## Lidxumy, body <br> No\& MI MO. 2162

AERONAUTICAL RESEARCH COUNCIL REPORTS AND MEMORANDA

# A Theoretical Discussion of Wings with Leading-edge Suction By <br> M. J. Lighthill, B.A., of the Aerodynamics Division, N.P.L. 

## Crows Copyright Reserved

LONDON: HIS MAJESTY'S STATIONERY OFFICE
Price es. od. net

# RATIONAL AERONAUTIC:L LIMULISHMENT A Theoretical Discussion $b \in W /$ ang with Leading-edge Sütion <br> By <br> M. J. Lighthill, B.A., of the Aerodynamics Division, N.P.L. 



1. Introduction and Summary.-Suction slots on wings are of two kinds: those into which only the boundary layer is sucked away, and those which also receive a considerable portion of the free air outside the boundary layer.
The purpose of the former is to overcome a discontinuous drop in the velocity at the surface of the aerofoil and so obviate the need for extended regions of adverse pressure gradient where transition or separation may occur unpleasantly soon. This requires special design of the aerofoil (to have a discontinuity in velocity at some point or points) and conversely an aerofoil so designed essentially requires to have such slots at these points and nowhere else. Hitherto their position has generally been well to the rear of the aerofoil and the aim has been to make the velocity non-decreasing as far as the slots on both surfaces for as wide a range of $C_{L}$ as possible.

The purpose of the second kind of suction slot is to eliminate large adverse pressure gradients occurring immediately behind it, by the action of sink effect. Such a slot could be placed anywhere on any wing, and would always have this effect. Naturally the most satisfactory position is near the summit of any large suction peak. These occur most frequently and with the greatest detriment near the leading edge at high lifts. Hence a slot suitably placed in the forward region may be expected to increase the maximum lift of some wings.

This report will study the use of slots near the leading edge ; both when they act solely by sink effect, and when the ideas of the two foregoing paragraphs are combined and the same slots used for both purposes (obviously the most economical method). The following discussion is based on the theory of R. \& M. 21121, of which at least $\S 1$ should be read and the rest lightly skimmed before the reader goes any further.
2. Slots as Sinks.—A sink of strength $2 \pi m$ on the boundary of the unit circle (at the point $\left.e^{i f}\right)$ possesses the complex potential

$$
\begin{equation*}
w(\zeta)=-m\left\{2 \log \left(\zeta-e^{i \beta}\right)-\log \zeta\right\} . \tag{1}
\end{equation*}
$$

That this corresponds physically to a slot is seen from the fact that the flux through a small semi-circle in the fluid with $e^{i p}$ as centre is $2 \pi m$, the change in the imaginary part of $w$ as the semi-circle is traversed.

Combining it with the potential (1) of R. \& M. 2112¹, we obtain

$$
\begin{equation*}
\frac{d w}{d \zeta}=e^{-i \alpha}-\frac{e^{i \alpha}}{\zeta^{2}}+\frac{i \chi}{\zeta}-m\left(\frac{2}{\zeta-e^{i \beta}}-\frac{1}{\zeta}\right) . \quad . \quad . . \quad . . \quad . . \tag{2}
\end{equation*}
$$

When $\zeta=e^{i \theta}$, this becomes

$$
\begin{equation*}
4 i e^{-i \theta} \cos \left(\frac{1}{2} \theta-\alpha\right) \sin \frac{1}{2} \theta+m i e^{-i \theta} \cot \frac{1}{2}(\theta-\beta) . \tag{3}
\end{equation*}
$$

But $|d z / d \zeta|=\left(2 / q_{0}\right) \sin \theta$, by (4) of R. \& M. 21121. Hence the velocity in the aerofoil plane is

$$
\begin{equation*}
\left|\frac{d w}{d z}\right|=q_{0}\left(\frac{\cos \left(\frac{1}{2} \theta-\alpha\right)}{\cos \frac{1}{2} \theta}+\frac{1}{2} m \operatorname{cosec} \theta \cot \frac{1}{2}(\theta-\beta)\right) . \tag{4}
\end{equation*}
$$

The parts due to incidence and sink effect are independent of one another.


As $\theta \rightarrow \beta$ the sink effect portion tends to infinity, and very close to $\beta$ on one side or other (the $\theta<\beta$ side if $0 \leqslant \beta \leqslant \pi$ and $0 \leqslant \alpha$ for example) there is a stagnation point. This corresponds with the facts for a slot of finite width, as is illustrated in the adjoining sketch.

Expression (4) should give an accurate picture of the state of affairs past the stagnation point.

