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Summary.—So far, little is known of the effect of camber or twist on the pressure distribution and drag of a wing flying at supersonic speeds, but with subsonic leading edges. According to the linear theory, for a subsonic leading edge, there is a singularity in the perturbation velocity component normal to the edge. Associated with this singularity is an infinite (or very large) suction over the sharp leading edge, as in subsonic flow.

The present investigation was undertaken with a view to finding the shape of a curved wing, such that the thrust loading on the leading edges, particularly near the wing tips, is removed or modified. The shapes of two groups of such wings have been found :—

(1) For the first group, when the wings are at design incidence, there are no leading-edge pressure singularities, and therefore no leading-edge thrust. The pressure difference is finite and positive everywhere on the wing, and decreases to zero on the leading edges.

(2) For wings of the second group, the leading-edge singularity is modified so that its strength increases along the edge from zero at the apex to a maximum, and then decreases to zero, after which it would become negative. The effect of additional incidence is to increase the local lift everywhere and to move the positions of maximum and zero singularity strength further downstream.

In this report, it is also shown how the shapes of wings of the second group can be determined to satisfy certain requirements with respect to camber and twist, or the magnitude of aerodynamic characteristics.

The lift, the induced drag, and the pitching-moment coefficients for some wings of triangular plan form have been calculated, and the results are shown graphically.

1. *Introduction.*—When the leading edges of a flat delta wing, at incidence, at supersonic speeds, lie within the Mach cone of the vertex, the component of the free-stream velocity, normal to a leading edge, is less than the local sonic velocity, and the leading edge becomes 'subsonic.' According to the linear theory, for a subsonic leading edge, there is a singularity in the perturbation velocity component normal to the edge. Associated with this singularity is an infinite (or very large) suction over the sharp leading edge, as in subsonic flow. The component of this suction force in the free-stream direction tends to reduce the induced drag. The present report is an account of an investigation undertaken with a view to finding the shape of a curved wing (of negligible thickness) such that the thrust loading on the leading edges is removed or modified, while, at the same time, certain requirements with respect to camber and twist, or aerodynamic properties, are satisfied. By removing the suction peaks near the leading edge of the outboard sections of the wing, the associated adverse pressure gradients are reduced, thereby reducing the tendency for the boundary layer to separate. (Some preliminary results were given in Reference 6.)

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In Ref. 2 (R. & M. 2548), the linearised differential equation for supersonic flow is solved in a special system of curvilinear co-ordinates known as hyperboloido-conal co-ordinates, and in Ref. 1, two of these solutions are applied to the case of a thin delta wing (with subsonic leading edges) in steady supersonic flow, when the local incidence varies linearly in a spanwise or chordwise direction, the induced velocity potentials being given by $\varphi = CyX$ or $\varphi = CxX$, where $X \equiv \sqrt{(x^2-k^2y^2)}$, k is the cotangent of the apex semi-angle of the surface, and C is an arbitrary constant. x is measured downstream from the apex, y is measured to starboard and z is measured vertically upwards.

In this report, some general solutions are discussed and the results are applied to determine the shapes of certain thin surfaces with swept-back leading edges, over which the induced flow is given by the velocity potentials $\varphi = Cx^2X$, $\varphi = Cy^2X$, $\varphi = Cx^3X$ or $\varphi = Cxy^2X$. The surfaces are symmetrical with respect to the zx-plane, and are set symmetrically to the wind direction, the apex pointing against the stream. The solutions are only valid if the surfaces lie wholly within the Mach cone of the apex, therefore the Mach angle μ ($= \operatorname{cosec}^{-1} M$) is greater than the apex semi-angle γ.

These solutions are combined to give the shapes of two wings, for which, at design incidence the pressure is finite at all points on the wing, and decreases to zero at the leading edges. Other combinations of these solutions, the solution $\varphi = xX$ (Ref. 1), and the solution for the flat delta wing at incidence (Refs. 1 and 2) are shown to give wings for which the pressure is finite except at the leading edges, where, in general, it becomes infinite, but such that the strength of the singularity on a leading edge increases along the edge from zero at the apex to a maximum and then decreases until a point of zero pressure is reached, after which the strength would become negative. The total lift in each case is finite.

The above solutions are further combined to give a general solution, which may be used when there are certain conditions to be satisfied.

A number of numerical examples for special values of γ and M are given, and some examples of the pressure distributions at different incidences are shown graphically. The local spanwise lift distribution, the total lift, the induced drag, and the moment coefficients, and also the variation in camber and twist, have been calculated.

The mathematical work involved is mostly self-checking; wherever possible, the formulae have been checked by using at least two methods of derivation.

2. *Method of Solution.*—The co-ordinates used are the pseudo-orthogonal co-ordinates r, μ, ν used in Refs. 1 and 2, where

$$x = \frac{\beta r \mu \nu}{hk}, \quad y = \frac{r(\mu^2 - h^2)^{1/2}(\nu^2 - h^2)^{1/2}}{h\beta}, \quad z = \frac{r(\mu^2 - k^2)^{1/2}(k^2 - \nu^2)^{1/2}}{k\beta} \qquad (1)$$

$$\left.\begin{aligned} \beta^2 &= M^2 - 1 = \cot^2 \bar{\mu} = k^2 - h^2 \\ k^2 &= \cot^2 \gamma, \quad h^2 = \cot^2 \gamma - \cot^2 \bar{\mu} \end{aligned}\right\} \qquad (2)$$

It is assumed that the surfaces all lie close to the plane $\mu = k$, (or $z = 0$), and that the induced velocities on the surface are small and equal to the induced velocities on the plane. Therefore the relation between the shape of the surface and its induced velocity potential φ is of the form

$$\frac{\partial z}{\partial x} = \frac{1}{V}\left(\frac{\partial \varphi}{\partial z}\right)_{\mu=k}, \qquad (3)$$

where V is the stream velocity.

For the linearised theory, the pressure Δp on an element of the upper surface, and the pressure coefficient C_p are given by:

$$\Delta p = -\rho V\left(\frac{\partial \varphi}{\partial x}\right)_{\mu=k} \qquad (4)$$

$$C_p = \frac{2\Delta p}{\rho V^2} = -\frac{2}{V}\left(\frac{\partial\varphi}{\partial x}\right)_{\mu=k}, \quad \ldots \quad (5)$$

where ρ is the density of the free stream.

The linearised differential equation for the induced velocity potential φ, in terms of the co-ordinates r, μ, ν, is (R. & M. 2548[2])

$$(\mu^2 - \nu^2)\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) - \left((\mu^2 - h^2)(\mu^2 - k^2)\right)^{1/2}\frac{\partial}{\partial\mu}\left[\left((\mu^2 - h^2)(\mu^2 - k^2)\right)^{1/2}\frac{\partial\varphi}{\partial\mu}\right]$$

$$-\left((\nu^2 - h^2)(k^2 - \nu^2)\right)^{1/2}\frac{\partial}{\partial\nu}\left[\left((\nu^2 - h^2)(k^2 - \nu^2)\right)^{1/2}\frac{\partial\varphi}{\partial\nu}\right] = 0 \quad \ldots \quad (6)$$

and it has been shown, in Appendix V of Ref. 2, that a solution of equation (6) can be found of the form $\varphi = r^n f(\mu, \nu)$, where $f(\mu, \nu)$ is the product of two Lamé functions of μ, ν respectively, of degree n, n being a positive integer.

A standard Lamé function of degree n, $E_n(\mu)$, can be determined in $(2n + 1)$ different ways, and belongs to one of four classes K, L, M, N (Ref. 3). Assuming that $E_n(\mu)$ has been determined, there is a second solution of Lamé's equation, defined by: (References 1 and 3)

$$F_n(\mu) = E_n(\mu)\int_\mu^\infty \frac{dt}{[E_n(t)]^2\{|(t^2 - h^2)(t^2 - k^2)|\}^{1/2}}.$$

As stated in Ref. 2, the normal solutions of equation (6) of the form

$$\varphi = r^n F_n(\mu) E_n(\nu) \quad \ldots \quad (7)$$

have the property that $\varphi \to 0$ on approaching the Mach cone $x^2 - \beta^2(y^2 + z^2) = 0$. Also they are continuous inside the cone, except possibly across the triangular region $x^2 - k^2y^2 > 0$, $x > 0$, $z = 0$. Therefore, provided the requisite boundary conditions are satisfied, a function φ defined by (7) inside the cone $x^2 - \beta^2(y^2 + z^2) = 0$, $x > 0$, and by $\varphi = 0$ elsewhere, may serve as a solution to any particular problem related to a triangular aerofoil. In order that x, y, z (*cf.* equations (1), (2)) shall be expressed, in general, by a single set of values of r, μ, ν, it is assumed that $0 \leqslant r \leqslant +\infty$, and that μ ranges from $+\infty$ to k and back, the sign of $(\mu^2 - k^2)^{1/2}$ changing from positive to negative as μ passes through the value k; and ν ranges from k to h and back, the sign of $(\nu^2 - h^2)^{1/2}$ changing from positive to negative as ν passes through the value h.

For a lifting surface of negligible thickness, we require solutions such that φ is an odd function of z, and on the plane $z = 0$ $(\mu = k)$, is of the form $\varphi = f(x, y^2)(x^2 - k^2y^2)^{1/2}$, where $f(x, y^2)$ is a rational algebraic function of x and y^2. Our solutions are therefore based on Lamé functions of the M class, that is $E_n(\mu)$ is of the form

$$E_n(\mu) \equiv M_n(\mu) = (|\mu^2 - k^2|)^{1/2}(\mu^{n-1} + a_1\mu^{n-3} + \ldots) \equiv (|\mu^2 - k^2|)^{1/2} P_n(\mu) \quad \ldots \quad (8)$$

where $\quad P_n(\mu) = \mu^{n-1} + a_1\mu^{n-3} + \ldots,$

the last term in the expansion for $P_n(\mu)$ being $a_{(n-2)/2}\mu$ or $a_{(n-1)/2}$ according as n is even or odd; and therefore $F_n(\mu)$ is of the form

$$F_n(\mu) = (|\mu^2 - k^2|)^{1/2} P_n(\mu)\int_\mu^\infty \frac{dt}{[P_n(t)]^2\{|(t^2 - k^2)^3(t^2 - h^2)|\}^{1/2}}. \quad \ldots \quad (9)$$

It has been shown in Reference 1 that

$$\lim_{\mu \to k} F_n(\mu) = \frac{1}{\beta k P_n(k)}, \quad \ldots \quad (10)$$

and that, if $\varphi_n = C_n r_n F_n(\mu) E_n(\nu)$, then

$$\lim_{\mu \to k} \varphi_n = \frac{C_n r^n E_n(\nu)}{\beta k P_n(k)}, \qquad (11)$$

and

$$\lim_{\mu \to k} \left(\frac{\partial \varphi_n}{\partial z}\right) = C_n r^{n-1} P_n(\nu) \beta k P_n(k) \int_k^\infty \frac{d}{dt}\left[\frac{1}{t[P_n(t)]^2 (t^2 - h^2)^{1/2}}\right] \frac{dt}{(t^2 - k^2)^{1/2}}. \qquad (12)$$

The general solutions for odd and even values of n are discussed in sections 3 and 4, but the solutions which follow, in sections 6 and 7, for $n = 3$ and $n = 4$, are complete in themselves, and can be read without reference to sections 3 and 4.

3. *Solutions for $n = 2N + 1$, where N is a positive integer.*—For $n = 2N + 1$, there are $(N + 1)$ M-functions of the form

$$M_{2N+1}{}^m(\mu) = (|\mu^2 - k^2|)^{1/2} P_{2N+1}{}^m(\mu), \qquad (13)$$

where $$P_{2N+1}{}^m(\mu) = \mu^{2N} + a_{1,m}\mu^{2N-2} + \ldots + a_{N,m}, \qquad (14)$$

$$m = 1, 2, \ldots (N + 1).$$

We consider the solution

$$\varphi_m = C_{2N+1} r^{2N+1} F_{2N+1}{}^m(\mu) E_{2N+1}{}^m(\nu).$$

At the plane $z = 0$, $\mu \to k$, and

$$r^2 = (x^2 - \beta^2 y^2)/\beta^2, \quad r^2\nu^2 = h^2 x^2/\beta^2. \qquad (15)$$

Hence, using equation (11), it can be shown that

$$\lim_{\mu \to k} \varphi_m = \frac{C_{2N+1} r^{2N+1} (k^2 - \nu^2)^{1/2} P_{2N+1}{}^m(\nu)}{\beta k P_{2N+1}{}^m(k)}$$

$$= \frac{C_{2N+1}}{k\beta^{2N+1} P_{2N+1}{}^m(k)} \sum_{s=0}^{N} \left[a_{s,m} (h^2 x^2)^{N-s} (x^2 - \beta^2 y^2)^s \right] (x^2 - k^2 y^2)^{1/2}$$

$$= \frac{C_{2N+1}}{k\beta^{2N+1} P_{2N+1}{}^m(k)} \sum_{s=0}^{N} \left[A_s x^{2N-2s} y^{2s} \right] (x^2 - k^2 y^2)^{1/2} \qquad (16)$$

where A_s is a function of $(\beta, h, a_{1,m}, a_{2,m}, \ldots a_{N,m})$, and $a_{0,m} = 1$.

By constructing a potential

$$\phi_{2N+1} = \sum_{m=1}^{N+1} (\lambda_m \varphi_m)$$

where the λ's are constants determined by equating corresponding coefficients, we can obtain any potential of the form (16), where the constant coefficients A_s are given, and hence, by using equations (12) and (3), we can determine the shape of the surface corresponding to the given flow.

In practice, it is simplest to determine the $(N+1)$ surfaces on which the induced velocity potentials are of the form

$$\phi_{2N+1}{}^s = D_{2N+1} x^{2N-2s+2} y^{2s-2} (x^2 - k^2 y^2)^{1/2} \qquad (17)$$

$$s = 1, 2, \ldots (N+1),$$

D_{2N+1} being a constant, and then to combine these solutions.

To determine the shape of the surface, we require the value of $\left(\frac{\partial \varphi_m}{\partial z}\right)_{\mu=k}$. Using (12),

$$\left(\frac{\partial \varphi_m}{\partial z}\right)_{\mu=k} = C_{2N+1}\, r^{2N} P_{2N+1}{}^m(\nu)\beta k P_{2N+1}{}^m(k) \times$$

$$\int_k^\infty \frac{d}{dt}\left[\frac{1}{t[P_{2N+1}{}^m(t)]^2(t^2-h^2)^{1/2}}\right]\frac{dt}{(t^2-k^2)^{1/2}}. \quad \ldots \quad (18)$$

It is shown in Appendix III that the integral in (18) can be evaluated in terms of the complete elliptic integrals of the first and second kind of modulus h/k. Hence

$$\left(\frac{\partial \varphi_m}{\partial z}\right)_{\mu=k} = \sum_{s=1}^{N+1}\left[B_s x^{2N-2s+2} y^{2s-2}\right], \quad \ldots \quad (19)$$

B_s being constant, and, using (3),

$$z_m = \frac{1}{V}\sum_{s=1}^{N+1}\left[\frac{B_s}{2N-2s+3}x^{2N-2s+3}y^{2s-2}\right] + f(y), \quad \ldots \quad (20)$$

where $f(y)$ is a (small) arbitrary function of y.

The equation of the surface whose induced velocity potential is given by (17) is

$$z = \sum_{m=1}^{N+1}(\lambda_m z_m), \quad \ldots \quad (21)$$

the λ's having been chosen to give the correct potential.

From (16),

$$\left(\frac{\partial \varphi_m}{\partial x}\right)_{\mu=k} = \frac{C_{2N+1}}{k\beta^{2N+1}P_{2N+1}{}^m(k)}\sum_{s=0}^{N}\left[A_s x^{2N-2s-1}y^{2s}\left\{\frac{x^2}{X} + (2N-2s)X\right\}\right] \quad \ldots \quad (22)$$

where $X \equiv (x^2 - k^2y^2)^{1/2}$.

The pressure coefficient for the surface (20) can be evaluated from the formula

$$C_p = -\frac{2}{V}\left(\frac{\partial \phi_{2N+1}{}^s}{\partial x}\right)_{\mu=k} = -\frac{2}{V}\sum_{m=1}^{N+1}\left[\lambda_m\left(\frac{\partial \varphi_m}{\partial x}\right)_{\mu=k}\right] \quad \ldots \quad (23)$$

4. *Solutions for $n = 2N$, where N is a Positive Integer.*—For $n = 2N$, there are N M-functions of the form

$$M_{2N}{}^m(\mu) = (|\mu^2 - k^2|)^{1/2}P_{2N}{}^m(\mu) \quad \ldots \quad (24)$$

where $$P_{2N}{}^m(\mu) = \mu^{2N-1} + b_{1,m}\mu^{2N-3} + \ldots + b_{N-1,m}\mu, \quad \ldots \quad (25)$$

$m = 1, 2, \ldots N$.

We consider the solution

$$\varphi_m = C_{2N}r^{2N}F_{2N}{}^m(\mu)E_{2N}{}^m(\nu).$$

Using (11) and (15), it can be shown that

$$\lim_{\mu\to k}\varphi_m = \frac{C_{2N}r^{2N}(k^2-\nu^2)^{1/2}P_{2N}{}^m(\nu)}{\beta k P_{2N}{}^m(k)}$$

$$= \frac{C_{2N}}{k\beta^{2N}P_{2N}{}^m(k)}\sum_{s=0}^{N-1}\left[A_s' x^{2N-1-2s}y^{2s}\right](x^2-k^2y^2)^{1/2} \quad \ldots \quad (26)$$

where A_s' is a function of β, h, $b_{1,m}$, $b_{2,m}$, $\ldots$ $b_{N-1,m}$.

By constructing a potential

$$\phi_{2N} = \sum_{m=1}^{N} (\lambda_m \varphi_m)$$

where the λ's are constants to be determined, we can obtain any potential of the form (26), where the constant coefficients A_s' are given, and hence we can determine the shape of the corresponding surface as in section 3. In particular, we can determine the N ' basic ' surfaces on which the induced velocity potentials are of the form

$$\phi_{2N}{}^{s} = D_{2N} x^{2N-2s+1} y^{2s-2} (x^2 - k^2 y^2)^{1/2} \quad \ldots \quad (27)$$

$s = 1, 2, \ldots N$, D_{2N} being a constant. These ' basic ' solutions can then be combined to give any solution of the form (26).

Using (12),

$$\left(\frac{\partial \varphi_m}{\partial z}\right)_{\mu=k} = C_{2N} \nu^{2N-1} P_{2N}{}^{m}(\nu) \beta k P_{2N}{}^{m}(k) \times$$

$$\cdot \int_k^\infty \frac{d}{dt}\left[\frac{1}{t[P_{2N}{}^{m}(t)]^2 (t^2 - h^2)^{1/2}}\right] \frac{dt}{(t^2 - k^2)^{1/2}} \cdot \quad \ldots \quad (28)$$

The value of the integral in (28) is found, in terms of the complete elliptic integrals of the first and second kind of modulus h/k, in Appendix III.

Hence $$\left(\frac{\partial \varphi_m}{\partial z}\right)_{\mu=k} = \sum_{s=1}^{N} \left[B_s' x^{2n-2s+1} y^{2s-2} \right], \quad \ldots \quad (29)$$

B_s' being a constant, and, using (3),

$$z_m = \frac{1}{V} \sum_{s=1}^{N} \left[\frac{B_s'}{2N - 2s + 2} x^{2N-2s+2} y^{2s-2} \right] + f(y) \quad \ldots \quad (30)$$

where $f(y)$ is a (small) arbitrary function of y.

The equation of the surface, whose induced velocity potential is given by (27), is

$$z = \sum_{m=1}^{N} (\lambda_m z_m), \quad \ldots \quad (31)$$

the λ's being chosen to give the correct potential.

From (26),

$$\left(\frac{\partial \varphi_m}{\partial x}\right)_{\mu=k} = \frac{C_{2N}}{k \beta^{2N} P_{2N}{}^{m}(k)} \sum_{s=0}^{N-1} \left[A_s' x^{2N-2s-2} y^{2s} \left\{ \frac{x^2}{X} + (2N - 2s - 1) X \right\} \right] \cdot \quad \ldots \quad (32)$$

The pressure coefficient for surface (30) can be evaluated from the formula

$$C_p = -\frac{2}{V} \left(\frac{\partial \phi_{2N}{}^{s}}{\partial x}\right)_{\mu=k} = -\frac{2}{V} \sum_{m=1}^{N} \left[\lambda_m \left(\frac{\partial \phi_m}{\partial x}\right)_{\mu=k} \right]. \quad \ldots \quad (33)$$

In section **5**, the solutions for $n = 1$ and $n = 2$ are quoted from Refs. **1** and **2** ; and in sections 6 and 7, the ' basic ' solutions for $n = 3$ and $n = 4$ are found. For purposes of reference, the results are tabulated in Appendix VI.

5. *Solutions for $n = 1$ and $n = 2$.*—The solutions for $n = 1$ and $n = 2$ are both given in Ref. 2, and the results, which are used later in this report, are quoted below (with a slightly different notation) :

For $n = 1$, the induced velocity potential is

$$\phi_1 = \frac{\beta V\delta}{E(\varkappa)} \nu(\mu^2 - k^2)^{1/2}(k^2 - \nu^2)^{1/2} \int_\mu^\infty \frac{dt}{(t^2 - k^2)^{3/2}(t^2 - h^2)^{1/2}}, \quad \ldots \quad (34)$$

where $E(\varkappa)$ is the complete elliptic integral of the second kind, modulus $\varkappa(= h/k)$, gives the flow past the flat delta wing, at small incidence δ, whose equation is

$$z \equiv z_1 = -\delta x + f(y) \quad \ldots \quad (35)$$

where $f(y)$ is a (small) arbitrary function of y.

On the wing,

$$(\phi_1)_{\mu=k} = \frac{V\delta}{kE(\varkappa)}(x^2 - k^2y^2)^{1/2}$$

$$\equiv \frac{V\delta}{kE(\varkappa)} X, \quad \ldots \quad (36)$$

and

$$\left(\frac{\partial \phi_1}{\partial x}\right)_{\mu=k} = \frac{V\delta}{kE(\varkappa)}\frac{x}{X}. \quad \ldots \quad (37)$$

For $n = 2$, the induced velocity potential is

$$\phi_2 = \frac{\beta^2 V\delta}{dkE(\varkappa)} \nu^2\mu\nu(\mu^2 - k^2)^{1/2}(k^2 - \nu^2)^{1/2} \int_\mu^\infty \frac{dt}{t^2(t^2 - k^2)^{3/2}(t^2 - h^2)^{1/2}} \quad (38)$$

where δ, d are constants (δ small and non-dimensional), gives the flow past the triangular surface whose equation is

$$z \equiv z_2 = -\frac{\delta}{d} f_1(\tan\gamma/\tan\bar{\mu})x^2 + f(y), \quad \ldots \quad (39)$$

where $f_1(\tan\gamma/\tan\bar{\mu}) = \dfrac{1}{2\varkappa^2 E(\varkappa)}\left[(2\varkappa^2 - 1)E(\varkappa) + (1 - \varkappa^2)K(\varkappa)\right], \quad \ldots \quad (40)$

$\varkappa^2 = h^2/k^2 = 1 - \tan^2\gamma/\tan^2\bar{\mu}$, $K(\varkappa)$, $E(\varkappa)$ are the complete elliptic integrals of the first and second kind with modulus $\varkappa$, and $f(y)$ is a (small) arbitrary function of y.

On the surface,

$$(\phi_2)_{\mu=k} = \frac{V\delta}{dkE(\varkappa)} xX \quad \ldots \quad (41)$$

and $$\left(\frac{\partial \phi_2}{\partial x}\right)_{\mu=k} = \frac{V\delta}{dkE(\varkappa)}\left(\frac{x^2}{X} + X\right). \quad \ldots \quad (42)$$

It can be shown that as $\varkappa \to 0$, $f_1 \to 0 \cdot 75$. The values of f_1 are given in Appendix II and Fig. 1.

6. *Basic Solutions for $n = 3$.*—For $n = 3$, there are two M-functions, and we assume

$$E_3^m(\mu) \equiv M_3^m(\mu) = (|\mu^2 - k^2|)^{1/2} P_3^m(\mu) \quad \ldots \quad (43)$$

where $\quad P_3^m(\mu) = \mu^2 - a_m, \quad m = 1, 2. \quad \ldots \quad (44)$

Putting $N = 1$ and $c_r = a_m$ in equation (III, 4) of Appendix III, the equation giving the two values of a_m is

$$5a_m^2 - 2(2h^2 + k^2)a_m + h^2k^2 = 0. \quad \ldots \quad (45)$$

Therefore

$$a_1 + a_2 = \frac{2(2h^2 + k^2)}{5} \quad \text{and} \quad a_1a_2 = \frac{h^2k^2}{5} \quad \ldots \quad (46)$$

We first consider the solution

$$\varphi_m = C_3 r^3 F_3^m(\mu) E_3^m(\nu), \qquad (m = 1, 2)$$
$$= C_3 r^3 (\mu^2 - k^2)^{1/2} (k^2 - \nu^2)^{1/2} (\mu^2 - a_m)(\nu^2 - a_m) \times$$
$$\int_\mu^\infty \frac{dt}{(t^2 - a_m)^2 (t^2 - k^2)^{3/2} (t^2 - h^2)^{1/2}}, \quad m = 1, 2. \qquad (47)$$

At the plane $z = 0$, $\mu \to k$, and

$$r^2 = (x^2 - \beta^2 y^2)/\beta^2, \quad r^2 \nu^2 = h^2 x^2/\beta^2. \qquad (48)$$

Hence, using equation (11), it can be shown that

$$\lim_{\mu \to k} \varphi_m = \frac{C_3 r^3 (k^2 - \nu^2)^{1/2} (\nu^2 - a_m)}{\beta k (k^2 - a_m)}$$

$$= \frac{C_3[(h^2 - a_m)x^2 + \beta^2 a_m y^2](x^2 - k^2 y^2)^{1/2}}{\beta^3 k (k^2 - a_m)} \qquad (49)$$

Our two basic solutions of the form (17) are

$$\phi_3^1 = D_3 x^2 X \text{ and } \phi_3^2 = D_3' y^2 X$$

where $X \equiv (x^2 - k^2 y^2)^{1/2}$, and D_3, D_3' are constants.

We construct a potential

$$\phi_3^s = \lambda_1 \varphi_1 + \lambda_2 \varphi_2, \quad s = 1, 2$$

where λ_1, λ_2 are constants to be determined.

It is slightly easier to construct the potential ϕ_3^2 first.

For the solution ϕ_3^2, equating the corresponding coefficients, and using (46), we find that

$$\frac{\lambda_1}{\lambda_2} = -\frac{6h^2k^2 - 5a_2k^2 - 5a_1h^2}{6h^2k^2 - 5a_1k^2 - 5a_2h^2}.$$

We therefore construct the potential

$$\phi_3^2 = 6h^2k^2(\varphi_1 - \varphi_2) - 5k^2(a_2\varphi_1 - a_1\varphi_2) - 5h^2(a_1\varphi_1 - a_2\varphi_2) \qquad (50)$$

which gives

$$(\phi_3^2)_{\mu=k} = \frac{5C_3h^2(a_1 - a_2)}{k\beta} y^2 X. \qquad (51)$$

For the solution ϕ_3^1, we construct the potential

$$\phi_3^1 = k^2\phi_3^2 - 3\beta^2h^2k^2(\varphi_1 - \varphi_2) \qquad (52)$$

which gives

$$(\phi_3^1)_{\mu=k} = \frac{5C_3(a_1 - a_2)h^2}{k\beta} x^2 X. \qquad (53)$$

The values of $\varphi_1 - \varphi_2$, $a_2\varphi_1 - a_1\varphi_2$, $a_1\varphi_1 - a_2\varphi_2$, when $\mu \to k$, are given in Appendix IV.

We choose the arbitrary constant C_3 so that

$$\frac{5C_3(a_1 - a_2)h^2}{\beta} = \frac{V\delta}{d^2E(\varkappa)}$$

where δ, d are constants, δ being small and non-dimensional, and $E(\varkappa)$ is the complete elliptic integral of the second kind, with modulus $\varkappa = h/k$. (C_3 is written in this form for the purpose of later constructions.)

where $\qquad P_4^m(\mu) = \mu(\mu^2 - a_m), \quad m = 1, 2. \qquad \qquad (61)$

Putting $N = 2$ and $d_r = a_m$ in equation (III,5) of Appendix III, the equation giving the two values of a_m is

$$7a_m^2 - (6h^2 + 4k^2)a_m + 3h^2k^2 = 0 \qquad (62)$$

therefore

$$a_1 + a_2 = \frac{6h^2 + 4k^2}{7}, \; a_1a_2 = \frac{3h^2k^2}{7} . \qquad (63)$$

We first consider the solution

$$\begin{aligned} \varphi_m &= C_4r^4F_4^m(\mu)E_4^m(\nu) \quad (m = 1, 2) \\ &= C_4r^4(\mu^2 - k^2)^{1/2}(k^2 - \nu^2)^{1/2}\mu\nu(\mu^2 - a_m)(\nu^2 - a_m) \times \\ &\int_\mu^\infty \frac{dt}{t^2(t^2 - a_m)^2(t^2 - k^2)^{3/2}(t^2 - h^2)^{1/2}} . \end{aligned} \qquad (64)$$

Using (11) and (48), it can be shown that

$$\begin{aligned} \lim_{\mu \to k} \varphi_m &= \frac{C_4r^4(k^2 - \nu^2)^{1/2}\nu(\nu^2 - a_m)}{\beta k^2(k^2 - a_m)} \\ &= \frac{C_4hx[(h^2 - a_m)x^2 + \beta^2a_my^2](x^2 - k^2y^2)^{1/2}}{\beta^4k^2(k^2 - a_m)} . \end{aligned} \qquad (65)$$

Our two basic solutions of the form (27) are $\phi_4^1 = D_4x^3X$, and $\phi_4^2 = D_4'xy^2X$, where $X = (x^2 - k^2y^2)^{1/2}$, and D_4, D_4' are constants.

We construct a potential

$$\phi_4^s = \lambda_1\varphi_1 + \lambda_2\varphi_2, \qquad s = 1, 2$$

where λ_1, λ_2 are constants to be determined.

For the solution ϕ_4^2, equating the corresponding coefficients and using (63), we find that

$$\frac{\lambda_1}{\lambda_2} = -\frac{10h^2k^2 - 7a_2k^2 - 7a_1h^2}{10h^2k^2 - 7a_1k^2 - 7a_2h^2} .$$

We therefore construct the potential

$$\phi_4^2 = 10h^2k^2(\varphi_1 - \varphi_2) - 7k^2(a_2\varphi_1 - a_1\varphi_2) - 7h^2(a_1\varphi_1 - a_2\varphi_2) \qquad (66)$$

which gives

$$(\phi_4^2)_{\mu=k} = \frac{7C_4h^3(a_1 - a_2)}{\beta^2k^2} xy^2X . \qquad (67)$$

For the solution ϕ_4^1, we construct the potential

$$\phi_4^1 = k^2\phi_4^2 - 3\beta^2h^2k^2(\varphi_1 - \varphi_2), \qquad (68)$$

which gives

$$(\phi_4^1)_{\mu=k} = \frac{7C_4h^3(a_1 - a_2)}{\beta^2k^2} x^3X . \qquad (69)$$

The values of $\varphi_1 - \varphi_2$, $a_2\varphi_1 - a_1\varphi_2$, $a_1\varphi_1 - a_2\varphi_2$, when $\mu \to k$, are given in Appendix IV.

We choose the arbitrary constant C_4 so that

$$\frac{7C_4(a_1 - a_2)h^3}{\beta^2k} = \frac{V\delta}{d^2E(\varkappa)}$$

where δ, d are constants, δ being small and non-dimensional. (*cf.* choice of C_3 in section 6.)

