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Structural Representation in Aeroelastic Calculations

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Summary

The practicability of allowing approximately for the higher-frequency normal modes of a structure by using a residual flexibility matrix is examined somewhat philosophically. There appears to be a better method of approximation, which retains the concept of residual flexibility, and arguments in favour of it are given.

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1. Introduction

The study of the dynamical behaviour of a deformable aircraft inevitably involves the semi-rigid representation of the flexibility of the aircraft. There are two popular representations; in the one the aircraft deflection is described in terms of the deflections under discrete loads acting one at a time, and in the other these are replaced by the deflections of the undamped aircraft when oscillating at its natural frequencies. Calculations of the second of these—the normal modes—often use a matrix made up from the first—the flexibility matrix—as data, and if complete numerical accuracy could be achieved there would be no difference in the results of aeroelastic calculations using either the flexibility matrix or the complete set of normal modes derived from it.

The most obvious lure of normal modes is perhaps the possibility of obtaining good approximations by using a truncated set and thus decreasing the number of equations to be solved. In a recent report[1] Taylor has suggested that aeroelastic calculations be made using a truncated set of normal modes and making up for their inadequacy by including a residual flexibility matrix[2] which allows for the further static flexibility of the structure. This present Report has been written in the belief that the order of a flexibility matrix is often no true measure of its accuracy, and that the deficiencies of the normal mode approach are only important when part of the loading is discrete. Based on these beliefs, a completely modal approach which can include the effects of residual flexibility, is suggested as the most economic.

The concept of residual flexibility is first examined and then applied to a typical set of equations without recourse to the use of a 'free-free flexibility matrix'[1,2,3] which is a popular artifice for using data for a constrained structure in equations for its unconstrained motion. It is an artifice however which couples the inertia and stiffness data for the system at an early stage, obscuring to some extent the physics of subsequent operations. Finally, the merits of an equivalent completely modal approach are expounded.

2. Residual Flexibility Matrices

2.1 Derivation

Consider the free vibrations of a structure that is sufficiently constrained for a flexibility matrix to exist. Its motion can be described by the matrix equation

$$(-\omega^2 M + F^{-1})q_f = 0, \tag{1}$$

where M and F are positive definite symmetric matrices. For example, if the structure were a cantilever beam, the matrix M could be a diagonal matrix of discrete masses representing its inertia, in which case q_f would be a column matrix of the deflections of the masses and F would be the familiar matrix of flexibility coefficients for single loads at each of the mass points in turn. The inverse of the flexibility matrix, F^{-1}, is the stiffness matrix for the sets of modes ('stiffness matrix' modes for the constrained system), in each of which every loading point but one has no deflection, the odd one having unit deflection.

Let q_1, q_2 be column matrices of the generalised coordinates of the normal modes; q_2 referring to the modes of higher frequency. Then the deflections of the masses can be written

$$q_f = (U_1, U_2)\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \tag{2}$$

where the columns of the square matrix (U_1, U_2) are the modal vectors. From the orthogonal properties of normal modes

$$\begin{pmatrix} U_1' \\ U_2' \end{pmatrix} F^{-1}(U_1, U_2) = \begin{pmatrix} E_{11} & 0 \\ 0 & E_{22} \end{pmatrix}, \tag{3}$$

where E_{11}, E_{22} are diagonal matrices whose elements are the stiffness coefficients of the normal modes. Premultiplying equation (3) by $\begin{pmatrix} U_1' \\ U_2' \end{pmatrix}^{-1}$ and postmultiplying by $(U_1, U_2)^{-1}$ gives

$$F^{-1} = \begin{pmatrix} U_1' \\ U_2' \end{pmatrix}^{-1} \begin{pmatrix} E_{11} & 0 \\ 0 & E_{22} \end{pmatrix} (U_1, U_2)^{-1}. \tag{4}$$

Equation (4) inverted is

$$F = (U_1, U_2)\begin{pmatrix} E_{11}^{-1} & 0 \\ 0 & E_{22}^{-1} \end{pmatrix} \begin{pmatrix} U_1' \\ U_2' \end{pmatrix} \tag{5}$$

from which

$$F = U_1 E_{11}^{-1} U_1' + U_2 E_{22}^{-1} U_2'. \tag{6}$$

If the system is approximated to by the coordinates q_1 alone,

$$F_R = U_2 E_{22}^{-1} U_2' = F - U_1 E_{11}^{-1} U_1' \tag{7}$$

can be looked on as the 'residual flexibility'. The matrix F_R is singular, having the same rank as E_{22}. The definition above of F_R is the definition of a residual flexibility matrix used below.

