

A General Treatment of Static Longitudinal Stability with Propellers, with Application to Single- Engined Aircraft
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# A General Treatment of Static Longitudinal Stability with Propellers, with Application to Single-Engined Aircraft 

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Summary.-A general method of treatment of stick-fixed static longitudinal stability with propellers is given, distortion and compressibility effects being neglected.

Model full-throttle data on some single-engined fighters are analysed for the flaps-up condition to establish a basis of estimation of effect of propeller on stability for this type of design.

The general effect of propellers on manœuvre point, more particularly the effect on $H_{m}-K_{n}$, is considered in an appendix.

## Conclusions

(1) The method given of stability analysis for single or multi-engined aeroplanes should prove simpler than earlier methods.
(2) From analysis of model tests on single-engined fighters tentative empirical factors have been obtained for estimating the full throttle stability in terms of that without propeller, for the flaps-up condition :-
(a) To estimate values of $C_{b}$ (for aeroplane less tail) it seems sufficient merely to add the appropriate component of force on the propeller, calculated as if the propeller were acting alone, to the $C_{L}$ without propeller.
(b) The model results indicate that, excluding the effect of thrust moment, the stability without tail is better at full throttle than for $T_{c}=0$ over an incidence range including normal cruise and climb. This favourable effect of full throttle is attributed mainly to change of wing $C_{m 0}$ due to velocity increase in the slipstream and may be as much as $0.04 \bar{c}$ in neutral point position for climbing flight. It may be estimated very roughly by the method given in section 4.2 , which expresses the effect as an equivalent change of thrust-line height: viz.

$$
\text { Effective } z_{p}-\text { actual } z_{p} \bumpeq \frac{8}{\pi} \times \frac{S_{s} c_{s}}{2 D^{2} c} \times C_{m 0 s}
$$

(c) For slope of tail-lift curve we suggest the multiplying factor $1+1.5 T_{c}$ as giving the effect of the slipstre 2 m
(d) For downwash derivative at the tail the data give

$$
\frac{(1-d \varepsilon / d \alpha)_{\text {Paul tirotile }}}{(1-d \varepsilon / d \alpha)_{T c=0}} \bumpeq 1-6 \cdot 2 T_{c}
$$

Taken in conjunction with the result of Ref. 2 this gives

$$
\frac{(1-d \varepsilon / d \alpha)_{\text {Full tirottle }}}{(1-d \varepsilon / d \alpha)_{\text {No propeller }}} \simeq\left(1-1.4 \frac{d N c}{d \theta}\right)\left(1-6.2 T_{c}\right)
$$

These formulae should not be used for values of $T_{c}$ greater than about $0 \cdot 1$.
The effect on stability of downwash change due to the propeller is very much greater than the effect of variation of the velocity factor $R$ from unity.

[^0](3) The algebra of the Appendix shows that at high speed the difference of effects of propeller on manœuvre and stability margins should be small. At all speeds it will be algebraically greater for large than for small aircraft of the same geometry and the difference will increase with reduction of speed, at constant throttle.

When manœuvre point is required from model tests, these should be made at a number of values of $T_{c}$, the same values being taken at all incidences instead of using single $T_{c}$-values or non-overlapping $T_{c}$-ranges at the different incidences.

1. Introduction.-Attempts have been made over a period of many years to establish general methods of estimation of the effect of propellers on longitudinal stability for multi-engined aircraft.

Bryant and McMillan (R. \& M. 2310 ${ }^{1}$ ) tackled the problem for the twin-engined aircraft, by carrying out a systematic programme of tests in the National Physical Laboratory Duplex Wind Tunnel on a model having the general proportions of the Blenheim. Among the quantities varied were tail span and height and propeller blade angle. Measurements of lift, drag and pitching moments were made over a range of values of wing incidence $\alpha$ and propeller thrust coefficients $T_{c}$ for the model with and without tail, in the former case with the tail set at various angles. In addition, to give an idea of the physical nature of the slipstream a number of total-head surveys were made in the tailplane region for various combinations of $\alpha$ and $T_{c}$.

From these experimental results is devised a method of estimating effect of propellers on stability for twin-engined aircraft, the algebra of this method being applicable to aircraft other than twin-engined, though the numerical content is not. In spite of the generality of this algebra its complication renders it difficult of application, and this report presents a simpler treatment (section 3), which includes moreover the effect of the force on the propeller normal to its axis (the so-called propeller ' fin effect').

At the same time it was felt that sufficient power-on model tests existed on single-engined aircraft to enable a tentative method of estimation of the effect of the propeller on stability to be established. This effect was investigated by the present author in Ref. 2 for single-engined aircraft with propeller at zero thrust, so that the numerical part of the present report (section 4) is an extension of Ref. 2 and utilises the same model tests.

## 2. Notation.

| $\bar{c}$ | length of wing mean chord $=$ gross area $\div$ span |
| :---: | :---: |
| $C_{m}$ | pitching-moment coefficient about centre of gravity ( $h, k$ ) of the aeroplane |
|  | pitching-moment coefficients of aeroplane less tail about the aerodynamic centre ( $\left.h_{0}, 0\right)$ without tail, the c.g. $(h, k)$ and the point $\left(h_{0}, k\right)$ respectively. We assume $C_{m 0}$ and $h_{0}$ are unaffected by the slipstream. (See however the discussion of section 4.2) |
| , $k$ | neutral point position, $H_{n}=h_{n}-h=$ c.g. margin |
| , $k$ | manœuvre point position, $H_{m}=h_{m}-h=$ manœuvre margin |
|  | static stability m |

The dimensionless co-ordinates $h, h_{0}, h_{n}, h_{m}$ and $k$ are referred to axes through the leading edge of the wing standard mean chord perpendicular to and along this chord and are ratios of actual lengths to the mean chord length $c$; $\vec{k}$ is positive if below the chord
$C_{L} \quad$ lift coefficient of aeroplane less tail assumed nearly equal to the lift coefficient of the aeroplane
$T_{c}, N_{c}=\frac{\text { propeller thrust, basic normal force }}{\rho V^{2} D^{2}}$, the normal force being measured positive upwards
$x N_{c}$ value of normal force on propeller when effects of wing and body interferences are included
$z_{p} \bar{c} \quad$ distance of the point ( $h_{0}, \bar{k}$ ) above the propeller thrust line in terms of $\bar{c}$
$x_{p} \bar{c}$ distance of the same point behind the propeller centre, measuring parallel to the thrust line
$\gamma=\frac{2 D^{2}}{S} \times z_{p}{ }^{\text {(trrust moment) }}+\frac{8}{\pi} \times \frac{S_{s} c_{s}}{S \bar{c}} C_{m_{s} s}$ (sipstrieam efecect)
$C_{m 0 s} \quad C_{m 0}$ of part of wing in slipstream, when there is no slipstream
$S_{s}$ gross wing area in slipstream
$c_{s}$ mean chord of part of wing in slipstream
$\delta=\frac{2 D^{2}}{S} \times x_{p} \times x$
$a, a_{1}, a_{2}$ lift coefficient derivatives of aeroplane less tail, tail elevators respectivelywithout propellers in all cases
$R_{w} a, R_{T} a_{1}, R_{T} a_{2}$ derivatives corresponding to $a, a_{1}, a_{2}$ but with propellers, for the steady flight condition in which $T_{c}$ varies with $\alpha$. We have assumed that the ratio of the lift derivatives of tail and elevators is the same with and without propellers
$R=R_{T} / R_{w}$
$S$ gross wing area
$\varepsilon$ mean downwash over the tailplane
$\bar{V} \quad$ tail volume coefficient, using tail arm to ( $\left.h_{0}, k\right)$
$\theta$ propeller thrust-line incidence, in radians
$\Delta$. change due to installation of propellers
$\mu_{1} \quad$ relative density coefficient $\frac{\omega}{g \dot{\rho} \times \text { tail arm }}$
Primes denote total differentiation with respect to $C_{L}$ for any given flight condition: thus $\left(\frac{R_{T}}{C_{L}}\right)^{\prime}=\frac{d}{d C_{L}}\left(\frac{R_{T}}{C_{L}}\right)$, and so on. The word' 'trim ' implies $C_{m}=0$.
3. Algebraic Treatment of Static Stability with Propellers.-The notation above and the theory which follows consider specifically the case of the single-engined aeroplane, but the method is evidently also applicable to multi-engined types.

