N.A.E.

R. & M. No. 2805 (10,675) A.R.C. Technical Report



# MINISTRY OF SUPPLY

# AERONAUTICAL RESEARCH COUNCIL REPORTS AND MEMORANDA

# Evaporation of Drops of Liquid

By

J. K. HARDY

Crown Copyright Reserved

### LONDON: HER MAJESTY'S STATIONERY OFFICE

1953 THREE SHILLINGS NET

# Evaporation of Drops of Liquid

By

### J. K. HARDY

Communicated by the Principal Director of Scientific Research (Air), Ministry of Supply

> Reports and Memoranda No. 2805\* March, 1947

Summary.—An analysis has been made of the processes which follow when a drop of liquid is subjected to a sudden change in the condition of the air in which it is suspended. Equations are given from which either the temperature of the drop, or the rate at which it will evaporate, can be calculated.

Introduction.—The cooling of small drops of liquid was considered in 1942 as part of the problem of creating, in the laboratory, conditions of icing similar to those which occur naturally. In order to create these conditions, water is sprayed into air the temperature of which is below 0 deg C. This had to be arranged so that there was time for the drops to reach a temperature which was practically the same as that of the air.

The rate at which the temperature of a drop will change is determined by the rate at which heat is transferred to the air. Heat is transferred both by convection and by evaporation, and this process, broadly, is the subject of this Report. The problems which will be considered are, the temperature of a drop, and the rate at which it will diminish by evaporation, or in different circumstances will grow by condensation.

When this problem was analysed it was not known to the author that the ground, in part, had already been covered. The theory of the process of evaporation was given by Maxwell and by Stefan also in the 1870's. It was the subject of numerous experiments by Winkleman in the course of the decade following. Since then, interest in the subject has been sporadic with the result that the literature is scattered and is difficult to trace. It seems useful, therefore, to widen the scope of this report by the addition of an Appendix giving data both on heat transfer and on evaporation over the entire range of Reynolds number. This, for the most part, has been deduced from the experiments of Frössling.

1

(23726)

<sup>\*</sup> R.A.E. Dept. Mech. Eng. 1, received 19th June, 1947.

#### LIST OF SYMBOLS

a Radius of drop, ft

 $C_p$  Specific heat of air at constant pressure, CHU/lb/deg C

 $C_w$  ,, ,, liquid, CHU/lb/deg C

*d* Diameter of drop, ft

D Diffusivity of vapour, ft<sup>2</sup>/sec

*e* Vapour pressure, mm of mercury

*H* Rate of transfer of heat by convection, CHU/sec

k Thermal conductivity of air, CHU/sec/sq ft/deg C/ft

L Latent heat of vapourization, CHU/lb

 $M_a$  Molecular weight of air (28.96)

 $M_e$  Molecular weight of vapour

n Concentration of vapour per unit volume, lb/cu ft

P Barometric pressure, mm of mercury

Pr Prandtl number  $\left(\frac{\mu'C_p}{k}\right)$ 

r Radial distance from centre of drop, ft

*R* Reynolds number

t Temperature, deg C.

Tr Taylor number  $\left(\frac{\mu}{D\rho'}\right)$ 

V Velocity, ft/sec

W Rate of evaporation of water, lb/sec

x Multiplying factor to allow for evaporation (equation 8)

 $\mu'$  Viscosity of air, lb/sec/ft

 $\rho'$  Weight density of air, lb/cu ft

 $\rho_w$ , ,, ,, liquid, lb/cu ft

 $\tau$  Time

Subscripts

*s* Conditions at surface of drop

0 Conditions in main body of air

1 Conditions at equilibrium temperature

 $\mathbf{2}$ 

#### ANALYSIS

When liquid is sprayed into air at a different temperature, the rate at which the temperature of the drops will change depends upon the rate at which the heat is transferred to the air both by convection and by evaporation. The exchange of heat by radiation is so small that it can be neglected, except perhaps in cases when combustion is involved. The drops will be assumed to be exactly spherical.

