIG. 4.</u>



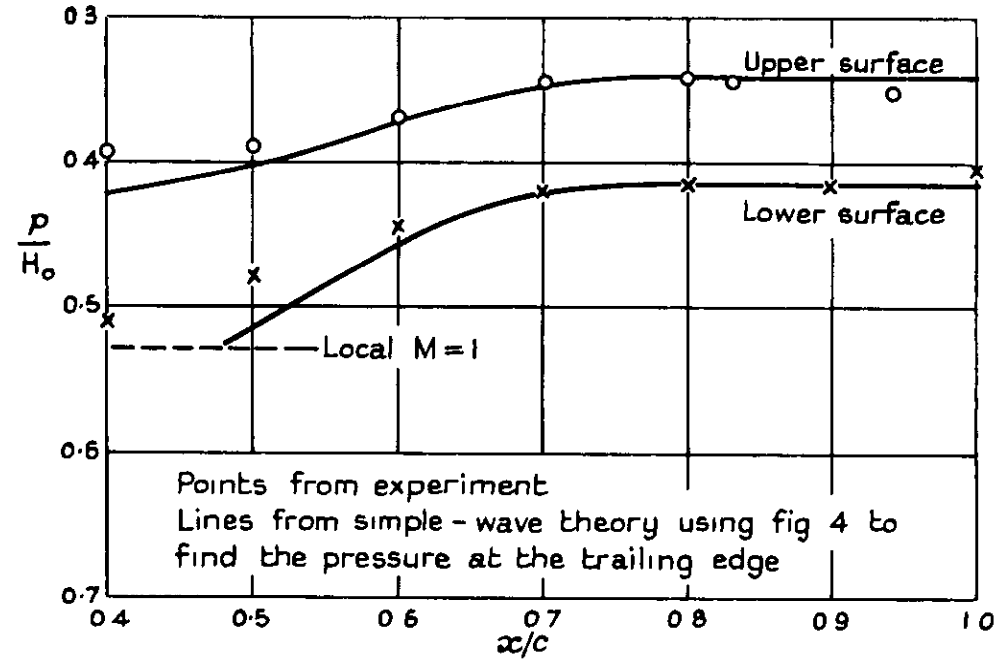
4% RAE 104
$\alpha = 0°$ <
6% RAE 104
$\alpha = 0°$ ×, $\alpha = 1°$ ⅄
$\alpha = 2°$ △, $\alpha = 3°$ Y
10% RAE 104
$\alpha = 0°$ ○, $\alpha = 2°$ ▽
10% RAE 102 with 25% plain flap
$\alpha = 0°$ $\eta = 0°$ □
$\alpha = 2°$ $\eta = 0°$ ◇
$\alpha = -0{\cdot}13°$ $\eta = -2°$ ✱
$\alpha = -0{\cdot}13°$ $\eta = -4°$ +
$\alpha = -2{\cdot}13°$ $\eta = -2°$ ▲
$\alpha = -2\ 13°$ $\eta = -4°$ ⊕
$\alpha = 1{\cdot}87°$ $\eta = -2°$ ■
$\alpha = 1{\cdot}87°$ $\eta = -4°$ ▼
Bulges on a wind tunnel wall †
Symmetrical double wedges
calculated (ref 3) v
measured ∧

<u>The experimental relationship between trailing-edge Mach number and deflection angle.</u>

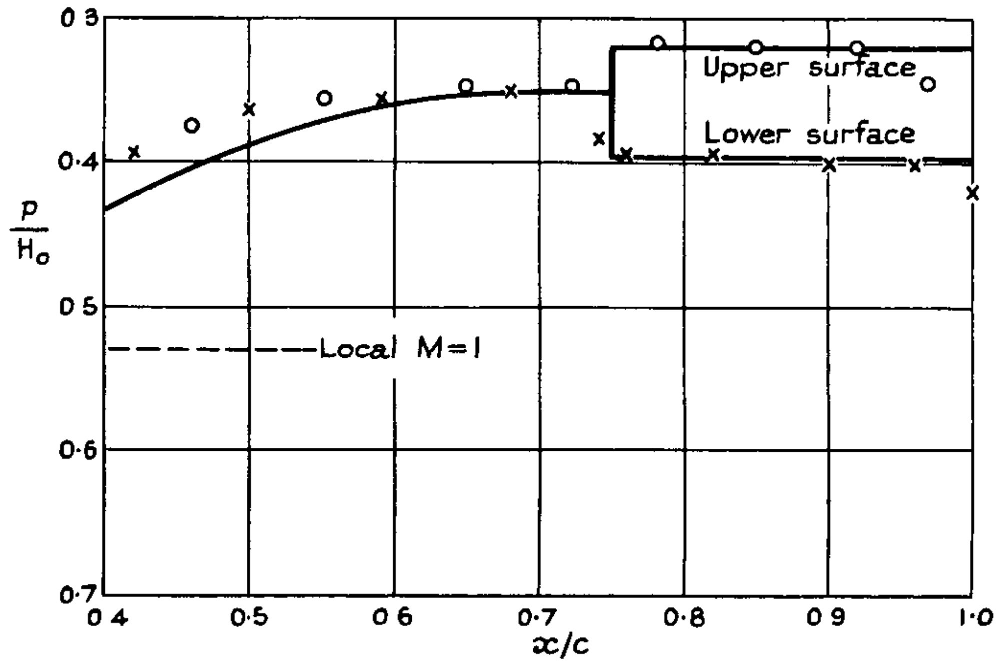
(The measurements were made at free-stream Mach numbers between 0·90 and 0 97)

EEM.

FIG. 5.



