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A Theoretical Study of Height Control in Flight Close to the Ground as Affected by Elevator Lift and Cockpit Position

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Summary.

During the final landing approach pilots are often observed to attempt tight control of flight path by coarse elevator usage. It is shown by theoretical analysis that this form of control is inherently conducive to instability and that adverse elevator lift is detrimental in this situation. However, if the pilot is located in a cockpit far forward of the centre of gravity of the aircraft, he perceives 'false' motion cues which tend to reduce the possibility of this form of pilot-induced oscillation.

*Replaces R.A.E. Technical Report 69 097—A.R.C. 31 488.

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1. *Introduction.*

Control of aircraft height and vertical velocity is generally exercised *via* the phugoid mode of longitudinal motion. The stability or otherwise of height control is then determined by the stability of this mode. Since the phugoid is a relatively slow mode this implies that it affords no short term control over height, and this is indeed reflected in general piloting practice.

There are, however, some special circumstances when the pilot wishes to impose much tighter control on the vertical motion of the aircraft, the most obvious example being the landing approach. Indeed, considerations of this manoeuvre have led to the discovery of what is now generally known as the speed stability mode[1], which is a degenerate form of the phugoid when the assumption is made that the pilot suppresses height variations—which are an essential feature of the mechanisms of the phugoid—by suitable elevator control. Observations of flying practice have confirmed that this assumption is a reasonable (even though not perfect) approximation to piloting technique in the landing approach, and that the consequent emergence of the speed stability mode is also reflected in practice.

It is generally understood that the assumption of constant height implied in the formulation of the speed stability theory need not be taken too literally. In fact it can easily be shown that all that is required for the speed stability mode to exist is that the pilot maintains a constant mean height, or flight path, short term variations are of no great significance to the relatively long term speed stability mode.

Nevertheless, a form of elevator height control is an essential assumption in this situation and it is perhaps surprising that the feasibility of such a control mode has never been seriously questioned. It will be shown in this report that there is in fact only very limited scope for pilots to exercise tight and at the same time stable control of aircraft height or flight path and that this depends critically on the position of the cockpit in the aircraft and is further influenced by adverse elevator lift.

There is, as a result, a distinct possibility in certain cases that by attempting to use the elevator for close control of height the pilot may not only transform an originally inoccuous phugoid into a speed divergence but at the same time generate an unstable oscillatory short period mode (P.I.O.).

There is perhaps another part of landing control where the latter problem is particularly acute. If the landing flare does not terminate in an immediate touchdown, the aircraft will 'float' over the runway, sometimes for a considerable period. In Refs. 2 and 3 it has been noted that pilots will not immediately try to correct this situation in the expectation that speed losses will cause the aircraft to settle down. However, if after 9–10 seconds from flare initiation this has not occurred—a slight increase in headwind may have this effect—the pilot will become 'uneasy' and begin to use elevator control to force a touchdown. Both in simulators and in real flight this can be observed frequently to lead to fairly coarse control usage and to an oscillatory height variation which has the appearance of a pilot induced oscillation. Figs. 1 and 2 are typical examples of such landings. For the full scale flight tests unfortunately no $\dot{H}$ records are available, but the elevator trace is by itself illuminating.

The present investigation was made to test whether a height control law likely to be used by the pilot is in fact a possible cause for a potential instability and if this is so to elucidate the factors which have a major influence on this phenomenon. It must be noted that in general flight the pilot is more likely to control pitch attitude so that the case considered here must be treated as a special one, relevant only in flight very close to the ground, when height becomes a dominant and readily perceived quantity. We can see from Section 2 that pitch control is exactly equivalent to the effect forward location of the cockpit has in the present case. By analogy the results of the present study indirectly show that pitch control is strongly stabilizing and clearly beneficial.

2. *The Assumed Elevator Control Law.*

The principal motion parameters a pilot might utilize as control stimuli are height deviations and pitch attitude. Since we are concerned here with a situation in which flight path control becomes the dominant preoccupation we assume the pilot concentrates on height, applying elevator according to

$$\eta = k_1 \, \Delta H + k_2 \, \Delta \dot{H} \,. \tag{1}$$

If we write this as a transfer function

$$\eta = k_1 \, \Delta H \left(1 + \frac{k_2}{k_1} s \right) \tag{1a}$$

it is seen that the $\dot{H}$ term can be interpreted as a first order lead with the time constant $\tau = k_2/k_1$.

Since the pilot is located at a position forward of the centre of gravity of the aircraft, the height he perceives (H_P) differs from that of the aircraft at its centre of gravity (H_A). From Fig. 3

$$H_P = H_A + x_P \, \theta \tag{2}$$

where x_P is the distance between the cockpit and the aircraft centre of gravity. Since the flight path angle

$$\gamma = (\theta - \alpha) = \frac{\dot{H}_A}{V} \tag{3}$$

we can express equation (2) in terms of the usual aircraft motion parameters as

$$H_P = V \int (\theta - \alpha)\, dt + x_P\, \theta \tag{4}$$

and by differentiation

$$\dot{H}_P = (\theta - \alpha)\, V + x_P\, \dot{\theta}\,. \tag{5}$$

With these relationships the control equation (1) becomes:

$$\eta = k_1\, V \int \theta\, dt + k_1\, x_P\, \theta + k_2\, V\, \theta + k_2\, x_P\, \dot{\theta} - k_1\, V \int \alpha\, dt - k_2\, V\, \alpha\,. \tag{6}$$

Equations (4–6) indicate that in its effect forward location of the cockpit ($x_p > 0$) is equivalent to the pilot applying elevator in response to pitch attitude. It follows that a pitch control law added to the height control law here discussed can be represented by an appropriate increase in x_p above its actual value observing the equivalences $\partial\eta/\partial\theta \equiv k_1\, x_p$ and $\partial\eta/\partial\dot{\theta} \equiv k_2\, x_p$.