We write for the pitching-moment coefficient at zero elevator tab angle

$$
C_{n}=C_{n ; 0}+\left(\hbar-h_{0}\right) C_{L}+k\left(C_{D 0}-C_{L}^{2} / 6\right)+\gamma T_{c}+\delta N_{c}
$$

$$
\begin{aligned}
& -R_{T} \bar{V}\left\{a_{1}\left(\alpha+\eta_{T}-\varepsilon\right)+a_{2} \eta\right\} \\
& \text { approximation, the correct ex] } \\
& \{\text { no-lift angle of aeroplane less tail) }\}
\end{aligned}
$$

From the definition of $C_{m * v}$ and $C_{w w o}$ we also have

$$
\begin{aligned}
C_{m u} & =C_{m 0}+\left(h-h_{0}\right) C_{L}+k\left(C_{D 0}-C_{L}^{2} / 6\right)+\gamma T_{c}+\delta N_{c} \\
C_{m u 0} & =C_{m 0}+k\left(C_{D 0}-C_{L}^{2} / 6\right)+\gamma T_{c}+\delta N_{c} .
\end{aligned}
$$

Differentiating (1) with respect to $C_{L}$ at constant $\eta$ and imposing the trim condition $C_{m}=0$ after differentiation gives

$$
\begin{align*}
\left(\frac{d C_{m}}{d C_{L}}\right)_{\text {trim }}= & \left(\frac{d}{d C_{L}}-\frac{1}{R_{T}} \frac{d R_{T}}{d C_{L}}\right)\left\{C_{m 0}+\left(h-h_{0}\right) C_{L}+k\left(C_{D 0}-\frac{C_{L}^{2}}{6}\right)\right. \\
& \left.+\gamma T_{c}+\delta N_{c}\right\}-\frac{\bar{V} a_{1}}{a} \frac{R_{T}}{R_{w}}\left(1-\frac{d \varepsilon}{d \alpha}\right) . \ldots \ldots \tag{2}
\end{align*} .
$$

So far we have followed the procedure of Bryant's R. \& M. 2310 . If we now write

$$
\begin{equation*}
\left(\frac{d}{d C_{L}}-\frac{1}{R_{T}} \frac{d R_{T}}{d C_{L}}\right) f=R_{T} \frac{d}{d C_{L}}\left(\frac{f}{R_{T}}\right)=R_{T}\left(\frac{f}{R_{T}}\right)^{\prime}, \text { where } f \text { is any function of } C_{L}, \text { equation } \tag{2}
\end{equation*}
$$

takes the form

$$
\begin{align*}
\frac{1}{R_{T}}\left(\frac{d C_{m}}{d C_{L}}\right)_{\operatorname{trim}}= & \left(C_{m 0}+k C_{D 0}\right)\left(\frac{1}{R_{T}}\right)^{\prime}+\left\{\left(h-h_{0}\right) \times \frac{C_{L}}{R_{T}}\right\}^{\prime}-\frac{k}{\overline{6}}\left(\frac{C_{L}{ }^{2}}{R_{T}}\right)^{\prime} . \\
& +\gamma\left(\frac{T_{c}}{R_{T}}\right)^{\prime}+\delta\left(\frac{N_{c}}{R_{T}}\right)^{\prime}-\frac{1}{R_{w}} \times \bar{V} \frac{a_{1}}{a}\left(1-\frac{d \varepsilon}{d \alpha}\right) . \tag{3}
\end{align*}
$$

This result could have been obtained more simply by dividing equation (1) by $R_{T}$. Then differentiate and impose the trim condition $C_{m}=0$. It should be clear that the differentations indicated by the dashes are complete, not partial. From a graphical standpoint if $T_{c} / R_{T}$, say, is plotted against $C_{L}$ for a particular flight condition then $\left(T_{d} / R_{T}\right)^{\prime}$ is the slope of the resulting curve.

Now the longitudinal c.g. position $h$ is certainly independent of $C_{L}$ or $T_{c}$ : if $C_{m 0}$ and $h_{0}$ are also*, then the second term of (3) becomes $\left(h-h_{0}\right)\left(C_{L} / R_{T}\right)$.

Noting that by definition of $h_{n},\left(d C_{n d} d C_{L}\right)_{\text {trim }}=0$ when $h=h_{n}$, we obtain

$$
\begin{aligned}
\left(h_{0}-h_{n}\right)\left(\frac{C_{L}}{R_{T}}\right)^{\prime}= & \left(C_{m o}+k C_{D o}\right)\left(\frac{1}{R_{T}}\right)^{\prime}-\frac{k}{6}\left(\frac{C_{L}{ }^{2}}{R_{T}}\right)^{\prime}+\gamma\left(\frac{T_{c}}{R_{T}}\right)^{\prime} \\
& +\delta\left(\frac{N_{c}}{R_{T}}\right)^{\prime}-\frac{1}{R_{w}} \bar{V} \frac{a_{1}}{a}\left(1-\frac{d \varepsilon}{\overline{d \alpha}}\right)
\end{aligned}
$$

$$
\begin{align*}
& =\left(\frac{C_{m w 0}}{R_{T}}\right)^{\prime}-\frac{1}{R_{w}} \cdot \bar{V} \frac{a_{1}}{a}\left(1-\frac{d \varepsilon}{d \alpha}\right)  \tag{4a}\\
& =\left(h-h_{n}\right)\left(\frac{C_{L}}{R_{T}}\right)^{\prime}=\vec{V} a_{2} \frac{d \eta_{\text {trim }}}{d C_{L}}
\end{align*}
$$

$$
=\left(C_{m 0}+k C_{D 0}\right)\left(\frac{1}{R_{T}}\right)^{\prime}+\left(h-h_{0}\right)\left(\frac{C_{L}}{R_{T}}\right)^{\prime}-\frac{k}{6}\left(\frac{C_{L}^{2}}{R_{T}}\right)^{\prime}+\text { etc. }
$$

$$
\begin{equation*}
=\left(\frac{C_{m m b}}{R_{T}}\right)^{\prime}-\frac{1}{R_{w v}} \vec{V} \frac{a_{1}}{a}\left(1-\frac{d \varepsilon}{d \alpha}\right) \quad . . \quad . . \quad . . \quad . \tag{4b}
\end{equation*}
$$

[^1]We may also show that

$$
\begin{equation*}
\left(\frac{d C_{m}}{d C_{L}}\right)_{\text {trim }}^{\text {due to tail }}=-R_{T}{ }^{\prime} \frac{C_{\text {miw }}}{R_{T}}-R \bar{V} \frac{a_{1}}{\bar{a}}\left(1-\frac{d \varepsilon}{d \alpha}\right) . \quad . \quad . . \quad . \tag{5}
\end{equation*}
$$

Evidently the contribution $R_{T}\left(\frac{C_{m u v}}{R_{T}}\right)^{\prime}$ to $\left(\frac{d C_{m}}{d C_{L}}\right)_{\text {trim }}$ given by equation (4b) is made up of
(a) $C_{m a v}{ }^{\prime}$, the contribution of aeroplane less tail, which is quite independent of $R_{T}$, and
(b) $-R_{T}{ }^{\prime} \frac{C_{\text {mrv }}}{R_{T}}$ due to the tail, arising from the variation of $R_{T}$ with $C_{L}$.

