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By W. J. G. PINSKER
Aerodynamics Dept., R.A.E., Bedford

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Summary.

Free-flight models suspended by cable from a helicopter have been observed to experience unstable lateral pendular oscillations when towed above a certain speed. A simplified mathematical model of this configuration, considering only roll angle and lateral displacement of the model, has produced a result in agreement with flight observation. A criterion is given for the prediction of the critical speed, and design parameters are discussed which will help to increase the speed range at which tow is possible free from instability in this mode.

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*Replaces R.A.E. Technical Report No. 68 247—A.R.C. 31 273.

1. *Introduction.*

A widely used technique for the study of the low speed flying characteristics, and the spin, of new aircraft designs, is to drop dynamically similar models in free flight from a suitable aircraft, usually a helicopter. Prior to release the model is suspended by cable from the helicopter flying at a chosen height and at a speed identical to the speed at which the model will stabilise in free gliding flight in the initial trim configuration. Recently difficulties have been experienced during such a flight in that the model developed an unstable lateral pendular oscillation above a certain towing speed which led to a limit cycle oscillation of substantial amplitude. The present report presents a theoretical analysis of the stability of this phenomenon with a result which is in close agreement with the observed behaviour. A stability criterion will be derived which allows the critical towing speed to be predicted and parameters are established which will permit this critical speed to be raised.

It should be noted that the present report is only concerned with one particular form of instability to which a body towed by an aircraft may be subject. The earliest investigation into this stability of aero-tow was made by Glauert[1], restricting himself to non-lifting bodies but including the dynamics of the towing cable. Further studies of essentially non-lifting configurations are given in Refs. 2 to 6.

2. *The Towing Configuration.*

The flight model is rigidly attached to a carriage (bomb-slip) to which is pivoted a suspension arm (d) which in turn is attached to the towing cable of length l (Fig. 1). The suspension arm is pivoted to allow freedom in pitch but is rigid with respect to roll and yaw. The pivot is located as close as possible to the centre of gravity of the assembly model plus bomb-slip so that the suspension does not introduce pitching moments. The aerodynamic incidence of the model on tow is therefore entirely defined by its aerodynamic trim which is practically identical to free flight trim after release from the bomb-slip. This description is somewhat over-simplified, representing the design aim rather than exact conditions realised in practice. However, the theoretical model on which the present investigation is based, conforms with this simplified mechanism.

A further assumption is that the helicopter is in steady rectilinear level flight and unaffected by the reaction transmitted through the towing cable. The cable itself is assumed to be weightless and to have neither stiffness nor to be subject to aerodynamic forces. As a consequence the cable can be treated as a mathematical line, capable only of transmitting tension forces. A rough allowance for some of these effects can be made by crediting the model with a suitable portion of the cable weight and drag, but this is not attempted here.

We are only concerned with conditions where the lift on the model is significantly less than the weight of the model plus bomb-slip, i.e. the analysis does not cover the condition typical of glider tow, where the cable tension is determined largely by the drag of the towed vehicle.

The cable therefore constrains the vertical motion of the model which can in consequence be ignored in the analysis. The fore and aft response of the model is also ignored, as it represents a separate freedom which does not couple with the lateral motion unless higher order terms are induced. The model is therefore assumed to move like the helicopter at constant speed. With these simplifying assumptions we are then left with three freedoms to consider, lateral displacement y, model bank angle ϕ, and angle of yaw ψ. We make the further assumption that the model has substantial weather-cock-stability about the centre of gravity of the model which eliminates yawing as a separate freedom. It should be noted that the term model as used here is meant to describe the actual model plus all those parts of the suspension rigidly attached to it. Prior to release for a flight trial the model is normally towed at a speed equivalent to the trimmed free-flight speed, i.e. in a condition where the lift on the model equals its weight. In the towed condition, which is the subject of the present investigation, the weight of the model is increased by that of the carriage, so that the total weight of the assembly is still $W > L$, i.e. the condition for the validity of the analysis, $L/W < 1$, stated earlier, is still satisfied.

3. *Stability Analysis.*

3.1. *Geometric Relations.*

The system of axes and the nomenclature used to describe the motion of the model is shown in Fig. 2.

The analysis is restricted to small perturbations, hence the angles are small, so that, e.g. $\sin\delta = \delta$, $\sin\sigma = \sigma$ and $\cos\delta = \cos\sigma = 1{\cdot}0$. The towing angle μ, however, is not restricted by such limitations as it enters into the analysis only as a fixed parameter for each particular flight condition and speed.

Fig. 3 illustrates the force equilibrium defining the towing angle μ as:

$$\tan\mu = \frac{D}{W-L} \tag{1}$$

where D is the drag, L the lift acting on the model and W the weight of the total towed assembly. In this projection the cable and the suspension arm are in line (but not in projections perpendicular to this) since the arm is freely pivoted in pitch. If l is the cable length and d the length of the suspension arm, we obtain for their vertical and horizontal components the following relationships:

$$\frac{d_x}{d_z} = \frac{l_x}{l_z} = \tan\mu = \frac{D}{W-L} \tag{2}$$

also

$$\frac{d_z}{d} = \frac{l_z}{l} = \cos\mu = \frac{1}{\sqrt{1+\left(\frac{D}{W-L}\right)^2}}. \tag{3}$$

If we assume $l >> d$ and therefore ignore d we can establish from Fig. 2 the following relationships between the lateral displacement y of the model and the angles δ and σ:

$$y = -\delta\, l_z = -\sigma\, l_x. \tag{4}$$

As the model is constrained in the vertical plane by the cable, it moves laterally along an arc with the flight path of the helicopter as the axis. The lateral displacement y is therefore measured along this arc and the lateral forces Y are considered tangential to the arc.