2.2 Application

Consider a discrete unconstrained system under aerodynamic stiffness, C, and mechanical forcing, f, represented by the equation

$$\begin{pmatrix} U_r' \\ U_f' \end{pmatrix} M(U_r, U_f) \begin{pmatrix} \ddot{q}_r \\ \ddot{q}_f \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & F^{-1} \end{pmatrix} \begin{pmatrix} q_r \\ q_f \end{pmatrix} = \begin{pmatrix} U_r' \\ U_f' \end{pmatrix} C(U_r, U_f) \begin{pmatrix} q_r \\ q_f \end{pmatrix} + \begin{pmatrix} U_r' \\ U_f' \end{pmatrix} f. \tag{8}$$

The displacements of the points of the system to which M, C, F and f refer are given by

$$Z = (U_r, U_f) \begin{pmatrix} q_r \\ q_f \end{pmatrix}, \tag{9}$$

where q_r are the generalised coordinates of the normal rigid-body modes and the q_f are generalised coordinates which are normal to the q_r coordinates with respect to inertia and for which a non-singular flexibility matrix F exists.

Take as an example a simple beam and assume that a flexibility matrix has been determined for it when it is constrained by forces only. As mentioned above, the inverse of this flexibility matrix is the stiffness matrix for a set of modes in which each loading point has unit deflection in turn. If the mass points are coincident with the loading and constraint points, it is obvious that these modes will not be normal to the rigid-body modes with respect to inertia. A set of modes exists however which have the same distortions but which also have rigid-body deflections sufficient for them to be normal to further rigid-body deflections and q_f are the generalised coordinates of this set. Since the modes differ from the first modes only by rigid-body displacements they have the same stiffness matrix and hence the same flexibility matrix. The inertia matrix is different however since the absolute deflections of the masses in the two sets of modes are not the same. It should be noted that the aerodynamic stiffness matrix C is made up of the aerodynamic forces at all the loading and constraint points when there is unit deflection at each of them in turn.

Let $C_{fr} \equiv U_f' C U_r$ etc., $f_r \equiv U_r' f$ etc., and

$$q_f = (U_1, U_2) \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \tag{10}$$

where q_1, q_2 are generalised coordinates for the finite-frequency normal modes, q_2 referring to the modes of higher frequency.

Equation 8 can be written

$$\begin{bmatrix} A_{rr} & & \\ & A_{11} & \\ & & A_{22} \end{bmatrix} \begin{pmatrix} \ddot{q}_r \\ \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{bmatrix} 0 & & \\ & E_{11} & \\ & & E_{22} \end{bmatrix} \begin{pmatrix} q_r \\ q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & U_1' \\ 0 & U_2' \end{pmatrix} \left\{ \begin{pmatrix} C_{rr} & C_{rf} \\ C_{fr} & C_{ff} \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ 0 & U_1 & U_2 \end{pmatrix} \begin{pmatrix} q_r \\ q_1 \\ q_2 \end{pmatrix} + \begin{pmatrix} f_r \\ f_f \end{pmatrix} \right\} \tag{11}$$

If the accelerations of the system are low, $A_{22}\ddot{q}_2$ will be much smaller than $E_{22}q_2$ and the last submatrix equation of (11) can be approximated to by

$$E_{22}q_2 = U_2' \left\{ (C_{fr}, C_{ff}) \begin{pmatrix} q_r \\ U_1 q_1 + U_2 q_2 \end{pmatrix} + f_f \right\}. \tag{12}$$

which rearranged is

$$(E_{22} - U_2' C_{ff} U_2) q_2 = U_2' \left\{ (C_{fr}, C_{ff}) \begin{pmatrix} q_r \\ U_1 q_1 \end{pmatrix} + f_f \right\}. \tag{13}$$