In full-throttle flight the term (b) is small at high speed and may have either sign, but for cruise or climb $C_{m w}$ is usually positive and $R_{T}^{\prime}$ of order $0 \cdot 3$, so that the term is stabilizing. Since the factor $R$ by which the last term of equation (5) is multiplied also increases the stability,* it is evidently true to say that for cruising or climbing flight the total effect of the slipstream factors $R_{w}$ and $R_{T}$ is stabilizing. This is illustrated by the worked example of section 5 .

We will now show how $C_{L}, R_{w}, R_{T}, T_{c}, N_{c}$ and such derivatives as $\left(\frac{1}{R_{T}}\right)^{\prime},\left(\frac{C_{L}}{R_{T}}\right)^{\prime}$, etc., are evaluated :-
(i) The first step is to find $T_{c}$ as a function of $C_{L} \dagger$ for the power condition for which the stability is required and the appropriate aeroplane weight, etc. In doing this we may often use generalized propeller charts such as those of N.A.C.A. Reports $640^{8}, 658^{9}$, but considerable errors may result if the propeller blade plan-form is unconventional, as for example in the case of blades obtained from conventional ones of larger diameter by cutting off the tips.
(ii) We then establish $C_{L}$ as a function of $\alpha$ for this same power condition. It seems sufficient for this purpose simply to increase the lift coefficient without propellers by amount $\frac{2 D^{2}}{S}\left(T_{c} \sin \theta+N_{c} \cos \theta\right) \bumpeq \frac{2 D^{2}}{S} \theta_{\text {radi }}\left(T_{c}+\frac{d N_{c}}{d \theta}\right)^{\ddagger}$ for single-engined aeroplanes: even with two or more engines this approximation may be sufficient. This gives $R_{w} a$ and hence $R_{w}$; the matter is discussed in more detail in section 4.1 below.
(iii) $R_{T}$ is in general a function of $\alpha, C_{L}, T_{c}$, and geometrical parameters such as tail height, etc. This function has not yet been evaluated though Bryant (R. \& M. 2310), Falkner ${ }^{3}$ and others have investigated the effects of several of the variables, more particularly for twin-engined types.

For a particular aeroplane with given engine boost and r.p.m., flying at a given height, $R_{T}$ may be expressed, in theory at least, as a function of either $\alpha, C_{L}$ or $T_{c}$. Since the theoretical $R_{T}$ vs. $T_{c}$ relation for a tailplane completely immersed in a slipstream of uniform velocity is $R_{T}=1+\frac{8}{\pi} \mathrm{~T}_{c}$ it is evidently convenient to use $T_{c}$ as our variable. We shall obtain later (section 4.3) an empirical relation between $R_{T}$ and $T_{c}$ for single-engined types.
(iv) $x N_{0}$ is calculated as $x \frac{d N_{c}}{d \theta} \times \theta$ where, it is suggested, $\frac{d N_{o}}{d \theta}$ should be given roughly its high-speed value (say for $J=3 \cdot 0, T_{c}=0$ ) as calculated by Rumph's method ${ }^{4}$ and $x$ the value $\mathbf{1 . 3}$; the justification for all this is given in section 4.2 below. Since $\theta$ differs from $\alpha$ merely by a known constant we thus get $\kappa N_{c}$ in terms of $\alpha$.

[^2]As soon as corresponding values of $\alpha, C_{L}, T_{c}, N_{c}, R_{T}$ have been obtained for the given flight condition we may plot the terms $\left(C_{m 0}+k C_{D 0}\right)\left(1 / R_{T}\right)$, etc., against $C_{L}$ and get the separate stability contribution of each term (except the last) on the right-hand side of equation (3) as the slope of the appropriate curve. The overall effect of these terms will best be got by plotting the sum $C_{m o d} / R_{T}$ against $C_{L}$. The evaluation of $1-d \varepsilon / d \alpha$ in the last term is made by the method of section 4.4. below.

Values of $\left(\frac{d C_{m}}{d C_{L}}\right)_{\text {trim }}, h-h_{n}, \frac{d \eta_{\text {trim }}}{d C_{L}}$ may now be obtained by use of equations (4b).
4. Analysis of Model Data on Single-engined Aeroplanes.-The available data have been analysed to give the effect of propellers on
(a) slope of the lift curve for aeroplane less tail,
(b) pitching moment for aeroplane less tail,
(c) slope of tail lift-curve,
(d) downwash derivative at the tail.

Figs. 1 to 3 are small general arrangement drawings of the three fighter models only, and Fig. 4 shows the $T_{c}$ vs. $C_{L}$ relations used in reducing the model results to the flight so-called 'fullthrottle' condition.
4.1. Slope of Lift Curve for Aeroplane less Tail.- Fig. 5 shows $C_{L}$ against $\alpha$ from tests on five models, in each case
(a) including the contribution of the forces on the propeller,
(b) subtracting this contribution*, using the basic $N_{c}$,
(c) as measured without propeller.

For the three fighter designs it appears that there is little systematic difference of lift-curve slope between (b) and (c): this is not confirmed by the curves for the experimental types Supermarine S24/37 and Folland E28/40 where the slope for condition (b) is some 5 to 10 per cent greater than for (c).

Although the reason for this difference of results between the experimental and fighter designs is not clear we shall give more weight to the latter, as we are primarily concerned with fighters, and take it that the lift coefficient with propeller is to be got from that without propeller by adding $2 D^{2} / S \times \theta \times\left(T_{c}+\right.$ basic $\left.d N_{c} / d \theta\right)$. This implies that the mutual interference on lift between propeller and wing plus body does not vary with incidence: see the comments at the foot of Fig. 1.

Note that Ref. 5 (Smelt and Davies) gives for the $\Delta C_{L}$ due to slipstream effect on the wing the expression

$$
\begin{equation*}
\left(\Delta C_{L}\right) \text { Slipstream }=\frac{\text { area of wing in slipsteaim }}{\text { total wing area }} \times s \times\left\{\lambda C_{L 0}-0.6 a_{0} \psi\right\} . \quad \ldots \quad . . \tag{6}
\end{equation*}
$$

Where $1+s$ is velocity factor at wing centre of pressure in slipstream
$C_{L 0}$ lift coefficient of part of wing in slipstream, when there is no slipstream
$a_{0}$ two-dimensional lift-curve slope
$\psi$ angle of downwash of slipstream at the wing centre of pressure:
$\lambda$ is a factor which is about unity for modern aeroplanes, whether single or multi-engined.