3.2. *The Equations of Motion.*

The analysis will show that an adequate answer is obtained by ignoring sideslip, so we consider only this simplified case. Assuming infinite weather-cock-stability sideslip $\beta = 0$ and as a consequence the model is aligned in azimuth with the instantaneous flight direction, i.e.

$$\psi = \frac{\dot{y}}{V} \tag{5}$$

as shown in Fig. 5.

If $m = W/g$ is the mass of the model assembly we write the lateral force equation as

$$m\ddot{y} = \Sigma\, Y. \tag{6}$$

The equilibrium of rolling moments about the centre of gravity of the model assembly is given as

$$I_x \ddot{\phi} = \Sigma\, R \tag{7}$$

where R is used rather than L for the rolling moments to avoid confusion with L = lift. I_x is the inertia of the model assembly about its centre of gravity

$$I_x = m\, k_x^2. \tag{8}$$

To derive the contributions of the model weight, lift and drag to the sideforces ΣY and the rolling moments ΣR we consider their contributions in the vertical plane (Fig. 4) and in the horizontal plane (Fig. 5). From Fig. 4 we see that in this plane we get the contributions:

$$\Sigma Y_1 = W\delta + L(\phi - \delta). \tag{9}$$

The component of cable tension $F_z = (W-L)$ makes no contribution since we are considering forces perpendicular to the cable. It should be noted that the identity $F_z = W-L$ is only valid for small angles. In the interest of clarity large angles are represented in Fig. 4 when this relationship appears inaccurate.

From Fig. 5 we obtain a further contribution to ΣY from the drag term as

$$\Sigma Y_2 = -D\psi + F_x \sigma. \tag{10}$$

The horizontal component of the cable tension equals the drag, $F_x = D$. With the kinematic relationships given in equation (4) (for δ and σ) and equation (5) (for ψ) and adding equations (9) and (10) we get for equation (6) finally:

$$m\ddot{y} = (L-W)\frac{y}{l_z} - D\frac{y}{l_x} - D\frac{\dot{y}}{V} + L\phi. \tag{11}$$

Similarly we obtain from Fig. 4 the rolling-moment contributions

$$R = F_z(\delta - \phi)\,d_z = (W-L)\left(-\frac{y}{l_z} - \phi\right)d_z. \tag{12}$$

The horizontal component of the cable tension shown in Fig. 5 also makes a rolling moment contribution

$$R = F_x(\sigma - \psi)\,d_z = D\left(-\frac{y}{l_x} - \frac{\dot{y}}{V}\right)d_z. \tag{13}$$

The retention of this expression increases the complexity of the subsequent analysis, especially replacing the explicit expression (equation (25)) by a quadratic in V_0^2. A numerical check has shown that ignoring R_2 changes the results only very slightly and we shall therefore ignore it here. However, we also consider aerodynamic roll damping $L_p = \dfrac{\partial L}{\partial p}$ and with this term included we write equation (7) with equation (12) as

$$m k_x^2 \ddot{\phi} = -(W-L)\,d_z\frac{y}{l_z} - (W-L)\,d_z\phi + L_p\dot{\phi}. \tag{14}$$

The equations of motion of the model are therefore defined by equations (11) and (14) which we write in the form:

$$\left.\begin{aligned} -\ddot{y} - \frac{D}{W}\frac{g}{V}\dot{y} - g\left(1-\frac{L}{W}\right)\frac{1}{l_z}y - \frac{g}{l_x}\frac{D}{W}y + g\frac{L}{W}\phi &= 0 \\ -g\left(1-\frac{L}{W}\right)\frac{d_z}{k_x^2 l_z}y - \ddot{\phi} + \frac{L_p}{mk_x^2}\dot{\phi} - g\left(1-\frac{L}{W}\right)\frac{d_z}{k_x^2}\phi &= 0 \end{aligned}\right\} \tag{15}$$

This system of linear differential equations can be reduced to the stability determinant

$$\begin{vmatrix} \lambda^2+\frac{D}{W}\frac{g}{V}\lambda+g\left(1-\frac{L}{W}\right)\frac{1}{l_z}+\frac{D}{W}\frac{g}{l_x} & -g\frac{L}{W} \\ g\left(1-\frac{L}{W}\right)\frac{d_z}{k_x^2 l_z} & \lambda^2+\frac{1}{\tau}\lambda+g\left(1-\frac{L}{W}\right)\frac{d_z}{k_x^2} \end{vmatrix} = 0 \tag{16}$$

where the roll damping term has been represented by an equivalent first order time constant

$$\tau = -\frac{mk_x^2}{L_p}. \tag{17}$$

Equation (16) represents a quartic in λ:

$$\lambda^4+a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0 = 0$$

with the coefficients:

$$\left.\begin{aligned} a_3 &= \frac{1}{\tau}+\frac{g}{V}\frac{D}{W} \\ a_2 &= g\left(1-\frac{L}{W}\right)\left(\frac{d_z}{k_x^2}+\frac{1}{l_z}\right)+g\frac{D}{W}\left(\frac{1}{l_x}+\frac{1}{\tau V}\right) \\ a_1 &= g\left(1-\frac{L}{W}\right)\left(\frac{g}{V}\frac{D}{W}\frac{d_z}{k_x^2}+\frac{1}{l_z}\frac{1}{\tau}\right)+g\frac{D}{W}\frac{1}{l_x}\frac{1}{\tau} \\ a_0 &= g^2\left(1-\frac{L}{W}\right)\frac{d_z}{k_x^2}\left(\frac{1}{l_z}+\frac{1}{l_x}\frac{D}{W}\right). \end{aligned}\right\} \tag{18}$$