Premultiplying by $U_2 E_{22}^{-1}$ gives, remembering equation (7),

$$(I - F_R C_{ff}) U_2 q_2 = F_R \left\{ (C_{fr}, C_{ff}) \begin{pmatrix} q_r \\ U_1 q_1 \end{pmatrix} + f_f \right\}. \tag{14}$$

and hence

$$\begin{pmatrix} q_r \\ U_1 q_1 + U_2 q_2 \end{pmatrix} = \begin{pmatrix} I & 0 \\ \bar{I} F_R C_{fr} & I + \bar{I} F_R C_{ff} \end{pmatrix} \begin{pmatrix} q_r \\ U_1 q_1 \end{pmatrix} + \begin{pmatrix} 0 \\ \bar{I} F_R f_f \end{pmatrix}, \tag{15}$$

where $\bar{I} \equiv (I - F_R C_{ff})^{-1}$.

Note that

$$I + \bar{I} F_R C_{ff} = \bar{I}. \tag{16}$$

(To prove this premultiply the equation by $\bar{I}^{-1}$).

From equations (11) and (15)

$$\begin{bmatrix} A_{rr} & \\ & A_{11} \end{bmatrix} \begin{pmatrix} \ddot{q}_r \\ \ddot{q}_1 \end{pmatrix} + \begin{pmatrix} 0 \\ E_{11} q_1 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & U_1' \end{pmatrix} \left\{ \begin{pmatrix} C_{rr} & C_{rf} \\ C_{fr} & C_{ff} \end{pmatrix} \left[\begin{pmatrix} I & 0 \\ \bar{I} F_R C_{fr} & \bar{I} \end{pmatrix} \begin{pmatrix} q_r \\ U_1 q_1 \end{pmatrix} + \begin{pmatrix} 0 \\ \bar{I} F_R f_f \end{pmatrix} \right] + \begin{pmatrix} f_r \\ f_f \end{pmatrix} \right\}. \tag{17}$$

Hence

$$A_{rr} \ddot{q}_r = (C_{rr} + C_{rf} \bar{I} F_R C_{fr}, \quad C_{rf} \bar{I}) \begin{pmatrix} q_r \\ U_1 q_1 \end{pmatrix} + f_r \tag{18}$$

and

$$A_{11} \ddot{q}_1 + E_{11} q_1 = U_1' \left\{ [(I + C_{ff} \bar{I} F_R) C_{fr}, \quad C_{ff} \bar{I}] \begin{pmatrix} q_r \\ U_1 q_1 \end{pmatrix} + (I + C_{ff} \bar{I} F_R) f_f \right\} \tag{19}$$

Note that $C_{ff} \bar{I} = C_{ff}(I - F_R C_{ff})^{-1} = (I - C_{ff} F_R)^{-1} C_{ff} = \bar{\bar{I}} C_{ff}$ (say) and that

$$I + \bar{\bar{I}} C_{ff} F_R = \bar{\bar{I}}. \tag{20}$$

Hence equation (19) can be written

$$A_{11} \ddot{q}_1 + E_{11} q_1 = U_1' \bar{\bar{I}} \left\{ (C_{fr}, \quad C_{ff}) \begin{pmatrix} q_r \\ U_1 q_1 \end{pmatrix} + f_f \right\}. \tag{21}$$

By ignoring the inertia forces in the less-grave normal modes, equation (11) has been replaced by equations (18) and (21).

Thus the full number of equations has been reduced to the number that have non-negligible inertia coefficients but the coefficients of these new equations are evaluated from matrices whose order is the same as that of the flexibility matrix. It can be seen from equation (7) that the residual flexibility matrix describes the flexibility that is left when the system is constrained so that it cannot deflect in its graver modes. Thus, even the simplest loading of the residual system will lead to a complicated deflection since the deflection can only be described in terms of the shapes of the higher-frequency modes.

Associated with this residual flexibility matrix is an aerodynamic stiffness matrix of the same order. Any increase in accuracy due to the inclusion of the residual flexibility is likely to be lost if this aerodynamic stiffness matrix is not accurate enough for the aerodynamic stiffnesses in the higher-frequency modes to be obtained. Thus a large number of collocation points will be needed in the application of the aerodynamic theory if the order of the matrices is at all large and, whether or not the necessary accuracy is achieved, the calculation will be expensive in time and effort.