In all applications in this report $\beta=\pi$ has been taken, so that (4) becomes

$$
\begin{equation*}
q=q_{0}\left(\frac{\cos \left(\frac{1}{2} \theta-\alpha\right)}{\cos \frac{1}{2} \theta}-\frac{1}{4} m \sec ^{2} \frac{1}{2} \theta\right) . \quad . \quad . . \quad . . \quad . \quad . \tag{5}
\end{equation*}
$$

A simple illustration is the following. In Appendix VI of R. \& M. $2112^{1}$ a low-drag aerofoil was designed with an exceptionally high " $C_{L}$ range" $(0 \cdot 266)$ for its thickness ( 13 per cent.). For this good quality however it suffers with regard to maximum lift, as Fig. 1 shows, where the velocity over the forward portion is plotted at incidence 10 deg., $C_{L}=1 \cdot 18$. Over this portion

$$
q_{0}=1 \cdot 2034 \frac{\cos \frac{1}{2} \theta}{\cos \left(\frac{1}{2} \theta-2 \cdot 29^{\circ}\right)}
$$

Hence

$$
q_{10^{\circ}}=1 \cdot 2034 \frac{\cos \left(\frac{1}{2} \theta-10^{\circ}\right)}{\cos \left(\frac{1}{2} \theta-2 \cdot 29^{\circ}\right)}
$$

which is the expression plotted and possesses a very steep decline from the leading edge, which is the point of maximum suction at this incidence. But, if we insert a suction slot at the leading edge and utilise the sink effect it produces, the velocity at incidence 10 deg. becomes

$$
\begin{equation*}
1 \cdot 2034\left\{\frac{\cos \left(\frac{1}{2} \theta-10^{\circ}\right)}{\cos \left(\frac{1}{2} \theta-2 \cdot 29^{\circ}\right)}-\frac{1}{4} m \sec \frac{1}{2} \theta \sec \left(\frac{1}{2} \theta-2 \cdot 29^{\circ}\right)\right\} . . . \quad . \tag{6}
\end{equation*}
$$

This is plotted for $m=0.04$ as a dotted line in Fig. 1, and the maximum adverse gradient is seen to be very moderate, so that with this amount of suction the maximum $C_{L}$ of the aerofoil must be $\geqslant 1 \cdot 18$. The strength of the sink is $2 \pi \cdot 0 \cdot 04$. But the chord of the aerofoil is $3 \cdot 689$, so that reduced with respect to this it is $2 \pi \cdot 0 \cdot 04 / 3 \cdot 689=0 \cdot 0681$. It would be produced by sucking 10 times the velocity at infinity through a slot of width $0 \cdot 00681$ chords. This would be possible (without choking) only at very low speeds : but the highest lifts are often only required at low speeds.

Another application is to wings for flight faster than sound. These have to be designed (see R. \& M. 1929²) very thin indeed and with very sharp leading edges. A good shape is the symmetrical biconvex, of thickness ratio 4 to 7 per cent. But this is not at all a favourable shape at low speeds: it has a low maximum lift and, even at moderate lifts, a high drag due to breakaway at the front on the upper surface.

An aerofoil of approximately this shape is given by the equation

$$
x=\left\{\begin{array}{l}
-\gamma \cos \theta(0<\theta<\pi)  \tag{7}\\
+\gamma \cos \theta(-\pi<\theta<0)
\end{array}\right\} . \quad . \quad . \quad . \quad . . \quad . \quad .
$$

This gives $\log q_{u}$, the conjugate of $\chi$, as

$$
-\frac{2 \gamma}{\pi} \cos \theta \log \left|\cot \frac{1}{2} \theta\right|
$$

The method of R. \& M. $2112^{1}$ can be carried through to give the aerofoil shape. For $\gamma=6 \mathrm{deg}$. the ordinates and abscissae are given in Table 1, below, with the velocity distributions over the forward part at incidences 0, 5 and 10 deg. ( $C_{L}=0,0.57$ and 1.13). The sink effect for $2 \pi m=0 \cdot 1$ is also shown, and by inspection we find that this, removed from $q_{5^{\circ}}$, gives a conservative pressure gradient; but that three times as much must be subtracted from $q_{10^{\circ}}$ to give the same effect.