We had earlier made the assumption that the pivot of the suspension arm coincides with the centre of gravity of the model assembly and that therefore the model trim is independent of the tow, e.g. of the towing speed. This allows us to separate terms independent of speed, such as L/D from those varying with speed. Using also the relationships between l_z and l_x given in equation (2) to eliminate l_x as an independent parameter we can reduce equations (18) to

$$\left.\begin{aligned} a_3 &= \frac{1}{\tau}+\frac{g}{V}\frac{L/W}{L/D} \\ a_2 &= g\left(1-\frac{L}{W}\right)\left(\frac{d_z}{k_x^2}+\frac{2}{l_z}\right)+\frac{g}{\tau V}\frac{L/W}{L/D}, \\ a_1 &= g\left(1-\frac{L}{W}\right)\left(\frac{2}{\tau l_z}+\frac{g}{V}\frac{d_z}{k_x^2}\frac{L/W}{L/D}\right) \\ a_0 &= g^2\left(1-\frac{L}{W}\right)\frac{d_z}{k_x^2 l_z}\left(2-\frac{L}{W}\right). \end{aligned}\right\} \tag{19}$$

3.3. *Stability Criteria.*

It will be convenient to derive some simple criteria which permit one to establish the stability of a given towing configuration and especially whether there exists a critical speed above which the tow becomes unstable, because this is what occurred in practice.

The simplest criterion is that all the coefficients of the quartic must have the same sign. All the terms in equation (19) are positive by definition so that this criterion will only indicate instability if $L/W > 1$, but this condition invalidates the kinematic constraints already assumed in the analysis and is therefore not of practical value.

Routh's discriminant can be derived by arranging the coefficient of the quartic and derived factors in the following array

$$\begin{array}{ccc} 1 & a_2 & a_0 \\ a_3 & a_1 & 0 \\ U_1 & U_3 & U_5 \\ U_2 & U_4 & U_6 \\ V_1 & V_3 & \\ V_2 & V_4 & \end{array} \tag{20}$$

where

$$U_1 = \frac{a_2 a_3 - 1 a_1}{a_3} \qquad U_3 = \frac{a_0 a_3 - 1 \times 0}{a_3} \qquad U_5 = 0$$

$$U_2 = \frac{a_1 U_1 - a_3 U_3}{U_1} \qquad U_4 = 0$$

$$V_1 = \frac{U_3 U_2 - U_1 U_4}{U_2} \qquad V_3 = 0$$

$$V_2 = 0 \qquad V_4 = 0$$

The number of unstable roots equals the number of sign changes given by the sequence of factors in the first column of equation (20). As we have already seen the coefficients of the quartic are all positive so we must look for a negative sign in U_1, U_2, or V_1. It was found that the critical factor is U_2 which can be written in terms of the coefficients of the quartic as

$$U_2 = a_1 - a_3 a_0 \frac{1}{a_2 - \dfrac{a_1}{a_3}}. \tag{21}$$

For stability $U_2 > 0$.

Carrying out the necessary algebra we arrive at the following stability criterion:

$$\frac{2k_x^2}{d_z \tau} - \frac{g\, l_z}{V}\frac{L/W}{L/D} - \frac{\left(\frac{1}{\tau}+\frac{g}{V}\frac{L/W}{L/D}\right)\left(2-\frac{L}{W}\right)}{\left(1-\frac{L}{W}\right)\left(\frac{d_z}{k_x^2}+\frac{2}{l_z}\right)+\frac{1}{\tau V}\frac{L/W}{L/D} - \frac{\left(1-\frac{L}{W}\right)\left(\frac{2}{\tau l_z}+\frac{g}{V}\frac{d_z}{k_x^2}\frac{L/W}{L/D}\right)}{\frac{1}{\tau}+\frac{g}{V}\frac{L/W}{L/D}}} > 0\,. \tag{22}$$

To permit from this expression a critical speed to be directly calculated we introduce the speed-dependent terms

$$\frac{L}{W} = \mathscr{L}\,V^2 \quad \text{where} \quad \mathscr{L} = \tfrac{1}{2}\rho\frac{C_L}{W/S}$$

$$\frac{1}{\tau} = D_R\,V \quad \text{where} \quad D_R = -g\frac{\rho}{2}\frac{l_p}{W/S}\left(\frac{b}{k_x}\right)^2 \tag{23}$$

if
$$l_p = \frac{\partial C_l}{\partial\left(p\frac{b}{V}\right)}.$$

Introducing the parameter $A = \frac{g}{D_R}\frac{\mathscr{L}}{L/D}$ we get (24)

$$V_c^2 = \frac{1}{\mathscr{L}}\;\frac{2\dfrac{1+A}{2\frac{k_x^2}{d_z}+l_z A}+\dfrac{\frac{2}{l_z}+\frac{d_z}{k_x^2}A}{1+A}-\dfrac{d_z}{k_x^2}-\dfrac{2}{l_z}}{\dfrac{1+A}{2\frac{k_x^2}{d_z}+l_z A}+\dfrac{\frac{2}{l_z}+\frac{d_z}{k_x^2}A}{1+A}-\dfrac{d_z}{k_x^2}-\dfrac{2}{l_z}+\dfrac{D_R}{L/D}}\,. \tag{25}$$