Therefore, from (67) and (69),

$$\left(\frac{\partial \phi_4{}^2}{\partial x}\right)_{\mu=k} = \frac{V\delta}{kd^3E(\varkappa)}\, y^2\!\left(\frac{x^2}{X} + X\right) \qquad (70)$$

and

$$\left(\frac{\partial \phi_4{}^1}{\partial x}\right)_{\mu=k} = \frac{V\delta}{kd^3E(\varkappa)}\, x^2\!\left(\frac{x^2}{X} + 3X\right). \qquad (71)$$

The values of $\frac{\partial}{\partial z}(\varphi_1 - \varphi_2)$, $\frac{\partial}{\partial z}(a_2\varphi_1 - a_1\varphi_2)$, $\frac{\partial}{\partial z}(a_1\varphi_1 - a_2\varphi_2)$, when $\mu \to k$, are given in Appendix V. Hence it can be shown that

$$\left(\frac{\partial \phi_4{}^2}{\partial z}\right)_{\mu=k} = \frac{V\delta}{d^3k^2}\left[f_8\!\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right)x^3 - f_9\!\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right)xy^2\right] \qquad (72)$$

where

$$f_8\!\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right) = [(8 - 3\varkappa^2 - 2\varkappa^4)E(\varkappa) - (1 - \varkappa^2)(8 + \varkappa^2)K(\varkappa)] \Big/ (6\varkappa^6E(\varkappa)),$$

and

$$f_9\!\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right) = [(1 - \varkappa^2)(8 - 13\varkappa^2 + 2\varkappa^4)K(\varkappa) - (8 - 17\varkappa^2 + 7\varkappa^4 - 4\varkappa^6)E(\varkappa)] \Big/ (2\varkappa^6E(\varkappa))$$

f_8, f_9 are > 0 for $0 \leqslant \varkappa \leqslant 1$, and when $\varkappa \to 0$, $f_8 \to 0{\cdot}15625$, $f_9 \to 2{\cdot}34375$.
The values of f_8, f_9 are given in Appendix II and Figs. 2 and 3.
Therefore, from (3) and (72), the velocity potential $\phi_4{}^2$ gives the flow over the surface

$$z \equiv z_{4,2} = \frac{\delta}{d^3k^2}\left(\tfrac{1}{4}f_8x^4 - \tfrac{1}{2}f_9k^2x^2y^2\right) + f(y) \qquad (73)$$

where $f(y)$ is a (small) arbitrary function of y.
Similarly

$$\left(\frac{\partial \phi_4{}^1}{\partial z}\right)_{\mu=k} = -\frac{V\delta}{d^3}\left[f_{12}\!\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right)x^3 - f_{13}\!\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right)k^2xy^2\right]. \qquad (74)$$

where

$$f_{12}\!\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right) = [(1 - \varkappa^2)(8 + 7\varkappa^2 + 12\varkappa^4)K(\varkappa) - (8 + 3\varkappa^2 + 7\varkappa^4 - 24\varkappa^6)E(\varkappa)] \Big/ (6\varkappa^6E(\varkappa))$$

and

$$f_{13}\!\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right) = [(8 - 11\varkappa^2 + \varkappa^4 + 2\varkappa^6)E(\varkappa) - (1 - \varkappa^2)(8 - 7\varkappa^2 - \varkappa^4)K(\varkappa)] \Big/ (2\varkappa^6E(\varkappa)).$$

f_{12}, f_{13} are $\geqslant 0$ for $0 \leqslant \varkappa \leqslant 1$, and when $\varkappa \to 0$, $f_{12} \to 2{\cdot}65625$, $f_{13} \to 0{\cdot}46875$.

From (3) and (74), the velocity potential $\phi_4{}^1$ gives the induced flow over the surface

$$z \equiv z_{4,1} = -\frac{\delta}{d^3}\left(\tfrac{1}{4}f_{12}x^4 - \tfrac{1}{2}f_{13}k^2x^2y^2\right) + f(y) \quad \ldots \quad (75)$$

where $f(y)$ is a (small) arbitrary function of y.

The basic solutions found in sections 5, 6, 7 will now be combined to determine the shape of a surface with swept-back leading edges, such that either:

(i) there are no pressure singularities, the pressure becoming zero on the leading edges; or

(ii) although the pressure, in general, becomes infinite on the leading edges, the strength of the pressure singularity increases from zero at the apex to a maximum value at some point on the leading edge, and then decreases to zero at a point on the leading edge further downstream.

There are five independent solutions giving surfaces of the two types considered. It will be shown how these five solutions can be combined to give the shapes of swept-back wings of type (ii), satisfying certain requirements with respect to camber and twist, or with respect to the aerodynamic characteristics.

8. *Two Wings Having no Pressure Singularities, at Design Incidence.*—By combining the two solutions given in section 6, or those given in section 7, it is possible to determine the shape of a thin wing with swept-back leading edges, which, at design incidence, has no pressure singularities, the pressure becoming zero at the leading edges.

(i) Using the basic solutions for $n = 3$, we construct the velocity potential

$$\psi_3 = \phi_3{}^1 - k^2\phi_3{}^2 = -3\beta^2h^2k^2(\varphi_1 - \varphi_2)_{n=3} . \quad \ldots \quad (76)$$

Therefore, from Appendix IV, (IV,1),

$$(\psi_3)_{\mu=k} = \frac{V\delta}{d^2kE(\varkappa)} X^3 . \quad \ldots \quad (77)$$

where $X \equiv (x^2 - k^2y^2)^{1/2}$; and

$$\left(\frac{\partial\psi_3}{\partial x}\right)_{\mu=k} = \frac{3V\delta}{d^2kE(\varkappa)} xX . \quad \ldots \quad (78)$$

Using (59) and (57), the shape of the wing is given by

$$z = z_{3,1} - k^2z_{3,2} =$$

$$-\frac{\delta}{d^2}\left[f_4\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right)x^3 - f_5\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right)k^2xy^2\right] + f(y) \quad \ldots \quad (79)$$

where $f_4 = \frac{1}{3}(f_6 + f_2)$ and $f_5 = f_7 + f_3$.

Using (5), the pressure coefficient for the upper surface of the wing (79) at design incidence is

$$C_{p0} = \frac{-6\delta}{d^2kE(\varkappa)} xX . \quad \ldots \quad (80)$$

The spanwise lift distribution $l(y)$ is obtained by integrating $-\rho V^2C_{p0}$ along the chords of the wing. Thus for all chords lying outside the Mach cones of the trailing edge,

$$l(y) = \int_{ky}^{x_1} -\rho V^2C_{p0}\,dx = 2\rho V[\psi_3(x,y)]_{z=0}$$

$$= \frac{2\rho V^2\delta}{d^2kE(\varkappa)}(x_1{}^2 - k^2y^2)^{3/2} \quad \ldots \quad (81)$$

where $x = x_1(y)$ defines the trailing edge.

For a triangular wing of maximum chord c, $x_1(y) = c$, and it can be shown that the design lift coefficient, based on area, is

$$C_{L0} = \frac{2\pi\delta}{kE(\varkappa)} \left(\tfrac{3}{4}\sigma^2\right), \quad \ldots \quad (82)$$

where $\sigma \equiv c/d$.

For a (small) additional incidence α, the pressure coefficient C_p can be found by superimposing the solution for the flat delta wing at incidence α (*see* section 5), Thus

$$\left.\begin{aligned} C_p &= \frac{-2x}{kE(\varkappa)}\left[\frac{\alpha}{X} + \frac{3\delta}{d^2}X\right] \\ \text{and} \quad C_L &= \frac{2\pi}{kE(\varkappa)}\left[\alpha + \tfrac{3}{4}\delta\sigma^2\right] \end{aligned}\right\} \quad \ldots \quad (83)$$

(ii) Using the basic solutions for $n = 4$, we construct the velocity potential

$$\psi_4 = \phi_4^1 - k^2\phi_4^2 = -3\beta^2h^2k^2(\varphi_1 - \varphi_2)_{n=4} \quad \ldots \quad (84)$$

Hence, from Appendix IV (IV,4)

$$\left(\psi_4\right)_{\mu=k} = \frac{V\delta}{d^3kE(\varkappa)} xX^3 \quad \ldots \quad (85)$$

and

$$\left(\frac{\partial\psi_4}{\partial x}\right)_{\mu=k} = \frac{V\delta}{d^3kE(\varkappa)}(X^3 + 3x^2X)$$

$$= \frac{V\delta}{d^3kE(\varkappa)}(4x^2 - k^2y^2)X \quad \ldots \quad (86)$$

Using (75) and (73), the shape of the wing is given by

$$z = z_{4,1} - k^2z_{4,2} = \frac{-\delta}{d^3}\left[\tfrac{1}{4}f_{10}\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right)x^4 - \tfrac{1}{2}f_{11}\left(\frac{\tan\gamma}{\tan\bar{\mu}}\right)k^2x^2y^2\right] + f(y) \quad \ldots \quad (87)$$

where $f_{10} = f_8 + f_{12}$, and $f_{11} = f_9 + f_{13}$.

The pressure coefficient, at design incidence, is

$$C_{p0} = -\frac{2\delta}{d^3kE(\varkappa)}(4x^2 - k^2y^2)X. \quad \ldots \quad (88)$$

For chords outside the Mach cones of the trailing edge ($x = x_1(y)$), the spanwise lift distribution is given by

$$l(y) = 2\rho V[\psi_4(x_1, y)]_{z=0}$$

$$= \frac{2\rho V^2\delta}{d^3kE(\varkappa)} x_1(x_1^2 - k^2y^2)^{3/2}. \quad \ldots \quad (89)$$

The design lift coefficient for a triangular wing of maximum chord c $[x_1(y) = c]$ is

$$C_{L0} = \frac{2\pi\delta}{kE(\kappa)} \left(\tfrac{3}{4}\sigma^3\right) \qquad (90)$$

here $\sigma = c/d$.

For the triangular wings given in this section, d is a constant proportional to the maximum chord c of the wing, but since these solutions are used in later constructions, it is convenient to write $d/c = 1/\sigma$ rather than 1.

The shape of and pressure distribution on wings (i) and (ii) for special values of γ and M, $(\sigma = 1)$ are shown in Figs. 6 and 7.

9. *Wings with the Strength of the Leading Edge Singularities Reaching a Maximum, and then Decreasing to Zero at Some Point on the Leading Edge.*—By combining, in various ways, the basic solutions for $n = 1, 2, 3, 4$, the shape of a wing, with swept-back leading edges, can be determined, such that, although the pressure, in general, becomes infinite on the leading edges, the strength of the pressure singularity increases along the edge from zero at the apex to a maximum, and then decreases to zero at some point on the edge further downstream. Three elementary solutions of this type are discussed in this section.

It is clear that there are only five independent solutions giving surfaces of the type considered in this section or in section 8. Therefore further solutions of this type, and also a general solution involving four additional arbitrary constants, are found by combining the five solutions ((i) to (v)) given in this and the previous section.

A general solution is given at the end of this section. For reference purposes, the functions $f_1, f_2, \ldots f_{13}$, which appear in the solutions, are given in Appendices I and II, and in Figs. 1 to 4, and the basic solutions $\phi_1, \phi_2, \phi_3^1, \ldots$ are tabulated in Appendix VI.

For the wings with leading-edge singularities, d is an arbitrary length, which is equal to the chordwise distance (in the free-stream direction) behind the apex, of the point of zero pressure on a leading edge. For a triangular wing, it is not, in general, proportional to the maximum chord c. In the formulae which follow, d/c is written as $1/\sigma$ (the point of zero pressure is on the wing if $\sigma \geqslant 1$, or downstream of the wing tips if $\sigma < 1$).

It is convenient, at this stage, to introduce non-dimensional co-ordinates

$$x' = x\sigma/c, \quad y' = y\sigma/c, \quad z' = z\sigma/c.$$

The elementary solutions chosen are as follows :

(iii) Using the basic solutions ϕ_1, ϕ_2 we construct the induced velocity potential

$$\Phi_2 = \phi_1 - \phi_2, \qquad (91)$$

for which

$$(\Phi_2)_{\mu=k} = \frac{V\delta c}{\sigma k E(\kappa)} (1 - x')X', \qquad (92)$$

where $X' \equiv (x'^2 - k^2 y'^2)^{1/2}$; and the pressure coefficient is

$$C_{p0} = -\frac{2}{V}\left(\frac{\partial \Phi_2}{\partial x}\right)_{\mu=k} = -\frac{2\delta}{kE(\kappa)}\left[\frac{x'(1-x')}{X'} - X'\right] \qquad (93)$$

at all points of the wing outside the Mach cones of points on the trailing edge.

On the leading edges of the wing, $X' = 0$ and $C_{p0} \to \dfrac{-(2/V)P}{(x' - k|y'|)^{1/2}}$, where P is the strength of the singularity in the axial velocity $\left(\dfrac{\partial\phi}{\partial x}\right)_{\mu=k}$. When $x' = 0$ or 1, $P = 0$, and $C_{p0} \to \dfrac{-2\delta}{kE(\varkappa)}$ and 0 respectively.

From (93)

$$P = \frac{V\delta}{kE(\varkappa)}(1 - x')\left(\frac{x'}{2}\right)^{1/2} \qquad (94)$$

and P increases from 0 to $\dfrac{V\delta}{2kE(\varkappa)}\left(\dfrac{2}{3}\right)^{3/2}$ as x' increases from 0 to $\frac{1}{3}$, and decreases to 0 as x' increases to 1.

Using (35) and (39), the shape of the wing, at design incidence, is given by

$$z' = z_1' - z_2' = \delta(f_1 x'^2 - x') + f(y') \qquad (95)$$

where $f(y')$ is a (small) arbitrary function of y'.

For chords lying entirely outside the Mach cones of the trailing edge, the spanwise lift distribution is given by

$$l(y) = \frac{2\rho V^2 c\delta}{\sigma kE(\varkappa)}(1 - x_1')X_1' \qquad (96)$$

where $x' = x_1'(y)$ defines the trailing edge, and $X_1' = (x_1'^2 - k^2 y'^2)^{1/2}$.

For a triangular wing of maximum chord c, the design lift coefficient is

$$C_{L0} = \frac{2\pi\delta}{kE(\varkappa)}(1 - \sigma)\,. \qquad (97)$$

(iv) Using the basic solutions ϕ_1, ϕ_3^1, we construct the induced velocity potential

$$\Phi_3^1 = \phi_1 - \phi_3^1 \qquad (98)$$

for which

$$(\Phi_3^1)_{\mu=k} = \frac{Vc\delta}{\sigma kE(\varkappa)}(1 - x'^2)X', \qquad (99)$$

and the pressure coefficient is

$$C_{p0} = -\frac{2\delta}{kE(\varkappa)}\left[\frac{x'(1 - x'^2)}{X'} - 2x'X'\right]. \qquad (100)$$

The strength of the pole on a leading edge is

$$P = \frac{V\delta}{kE(\varkappa)}(1 - x'^2)\left(\frac{x'}{2}\right)^{1/2} \qquad (101)$$

and P increases from 0 to a maximum as x' increases from 0 to $\sqrt{(\frac{1}{5},)}$ and decreases to 0 as x' increases to 1.

Using (35) and (59), the shape of the wing, at design incidence, is given by

$$z' = z_1' - z_{3,1}' = \delta(-x' + \tfrac{1}{3}f_6 x'^3 - f_7 k^2 x' y'^2) + f(y'). \qquad (102)$$

The spanwise lift distribution is given by

$$l(y) = \frac{2\rho V^2 c\delta}{\sigma kE(\varkappa)}(1 - x_1'^2)X' \qquad (103)$$

and for a triangular wing of maximum chord c, the design lift coefficient is

$$C_{L0} = \frac{2\pi\delta}{kE(\varkappa)} (1 - \sigma^2). \qquad (104)$$

(v) Using the basic solutions ϕ_1, $\phi_4^{\,1}$, we construct the induced velocity potential

$$\Phi_4^{\,1} = \phi_1 - \phi_4^{\,1} \qquad (105)$$

for which

$$(\Phi_4^{\,1})_{\mu=k} = \frac{Vc\delta}{\sigma k E(\varkappa)} (1 - x'^3)X' \qquad (106)$$

and the pressure coefficient is

$$C_{p0} = -\frac{2\delta}{kE(\varkappa)}\left[\frac{x'(1 - x'^3)}{X'} - 3x'^2X'\right]. \qquad (107)$$

The strength of the pole on a leading edge is

$$P = \frac{V\delta}{kE(\varkappa)} (1 - x'^3)\left(\frac{x'}{2}\right)^{1/2} \qquad (108)$$

and P increases from 0 to a maximum as x' increases from 0 to $(\frac{1}{7})^{1/3}$, and decreases to 0 as x' increases to 1.

Using (35) and (75), the shape of the wing, at design incidence, is given by

$$z' = z_1' - z_{4,1}' = \delta(-x' + \tfrac{1}{4}f_{12}x'^4 - \tfrac{1}{2}f_{13}k^2x'^2y'^2) + f(y'). \qquad (109)$$

The spanwise lift distribution is given by

$$l(y) = \frac{2\rho V^2c\delta}{\sigma kE(\varkappa)} (1 - x_1'^3)X_1' \qquad (110)$$

and, for a triangular wing of maximum chord c, the design lift coefficient is

$$C_{L0} = \frac{2\pi\delta}{kE(\varkappa)} (1 - \sigma^3). \qquad (111)$$

Some further simple examples of wings of this type will now be given. These can be obtained directly from the 'basic' solutions, or by combining some of the solutions (i) to (v) given above.

(vi) The velocity potential

$$\Phi_3^{\,2} = \Phi_3^{\,1} + \psi_3 = \phi_1 - k^2\phi_3^{\,2} \qquad (112)$$

gives

$$(\Phi_3^{\,2})_{\mu=k} = \frac{Vc\delta}{\sigma kE(\varkappa)} (1 - k^2y'^2)X' \qquad (113)$$

$$C_{p0} = \frac{-2\delta}{kE(\varkappa)}\,\frac{x'(1 - k^2y'^2)}{X'} \qquad (114)$$

$$z' = \delta(-x' - \tfrac{1}{3}f_2x'^3 + f_3k^2x'y'^2) + f(y'). \qquad (115)$$

(vii) The velocity potential

$$\Phi_4^{\,2} = \Phi_4^{\,1} + \psi_4 = \phi_1 - k^2\phi_4^{\,2} \qquad (116)$$

gives

$$(\Phi_4^{\,2})_{\mu=k} = \frac{Vc\delta}{\sigma kE(\varkappa)} (1 - k^2x'y'^2)X' \qquad (117)$$

$$C_{p0} = -\frac{2\delta}{kE(\varkappa)}\left[\frac{x'(1 - k^2x'y'^2)}{X'} - k^2y'X'\right] \quad \ldots \quad (118)$$

$$z' = \delta(-x' - \tfrac{1}{4}f_8x'^4 + \tfrac{1}{2}f_9k^2x'^2y'^2) + f(y'). \quad \ldots \quad (119)$$

(viii) The velocity potential

$$\Phi_{3,4}^{1,2} = \psi_4 + \Phi_4^1 - \Phi_3^1 = \phi_3^1 - k^2\phi_4^2 \quad \ldots \quad (120)$$

gives

$$(\Phi_{3,4}^{1,2})_{\mu=k} = \frac{Vc\delta}{\sigma kE(\varkappa)}\, x'(x' - k^2y'^2)X' \quad \ldots \quad (121)$$

$$C_{p0} = \frac{-2\delta}{kE(\varkappa)}\left[\frac{x'^2(x' - k^2y'^2)}{X'} + (2x' - k^2y'^2)X'\right] \quad \ldots \quad (122)$$

$$z' = \delta(-\tfrac{1}{3}f_6x'^3 + f_7k^2x'y'^2 - \tfrac{1}{4}f_8x'^4 + \tfrac{1}{2}f_9k^2x'^2y'^2) + f(y'). \quad \ldots \quad (123)$$

In each case, the effect on the pressure distribution, of a (small) additional incidence α, can be found by superposing the solution for a flat delta wing at incidence α (*see* section 5), that is by adding the terms

$$\left.\begin{aligned} \Delta(C_p) &= -\frac{2\alpha x'}{kE(\varkappa)X'} \\ \Delta(C_L) &= \frac{2\pi\alpha}{kE(\varkappa)} \end{aligned}\right\} \quad \ldots \quad (124)$$

to C_{p0} and C_{L0} respectively.

Some examples of the shape of and pressure distribution on wings of shape (iii), (vi), (vii), (viii) are shown in Figs. 14, 9, 10, 15.

9.1. *General Solution.*—Since we are using the linear theory of supersonic flow, a general solution for wings of the type considered can be obtained by combining the five independent solutions (i) to (v). In practice, it may be necessary to satisfy certain requirements with respect to the shape of the wing, and it is therefore useful to write down the following five solutions :

Induced velocity potential	*Shape of surface*
$\Omega_1 = \Phi_2$	$z' = \delta[-x' + f_1x'^2]$
$\Omega_2 = f_5\Phi_3^1 + f_7\psi_3$	$z' = \delta[-f_5x' + (\tfrac{1}{3}f_6f_5 - f_4f_7)x'^3]$
$\Omega_3 = f_{11}\Phi_4^1 + f_{13}\psi_4$	$z' = \delta[-f_{11}x' + \tfrac{1}{4}(f_{11}f_{12} - f_{10}f_{13})x'^4]$
$\Omega_4 = f_4\Phi_3^1 + \tfrac{1}{3}f_6\psi_3$	$z' = \delta[-f_4x' + (\tfrac{1}{3}f_6f_5 - f_4f_7)k^2x'y'^2]$
$\Omega_5 = f_{10}\Phi_4^1 + f_{12}\psi_4$	$z' = \delta[-f_{10}x' + \tfrac{1}{2}(f_{11}f_{12} - f_{10}f_{13})k^2x'^2y'^2]$. .. (125)

In each case, a (small) arbitrary function of y' can be added to the expression for z'.

We shall take as a general solution

$$\Omega = \sum_{s=1}^{5} (A_s\Omega_s), \quad \ldots \quad (126)$$

where A_s is an arbitrary constant.

Regarding A_1 as a scale factor, the solution contains the four arbitrary parameters A_s/A_1, $s = 2, 3, 4, 5$.

The shape of the corresponding surface is given by

$$z' = ax' + bx'^2 + d_1x'^3 + fx'^4 + gk^2x'y'^2 + h_1k^2x'^2y'^2 + f(y') \quad \ldots \quad (127)$$

where

$$\left.\begin{aligned} a &= -\delta(A_1 + A_2f_5 + A_3f_{11} + A_4f_4 + A_5f_{10}) \\ b &= \delta A_1 f_1 \\ \frac{d_1}{A_2} &= \frac{g}{A_4} = \delta(\tfrac{1}{3}f_6f_5 - f_4f_7) \\ \frac{4f}{A_3} &= \frac{2h_1}{A_5} = \delta(f_{11}f_{12} - f_{10}f_{13}) \end{aligned}\right\} \quad \ldots \quad (128)$$

(126) may also be written

$$\Omega = A\Phi_2 + B\Phi_3^1 + C\psi_3 + D\Phi_4^1 + E\psi_4 \quad \ldots \quad (129)$$

where

$$\left.\begin{aligned} A &= A_1 \\ B &= A_2f_5 + A_4f_4 \\ C &= A_2f_7 + \tfrac{1}{3}A_4f_6 \\ D &= A_3f_{11} + A_5f_{10} \\ E &= A_3f_{13} + A_5f_{12} \end{aligned}\right\} \quad \ldots \quad (130)$$

Hence, using the results (i) to (v), it can be shown that the corresponding pressure coefficient C_{p0}, the local spanwise lift coefficient C_{l0}, and the design lift coefficient C_{L0} for a triangular wing of maximum chord c are given by :

$$-C_{p0} = \frac{2\delta}{kE(\varkappa)} A\left[\left\{\frac{x'(1-x')}{X'} - X'\right\} + B\left\{\frac{x'(1-x'^2)}{X'} - 2x'X'\right\}\right.$$

$$+ C(3x'X') + D\left\{\frac{x'(1-x'^3)}{X'} - 3x'^2X'\right\}$$

$$\left. + E\left\{X'^3 + 3x'^2X'\right\}\right] ; \quad \ldots \quad (131)$$

$$C_{l0} = \frac{4\delta}{kE(\varkappa)}\left(\frac{\sigma + ky'}{\sigma - ky'}\right)^{1/2}\left[A(1-\sigma) + B(1-\sigma^2) + D(1-\sigma^3)\right.$$

$$\left. + (C + E\sigma)(\sigma^2 - k^2y'^2)\right], \quad (y' \geqslant 0) ; \quad \ldots \quad (132)$$

$$C_{L0} = \frac{2\pi\delta}{kE(\varkappa)}\left[A(1-\sigma) + B(1-\sigma^2) + D(1-\sigma^3) + \tfrac{3}{4}(C + E\sigma)\sigma^2\right]. \quad (133)$$

10. *Formulae for the Pitching-Moment Coefficients.*—The centre of pressure for a flat delta wing is at two-thirds the maximum chord from the vertex. For the wings considered in this report (maximum chord c), the pitching moment is taken about this chordwise position. The corresponding moment coefficient is given by

$$C_{M0} = -\frac{8k}{c^3}\int_0^c \int_0^{x/k} C_{p0}(\tfrac{2}{3}c - x)\, dy\, dx$$

$$= -\frac{8k}{\sigma^3}\int_0^\sigma \int_0^{x'/k} C_{p0}(\tfrac{2}{3}\sigma - x')\, dy'\, dx'. \quad \ldots \quad (134)$$

The formulae for surfaces (i) to (v) are as follows :

Surface	*Velocity Potential*	$\frac{kE(\varkappa)}{2\pi\delta} C_{M0}$
(i)	ψ_3	$-\frac{1}{5}\sigma^2$
(ii)	ψ_4	$-\frac{1}{4}\sigma^3$
(iii)	Φ_2	$\frac{1}{6}\sigma$
(iv)	Φ_3^1	$\frac{4}{15}\sigma^2$
(v)	Φ_4^1	$\frac{1}{3}\sigma^3$

The general formula is :

$$C_{M0} = \frac{2\pi\delta}{kE(\varkappa)}\left[A(\tfrac{1}{6}\sigma) + (\tfrac{4}{15}B - \tfrac{1}{5}C)\sigma^2 + (\tfrac{1}{3}D - \tfrac{1}{4}E)\sigma^3\right] \quad \ldots \quad (135)$$

where $A \ldots . E$ are given by (130).

It is thus possible to choose the constants $A \ldots . E$, so that, as one condition, C_{M0} has any given value.

11. *The Formulae for the Induced Drag at Design Incidence* (*Refs.* 2 *and* 4).—The drag on a body in fluid flow is the resultant in the free-stream direction of all the pressure forces acting on the body. In the linearised theory of supersonic (inviscid) flow the total drag, due to lift (usually termed the 'induced drag') is taken as the resultant axial force due to the lifting pressure distribution.

For a wing with no pressure singularities on the leading edges, the induced drag D_i is equal to the axial component of the pressure integral, D_p, and is given by

$$D_i = D_p = \rho V^2 \int\int C_{p0} \frac{\partial z}{\partial x}\, dx\, dy \quad \ldots \quad (136)$$

where the integral is taken over the surface of the wing. The corresponding drag coefficient is $C_{Di} = \frac{D_i}{(\frac{1}{2}\rho V^2 S)}$, where S is the area of the wing.

But, for a wing with pressure singularities on the leading edges, according to the linear theory, there is an infinite suction force or leading-edge thrust, determined by the strength of the singularity, as in subsonic flow. The component D_s of this suction force in the free-stream direction tends to reduce the induced drag, and the resultant induced drag is given by

$$D_i = D_p - D_s\ , \quad \ldots \quad (137)$$

the corresponding drag coefficient being given by

$$C_{Di} = \frac{D_i}{(\frac{1}{2}\rho V^2 S)} = C_{Dp} - C_{Ds}\,. \quad \ldots \qquad (138)$$

Using the result given in Appendix IV, Ref. 2, the longitudinal component of the suction force per unit length of a leading edge is :

$$\begin{aligned} T &= \pi\rho P^2(\cot^2\gamma - \cot^2\bar{\mu})^{1/2}\sin\gamma \\ &= \pi\rho P^2(k^2 - \beta^2)^{1/2}\sin\gamma \quad \ldots \qquad (139) \end{aligned}$$

where P is the strength of the singularity in the axial velocity on the leading edge (*cf.* after (93)) ; and hence

$$D_s = 2\int_0^{x_1} T\frac{dx}{\cos\gamma} = 2\pi\rho\,\frac{(k^2 - \beta^2)^{1/2}}{k}\int_0^{x_1} P^2\,dx \quad \ldots \qquad (140)$$

where x_1 is the chordwise distance of the wing tip from the apex.

For a given spanwise lift distribution, the trailing vortex field in regions far behind the aerofoil is the same in supersonic as in subsonic flow (Ref. 2). It is therefore convenient to subdivide the induced drag into vortex drag, which is the same for supersonic as for subsonic flow, and induced wave drag, which appears only in supersonic flow. Thus the vortex-drag coefficient C_{Dv}, for a wing of aspect ratio A, is $\varepsilon C_L^2/\pi A$ (Ref. 5), where ε depends upon the spanwise lift distribution of the aerofoil ; and the induced wave-drag coefficient is :

$$\begin{aligned} C_{Dw} &= C_{Di} - \varepsilon C_L{}^2/(\pi A) \\ &= C_{Dp} - C_{Ds} - \varepsilon C_L^2/(\pi A). \quad \ldots \qquad (141) \end{aligned}$$

To find the value of ε, the spanwise lift distribution $l(y)$ must be expressed as a Fourier series of the form $\sum_{n=1}^{\infty} (A_n \sin n\theta)$, by putting $ky = -c\cos\theta$, $(0 \leqslant |\theta| \leqslant \pi,\ -c \leqslant ky \leqslant +c)$.

Then

$$\varepsilon = \frac{\Sigma(nA_n{}^2)}{A_1{}^2}\,. \quad \ldots \qquad (142)$$

(Ref. 5)

Note: The integrands in (136) and (140) are not linear, and therefore the drag coefficients for the separate surfaces (i) to (v) cannot be linearly superimposed to obtain a general solution.

If the velocity potential Ω is given by (129), and the corresponding surface by (127), the general formulae for the drag coefficients are as follows : (Using the non-dimensional co-ordinates $x' = x\sigma/c$, $y' = y\sigma/c$, $z' = z\sigma/c$)

The pressure drag coefficient is :

$$\begin{aligned} C_{Dp} &= \frac{4k}{\sigma^2}\int_{y'=0}^{\frac{\sigma}{k}}\int_{x'=ky'}^{\sigma} C_{p0}(a + 2bx' + 3d_1x'^2 + 4fx'^3 + gk^2y'^2 + 2h_1k^2x'y'^2)\,dx'\,dy' \\ &= \frac{8\pi\delta}{kE(\varkappa)}\left[AP_1 + BP_2 + CP_3 + DP_4 + EP_5\right] \quad \ldots \qquad (143) \end{aligned}$$

where $A \ldots\ldots E$ are given by (130), and

$$\begin{aligned} P_1 = &-[\tfrac{1}{4}a - (\tfrac{1}{4}a - \tfrac{1}{3}b)\sigma - (\tfrac{3}{8}b - \tfrac{3}{8}d_1 - \tfrac{1}{16}g)\sigma^2 \\ &- (\tfrac{9}{20}d_1 - \tfrac{2}{5}f + \tfrac{1}{16}g - \tfrac{1}{10}h_1)\sigma^3 - (\tfrac{1}{2}f + \tfrac{5}{48}h_1)\sigma^4], \end{aligned}$$

$$
\begin{aligned}
P_2 &= -\ [\tfrac{1}{4}a + \tfrac{1}{3}b\sigma - (\tfrac{1}{4}a - \tfrac{3}{8}d_1 - \tfrac{1}{16}g)\sigma^2 - (\tfrac{2}{5}b - \tfrac{2}{5}f - \tfrac{1}{10}h_1)\sigma^3 \\
&\quad - (\tfrac{1}{2}d_1 + \tfrac{1}{16}g)\sigma^4 - (\tfrac{4}{7}f + \tfrac{3}{28}h_1)\sigma^5], \\
P_3 &= -\ 3\sigma^2[\tfrac{1}{16}a + \tfrac{1}{10}b\sigma + (\tfrac{1}{8}d_1 + \tfrac{1}{96}g)\sigma^2 + (\tfrac{1}{7}f + \tfrac{1}{56}h_1)\sigma^3], \\
P_4 &= -\ [\tfrac{1}{4}a + \tfrac{1}{3}b\sigma + (\tfrac{3}{8}d_1 + \tfrac{1}{16}g)\sigma^2 - (\tfrac{1}{4}a - \tfrac{2}{5}f - \tfrac{1}{10}h_1)\sigma^3 \\
&\quad - \tfrac{5}{12}b\sigma^4 - (\tfrac{15}{28}d_1 + \tfrac{1}{16}g)\sigma^5 - (\tfrac{5}{8}f + \tfrac{7}{64}h_1)\sigma^6], \\
P_5 &= -\ \sigma^3[\tfrac{3}{16}a + \tfrac{5}{16}b\sigma + (\tfrac{45}{112}d_1 + \tfrac{1}{32}g)\sigma^2 \\
&\quad + (\tfrac{15}{32}f + \tfrac{7}{128}h_1)\sigma^3] \,. \qquad \qquad (144)
\end{aligned}
$$

The suction-drag coefficient is :

$$
-C_{D_s} = -\frac{2\pi\delta^2(k^2 - \beta^2)^{1/2}}{\sigma^2 k^2\,[E(\varkappa)]^2}\int_0^\sigma x'[A(1 - x') + B(1 - x'^2) + D(1 - x'^3)]^2 dx' \,. \qquad (145)
$$

The total induced-drag coefficient is

$$C_{Di} = C_{Dp} - C_{Ds} \,.$$

The vortex-drag coefficient is

$$C_{Dv} = \varepsilon C_L{}^2/(\pi A), \quad \text{where}$$

$$
\varepsilon = 1 + \frac{\tfrac{3}{16}\sigma^4(C + E\sigma)^2}{[A(1-\sigma) + B(1-\sigma^2) + D(1-\sigma^3) + \tfrac{3}{4}\sigma^2(C + E\sigma)]^2} \,. \qquad (146)
$$

The induced wave-drag coefficient is

$$C_{Dw} = C_{Di} - C_{Dv} = C_{Dp} - C_{Ds} - C_{Dv} \,.$$

The above formulae give the drag coefficients at design incidence. A formula for the total induced drag coefficient at any incidence is given at the end of section 12.