3. *Stability Analysis.*

Since we are concerned here especially with the short term response of the aircraft, the phugoid is ignored by assuming constant speed. Ignoring also the minor terms $L_{\dot{\alpha}}$ and L_q, the longitudinal motion of the aircraft is then defined by:

$$L_\alpha\, \alpha - m\, V\, \dot{\theta} + m\, V\, \dot{\alpha} = -L_\eta\, \eta \tag{7}$$

$$M_\alpha\, \alpha + M_q\, q + M_{\dot{w}}\, \dot{\alpha} - B\, \ddot{\theta} = -M_\eta\, \eta\,. \tag{8}$$

It is convenient to express the elevator pitching moment in terms of elevator lift L_η and the effective elevator moment arm x_η (negative for a rear control) as

$$M_\eta = L_\eta\, x_\eta\,. \tag{9}$$

The elevator moment arm is represented by the factor

$$\xi_\eta = \frac{x_\eta}{k_y^2} \tag{10}$$

where k_y is the inertia radius in pitch.
Introducing the aerodynamic derivatives in the form

$$\mathcal{M}_\alpha = \frac{M_\alpha}{B} \qquad \mathcal{M}_q = \frac{M_q}{B} \qquad \mathcal{M}_{\dot{\alpha}} = \frac{M_{\dot{w}}}{B} \qquad \mathcal{L}_\alpha = \frac{L_\alpha}{mV} \qquad \mathcal{M}_\eta = \frac{M_\eta}{B} \tag{11}$$

and substituting equation (6) for η in equations (7) and (8), the stability determinant of the longitudinal motion becomes:

$$\begin{array}{cc} \int\alpha & \int\theta \end{array}$$

$$\begin{vmatrix} -D^2+D\left(\mathcal{L}_\alpha-\dfrac{\mathcal{M}_\eta}{\xi_\eta}\right)-k_1\dfrac{\mathcal{M}_\eta}{\xi_\eta} & \left\{\begin{array}{l} -D^2\left(1-k_2\dfrac{\mathcal{M}_\eta}{\xi_\eta}\dfrac{x_P}{V}\right)+k_1\dfrac{\mathcal{M}_\eta}{\xi_\eta} \\ +D\dfrac{\mathcal{M}_\eta}{\xi_\eta}\left(\dfrac{x_P}{V}k_1+k_2\right) \end{array}\right\} \\ \left\{\begin{array}{l} D^2\mathcal{M}_{\dot\alpha}+D\left(\mathcal{M}_\alpha-k_2\mathcal{M}_\eta V\,D\right) \\ -k_1\mathcal{M}_\eta V \end{array}\right\} & \left\{\begin{array}{l} -D^3+D^2\left(\mathcal{M}_q-k_2\mathcal{M}_\eta x_P\right) \\ +D\mathcal{M}_\eta\left(k_1x_P+k_2V\right) \\ +k_1\mathcal{M}_\eta V \end{array}\right\} \end{vmatrix} = 0\,. \qquad (12)$$

This determinant has as a solution a quintic in D

$$D^5+b_4D^4+b_3D^3+b_2D^2+b_1D+b_0=0\,. \qquad (13)$$

The last coefficient b_0 contains two equal terms with opposite sign, making it zero. Elimination of this zero root reduces equation (13) to a quartic

$$D^4+a_3D^3+a_2D^2+a_1D+a_0=0 \qquad (14)$$

with the coefficients:

$$\left.\begin{aligned} a_3 &= \mathcal{L}_\alpha-\mathcal{M}_q-\mathcal{M}_{\dot\alpha}-K_2\left(\frac{1}{\xi_\eta}-\frac{\mathcal{M}_{\dot\alpha}}{\xi_\eta}\frac{x_P}{V}+x_P\right) \\ a_2 &= -\mathcal{M}_\alpha-\mathcal{L}_\alpha\mathcal{M}_q-K_1\left(\frac{1}{\xi_\eta}-\frac{\mathcal{M}_{\dot\alpha}}{\xi_\eta}\frac{x_P}{V}+x_P\right)+K_2\left\{\frac{1}{\xi_\eta}\left(\mathcal{M}_q+\mathcal{M}_{\dot\alpha}+\mathcal{M}_\alpha\frac{x_P}{V}\right)-\mathcal{L}_\alpha x_P\right\} \\ a_1 &= K_1\left\{\frac{1}{\xi_\eta}\left(\mathcal{M}_q+\mathcal{M}_{\dot\alpha}+\mathcal{M}_\alpha\frac{x_P}{V}\right)-\mathcal{L}_\alpha x_P\right\}+K_2\left(\frac{\mathcal{M}_\alpha}{\xi_\eta}-\mathcal{L}_\alpha V\right) \\ a_0 &= K_1\left(\frac{\mathcal{M}_\alpha}{\xi_\eta}-\mathcal{L}_\alpha V\right) \end{aligned}\right\} \qquad (15)$$

where the control gains are represented in the form

$$\left.\begin{aligned} K_1 &= k_1\mathcal{M}_\eta \\ K_2 &= k_2\mathcal{M}_\eta\,. \end{aligned}\right\} \qquad (16)$$

Since normally $\mathscr{L}_\alpha$, K_1 and K_2 are positive and $\mathscr{M}_\alpha, \mathscr{M}_q, \mathscr{M}_{\dot{w}}, \mathscr{M}_\eta$, ξ_η are negative the coefficients $a_3 - a_0$ contain all positive terms apart from those associated with K_1 and K_2 containing the elevator arm ξ_η as a factor. These terms represent the effect of adverse elevator lift and it is readily apparent that—if sufficiently large—these can lead to instability.

It can be shown that the system defined by equation (15) will be stable if

$$a_0 > 0; \qquad a_1 > 0; \qquad a_2 > 0; \qquad a_3 > 0 \tag{17}$$

and if the Routh discriminant

$$a_1^2 - a_1 a_2 a_3 + a_0 a_3^2 < 0 \tag{18}$$

According to Glauert, failure to satisfy $a_0 > 0$ indicates a divergence, whereas equation (18) determines oscillatory stability.