[^3]If we take the propeller to be 0.7 diameters ahead of the local wing centre of pressure we get $s \bumpeq 1 \cdot 8 b$, where $(1+2 b)^{2}=1+\frac{8}{\pi} T_{c}$. For $T_{c}=0 \cdot 1$ (a typical value for climb) this gives $s=0 \cdot 10$. Taking also $\frac{d C_{L 0}}{d \alpha} \bumpeq \frac{d C_{L}}{d \alpha}$ we get

$$
\frac{d}{d \alpha}\left(\Delta C_{L}\right)_{\text {slipstream }}=\frac{\text { wing area in slipstream }}{\text { total wing area }} \times 0 \cdot 1\left\{1-0 \cdot 6 \frac{a_{0}}{a} \frac{d \psi}{d \alpha}\right\} a .
$$

The term in curly brackets is of order 0.8 and so the fractional increase in lift-curve slope due to slipstream is, very roughly,

$$
0.08 \times \frac{\text { wing area in slipstream }}{\text { total wing area }} .
$$

We can now see that, for single-engined designs at least, where the fraction of wing area in the slipstream will be only of order $0 \cdot 3$, Ref. 5 would only predict some 2 to 3 per cent increase in lift-curve slope at the $C_{L}$ corresponding to $T_{c}=0 \cdot 1$.

Ignoring this increase therefore seems to be justified for single-engined aeroplanes.
The increase in lift-curve slope due to the direct forces on the propeller is of order 10 per cent for the three fighters of Fig. 5 and appears to be almost constant over a $C_{L}$-range including high speed and climb.
4.2. Pitching Moment due to Propeller Normal Force plus Slipstream Effect on Aeroplane less Tail.-These effects have been investigated for $T_{c} \bumpeq 0^{6,7}$. The main conclusions are :-
(a) The normal force on propeller alone is predictable with quite good accuracy by Rumph's method. ${ }^{4}$
(b) For the combination of propeller with aeroplane less tail on single-engined fighter designs the effective value of rate of change with incidence of propeller normal force (i.e., that which would give the total observed change in $d C_{m} / d C_{L}$ ) is of order 30 per cent more than for the propeller alone $\left(\varkappa \bumpeq 1 \cdot 3\right.$ for $\left.T_{c} \bumpeq 0\right)$. The conception of attributing all this change in $d C_{m} / d C_{L}$ to the propeller (with a factor to allow for effect of the wing on the propeller normal force, commonly supposed to arise from the upwash at the propeller caused by the wing), is, of course, not strictly correct. The effect of the propeller slipstream on the wing plus body must also be considered, in general. However, the use of the factor $x$ may be justified as an empiricism, bearing in mind that it includes the slipstream effect.

We will now discuss the effects corresponding to (a) and (b) above but at full throttle instead of $T_{c} \bumpeq 0$.
(i) Normal forces have been measured at full throttle in a good many cases and generally speaking they are somewhat greater than at $T_{c}=0$. Figs. 6a, 6b, 6c, illustrate this; all these curves have been drawn for blade angle of 50 deg at $0 \cdot 7 R$.
(ii) Figs. 7a, 7b, 7c, show the increase in $C_{75}$ due to propeller (excluding effect of thrust moment) against incidence for the aeroplane less tail, again for $T_{c}=0$ and full throttle. The propellers of these models are those to which Fig. 6 applies.

The increase of stability for the full throttle condition over that for $T_{c}=0$ is shown clearly in Fig. 7. The difference increases from roughly zero at $T_{c}=0$ to quite considerable values in the climb region, though it may change sign quickly at still higher thrusts.

Estimates of the change in $C_{m}$ of aeroplane less tail in passing from $T_{c}=0$ to full throttle have been made on the basis of the formula $\left(\Delta C_{m}\right)_{\text {full throttle }}-\left(\Delta C_{m}\right)_{r_{c}=0} \bumpeq+\frac{8}{\pi} \times \frac{S^{s} c_{s}}{S \bar{c}} \times C_{m 0 s} \times T_{c}$ (see section 2 for definitions of symbols $S_{s}, c_{s}, C_{m 0 s}$ ), values of $C_{m 0 s}$ being calculated from thin aerofoil theory, and the effect of thrust moment being ignored for the moment.

Starting with $C_{m}$, $\alpha$ curves for $T_{c}=0$ the corresponding curves were estimated for full throttle and are shown in Figs. 7. It will be seen that, up to climb $T_{c}$ 's at least, the estimated full throttle curves show some measure of agreement with experiment, and we suggest that the above method of allowing for stabilizing effect of the slipstream on the wing plus body be used until an improved one can be developed. Note that we have made no attempt to allow for any change in propeller normal force from $T_{c}=0$ to full throttle ; the method is in fact semi-empirical.

The allowance for slipstream effect is equivalent to changing the thrust line height from its true value of $z_{p}$ (distance below $\left(h_{0}, k\right)$ ) to an effective value $z_{p}+\frac{8}{\pi} \cdot \frac{S_{s} c_{s}}{2 D^{2} \bar{c}} \cdot C_{m 0 s}$ (see the expression for $\gamma$ in section 2 above).
4.3. Slope of Tail Lift-Curve.--In order to reduce scatter due to experimental errors we have investigated the tail lift-curve slope relative to its value at $T_{c}=0$, not relative to the value without propellers. A separate investigation has shown that a propeller running at $T_{c}=0$ has no systematic effect on $a_{1}$.

Fig. 8 shows the results obtained for four fighter designs, $R_{T}$ being determined as the ratio of $d C_{n} d_{\eta}$ for full throttle and $T_{c}=0$ at the same wing incidence. The scatter when $R_{t}$ is plotted against $T_{c}$ is considerable, but some of it is probably due to errors in the model results. We should expect $R_{T}$ to vary with the parameter propeller diameter $\div$ tail span: values of this ratio for the four models are :-

| Typhoon | Tempest II | F1/43 (5-blader) | F1/43 (contra-propeller) |
| :---: | :---: | :---: | :---: |
| 1.072 | 0.88 | 1.063 | $1 \cdot 046$ |

As a tentative value of $R_{T}$ to be used for estimates we suggest $1+1 \cdot 5 T_{c}$.
4.4. Downwash Derivative at the Tail.-No suitable complete model tests at full throttle have been made for more than one tail setting and so to get downwash we are forced to various subterfuges.
(a) By what we shall call the direct method we can find downwash values by assuming that the ratio $d C_{m} / d \eta \div d C_{m} / d \eta_{T}$, which can usually be determined without propellers, is the same with propellers.
(b) An indirect method whereby we can get $1-d \varepsilon / d \alpha$ is suggested by equation (5) of section 3 above.

For the Typhoon and F1/43 (5-blader) both methods may be applied but for the Tempest II $d C_{m} / d \eta \div d C_{w} d d \eta_{T}$ cannot be found and so only the indirect method is available; even then we only get $R \bar{V} \frac{a_{1}}{a}\left(1-\frac{d \varepsilon}{d \alpha}\right)$, from which $1-d \varepsilon / d \alpha$ cannot be found, $a_{1}$ being unknown. However we can find the ratio of $1-d \varepsilon / d \alpha$ for the full throttle and $T_{c}=0$ conditions, and this is plotted against $T_{c}$ in Fig. 9 for all cases. For the Typhoon and F1/43 there are differences in the value obtained by the direct and indirect methods, due partly to the difficulty of correctly reading $C_{m}$ vs. $C_{L}$ and $\varepsilon$ vs. $\alpha$ slopes when curvatures are large, as they often are at full throttle.