When $\sigma \rightarrow 0$ (that is $d/c \rightarrow \infty$), camber and twist tend to vanish, and the wing tends to become a flat delta wing, at incidence. It has been verified that, when $\sigma \rightarrow 0$ (expressing the results in a form not involving the scale factor δ),

$$
\begin{aligned}
&C_{Dp}/(C_{L0}{}^2/\pi A) \longrightarrow 2E(\varkappa), \\
&C_{Dp}/(C_{L0}{}^2/\pi A) \longrightarrow (k^2 - \beta^2)^{1/2}/k = (1 - \tan^2\gamma \,.\, \cot^2\bar{\mu})^{1/2}, \\
&C_{Dv}/(C_{L0}{}^2/\pi A) \longrightarrow 1
\end{aligned}
$$

which are the results for the flat delta wing.

Some numerical examples of the total induced drag and the induced wave drag, for different values of σ, are shown in Figs. 6 to 17.

12. *Numerical Examples.*—Some numerical results, for specified values of γ and M, for wings of triangular plan form, are shown in Figs. 6 to 17. Formulae giving the shape of the wing, and the pressure distribution on the wing, are given below. Some notes on the choice of the arbitrary constants in the general solutions are given in examples (xiv), (xv), (xvi) at the end of this section.

The *non-dimensional co-ordinates* $x' = x\sigma/c$, $y' = y\sigma/c$, $z' = z\sigma/c$ *are used,* where $\sigma = c/d$, and c is the maximum chord of the wing ($1/\sigma$ measures the distance in maximum chord lengths, of the position of zero leading-edge pressure from the apex of the wing, except in examples (i) and (ii), where $\sigma = 1$. *See* Fig. 5). Since these co-ordinates are used throughout the numerical examples, the *dashes are dropped* in this section and in the figures. In each case (y) is chosen so that $z = 0$ on the leading edges.

Some numerical values of the pitching-moment coefficients and the drag coefficients for different values of σ or for a given σ and different lift coefficients, are given. When $\sigma \to 0$, these values tend to those for the corresponding flat delta wing, at incidence.

The formulae for the local spanwise lift, total lift, and drag coefficients are given in Appendix VII; the spanwise lift coefficient, for given σ and C_{L0}, are plotted against y, and the total induced-drag and wave-drag coefficients (in a form not involving the scale factor δ), against $1/\sigma$.

The examples given below, and shown in Figs. 6 to 17, are grouped under the headings:

(1) wings with no leading-edge singularities;
(2) mainly twisted wings;
(3) mainly cambered wings;
(4) cambered and twisted wings.

Figs. 6 and 7 show the effect of removing the leading-edge pressure singularities; Figs. 8 to 11 are cases of almost pure twist; Figs. 12 and 13 are cases of camber and small twist; and Figs. 14 to 17 show the effect of combined camber and twist.

Figs. 18 and 19 show application to wings of the solutions given in Figs. 8 and 14 respectively. These are constructed by selecting a definite plan form, which is then regarded as the front part of one of the curved plates. No allowance is made for tip loss in these calculations.

Figs. 20 to 22 show examples of wings which satisfy the following additional conditions: (for given γ and M, and $\sigma = 1$):

In Fig. 20, $C_{L0} = 0{\cdot}1$, $C_{M0} = 0$ (only one of a number of solutions which would satisfy these conditions).

In Fig. 21, $C_{L0} = 0{\cdot}1$, zero camber and positive incidence at the root, positive camber elsewhere (not the only solution).

In Fig. 22, $C_{L0} = 0{\cdot}1$, zero camber and positive incidence at the root, minimum induced drag (using the solutions for $n = 1, 2, 3, 4$ given in this report). The solution is completely determined by these conditions.

The numbers (i), (ii)......(viii) of the examples correspond to those in sections 8, 9 of the report. For each example, a short table showing the values of the drag coefficients C_{Dw}, C_{Dv}, C_{Di} for given σ (taken as 1 in most cases), and different values of C_{L0}, and also those for the corresponding flat delta wing, at incidence, is given.

(1) *Wings with no leading-edge singularities*

(i) (*cf.* equations (76) to (83)) (Fig. 6)

$$\gamma = 45°, \quad M = 1{\cdot}166, \quad (\sigma = 1)$$
$$z = \delta[-\,0{\cdot}658x^3 + 2{\cdot}525xy^2] + f(y)\,;$$

(in non-dimensional co-ordinates)

$$-\,C_{p0} = 4{\cdot}7011\delta x(x^2 - y^2)^{1/2}$$

	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
	$-4/15$	$4/3$	$1{\cdot}717$	$3{\cdot}050$
For the corresponding flat delta wing, at any incidence.	0	1	$0{\cdot}753$	$1{\cdot}753$

Drag coefficients, at design incidence				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dw}	C_{Dv}	C_{Di}	C_{Dw}	C_{Dv}	C_{Di}
0·025	0·000085	0·000066	0·00015	0·000037	0·00005	0·000087
0·05	0·00034	0·00027	0·00061	0·00015	0·0002	0·00035
0·075	0·00077	0·00060	0·0014	0·00034	0·00044	0·00078
0·1	0·0014	0·0010	0·0024	0·00060	0·0008	0·0014
0·15	0·0031	0·0024	0·0055	0·0013	0·0018	0·0031
0·2	0·0054	0·0043	0·0097	0·0024	0·0032	0·0056

(ii) (*cf.* equations (84) to (90)) (Fig. 7)

$\gamma = 30°, \qquad M = 1\cdot852, \qquad (\sigma = 1)$

$z = \delta[-0\cdot678x^4 + 4\cdot223x^2y^2] + f(y)$;

$-C_{p0} = 0\cdot7732\delta(4x^2 - 3y^2)(x^2 - 3y^2)^{1/2}$

	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
	$-1/3$	4/3	3·783	5·116
For the flat delta wing at any incidence	0	1	1·550	2·550

Drag coefficients, at design incidence				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dw}	C_{Dv}	C_{Di}	C_{Dw}	C_{Dv}	C_{Di}
0·025	0·00033	0·00011	0·00044	0·00013	0·00009	0·00022
0·05	0·0013	0·00045	0·0018	0·00053	0·00035	0·00088
0·075	0·0029	0·0011	0·0040	0·0012	0·0008	0·0020
0·1	0·0052	0·0019	0·0071	0·0021	0·0014	0·0035
0·15	0·012	0·004	0·016	0·0048	0·0031	0·0079
0·2	0·021	0·007	0·028	0·0086	0·0055	0·0141

(2) *Twisted wings* (small camber)

(vi) (*cf.* equations (112) to (115))

(vi *a*) $\gamma = 45°, \qquad M = 1\cdot345$

$z = \delta[-x - 0\cdot069x^3 + 2\cdot141xy^2] + f(y)$;

$-C_{p0} = 1\cdot3393\delta x(1 - y^2)(x^2 - y^2)^{-1/2}$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
1/2		$C_{L0} = 0$		
3/4	0·214	2·920	1·771	4·691
1	0·088	1·333	1·337	2·670
5/4	0·050	1·109	1·375	2·484
3/2	0·034	1·047	1·419	2·466
Flat delta wing at any incidence	0	1	1·550	2·550

Drag coefficients, at design incidence, when $\sigma = 1$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·000066	0·000066	0·00013	0·000050	0·000075	0·000125
0·05	0·00026	0·00027	0·00053	0·00020	0·00030	0·00050
0·075	0·00060	0·00060	0·0012	0·00045	0·00068	0·00113
0·1	0·0010	0·0011	0·0021	0·0008	0·0012	0·0020
0·15	0·0024	0·0024	0·0048	0·0018	0·0027	0·0045
0·2	0·0042	0·0043	0·0085	0·0032	0·0048	0·0080

(vi *b*) $\gamma = 45°$, $M = 1·281$ (Fig. 9)

$$z = \delta\,(-x - 0·0767x^3 + 2·227xy^2) + f(y)\,;$$

$$-C_{p0} = 1·4103\delta x(1 - y^2)(x^2 - y^2)^{-1/2}$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
1/2		$C_{L0} = 0$		
3/4	0·214	2·920	1·405	4·325
1	0·088	1·333	1·076	2·409
5/4	0·050	1·109	1·104	2·213
3/2	0·034	1·047	1·137	2·184
Flat delta wing at any incidence	0	1	1·236	2·236

Drag coefficients at design incidence, when $\sigma = 1$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·00007	0·00005	0·00012	0·000050	0·000061	0·00011
0·05	0·00027	0·00020	0·00047	0·00020	0·00024	0·00044
0·075	0·0006	0·0005	0·0011	0·00045	0·00055	0·0010
0·1	0·0010	0·0008	0·0019	0·00080	0·00098	0·0018
0·15	0·0024	0·0019	0·0043	0·0018	0·0022	0·0040
0·2	0·004	0·003	0·007	0·0032	0·0039	0·0071

(vii) (*cf.* equations (116) to (119)) (Fig. 10)

$\gamma = 60°$, $M = 1·13$

$$z = \delta\,[-x - 0·044x^4 + 0·398x^2y^2] + f(y)\,;$$

$$-C_{p0} = 2·3198\delta[x(1 - \tfrac{1}{3}xy^2)(x^2 - \tfrac{1}{3}y^2)^{-1/2} - \tfrac{1}{3}y^2(x^2 - \tfrac{1}{3}y^2)^{1/2}]$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
3/4	0·484	7·347	4·900	12·247
1	0·110	1·333	1·381	2·714
5/4	0·048	1·065	1·406	2·471
3/2	0·026	1·019	1·459	2·478
2	0·010	1·003	1·510	2·513
Flat delta wing at any incidence	0	1	1·550	2·550

Drag coefficients at design incidence, for $\sigma = 1$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·000038	0·000040	0·000078	0·000029	0·000044	0·000073
0·05	0·00015	0·00016	0·00031	0·00011	0·00018	0·00029
0·075	0·00034	0·00036	0·00070	0·00026	0·00040	0·00066
0·1	0·00061	0·00063	0·0012	0·00046	0·00071	0·0012
0·15	0·0014	0·0014	0·0028	0·0010	0·0016	0·0026
0·2	0·0025	0·0025	0·0050	0·0018	0·0029	0·0047

(xii) [General solution (126), with $A_1 = A_2 = A_3 = 0,\ A_4 = 1,\ A_5 = -0{\cdot}1$]

$\gamma = 45°, \quad M = 1{\cdot}281 \quad$ (Fig. 11)

$$z = \delta[-0{\cdot}4481x + 1{\cdot}3956xy^2 - 0{\cdot}2896x^2y^2] + f(y)\ ;$$

$$-C_{p0} = 1{\cdot}41034\delta\left[(0{\cdot}4481x - 0{\cdot}7085x^3 + 0{\cdot}2604x^4)\frac{1}{X} + (0{\cdot}4782x - 0{\cdot}1815x^2 + 0{\cdot}2407y^2)X\right],\ \text{where } X \equiv (x^2 - y^2)^{1/2}$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}{}^2/\pi A)$	$C_{Dw}/(C_{L0}{}^2/\pi A)$	$C_{Di}/(C_{L0}{}^2/\pi A)$
1	0·122	1·333	1·052	2·385
5/4	0·078	1·129	1·078	2·207
3/2	0·054	1·061	1·111	2·172
2	0·030	1·019	1·158	2·177
Flat delta wing at any incidence	0	1	1·236	2·236

Drag coefficients at design incidence, for $\sigma = 1$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·00007	0·00005	0·00012	0·00005	0·00006	0·00011
0·05	0·00026	0·00021	0·00047	0·0002	0·00024	0·00044
0·075	0·00060	0·00047	0·00107	0·00045	0·00055	0·0010
0·1	0·00106	0·00084	0·00190	0·0008	0·00098	0·0018
0·15	0·00239	0·00188	0·00427	0·0018	0·0022	0·0040
0·2	0·00424	0·00335	0·00759	0·0032	0·0039	0·0071

(3) *Cambered wings* (with small twist)

(ix) [General solution (126), with $A_1{=}A_2{=}A_3{=}-1,\ A_4{=}4,\ A_5{=}1{\cdot}3$]

$\gamma = 45°, \quad M = 1{\cdot}281 \quad$ (Fig. 12)

$$z = \delta[-0{\cdot}0279x - 0{\cdot}7085x^2 - 1{\cdot}3956x^3 - 1{\cdot}4482x^4 + 5{\cdot}5824xy^2 + 3{\cdot}7653x^2y^2] + f(y)\ ;$$

$$-C_{p0} = 1{\cdot}41034\delta\left[(0{\cdot}02794x + x^2 - 0{\cdot}4597x^3 - 0{\cdot}5682x^4)\frac{1}{X} + (1 + 6{\cdot}2194x + 9{\cdot}2952x^2 - 2{\cdot}7500y^2)X\right],\ \text{where}$$

$$X \equiv (x^2 - y^2)^{1/2}$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
1/2	−0·298	1·501	3·726	5·227
3/4	−0·280	1·411	2·780	4·191
1	−0·264	1·333	2·547	3·880
5/4	−0·250	1·270	2·393	3·663
3/2	−0·240	1·221	2·272	3·493
2	−0·224	1·151	2·103	3·254
Flat delta wing at any incidence	0	1	1·236	2·236

Drag coefficients at design incidence, for $\sigma = 1$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·00007	0·00012	0·00019	0·00005	0·00006	0·00011
0·05	0·00026	0·00051	0·00077	0·00020	0·00024	0·00044
0·075	0·0006	0·0011	0·0017	0·00045	0·00055	0·0010
0·1	0·0011	0·0020	0·0031	0·0008	0·00098	0·0018
0·15	0·0023	0·0046	0·0069	0·0018	0·0022	0·0040
0·2	0·0042	0·0081	0·0123	0·0032	0·0039	0·0071

The camber at 4 different spanwise positions, when $\sigma = 1$, is given by :

y	0	1/4	1/2	3/4
Camber per cent	138δ	128δ	99δ	56δ

In Fig. 12, δ is taken equal to 0·01 ($C_{L0} = 0{\cdot}17$ when $\sigma = 1$).

The relation between twist, camber and lift (at design incidence) is
Twist/camber at root — camber at $\frac{3}{4}$ semi-span/C_{L0} = 0·012/0·48/0·1.

(x) [General solution (126), with $A_1=A_2=A_3= -1$, $A_4=10$, $A_5= -0{\cdot}2$]

$$\gamma = 45°, \qquad M = 1{\cdot}281 \qquad \text{(Fig. 13)}$$

$$z = \delta[-0{\cdot}3739x - 0{\cdot}7085x^2 - 1{\cdot}3956x^3 - 1{\cdot}4482x^4 + 13{\cdot}956xy^2 - 0{\cdot}5793x^2y^2] + f(y)\,,$$

$$-C_{p0} = 1{\cdot}41034\delta\left[0{\cdot}3732\,\frac{x}{X} + \frac{x^2}{X} - 4{\cdot}7107\,\frac{x^3}{X} + 3{\cdot}3375\,\frac{x^4}{X} + X + 9{\cdot}0886xX + 7{\cdot}4326x^2X - 0{\cdot}8599X^3\right]$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
1		1·333	2·553	3·886
3/2		1·358	2·124	3·482
1·69	−0·190	1·35	1·95	3·30
2	−0·160	1·273	1·805	3·078
Flat delta wing at any incidence	0	1	1·236	2·236

For this wing (x), on the leading edges at $x = 1$, *and* at $x = 0{\cdot}59$, $C_{p0} = 0$, and

$$0 < x < 0{\cdot}59, \qquad -C_{p0} = +\infty\ ;$$
$$0{\cdot}59 < x < 1\ , \qquad -C_{p0} = -\infty\ ;$$
$$x > 1\ , \qquad -C_{p0} = +\infty\ .$$

Drag coefficients at design incidence, for $\sigma = 0\cdot59$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·000067	0·000097	0·000164	0·00005	0·000061	0·00011
0·05	0·00026	0·00039	0·00065	0·0002	0·00024	0·00044
0·075	0·00060	0·00087	0·00147	0·00045	0·00055	0·0010
0·1	0·0011	0·0015	0·0026	0·00080	0·00098	0·0018
0·15	0·0024	0·0035	0·0059	0·0018	0·0022	0·0040
0·2	0·0043	0·0062	0·0105	0·0032	0·0039	0·0071

The camber at 4 different spanwise positions, when $\sigma = 0\cdot59$ (*i.e.*, $1/\sigma = 1\cdot69$), is given by :

y	0	1/8	1/4	3/8
Camber per cent	42δ	77δ	35δ	26δ

In Fig. 13, δ is taken equal to $0\cdot01$ ($C_{L0} = 0\cdot066$ when $\sigma = 0\cdot59$).

The relation between twist, camber and lift is :
Twist/camber at root — camber at $\frac{3}{4}$ semi-span/$C_{L0} = 0\cdot01/0\cdot2/0\cdot1$.

(4) *Wings with camber and twist*

(iii) (*cf.* equations (91) to (97)) (Fig. 14)

$$\gamma = 45°, \quad M = 1\cdot166$$

$$z = \delta\,[-x + 0\cdot658x^2] + f(y)\;;$$

$$-C_{p0} = 1\cdot56703\delta[x(1 - x)(x^2 - y^2)^{-1/2} - (x^2 - y^2)^{1/2}]$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
1		$C_{L0} = 0$		
5/4	0·66	1	2·210	3·210
3/2	0·34	1	0·895	1·895
2	0·16	1	0·677	1·677
Flat delta wing at any incidence	0	1	0·752	1·752

Drag coefficients at design incidence, when $\sigma = 2/3$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·00005	0·000044	0·000094	0·00005	0·000037	0·000087
0·05	0·0002	0·00018	0·00038	0·0002	0·00015	0·00035
0·075	0·00045	0·00040	0·00085	0·00045	0·00033	0·00078
0·1	0·0008	0·00071	0·0015	0·0008	0·00060	0·0014
0·15	0·0018	0·0016	0·0034	0·0018	0·0013	0·0031
0·2	0·0032	0·0028	0·0060	0·0032	0·0024	0·0056

(viii) *cf.* equations (120) to (123)

$$\gamma = 30°, \quad M = 1\cdot852 \quad \text{(Fig. 15)}$$

$$z = \delta[-0\cdot661x^3 + 0\cdot505xy^2 - 0\cdot044x^4 + 3\cdot575x^2y^2]$$

$$-C_{p0} = 0\cdot7732\delta[x^2(x - 3y^2)(x^2 - 3y^2)^{-1/2} + (2x - 3y^2)(x^2 - 3y^2)^{1/2}]$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$.
1	$-0{\cdot}244$	$1{\cdot}333$	$16{\cdot}030$	$17{\cdot}363$
5/4	$-0{\cdot}250$	$1{\cdot}188$	$11{\cdot}840$	$13{\cdot}028$
3/2	$-0{\cdot}254$	$1{\cdot}120$	$9{\cdot}396$	$10{\cdot}516$
2	$-0{\cdot}256$	$1{\cdot}061$	$6{\cdot}405$	$7{\cdot}466$
Flat delta wing at any incidence	0	1	$1{\cdot}551$	$2{\cdot}551$

Drag coefficients at design incidence, for $\sigma = 1$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·0001	0·0014	0·0015	0·00009	0·00013	0·00022
0·05	0·0005	0·0055	0·0060	0·000344	0·000534	0·00088
0·075	0·001	0·012	0·013	0·00078	0·0012	0·0020
0·1	0·002	0·022	0·024	0·0014	0·0021	0·0035
0·15	0·004	0·050	0·054	0·0031	0·0048	0·0079
0·2	0·0073	0·0883	0·096	0·0055	0·0086	0·0141

(xiii) [General solution (129), with $C = 4$, $B = 3$, $A = D = E = 0$]

$$\gamma = 45°, \qquad M = 1{\cdot}281 \qquad \text{(Fig. 16)}$$

$$z = \delta[-3x - 0{\cdot}9388x^3 + 9{\cdot}0553xy^2] + f(y)\ ;$$

$$-C_{p0} = 4{\cdot}23102\delta[x(1-x^2)(x^2-y^2)^{-1/2} + 2x(x^2-y^2)^{1/2}]$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
1/2	0	$6{\cdot}333$	$3{\cdot}282$	$9{\cdot}615$
1	0	$1{\cdot}333$	$1{\cdot}207$	$2{\cdot}540$
3/2	0	$1{\cdot}066$	$1{\cdot}179$	$2{\cdot}245$
2	0	$1{\cdot}021$	$1{\cdot}195$	$2{\cdot}216$
Flat delta wing at any incidence	0	1	$1{\cdot}236$	$2{\cdot}236$

The camber at 4 different spanwise positions, when $\sigma = 1$, is given by :

y	0	1/4	1/2	3/4
Camber per cent	36δ	33δ	26δ	15δ

In Fig. 16, δ is taken equal to $0{\cdot}01$ ($C_{L0} = 0{\cdot}13$ for $\sigma = 1$).

The relation between twist, camber and lift is :
Twist/camber at root — camber at $\frac{3}{4}$ semi-span $/C_{L0} = 0{\cdot}029/0{\cdot}155/0{\cdot}1$.

Drag coefficients at design incidence, for $\sigma = 1$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·00007	0·00006	0·00013	0·00005	0·00006	0·00011
0·05	0·00026	0·00024	0·0005	0·00020	0·00024	0·00044
0·075	0·00060	0·00054	0·0011	0·00045	0·00055	0·0010
0·1	0·00106	0·00096	0·0020	0·00080	0·00098	0·0018
0·15	0·00239	0·00216	0·0045	0·0018	0·0022	0·0040
0·2	0·0424	0·00384	0·0081	0·0032	0·0039	0·0071

(xi) [General solution (126) with $A_1 = A_2 = A_3 = 0$, $A_4 = 5$, $A_5 = -1$]

$\gamma = 45°$, $M = 1\cdot281$ (Fig. 17)

$$z = \delta\,[-0\cdot9387x + 6\cdot978xy^2 - 2\cdot8964x^2y^2] + f(y)$$

$$-C_{p0} = 1\cdot410034\delta\left[(0\cdot9387x - 3\cdot5425x^3 + 2\cdot6038x^4)\frac{1}{X} + (2\cdot391x - 1\cdot815x^2 + 2\cdot4066y^2)X\right],\ X \equiv (x^2 - y^2)^{1/2}$$

$1/\sigma$	C_{M0}/C_{L0}	$C_{Dv}/(C_{L0}^2/\pi A)$	$C_{Dw}/(C_{L0}^2/\pi A)$	$C_{Di}/(C_{L0}^2/\pi A)$
3/4	$-0\cdot1$	1·003	1·895	2·898
1	$+0\cdot082$	1·333	1·108	2·441
5/4	$+0\cdot106$	1·328	1·052	2·380
Flat delta wing at any incidence	0	1	1·236	2·236

Drag coefficients at design incidence, for $\sigma = 1$				For the corresponding flat delta wing, at incidence		
C_{L0}	C_{Dv}	C_{Dw}	C_{Di}	C_{Dv}	C_{Dw}	C_{Di}
0·025	0·00007	0·00005	0·00012	0·00005	0·00006	0·00011
0·05	0·00027	0·00022	0·00049	0·00020	0·00024	0·00044
0·075	0·0006	0·0005	0·0011	0·00045	0·00055	0·0010
0·1	0·0010	0·0009	0·0019	0·00080	0·00098	0·0018
0·15	0·0024	0·0020	0·0044	0·0018	0·0022	0·0040
0·2	0·0042	0·0036	0·0078	0·0032	0·0039	0·0071

The camber at 4 different spanwise positions, when $\sigma = 1$, is given by :

y	0	1/4	1/2	3/4
Camber per cent	0	$3\cdot39\delta$	$9\cdot05\delta$	$10\cdot18\delta$

In Fig. 17, δ is taken equal to $0\cdot1$ ($C_{L0} = 0\cdot25$ for $\sigma = 1$).

The relation between twist, camber and lift is :
Twist/camber at $\frac{3}{4}$ semi-span — camber at root/C_{L0} = 0·08/0·41/0·1.

Three wings satisfying given conditions

In each of the following three examples, $\sigma \equiv c/d = 1$, $\gamma = 30°$, $M = 1\cdot442$.

(xiv) (Fig. 20)

Given conditions : $C_{L0} = 0\cdot1$, $C_{M0} = 0$.

[Other conditions could be also satisfied, *cf.* (133), (135)].

One possible solution is found by putting $A = D = E = 0$, $C = 4$, $B = 3$ in the general solution (129). (*cf.* (135).)

$C_{L0} = 0\cdot1$ gives $\delta = 0\cdot0117277$, and hence

$$z = \delta[-3x - 0\cdot9446x^3 + 29\cdot3694xy^2] + f(y),\ \text{and}$$

$$-C_{p0} = 0\cdot0318309\left[(x - x^3)\frac{1}{X} + 2xX\right],\ X \equiv (x^2 - 3y^2)^{1/2}$$

Drag coefficients at design incidence	*For the corresponding flat delta wing, at incidence*
$C_{Dv} = 0 \cdot 00184$	$C_{Dv} = 0 \cdot 00138$
$C_{Dw} = 0 \cdot 00106$	$C_{Dw} = 0 \cdot 00104$
$C_{Di} = 0 \cdot 00290$	$C_{Di} = 0 \cdot 00242$

The camber at 3 different spanwise positions is :

y	0	1/4	1/2
Camber per cent	0·426	0·339	0·104

The relation between twist, camber and lift is :

Twist/Camber at root — camber at $\frac{3}{4}$ semi-span/C_{L0} = 0·05/0·24/0·1.

(xv) (Fig. 21)

Given conditions :

(1) zero camber at the root,
(2) positive (or approximately zero) camber elsewhere,
(3) positive incidence at the root,
(4) $C_{L0} = 0 \cdot 1$.

Using (126), (127), (128), condition (1) gives $A_1 = A_2 = A_3 = 0$. The curvature of a section parallel to the x-axis is approximately equal to $\partial^2 z/\partial x^2 = 2hk^2y^2$. Hence, for positive camber, $h < 0$, and therefore $A_5 < 0$.

Condition (3) gives $a < 0$, or, taking $A_5 = -1$, $A_4 f_4 > f_{10}$.

We should also ensure that the strength of the pressure singularity on a leading edge is $\geqslant 0$ for $x \leqslant 1$. This leads to a second inequality to be satisfied by A_4. A value satisfying both inequalities is $A_4 = 5 \cdot 5$

$$C_{L0} = 0 \cdot 1 \text{ then gives } \delta = 0 \cdot 0467782.$$

Hence

$$C_{M0} = 0 \cdot 012 \quad \text{and}$$

$$z = \delta[-1 \cdot 2741x + 22 \cdot 3396xy^2 - 7 \cdot 9396x^2y^2] + f(y),$$

$$-C_{p0} = 0 \cdot 04232 \left[(1 \cdot 2741x - 3 \cdot 6217x^3 + 2 \cdot 3476x^4)\frac{1}{X} + (2 \cdot 0482x - 1 \cdot 3348x^2 + 6 \cdot 2832y^2)X \right], \quad X \equiv (x^2 - 3y^2)^{1/2}$$

Drag coefficients at design incidence	*For the corresponding flat delta wing, at incidence*
$C_{Dv} = 0 \cdot 00184$	$C_{Dv} = 0 \cdot 00138$
$C_{Dw} = 0 \cdot 00087$	$C_{Dw} = 0 \cdot 00104$
$C_{Di} = 0 \cdot 00271$	$C_{Di} = 0 \cdot 00242$

The camber at 3 different spanwise positions is :

y	0	$1/(2\sqrt{3})$	$3/(4\sqrt{3})$
Camber per cent	0	0·387	0·435

The relation between twist, camber and lift is :
Twist/Camber at $\frac{3}{4}$ semi-span — camber at root/C_{L0} = 0·07/0·43/0·1.

(xvi) (Fig. 22)

Given conditions :

(1) zero camber at the root,

(2) $C_{L0} = 0\cdot 1$,

(3) minimum induced drag, with conditions (1), (2), (using the solutions for $n = 1, 2, 3, 4$).

Using (126), (127), (128), condition (1) gives $A_1 = A_2 = A_3 = 0$. The condition $C_{L0} = 0\cdot 1$ gives a relation between $A_4\delta$ and $A_5\delta$, and hence the drag coefficient C_{Di} can be expressed as a function of $A_5\delta$ or of $A_4\delta$. It is found that C_{Di} is least when $A_4\delta = 0\cdot 171378$, $A_5\delta = -0\cdot 023669$.

For these values, $C_{M0} = 0\cdot 013$, and

$$z = -0\cdot 0573x + 0\cdot 6961xy^2 - 0\cdot 1879x^2y^2 + f(y),$$

$$-C_{p0} = 0\cdot 904725\left[(0\cdot 0573x - 0\cdot 1129x^2 + 0\cdot 0556x^3)\frac{1}{X} + (0\cdot 0638x - 0\cdot 0316x^2 + 0\cdot 1487y^2)X\right], \quad X \equiv (x^2 - 3y^2)^{1/2}$$

Drag coefficients at design incidence	*For the corresponding flat delta wing, at incidence*
$C_{Dv} = 0\cdot 00184$	$C_{Dv} = 0\cdot 00138$
$C_{Dw} = 0\cdot 00085$	$C_{Dw} = 0\cdot 00104$
$C_{Di} = 0\cdot 00269$	$C_{Di} = 0\cdot 00242$

The total induced-drag coefficient at any (small) incidence

The total induced-drag coefficient, at any incidence, can be expressed in terms of the design lift coefficient C_{L0} and the lift coefficient ΔC_L due to additional incidence α.

If C_p, C_{p0} are the pressure coefficients, and C_L, C_{L0} the lift coefficients at additional incidence α and design incidence respectively,

$$\Delta C_p \equiv C_p - C_{p0} = -\frac{2\alpha}{kE(\varkappa)}\,\frac{x}{X},$$

$$\Delta C_L \equiv C_L - C_{L0} = \frac{2\pi}{kE(\varkappa)}\,\alpha,$$

and, for a chosen wing, the scale factor δ is proportional to C_{L0}, the relation between δ and C_{L0} being given by (133).