2.3 Criticism

Flexibility matrices are generally ill-conditioned. This is essentially the result of the loadings used in their determination, and these are about the worst possible from this point of view. If the deflection of the structure under a discrete load is regarded as an arbitrary mode, the column of the deflections at the loading points can be expressed as $z_{\text{arby}} = \sum_{r=1}^{\infty} \alpha_r z_r$ where z_r is a column of the deflections in the rth normal mode and α_r is a scalar. Since the load is single, it is likely that α_1 will be the largest α and α_r will decrease as r increases. In a sense the usual flexibility matrix contains too much information on the amount of energy needed to distort the structure into a simple shape and too little about the more complicated shapes. The result of this is that what is left of the flexibility matrix gets less and less accurate as the graver modes are extracted. Minhinnick has suggested[4] that about one significant decimal figure is lost when each normal mode is found. Thus the number of significant figures to which the flexibility coefficients are determined certainly puts an upper limit on the number of normal modes that can be found accurately. When this limit is reached the residual flexibility matrix is dross as far as the normal mode type of calculation is concerned and if the flexibility matrix has been determined by tests, its residue is likely to consist largely of the expression of what are actually physical impossibilities. In these circumstances the author believes it impolitic to include the residual flexibility in the data for further calculations.

Were flexibility coefficients commonly found for multiple loads, the position might be better, for in this case it would be possible to arrange the loads so that a different α_r was largest at each loading condition. In still-air resonance tests an attempt is made to make all the α_r save one zero.

3. Calculation of Normal Modes

In the above, 'free-free flexibility matrices' have been avoided since their use introduces an unnecessary sophistication.

For completeness, a method of deriving the normal modes of a free-free structure directly from the simple flexibility matrix is reiterated below.

The equation for the finite-frequency normal modes of the typical system is, following equation (8),

$$(\omega^2 U_f' M U_f - F^{-1}) q_f = (0), \tag{22}$$

with

$$U_r' M U_f = (0) \tag{23}$$

by the definition of q_f.

Put

$$U_f = \bar{U}_f + U_r Q, \tag{24}$$

i.e., the deflections in the 'stiffness matrix' modes for the free-free system, U_f, are the sums of the deflections in the 'stiffness matrix' modes for the constrained system (*see* Section 2.1), $\bar{U}_f$, and the rigid-body deflections necessary for orthogonality, $U_r Q$. Substituting for U_f from equation (24) in equation (23) gives

$$U_r' M(\bar{U}_f + U_r Q) = (0), \tag{25}$$

which rearranged is

$$Q = -A_{rr}^{-1} U_r' M \bar{U}_f. \tag{26}$$

From equations (24) and (26)

$$U_f = (I - U_r A_{rr}^{-1} U_r' M) \bar{U}_f. \tag{27}$$

Substituting for U_f in equation (22) gives

$$\{\omega^2 \bar{U}_f'(M - M U_r A_{rr}^{-1} U_r' M) \bar{U}_f\} q_f = F^{-1} q_f. \tag{28}$$

The best method of solving equation (28) generally is to replace F by the product of a lower triangular matrix and its transpose, i.e.

$$F = L_F L_F', \tag{29}$$

and

$$L_F' F^{-1} L_F = I. \tag{30}$$

Putting $q_f = L_F q_F$ and premultiplying by L_F' equation (28) can be written

$$\omega^2 L_F' \bar{U}_f'(M - M U_r A_{rr}^{-1} U_r' M) \bar{U}_f L_F q_F = q_F. \tag{31}$$

Library computer programs based on this method are readily available. The above method cannot be applied if the matrix F is not positive definite as it should be from physical considerations since the triangular matrices are then either indeterminate or complex. If the deficiency is due to numerical difficulties a possible method ot overcoming it is to replace the flexibility matrix by its positive latent roots, Λ_L, and their vectors, U_L.

Let

$$F U_L = U_L \Lambda_L \tag{32}$$

where U_L has more rows than columns, but $U_L' U_L = I$, and Λ_L is a diagonal matrix.