The turning-up of the curves at values of $T_{c}$ of order $0 \cdot 1$ may be associated with the much earlier stall of the model propeller (blade angle 50 deg ) than would occur in flight (constant r.p.m.) : we suggest the tentative relation

$$
\frac{(1-d \varepsilon / d \alpha)_{\text {fult throttle }}}{(1-d \varepsilon / d \alpha)_{T_{c}=0}}=1-6 \cdot 2 T_{c}
$$

This should hold up to values of $T_{c}$ of at least 0.1 and may therefore be applied to the climb condition.

In Ref. 2 the approximate relation

$$
\frac{(1-d \varepsilon / d \alpha)_{T_{c}=0}}{(1-d \varepsilon / d \alpha)_{\text {nop propeller }}}=1-1 \cdot 4 \frac{d N_{c}}{d \theta}
$$

was given.
Hence

$$
\frac{(1-d \varepsilon / d \alpha)_{\text {full throtle }}}{(1-d \varepsilon / d \alpha)_{\text {nop popedeler }}} \bumpeq\left(1-1 \cdot 4 \frac{d N_{o}}{d \theta}\right)\left(1-6 \cdot 2 T_{c}\right),
$$

where $d N_{c} / d \theta$ is the value for $T_{c}=0$ and $J \bumpeq 3$.
It must be emphasised that this relation has been deduced from tests on only a few singleengined, rather similar, fighter designs and must therefore be used with caution. In particular it should not be applied with values of $T_{c}$ much greater than $0 \cdot 1$.

The form of the relation is very convenient, for in conjunction with the formula $R_{T} \bumpeq 1+1 \cdot 5 T_{c}$ (section 4.3) and the method given in section 4.1 for estimating $R_{w}$ we can very quickly find

$$
\left\{R \bar{V} \frac{a_{1}}{a}\left(1-\frac{d \varepsilon}{d \alpha}\right)\right\}_{\text {full throttle }} \text { from the value of }\left\{\bar{V} \frac{a_{1}}{a}\left(1-\frac{d \varepsilon}{d \alpha}\right)\right\}_{\text {no propeller }} \text {. }
$$

5. Illustrative Example.-Consider a hypothetical single-engined fighter with the following geometrical and aerodynamic characteristics:-

$$
\begin{aligned}
& C_{m: 0}=-0 \cdot 02, h=0 \cdot 25, h_{0}=0 \cdot 20, k=-0 \cdot 1, C_{D 0}=0 \cdot 015, \frac{2 D^{2}}{S}=1 \cdot 2 \\
& \text { effective } z_{p}\left(\text { i.e., actual } z_{p}+\frac{8}{\pi} \frac{S_{s} c_{s}}{2 D^{2} \bar{c}} \times C_{m 0 \mathrm{~s}}\right)=-0 \cdot 1, x_{p}=1 \cdot 3 \\
& d N_{c} / d \theta=0 \cdot 2, x=1 \cdot 3, a=4 \cdot 0, a_{1}=3 \cdot 0, a_{2}=2 \cdot 0, \bar{V}=0 \cdot 5, \theta=\alpha-2 \mathrm{deg}
\end{aligned}
$$

No-lift angle of aeroplane less tail less propeller $=2 \mathrm{deg}, d \varepsilon / d \alpha=0.4$ and so tailplane contribution to $-d C_{m} / d C_{L}=0.225$, without propeller. Also, from the above values of $z_{p}$ and $x_{p}, \gamma=-0 \cdot 12$, $\delta=+2 \cdot 03$.

Fig. 10a shows $T_{c}$ vs $\alpha$ and $C_{L}$ vs $\alpha$ curves, the latter with and without propeller: the estimated lift curve with propeller is nearly linear in the range under consideration ( $\alpha=-2$ deg to +8 deg ).

Fig. 10b gives plots against $C_{L}$ of the terms $\left(C_{m 0}+k C_{D 0}\right)\left(1 / R_{T}\right)$, etc., and of their sum $C_{\text {miod }} / R_{T}$. Fig. 10c shows $R_{T}$ plotted against $C_{L}$.
Table 1 shows the paper-work necessary to find $-d C_{m} / d C_{L}, h-h_{n}, d \eta / d C_{L}$ and also $-\left(d C_{m} / d C_{\bar{L}}\right)_{\text {tail }},-\left(d C_{m} / d C_{L}\right)_{\text {due to propeller }}$ : the latter has been split up into three parts :-
(a) Due to direct forces on the propeller: $-\left(\Delta C_{m w}\right)^{\prime}$
(b) Due to $R$ not being unity: $(R-1) \bar{V} \frac{a_{1}}{a}\left(1-\frac{\dot{d} \varepsilon}{d \alpha}\right)_{\text {full throttle }}+R_{T}{ }^{\prime} \cdot \frac{C_{m \omega}}{R_{T}}$
(c) Due to downwash change, if $R=1: \bar{V} \frac{a_{1}}{a} \Delta\left(1-\frac{d \varepsilon}{d \alpha}\right)$.

Table 1 gives a general idea of how the calculations may be made and of the amount of work involved: Table 2 summarises the more important (asterisked) columns of Table 1. Note that variation of $R$ (from the value unity) produces stability changes which, although appreciable, are quite small compared with the effects of direct forces on and downwash due to the propeller.
6. Conclusions.-(i) The method given of stability analysis for single or multi-engined aeroplanes should prove simpler than earlier methods.
(ii) From analysis of model tests on single-engined fighters tentative empirical factors have been obtained for estimating the full throttle stability in terms of that without propeller:--
(a) To estimate values of $C_{L}$ (for aeroplane less tail) it seems sufficient merely to add the appropriate components of direct propeller forces, calculated as if the propeller were acting alone, to the $C_{L}$ without propeller.
(b) The model results indicate that without tail and excluding the effect of thrust moment the stability is better at full throttle than for $T_{c}=0$ over an incidence range including cruise and climb. This favourable effect of full throttle is attributed mainly to change of wing $C_{m 0}$ due to velocity increase in the slipstream and may be as much as $0 \cdot 04 \bar{c}$ in the neutral point position in the climb region. It may be estimated very roughly by the method given in section 4.2, which expresses the effect as an equivalent change of thrust-line height: viz.,

$$
\text { Effective } z_{p}-\text { actual } z_{p} \bumpeq \frac{8}{\pi} \times \frac{S_{s} c_{s}}{2 D^{2} \bar{c}} \times C_{m 0 s} .
$$

(c) For slope of tail lift curve we suggest the factor $1+1 \cdot 5 T_{c}$ as giving the effect of the slipstream.
(d) For downwash derivative at the tail the data give

$$
\frac{\left(1-\frac{d \varepsilon}{d \alpha}\right)_{\text {tull throttle }}}{\left(1-\frac{d \varepsilon}{d \alpha}\right)_{T_{c}=0}} \bumpeq 1-6 \cdot 2 T_{c}
$$

Taken in conjunction with the result of Ref. 2 this gives

$$
\frac{\left(1-\frac{d \varepsilon}{d \alpha}\right)_{\text {full throttle }}}{\left(1-\frac{d \varepsilon}{d \alpha}\right)_{\text {io propeller }}} \bumpeq\left(1-1 \cdot 4 \frac{d N_{c}}{d \theta}\right)\left(1-6 \cdot 2 T_{c}\right)
$$

These formulae should not be used for values of $T_{c}$ greater than about $0 \cdot 1$.
The effect on stability of downwash change due to the propeller is very much greater than the effect of variation of the velocity factor $R$ from unity.
(iii) The algebra of the Appendix shows that at high speed the difference of effects of propeller on manœuvire and stability margins should be small. At all speeds it will be algebraically greater for large than for small aircraft of the same geometry and the difference will increase with reduction of speed, at constant throttle.