The total induced-drag coefficient at additional incidence α is

$$C_D \equiv C_{Di} + \Delta C_{Di} = \frac{4k}{\sigma^2}\int_{y=0}^{\sigma/k}\int_{x=ky}^{\sigma}(C_{p0} + \Delta C_p)\left(\frac{\partial z}{\partial x} - \alpha\right)dx\,dy$$

$$-\frac{2\pi(k^2 - \beta^2)^{1/2}}{\sigma^2k^2[E(\varkappa)]^2}\int_0^{\sigma} x\left[\delta\,\{A(1 - x) + B(1 - x^2) + D(1 - x^3)\} + \alpha\right]^2 dx$$

$$= p_1C_{L0}^2 + p_2C_{L0}(\Delta C_L) + p_3(\Delta C_L)^2, \quad \ldots \quad (147)$$

where

$$p_1 = \frac{C_{Di}}{C_{L0}^2} ,$$

$$p_2 = \frac{kE(\varkappa)}{2\pi} - \left[\frac{kE(\varkappa)}{2\pi} \{a' + \tfrac{4}{3}b'\sigma + \tfrac{3}{2}d_1'\sigma^2 + \tfrac{8}{5}f'\sigma^3 + \tfrac{1}{4}g'\sigma^2 + \tfrac{2}{5}h_1'\sigma^3\} + \frac{(k^2 - \beta^2)^{1/2}}{2\pi} \{(A + B + D) - \tfrac{2}{3}A\sigma - \tfrac{1}{2}B\sigma^2 - \tfrac{2}{3}D\sigma^3\} \right] \Big/ [A(1 - \sigma) + B(1 - \sigma^2) + D(1 - \sigma^3) + \tfrac{3}{4}(C + E\sigma)\sigma^2],$$

where $a' = a/\delta$, $b' = b/\delta$, etc.,

$$p_3 = \frac{kE(\varkappa)}{2\pi} - \frac{k\varkappa}{4\pi} . \qquad [\text{Since } (k^2 - \beta^2)^{1/2} = h = k\varkappa]$$

(147) may also be written in the form

$$\frac{C_D}{C_L^2} = p_1 \frac{C_{L0}^2}{C_L^2} + p_2 \frac{C_{L0}}{C_L}\left(1 - \frac{C_{L0}}{C_L}\right) + p_3 \left(1 - \frac{C_{L0}}{C_L}\right)^2 . \quad .. \quad .. \quad (148)$$

The values of p_1, p_2, p_3 have been calculated for surfaces (i) to (xvi) for specified γ and M; in each case σ is chosen so that the point of zero pressure on a leading edge is at the wing tip, that is $\sigma = 1$, except for surface (x), where $\sigma = 0 \cdot 59$. The resulting formulae for the induced-drag coefficients are given below :

Surface	$C_D \equiv C_{Di} + \Delta C_{Di}$
(i)	$0 \cdot 2427 C_{L0}^2 + 0 \cdot 2995 C_{L0}(\Delta C_L) + 0 \cdot 1395 (\Delta C_L)^2$
(ii)	$0 \cdot 7052 C_{L0}^2 + 0 \cdot 6980 C_{L0}(\Delta C_L) + 0 \cdot 3516 (\Delta C_L)^2$
(vi *a*)	$0 \cdot 2125 C_{L0}^2 + 0 \cdot 2722 C_{L0}(\Delta C_L) + 0 \cdot 1395 (\Delta C_L)^2$
(vi *b*)	$0 \cdot 3628 C_{L0}^2 + 0 \cdot 3300 C_{L0}(\Delta C_L) + 0 \cdot 1780 (\Delta C_L)^2$
(vii)	$0 \cdot 1247 C_{L0}^2 + 0 \cdot 2136 C_{L0}(\Delta C_L) + 0 \cdot 1172 (\Delta C_L)^2$
(xii)	$0 \cdot 1898 C_{L0}^2 + 0 \cdot 3267 C_{L0}(\Delta C_L) + 0 \cdot 1780 (\Delta C_L)^2$
(ix)	$0 \cdot 3087 C_{L0}^2 + 0 \cdot 3654 C_{L0}(\Delta C_L) + 0 \cdot 1780 (\Delta C_L)^2$
(x) ($\sigma = 0 \cdot 59$)	$0 \cdot 2629 C_{L0}^2 + 0 \cdot 3586 C_{L0}(\Delta C_L) + 0 \cdot 1780 (\Delta C_L)^2$
(iii)	$0 \cdot 5403 \delta^2 - 0 \cdot 0863 \delta (\Delta C_L) + 0 \cdot 1395 (\Delta C_L)^2$ (for this surface for $\sigma = 1$, $C_{L0} = 0$, δ is proportional to the design incidence)
(viii)	$2 \cdot 3932 C_{L0}^2 + 0 \cdot 6938 C_{L0}(\Delta C_L) + 0 \cdot 3516 (\Delta C_L)^2$
(xiii)	$0 \cdot 2022 C_{L0}^2 + 0 \cdot 3393 C_{L0}(\Delta C_L) + 0 \cdot 1780 (\Delta C_L)^2$
(xi)	$0 \cdot 1942 C_{L0}^2 + 0 \cdot 3315 C_{L0}(\Delta C_L) + 0 \cdot 1780 (\Delta C_L)^2$
(xiv)	$0 \cdot 2899 C_{L0}^2 + 0 \cdot 4725 C_{L0}(\Delta C_L) + 0 \cdot 2416 (\Delta C_L)^2$
(xv)	$0 \cdot 2713 C_{L0}^2 + 0 \cdot 4542 C_{L0}(\Delta C_L) + 0 \cdot 2416 (\Delta C_L)^2$
(xvi)	$0 \cdot 2694 C_{L0}^2 + 0 \cdot 4505 C_{L0}(\Delta C_L) + 0 \cdot 2416 (\Delta C_L)^2$

The above formulae hold for all (small) positive or negative values of ΔC_L. It can be verified that the condition $p_2^2 - 4p_1p_3 < 0$ for positive drag is satisfied in each case. A table of values of C_D, and the corresponding values for the flat delta wing is given below.

				The corresponding flat delta wing
Surface	C_{L0}	C_L	$C_D \equiv C_{Di} + \Delta C_{Di}$	C_D
(i)	0·1	0·05	0·00128	0·00035
		0·1	0·00243	0·00139
		0·15	0·00427	0·00314
		0·2	0·00682	0·00558
(ii)	0·1	0·05	0·00444	0·00088
		0·1	0·00705	0·00352
		0·15	0·01142	0·00791
		0·2	0·01755	0·01406
(vi *a*)	0·1	0·05	0·00111	0·00035
		0·1	0·00212	0·00139
		0·15	0·00383	0·00314
		0·2	0·00624	0·00558
(vi *b*)	0·1	0·05	0·00242	0·00044
		0·1	0·00363	0·00178
		0·15	0·00572	0·00400
		0·2	0·00871	0·00712
(vii)	0·1	0·05	0·00047	0·00029
		0·1	0·00125	0·00117
		0·15	0·00261	0·00264
		0·2	0·00456	0·00469
(xii)	0·1	0·05	0·00071	0·00044
		0·1	0·00190	0·00178
		0·15	0·00398	0·00400
		0·2	0·00695	0·00712
(ix)	0·1	0·05	0·00170	0·00044
		0·1	0·00309	0·00178
		0·15	0·00536	0·00400
		0·2	0·00852	0·00712
(xi)	0·1	0·05	0·00128	0·00044
		0·1	0·00263	0·00178
		0·15	0·00487	0·00400
		0·2	0·00800	0·00712
(iii)	0	0·05	0·00532	0·00035
	(δ=0·01)	0·1	0·00593	0·00139
		0·15	0·00725	0·00314
		0·2	0·00926	0·00558
(viii)	0·1	0·05	0·02134	0·00088
		0·1	0·02393	0·00351
		0·15	0·02828	0·00791
		0·2	0·03439	0·01406

Surface	C_{L0}	C_L	$C_D = C_{Di} + \Delta C_{Di}$	The corresponding flat delta wing C_D
(xiii)	0·1	0·05	0·00077	0·00044
		0·1	0·00202	0·00178
		0·15	0·00416	0·00400
		0·2	0·00719	0·00712
(xi)	0·1	0·05	0·00073	0·00044
		0·1	0·00194	0·00178
		0·15	0·00404	0·00400
		0·2	0·00704	0·00712
(xiv)	0·1	0·05	0·00114	0·00060
		0·1	0·00290	0·00242
		0·15	0·00587	0·00544
		0·2	0·01004	0·00966
(xv)	0·1	0·05	0·00105	0·00060
		0·1	0·00271	0·00242
		0·15	0·00559	0·00544
		0·2	0·00967	0·00966
(xvi)	0·1	0·05	0·00105	0·00060
		0·1	0·00269	0·00242
		0·15	0·00555	0·00544
		0·2	0·00961	0·00966

For $\sigma = 1$ and $C_L > C_{L0}$, it can be shown that C_D is less than the induced-drag coefficient C_{Di} of the same surface (at design incidence) designed for lift coefficient C_L, *e.g.*, for (xvi), if $C_{L0} = 0{\cdot}1$ and $C_L = 0{\cdot}2$, $C_D = 0{\cdot}00961$; and if $C_{L0} = C_L = 0{\cdot}2$, $C_D = C_{Di} = 0{\cdot}01076$.

It can also be shown, that for $C_L > C_{L0}$ ($\sigma = 1$), if $p_2 - 2p_3 > 0$, C_D is greater than the corresponding C_D for the flat delta wing for all C_L; and if $p_2 - 2p_3 < 0$, C_D is less than the corresponding C_D for the flat delta wing when $C_L > \dfrac{p_1 + p_3 - p_2}{2p_3 - p_2} C_{L0}$ ($p_1 + p_3 - p_2$ is always > 0), *e.g.*, for (xvi), when $C_L > 0{\cdot}185$.

13. *Conclusion.*—Solutions of the linearised supersonic flow equations in terms of the Lamé functions of the M-class, of degree 1, 2, 3, 4 have been found, and have been combined to give a general solution for the velocity potential of the supersonic flow over swept-back wings, with modified pressure singularities on the leading edges. The solutions have been chosen so that the strength of these singularities decreases towards the wing tips. By removing the suction peaks near the leading edges of the outboard sections of the wing, the associated adverse pressure gradients are reduced, thereby reducing the tendency for the boundary layer to separate.

A number of examples for specified values of the apex angle γ and the Mach number M, have been worked out, and the corresponding lift, induced drag and pitching-moment coefficients calculated. The effect of additional incidence on the total induced drag coefficient has also been calculated (*cf.* end of section 12).

For the wings with no leading-edge singularities, the total induced drag is considerably higher than for the corresponding flat delta wing; but for the wings with leading-edge singularities, decreasing towards the wing tips, in some cases, for $1/\sigma$ greater than some value (> 1) (that is a value for which the point of zero pressure on a leading edge is downstream of the wing tip), the total induced drag is less than that for the corresponding flat delta wing (*cf.* Figs. 10b, 11b, 14b, 16b). An example is given (Fig. 22) of a wing designed for a given lift coefficient and minimum total induced drag, with the condition that there is no camber at the root.

An attempt was made to find a solution giving a surface with increasing camber towards the wing tips and little or no twist. Examination of the conditions required showed that this is not possible, at any rate with the general solution so far found. But solutions giving surfaces with increasing (or constant) camber and some twist can be found (*cf.* Fig. 17).

By including solutions for higher values of n, a still more general solution could be found. The complexity of the algebra increases with the value of n, but the basic solutions, once found, contribute to an unlimited number of other solutions.

The examples of wings given in this report are designed, in each case, for a specified Mach number. The effect of a change of Mach number on the aerodynamic characteristics of the wing is being considered, and the results will be given in another report.

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LIST OF SYMBOLS

γ	Apex semi-angle
c	Maximum chord of a triangular wing
d	Chordwise distance behind the apex of the point of zero pressure on a leading edge
or d	An arbitrary length, for wings with no pressure singularities ((i), (ii))
$1/\sigma \equiv d/c$	Chordwise distance, in maximum chord lengths, behind the apex of the point of zero pressure on a leading edge
or $1/\sigma \equiv d/c$	An arbitrary constant, for wings with no pressure singularities ((i), (ii))
S	Area of triangular wing
x	Chordwise co-ordinate (measured downstream from the apex)
y	Spanwise co-ordinate (positive to starboard)
z	Normal co-ordinate (positive upwards)
$x' = x\sigma/c$, $y' = y\sigma/c$, $z' = z\sigma/c$	Non-dimensional co-ordinates (The dashes are dropped in the numerical examples)
r, μ, ν	*cf.* equations (1), (2)

δ	A small dimensionless constant (proportional to the design lift coefficient C_{L0})
α	Angle of incidence (measured in radians)
X	$(x^2 - k^2y^2)^{1/2}$
X_1	$(x_1^2 - k^2y^2)^{1/2}$
$x = x_1(y)$	Defines the trailing edge of the wing
ρ	Free-stream density
V	Free-stream velocity
$\bar{\mu}$	Mach angle
M	Mach number
β	$(M^2 - 1)^{1/2}$
k	$\cot \gamma$
h	$(\cot^2 \gamma - \cot^2 \bar{\mu})^{1/2} = (k^2 - \beta^2)^{1/2}$
$\varkappa$	h/k
$f_1, f_2, \ldots f_{13}$	Functions of $(\tan \gamma/\tan \bar{\mu})$ given in Appendices I, II
A_s	*cf.* equation (126)
A,B,C,D,E	*cf.* equation (130)
a,b,d,f,g,h	*cf.* equations (127), (128)
$E_n(\mu)$	Standard Lamé function of degree n
$F_n(\mu)$	Lamé function of the second kind of degree n
$M_n(\mu)$	Standard Lamé function of degree n, of the M-class
$P_n(\mu)$	$M_n(\mu)/(\mu^2 - k^2)^{1/2}$
$K(\varkappa)$	Complete elliptic integral of the first kind, modulus $\varkappa$
$E(\varkappa)$	Complete elliptic integral of the second kind, modulus $\varkappa$
$c_r(r=1, 2, \ldots \overline{2N+1})$	A zero of $P_{2N+1}(\mu)$
$d_r(r=1, 2, \ldots 2N)$	A zero of $P_{2N}(\mu)$
H	$V\delta/(kE(\varkappa))$ (in Appendix VI)
P	Strength of singularity in axial velocity on a leading edge
φ	Velocity potential, *cf.* equation (3), etc.
ϕ	Velocity potential, *cf.* equations (17), (27), and sections 5, 6, 7
Φ	Velocity potential, *cf.* section 9, etc.
ψ	Velocity potential, *cf.* section 8, etc.
Ω	Velocity potential, *cf.* equations (126), (129)
Δp	Pressure on an element of the upper surface of the wing
C_p	Pressure coefficient
C_{p0}	Design pressure coefficient
$l(y)$	Spanwise lift distribution

C_{l0}	Local spanwise lift coefficient at design incidence
C_L	Lift coefficient (based on area)
C_{L0}	Design lift coefficient
C_{M0}	Pitching-moment coefficient
D_p	Pressure integral
D_s	Suction force at leading edge
$D_i = D_p - D_s$	Total drag due to lift (induced drag), at design incidence
C_{Di}	Total induced-drag coefficient, at design incidence, based on area
C_{Dv}	Vortex drag coefficient, at design incidence, based on area
$C_{Dw} = C_{Di} - C_{Dv}$	Induced wave-drag coefficient, at design incidence, based on area
ΔC_p	Pressure coefficient due to additional incidence α
$\Delta C_L = C_L - C_{L0}$	Lift coefficient due to additional incidence α
ΔC_{Di}	Total induced-drag coefficient due to additional incidence α
p_1, p_2, p_3	*cf.* equation (147)
$C_D \equiv C_{Di} + \Delta C_{Di}$	Total induced-drag coefficient at (additional) incidence α
$\bar{P}$	*cf.* Appendix VII.

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APPENDIX I

The functions $f_1(\tan\gamma/\tan\bar{\mu})$, $f_2(\tan\gamma/\tan\bar{\mu})$, ... $f_{13}(\tan\gamma/\tan\bar{\mu})$.

$\varkappa^2 = 1 - (\tan^2\gamma/\tan^2\bar{\mu})$; K, E are written for $K(\varkappa)$, $E(\varkappa)$.

Solutions in which the functions occur are given in brackets [ ].

$$f_1 = \{(2\varkappa^2 - 1)E + (1 - \varkappa^2)K\}/(2\varkappa^2 E) \qquad [\phi_3]$$

$$\left.\begin{aligned} f_2 &= \{(2 - \varkappa^2)E - 2(1 - \varkappa^2)K\}/(2\varkappa^4 E) \\ f_3 &= \{(1 - \varkappa^2)(2 - 5\varkappa^2)K - 2(1 - 3\varkappa^2 - \varkappa^4)E\}/(2\varkappa^4 E) \end{aligned}\right\} \quad [\phi_3^{\,3}]$$

$$\left.\begin{aligned} f_4 &= \{(2\varkappa^2 - 1)E + (1 - \varkappa^2)K\}/(2\varkappa^2 E) = f_1 \\ f_5 &= 3\{(1 + \varkappa^2)E - (1 - \varkappa^2)K\}/(2\varkappa^2 E) \end{aligned}\right\} \quad [\psi_3]$$

$$\left.\begin{aligned} f_6 &= \{(1 - \varkappa^2)(2 + 3\varkappa^2)K - 2(1 + \varkappa^2 - 3\varkappa^4)E\}/(2\varkappa^4 E) \\ f_7 &= \{(2 - 3\varkappa^2 + \varkappa^4)E - 2(1 - \varkappa^2)^2 K\}/(2\varkappa^4 E) \end{aligned}\right\} \quad [\phi_3^{\,1}]$$

$$\left.\begin{aligned} f_8 &= \{(8 - 3\varkappa^2 - 2\varkappa^4)E - (1 - \varkappa^2)(8 + \varkappa^2)K\}/(6\varkappa^6 E) \\ f_9 &= \{(1 - \varkappa^2)(8 - 13\varkappa^2 + 2\varkappa^4)K \\ &\quad - (8 - 17\varkappa^2 + 7\varkappa^4 - 4\varkappa^6)E\}/(2\varkappa^6 E) \end{aligned}\right\} \quad [\phi_4^{\,2}]$$

$$\left.\begin{aligned} f_{10} &= \{2(1 - \varkappa^2)(1 + 2\varkappa^2)K - (2 + 3\varkappa^2 - 8\varkappa^4)E\}/(2\varkappa^4 E) \\ f_{11} &= 3\{2(1 - \varkappa^2 + \varkappa^4)E - (1 - \varkappa^2)(2 - \varkappa^2)K\}/(2\varkappa^4 E) \end{aligned}\right\} \quad [\psi_4]$$

$$\left.\begin{aligned} f_{12} &= \{(1 - \varkappa^2)(8 + 7\varkappa^2 + 12\varkappa^4) \\ &\quad - (8 + 3\varkappa^2 + 7\varkappa^4 - 24\varkappa^6)\}/(6\varkappa^6 E) \\ f_{13} &= \{(8 - 11\varkappa^2 + \varkappa^4 + 2\varkappa^6)E \\ &\quad - (1 - \varkappa^2)(8 - 7\varkappa^2 - \varkappa^4)K\}/(2\varkappa^6 E) \end{aligned}\right\} \quad [\phi_4^{\,1}]$$

APPENDIX II

The Functions f_1, f_2, ... f_{13}. Numerical Values

$\frac{\tan \gamma}{\tan \bar{\mu}}$	$\varkappa^2$	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}
0	1	0·5	0·5	3·0	0·5	3·0	1·0	0	0·5	3·0	1·5	3·0	1·0	0
0·1	0·99	0·5135	0·4781	2·9552	0·5135	2·9600	1·0624	0·0048	0·4711	2·9602	1·5751	2·9744	1·1040	0·0142
0·2	0·96	0·5390	0·4396	2·8655	0·5390	2·8831	1·1774	0·0176	0·4234	2·8850	1·7163	2·9358	1·2929	0·0508
0·3	0·91	0·5690	0·3977	2·7570	0·5690	2·7929	1·3093	0·0359	0·3742	2·7992	1·8786	2·9002	1·5044	0·1010
0·4	0·84	0·6000	0·3571	2·6428	0·6000	2·6999	1·4429	0·0571	0·3287	2·7135	2·0430	2·8713	1·7143	0·1578
0·5	0·75	0·6300	0·3197	2·5298	0·6300	2·6097	1·5703	0·0799	0·2884	2·6332	2·2007	2·8495	1·9123	0·2163
0·6	0·64	0·6585	0·2861	2·4217	0·6585	2·5247	1·6894	0·1030	0·2532	2·5603	2·3476	2·8338	2·0944	0·2735
0·7	0·51	0·6845	0·2566	2·3206	0·6845	2·4463	1·7969	0·1257	0·2234	2·4951	2·4817	2·8235	2·2583	0·3284
0·8	0·36	0·7085	0·2303	2·2270	0·7085	2·3743	1·8952	0·1473	0·1972	2·4381	2·6038	2·8167	2·4066	0·3786
0·9	0·19	0·7300	0·2080	2·1408	0·7300	2·3093	1·9820	0·1685	0·1760	2·3850	2·7125	2·8146	2·5365	0·4306
1·0	0	0·7500	0·1875	2·0625	0·7500	2·2500	2·0625	0·1875	0·1562	2·3437	2·8125	2·8125	2·6562	0·4687

APPENDIX III

Integration of $I \equiv \int_k^\infty \frac{d}{dt}\left[\frac{1}{t\,[P_n(t)]^2(t^2 - h^2)^{1/2}}\right]\frac{dt}{(t^2 - k^2)^{1/2}}$

$M_n(\mu) = (|\mu^2 - k^2|)^{1/2}P_n(\mu)$ is a solution of Lamé's equation, and it is easy to show that the differential equation satisfied by $P_n(\mu)$ is

$$(\mu^2 - h^2)(\mu^2 - k^2)\frac{d^2P}{d\mu^2} + \mu(4\mu^2 - 3h^2 - k^2)\frac{dP}{d\mu}$$
$$+ [p(h^2 + k^2) - h^2 - (n^2 + n-2)\mu^2]P = 0. \quad \text{(III,1)}$$

The roots of $P_n(\mu) = 0$ are all real and unequal, and not equal to $\pm h$ or $\pm k$ (Ref. 3), therefore $P_n(\mu)$ can be expressed in the form

$$P_n(\mu) \equiv P_{2N+1}(\mu) = \prod_{r=1}^{N} (\mu^2 - c_r), \text{ if } n \text{ is odd,} \quad \text{(III,2)}$$

where the c_r's are real and unequal ; and

$$P_n(\mu) \equiv P_{2N}\mu = \mu \prod_{r=1}^{N-1} (\mu^2 - d_r), \text{ if } n \text{ is even,} \quad \text{(III,3)}$$

where the d_r's are real and unequal, N being a positive integer.

Substituting (III,2) in (III,1), and putting $\mu^2 = c_r$, it can be shown, after some simplification, that

$$\frac{1}{2c_r}\left\{5 + \frac{h^2}{c_r - h^2} + \frac{3k^2}{c_r - k^2}\right\} + 2\sum_{s=1}^{N}\left(\frac{1}{c_r - c_s}\right) = 0. \quad \text{(III,4)}$$
$$s \neq r.$$

Similarly, by substituting (III,3) in (III,1), and putting $\mu^2 = d_r$, it can be shown that

$$\frac{1}{2d_r}\left\{7 + \frac{h^2}{d_r - h^2} + \frac{3k^2}{d_r - k^2}\right\} + 2\sum_{s=1}^{N-1}\left(\frac{1}{d_r - d_s}\right) = 0. \quad \text{(III,5)}$$
$$s \neq r.$$

For $n = 2N+1$,

$$\frac{1}{[P_n(t)]^2} \equiv \frac{1}{[P_{2N+1}(t)]^2} = \frac{1}{\prod_{r=1}^{N} (t^2 - c_r)^2}$$
$$= \sum_{r=1}^{N}\left[\frac{A_r^2}{(t^2 - c_r)^2} + \frac{2A_r}{t^2 - c_r}\sum_{s=1}^{N}\left(\frac{A_s}{c_r - c_s}\right)\right], \quad s \neq r \quad \text{(III,6)}$$

where $A_r = \dfrac{1}{P_{2N+1}'(c_r)}$, the dash indicating differentiation with respect to the argument c_r, and $P_{2N+1}(c_r) \equiv [P_{2N+1}(t)]_{t^2=c_r}$

Therefore

$$(I)_{n=2N+1} = \sum_{r=1}^{N}\left[A_r^2\int_k^\infty \frac{d}{dt}\left(\frac{1}{t(t^2 - c_r)^2(t^2 - h^2)^{1/2}}\right)\frac{dt}{(t^2 - k^2)^{1/2}}\right.$$
$$\left. + 2A_r\sum_{s=1}^{N}\left(\frac{A_s}{c_r - c_s}\right)\left\{\int_k^\infty \frac{d}{dt}\left(\frac{1}{t(t^2 - c_r)(t^2 - h^2)^{1/2}}\right)\frac{dt}{(t^2 - k^2)^{1/2}}\right\}\right]$$
$$= \sum_{r=1}^{N}\left[A_r^2\left\{I_1 + \frac{2}{A_r}I_2\sum_{s=1}^{N}\left(\frac{A_s}{c_r - c_s}\right)\right\}\right], \quad s \neq r. \quad \text{(III,7)}$$

To evaluate I_1 and I_2, we put $t = k \operatorname{ns} u$, where $\operatorname{ns} u$ is a Jacobian elliptic function of modulus h/k, and write $h/k = \varkappa$, $c_r/k^2 = \varepsilon_r^2$. The first and second complete elliptic integrals, with modulus $\varkappa$, are denoted by K, E respectively.

It can be shown that

$$
\begin{aligned}
k^7 I_1 &= -\int_0^{K(\varkappa)} \frac{d}{du}\left[\frac{\operatorname{sn}^6 u}{(1-\varepsilon_r^2 \operatorname{sn}^2 u)^2 \operatorname{dn} u}\right] \frac{\operatorname{sn} u}{\operatorname{cn} u}\, du \\
&= \int_0^{K(\varkappa)} \frac{\operatorname{sn}^6 u}{(1-\varepsilon^2 \operatorname{sn}^2 u)^2 \operatorname{cn}^2 u} - \left[\frac{\operatorname{sn}^7 u}{\operatorname{cn} u \operatorname{dn} u (1-\varepsilon^2 \operatorname{sn}^2 u)^2}\right]_{u=K} \\
&= -\frac{1}{2\varepsilon_r^4(\varepsilon_r^2-1)}\left(5+\frac{3}{\varepsilon_r^2-1}+\frac{\varkappa^2}{\varepsilon_r^2-\varkappa^2}\right)\int_0^K \frac{du}{1-\varepsilon_r^2 \operatorname{sn}^2 u} \\
&\quad -\left(\frac{2\varepsilon_r^2(\varepsilon_r^2-\varkappa^2)+(1-\varkappa^2)}{2\varepsilon_r^2(\varepsilon_r^2-1)^2(1-\varkappa^2)(\varepsilon_r^2-\varkappa^2)}\right)E + \frac{4\varepsilon_r^2-1}{2\varepsilon_r^4(\varepsilon_r^2-1)^2} K \qquad \text{(III,8)}
\end{aligned}
$$

and

$$
\begin{aligned}
k^5 I_2 &= -\int_0^{K(\varkappa)} \frac{d}{du}\left[\frac{\operatorname{sn}^4 u}{(1-\varepsilon_r^2 \operatorname{sn}^2 u)\operatorname{dn} u}\right]\frac{\operatorname{sn} u}{\operatorname{cn} u}\, du \\
&= \int_0^K \frac{\operatorname{sn}^4 u\, du}{(1-\varepsilon_r^2 \operatorname{sn}^2 u)\operatorname{cn}^2 u} - \left[\frac{\operatorname{sn}^5 u}{\operatorname{cn} u \operatorname{dn} u}\left(\frac{1}{1-\varepsilon_r^2 \operatorname{sn}^2 u}\right)\right]_{u=K} \\
&= \frac{1}{\varepsilon_r^2(\varepsilon_r^2-1)}\int_0^K \frac{du}{1-\varepsilon_r^2 \operatorname{sn}^2 u} - \frac{1}{\varepsilon_r^2(\varepsilon_r^2-1)} K + \frac{1}{(\varepsilon_r^2-1)(1-\varkappa^2)} E. \qquad \text{(III,9)}
\end{aligned}
$$

Substituting (III,8), (III,9) in (III,7), the coefficient of

$$\frac{A_r^2}{k^7 \varepsilon_r^2(\varepsilon_r^2-1)}\int_0^K \frac{du}{1-\varepsilon_r^2 \operatorname{sn}^2 u}, \text{ in the expression for } I, \text{ is}$$

$$
\begin{aligned}
&-\frac{1}{2\varepsilon_r^2}\left(5+\frac{3}{\varepsilon_r^2-1}+\frac{\varkappa^2}{\varepsilon_r^2-\varkappa^2}\right)+\frac{2k^2}{A_r}\sum_{s=1}^{N}\frac{A_s}{c_r-c_s} \qquad (s \neq r) \\
&= -k^2\left[\frac{1}{2c_r}\left(5+\frac{3k^2}{c_r-k^2}+\frac{h^2}{c_r-h^2}\right)+2\sum_{s=1}^{N}\left(\frac{1}{c_r-c_s}\right)\right] \\
&= 0, \text{ by (III,4).}
\end{aligned}
$$

Hence it can be shown that

$$
\begin{aligned}
(I)_{n=2N+1} &= \frac{1}{k}\sum_{r=1}^{N}\left[\frac{1}{[P_{2N+1}'(c_r)]^2}\left\{\frac{k^2\beta^2-2c_r(h^2-c_r)}{2\beta^2 c_r(k^2-c_r)^2(h^2-c_r)} E\right.\right. \\
&\quad \left.\left. + \frac{4c_r-k^2}{2c_r^2(k^2-c_r)^2} K - 2\sum_{s=1}^{N}\left(\frac{1}{c_r-c_s}\right)\left(\frac{1}{k^2-c_r}\right)\left(\frac{1}{c_r}K-\frac{1}{\beta^2}E\right)\right\}\right] \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad s \neq r \\
&= \frac{1}{k}\sum_{r=1}^{N}\left[\frac{1}{[P_{2N+1}'(c_r)]^2 2c_r}\left(\frac{1}{k^2-c_r}\right)\left\{\frac{k^2\beta^2-2c_r(h^2-c_r)}{\beta^2(k^2-c_r)(h^2-c_r)} E\right.\right. \\
&\quad \left.\left. + \frac{4c_r-k^2}{c_r(k^2-c_r)} K + \left(5-\frac{h^2}{h^2-c_r}-\frac{3k^2}{k^2-c_r}\right)\left(\frac{1}{c_r}K-\frac{1}{\beta^2}E\right)\right\}\right], \qquad \text{(III,10)}
\end{aligned}
$$

using the relation (III,4).
For $n = 2N$,

$$\frac{1}{[P_n(t)]^2} \equiv \frac{1}{[P_{2N}(t)]^2} = \frac{1}{t^2 \prod_{r=1}^{N-1} (t^2 - d_r)^2} .$$

Therefore

$$(I)_{n=2N} = \sum_{r=1}^{N-1} \left[B_r^2 \left\{ I_3 + \frac{2I_4}{B_r} \sum_{s=1}^{N-1} \left(\frac{B_s}{d_r - d_s} \right) \right\} \right] , \qquad s \neq r \qquad \ldots \quad \ldots \quad \text{(III,11)}$$

where

$$B_r = \frac{1}{\left(\frac{P_{2N}}{t}\right)'(d_r)} , \quad I_3 = \int_k^\infty \frac{d}{dt} \left(\frac{1}{t^3 (t^2 - d_r)^2 (t^2 - h^2)^{1/2}} \right) \frac{dt}{(t^2 - k^2)^{1/2}}$$

and

$$I_4 = \int_k^\infty \frac{d}{dt} \left(\frac{1}{t^3 (t^2 - d_r)(t^2 - h^2)^{1/2}} \right) \frac{dt}{(t^2 - k^2)^{1/2}} .$$

$$\left[\left(\frac{P_{2N}}{t}\right)'(d_r) \text{ is written for } \left\{ \frac{d}{d(t^2)} \frac{P_{2N}(t)}{t} \right\}_{t^2 = d_r} \right] .$$