*Rate of Transfer of Heat by Convection*.—If the motion of the drop through the air is such that the flow is viscous, the rate of transfer of heat by convection can be calculated. The limits of Reynolds number to which this applies are the same as for Stoke's law of resistance, namely from 0 to 1.

The rate of conduction of heat from the surface of a sphere is given by the equation

$$H = 4\pi r^2 k \frac{dt}{dr} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (1)$$

in which r is the distance radially from the centre of the sphere. For a sphere radius a, the rate of transfer of heat from the surface is given by integration of (1), so that

$$\int_{a}^{\infty}\frac{H}{r^{2}}\,dr=\int_{t_{s}}^{t_{0}}4\pi k\,dt.$$

On integration this becomes

The rate of transfer of heat from a drop by convection may be calculated from this equation.

If there are a number of drops, as in a spray, the limits of integration, strictly, should be finite. Taking half the distance between the drops as the limit of integration, and this as y/2 diameters, gives

$$H = 4\pi ak \frac{y}{y-1} (t_0 - t_a)$$

with  $t_a$  the temperature of the air at half-distance. Substituting from (2) gives

$$t_a = t_0 + \frac{1}{y} (t_s - t_0).$$

For a distance apart of 100 diameters the difference as compared with a single drop will be less than 1 per cent.

Rate of Evaporation.—The rate of evaporation from a drop may be found by a process similar to that for convection. The equation which gives the rate of evaporation, corresponding to (1), is

$$W = 4\pi r^2 D \frac{dn}{dr} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (3)$$

in which D is the diffusivity of the vapour, and n the concentration of vapour per unit volume of air. Integration, as for (2), gives

$$W = 4\pi a D(n_s - n_0). \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (4)$$

The value of n, from the equation of state, is

$$n = rac{e}{P-e} rac{M_e}{M_a} 
ho' \; .$$

In this, e is the vapour pressure, P the barometric pressure, and  $M_e$ ,  $M_a$  the molecular weights of the vapour and air respectively.

Substitution in (4) gives

If the vapour pressure of the liquid is small relative to the barometric pressure, as is the case for liquids of low volatility, P may be written for P - e, so that

The rate of evaporation from a drop may be calculated from this equation. The value of  $e_i$  is the vapour pressure at saturation at the temperature of the surface;  $e_0$  is the vapour pressure in the body of the air.

*Rate of Cooling of Drops.*—The problem to be considered now is the rate at which the temperature of a drop will change after it is projected into air at a lower temperature. It is necessary, first to be clear as to the nature of the processes involved.

The air, in general, will not be saturated with vapour so that the terminal temperature of the drop will not be the same as that of the air. The drop reaches a steady temperature such that the rate at which heat is lost by evaporation is balanced exactly by that gained by convection. The equilibrium temperature, as it will be termed, may be calculated from equation (15) which is given later. The equilibrium temperature is lower than the temperature of the air so that there is a reversal in direction of transfer by convection during the process of cooling. Convection and evaporation act together until the temperature of the drop reaches that of the air; below this temperature, evaporation is opposed by convection. There is similar reversal when a drop is heated, with the difference that it is the direction of evaporation which is reversed. This reversal occurs at the temperature of the dew point. The equations which follow may be applied irrespective of whether the process involves evaporation or condensation, attention to signs being all that is necessary.

At any stage in the process, the rate of change of temperature of a drop is determined by the rate at which heat is transferred to the air by convection and by evaporation. The rates of transfer are given by equations (2) and (6), so that

$$\frac{4}{3}\pi a^{3}\rho_{w}'C_{w}t_{s} = 4\pi a \left[k(t_{s}-t_{0}) + LD\rho'\left(\frac{e_{s}-e_{0}}{P}\right)\frac{M_{e}}{M_{a}}\right]$$

The period,  $\tau$ , required for the temperature of a drop to change from  $t_{s,0}$ , initially, to  $t_{s,1}$  is given by integrating this equation so that,

$$\tau = \frac{a^2 \rho_w' C_w}{3} \int_{t_s 0}^{t_s 1} \frac{1}{k(t_s - t_0) + LD\rho' \left(\frac{e_s - e_0}{P}\right) \frac{M_r}{M_a}} dt_s . \qquad (7)$$

Integration is complicated by the form of the relation between vapour pressure and temperature, and this in general, necessitates a graphical process.