When manœuvre point is'required from model tests, these should be made at a number of values of $T_{c}$, the same values being taken at all incidences instead of using single $T_{c}$-values or non-overlapping $T_{c}$-ranges at the different incidences.

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## APPENDIX

Relative Effect of Propellers on the Mancouvre Margin $H_{m}$ and on $K_{n}=-d C_{m} / d C_{L}$
We shall only consider this with $\left(\partial R_{T} / \partial \alpha\right)_{T_{c} \text { const. }}=0$.
Under these conditions the manoeuvre margin is defined by

$$
\begin{equation*}
\left(\frac{\partial C_{m}}{\partial C_{L}}\right)_{V \text { const. }}-\frac{R_{T} \bar{V} a_{1}}{2 \mu_{1}}=h-h_{m}\left(\text { or }-H_{m}\right) . \quad . \quad . \quad \ldots \tag{8}
\end{equation*}
$$

Since the speed $V$ is constant, as well as the engine power, so is $T_{c}$ and we may replace ' $V$ const.' by ' $T_{c}$ const.' in the above. Note that $J$ and the propeller blade angle are fixed during the manœuvre; we have assumed that $R_{T}$ is also fixed. The derivative $\partial C_{m} / \partial C_{L}$ is to be taken where $C_{m}=0$ and the $T_{c}$ will be that corresponding to straight flight at the speed in question.

Now from equation (1) of section 3 we get

$$
\begin{align*}
\left(\frac{\partial C_{m}}{\partial C_{L}}\right)_{T_{c} \text { const. }}= & \frac{\partial}{\partial C_{L}}\left\{C_{m 0}+\left(h-h_{0}\right) C_{L}+k\left(C_{D_{0}}-\frac{C_{L}^{2}}{6}\right)+\gamma T_{c}+\delta N_{c}\right\}_{T_{c} \text { const. }} \\
& -\frac{R_{T}}{R_{u i 1}} \bar{V} \frac{a_{1}}{a}\left(1-\frac{\partial \varepsilon}{\partial \alpha}\right)_{T_{c} \text { const. }} \quad \cdots \quad \cdots \quad \ldots \tag{9}
\end{align*}
$$

where $R_{w i}$ is the value of $R_{w}$ taken at constant $T_{c}$.
Before taking partial derivatives $C_{m}, C_{w_{0} 0}, N_{c}$ are to be expressed as functions of $C_{L}$ and $T_{c}$, $\varepsilon$ as a function of $\alpha$ and $T_{c}$. This being understood, we shall from now on omit the suffix ' $T_{c}$ const.'

Equation (9) now gives

$$
\frac{\partial C_{m}}{\partial C_{L}}=h-h_{0}-\frac{k}{3} C_{L}+\delta \frac{\partial N_{c}}{\partial C_{L}}-\frac{R_{T} \bar{V}}{R_{w 1}} \frac{a_{1}}{a}\left(1-\frac{\partial \varepsilon}{\partial \alpha}\right) .
$$

The manoeuvre margin $H_{m}$ is given by

$$
\begin{aligned}
H_{m} & =-\frac{\partial C_{m}}{\partial C_{L}}+\frac{R_{T} \bar{V} a_{1}}{2 \mu_{1}} \text { (see equation (8)) } \\
\text { i.e., } \quad H_{m}+h & =h_{0}+\frac{k}{3} C_{L}-\delta \frac{\partial N_{c}}{\partial C_{L}}+\frac{R_{T} \bar{V}}{R_{v 1}} \frac{a_{1}}{a}\left(1-\frac{\partial \varepsilon}{\partial \alpha}\right)+\frac{R_{T} \bar{V} a_{1}}{2 \mu_{1}} .
\end{aligned}
$$

So, using now suffix ' 0 ' to denote absence of propellers,

$$
\begin{equation*}
\Delta H_{m}=-\delta \frac{\partial N_{c}}{\partial C_{L}}+\bar{V} \frac{a_{1}}{a}\left[\frac{R_{T}}{R_{i \dot{1}}}\left(1-\frac{\partial \varepsilon}{\partial \alpha}\right)-\left(1-\frac{\partial \varepsilon}{\partial \alpha}\right)_{0}\right]+\frac{\bar{V} a_{1}}{2 \mu_{1}}\left(R_{T}-1\right) \ldots \tag{10a}
\end{equation*}
$$

It is more profitable to compare $\Delta H_{m}$ with $\Delta\left(d C_{m} / d C_{L}\right)=\Delta K_{n}$ than with $\Delta H_{n}$ :-

$$
\begin{equation*}
\Delta K_{n}=-\gamma \frac{d T_{c}}{d C_{L}}-\delta \frac{d N_{c}}{d C_{L}}+\frac{\bar{V} a_{1}}{a}\left[\frac{R_{T}}{R_{w}}\left(1-\frac{d \varepsilon}{d \alpha}\right)-\left(1-\frac{d \varepsilon}{d \alpha}\right)_{0}\right]+\frac{R_{T}^{\prime}}{R_{T}} C_{m w} \ldots \tag{10b}
\end{equation*}
$$

This follows from equation (4b), remembering that $R_{T}\left(C_{m w} / R_{T}\right)^{\prime}=C_{m i v}{ }^{\prime}-R_{T}{ }^{\prime} / R_{T} \times C_{m w}$. Of course $(\partial \varepsilon / \partial \alpha)_{0}=(d \varepsilon / d \alpha)_{0}$, both derivatives applying to the ' no propeller' case. If further we take $\partial N_{c} / \partial C_{L} \bumpeq d N_{c} / d C_{L}, R_{w i} \bumpeq R_{w}$ (there is some justification for these approximations) we get

$$
\begin{equation*}
\Delta H_{m} \doteq \Delta K_{n} \bumpeq \gamma \frac{d T_{c}}{d C_{L}}-\Delta \frac{a_{1}}{a} R\left(\frac{d \varepsilon}{d \alpha}-\frac{\partial \varepsilon}{\partial \alpha}\right)+\frac{\bar{V} a_{1}}{2 \mu_{1}}\left(R_{T}-1\right)-\frac{R_{T}{ }^{\prime}}{R_{T}} C_{m \omega} . \quad . \tag{10c}
\end{equation*}
$$

Of the terms on the right-hand side the first is small at high speed but may be considerable in the climb region and of either sign, according to the sign of $\gamma$. The second term is positive and will increase with reduction of speed; unfortunately the single-engined model tests analysed in this report were made with $T_{c}$ varying with $\alpha$ according to a fixed-throttle steady-flight condition and so give no data on $\partial \varepsilon / \partial \alpha$. This emphasises that to find manoeuvre point from model tests these tests should be made at a number of fixed values of $T_{c}$, the same value of $T_{c}$ being taken at all incidences instead of using single $T_{o}$-values or non-overlapping $T_{c}$-ranges at the different incidences.
$\mu_{1}$ may vary from values less than ten for very large aircraft flying at low height to more than a hundred for small fighters at high altitude ; in the first case the third term of equation (10c) is of order $0.1\left(R_{T}-1\right)$, in the second case $0.01\left(R_{T}-1\right)$; again the term is very small at high speed, where $R_{T} \bumpeq 1$, and increases as the speed falls.