If a_1 and a_3 have the same sign the boundary of oscillatory stability is defined by

$$a_1^2 - a_1 a_2 a_3 + a_0 a_3^2 = 0 \tag{19}$$

and this expression establishes a relationship between K_1 and K_2 values at the boundary. For this purpose we write the coefficients of the quartic (15) in the form

$$\left.\begin{aligned} a_3 &= a_3 \\ a_2 &= a_{20} + K_1 a_{2K} \\ a_1 &= a_{10} + K_1 a_{1K} \\ a_0 &= K_1 a_{0K}. \end{aligned}\right\} \tag{20}$$

With these coefficients equation (19) gives a quadratic in K_1:

$$K_1^2 (a_{1K} - a_3 a_{2K} a_{1K}) + K_1 (2a_{10} a_{1K} - a_3 a_{20} a_{1K} - a_3 a_{2K} a_{10} + a_3^2 a_{0K} + a_{10}^2 - a_3 a_{10} a_{20} = 0 \tag{21}$$

From equations (15) and (20) the coefficients in this equation are given as:

$$\left.\begin{aligned} a_3 &= \mathscr{L}_\alpha - \mathscr{M}_q - \mathscr{M}_{\dot{\alpha}} - K_2\left(\frac{1}{\xi_\eta} - \frac{\mathscr{M}_{\dot{\alpha}}}{\xi_\eta}\frac{x_P}{V} + x_P\right) \\ a_{20} &= -\mathscr{M}_\alpha - \mathscr{L}_\alpha \mathscr{M}_q + K_2\left\{\frac{1}{\xi}\left(\mathscr{M}_q + \mathscr{M}_{\dot{\alpha}} + \mathscr{M}_\alpha \frac{x_P}{V}\right) - \mathscr{L}_\alpha x_P\right\} \\ a_{2K} &= -\left(\frac{1}{\xi_\eta} - \frac{\mathscr{M}_{\dot{\alpha}}}{\xi_\eta}\frac{x_P}{V} + x_P\right) \\ a_{10} &= K_2\left(\frac{\mathscr{M}_\alpha}{\xi_\eta} - \mathscr{L}_\alpha V\right) \\ a_{1K} &= \frac{1}{\xi_\eta}\left(\mathscr{M}_q + \mathscr{M}_{\dot{\alpha}} + \mathscr{M}_\alpha \frac{x_P}{V}\right) - \mathscr{L}_\alpha x_P \\ a_{0K} &= \frac{\mathscr{M}_\alpha}{\xi_\eta} - \mathscr{L}_\alpha V. \end{aligned}\right\} \tag{22}$$

If adverse elevator lift is to be ignored we make $\xi_\eta = \infty$ in equations (12), (15) and (22).

4. *Numerical Examples.*

We consider two basic aircraft in low speed flight, and in each the forward location of the cockpit (x_P) is varied systematically. In one of the examples the effect of elevator lift is also investigated separately.

The first example is typical of large tailless supersonic transport design, the second represents a more conventional subsonic aircraft. They are defined by the aerodynamic, inertial and geometric quantities listed in Table 1.

Stability boundaries have been calculated from the two conditions defined in equations (17) and (18) and in all cases considered these follow one of the two basic patterns illustrated in Fig. 4. The condition $a_0 > 0$ establishes a boundary for divergence at $K_1 = 0$. This boundary simply states that $\partial\eta/\partial H$ must be positive, a trivial conclusion. The boundaries derived from the discriminant equation (18), however, are far less obvious and contribute the more interesting results of the present analysis. In Type I, this boundary has two distinct branches. One passes through the origin of the $K_1 - K_2$ graphs, leaving only a small segment in the positive quadrant for small values of K_1 and modest values of K_2 as a region of stable control. There is a further stable region, bounded by the second branch which contains the range of very large values of K_1. The associated gains in $\partial\eta/\partial H$ are, however, impractically large so that this region is of no practical significance.

The situation arising from the stability graph designated as Type II, leaves practically all the positive quadrant stable except for a small region with small values of K_2 and modest values of K_1.

The specific results obtained for the two aircraft chosen as examples are shown in Figs. 5 and 6. The first example, aircraft A of Table 1, features strong adverse elevator lift and to isolate the effect of its contribution, the stability boundaries have been calculated for this aircraft with and without this term included. The results of Fig. 5a show that the regime available for stable control increases as the pilot is moved forward in the aircraft.

If the pilot were to occupy a position coincident with the centre of gravity ($x_P = 0$), he can only maintain stable control by restricting the gain in height control $\partial\eta/\partial H$ to about 0·1°/ft and even then only by using considerable phase lead, i.e. $\partial\eta/\partial\dot{H}$. With more forward location of the cockpit, the region permitting stable height control is rapidly widening, although in every case pure height control $\partial\eta/\partial H$ will always result in a divergent oscillation.

Fig. 5b shows that removal of the elevator lift effect substantially improves the situation. A most interesting observation is that with a far forward cockpit location, $x_P = 160$ ft for instance, even pure height control without any lead becomes possible if a sufficient gain is used, although with a lesser gain instability would result.

For the aircraft chosen for the second example (B) (Fig. 6) we get a very similar picture. The closer to the centre of gravity the pilot is located, the more restricted is the height control activity he can engage in without provoking a 'pilot induced oscillation'.

For this latter example the effect of variations in static longitudinal stability have also been studied with the result shown in Fig. 7. It is seen that increasing $m_w \propto \mathscr{M}_\alpha$ improves the situation by widening the range of gains permitting stable control.

The present analysis has been made with particular reference to flight conditions in which a pilot can be assumed to exercise tight height control. When one wishes to apply these results to the height (or flightpath) control mode of an autopilot, it should be appreciated that the height control gains used in an autopilot are very small by comparison with the ranges considered here; values of $\partial\eta/\partial H$ of 1/30°/ft are typical and operate mainly *via* the phugoid mode. Secondly autopilots invariably employ in addition to the height gains considered here pitch attitude feedback in order to ensure stability. This term has not been considered in the present analysis, because it would appear unlikely that the human pilot would be capable of mentally summing height and attitude information in a situation demanding tight control.