TABLE 1

| $\begin{gathered} \theta \\ (\mathrm{deg} .) \end{gathered}$ | $X$ | Y | $q_{0}$ | $q_{5}{ }^{\circ}$ | $q_{10}{ }^{\circ}$ | $\frac{0 \cdot 1}{8 \pi} q_{0} \sec ^{2} \frac{1}{2} \theta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 180 | 0 | 0 | 0 | $\infty$ | $\infty$ | $\infty$ |
| 170 | $0 \cdot 0088$ | $0 \cdot 0009$ | 0.911 | 1.815 | $2 \cdot 705$ | $0 \cdot 477$ |
| 160 | $0 \cdot 0335$ | $0 \cdot 0034$ | 0.959 | $1 \cdot 429$ | $1 \cdot 889$ | $0 \cdot 127$ |
| 150 | $0 \cdot 0724$ | $0 \cdot 0071$ | 0.991 | $1 \cdot 310$ | $1 \cdot 618$ | $0 \cdot 059$ |
| 140 | $0 \cdot 1238$ | $0 \cdot 0115$ | $1 \cdot 015$ | 1-255 | $1 \cdot 484$ | 0.035 |
| 130 | $0 \cdot 1858$ | $0 \cdot 0161$ | 1.035 | $1 \cdot 224$ | $1 \cdot 405$ | 0.023 |
| 120 | $0 \cdot 2565$ | $0 \cdot 0204$ | $1 \cdot 048$ | $1 \cdot 203$ | $1 \cdot 348$ | 0.017 |
| 110 | $0 \cdot 3340$ | 0.0238 | 1.061 | 1-189 | $1 \cdot 307$ | 0.013 |
| 100 | $0 \cdot 4159$ | $0 \cdot 0260$ | 1.067 | $1 \cdot 174$ | $1 \cdot 272$ | $0 \cdot 010$ |
| 90 | $0 \cdot 5000$ | $0 \cdot 0268$ | 1.069 | $1 \cdot 158$ | $1 \cdot 239$ | 0.008 |

The shape and velocity distributions are plotted in Fig. 2. Reduced with respect to the chord the strength is $0 \cdot 1 / 3 \cdot 864=0 \cdot 02588$, which corresponds to a flow of 10 times the velocity at infinity through a slot of width $0 \cdot 0026$ chords. The higher value (three times this) is therefore practically unattainable. But we can expect the maximum lift to be at least higher by use of suction, and, more important, the range of $C_{L}$ without stalling drag will be very much greater.

But the most important application of leading-edge suction will be to cambered aerofoils with a discontinuity in velocity at the slot independent of the quantity sucked away. These are considered in the next section.
3. Wings with Discontinuous Velocity at the Leading Edge.-In R. \& M. 21121, Appendix VIII, a symmetrical aerofoil with two leading-edge slots (for boundary-layer suction) was designed, and the shape and velocity distribution shown in Fig. 7. But it is undesirable to use two slots where one might serve and it is wasteful to construct an aerofoil which, in addition to a large maximum lift, possesses a minimum lift of equal magnitude and opposite sign which could never be used. Both these arguments point to the need for making aerofoils of this type cambered.

It also seems clear that the greatest advantage will be obtained if the discontinuity at the slot is as large as possible ; in fact if for positive incidence the lower surface velocity at the slot is infinite (in the mathematics at any rate) and the upper surface velocity quite low. Finally we want, over as large a range of $C_{L}$ as possible, no large adverse velocity gradients. These considerations led the following distribution to be chosen :-

$$
\log q_{0}=\left\{\begin{array}{l}
\log \cos \frac{1}{2} \theta  \tag{8}\\
-\log \cos \left(\frac{1}{2} \theta-\alpha\right) \\
+k+a(1-\cos \theta) \\
+k+b(1-\cos \theta)
\end{array}\right.
$$

$$
\left.\begin{array}{l}
(0<\theta<\pi) \\
(2 \alpha<\theta<\pi) \\
(0<\theta<\pi) \\
(-\pi<\theta<0)
\end{array}\right\}
$$

At incidence $\alpha$ on the upper surface the velocity sidles down at a fairly even pace along the whole chord, while at zero incidence the same is true (at a slower pace) for the lower surface. For all positive incidences the velocity rises from its stagnation point on the lower surface to an infinite value just before the slot.

To obtain the shape we need the conjugate of (8) and in particular the integral

$$
\begin{align*}
& f(\theta, \alpha)=\frac{1}{2 \pi} \int_{0}^{\pi} \log \cos \frac{1}{2} t \cdot \cot \frac{1}{2}(0-t) \cdot d t \\
& \quad-\frac{1}{2 \pi} \int_{2 a}^{\pi} \log \cos \left(\frac{1}{2} t-\alpha\right) \cdot \cot \frac{1}{2}(\theta-t) \cdot d t . \quad \ldots \tag{9}
\end{align*}
$$

Now it can be shown that

$$
\begin{align*}
\frac{\partial f}{\partial \theta}= & \frac{1}{2 \pi}\left[\alpha-\tan \frac{1}{2} \theta\left(\log \tan \frac{1}{2} \theta+\log \sin \alpha\right)\right. \\
& \left.-\tan \left(\frac{1}{2} \theta-\alpha\right)\left\{\log \cos \frac{1}{2} \theta-\log \sin \alpha-\log \sin \left(\frac{1}{2} \theta-\alpha\right)\right\}\right] \tag{10}
\end{align*}
$$

Hence, if we assume $f(0, \alpha)=0$, as we may, since an arbitrary constant present in $\chi$ does not matter, we can integrate ( 10 ) numerically to obtain $f(0, \alpha)$. The result for $\alpha=10,15$ and 20 deg . is given in Table 2, below.