A solution of equation (7) may be obtained by the use of a factor to allow for evaporation, the rate of convection being multiplied by a factor x. This is inexact because the value of x varies with the temperature of the drop, but a calculation can be made in steps, the intervals of temperature being sufficiently small to give the accuracy required. The value of x is the mean for the interval, and is given by the equation

The value of  $e_s$  is for saturation at the temperature  $t_s$ . In the case of water vapour, equation (8) can be simplified because the diffusivity of the vapour in air is almost exactly equal to that of heat, so that

$$D = \frac{k}{\rho' C_p}$$

The value of  $M_e/M_a$  is 0.622; so for water vapour

Equation (7) when x is substituted becomes,

$$\tau = \frac{a^2 \rho'_w C_w}{3k} \int_{t_{s\,0}}^{t_{s\,1}} \frac{1}{x \ (t_s - t_0)} \ dt_s \ . \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (10)$$

Integration gives

Rate of Evaporation of a Drop.—In calculating the rate at which a drop evaporates, it is convenient to separate the period of cooling, or heating, from the period in which evaporation proceeds at a steady temperature.

Evaporation during Period of Cooling.—The amount of water which evaporates during the process of cooling may be calculated either by graphical integration of the rate of evaporation, or quite simply but approximately from the change in temperature of the drop, by use of the factor x.

In the graphical method, it is necessary first to calculate the change of vapour pressure at the surface of the drop with time, this is done by calculating the change of temperature from (7). Next, the rate of evaporation is calculated from (6), and the results are plotted against time and integrated. This gives the total amount of water evaporated.

In the alternative method, the amount of water evaporated is calculated from the total amount of heat which the drop has lost to the air. Of the total, the fraction x - 1 is lost by evaporation so that the weight of water evaporated is

$$w = \frac{4}{3}\bar{a}^{3}\rho_{w}'C_{w}\left(\frac{x-1}{L}\right)(t_{s\,0}-t_{s\,1}) \qquad \dots \qquad \dots \qquad (12)$$

in this,  $t_{s0}$  is the initial and  $t_{s1}$  the equilibrium temperature of the drop, and  $\bar{a}^3$  is the mean of the cube of the radius. The value of  $t_{s1}$  is given by equation (15).

Equating change in size to weight evaporated gives

$$a_0^{3} - a_1^{3} = \frac{C_w}{L} (x - 1)(t_{s 0} - t_{s 1})\overline{a}^{3} . \qquad (13)$$

It is evident that the change in size is so small that  $a_0^3$  may be substituted for  $\bar{a}^3$ , unless the change in temperature is very large, so that

*Evaporation at constant temperature.*—The temperature assumed by the drop, finally, is such that the rate of loss of heat by evaporation is balanced exactly by the rate of gain by convection. It follows, from equations (2) and (6), that

or

This equation is identical with equation (12) of Ref. 1, which is the psychrometric equation in its general form. In the case of water, the value of  $t_{s_1}$  can be found from the psychrometric tables as an alternative to calculation from equation (15). This is conditional on the drop being of a size such that the vapour pressure at the surface is unaffected by the curvature. The effect of size of drop on vapour pressure will be discussed later.