It should be noted that equation (25) still contains two terms which are themselves dependent on speed namely d_z and l_z, the vertical component of the suspension arm and of the cable. From equation (3) we see that they are related to the absolute lengths d and l by

$$\frac{d_z}{d} = \frac{l_z}{l} = \frac{1}{\sqrt{1+\left(\dfrac{D/L}{\frac{1}{\mathscr{L}V^2}-1}\right)^2}} = \cos\mu\,. \tag{26}$$

The introduction of this relationship into equation (25) would lead to a rather unmanageable expression and has therefore been abandoned. Strictly speaking therefore the solution of equation (25) should be carried out iteratively, i.e. a first answer for V_c should be used to derive new values of d_z and l_z from equation (26) and so on. In cases where V_c is relatively low in relation to the speed where $L = W$, the angle μ (Fig. 3) should be small enough to equate $\cos\mu = 1$ and therefore assume $d_z = d$ and $l_z = l$.

4. *Results.*

The analysis developed in Section 3 has been tested against a case where difficulties were experienced when a particular free-flight model was towed at a speed greater than approximately 35 knots.

The relevant data for the model and the suspension system are

$$L/D = 3 \qquad \mathscr{L} = 1/6160$$

$$D_R = 0{\cdot}236 \qquad k_x^2 = 0{\cdot}64 \text{ ft}^2$$

$$d = 1{\cdot}25 \text{ ft} \qquad l = 100 \text{ ft}.$$

The stability roots of equation (18) have been computed for a range of speeds well beyond that at which the model went into an unstably lateral swinging oscillation in flight. The speed at which for this model $L = W$ is 46·4 knots. The solution is shown in Fig. 6. At low speed there are two oscillatory modes, a well damped high frequency roll oscillation which, at $V > 31$ knots is over-critically damped and degenetates into two subsidences. These modes are of no interest here. The second oscillatory mode with a period of approximately 8 seconds is mildly damped up to $V_c = 36$ knots and becomes rapidly unstable above this speed. This mode closely resembles the model behaviour observed in flight. A more precise comparison is not possible as the motion of the model was only noted by visual observation and since it developed to an unpleasantly large amplitude ($\delta \approx \pm 30°$) the condition was not maintained. Stability was regained by reducing speed.

It should be noted that in this computation d_z and l_z were correctly calculated for each speed, so that this solution is correct within the general assumptions of the analysis.

Assuming $d_z = d$ and $l_z = l$, the critical speed V_c was then established from equation (25) and gave a value of 36·4 knots, practically identical with that predicted in Fig. 6.

This equation is simple enough to lend itself to some investigation of the effects of those parameters which are under the control of the experimenter. It was found by some exploratory work, not sufficiently formalised to be presented here, that the instability of the lateral swinging mode is the result of aerodynamic roll damping lagging the bank response of the model in such a sense that the lift vector controlled by the bank angle 'drives' the model unstable. The only positive damping term is the component of the drag opposing the lateral swing. It is to be expected therefore that by increasing the model drag, i.e. the ratio D/L, the critical speed could be increased and *vice versa.*

Holding the other characteristics of the towing configuration constant D/L was therefore varied over a range including the datum condition. The result on critical speed V_c is shown in Fig. 7 and confirms the the above observation.

Next the length of the suspension arm, or more precisely d_z was similarly varied with the result shown in Fig. 8. It is seen that lengthening the arm is beneficial. Reducing the length of the arm below $(d_z/k_x^2) < 0{\cdot}2$ also improves stability but this regime is not of great practical interest for the type of configuration considered here.

The effect of cable length in terms of l_z was also investigated with the result given in Fig. 9. It is seen that lengthening the towing cable will increase the stable speed range and *vice versa.* If the cable length were reduced to 1 ft, the model would be unstable at all speeds according to these calculations. However, the analysis is invalid for these small values of l_z, because we had earlier made the assumption that $d_z << l_z$, which is clearly not the case here. It is worth noting that the remedy suggested by the present analysis of increasing the cable length to improve stability is at variance with the results of the various analyses made of the behaviour of non-lifting towed bodies in Refs. 1 to 6. These all predict reduction of cable length as beneficial. All these studies are concerned with aerodynamic terms ignored in the present theory and they can be expected to become important in conditions where those treated here have become relatively small. This may happen for instance if long suspension cables are considered, where the present theory predicts improved stability, but in practice other effects may predominate and lower the damping.

Finally the effect of variation in the weight of the model assembly was studied and, as Fig. 10 shows, increasing weight is the most powerful of the parameters investigated for increasing the range of speeds at which the model can be towed without encountering instability. This is a particularly useful result because it should be readily possible to add weight to the model carriage without affecting the weight of the model itself which is fixed by considerations of dynamic similarity.

5. *Discussion and Conclusions.*

Remembering that only one single-flight observation is available to test the validity of the present theory, this evidence agrees with the results derived here. The presented theory of the stability of suspended flight models suggests that this method of towing results in an unstable lateral swinging oscillation above a certain critical speed which can be raised by increasing the length of the towing cable, l, the length of the suspension arm d, the drag of the assembly of model plus carriage and most powerfully by increasing the weight of the carriage to which the model is attached during tow. The latter results is of particular practical value because this weight increase need not affect the configuration of the actual model. It may also be possible to add a drag-producing device to the carriage without affecting the aerodynamic properties of the model after release.