Equation (28), written in terms of the flexibility matrix rather than its inverse and with $\overline{M}$ substituted for $(M - MU_r A_{rr}^{-1} U_r' M)$, is

$$\omega^2 F \overline{U}_f' \overline{M} \overline{U}_f q_f = q_f. \tag{33}$$

Putting $q_f = U_L \Lambda_L^{\frac{1}{2}} q_L$ and premultiplying by $\Lambda_L^{-\frac{1}{2}} U_L'$ equation (33) becomes

$$\omega^2 \Lambda_L^{-\frac{1}{2}} U_L' F \overline{U}_f' \overline{M} \overline{U}_f U_L \Lambda_L^{\frac{1}{2}} q_L = q_L. \tag{34}$$

Substituting for $U_L' F$ using the transpose of equation (32), with the assumption that F is at least symmetric,

$$\omega^2 \Lambda_L^{\frac{1}{2}} U_L' \overline{U}_f' \overline{M} \overline{U}_f U_L \Lambda_L^{\frac{1}{2}} q_L = q_L, \tag{35}$$

which is of the same form as equation (31) but of lower order.

The above results are directly applicable to equation (8) *et seq.* U_f is given by equation (27) and if the columns of U_1 are the latent vectors pertaining to the graver modes of frequencies $\lceil \omega_1 \rfloor$ so that

$$q_f = U_1 q_1, \text{ cf. equation (10)} \tag{36}$$

from equation (28),

$$A_{11} = U_1' \overline{U}_f' \overline{M} \overline{U}_f U_1 \tag{37}$$

and

$$E_{11} = \lceil \omega^2 \rfloor A_{11} \tag{38}$$

and from equation (7),

$$F_R = F - U_1 E_{11}^{-1} U_1'. \tag{39}$$

However, the next part of the evaluation of the coefficients of equations (18) and (21) involves a number of operations on matrices probably large in order. Much the simpler approach is to calculate more normal modes and allow for their flexibility by the method described below.

4. Suggested Structural Representation

No problem involving a complex structure can be solved exactly. Most methods of approximation can be regarded as the replacement of the infinite degree of flexibility of the structure by flexibility in a finite number of necessarily arbitrary semi-rigid modes. In the case of flexibility coefficients, the arbitrary modes are the deflections of the structure under discrete loads, in the case of stiffness coefficients they are the deflections under positional constraints and in the case of normal modes they are the deflections under normal loading distributions which are the product of the mass of the structure and the deflections themselves.

Whether a particular set of arbitrary modes is suitable for use in a calculation depends on whether the displacements, calculated as the result of the combination of arbitrary loading distributions taken to be the equivalent of the actual loading distributions, are close enough to the actual displacements.

The number of normal modes needed for accurate representation is not all that easy to pontify upon. In one case in which aerodynamic loads occur, i.e., flutter, for a number of years it has been assumed with apparent success that they could be represented by a limited number of normal inertia loadings, i.e., loading proportional to the local mass multiplied by the displacement in the mode, and it seems reasonable that aerodynamic loads, being reasonably well distributed, can be economically represented by normal inertia loadings in other types of calculation. Whilst the accuracy of a calculation probably increases more or less evenly, up to a point, as the number of well distributed flexibility coefficients used to describe the structure is increased, the increase in accuracy when the number of normal modes included is increased depends to a large extent on the shapes of the normal modes added. Quite often a good approximation can be obtained using only one or two normal modes—not necessarily the gravest. In general, however, all the modes down to the fundamental are included. It is essential that the normal modes should include all those likely to be an important part of the actual deflection. Thus, if a divergence calculation is made on a wing whose torsion mode is third in order of frequency, it is useless to make a calculation based on only the first two modes.

Normal modes are less efficient as a means of structural representation when the loading is discrete. This is a consequence of the large number of normal inertia loadings needed to represent a discrete load. For such loads flexibility matrices seem more appropriate since they themselves are derived from point loads. For calculations in which both distributed and discrete loads are involved, the optimum treatment is probably to use both normal and arbitrary modes, the arbitrary modes being given by the columns of the flexibility matrix for the points at which the discrete loads act. This would have the disadvantage of introducing structural couplings into the equations and worsen their condition but some idea of the possible advantages is given by the example quoted by Schwendler and MacNeal[2] of a cantilevered torsion bar, with an oscillating load at its free end at half the frequency of the fundamental mode. The exact mechanical admittance (θ_{tip}/T) of this system is 1·27324; the admittance when only the fundamental mode is involved is 1·08076 which becomes 1·27019 when residual flexibility is added but the admittance when only the fundamental and a mode of linear twist between root and tip are included is 1·27321. The author intends to test this artifice more fully in the case of the allied problem of the flutter of wings with stores whose mass can be varied. The discrete-load modes could be modified to modes that are normal to each other and the true normal modes, either with respect to inertia only or to both inertia and stiffness, but this should be done only if they cannot be included otherwise.