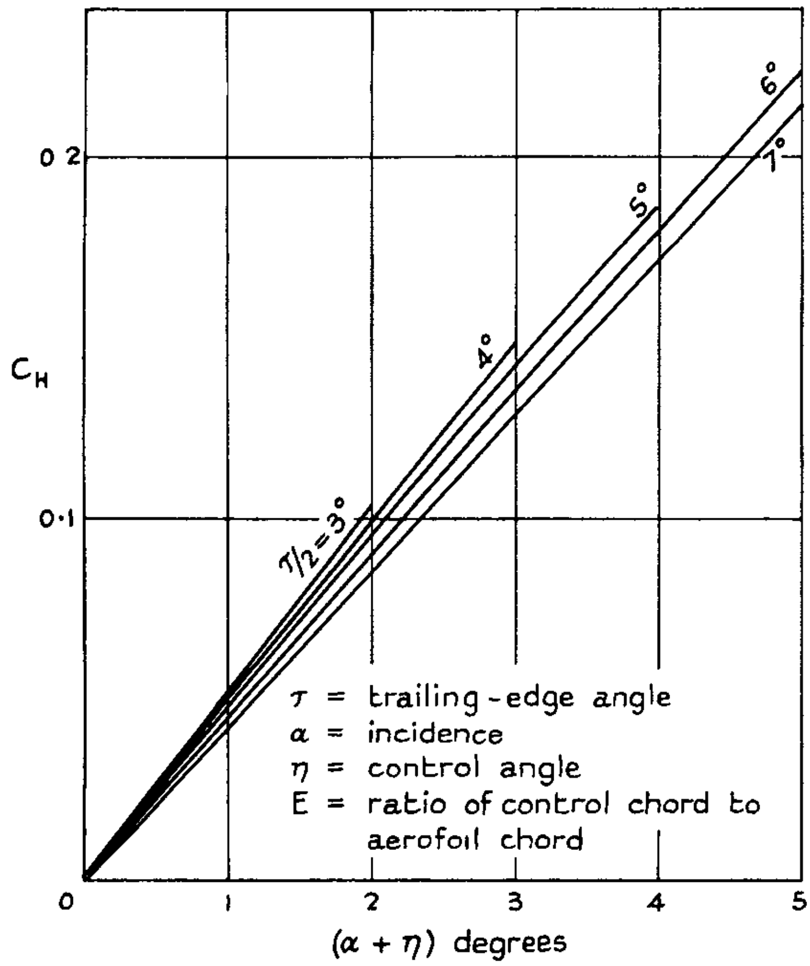
(a) 6% thick RAE 104 section at $\alpha = 2°$



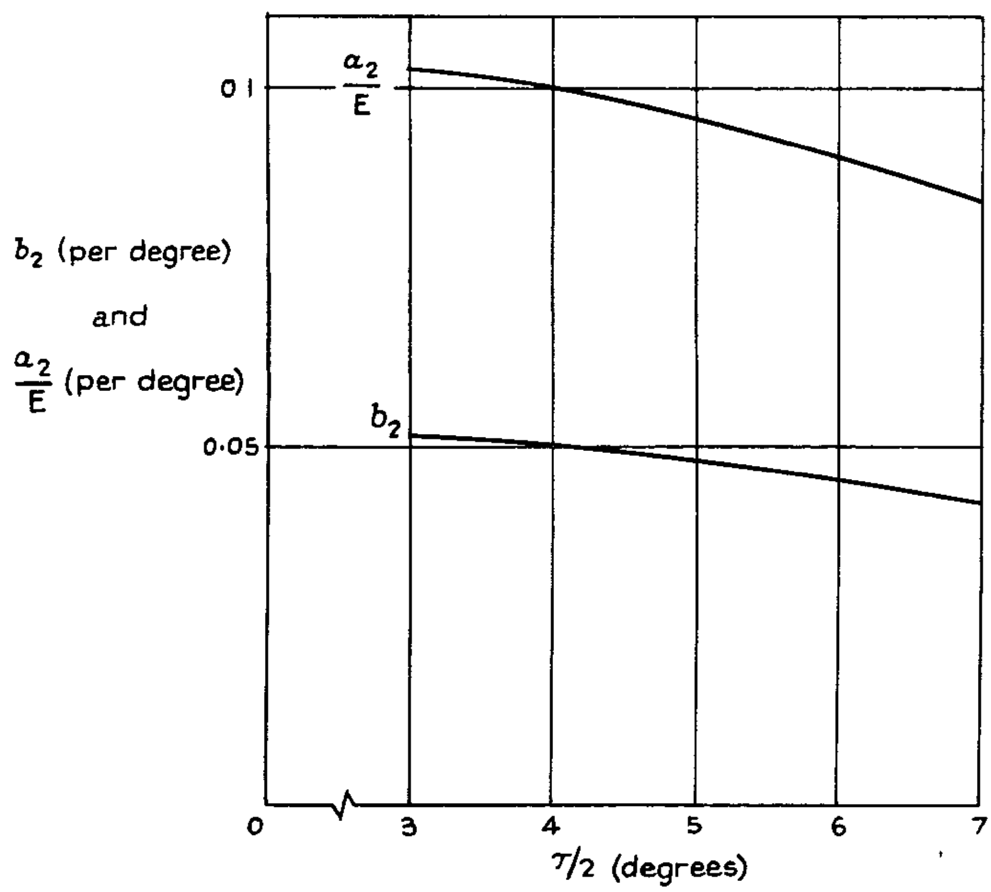
(b) 10% thick RAE 102 section with 25% flap at $\alpha = 0°$, $\eta = 2°$

Comparison between the measured and calculated pressures over the rear of two aerofoils.

FIG. 6.



(a) Hinge - moment coefficient



(b) Values of a_2 and b_2

Characteristics near $M_o = 1$ of plain two-dimensional controls with straight surfaces.

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