It must be remembered, however that the analysis presented here is an approximation, retaining only terms specifically associated with lift. The results are therefore applicable only to towed bodies carrying substantial lift and having sufficient directional stability for sideslip to be negligible. No experimental evidence is available to permit the limits of the velocity of these approximations to be defined.

LIST OF SYMBOLS

A	Parameter defined in equation (24)
a	Coefficients of the stability quartic
b (ft)	Reference length of model (span)
d (ft)	Length of model suspension arm
d_z (ft)	Vertical component of d
d_x (ft)	Horizontal component of d
D (lb)	Drag
D_R	Roll damping parameter (equation (23))
F (lb)	Cable tension
F_z (lb)	Vertical component of F
F_x (lb)	Horizontal component of F
g (ft/sec^2)	Gravity
$I_x = K_x^2 m$	Roll inertia
k_x (ft)	Inertia radius in roll of model
L (lb)	Lift
$\mathscr{L}$	Lift factor (equation (23))
l (ft)	Cable length
l_z (ft)	Vertical component of l
l_x (ft)	Horizontal component of l
$l_p = \dfrac{\partial C_l}{\partial\left(p\dfrac{b}{V}\right)}$	Non-dimensional roll damping derivative
$L_p = \dfrac{\partial L}{\partial p}$	Dimensional roll damping derivative
$m = W/g$	Model mass
R (lb ft)	Rolling moment about model centre of gravity
S (ft^2)	Wing area of model
U	Equation (20)
V (ft/sec)	Airspeed
V_c (ft/sec)	Critical speed
W (lb)	Model weight
y (ft)	Lateral displacement of model
Y (lb)	Sideforce acting perpendicular to suspension cable
τ (sec)	Equivalent roll damping time constant

λ (sec^{-1})	Stability root
ω (rad/sec)	Angular frequency
δ	Pendulum angle (in the vertical plane)
σ	Pendulum angle (in the horizontal plane)
μ	Cable towing angle
ψ	angle of yaw
ϕ	Angle of roll
ρ	Air density

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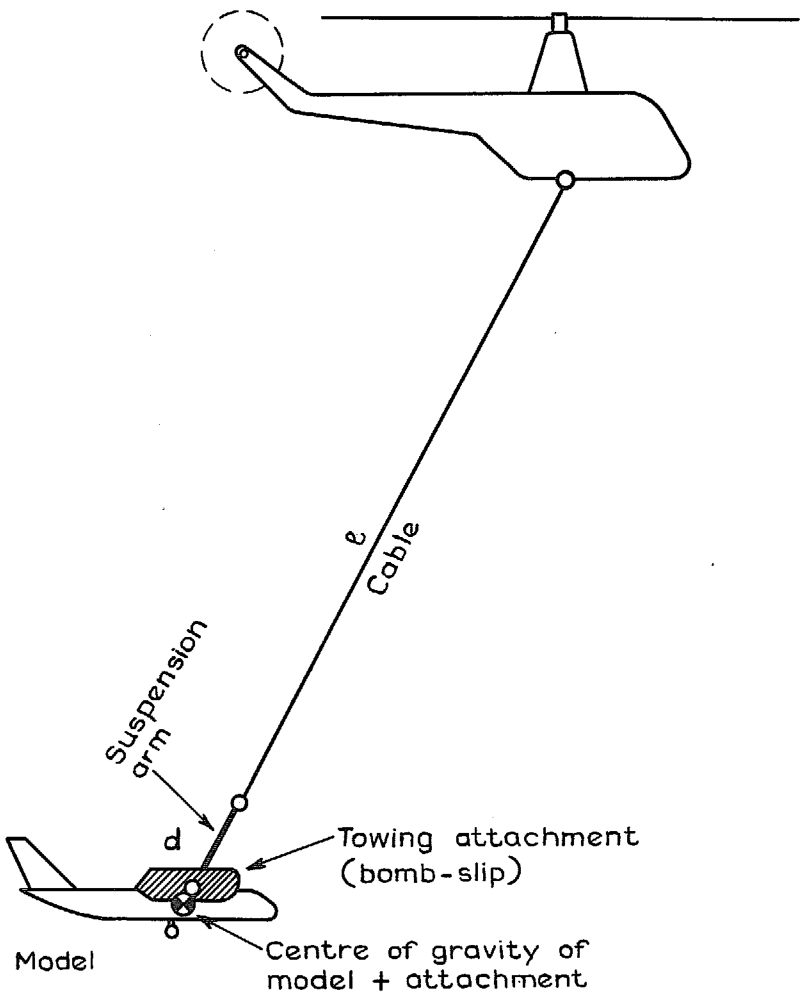

FIG. 1. Towing configuration.