Again using the substitution $t = k \operatorname{ns} u$, and writing $d_r/k = \delta_r^2$, it can be shown that

$$k^9 I_3 = -\frac{1}{2\delta_r^6 (\delta_r^2 - 1)} \left(7 + \frac{3}{\delta_r^2 - 1} + \frac{\varkappa^2}{\delta_r^2 - \varkappa^2} \int_0^K \frac{du}{1 - \delta_r^2 \operatorname{sn}^2 u} \right.$$
$$+ \frac{3\varkappa^2 (2\delta_r^2 - 1) - 2\delta_r^2 (\delta_r^2 - 1)^2}{2\delta_r^6 \varkappa^2 (\delta_r^2 - 1)^2} K + \left(\frac{1}{\delta_r^4 \varkappa^2} - \frac{1}{(1 - \varkappa^2)(\delta_r^2 - 1)^2} \right.$$
$$\left. - \frac{1}{2\delta_r^4 (\delta_r^2 - 1)^2 (\delta_r^2 - \varkappa^2)} \right) E \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(III,12)}$$

and

$$k^7 I_4 = \frac{\varkappa^2 (2\delta_r^2 - 1) - (\delta_r^2 - 1)}{\varkappa^2 \delta_r^2 (\delta_r^2 - 1)(1 - \varkappa^2)} E + \frac{\delta_r^2 (\delta_r^2 - 1) - \varkappa^2}{\varkappa^2 \delta_r^4 (\delta_r^2 - 1)} K$$
$$+ \frac{1}{\delta_r^4 (\delta_r^2 - 1)} \int_0^K \frac{du}{1 - \delta_r^2 \operatorname{sn}^2 u} . \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(III,13)}$$

Substituting (III,12) and (III,13) in (III,11), the coefficient of

$$\frac{B_r^2}{k^9 \delta_r^4 (\delta_r^2 - 1)} \int_0^K \frac{du}{1 - \delta_r^2 \operatorname{sn}^2 u} \text{, in the expression for } I \text{, is}$$

$$-\frac{1}{2\delta_r^2} \left(7 + \frac{3}{\delta_r^2 - 1} + \frac{\varkappa^2}{\delta_r^2 - \varkappa^2} \right) - 2 \sum_{s=1}^{N-1} \left(\frac{1}{\delta_r^2 - \delta_s^2} \right) , \quad s \neq r$$

$$= 0 \qquad \text{by (III,5).}$$

Hence it can be shown that

$$(I)_{n=2N} = \frac{1}{k^3}\sum_{r=1}^{N-1}\left[\frac{1}{\left[\left(\frac{P_{2N}}{t}\right)'(d_r)\right]^2}\left\{\left(\frac{1}{h^2 d_r^2} - \frac{1}{\beta^2(k^2-d_r)^2}\right.\right.\right.$$

$$+ \left.\frac{k^4}{2d_r^2(k^2-d_r)^2(h^2-d_r)}\right)E - \frac{3h^2k^2(k^2-2d_r)+2d_r(k^2-d_r)^2}{2h^2d_r^3(k^2-d_r)^2}K$$

$$+ \frac{1}{2d_r}\left(7+\frac{h^2}{d_r-h^2}+\frac{3k^2}{d_r-k^2}\right)\left(\frac{h^2+d_r(k^2-d_r)}{h^2d_r^2(k^2-d_r)}K\right.$$

$$\left.\left.\left. + \frac{h^2(k^2-2d_r)-(k^2-d_r)}{\beta^2h^2d_r(k^2-d_r)}E\right)\right\}\right]. \quad \text{(III,14)}$$

APPENDIX IV

The values of $\varphi_1 - \varphi_2$, $a_2\varphi_1 - a_1\varphi_2$, $a_1\varphi_1, - a_2\varphi_2$, *when* $\mu \longrightarrow k$, *for* $n = 3$ *and* $n = 4$.
(Cartesian co-ordinates x, y, z)

For $n = 3$, when $\mu \longrightarrow k$,

$$\varphi_1 - \varphi_2 = -\frac{V\delta}{3\beta^2d^2h^2k^3E(\varkappa)}(x^2-k^2y^2)^{3/2} \quad \text{(IV,1)}$$

$$a_2\varphi_1 - a_1\varphi_2 = -\frac{V\delta}{15\beta^2d^2k^3E(\varkappa)}(4x^2-k^2y^2)(x^2-k^2y^2)^{1/2} \quad \text{(IV,2)}$$

$$a_1\varphi_1 - a_2\varphi_2 = -\frac{V\delta}{15\beta^2d^2h^2kE(\varkappa)}[2x^2-(2k^2+3h^2)y^2](x^2-k^2y^2)^{1/2}. \quad \text{(IV,3)}$$

For $n = 4$, when $\mu \longrightarrow k$,

$$\varphi_1 - \varphi_2 = \frac{-V\delta}{3\beta^2d^3h^2k^3E(\varkappa)}x(x^2-k^2y^2)^{3/2}. \quad \text{(IV,4)}$$

$$a_2\varphi_1 - a_1\varphi_2 = \frac{-V\delta}{7\beta^2d^3k^3E(\varkappa)}(2x^2-k^2y^2)(x^2-k^2y^2)^{1/2} \quad \text{(IV,5)}$$

$$a_1\varphi_1 - a_2\varphi_2 = \frac{-V\delta}{21\beta^2d^3h^2kE(\varkappa)}[4x^2-(3h^2+4k^2)y^2](x^2-k^2y^2)^{1/2}. \quad \text{(IV,6)}$$

APPENDIX V

The values of $\frac{\partial}{\partial z}(\varphi_1 - \varphi_2)$, $\frac{\partial}{\partial z}(a_2\varphi_1 - a_1\varphi_2)$, $\frac{\partial}{\partial z}(a_1\varphi_1 - a_2\varphi_2)$, *when* $\mu \longrightarrow k$, *for* $n = 3$ *and* $n = 4$.

(Cartesian co-ordinates x, y, z)

For $n = 3$, when $\mu \longrightarrow k$,

$$\frac{\partial}{\partial z}(\varphi_1 - \varphi_2) = \frac{V\delta}{2\beta^2 d^2 h^4 k^2 E(\varkappa)}\Big[\{(2h^2 - k^2)E(\varkappa) + \beta^2 K(\varkappa)\}x^2 - \{(k^2 + h^2)E(\varkappa) - \beta^2 K(\varkappa)\}k^2y^2\Big]$$

$$\frac{\partial}{\partial z}(a_2\varphi_1 - a_1\varphi_2) = \frac{V\delta}{10\beta^2 d^2 h^4 k^2 E(\varkappa)}\Big[\{2\beta^2(2h^2 + k^2)K(\varkappa) - (2k^4 + 3k^2h^2 - 8h^4)E(\varkappa)\}x^2 + \{\beta^2(2k^2 - h^2)K(\varkappa) - 2(k^4 - k^2h^2 + h^4)E(\varkappa)\}k^2y^2\Big]$$

$$\frac{\partial}{\partial z}(a_1\varphi_1 - a_2\varphi_2) = \frac{V\delta}{10\beta^2 d^2 h^2 E(\varkappa)}\Big[3E(\varkappa)x^2 - \{2(4k^2 + h^2)E(\varkappa) - 5\beta^2 K(\varkappa)\}y^2\Big].$$

For $n = 4$, when $\mu \longrightarrow k$,

$$\frac{\partial}{\partial z}(\varphi_1 - \varphi_2) = \frac{V\delta}{6\beta^2 d^3 h^6 k^2 E(\varkappa)}\Big[\{2\beta^2(k^2 + 2h^2)K(\varkappa) + (8h^4 - 3h^2k^2 - 2k^4)E(\varkappa)\}x^3 - 3\{\beta^2(h^2 - 2k^2)K(\varkappa) + 2(k^4 - h^2k^2 + h^4)E(\varkappa)\}k^2xy^2\Big]$$

$$\frac{\partial}{\partial z}(a_2\varphi_1 - a_1\varphi_2) = \frac{V\delta}{42\beta^2 d^3 h^6 k^2 E(\varkappa)}\Big[\{\beta^2(24h^4 + 13h^2k^2 + 8k^4)K(\varkappa) + (48h^6 - 16h^4k^2 - 9h^2k^4 - 8k^6)E(\varkappa)\}x^3 + 3\{\beta^2(8k^4 - h^2k^2 - 4h^4)K(\varkappa) - (8k^6 - 5h^2k^2 - 5h^4k^2 + 8h^6)E(\varkappa)\}k^2xy^2\Big]$$

$$\frac{\partial}{\partial z}(a_1\varphi_1 - a_2\varphi_2) = \frac{V\delta}{14\beta^2 d^3 h^4 E(\varkappa)}\Big[\{5\beta^2 K(\varkappa) + 5(2h^2 - k^2)E(\varkappa)\}x^3 + \{\beta^2(9k^2 - 2h^2)K(\varkappa) - (9k^4 + h^2k^2 + 4h^4)E(\varkappa)\}xy^2\Big].$$

APPENDIX VI

Basic solutions for $n = 1, 2, 3, 4, (5, 6)$. $\quad H \equiv \dfrac{V\delta}{kE(\varkappa)}$, $X \equiv (x^2 - k^2y^2)^{1/2}$, $\varkappa = h/k$

Non-dimensional co-ordinates, $x' = x\sigma/c$, etc. (The dashes are dropped)

n	No. of M solutions	φ_m	$(\phi_n^s)_{\mu=k}$	$\dfrac{1}{H}\left(\dfrac{\partial \phi_n^s}{\partial x}\right)_{\mu=k}$	$z_{n,s}$
1	1	$C_1 r F_1(\mu) E_1(\nu)$	$\phi_1 = HX$	$\dfrac{x}{X}$	$-\delta x$
2	1	$C_2 r^2 F_2(\mu) E_2(\nu)$	$\phi_2 = HxX$	$\dfrac{x^2}{X} + X$	$-\delta f_1 x^2$
3	2	$C_3 r^3 F_3^m(\mu) E_3^m(\nu)$	$\phi_3^1 = Hx^2X$	$\dfrac{x^3}{X} + 2x\,X$	$-\delta(\frac{1}{3}f_6x^3 - f_7k^2xy^2)$
			$\phi_3^2 = Hy^2X$	$\dfrac{y^2x}{X}$	$\dfrac{\delta}{k^2}(\frac{1}{3}f_2x^3 - f_3k^2xy^2)$
4	2	$C_4 r^4 F_4^m(\mu) E_4^m(\nu)$	$\phi_4^1 = Hx^3X$	$\dfrac{x^4}{X} + 3x^2X$	$-\delta(\frac{1}{4}f_{12}x^4 - \frac{1}{2}f_{13}k^2x^2y^2)$
			$\phi_4^2 = Hxy^2X$	$y^2\left(\dfrac{x^2}{X} + X\right)$	$\dfrac{\delta}{k^2}(\frac{1}{4}f_8x^4 - \frac{1}{2}f_9k^2x^2y^2)$
5	3	$C_5 r^5 F_5^m(\mu) E_5^m(\nu)$	$\phi_5^1 = Hx^4X$	$\dfrac{x^5}{X} + 4x^3X$	
			$\phi_5^2 = Hx^2y^2X$	$y^2x\left(\dfrac{x^2}{X} + 2X\right)$	
			$\phi_5^3 = Hy^4X$	$\dfrac{xy^4}{X}$	

APPENDIX VI—*continued*

Basic solutions for $n = 1, 2, 3, 4, (5, 6)$. $H \equiv \dfrac{V\delta}{kE(\varkappa)}$, $X \equiv (x^2 - k^2y^2)^{1/2}$, $\varkappa = h/k$

Non-dimensional co-ordinates, $x' = x\sigma/c$, *etc.* (The dashes are dropped)

n	No. of M solutions	φ_m	$(\phi_n^s)_{\mu=k}$	$\frac{1}{H}\left(\frac{\partial\phi_n^s}{\partial x}\right)_{\mu=k}$	$z_{n,s}$
6	3	$C_6r^6F_6^m(\mu)E_6^m(\nu)$	$\phi_6^1 = Hx^5X$	$\frac{x^6}{X} + 5x^4X$	
			$\phi_6^2 = Hx^3y^2X$	$x^2y^2\left(\frac{x^2}{X} + 3X\right)$	
			$\phi_6^3 = Hxy^4X$	$y^4\left(\frac{x^2}{X} + X\right)$	

APPENDIX VII

Formulae for the Local Spanwise Lift, the Total Lift, the Induced Drag, and the Pitching-Moment Coefficients for the Surfaces Shown in Figs. 6 to 17

The numbers (i), (ii), ... of the surfaces correspond to those in the text (section 12).

Non-dimensional co-ordinates $x' = x\sigma/c$, $y' = y\sigma/c$, $z' = z\sigma/c$. The dashes are dropped.

(i) $$C_{l0} = \frac{4\delta}{kE(\varkappa)}(1 - ky)^{1/2}(1 + ky)^{3/2}$$ (Fig. 6)

$$C_{L0} = \frac{3\pi\delta}{2kE(\varkappa)}, \qquad C_{M0} = \frac{-2}{5}\frac{\pi\delta}{kE(\varkappa)}$$

$$C_{Dp}/(C_{L0}^2/\pi A) = \tfrac{4}{9}E(\varkappa)(12f_4 - f_5)$$

$$C_{Ds} = 0, \qquad \varepsilon \equiv C_{Dv}/(C_{L0}^2/\pi A) = \tfrac{4}{3}$$

(ii) $$C_{l0} = \frac{4\delta}{kE(\varkappa)}(1 - ky)^{1/2}(1 + ky)^{3/2}$$ (Fig. 7)

$$C_{L0} = \frac{3\pi\delta}{2kE(\varkappa)}, \qquad C_{M0} = -\tfrac{1}{2}\frac{\pi\delta}{kE(\varkappa)}$$

$$C_{Dp}/(C_{L0}^2/\pi A) = \tfrac{1}{18}E(\varkappa)(30f_{10} - 7f_{11})$$

$$C_{Ds} = 0, \qquad C_{Dv}/(C_{L0}^2/\pi A) = \tfrac{4}{3}$$

(vi) $$C_{l0} = \frac{4\delta}{kE(\varkappa)}\left(\frac{\sigma + ky}{\sigma - ky}\right)^{1/2}(1 - k^2y^2)$$ (Figs. 8, 9)

$$C_{L0} = \frac{2\pi\delta}{kE(\varkappa)}(1 - \tfrac{1}{4}\sigma^2), \qquad C_{M0} = \tfrac{2}{15}\frac{\pi\delta}{kE(\varkappa)}\sigma^2$$

$$C_{Dp}/(C_{L0}^2/\pi A) = 2E(\varkappa)[1 - (\tfrac{1}{4} - \tfrac{1}{2}f_2 + \tfrac{1}{4}f_3)\sigma^2 - (\tfrac{1}{6}f_2 - \tfrac{1}{8}f_3)\sigma^4]/(1 - \tfrac{1}{4}\sigma^2)^2$$

$$C_{Ds}/(C_{L0}^2/\pi A) = \frac{(k^2 - \beta^2)^{1/2}}{k}(1 - \sigma^2 + \tfrac{1}{3}\sigma^4)/(1 - \tfrac{1}{4}\sigma^2)^2$$

$$C_{Dv}/(C_{L0}^2/\pi A) \equiv \varepsilon = 1 + 3\sigma^4/(4 - \sigma^2)^2$$

(vii) $$C_{l0} = \frac{4\delta}{kE(\varkappa)}\left(\frac{\sigma + ky}{\sigma - ky}\right)^{1/2}(1 - \sigma k^2y^2)$$ (Fig. 10)

$$C_{L0} = \frac{2\pi\delta}{kE(\varkappa)}(1 - \tfrac{1}{4}\sigma^3), \quad C_{M0} = \tfrac{1}{12}\frac{\pi\delta}{kE(\varkappa)}\sigma^3$$

$$C_{Dp}/(C_{L0}^2/\pi A) = 2E(\varkappa)\frac{[1 - (\tfrac{1}{4} - \tfrac{2}{5}f_8 + \tfrac{1}{5}f_9)\sigma^3 - (\tfrac{5}{32}f_8 - \tfrac{7}{64}f_9)\sigma^6]}{(1 - \tfrac{1}{4}\sigma^3)^2}$$

$$C_{Ds}/(C_{L0}^2/\pi A) = \frac{(k^2 - \beta^2)^{1/2}}{k}(1 - \tfrac{4}{5}\sigma^3 + \tfrac{1}{4}\sigma^6)(1 - \tfrac{1}{4}\sigma^3)^2$$

$$C_{Dv}/(C_{L0}^2/\pi A) \equiv \varepsilon = 1 + 3\sigma^6/(4 - \sigma^3)^2$$

(xii) $\gamma = 45°$, $M = 1\cdot 281$ (Fig. 11)

$$C_{l0} = \frac{4\delta}{E(\varkappa)}\left(\frac{\sigma + y}{\sigma - y}\right)^{1/2} [0\cdot 7085(1 - \sigma^2) - 0\cdot 2604(1 - \sigma^3)$$
$$+ (0\cdot 6317 - 0\cdot 2407\sigma)(\sigma^2 - y^2)]$$

$$C_{L0} = \frac{2\pi\delta}{E(\varkappa)} [0\cdot 4481 - 0\cdot 2347\sigma^2 + 0\cdot 0799\sigma^3]$$

$$C_{M0} = -\frac{2\pi\delta}{E(\varkappa)} [0\cdot 06260\sigma^2 - 0\cdot 02662\sigma^3]$$

$C_{Dp}/(C_{L0}^2/\pi A) = 11\cdot 3448\bar{P}/C^2$, where :

$$\bar{P} = 0\cdot 7085P_2 + 0\cdot 6317P_3 - 0\cdot 2604P_4 - 0\cdot 2407P_5$$
$$P_2 = 0\cdot 11203 - 0\cdot 19925\sigma^2 + 0\cdot 02896\sigma^3 + 0\cdot 08722\sigma^4 - 0\cdot 03103\sigma^5$$
$$P_3 = 3\sigma^2[0\cdot 02801 - 0\cdot 01454\sigma^2 + 0\cdot 00517\sigma^3]$$
$$P_4 = 0\cdot 11203 - 0\cdot 08722\sigma^2 - 0\cdot 08307\sigma^3 + 0\cdot 08722\sigma^5 - 0\cdot 03168\sigma^6$$
$$P_5 = \sigma^3[0\cdot 08402 - 0\cdot 04361\sigma^2 + 0\cdot 01584\sigma^3]$$

$$C \equiv \frac{E(\varkappa)}{2\pi\delta} C_{L0}$$

$$C_{Ds}/(C_{L0}^2/\pi A) = \frac{0\cdot 6}{C^2} [0\cdot 200811 - 0\cdot 317493\sigma^2 + 0\cdot 093345\sigma^3$$
$$+ 0\cdot 167324\sigma^4 - 0\cdot 105417\sigma^5 + 0\cdot 016949\sigma^6]$$

$$C_{Dv}/(C_{L0}^2/\pi A) \equiv \varepsilon = 1 + \frac{3\,(0\cdot 157925\sigma^2 - 0\cdot 060165\sigma^3)^2}{(0\cdot 44812 - 0\cdot 234725\sigma^2 + 0\cdot 079885\sigma^3)^2}$$

(ix) $\gamma = 45°$, $M = 1\cdot 281$ (Fig. 12)

$$C_{l0} \equiv \frac{4\delta}{E(\varkappa)}\left(\frac{\sigma + y}{\sigma - y}\right)^{1/2} [-(1 - \sigma) + 0\cdot 4597(1 - \sigma^2)$$
$$+ 0\cdot 56824(1 - \sigma^3) + (2\cdot 3796 + 2\cdot 7500\sigma)(\sigma^2 - y^2)]$$

$$C_{L0} = \frac{2\pi\delta}{E(\varkappa)} [0\cdot 02794 + \sigma + 1\cdot 325\sigma^2 + 1\cdot 4942\sigma^3]$$

$$C_{M0} = \frac{2\pi\delta}{E(\varkappa)} [-\tfrac{1}{6}\sigma - 0\cdot 353334\sigma^2 - 0\cdot 498082\sigma^3]$$

$C_{Dp}/(C_{L0}^2/\pi A) = 11\cdot 3448\bar{P}/C^2$, where :

$$\bar{P} = -P_1 + 0\cdot 4597P_2 + 2\cdot 3796P_3 + 0\cdot 5682P_4 + 2\cdot 7500P_5$$
$$P_1 = 0\cdot 00697 + 0\cdot 22920\sigma - 0\cdot 09124\sigma^2 - 0\cdot 07637\sigma^3 - 0\cdot 33188\sigma^4$$
$$P_2 = 0\cdot 00697 + 0\cdot 23617\sigma + 0\cdot 16748\sigma^2 - 0\cdot 08065\sigma^3 - 0\cdot 34890\sigma^4 - 0\cdot 42412\sigma^5$$

$$\bar{P}_3 = 3\sigma^2[0\cdot 00174 + 0\cdot 07085\sigma + 0\cdot 11630\sigma^2 + 0\cdot 13965\sigma^3]$$

$$P_4 = 0\cdot 00697 + 0\cdot 23617\sigma + 0\cdot 17445\sigma^2 + 0\cdot 19578\sigma^3 - 0\cdot 29521\sigma^4 - 0\cdot 39874\sigma^5 - 0\cdot 49329\sigma^6$$

$$P_5 = \sigma^3[0\cdot 00523 + 0\cdot 22141\sigma + 0\cdot 38628\sigma^2 + 0\cdot 47293\sigma^3]$$

$$C = \frac{E(\varkappa)}{2\pi\delta} C_{L0}$$

$$C_{Ds}/(C_{L0}^2/\pi A) = \frac{0\cdot 6}{C^2}[0\cdot 000781 + 0\cdot 037253\sigma + 0\cdot 487156\sigma^2 - 0\cdot 380461\sigma^3 - 0\cdot 308385\sigma^4 + 0\cdot 149269\sigma^5 + 0\cdot 080724\sigma^6]$$

$$C_{Dv}/(C_{L0}^2/\pi A) \equiv \varepsilon = 1 + \frac{3}{C^2}(0\cdot 5949\sigma^2 + 0\cdot 6875\sigma^3)^2$$

(x) $\gamma = 45°$, $M = 1\cdot 281$ (Fig. 13)

$$C_{l0} = \frac{4\delta}{E(\varkappa)}\left(\frac{\sigma + y}{\sigma - y}\right)^{1/2}[-(1-\sigma) + 4\cdot 7107(1-\sigma^2) - 3\cdot 3375(1-\sigma^3) + (6\cdot 17 - 0\cdot 8599\sigma)(\sigma^2 - y^2)]$$

$$C_{L0} = \frac{2\pi\delta}{E(\varkappa)}[0\cdot 3732 + \sigma - 0\cdot 0832\sigma^2 + 2\cdot 6925\sigma^3]$$

$$C_{M0} = \frac{2\pi\delta}{E(\varkappa)}[-\tfrac{1}{6}\sigma + 0\cdot 2219\sigma^2 + 0\cdot 8975\sigma^3]$$

$$C_{Dp}/(C_{L0}^2/\pi A) = 11\cdot 3448\bar{P}/C^2,$$ where :

$$\bar{P} = -P_1 + 4\cdot 7107P_2 + 6\cdot 17P_3 - 3\cdot 3375P_4 - 0\cdot 8599P_5$$

$$P_1 = 0\cdot 09347 + 0\cdot 1427\sigma - 0\cdot 61459\sigma^2 + 0\cdot 88144\sigma^3 - 0\cdot 78444\sigma^4$$

$$P_2 = 0\cdot 09347 + 0\cdot 23617\sigma - 0\cdot 44237\sigma^2 + 0\cdot 35381\sigma^3 + 0\cdot 17445\sigma^4 - 0\cdot 88960\sigma^5$$

$$P_3 = 3\sigma^2[0\cdot 02337 + 0\cdot 07085\sigma + 0\cdot 02908\sigma^2 + 0\cdot 21723\sigma^3]$$

$$P_4 = 0\cdot 09347 + 0\cdot 23617\sigma - 0\cdot 34890\sigma^2 + 0\cdot 54374\sigma^3 - 0\cdot 29521\sigma^4 + 0\cdot 12461\sigma^5 - 0\cdot 96849\sigma^6$$

$$P_5 = \sigma^3[0\cdot 07011 + 0\cdot 22141\sigma + 0\cdot 12461\sigma^2 + 0\cdot 71052\sigma^3]$$

$$C = \frac{E(\varkappa)}{2\pi\delta} C_{L0}$$

$$C_{Ds}/(C_{L0}^2/\pi A) = \frac{0\cdot 6}{C^2}[0\cdot 139308 + 0\cdot 497653\sigma - 1\cdot 258220\sigma^2 - 2\cdot 772021\sigma^3 + 9\cdot 621871\sigma^4 - 8\cdot 983870\sigma^5 + 2\cdot 784650\sigma^6]$$

$$C_{Dv}/(C_{L0}^2/\pi A) \equiv \varepsilon = 1 + \frac{3}{C^2}(1\cdot 5425\sigma^2 - 0\cdot 2150\sigma^3)^2$$

(iii) $\gamma = 45°, \qquad M = 1\cdot166$ (1

$$C_{l\,0} = \frac{4\delta}{E(\varkappa)} \left(\frac{\sigma + y}{\sigma - y}\right)^{1/2} (1 - \sigma)$$

$$C_{L\,0} = \frac{2\pi\delta}{E(\varkappa)} (1 - \sigma) \quad , \quad C_{M\,0} = \tfrac{1}{3} \frac{\pi\delta}{E(\varkappa)} \sigma$$

$$C_{D\,p}/(C_{L\,0}{}^2/\pi A) = 2E(\varkappa)[1 - (1 + \tfrac{4}{3}f_1)\sigma + \tfrac{3}{2}f_1\sigma^2]/(1 - \sigma)^2$$

$$C_{D\,s}/(C_{L\,0}{}^2/\pi A) = (2 - M^2)^{1/2}(1 - \tfrac{4}{3}\sigma + \tfrac{1}{2}\sigma^2)/(1 - \sigma)^2$$

$$C_{D\,v}/(C_{L\,0}{}^2/\pi A) \equiv \varepsilon = 1$$

(viii) $\gamma = 30°, \qquad M = 1\cdot852$ (1

$$C_{l\,0} = \frac{4\delta}{kE(\varkappa)} \sigma \left(\frac{\sigma + ky}{\sigma - ky}\right)^{1/2} (\sigma - k^2y^2)$$

$$C_{L\,0} = \frac{2\pi\delta}{kE(\varkappa)} \sigma^2(1 - \tfrac{1}{4}\sigma)$$

$$C_{M\,0} = -\frac{2\pi\delta}{kE(\varkappa)} \sigma^2(\tfrac{4}{15} - \tfrac{1}{12}\sigma)$$

$$C_{D\,p}/(C_{L\,0}{}^2/\pi A) = 2E(\varkappa)[(\tfrac{2}{3}f_6 - \tfrac{1}{4}f_7) + (\tfrac{1987}{1792}f_6 - \tfrac{1}{56}f_7 + \tfrac{29}{56}f_8 - \tfrac{3}{14}f_9)\sigma$$
$$+ (\tfrac{7}{64}f_9 - \tfrac{5}{32}f_8)\sigma^2]/(1 - \tfrac{1}{4}\sigma)^2$$

$$C_{D\,s}/(C_{L\,0}{}^2/\pi A) = \left(\frac{4 - M^2}{3}\right)^{1/2} [1 - \tfrac{4}{5}\sigma - \tfrac{1}{3}\sigma^2 + \tfrac{1}{4}\sigma^4]/[\sigma^2(1 - \tfrac{1}{4}\sigma)^2]$$

$$C_{D\,v}/(C_{L\,0}{}^2/\pi A) \equiv \varepsilon = 1 + 3\sigma^2/(4 - \sigma)^2$$

(xiii) $\gamma = 45°, \qquad M = 1\cdot281$ (1

$$C_{l\,0} = \frac{4\delta}{E(\varkappa)} \left(\frac{\sigma + y}{\sigma - y}\right)^{1/2} (3 + \sigma^2 - 4y^2)$$

$$C_{L\,0} = \frac{2\pi\delta}{E(\varkappa)} (3) = 13\cdot29214\delta, \quad C_{M\,0} = 0$$

$$C_{D\,p}/(C_{L\,0}{}^2/\pi A) = 1\cdot26053[2\cdot25 - 0\cdot641718\sigma^2 + 0\cdot565956\sigma^4]$$

$$C_{D\,s}/(C_{L\,0}{}^2/\pi A) = 0\cdot6(1 - \sigma^2 + \tfrac{1}{3}\sigma^4)$$

$$C_{D\,v}/(C_{L\,0}{}^2/\pi A) \equiv \varepsilon = 1 + \tfrac{1}{3}\sigma^4$$

(xi) $\gamma = 45° \quad , \quad M = 1\cdot281$ (F

$$C_{l\,0} = \frac{4\delta}{E(\varkappa)} \left(\frac{\sigma + y}{\sigma - y}\right)^{1/2} [3\cdot5425(1 - \sigma^2) - 2\cdot6038(1 - \sigma^3)$$
$$+ (3\cdot1587 - 2\cdot4066\sigma)(\sigma^2 - y^2)]$$

$$C_{L0} = \frac{2\pi\delta}{E(\varkappa)}[0\cdot9387 - 1\cdot1735\,\sigma^2 + 0\cdot7988\,\sigma^3]$$

$$C_{M0} = \frac{2\pi\delta}{E(\varkappa)}[0\cdot3129\sigma^2 - 0\cdot2662\sigma^3]$$

$$C_{Dp}/(C_{L0}^2/\pi A) = \frac{2\cdot8362}{C^2}[0\cdot88116 - 2\cdot73910\sigma^2 + 1\cdot83742\sigma^3$$

$$+ 3\cdot42474\sigma^4 - 4\cdot88008\sigma^5 + 1\cdot77468\sigma^6]$$

$$C_{Ds}/(C_{L0}^2/\pi A) = \frac{0\cdot6}{C^2}[0\cdot88116 - 3\cdot32534\sigma^2 + 1\cdot95535\sigma^3$$

$$+ 4\cdot18310\sigma^4 - 5\cdot27083\sigma^5 + 1\cdot69495\sigma^6]$$

$$C_{Dv}/(C_{L0}^2/\pi A) \equiv \varepsilon = 1 + \frac{3}{16C^2}\sigma^4(3\cdot1587 - 2\cdot4066\sigma)^2,$$

where

$$C = \frac{E(\varkappa)}{2\pi\delta}C_{L0} \quad .$$

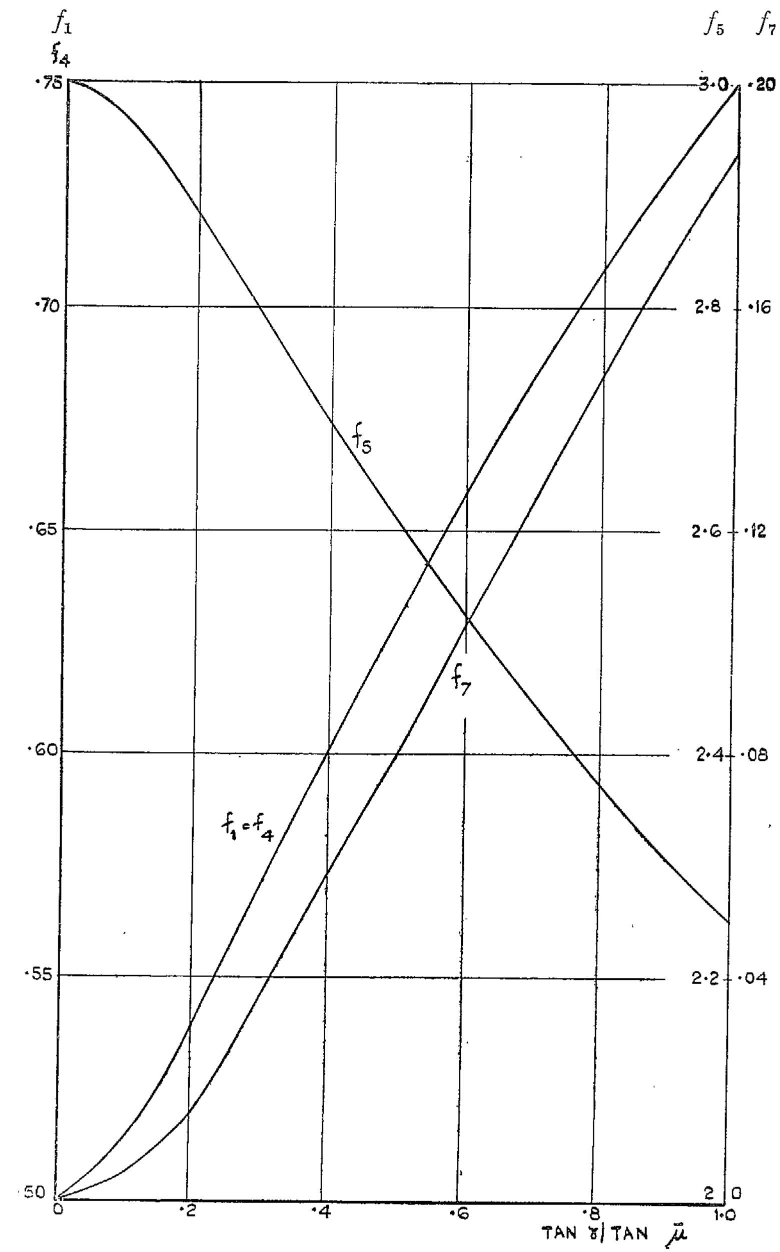


FIG. 1. The functions $f_1(\tan\gamma/\tan\bar{\mu})$, $f_4(\tan\gamma/\tan\bar{\mu})$, $f_5(\tan\gamma, \tan\bar{\mu})$, $f_7(\tan\gamma/\tan\bar{\mu})$.