TABLE 2

| $\stackrel{0}{(\mathrm{deg} .)}$ | $\begin{gathered} \frac{180}{\pi} f\left(0,10^{\circ}\right) \\ (\text { deg. }) \end{gathered}$ | $\begin{gathered} \frac{180}{\pi} f\left(0,15^{\circ}\right) \\ (\text { deg. }) \end{gathered}$ | $\begin{gathered} \frac{180}{\pi} f\left(\theta, 20^{\circ}\right) \\ \quad \text { (deg.) } \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| 180 | - $-\infty$ | $-\infty$ | $-\infty$ |
| 170 | $14 \cdot 10$ | $10 \cdot 21$ | $6 \cdot 27$ |
| 160 | $16 \cdot 62$ | $16 \cdot 92$ | $16 \cdot 83$ |
| 150 | $15 \cdot 57$ | $17 \cdot 59$ | $18 \cdot 87$ |
| 140 | $14 \cdot 34$ | $17 \cdot 02$ | 18.99 |
| 130 | $13 \cdot 12$ | $16 \cdot 09$ | $18 \cdot 44$ |
| 120 | $12 \cdot 00$ | $15 \cdot 08$ | $17 \cdot 62$ |
| 110 | $10 \cdot 96$ | 14.05 | $16 \cdot 66$ |
| 100 | $10 \cdot 01$ | $13 \cdot 03$ | $15 \cdot 65$ |
| 90 | $9 \cdot 10$ | $12 \cdot 02$ | 14.58 |
| 80 | $8 \cdot 24$ | 11.02 | $13 \cdot 48$ |
| 70 | $7 \cdot 40$ | $9 \cdot 99$ | $12 \cdot 31$ |
| 60 | $6 \cdot 56$ | $8 \cdot 94$ | $11 \cdot 07$ |
| 50 | $5 \cdot 71$ | $7 \cdot 83$ | $9 \cdot 70$ |
| 40 | $4 \cdot 83$ | $6 \cdot 64$ | $8 \cdot 13$ |
| 30 | $3 \cdot 87$ | $5 \cdot 25$ | $6 \cdot 19$ |
| 20 | $2 \cdot 76$ | $3 \cdot 54$ | $4 \cdot 09$ |
| 10 | $1 \cdot 38$ | 1.71 | 1.96 |
| 0 | 0 | 0 | 0 |
| - 10 | $-1.18$ | - 1.41 | - 1.63 |
| - 20 | - $2 \cdot 08$ | $-2 \cdot 64$ | - 3.06 |
| - 30 | - 2.99 | - $3 \cdot 79$ | - $4 \cdot 40$ |
| - 40 | $-3 \cdot 86$ | - $4 \cdot 88$ | - $5 \cdot 70$ |
| - 50 | - $4 \cdot 73$ | $-5.92$ | -6.99 |
| - 60 | - $5 \cdot 61$ | - $7 \cdot 09$ | -8.30 $-\quad 9.67$ |
| - 70 -80 | - 6.53 | - $8 \cdot 22$ | - $9 \cdot 67$ |
| - 80 -90 | -7.52 -8.60 | P -9.51 -10.83 | 1 11.11 -12.70 |
| -90 | -8.60 -9.81 | - 10.83 -12.39 | - 12.70 -14.46 |
| - 100 | -9.81 -11.21 | -12.39 -14.11 | - $14 \cdot 46$ |
| - 110 | - 11.21 | - $14 \cdot 11$ | - 16.50 |
| - 120 | - 12.89 | - 16.25 | - 18.92 |
| - 130 | - $15 \cdot 00$ | - 18.83 | - 21.94 |
| - 140 | - 17.80 | - 22.34 | - 25.92 |
| - 150 | - 21.87 | - 27.30 | - 31.62 |
| - 160 | - 28.67 | - $35 \cdot 59$ | - 40.98 |
| - 170 | $-44 \cdot 47$ | - 53.91 | - $62 \cdot 11$ |
| - 180 | $-\infty$ | - $\infty$ | - $\infty$ |

The $\chi$ corresponding to (8) is then

$$
\begin{equation*}
f(\theta, \alpha)+\frac{a-b}{\pi}(1-\cos \theta) \log \tan \frac{1}{2} \theta-\frac{1}{2}(a+b) \sin \theta . \tag{11}
\end{equation*}
$$