The last term - $\left(R_{T}^{\prime} / R_{T}\right) C_{m w}$ is usually very small and positive at high speed, negative for climb; for the latter condition it is fairly sensitive to c.g. position, via the term ( $h-h_{0}$ ) $C_{L}$ of $C_{i m w}$. Column (S) of Table 1 gives values of $R_{T}{ }^{\prime} / R_{T} \times C_{m w}$ for the hypothetical aeroplane of section 5, which has $h-h_{0}=0.05$.

To sum up, we can say that $\Delta H_{m}-\Delta \dot{K}_{n}$ should be small but not necessarily negligible at high speed. At all speeds it will be algebraically greater for large than for similar small aircraft and the difference will increase with reduction of speed.

TABLE 1
Calculation of Full-Throttle Stability for a Hypothetical Single-engined Fighter

|  |  |  |  |  |  |  |  |  |  |  | (A) | (B) | (C) | (D) | (E) |  | (F) | (G) | (H) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\alpha}{\operatorname{deg}}$ | $\begin{gathered} \theta \\ \operatorname{deg} \end{gathered}$ | $T_{\text {c }}$ | $\underset{d}{T_{c}+}+$ | $T C_{L}$ | $\begin{gathered} C_{L} \\ \text { (No } \\ \text { prop.) } \end{gathered}$ | $\underset{\substack{\text { Full } \\ C_{L}}}{\text { Throttle }}$ |  | $R_{T}=$ <br> $1+1.5$ | $R=$ $R_{R}=$ $\overline{R_{w}}$ | $N_{c}$ | $\frac{\gamma T_{c}}{R_{T}}$. | $\frac{\delta N_{e}}{R_{T}}$ | $\frac{C_{m 0}+k C_{p 0}}{R_{T}}$ | $\frac{\left(h-h_{0}\right) C_{K}}{R_{T}}$ | $-\frac{k}{6} \frac{C_{L}{ }^{2}}{R_{T}}$ | $1-6 \cdot 2 T_{c}$ | $\begin{gathered} R \bar{V} \frac{a_{1}}{a} \\ (1-d \varepsilon / d \alpha) \\ \text { Full } \\ \text { throttle } \end{gathered}$ | $\begin{aligned} & (\mathrm{A})+(\mathrm{B}) \\ & +(\mathrm{C})+ \\ & (\mathrm{D})+(\mathrm{E}) \end{aligned}$ | $\frac{d(\mathrm{G})}{d C_{L}}$ |
| -2 | -4 | 0 | $0 \cdot 200$ | $-0.017$ | 0 | $-0.017$ | 11 | $1 \cdot 00$ | $0 \cdot 930$. | -0.014 | 0 | -0.0283 | -0:0215 | -0.0008 | 0 | $1 \cdot 00$ | $0 \cdot 1508$ | -0.0506 | $0 \cdot 141_{5}$ |
| 0 | -2 | 0.011 | $0 \cdot 211$ | -0.009 | 0.140 | $0 \cdot 131$ | $\stackrel{+}{\dot{+}}$ | $1 \cdot 016$ | 0.945 | $-0.007$ | $-0.0012$ | -0.0140 | -0.0212 | $0 \cdot 0064$ | $0 \cdot 0003$ | 0.932 | $0 \cdot 1424$ | -0.0297 | $0 \cdot 138$ |
| 2 | 0 | 0.034 | 0.234 | 0 | $0 \cdot 279$ | 0.279 | $\because$ | 1.051 | $0 \cdot 978$ | 0 | $-0.0039$ | 0 | -0.0205 | $0 \cdot 0132$ | $0 \cdot 0012$ | $0 \cdot 790$ | $0 \cdot 1251$ | $-0.0100$ | $0 \cdot 130$ |
| 4 | 2 | 0.062 | $0 \cdot 262$ | 0.011 | $0 \cdot 419$ | 0:430 | $\dot{+}$ | 1.093 | 1.018 | $0 \cdot 007$ | -0.0068 | $0 \cdot 0130$ | -0.0196 | 0.0196 | $0 \cdot 0028$ | $0 \cdot 616$ | $0 \cdot 1018$ | $+0 \cdot 0090$ | $0 \cdot 124$ |
| 6 | 4 | 0.093 | $0 \cdot 293$ | 0.024 | $0 \cdot 558$ | 0.582 | $\stackrel{3}{4}$ | 1:139 | $1 \cdot 059$ | $0 \cdot 014$ | -0.0098 | 0.0250 | -0.0189 | $0 \cdot 0256$ | 0.0049 | $0 \cdot 424$ | 0.0729 | $0 \cdot 0268$ | $0 \cdot 113$ |
| 8 | 6 | 0.125 | $0 \cdot 325$ | $0 \cdot 041$ | $0 \cdot 698$ | 0.739 | 苞 | $1 \cdot 187$ | 1-103 | $0 \cdot 021$ | -0.0126 | $0 \cdot 0359$ | -0.0181 | 0.0312 | $0 \cdot 0076$ | $0 \cdot 225$ | $0 \cdot 0403$ | $0 \cdot 0440$ | $0 \cdot 107$ |