For the reader more familiar with the servo control approach, root locus plots have also been constructed for a number of the more interesting cases covered in the previous analysis. They are presented in Figs. 9 and 10. In order to apply this technique the pilot control equation (1) has been expressed in transfer function form (equation (1a)):

$$\frac{\eta}{H}(s) = k_2\left(s+\frac{k_1}{k_2}\right) \tag{23}$$

and the feedback gain then becomes k_2. When a root locus is derived for a particular value of lead k_1/k_2, this corresponds to a traverse of the positive quadrant of the $k_1 - k_2$ plane along a radial ray as indicated in Fig. 8. The open loop transfer function describing the system under discussion has the form

$$G(s)\,H(s) = \left(-\frac{\eta}{H_P}\right)(s)\,\frac{H_P}{\eta}(s) = K\,\frac{\left(s+\frac{k_1}{k_2}\right)(s^2+a\,s+b)}{s^2\,(s^2+2\,\zeta\,\omega_0\,s+\omega_0^2)} \tag{24}$$

K is the gain

$$K = k_2\,\{V\,\mathscr{L}_\eta + x_P\,(\mathscr{M}_\eta - \mathscr{L}_\eta \mathscr{M}_{\dot{\alpha}})\} \tag{25}$$

and the numerator polynomial of the aircraft transfer function is given by:

$$a = \frac{-V\,\mathscr{L}_\eta\,(\mathscr{M}_{\dot{\alpha}} + \mathscr{M}_q) + x_P\,(\mathscr{M}_\eta\,\mathscr{L}_\alpha - \mathscr{L}_\eta \mathscr{M}_\alpha)}{V\,\mathscr{L}_\eta + x_P\,(\mathscr{M}_\eta - \mathscr{L}_\eta \mathscr{M}_{\dot{\alpha}})} \tag{26}$$

$$b = \frac{V\,(\mathscr{M}_\eta\,\mathscr{L}_\alpha - \mathscr{L}_\eta \mathscr{M}_\alpha)}{V\,\mathscr{L}_\eta + x_P\,(\mathscr{M}_\eta - \mathscr{L}_\eta \mathscr{M}_{\dot{\alpha}})} \tag{27}$$

It is not proposed to discuss the root locus plots in detail since they only confirm the conclusions reached above. Those shown in Fig. 9 are for aircraft A with elevator lift included, Fig. 10 gives the results for the same aircraft without elevator lift. In each case two values of the control lead time constants (k_1/k_2) are considered, $k_1/k_2 = 0$ corresponding to pure rate of height control. The figure 2 shown beside poles at the origin denotes that there is a double pole corresponding to the s^2 term in the denominator of the open loop transfer function. If $k_1/k_2 = 0$, the first order factor in the numerator reduces to s which cancels one of the s's in the denominator. The results of the earlier analysis suggests, that for $k_1 = 0$, i.e. along the vertical axis there is a condition of neutral stability and this would correspond to a locus fixed at the origin of the root locus plot for all values of k_2 in this case. This result could be obtained if one omitted to cancel the s factors in the transfer function, so that in addition to the pole shown at the origin there would be a further pole at zero and these could be thought of as contributing a point locus. It is seen that in treating the open loop transfer function merely as an algebraic equation, one can in this particular circumstance in fact lose a physically important and real statement about the system behaviour if the two s factors are removed by an algebraically perfectly legitimate operation. The missing root corresponds to neutral height stability, a result which must be expected for $k_1 = 0_1$ i.e. if there is no height control. To complete the locus plot for all cases with $k_1/k_2 = 0$ we should therefore add a 'stationary' locus at $j\omega = \sigma = 0$.

5. *Discussions and Conclusions.*

Tight control of height or flight path has been studied as a condition particularly relevant to flight close to the ground. A situation where pilots are known to resort to coarse elevator usage for tight control of height exists in the latter stages of the landing approach and this is perhaps even more evident during a float which does not rapidly lead to a touchdown.

It is shown theoretically that pure height control $\partial\eta/\partial H$ by means of elevator will inevitably lead to an oscillatory instability which can be overcome by applying a sufficient amount of phase advance $\partial\eta/\partial\dot{H}$. However, even with proper control lead, the gains a pilot can employ in this way are severely limited, if the cockpit is located relatively close to the centre of gravity of the aircraft. Placing the cockpit further forward, as is a feature of the modern transport aircraft, gives the pilot additional—even though strictly false—height sensations, which help to make control in this mode more stable, permitting the use of larger elevator control gains. If the cockpit were to be placed at an extreme forward position, say 160 ft ahead of the centre of gravity in our examples, this would permit the pilot to use pure height control without any control lead provided a sufficiently large gain in $\partial\eta/\partial H$ is used. This result appears to contradict the general rules of servo control and is the result of the fact that the 'false' height sensations assisting the pilot in this situation provide in fact an effective lead. It may be noted that forward cockpit location in the present context is exactly equivalent to the pilot using pitch attitude control which would evidently be benificial to the stability of this mode.

Adverse elevator lift, as is obtained from the elevons of a tailless aircraft, has a detrimental effect on the stability of height control, but here too forward cockpit location is favourable.

There are perhaps two main conclusions one can draw. Attempts to control the flight path of an aircraft, as for instance during the period proceeding touchdown, but tightening elevator control are bound to lead to pilot induced oscillations unless substantial phase advance is used. But even pure $\dot{H}$-control will lead to instability unless the pilot is located far forward of the centre of gravity of the aircraft. The trend towards pilot induced instability is generally more powerful with elevators having strong adverse lift, i.e. with tailless aircraft and with aircraft flying with small longitudinal stability. To induce an aircraft persisting in an unduly prolonged float to settle down onto the runway, it would seem to be best not to engage in tight elevator control, but perhaps to 'inch' the stick gently forward, resisting any temptation to true closed loop control.

Another conclusion one might draw from the results of this analysis is the importance of pilot cockpit location and this may become important in landing simulations. To present the pilot with all significant clues, the effects of cockpit location must be faithfully represented.