The "conditions (7) " of R. \& M. $2112^{1}$ are

$$
\left.\begin{array}{r}
\frac{1}{2} \pi-\sin 2 \alpha \log \sin \alpha-\left(\frac{1}{2} \pi-\alpha\right) \cos 2 \alpha-\frac{1}{2} \sin 2 \alpha-\frac{1}{2} \pi a-\frac{1}{2} \pi b=0 \\
-\sin ^{2} \alpha(1+2 \log \sin \alpha)-\left(\frac{1}{2} \pi-\alpha\right) \sin 2 \alpha+2 a-2 b=0  \tag{12}\\
\int_{0}^{2 a} \log \sin \frac{1}{2} \theta-d \theta+2 k \pi+a \pi+b \pi=0
\end{array}\right\} .
$$

For $\alpha=10$ deg. these give $a=0 \cdot 3190, b=0 \cdot 1180, k=-0 \cdot 0659$. In Table 3

$$
\frac{1}{2} e^{k} \frac{d x}{d \theta}=e^{k} q_{0}^{-1} \sin \theta \cos \chi
$$

and

$$
\frac{1}{2} e^{k} \frac{d y}{d \theta}=e^{k} q_{0}^{-1} \sin \theta \sin \chi
$$

are tabulated. Both become zero at $\theta=-\pi$, since $q_{0}$ is finite there; but at $\theta=+\pi$ they are indeterminate, since $\sin \theta / q_{0}$ tends to a finite limit and $\chi$ to infinity. We integrate starting from the trailing edge, and go all the way to the leading edge on the lower surface, but stop short (owing to the indeterminacy) on the upper surface at $\theta=170 \mathrm{deg}$. The value at 180 deg . is of course that at -180 deg . The leading edge being $\left(x_{1}, y_{1}\right)$, we reduce, translate and rotate by the formulae

$$
\begin{equation*}
X=1-\frac{x x_{1}+y y_{1}}{x_{1}^{2}+y_{1}^{2}}, \quad Y=\frac{y x_{1}-y x_{1}}{x_{1}^{2}+y_{1}^{2}}, \quad . \quad . \quad . \quad . \tag{13}
\end{equation*}
$$

to make the chord the line joining $X=0, Y=0$ to $X=1, Y=0$. The velocity-distributions at incidence $0,5,10$ and 15 deg. are then calculated. The true chord is found to be $3 \cdot 792$, so these correspond to $C_{L}$ 's of $0,0 \cdot 578,1 \cdot 151$ and $1 \cdot 715$.

To obtain the sink effect due to sucking away four times the velocity at infinity through a slot of width 0.004 chords, we must have $2 \pi m=0.016 .3 \cdot 792$, so that $\frac{1}{4} m=0.002414$. The expression

$$
0 \cdot 002414 q_{0} \sec ^{2} \frac{1}{2} \theta
$$

is shown in the last column.

In Fig. 3 the shape and velocity-distributions are shown, with the sink effect put in (for $C_{L}=1.715$ only) as a dotted line, and a second dotted line indicating sink effect due to twice as much suction. The results are encouraging.