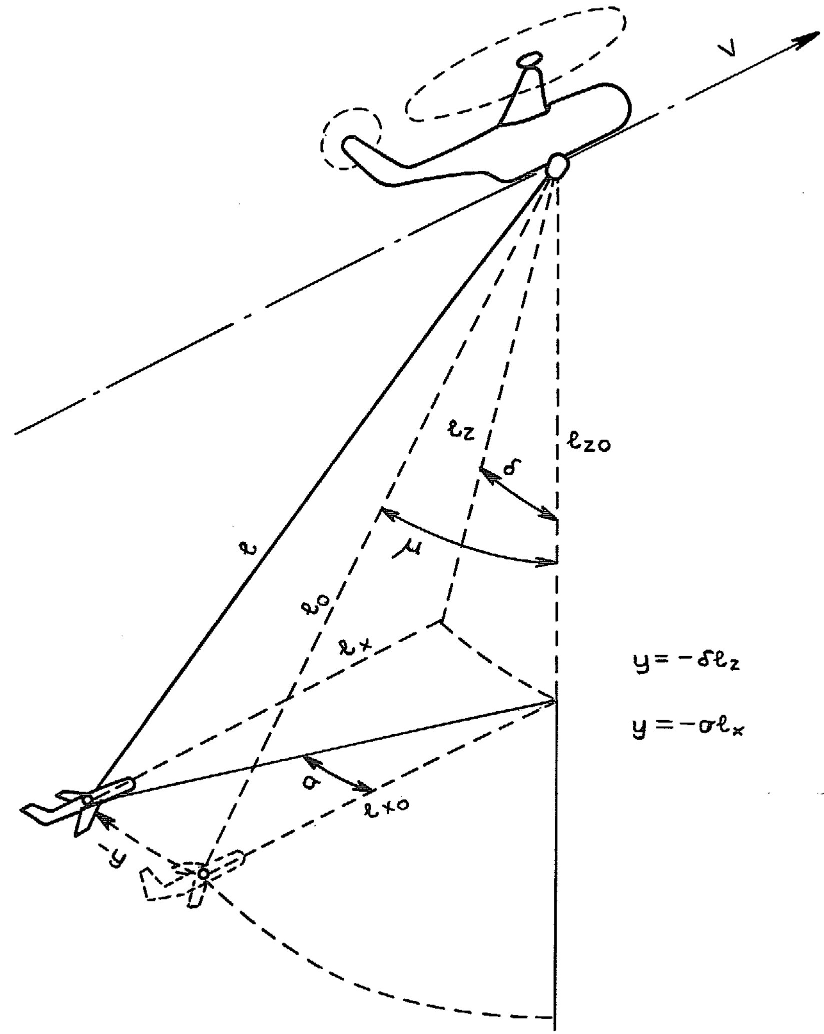

FIG. 2. Reference system and notation used to define lateral model motion.

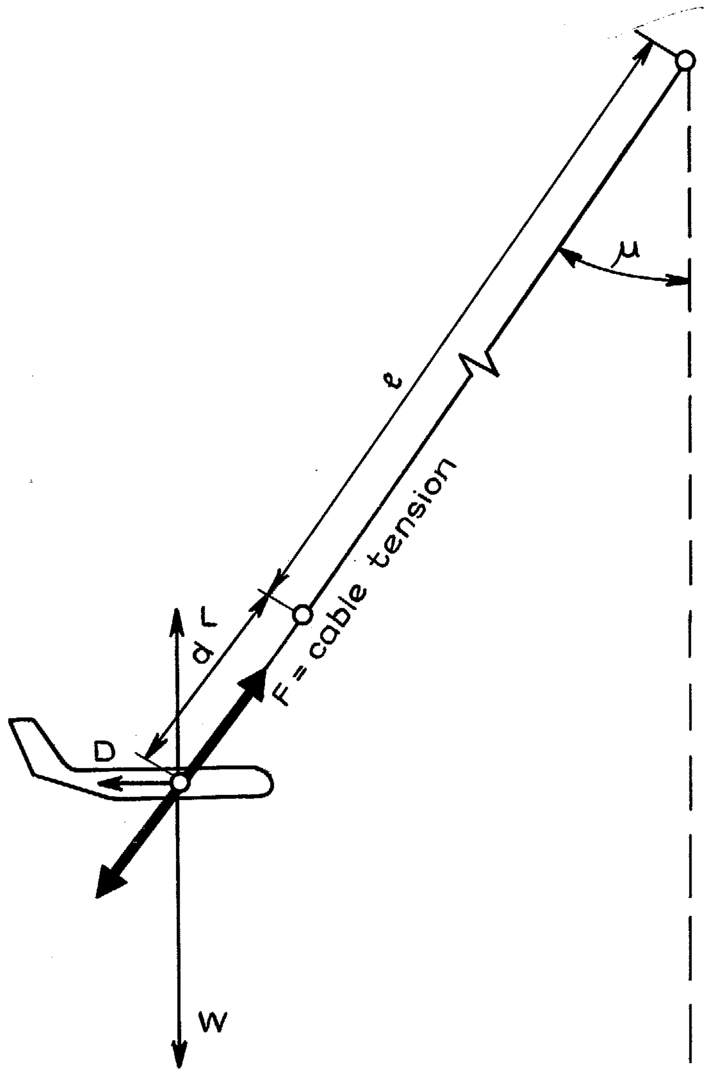

FIG. 3. Section through plane $l - l_x - l_z$ in Fig. 2 to illustrate force equilibrium between model and cable.

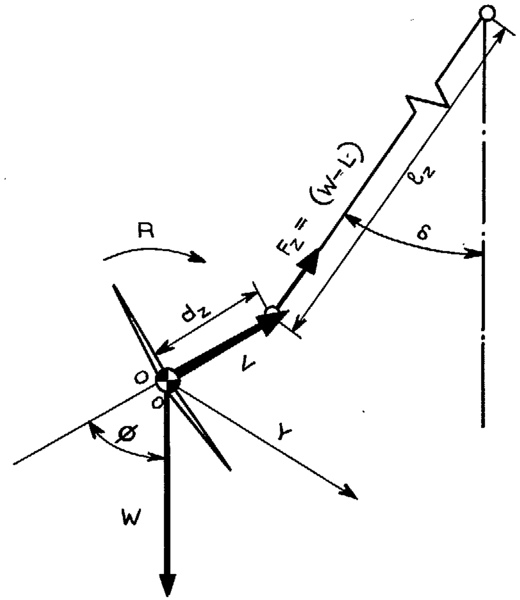

FIG. 4. Projection in the vertical plane normal to and viewed in the direction of flight to illustrate contributions to side force Y and rolling moment R.