LIST OF SYMBOLS

$B = m k_y^2$	Pitch inertia
D	Differential operator
g (ft/sec^2)	Gravitational acceleration
$G(s)$	Transfer function of controlled system
H (ft)	Height
H_A	Height of aircraft centre of gravity
$\dot{H}$ (ft/sec)	Vertical velocity
H_P (ft)	Height of pilot above ground
$H(s)$	Transfer function of control feedback
K	Gain constant in servo analysis
K_1	Height control gain constant } equation (16)
K_2	Rate of height control gain constant } equation (16)
k_y	Inertia radius in pitch
$k_1 = \frac{\partial \eta}{\partial H}$	Height control gain
$k_2 = \frac{\partial \eta}{\partial \dot{H}}$	Rate of height control gain
L	Lift
$L_\alpha = \frac{\partial L}{\partial \alpha}$	Lift slope
$L_\eta = \frac{\partial L}{\partial \eta}$	Elevator lift slope
M	Pitching moment
$M_\alpha = \frac{\partial M}{\partial \alpha}$	Static longitudinal stability
$M_\eta = \frac{\partial M}{\partial \eta}$	Elevator moment derivative
$M_q = \frac{\partial M}{\partial q}$	Damping derivatives
$M_{\dot{w}} = \frac{\partial M}{\partial \dot{\alpha}}$	Damping derivatives
m	Aircraft mass
$q = \dot{\theta}$	Rate of pitch
s	Laplace operator
T (sec)	Period of pitching oscillation

V (ft/sec)	Speed
x_P (ft)	Distance of pilot forward of c.g.
x_η (ft)	Effective elevator moment arm (positive for foreplane)
α	Incidence
γ	Angle of climb
θ	Pitch attitude
$\xi_\eta = \frac{x_\eta}{k_y^2}$ (ft^{-1})	Elevator moment arm factor
η	Elevator deflection
ω (sec^{-1})	Angular frequency or imaginary part of a root in the root locus plane
ω_0 (sec^{-1})	Undamped frequency of the pitching oscillation
ζ	Damping ratio of the pitching oscillation
σ (sec^{-1})	Real part of a root in the root locus plane

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2	W. J. G. Pinsker	The landing flare of large transport aircraft. A.R.C. R. & M. 3602 (1967).
3	Maurice D. White	Proposed analytical model for the final stages of landing a transport airplane. NASA Technical Note TR D-4438 (1968).

TABLE 1

Aerodynamic and Inertial Properties of the Two Example Aircraft.

Aircraft	A (SST)	B (Subsonic transport)
$\mathscr{M}_\alpha$ (sec^{-2})	−0·3	−0·68
$\mathscr{M}_q$ (sec^{-1})	−0·33	−0·7
$\mathscr{M}_{\dot{\alpha}}$ (sec^{-1})	−0·33	−0·37
$\mathscr{L}_\alpha$ (sec^{-1})	0·4	0·6
$\mathscr{M}_\eta$ (sec^{-2})	−0·3	−0·4
k_y (ft)	60	40
x_η (ft)	−50	−80
x_η/k_y	−0·835	−2·0
V (ft/sec)	250	220
$x_P \left(\frac{x_P}{k_y}\right)$	40 (0·666)	40 (1·0)
	80 (1·333)	80 (2·0)
	120 (2·0)	120 (3·0)
	160 (2·666)	160 (4·0)
Longitudinal short period characteristics		
Period T (sec)	16	10
ω_0	0·666	1·05
Damping ratio	0·8	0·8

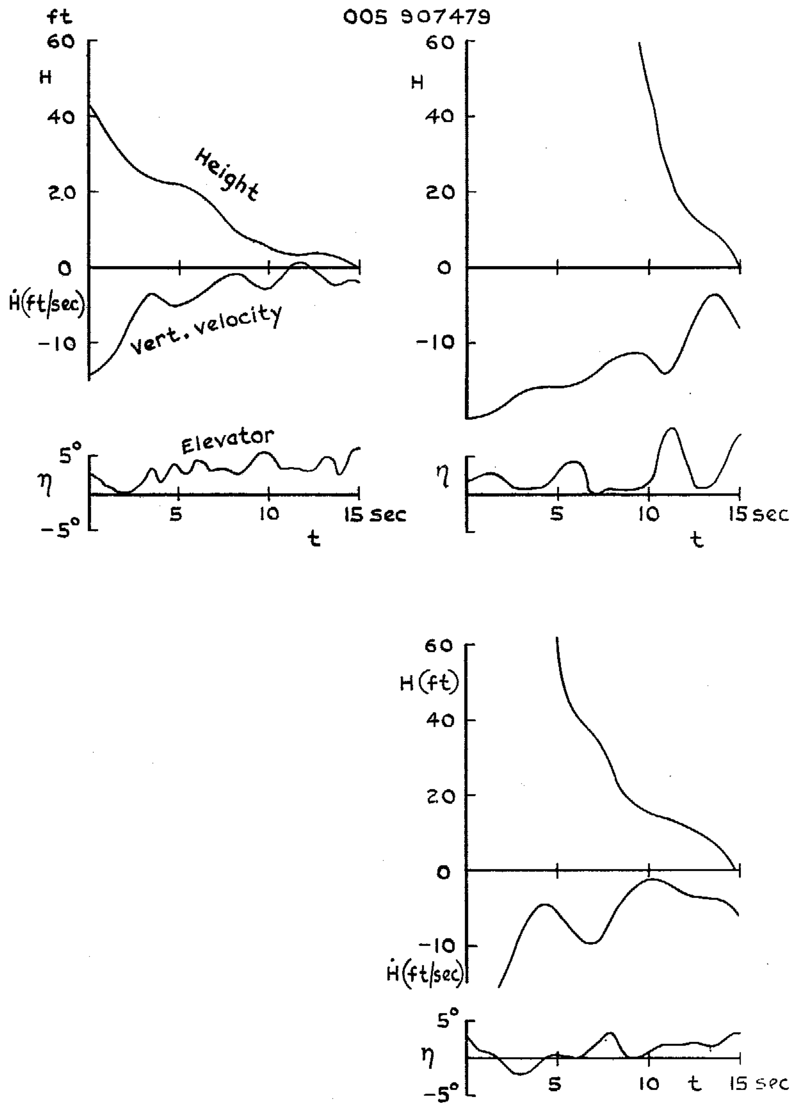


FIG. 1. Typical examples of pilot-induced oscillations, recorded during simulated landings of a large transport aircraft (RAE Aero flight simulator).