TABLE 3

| $\begin{gathered} 0 \\ (\mathrm{deg} .) \end{gathered}$ | $\frac{1}{2} e^{2} \frac{d x}{d \bar{\theta}}$ | $\frac{1}{2} e^{k} \frac{d y}{d \theta}$ | $\frac{27}{\pi} e^{k} x$ | $\frac{27}{x} e^{k} y$ | X | $Y$ | $q_{0}$ | $q_{5}{ }^{\circ}$ | $q_{10}{ }^{\circ}$ | $q_{15}$ | $\begin{aligned} & 0 \cdot 002414 \\ & q_{0} \sec ^{2} \frac{1}{2} 0 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 180 | - | - | $30 \cdot 507$ | $-0.791$ | 0 | 0 | O | 0.891 | $1 \cdot 772$ | $2 \cdot 641$ | $\infty$ |
| 170 | $0 \cdot 2379$ | $0 \cdot 1354$ | $29 \cdot 897$ | $-0.582$ | $0 \cdot 0202$ | $0 \cdot 0063$ | 0.594 | $1 \cdot 183$ | 1.763 | $2 \cdot 330$ | $0 \cdot 188$ |
| 160 | $0 \cdot 3297$ | 0. 1515 | $29 \cdot 049$ | $-0 \cdot 142$ | $0 \cdot 0483$ | $0 \cdot 0200$ | 0.882 | $1 \cdot 315$ | 1.738 | $2 \cdot 148$ | 0.070 |
| 150 | 0.4274 | 0. 1415 | 27.913 | +0.301 | $0 \cdot 0859$ | $0 \cdot 0336$ | 1.039 | $1 \cdot 374$ | 1-698 | $2 \cdot 008$ | 0.038 |
| 140 | $0 \cdot 5216$ | 0. 1189 | $26 \cdot 489$ | 0.694 | 0.1329 | $0 \cdot 0452$ | 1-125 | $1 \cdot 390$ | $1 \cdot 644$ | 1.887 | $0 \cdot 024$ |
| 130 | $0 \cdot 6096$ | $0 \cdot 0871$ | $24 \cdot 789$ | $1 \cdot 005$ | 0. 1888 | $0 \cdot 0540$ | 1-165 | $1 \cdot 378$ | 1.581 | 1.772 | 0.017 |
| 120 | $0 \cdot 6880$ | $0 \cdot 0503$ | $22 \cdot 841$ | $1 \cdot 212$ | $0 \cdot 2528$ | $0 \cdot 0591$ | 1-175 | $1 \cdot 348$ | $1 \cdot 511$ | 1-662 | $0 \cdot 012$ |
| 110 | 0.7549 | + $0 \cdot 0125$ | $20 \cdot 673$ | $1 \cdot 306$ | $0 \cdot 3239$ | 0.0603 | $1 \cdot 165$ | 1-306 | 1.436 | $1 \cdot 556$ | $0 \cdot 008$ |
| 100 | $0 \cdot 8069$ | $-0.0221$ | $18 \cdot 326$ | $1 \cdot 290$ | $0 \cdot 4008$ | $0 \cdot 0578$ | $1 \cdot 142$ | $1 \cdot 257$ | $1 \cdot 361$ | $1 \cdot 456$ | $0 \cdot 005$ |
| 90 | $0 \cdot 8405$ | -0.0502 | $15 \cdot 850$ | $1 \cdot 180$ | $0 \cdot 4818$ | $0 \cdot 0521$ | $1 \cdot 112$ | $1 \cdot 204$ | $1 \cdot 288$ | $1 \cdot 361$ | -_ |
| 80 | 0.8527 | -0.0689 | $13 \cdot 304$ | $0 \cdot 998$ | $0 \cdot 5650$ | $0 \cdot 0440$ | $1 \cdot 079$ | $1 \cdot 154$ | 1-220 | $1 \cdot 277$ |  |
| 70 | $0 \cdot 8393$ | -0.0768 | $10 \cdot 759$ | $0 \cdot 777$ | $0 \cdot 6482$ | $0 \cdot 0346$ | 1.044 | $1 \cdot 103$ | 1-155 | $1 \cdot 197$ |  |
| 60 | 0.7977 | $-0.0738$ | 8-297 | $0 \cdot 548$ | $0 \cdot 7287$ | $0 \cdot 0250$ | $1 \cdot 012$ | 1.059 | $1 \cdot 099$ | $1 \cdot 129$ |  |
| 50 | 0.7259 | $-0.0618$ | $6 \cdot 003$ | $0 \cdot 343$ | $0 \cdot 8037$ | $0 \cdot 0163$ | 0.985 | 1.021 | $1 \cdot 050$ | $1 \cdot 070$ | - |
| 40 | $0 \cdot 6235$ | -0.0446 | $3 \cdot 972$ | $0 \cdot 183$ | $0 \cdot 8700$ | $0 \cdot 0094$ | 0.963 | $0 \cdot 990$ | $1 \cdot 009$ | $1 \cdot 021$ | - |
| 30 | $0 \cdot 4934$ | $-0.0261$ | $2 \cdot 290$ | $0 \cdot 077$ | $0 \cdot 9251$ | $0 \cdot 0045$ | 0.948 | 0.966 | 0.977 | 0.981 |  |