$\stackrel{\rightharpoonup}{\omega}$

|  | (I) | (J) | (K) | (L) | (M) |  | (N) | (O) | (P) | (Q) | $(\mathrm{R})^{-}$ | (S) |  | (T) | (U) | (V) | (W) | (X) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \alpha \\ \operatorname{deg} \end{gathered}$ | $\underset{(\mathrm{H})}{R_{R} \times}$ | ${ }_{(1 \mathrm{~F})}^{(\mathrm{I})}$ | $\frac{-(J)}{R_{T} \vec{V} a_{2}}$ | $\left(\frac{C_{L}}{R_{T}}\right)$ | $R_{T} \times(\mathrm{L})$ $=\phi$ | $\frac{(\mathrm{J})}{(\mathrm{M})}$ | $\underset{-}{-\frac{1}{3} \frac{k}{3}, ~} \begin{gathered} \text { No } \\ \text { prop. } \end{gathered}$ | $\left(C_{m}\right)^{\prime}$ <br> No tail, <br> Nó prop. | $\begin{aligned} & 0.225 \\ & -(0) \end{aligned}$ | $(\mathrm{J})-$ | $R_{r}{ }^{\prime}$ | $\stackrel{(\mathrm{R})}{(\mathrm{G})} \times$ | $(\mathrm{F})+$ | $\frac{(1-d \varepsilon / d \alpha)_{f . t h}}{(1-d \varepsilon / d \alpha)_{\text {prop }}^{\text {no }}}$ | $\left[\begin{array}{c} -0 \cdot 225 \\ x \\ {[1-(T)]} \end{array}\right.$ | $\begin{gathered} 0 \cdot 225 \times \\ (\mathrm{T}) \end{gathered}$ | $\underset{\times(\mathrm{V})}{(\mathrm{R}-1)}$ | $\underset{(\mathrm{S})}{(\mathrm{W})+}$ | $\mid(\mathrm{Q})-(\mathrm{X})$ |
| -2 | $0 \cdot 1413$ | $0 \cdot 009$ | $-0.009$ | $1 \cdot 00$ | 1.00 | -0.009 | 0 | 0.050 | $0 \cdot 175$ | $-0 \cdot 166$ | 0 | 0 | 0.151 | 0.720 | $-0.063$ | $0 \cdot 162$ | -0.011 | $-0.011$ | -0.092 |
| 0 | $0 \cdot 140$ | 0.002 | $-0.002$ | 0.93 | $0 \cdot 945$ | -0.002 | 0.005 | 0.055 | $0 \cdot 170$ | -0.168 | -0.178 | $-0.0053$ | 0.137 | 0.671 | -0.674 | $0 \cdot 151$ | -0.008 | -0.013 | $-0.081$ |
| 2 | $0 \cdot 137$ | -0.012 | $+0.011$ | $0 \cdot 88$ | $0 \cdot 93$ | +0.013 | $0 \cdot 009$ | 0.059 | 0-166 | -0.178 | 0.278 | -0.0028 | $0 \cdot 122$ | 0.569 | -0.097 | $0 \cdot 128$ | -0.003 | -0.006 | $-0.075$ |
| 4 | $0 \cdot 1357$ | -0.034 | $0 \cdot 031$ | $0 \cdot 82$ | $0 \cdot 90$ | $0 \cdot 038$ | 0.014 | $0 \cdot 064$ | $0 \cdot 161$ | -0.195 | 0.292 | +0.0026 | $0 \cdot 104_{5}$ | $0.443_{5}$ | -0.125 | $0 \cdot 100$ | +0.002 | +0.005 | $-0.075$ |
| 6 | $0 \cdot 1291$ | -0.056 | $0 \cdot 049$ | $0 \cdot 76$ | $0 \cdot 87$ | $0 \cdot 064$ | 0.019 | 0.069 | $0 \cdot 156$ | -0.212 | -0.300 | $0 \cdot 0080$ | 0.081 | $0 \cdot 305$ | $-0.156$ | $0 \cdot 069$ | $0 \cdot 004$ | $0 \cdot 012$ | -0.068 |
| 8 | $0 \cdot 127$ | -0.087 | $0 \cdot 073$ | $0 \cdot 72$ | $0 \cdot 85_{5}$ | 0.102 | 0.025 | $0 \cdot 075$ | $0 \cdot 150$ | $-0.237$ | 0.316 | 0.0139 | 0.054 | 0.162 | -0.189 | 0:036 | $0 \cdot 004$ | $0 \cdot 018$ | -0.066 |

TABLE 2
Stability Data Abstracted from Table 1

| $\begin{gathered} \alpha \\ \operatorname{deg} \end{gathered}$ | To | Full throttle $C_{L}$ | $\left(-\frac{d C_{m}}{d C_{L}}\right)$ | $\frac{d \underline{\eta}}{d C_{L}}$ | $h-h_{n}$ | $\left(\begin{array}{c} -\frac{d C_{m}}{d C_{L}} \\ \text { Trim. } \\ \text { Due to tail. } \\ \text { With Prop } \end{array}\right.$ | $\left(-\frac{d C_{m}}{d C_{s}}\right)_{\text {trim }}$ Due to the Propeller |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | Direct force on prop.* | $\begin{gathered} \text { Variation } \\ \text { of } \\ R \end{gathered}$ | Downwash change | Total |
| -2 | 0 | -0.017 | 0.009 | -0.009 | -0.009 | $0 \cdot 151$ | -0.092 | -0.011 | -0.063 | $-0 \cdot 166$ |
| 0 | 0.011 | $0 \cdot 131$ | $0 \cdot 002$ | -0.002 | $-0.002$ | $0 \cdot 137$ | -0.081 | $-0.013$ | $-0.074$ | $-0.168$ |
| 2 | 0.034 | 0. 279 | -0.012 | 0.011 | 0.013 | 0.122 | -0.075 | -0.006 | -0.097 | -0.178 |
| 4 | $0 \cdot 062$ | $0 \cdot 430$ | -0.034 | $0 \cdot 031$ | $0 \cdot 038$ | 0-1045 | -0.075 | +0.005 | -0.125 | -0.195 |
| 6 | 0.093 | $0 \cdot 582$ | -0.056 | $0 \cdot 049$ | $0 \cdot 064$ | $0 \cdot 081$ | -0.068 | $0 \cdot 012$ | -0.156 | $-0.212$ |
| 8 | $0 \cdot 125$ | $0 \cdot 739$ | -0.087 | $0 \cdot 073$ | 0.102 | $0 \cdot 054$ | -0.066 | $0 \cdot 018$ | -0.189 | -0.237 |

* The favourable slipstream effect on wing + body is included with the thrust moment effect, which along with the effect of normal force on the propeller, makes up the values in this column. The value of $\frac{d C_{m}}{d C_{L}}$ or $h-h_{n}$ without propeller is $0 \cdot 175$.


Fig. 1. Typhoon.


Fig. 2. General arrangement of Tempest $I I$ model.



Fig. 4. $T_{\text {c }}$ vs. $C_{L}$ relations used to apply model results to condition of steady flight at 'full throttle ' (see Figs. 5 to 8).

* THE 5-bLADED PROPELLER IS OF SLIGHTLY (ABOUT $1.7 \%$ )

Fig. 3. Supermarine F1/43 (Spiteful).

i.e. Change in lift of model and propeller due to thelr mutual interference

Fig. 5. The mutual lift interference between propeller and aeroplane less tail for five single-engined aircraft at full throttle.


Fig. 6a. $N_{c}$ against $\theta$ for the Typhoon propeller.


Fig. 6b. $N_{c}$ against $\theta$ for the Tempest II propeller.


Fig. 6c. $\quad N_{c}$ against $\theta$ for the F1/43 5 -bladed propeller.


Fig. 7a. Typhoon with de Havilland 3-blader.


Fig. 7b. Tempest II (Centaurus) with 4-blader.


Fig. 7c. Supermarine F1/43 with 5 -blader.
Pitching moment due to propeller ' lift' force plus slipstream effect on aeroplane less tail, for three single-engined models.


Fig. 8. The slipstream factor $R_{T}$ for some single-engined fighters. ( $\alpha$ varying with $T_{c}$ according to full-throttle condition.)


Fig. 9. $(1-d \varepsilon / d \alpha)$ at full throttle relative to its value for $T_{c}=0$.


Figs. 10a and 10b. Hypothetical single-engined fighter. Full-throttle stability without tail.


Fig. 10c. $\quad R_{T}$ against $C_{L}$ for the hypothetical fighter design.

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[^0]:    * R.A.E. Report Aero. 1944, received 16th August, 1944.

[^1]:    * The effect of slipstream on wing $C_{m}$ is best included in the term $\gamma T_{c}$ (see section 4.2).

[^2]:    * Except at high speeds, $R_{T}>R_{w}$, i.e., $R>1$.
    $\dagger$ For conventional designs, in the range from dive to climb, we may ignore the small difference between $C_{L}$ (i.e., lift coefficient for aeroplane less tail) and trimmed lift coefficient.
    $\ddagger$ i.e., the lift coefficient corresponding to the direct forces on the propeller if acting alone.

[^3]:    * No such subtraction was necessary for the $\mathrm{S} 24 / 37$ and E28/40 designs which were tested with propeller supported free of the model. So for these two models we have curves (b) and (c) only.