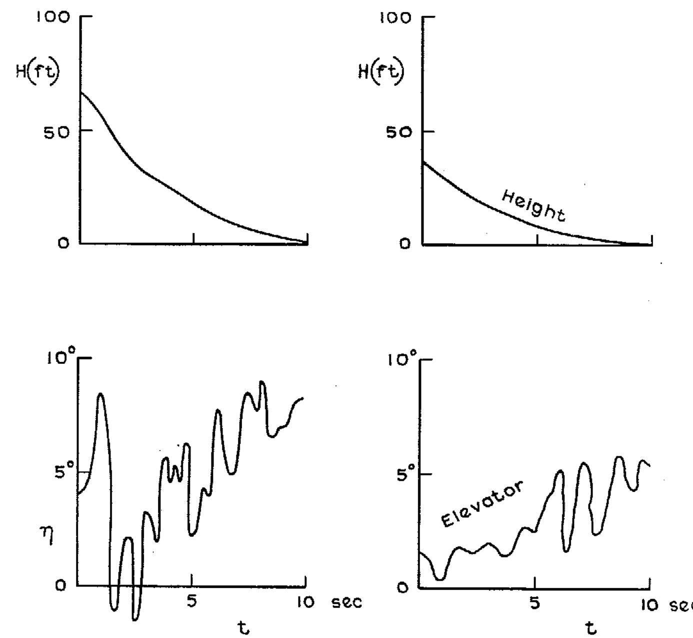


FIG. 2. Typical examples of pilot's oscillatory elevator activity just prior to touchdown. Recorded during flight trials at RAE Bedford.

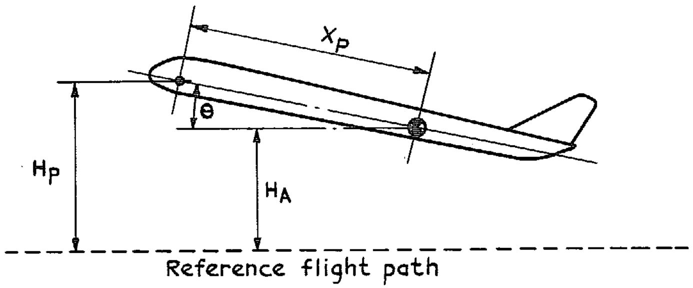


FIG. 3. Geometric relationship between height at gilots station and aircraft centre of gravity.

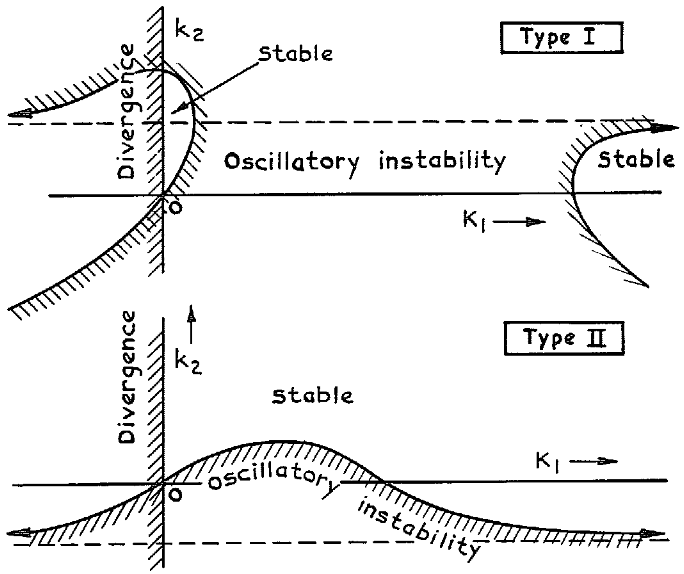


FIG. 4. The two basic forms of the stability boundaries for height control taken in the $K_1 - K_2$ plane.

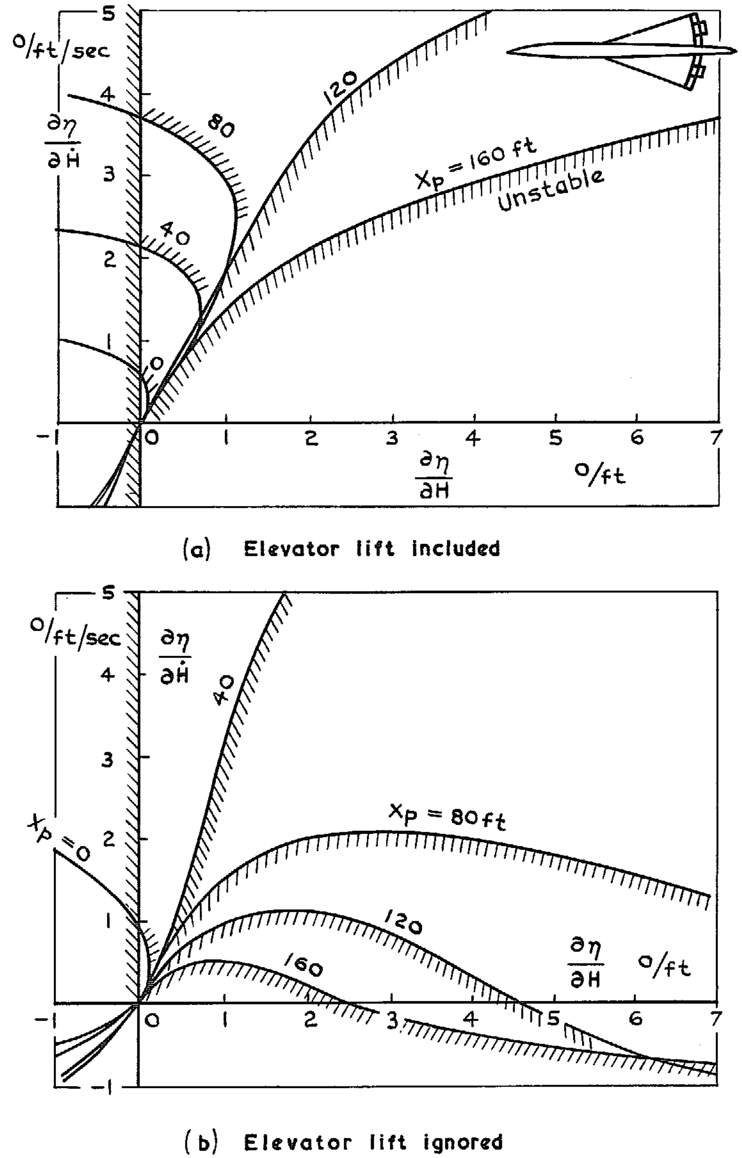


FIG. 5a & b. Stability boundaries for height control with elevator as a function of pilots location ahead of aircraft centre of gravity (X_p)