| 20 | $0 \cdot 3405$ | $-0.0113$ | 1.034 | $0 \cdot 022$ | $0 \cdot 9661$ | $0 \cdot 0016$ | 0.939 | 0.950 | 0.954 | $0 \cdot 950$ | - |
| 10 | $0 \cdot 1735$ | $-0.0028$ | $0 \cdot 261$ | $+0 \cdot 003$ | 0.9915 | $0 \cdot 0003$ | 0.937 | 0.941 | 0.937 | $0 \cdot 927$ | - |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0.936 | 0.932 | 0.922 | $0 \cdot 904$ |  |
| -10 | $0 \cdot 1733$ | $-0 \cdot 0027$ | $0 \cdot 261$ | $-0.003$ | 0.9915 | $0 \cdot 0001$ | 0.938 | 0.928 | $0 \cdot 910$ | $0 \cdot 885$ | - |
| -20 | $0 \cdot 3394$ | -0.0108 | 1.033 | -0.022 | $0 \cdot 9661$ | +0.0002 | 0.942 | 0.924 | 0.899 | $0 \cdot 867$ |  |
| $-30$ | $0 \cdot 4917$ | -0.0226 | $2 \cdot 284$ | -0.071 | $0 \cdot 9251$ | -0.0004 | 0.951 | 0.926 | 0.893 | $0 \cdot 853$ | - |
| -40 | 0.6242 | $-0.0362$ | $3 \cdot 963$ | $-0.159$ | $0 \cdot 8700$ | $-0.0018$ | 0.962 | 0.928 | 0.887 | 0.839 | - |
| --50 | $0 \cdot 7327$ | -0.0494 | $6 \cdot 005$ | -0.288 | $0 \cdot 8030$ | $-0 \cdot 0043$ | 0.977 | 0.933 | 0.883 | $0 \cdot 826$ | -- |
| --60 | $0 \cdot 8143$ | -0.0600 | $8 \cdot 332$ | $-0.453$ | $0 \cdot 7267$ | $-0 \cdot 0078$ | 0.992 | 0.939 | 0.878 | $0 \cdot 810$ | --- |
| - 70 | $0 \cdot 8670$ | $-0.0663$ | $10 \cdot 862$ | $-0.643$ | $0 \cdot 6436$ | -0.0118 | 1.013 | 0.947 | $0 \cdot 874$ | $0 \cdot 795$ | - |
| -80 | 0.8909 | $-0.0667$ | $13 \cdot 505$ | - 0.845 | 0. 5569 | -0.0162 | $1 \cdot 034$ | 0.953 | 0.866 | $0 \cdot 773$ |  |
| -90 | 0.8868 | $-0.0607$ | $16 \cdot 179$ | $-1.037$ | $0 \cdot 4691$ | $-0.0202$ | 1.054 | 0.958 | 0.855 | $0 \cdot 745$ |  |
| --100 | 0.8561 | -0.0491 | 18.800 | $-1.203$ | $0 \cdot 3831$ | -0.0234 | $1 \cdot 075$ | 0.959 | 0.836 | $0 \cdot 707$ | $0 \cdot 005$ |
| -110 | $0 \cdot 8016$ | -0.0323 | $21 \cdot 292$ | $-1 \cdot 327$ | $0 \cdot 3014$ | -0.0254 | 1.096 | 0.955 | $0 \cdot 808$ | $0 \cdot 654$ | $0 \cdot 008$ |
| $-120$ | $0 \cdot 7255$ | $-0.0123$ | $23 \cdot 588$ | $-1 \cdot 394$ | $0 \cdot 2261$ | -0.0256 | $1 \cdot 117$ | 0.944 | $0 \cdot 764$ | $0 \cdot 578$ | 0.012 |
| -130 | $0 \cdot 6311$ | $+0.0090$ | $25 \cdot 626$ | $-1 \cdot 399$ | $0 \cdot 1594$ | -0.0241 | $1 \cdot 137$ | 0.920 | $0 \cdot 693$ | 0.467 | $0 \cdot 017$ |
| -140 | 0.5212 | 0.0292 | $27 \cdot 359$ | $-1 \cdot 341$ | 0.1027 | -0.0207 | $1 \cdot 154$ | 0.873 | $0 \cdot 586$ | 0. 294 | $0 \cdot 025$ |
| -150 | 0.3986 | $0 \cdot 0461$ | $28 \cdot 741$ | $-1.227$ | $0 \cdot 0575$ | $-0.0158$ | $1 \cdot 167$ | 0.783 | $0 \cdot 393$ | 0 | 0.041 |
| -160 | 0. 2661 | $0 \cdot 0568$ | 29.740 | $-1.071$ | $0 \cdot 0249$ | -0.0098 | $1 \cdot 176$ | 0.591 |  | 0.591 | $0 \cdot 093$ |
| - 170 | 0. 1250 | $0 \cdot 0571$ | $30 \cdot 329$ | $-0.897$ | $0 \cdot 0058$ | $-0.0036$ | 1-183 |  | 1.183 | $2 \cdot 357$ | $0 \cdot 376$ |
| -180 | 0 | 0 | $30 \cdot 507$ | $-0.791$ | 0 | 0 | $1 \cdot 185$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |