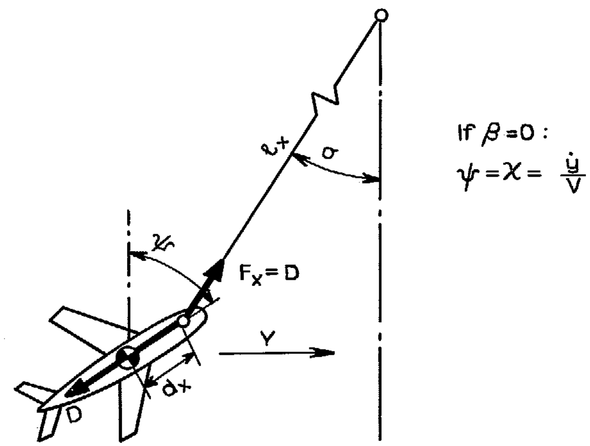

FIG. 5. Projection in the horizontal plane to illustrate force contributions to Y. (Yawing moments are ignored assuming $N_\beta = \infty$, i.e. $\beta = 0$).

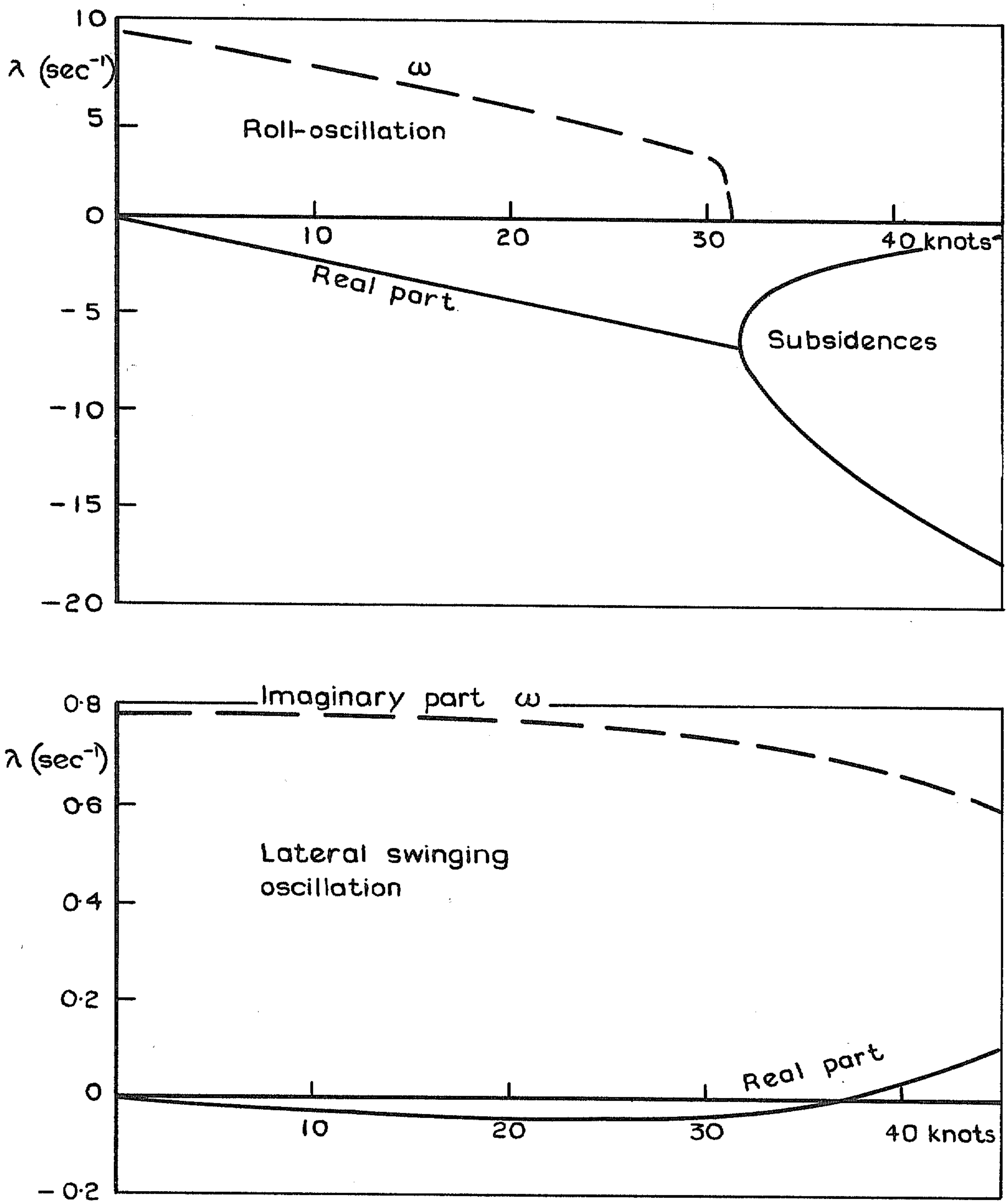

FIG. 6. Change with airspeed of the stability of the towed model considered as an example.