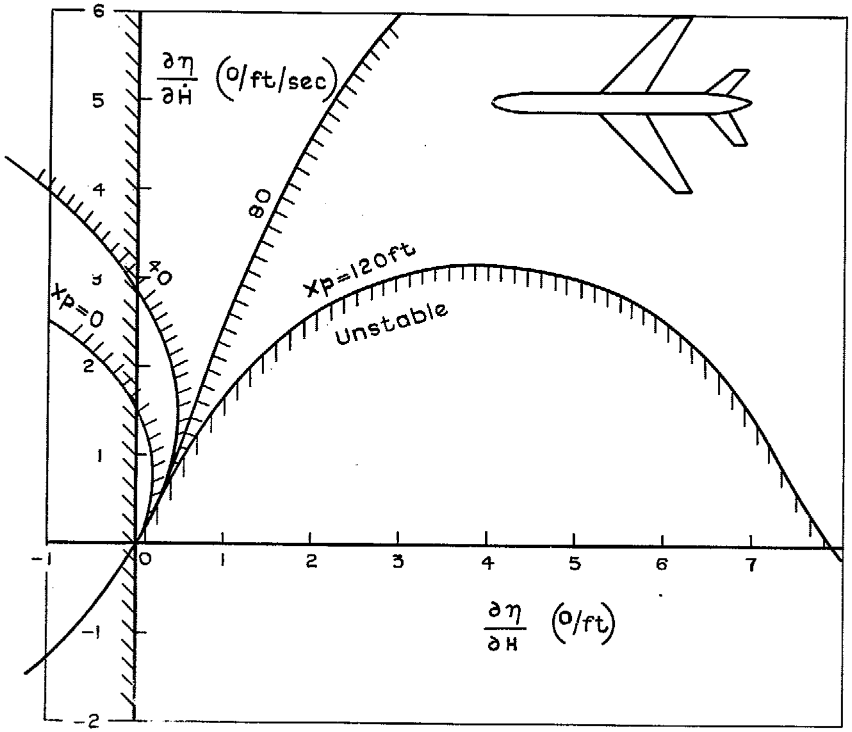


FIG. 6. Stability boundaries for aircraft 'B' elevator lift included but relatively small.

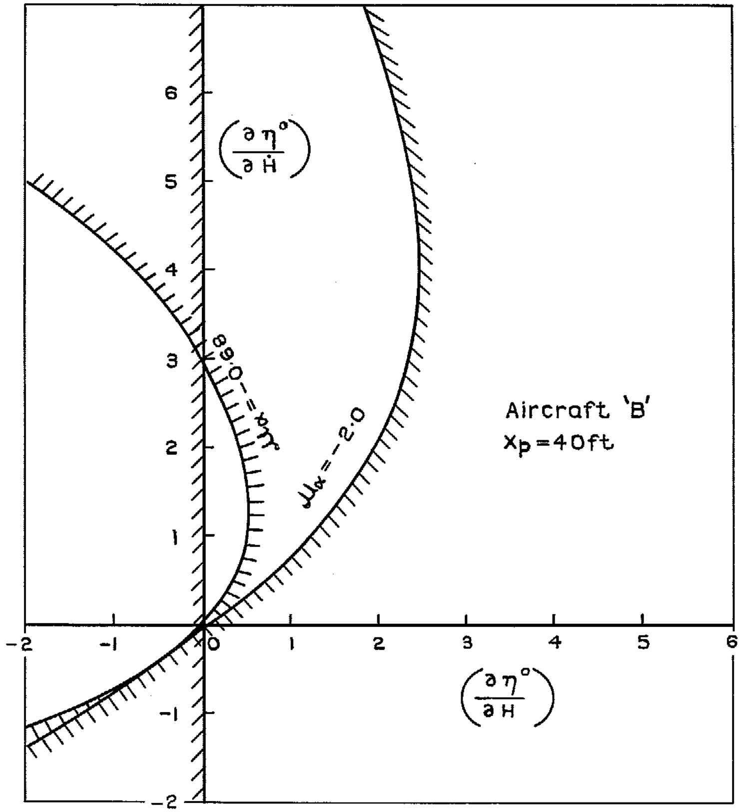
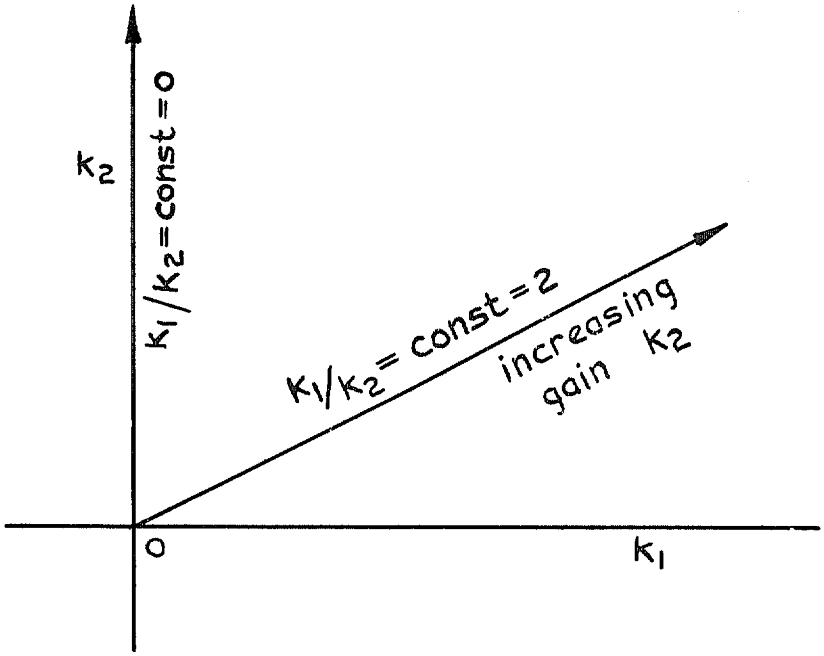


FIG. 7. Effect of static longitudinal stability on stability of height control.