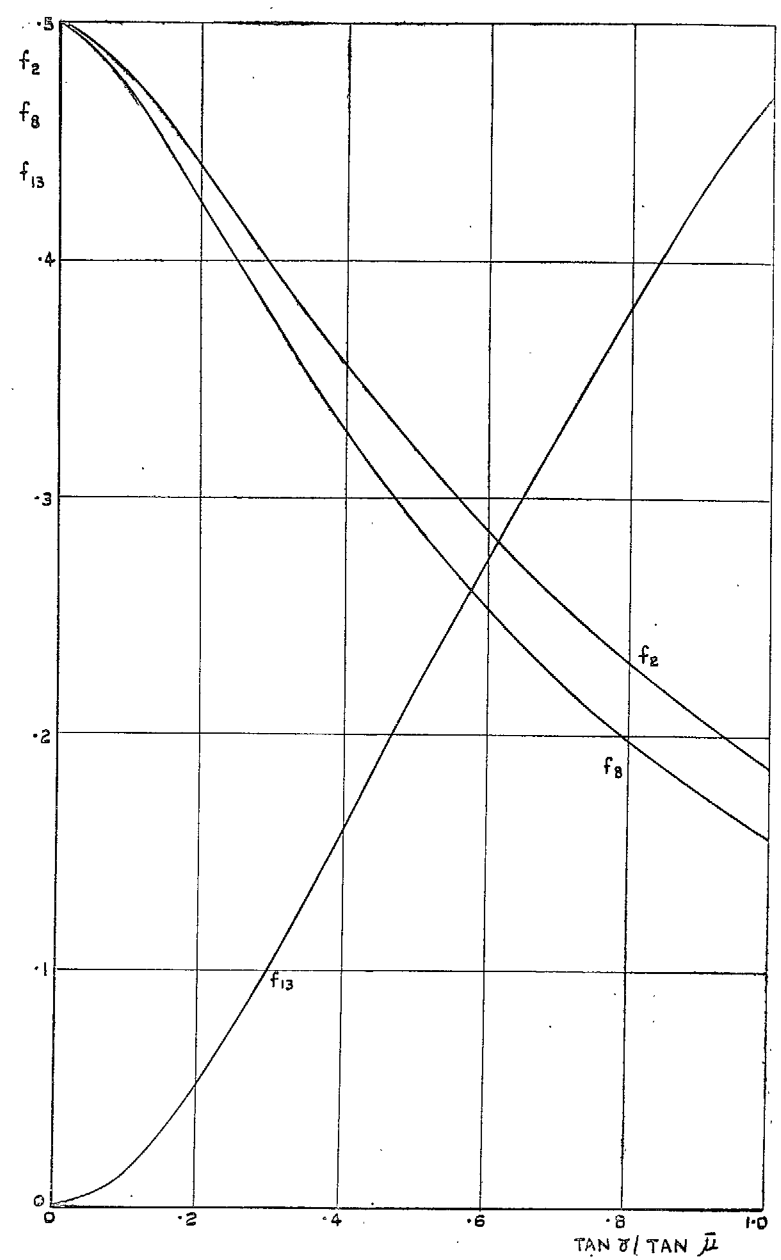


FIG. 2. The functions $f_2(\tan\gamma/\tan\bar{\mu})$, $f_8(\tan\gamma/\tan\bar{\mu})$, $f_{13}(\tan\gamma/\tan\bar{\mu})$.

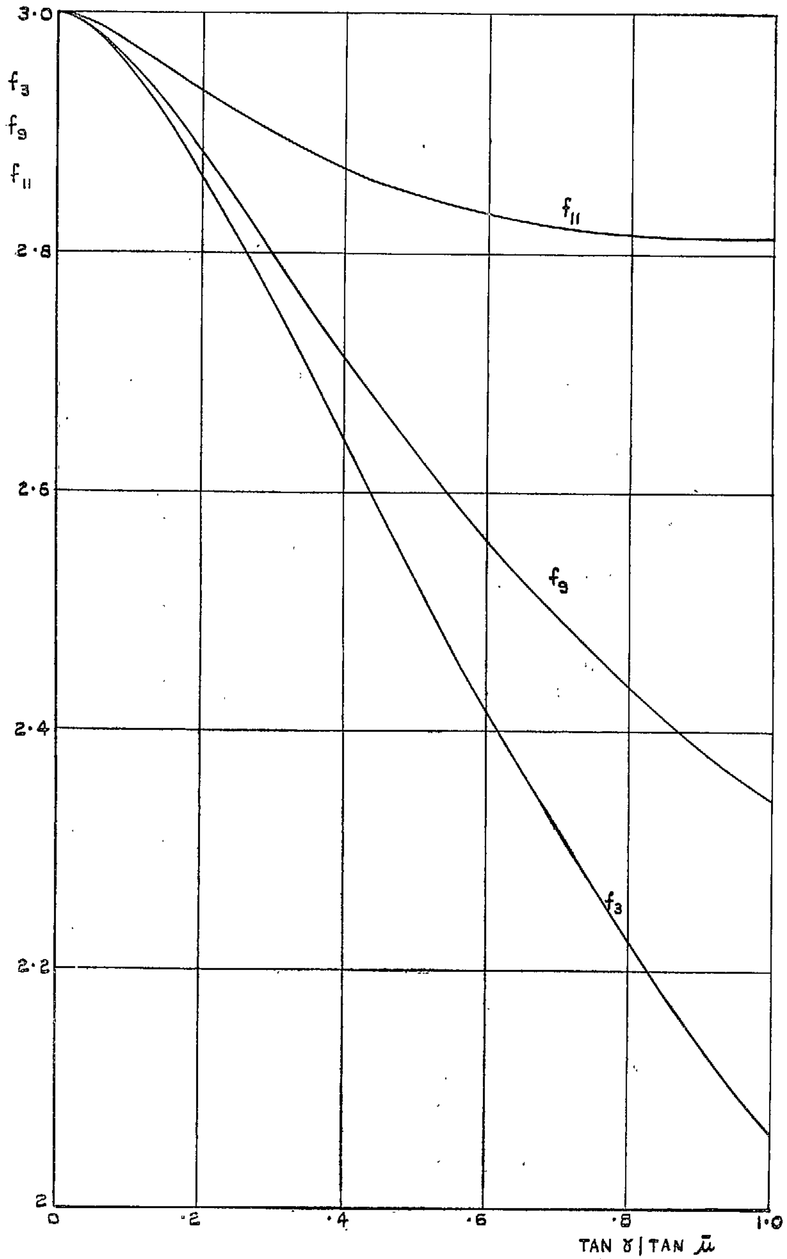


FIG. 3. The functions $f_3(\tan\gamma/\tan\bar{\mu})$, $f_9(\tan\gamma/\tan\bar{\mu})$, $f_{11}(\tan\gamma/\tan\bar{\mu})$.

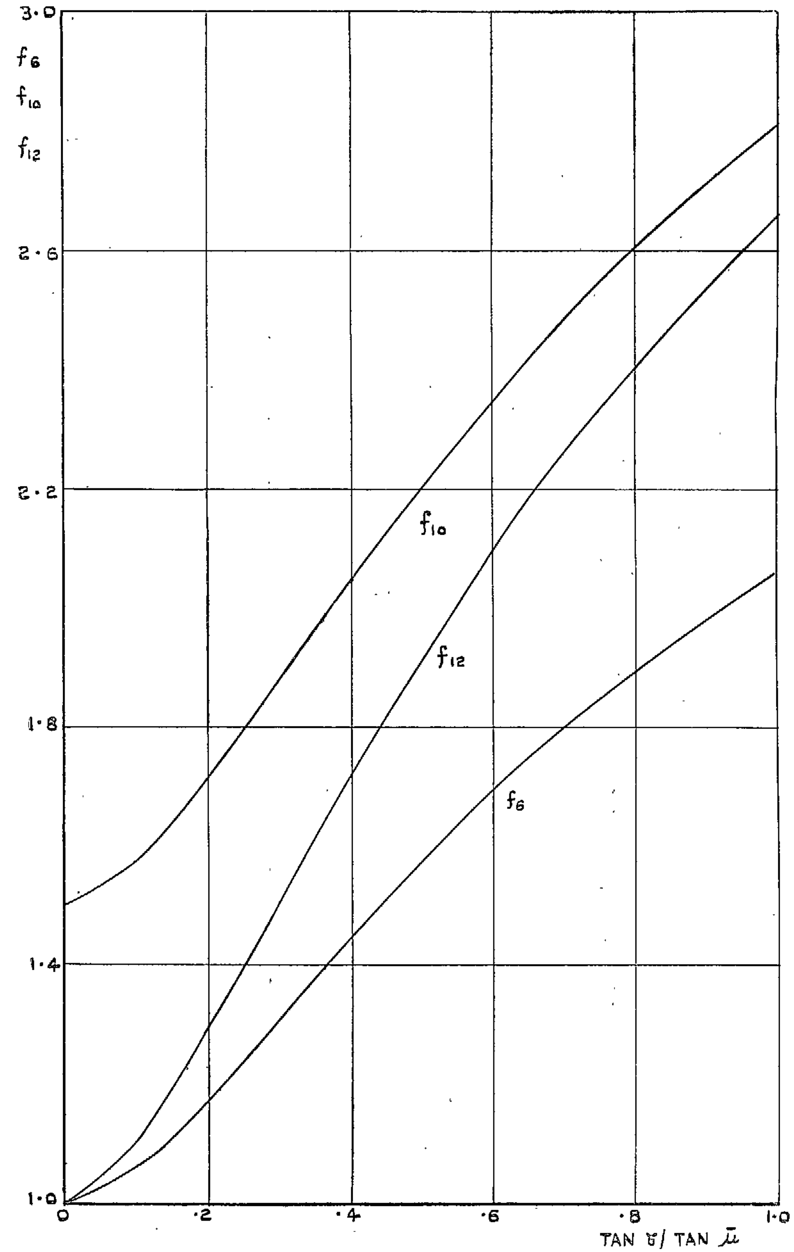


FIG. 4. The functions $f_6(\tan\gamma/\tan\bar{\mu})$, $f_{10}(\tan\gamma/\tan\bar{\mu})$, $f_{12}(\tan\gamma/\tan\bar{\mu})$.

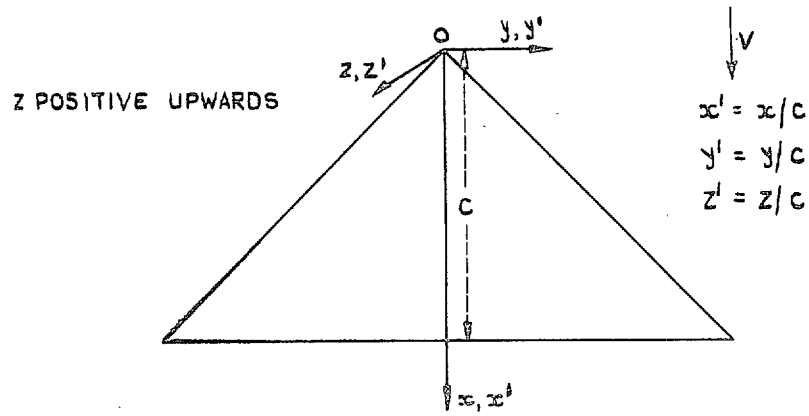
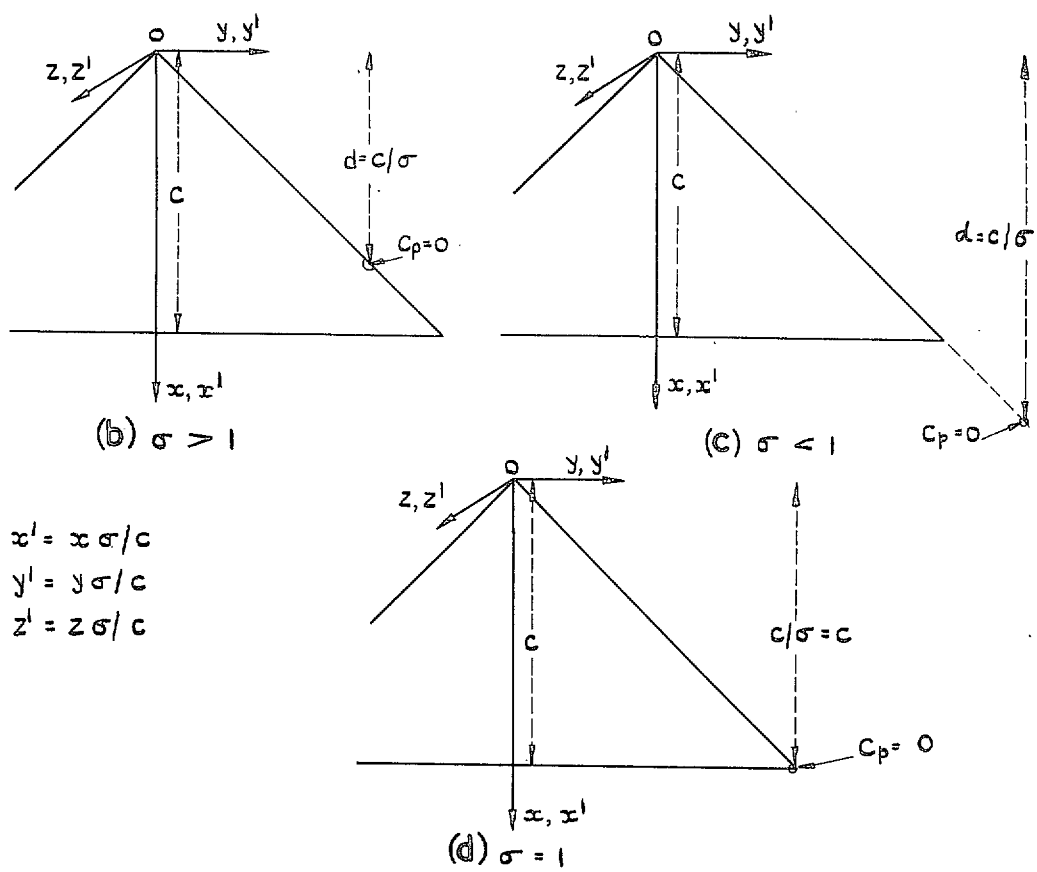


FIG. 5a. Notation for surfaces (i) and (ii) (Figs. 6 and 7).



FIGS. 5b, 5c and 5d. Notation for surfaces (iii) to (xvi) (Figs. 8 to 22).

Note : In FIGS. 6–22, x', y', z', are written x, y, z.

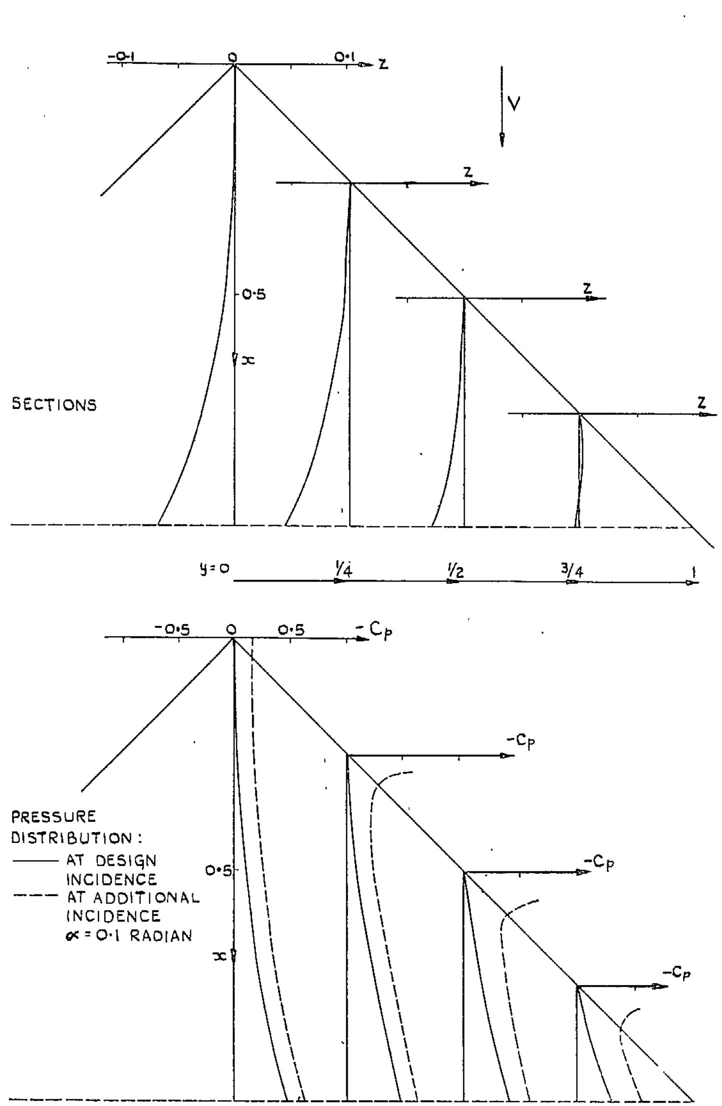


FIG. 6. Surface (i) for $\gamma = 45$ deg. $M = 1 \cdot 166$. Shape and pressure distribution.

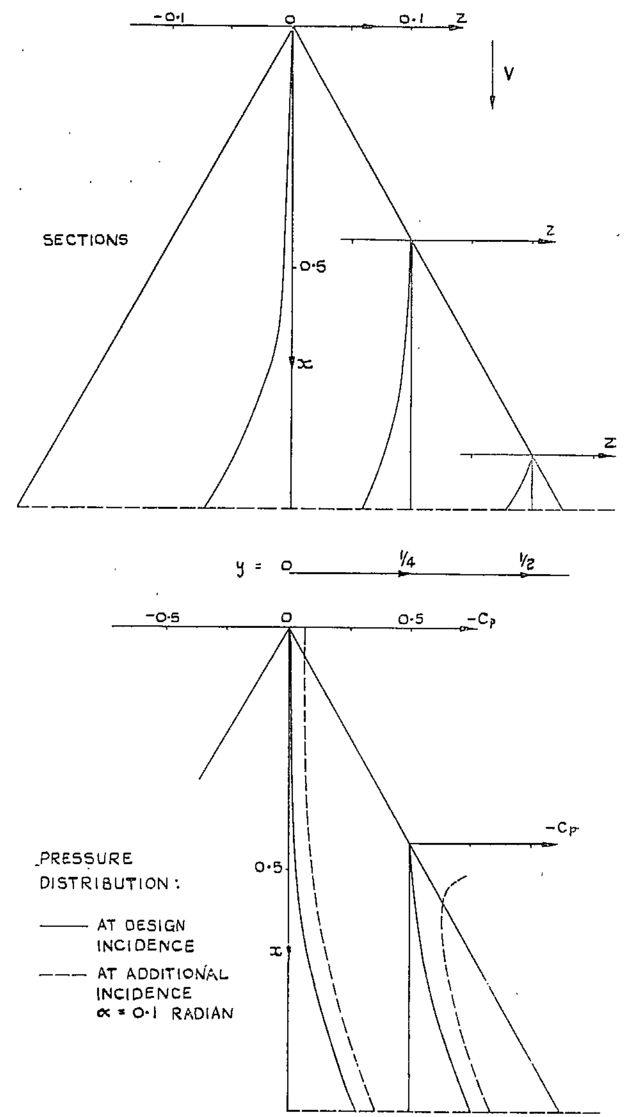


FIG. 7. Surface (ii) for $\gamma = 30$ deg. $M = 1 \cdot 852$. Shape and pressure distribution.

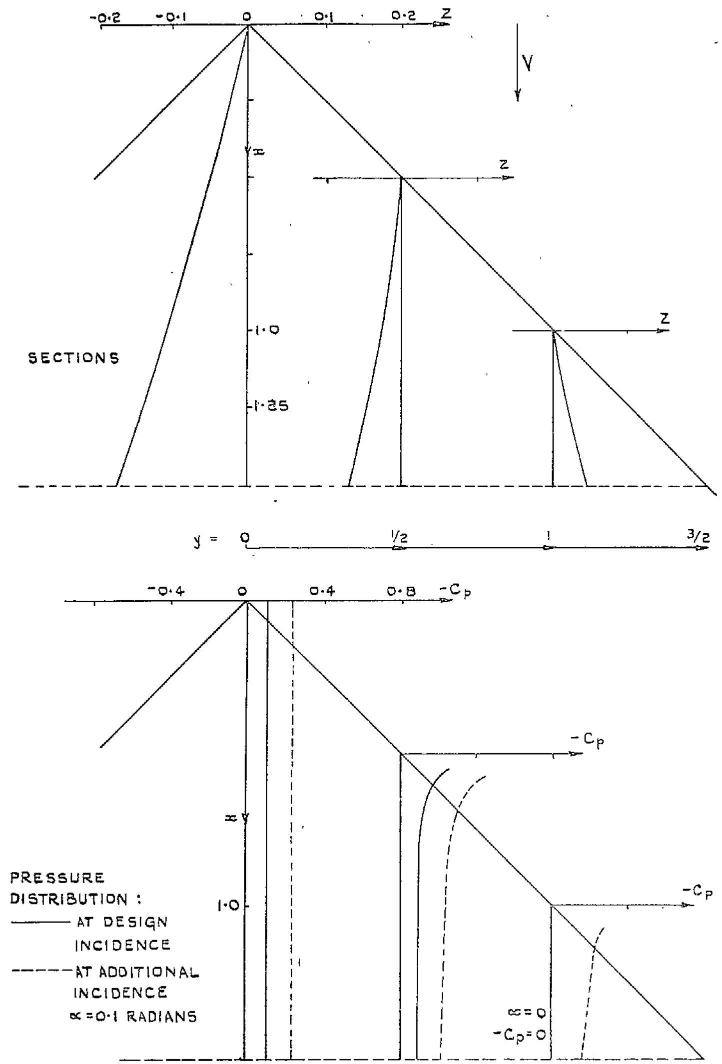


FIG. 8a. Surface (via) for $\gamma = 45$ deg. $M = 1 \cdot 345$. Shape and pressure distribution.

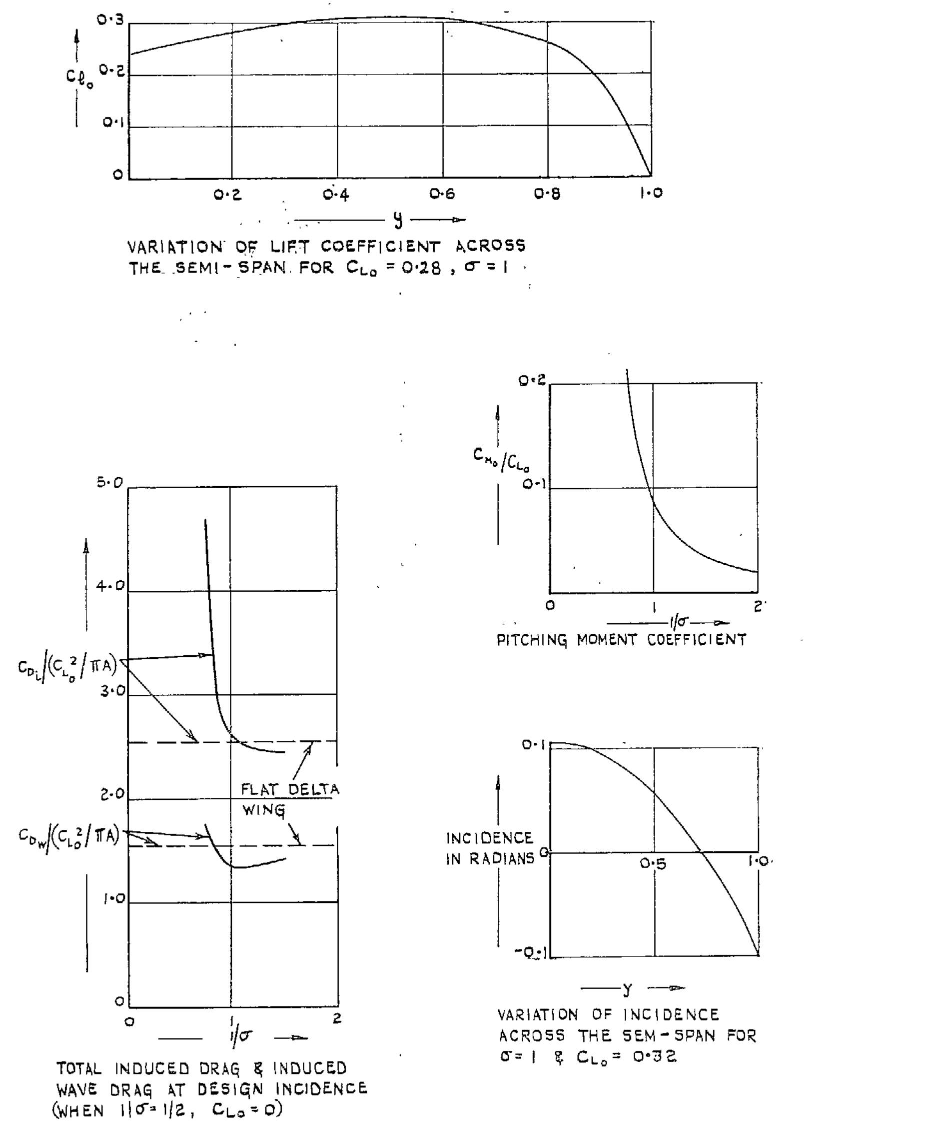


FIG. 8b. Delta wing (via) for $\gamma = 45$ deg. $M = 1 \cdot 345$.

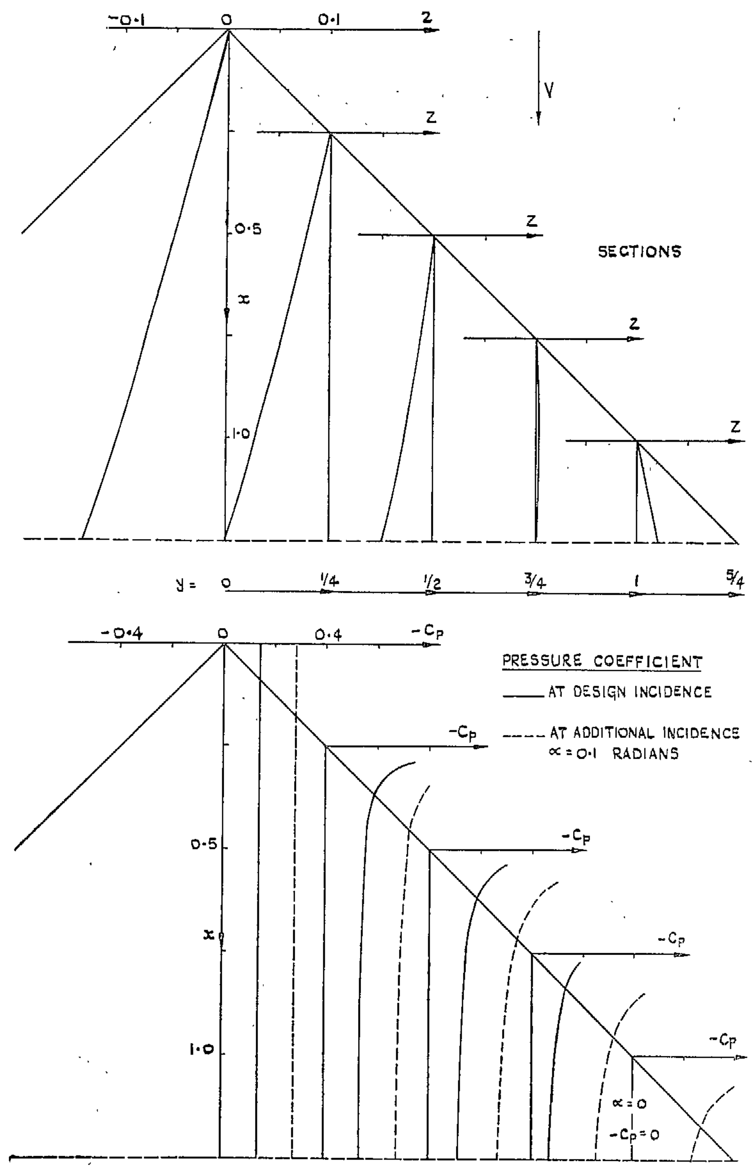


FIG. 9a. Surface (vi*b*) for $\gamma = 45$ deg. $M = 1 \cdot 281$.
Shape and pressure distribution.

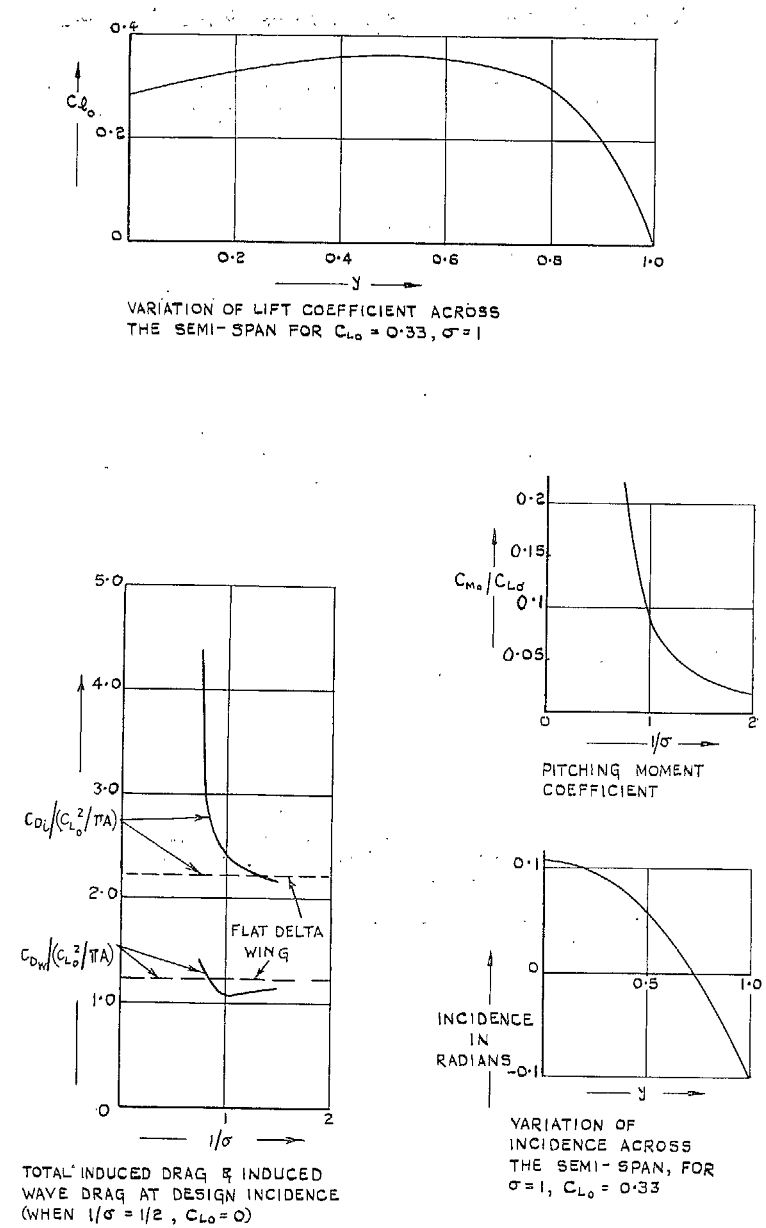


FIG. 9b. Delta wing (vi*b*) for $\gamma = 45$ deg. $M = 1 \cdot 281$.