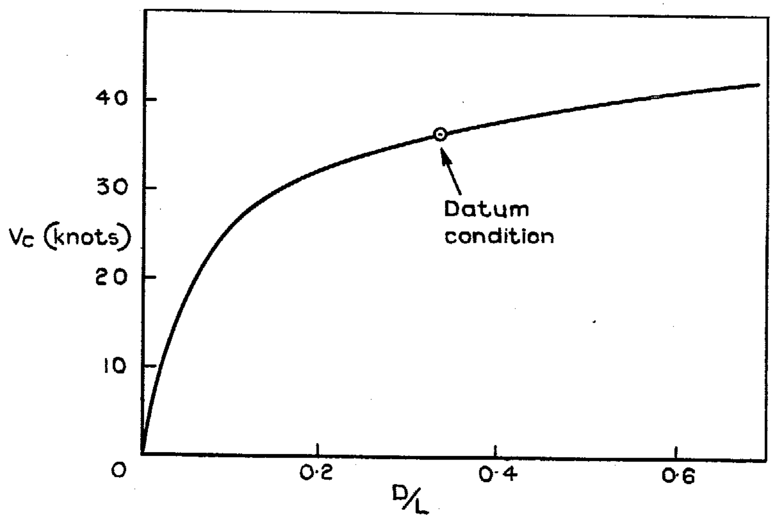

FIG. 7. Effect of model drag D on critical speed V_c.

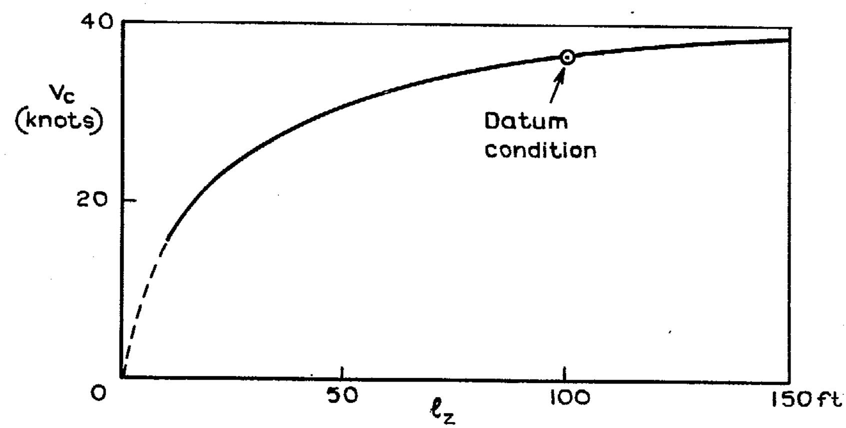

FIG. 9. Effect of cable length l_z on critical speed.

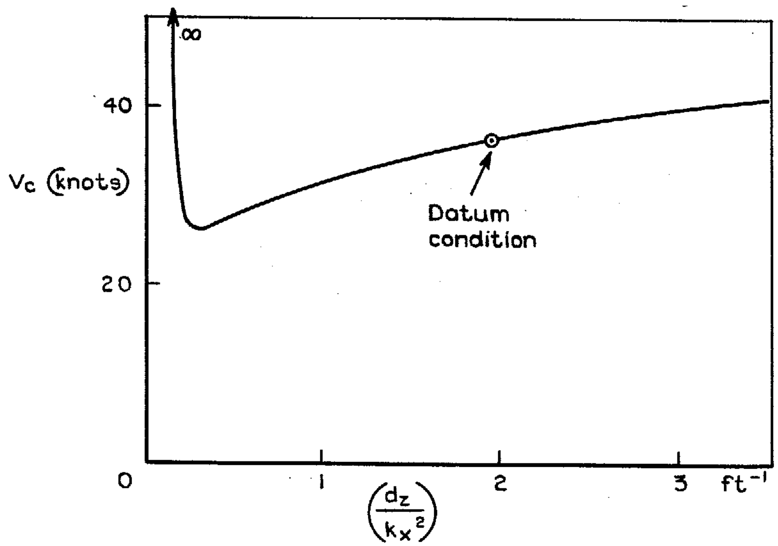

FIG. 8. Effect of length of suspension arm d_z on critical speed V_c.

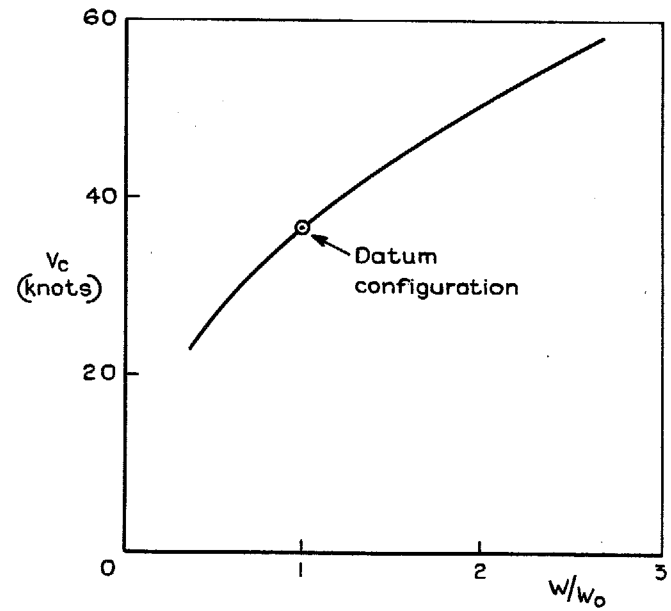

FIG. 10. Effect of weight of model assembly W on critical towing speed.

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