Pilot control function

$$H(s) = \frac{-\eta}{H_p}(s) = -K_2\left(s + \frac{K_1}{K_2}\right)$$

Open loop transfer function:

$$G(s)\,H(s) = \left(\frac{\eta}{H_p}\right)\left(\frac{H_p}{\eta}\right) =$$

$$-K\,\frac{\left(s + \frac{K_1}{K_2}\right)\left(s^2 + as + b\right)}{s^2\left(s^2 + 2\zeta_0\,\omega_0\,s + \omega^2\right)}$$

FIG. 8. Relationship between lines of constant K_1/K_2 in the stability graphs and the open loop transfer function.

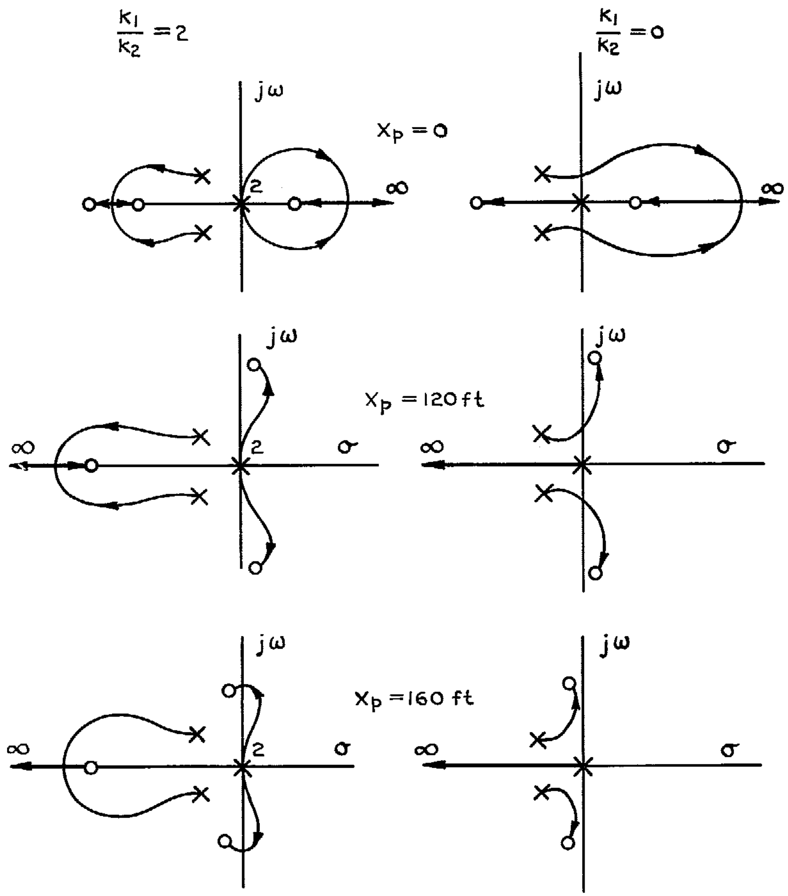


FIG. 9. Root locus plots for $K_2 0 \rightarrow \infty$ for constant values of k_1/k_2 aircraft 'A', elevator lift included.

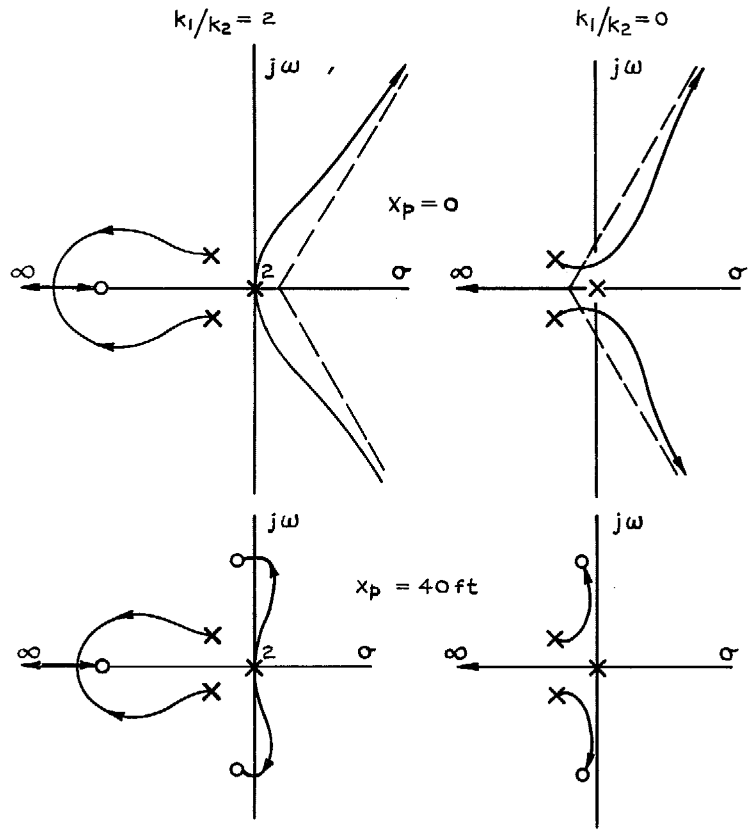


FIG. 10. Root locus plots for aircraft 'A' when elevator lift is ignored.

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