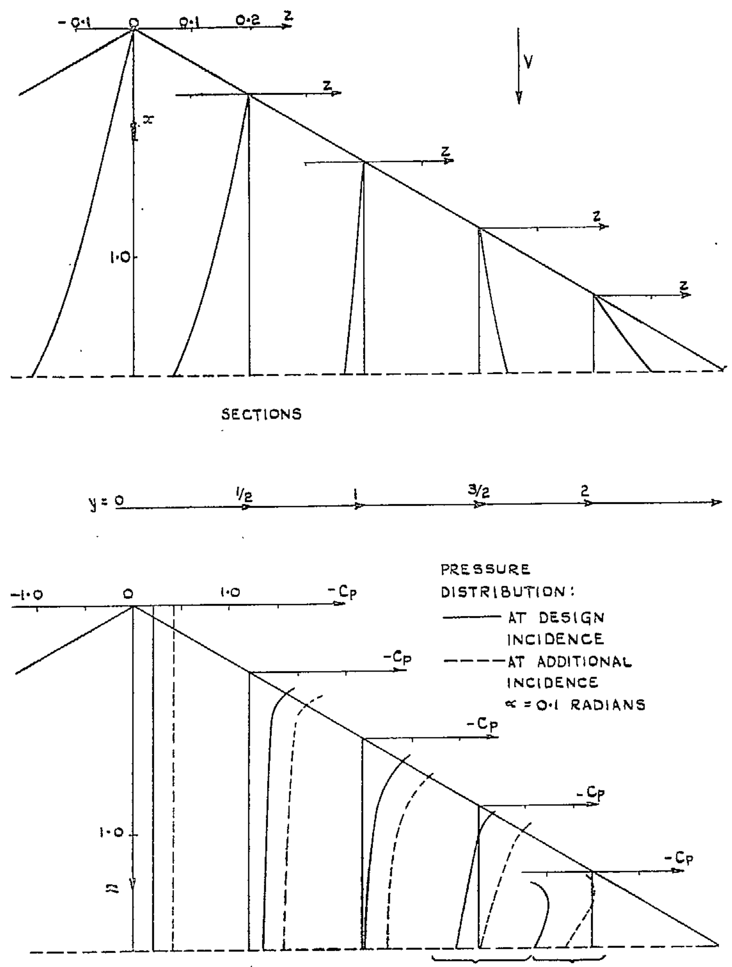


FIG. 10a. Surface (vii) for $\gamma = 60$ deg. $M = 1 \cdot 13$.
Shape and pressure distribution.

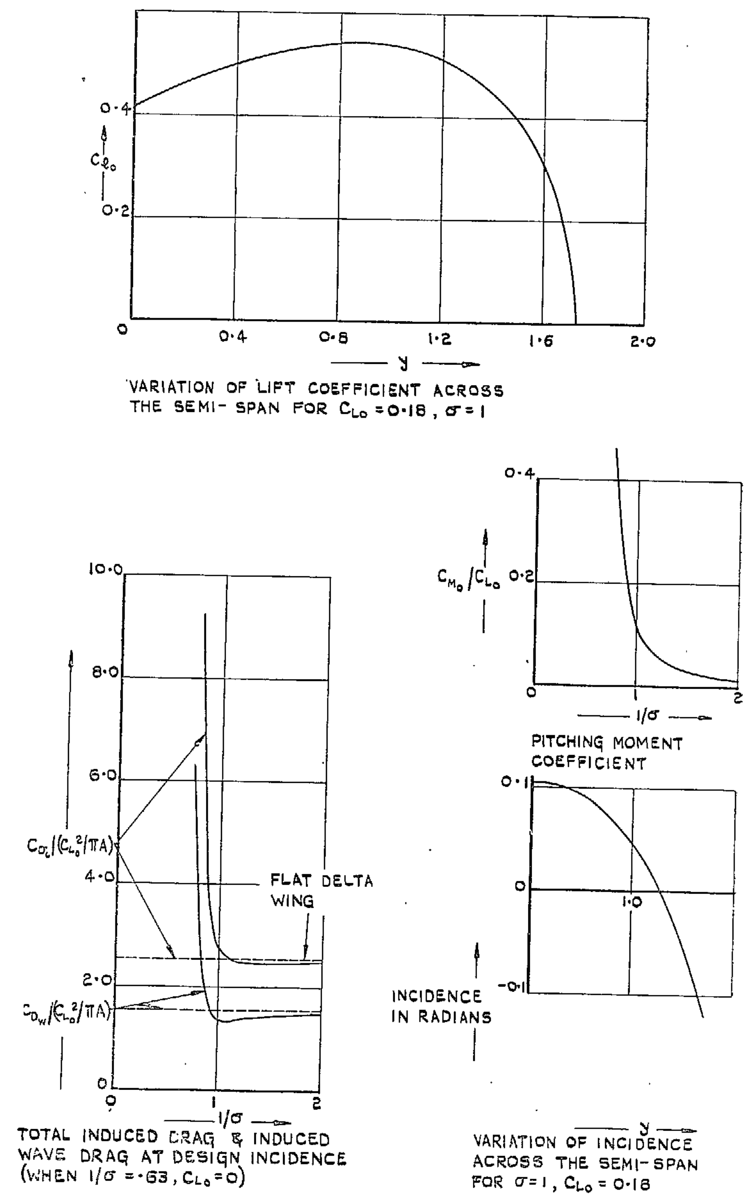


FIG. 10b. Delta wing (vii) for $\gamma = 60$ deg. $M = 1 \cdot 13$.

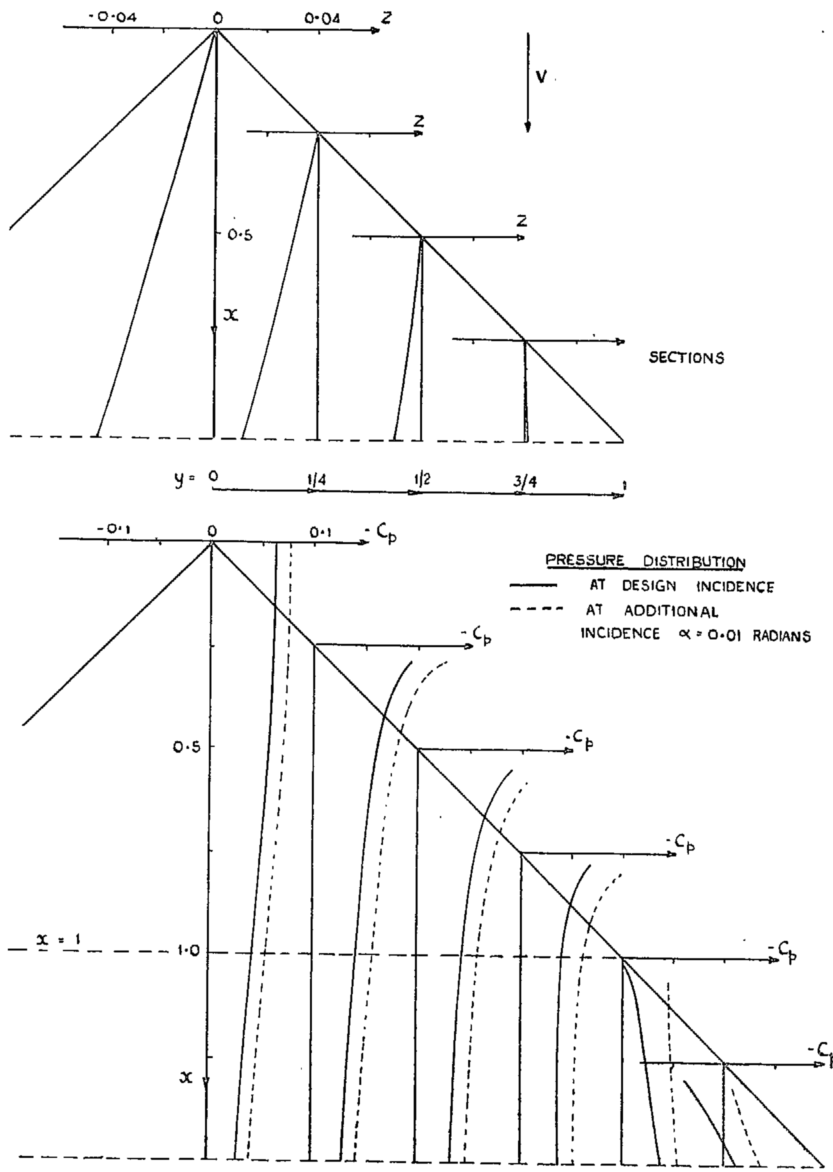


FIG. 11a. Surface (xii) for $\gamma = 45$ deg. $M = 1{\cdot}281$. Shape and pressure distribution.

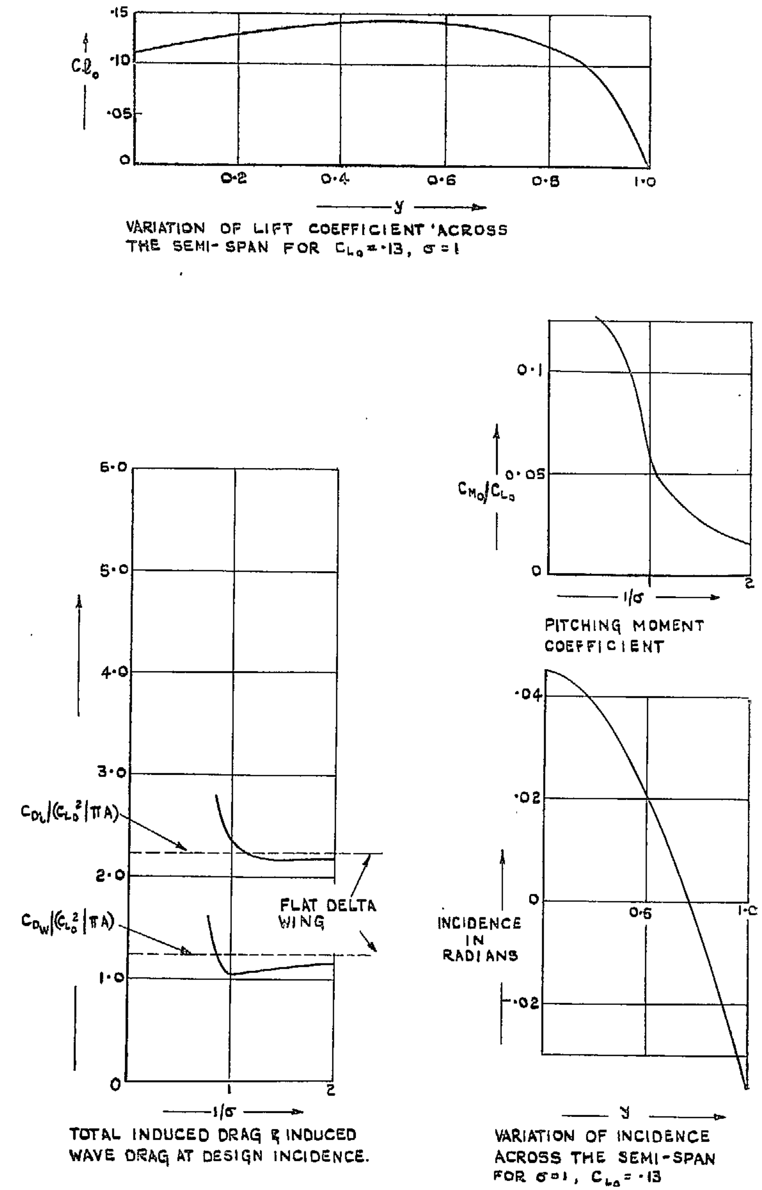


FIG. 11b. Delta wing (xii) for $\gamma = 45$ deg. $M = 1{\cdot}281$.

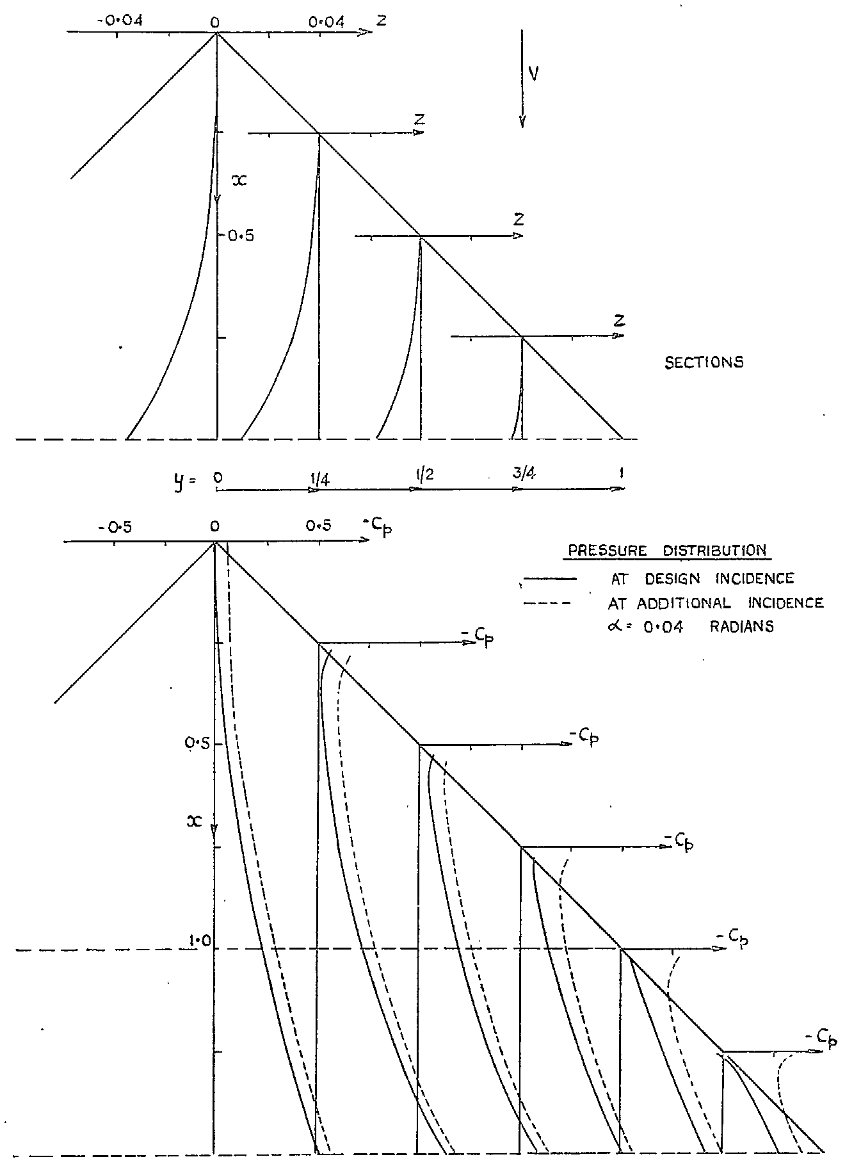


FIG. 12a. Surface (ix) for $\gamma = 45$ deg. $M = 1\cdot281$. Shape and pressure distribution.

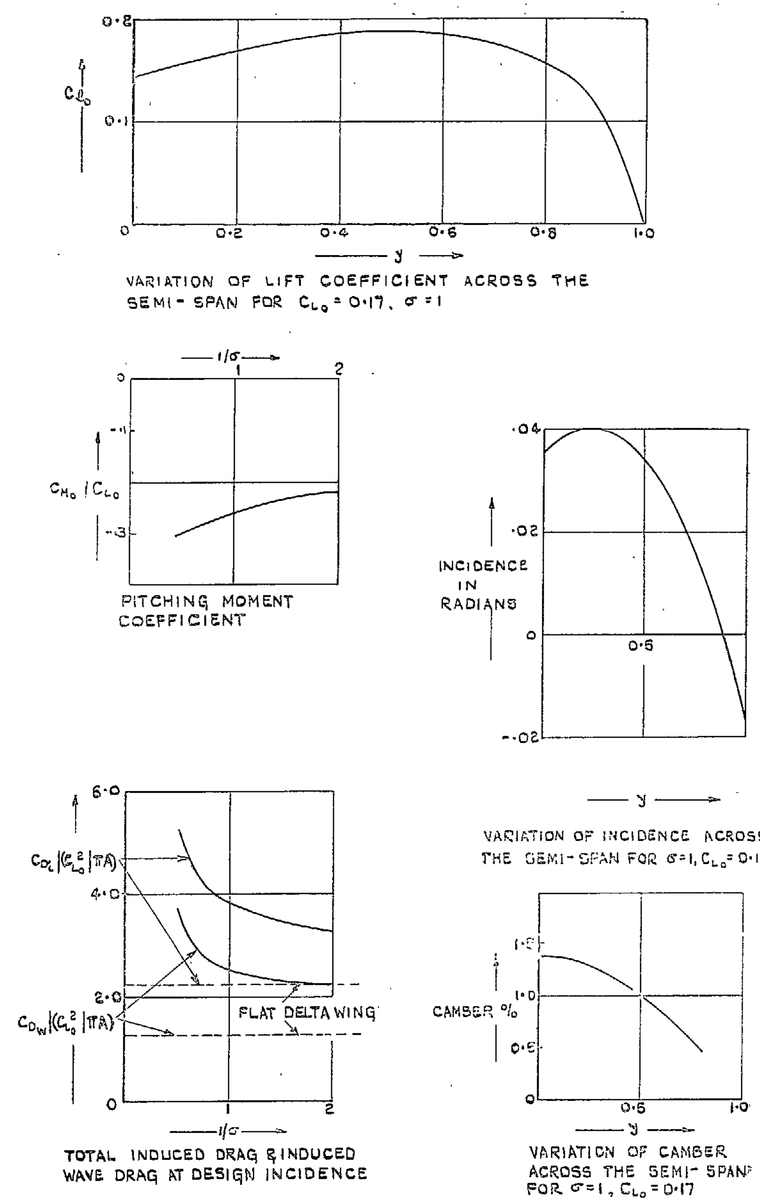


FIG. 12b. Delta wing (ix) for $\gamma = 45$ deg. $M = 1\cdot281$.

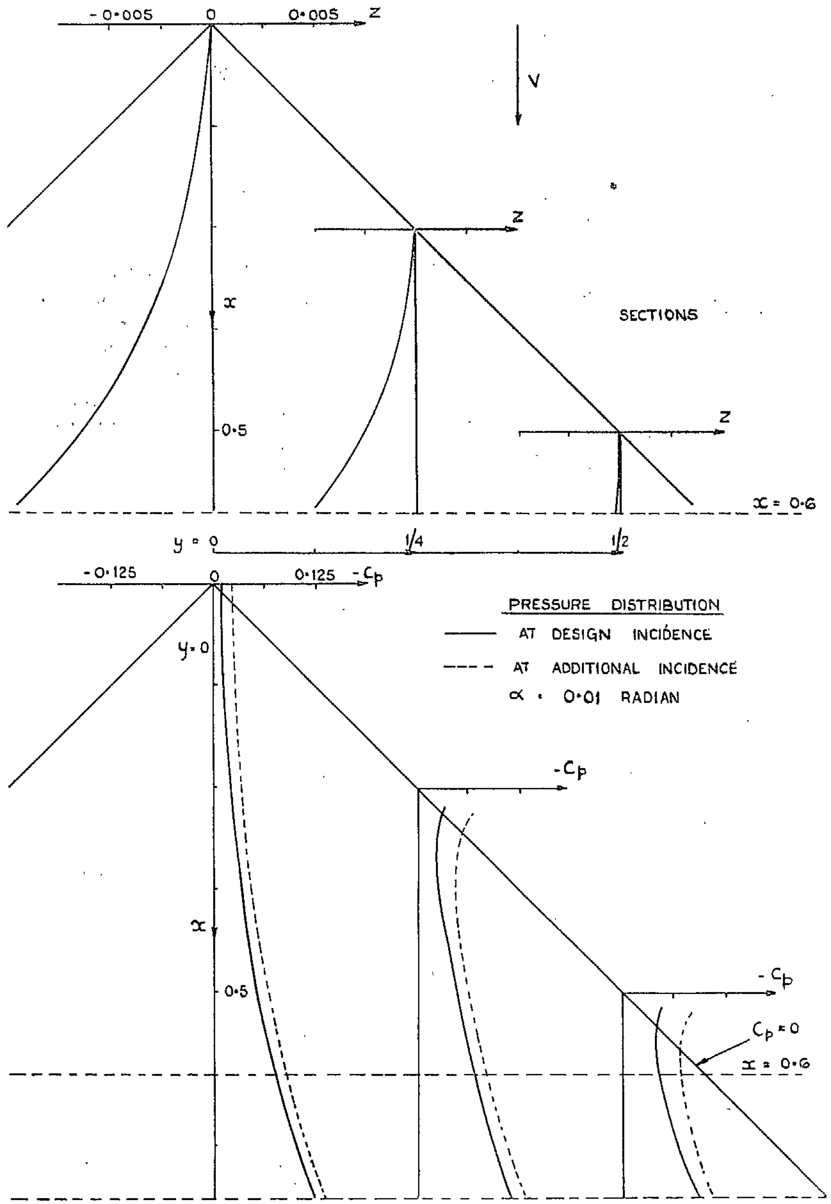


FIG. 13a. Surface (x) for $\gamma = 45$ deg. $M = 1 \cdot 281$. Shape and pressure distribution.

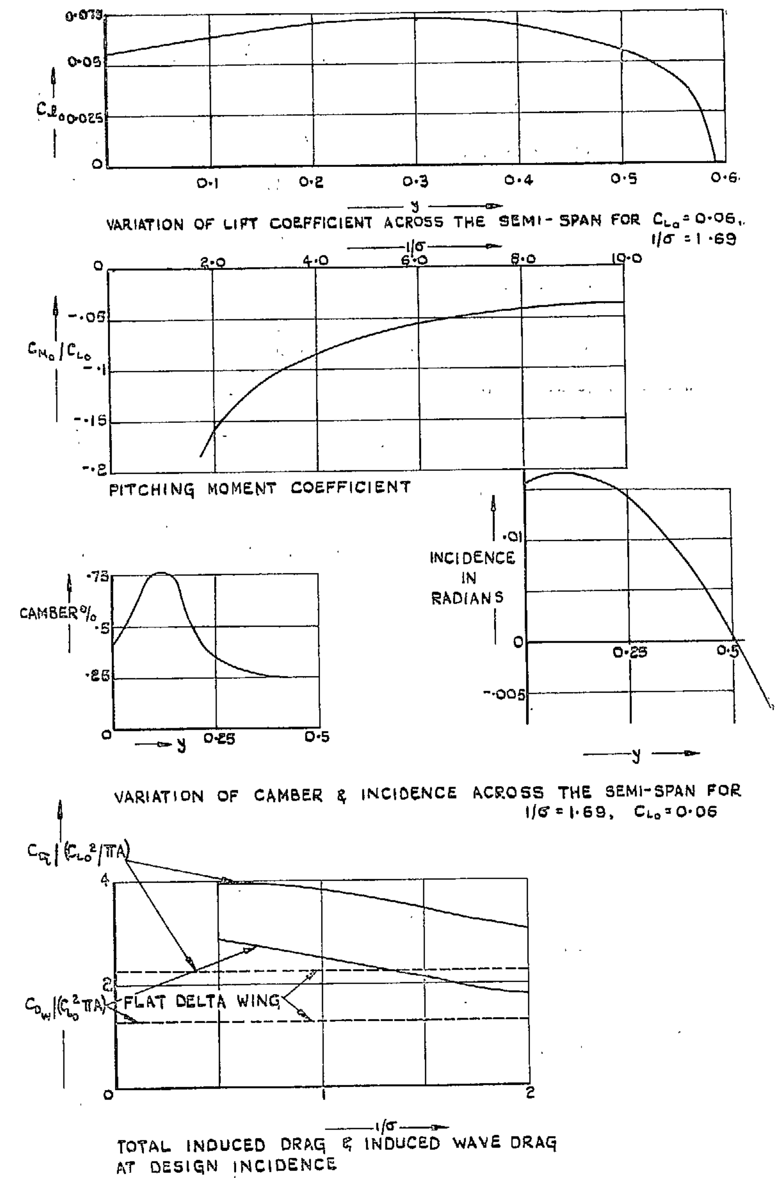


FIG. 13b. Delta wing (x) for $\gamma = 45$ deg. $M = 1 \cdot 281$.

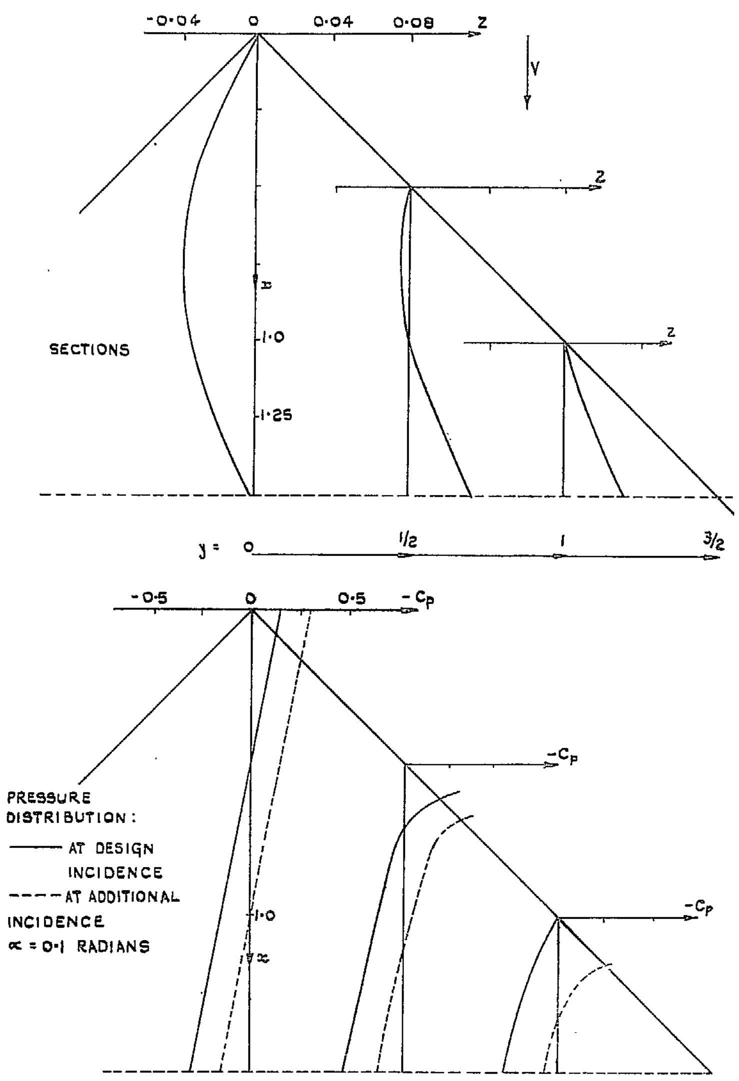


FIG. 14a. Surface (iii) for γ = 45 deg. M = 1·166. Shape and pressure distribution.

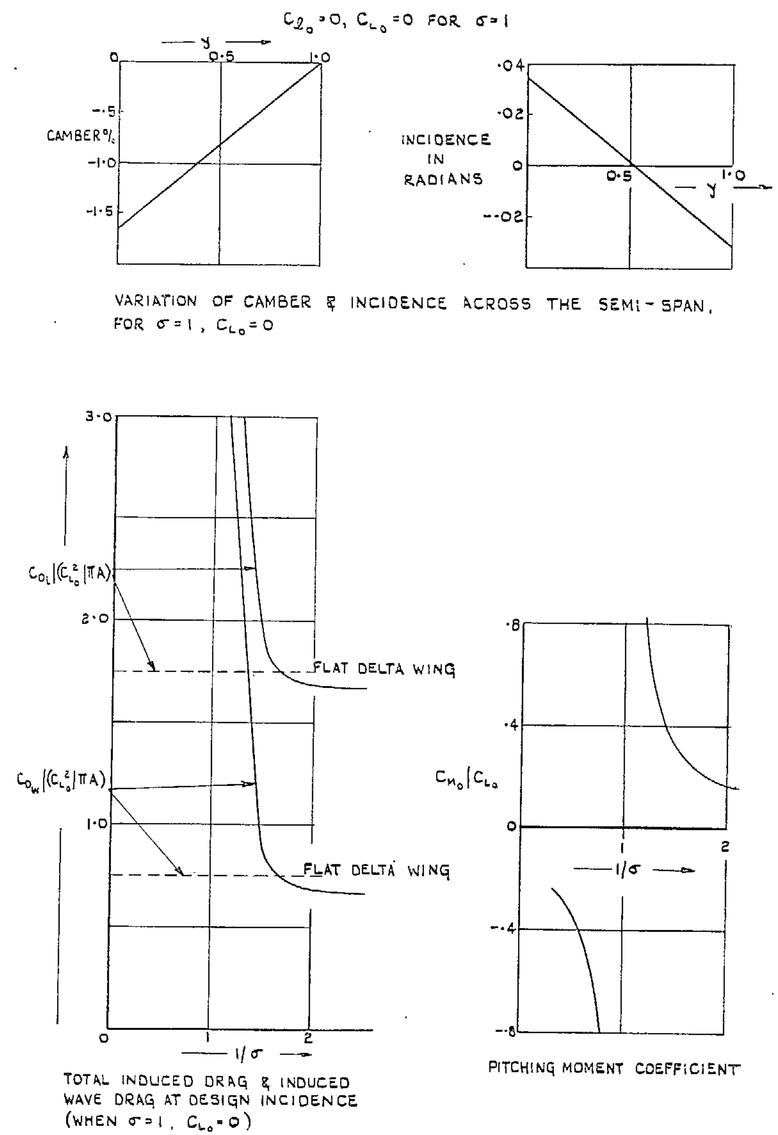


FIG. 14b. Delta wing (iii) for γ = 45 deg. M = 1·166.