Innumerable variations on the velocity-distribution (8) can be made. In Fig. 4 the result of modifying the lower surface for low drag is shown. We take

$$
\log q_{0}=\left\{\begin{array}{ll}
\log \cos \frac{1}{2} \theta & (0 \leqslant \theta \leqslant \pi)  \tag{14}\\
-\log \cos \left(\frac{1}{2} \theta-\alpha\right) & (2 \alpha \leqslant \theta \leqslant \pi) \\
+a-b \cos \theta+x & (0 \leqslant \theta \leqslant \pi) \\
+x & (\pi \leqslant \theta \leqslant 2 \pi-\beta) \\
+a-b+x & (-\beta \leqslant \theta \leqslant 0)
\end{array}\right\}
$$

This has a slot at $-\beta$, on the lower surface, where the boundary layer is sucked away, as well as the fundamental leading-edge slot. Choosing $\alpha=20$ deg., $\beta=40$ deg., we obtain, by methods exactly similar to those used before, an aerofoil 13 per cent. thick, whose properties are shown in Fig. 4.

Again, in Fig. 5 is shown the result of a more determined effort to combine high maximum lift, low drag and relatively low thickness. By taking

$$
\log q_{0}=\left\{\begin{array}{ll}
\log \cos \frac{1}{2} \theta & (0 \leqslant \theta \leqslant \pi) \\
-\log \cos \left(\frac{1}{2} \theta-\alpha\right) & (2 \alpha \leqslant \theta \leqslant \pi) \\
+k+a-\frac{1}{4} \cos \theta & \left(\frac{1}{2} \pi \leqslant \theta \leqslant \pi\right) \\
+k+a(1-\cos \theta) & \left(0 \leqslant \theta \leqslant \frac{1}{2} \pi\right) \\
+k+b & \left(\pi \leqslant \theta \leqslant \frac{3}{2} \pi\right) \\
+k+b(1-\cos \theta) & \left(\frac{3}{2} \pi \leqslant \theta \leqslant 2 \pi\right)
\end{array}\right\}
$$

the adverse velocity gradient is kept down over the forward part of the aerofoil, and at low $C_{L}$ 's there is a favourable one up to 0.5 chord on the lower surface and 0.35 chord on the upper surface. The aerofoil is found to be only $14 \cdot 2$ per cent. thick: yet the velocity-distribution with sink effect at $C_{L}=2.95$ augurs a maximum lift not far from this.

Mathematical Note.-The spiral at the leading edge in the aerofoils of this section is not the usual logarithmic (or " equiangular ') spiral $\chi=A \log s$, but one of the form $\chi=A(\log s)^{2}$. This is due to the infinite jump in $\log q_{0}$ there.

Conclusions.-Though nothing can be laid down for certain without experimental evidence, all the indications of theory are that leading-edge suction has a part to play in the development of high-speed aircraft. The aerofoil of Fig. 3 is to be tested in the National Physical Laboratory 4 ft . tunnel, and one similar to that of Fig. 2 in the $13 \mathrm{ft} . \times 9 \mathrm{ft}$. one, and this will greatly increase our information.

Acknowledgment.-My thanks are due to Mrs. N. A: Lighthill for the entire design of the acrofoil of Fig. 5.

## REFERENCES




Fig. 1. The Aerofoil of R. \& M. 21121, Fig. 5. Velocities over Forward Portion. Thickness 13 per cent.


Fig. 2. Supersonic Aerofoil, Thickness $5 \cdot 4$ per cent.

Fig. 3.-Thin Aerofoil with High Maximum Lift, Thickness 8.6 per cent.


Fig. 4. Thickness 13 per cent. Upper Surface Velocities Shown; Lower Surface Flat up to Slot at $C_{L}=0$.


Fig. 5. Thickness $14 \cdot 2$ per cent.

## Publications of the

 Aeronautical Research CommitreeTECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COMMITTEE－

1934－35 Vol．I．Aerodynamics．40s．（40s．8d．）
Vol．II．Seaplanes，Structures，Engines，Materials，etc． 40s．（40s．8d．）
1935－36 Vol．I．Aerodynamics．30s．（30s．7d．）
Vol．II．Structures，Flutter，Engines，Seaplanes，etc． 30s．（30s．7d．）
1936 Vol．I．Aerodynamics General，Performance， Airscrews，Flutter and Spinning． 40s．（40s．9d．）
Vol．III．Stability and Control，Structures，Seaplanes， Engines，etc．50s．（50s．rod．）
1937 Vol．I．Aerodynamics General，Performance， Airscrews，Flutter and Spinning． 40s．（40s．9d．）
Vol．III．Stability and Control，Structures，Seaplanes， Engines，etc．60s．（6is．）
ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COMMITTEE－

$$
\begin{aligned}
& \text { 1.933-34 } \quad \text { Is. } 6 d \text {. ( } 1 s_{0} 8 d \text { ) } \\
& \text { 1934-35 Is. 6d. ( } 1 s_{0} 8 d_{0} \text { ) } \\
& \text { April 1, } 1935 \text { to December } 3 \text { r, 1936. } 4 \text { s. ( } 4 \text { s. } 4 \text { d.) } \\
& 1937 \text { 2s. (2s. 2d.) } \\
& x 938 \text { Is. } 6 d \text { 。( } 1 s \text { 。8d.) }
\end{aligned}
$$

INDEXES TO THE TECHNICAL REPORTS OF THE ADVISORY COMMITTEE ON AERONAUTICS－

December I，1936－June 30，I． 939
Reports \＆Memoranda No．1850．is． 3 d．（ $1 s_{0} 5 d$ ．）
July 1，r939－June 30， 1945
Reports 8 c Memoranda No．1950．is．（is．2d．）
Prices in brackets include posiage．
Obtainable from

## His Majesty＇s Stationery Office

London W．C． $2:$ York House，Kingsway
［Post Orders－P．O．Box No．569，London S．E．r．］

Edinburgh 2：I3A Castle Street
Cardiff：il St．Andrew＇s Crescent or through any bookseller．