5. Advantages of Normal Modes

The overriding requirement of any calculation is that the results should be what they purport to be. It is far better to have no results than results that are wrong but not suspect. Thus every opportunity should be taken to apply checks, particularly those which involve some thought. Ocular examination of flexibility matrices is unrewarding, being limited simply to a check of symmetry and the absence of gross errors. The matrix can be checked for positive-definiteness but if it fails this check there is little that can be done to correct the fault on a physical basis with only the raw matrix as evidence. If the normal modes are found, the structural data are then available in a form which has enough character for a judgement of its accuracy to be made on past experience and physical intuition. If the flexibility matrix is not positive definite, one or more of the latent roots of the dynamical matrix will be negative and a decision can be made according to the order of the negative root and the shapes of the modes as to whether the fault is fundamental or due to numerical deficiencies such as round-off error. If the latter is the case it will be possible to continue into further calculation, if sufficient plausible modes have been obtained, without further tampering with the data. The results of still-air resonance tests are generally available in the case of actual aircraft for an overall check of the representation of the structure.

The inclusion of aerodynamic forces and residual flexibility is easier if the representation is by normal modes. Rewrite equation (8) as

$$\begin{bmatrix} -\omega^2 A_{11} + E_{11} & 0 \\ 0 & E_{22} \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} + \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}, \tag{40}$$

where $C_{rs} \equiv U_r' C U_s$, $f_r \equiv U_r' f$ and here the q_1 modes include the q_r modes. C_{rs} is the matrix of generalised aerodynamic stiffness based on the work done in the modes r by the aerodynamic forces which arise from deflections in the modes s. These can be calculated as best thought fit, bearing in mind the complexity of the modes, and it is possible for some of the elements of the matrices to be treated as negligible. It follows immediately that

$$(E_{22} - C_{22})q_2 = C_{21}q_1 + f_2 \tag{41}$$

and

$$(-\omega^2 A_{11} + E_{11})q_1 = \{C_{11} + C_{12}(E_{22} - C_{22})^{-1}C_{21}\}q_1 + f_1 + C_{12}(E_{22} - C_{22})^{-1}f_2 \tag{42}$$

which replace equations (13) and (18) respectively. $(E_{22} - C_{22})$ will be singular only if C_{22} is appropriate to a critical static divergence speed in the q_2 coordinates alone.

The representation of structural damping is always a difficulty. In calculations based on normal modes it is often assumed that the modes are orthogonal with respect to the damping as well as the stiffness and inertia. This is undoubtedly an oversimplification but on the face of it normal modes provide a better basis for representation of damping than influence matrices.

Finally some account should be taken of economy. A unified approach to flutter calculations on the one hand and stability and control problems on the other has long been advocated. To the flutter engineer, basically studying maintained oscillations at a single frequency, normal modes appear to have no real competitors as far as structural representation is concerned. Hence the normal modes of aircraft will be found in any case and their use as data in other aeroelastic calculations should not incur extra work.

6. Conclusions

It is advantageous to use normal modes rather than influence matrices for structural representations in aeroelastic calculations for the following reasons:

(a) it is easier to check the data and reject that part which is inaccurate,

(b) the data is presented in a comprehensible form and the characteristics of the degrees of freedom being allowed are known,

(c) the calculation of the aerodynamic coefficients is simplified,

(d) residual flexibility can be included easily and accurately,

(e) the order of the matrices involved in the calculation of the coefficients of the reduced simultaneous equations is decreased.

In cases where discrete loads are present, modes which are the deflections of the structure under static loads at the appropriate points should be added to the normal modes. Hence equations should be formulated in a way which allows account to be taken of structural couplings.

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