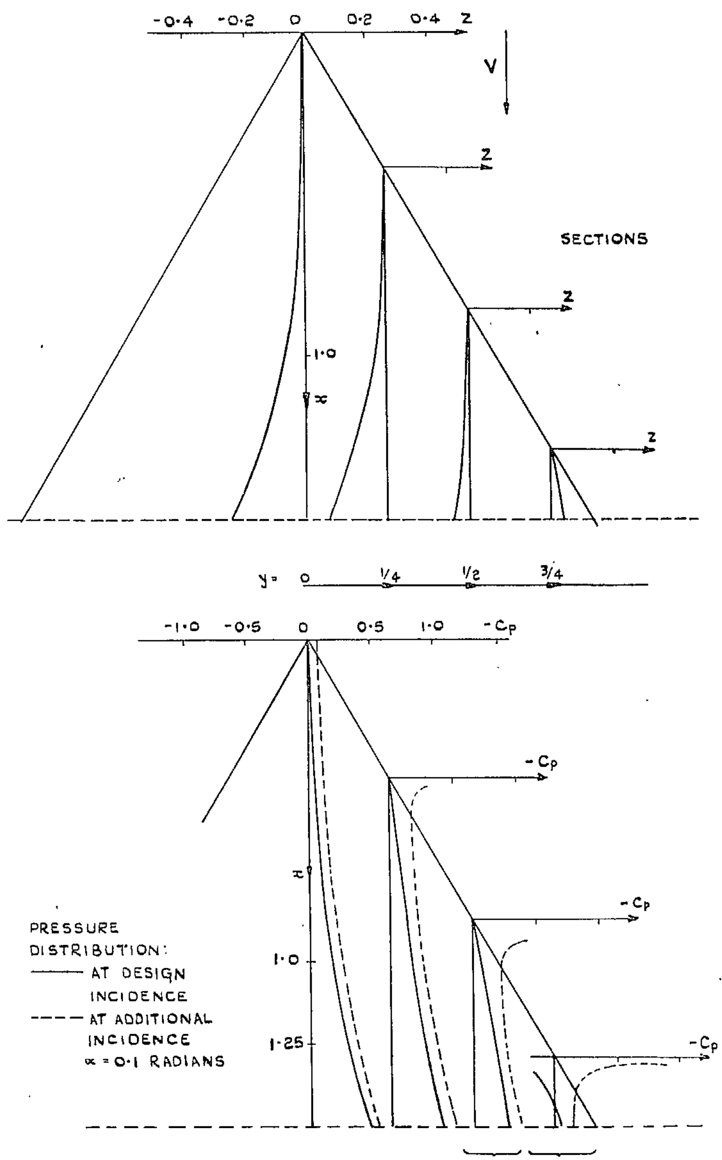


FIG. 15a. Surface (viii) for $\gamma = 30$ deg. $M = 1 \cdot 852$. Shape and pressure distribution.

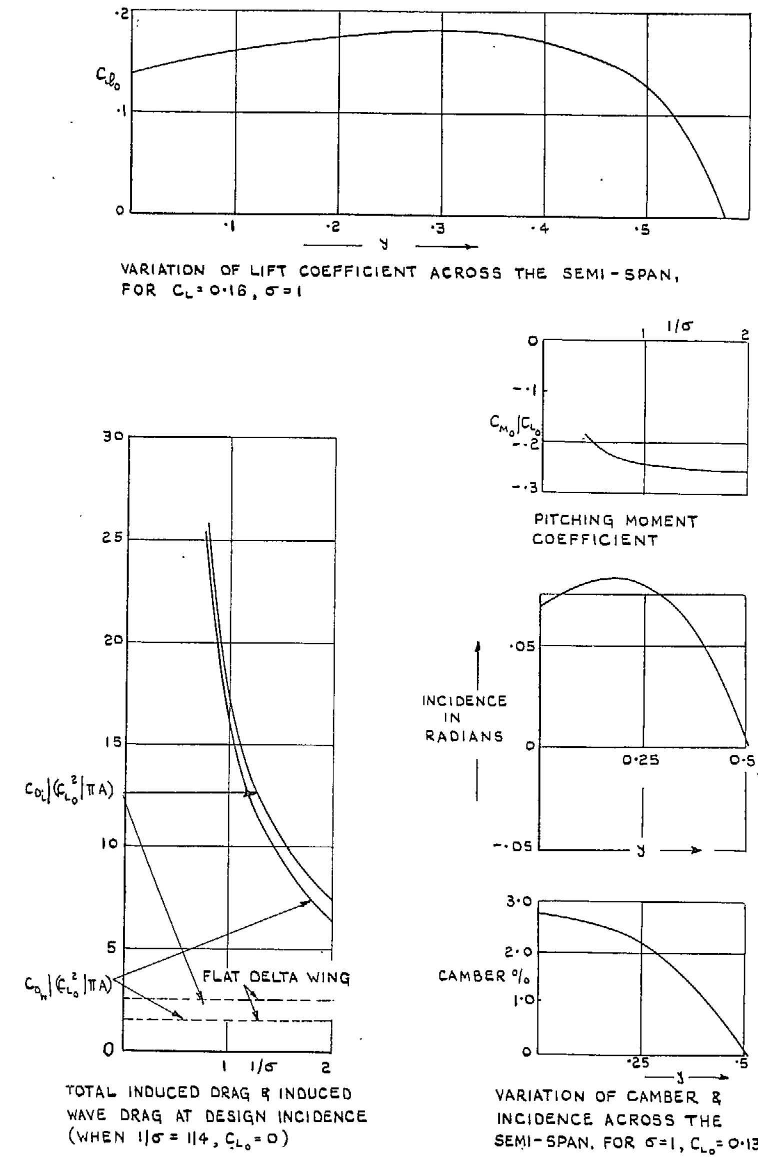


FIG. 15b. Delta wing (viii) for $\gamma = 30$ deg. $M = 1 \cdot 852$.

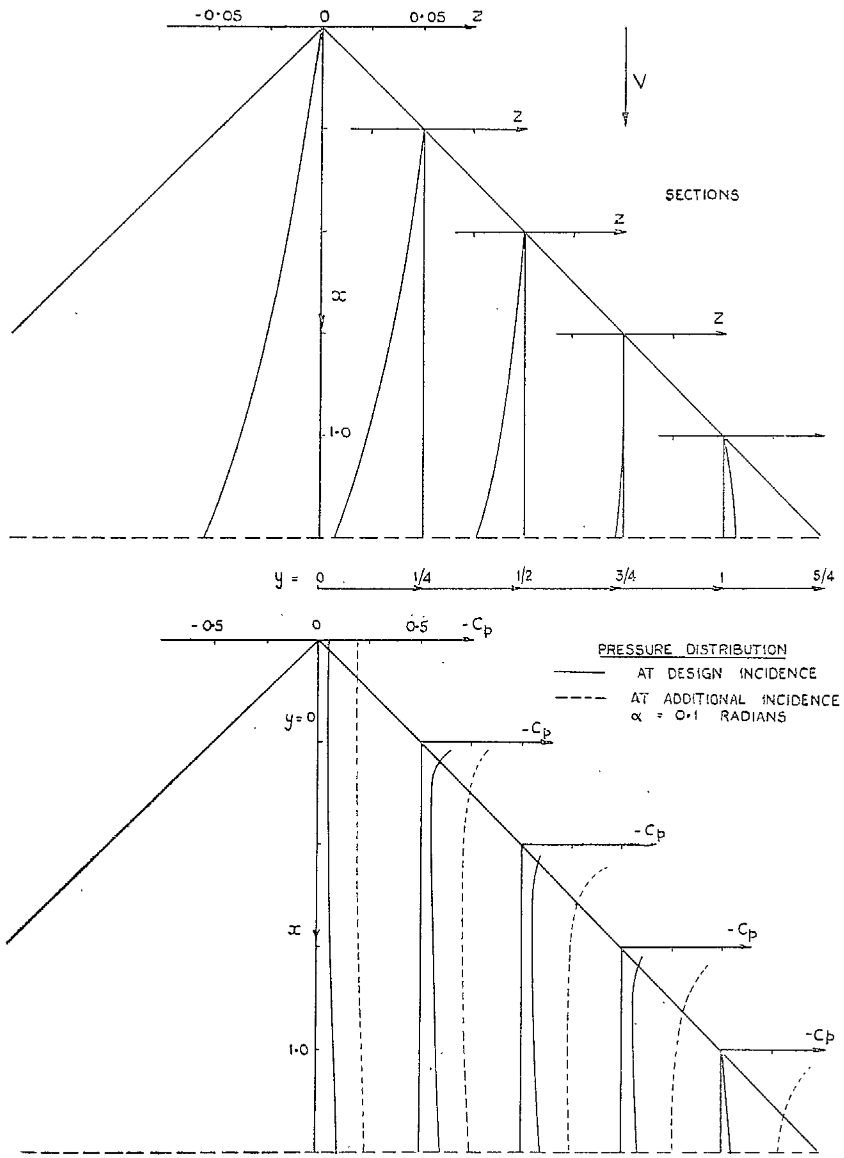


Fig. 16a. Surface (xiii) for $\gamma = 45$ deg. $M = 1 \cdot 281$. Shape and pressure distribution.

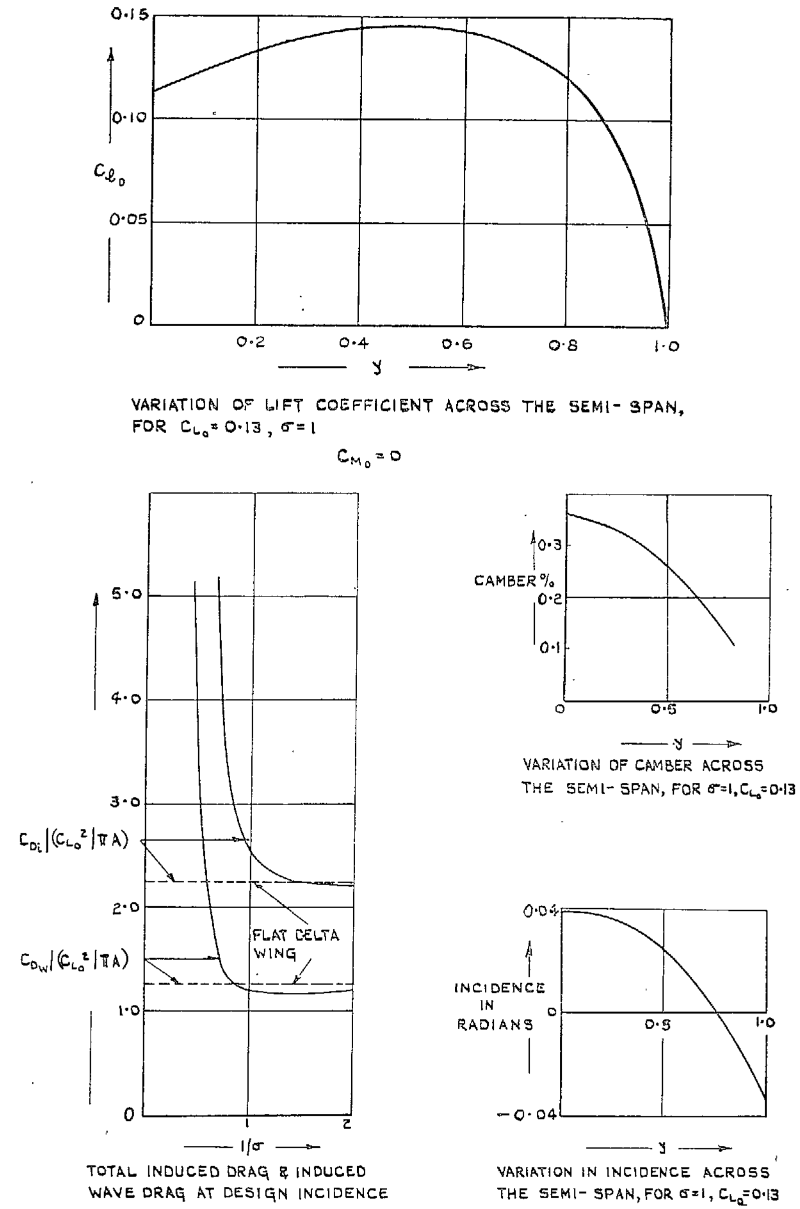


Fig. 16b. Delta wing (xiii) for $\gamma = 45$ deg. $M = 1 \cdot 281$.

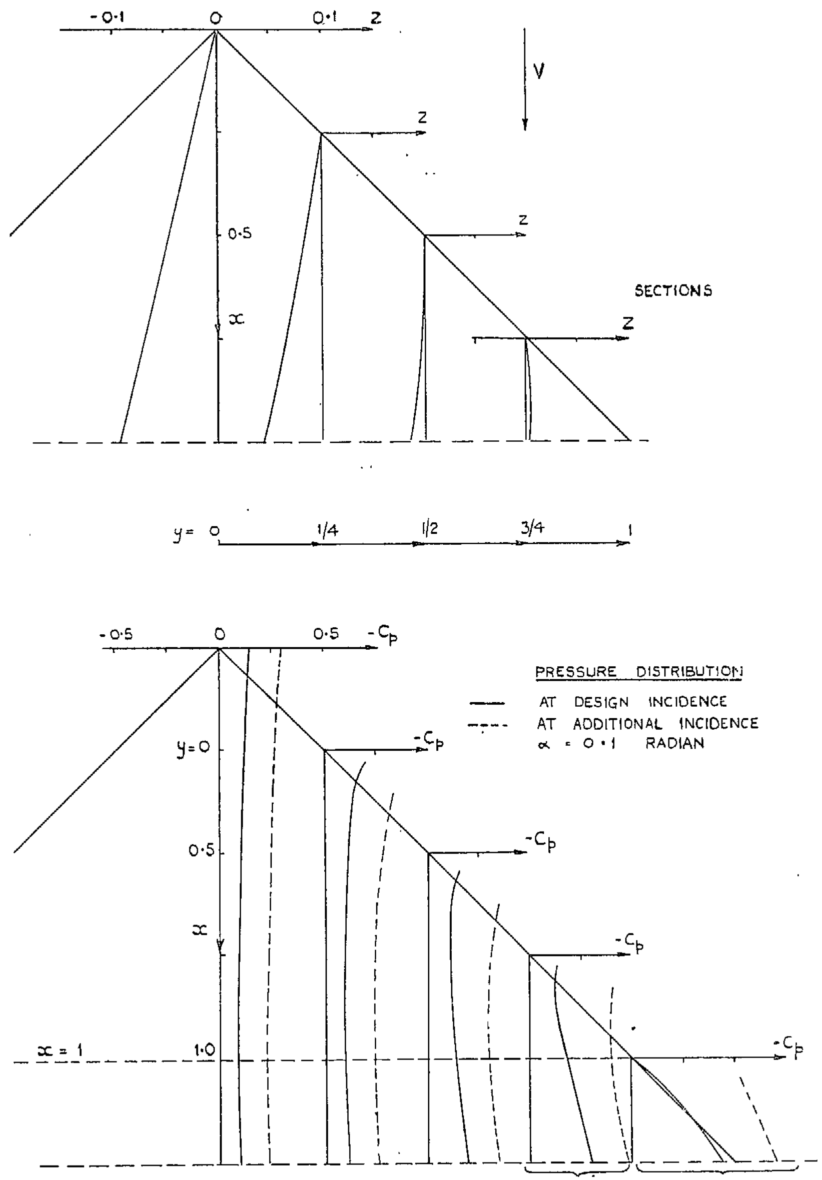


Fig. 17a. Surface (xi) for $\gamma = 45$ deg. $M = 1\cdot281$. Shape and pressure distribution.

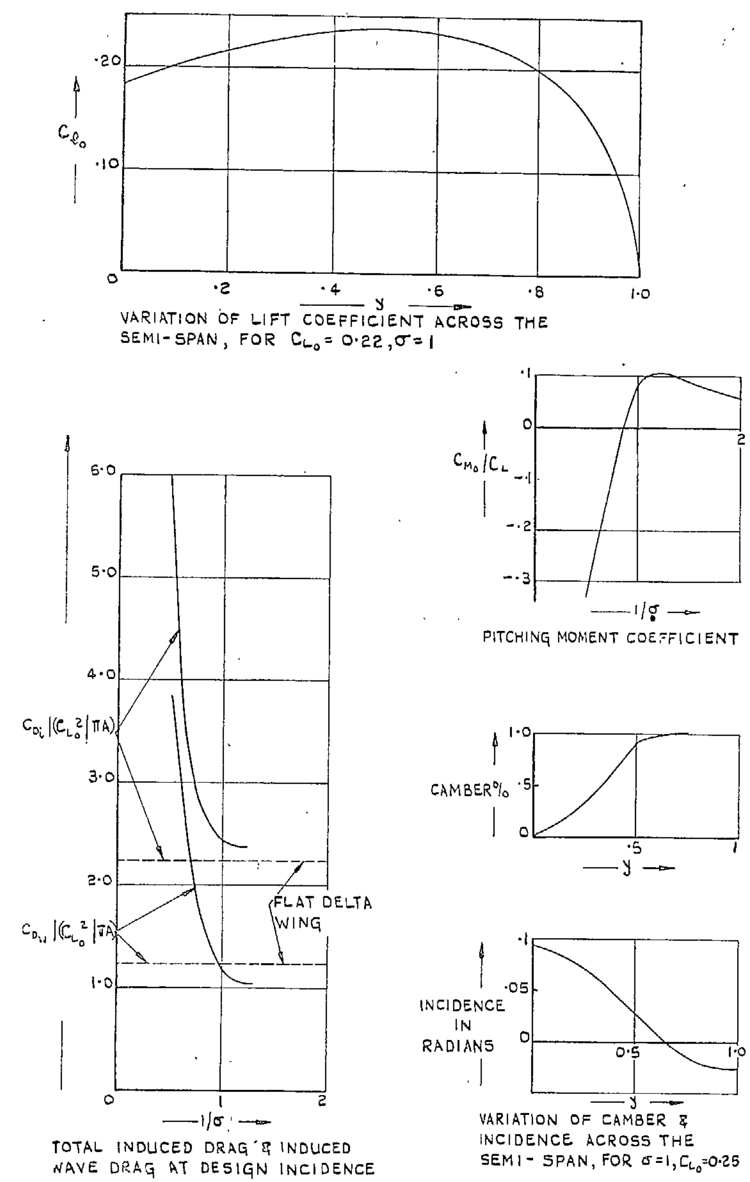


Fig. 17b. Delta wing (xi) for $\gamma = 45$ deg. $M = 1\cdot281$.

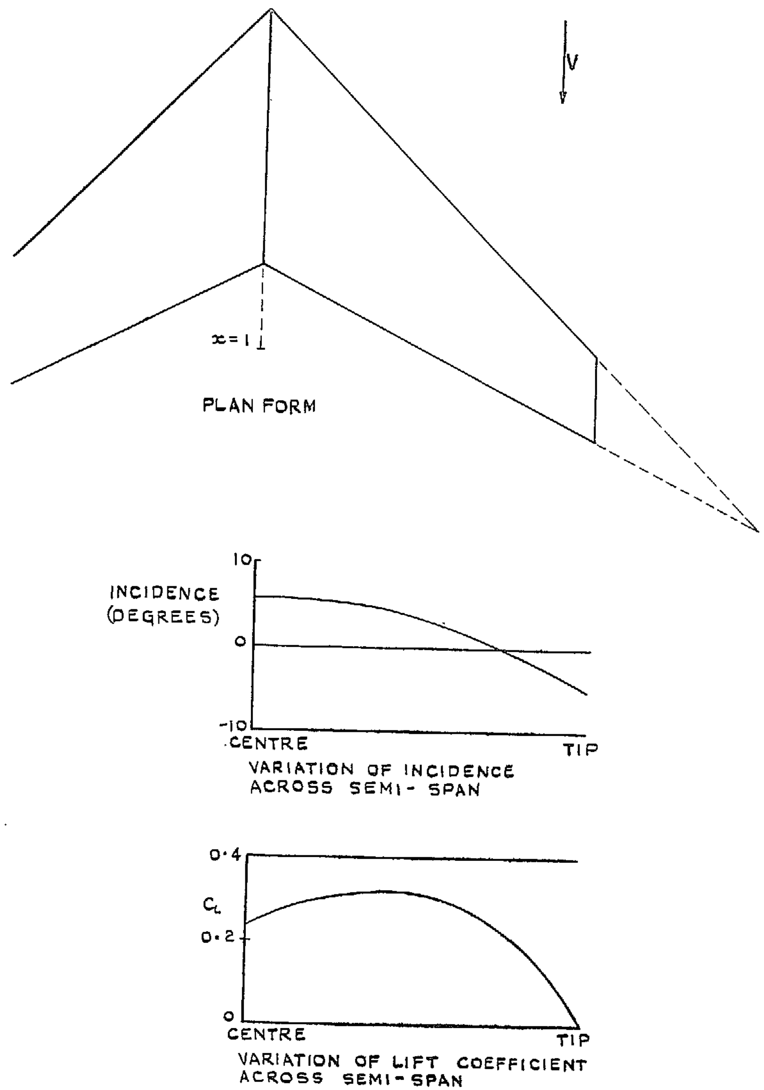


FIG. 18. Calculations for twisted wing of shape (vi), $\gamma = 45$ deg. $M = 1{\cdot}345$, $\alpha = 0$.

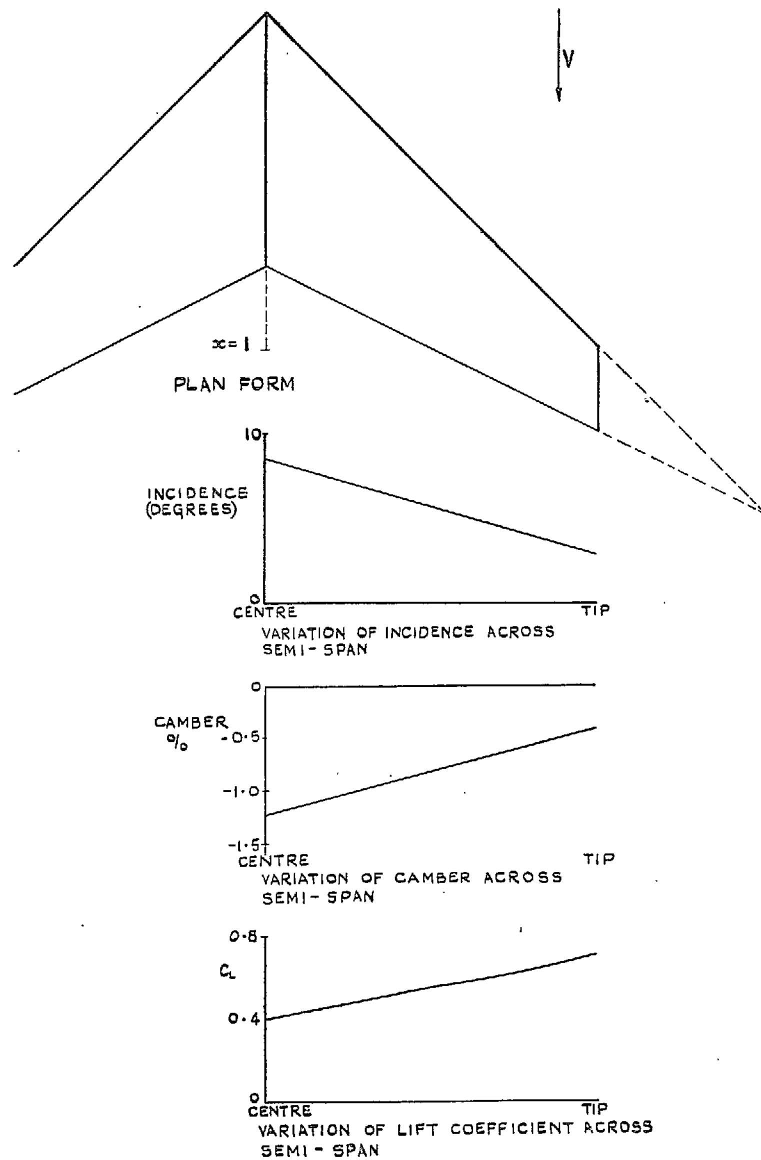


FIG. 19. Calculations for cambered and twisted wing of shape (iii), $\gamma = 45$ deg. $M = 1{\cdot}166$, $\alpha = 0{\cdot}1$.

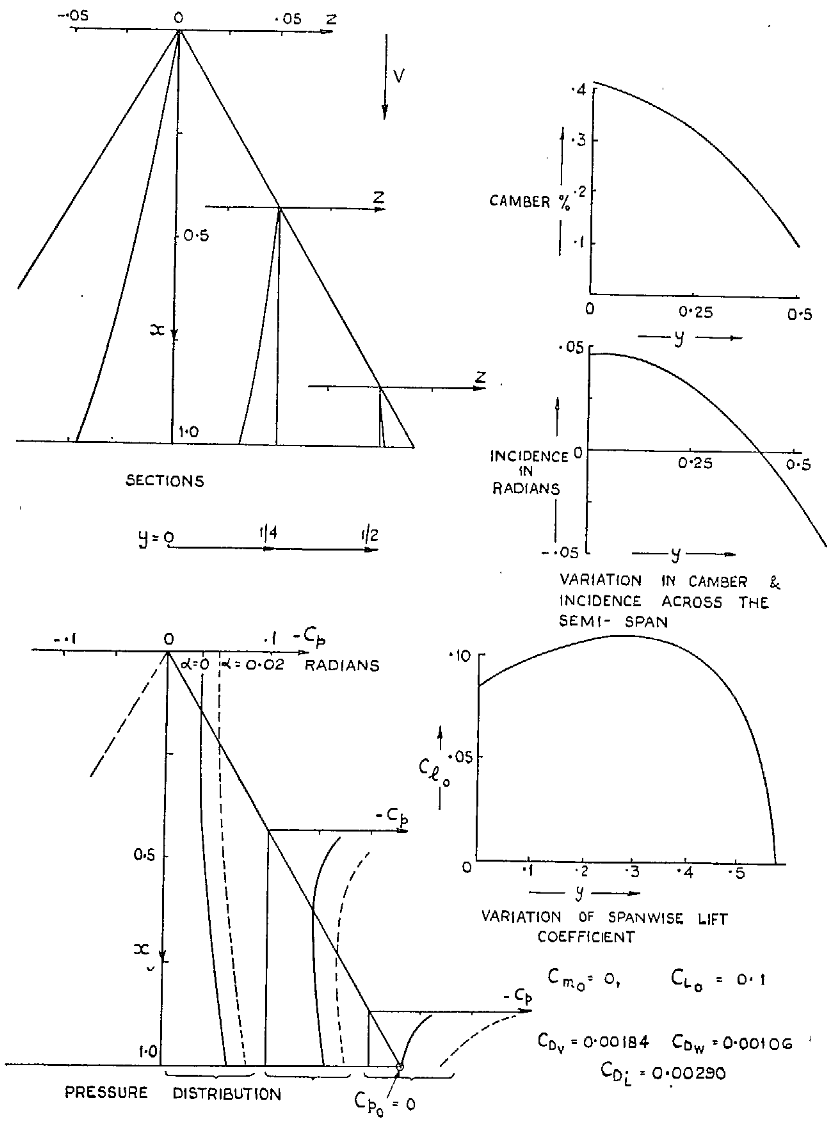


FIG. 20. Delta wing (xiv), $\gamma = 30$ deg. $M = 1 \cdot 442$. Shape and pressure distribution.

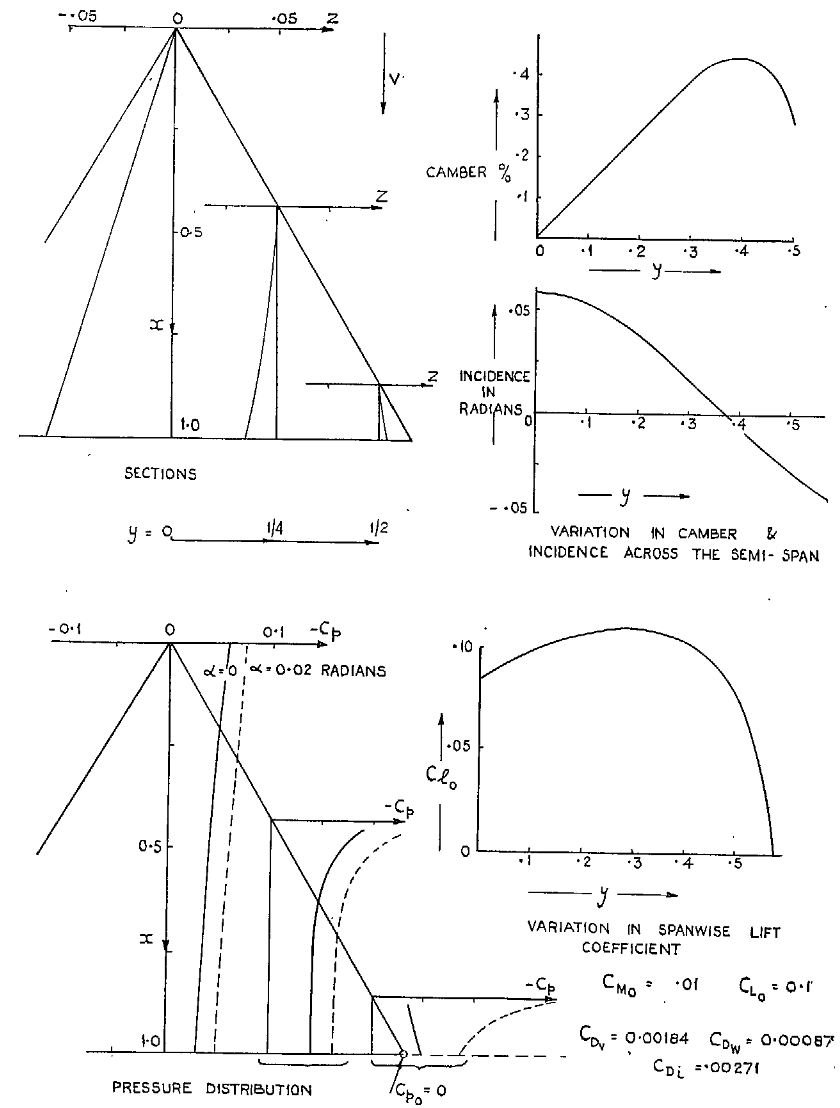


FIG. 21. Delta wing (xv), $\gamma = 30$ deg. $M = 1 \cdot 442$. Shape and pressure distribution.

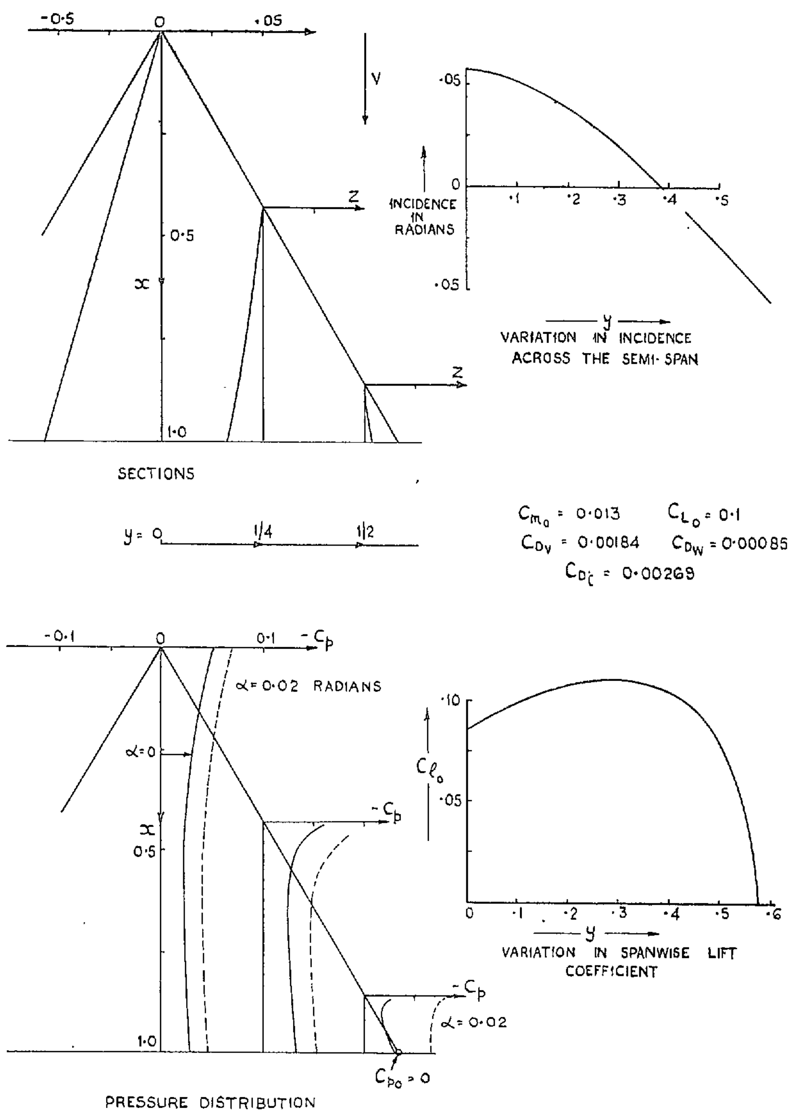


FIG. 22. Delta wing (xvi), γ = 30 deg. $M = 1\cdot442$. Shape and pressure distribution.

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