The rate of evaporation is related to the rate at which the size of the drop diminishes by the equation

$$W = 4\pi a^2 \rho_w' \dot{a}$$

which, by substituting for W from (6), becomes

$$\dot{a} = \frac{D\rho'}{a\rho_w'} \left(\frac{e_s - e_0}{P}\right) \frac{M_e}{M_a} \cdot \dots \quad \cdots \quad \dots \quad \dots \quad (16)$$

The solution of this equation, with the condition that  $a = a_1$  when  $\tau = 0$ , is

$$a^{2} = a_{1}^{2} - 2D \frac{\rho'}{\rho_{w}'} \left(\frac{e_{s} - e_{0}}{P}\right) \frac{M_{e}}{M_{a}} \tau. \qquad .. \qquad (17)$$

The rate at which a drop will evaporate, at a steady temperature, may be calculated from this equation.

It has been assumed throughout that the vapour pressure at the surface of the drop,  $e_s$ , is that for saturation at the temperature of the drop. For very small drops there is an increase in vapour pressure owing to the pressure generated in the drop by surface tension. Fuchs<sup>2</sup> claims that this may be neglected in comparison with the decrease which occurs as the diameter of the drop approaches the length of the mean free path of the molecules of the vapour. At normal atmospheric pressure this length is of the order of one micron. Experimental evidence is conflicting, and, it seems, there is no great error in using normal saturation pressure irrespective of size of drop.

Discussion.—A drop, when exposed to a change in environment, reaches equilibrium with great rapidity. The process is so rapid that the change in size, owing to evaporation, during the phase of equilibration is scarcely significant. This will be shown by two examples. The

first will be the cooling of the spray used to produce conditions of icing in a wind-tunnel. In this case it is important that the temperature of the drops in the working-section of the tunnel should be as nearly as possible that of the air. The second case, which shows the time required for a drop to evaporate, will be that of a drop which is suddenly exposed to air at a higher temperature. This may occur in the intake to a turbine engine, and it is important to know whether the drop will evaporate before it reaches the engine or whether it will strike some part of the engine and freeze.

Conditions of icing are produced by spraying water from a paint-gun, which gives drops with a characteristic size of 25 microns diameter. It will be assumed that the temperature of the drops as they leave the gun is + 5 deg C, that the air into which they are sprayed is at a temperature of - 10 deg C, and is saturated, and that the barometric pressure is 760 mm. The time required for the drops to reach a temperature of - 9 deg C, will be calculated.

The value of x, from equation (10), is 1.59 when the temperature of the drop is  $+ 5 \deg C$ , 1.46 when at  $-2 \deg C$  the mid-temperature, and 1.35 when at  $-9 \deg C$ . With the value 1.46, equation (11) gives the time to cool to  $-9 \deg C$  as 0.017 second. In order to find if the variation in the value of x is important, another calculation was made in two steps, from  $+ 5 \deg to - 2 \deg C$  and  $- 2 \deg to - 9 \deg C$ , with values of x appropriate to each. This gives 0.017 seconds also.

In order to show the rate at which a drop evaporates, the case will be considered of a drop which is projected into air at a higher temperature, the air being unsaturated. The size of drop will be taken as 20 microns diameter, and its temperature, initially as -20 deg C. The air will be taken as being at a temperature of 0 deg C, with the temperature of the dewpoint as -20 deg C, and the barometric pressure as 760 mm. These conditions would result from a sudden increase of 20 deg C in the temperature of the air.

The first step is to calculate the temperature finally reached by the drop, namely the equilibrium temperature. This is given by equation (15), which must be solved by trial because the value of  $e_{s1}$ , varies with  $t_{s1}$ . For the conditions chosen, the equilibrium temperature is -5.4 deg C.

The value of x initially is  $1 \cdot 0$ , and it diminishes progressively until, at equilibrium temperature, the value of x is zero. The time taken for the drop to reach equilibrium has been calculated from equation (11), in 8 steps which were chosen so as to keep the change in x small. The total period is  $0 \cdot 015$  second. The change in size of the drop during this period is given by equation (14). With the mean value of x, this gives  $a_1 = 0.996a_0$ —an insignificant change.

The rate of evaporation after the drop has reached equilibrium is given by equation (17). From this, the time required for the diameter to decrease to 5 microns is 0.91 second, and for complete evaporation is 0.98 second. Thus the total period, reckoned from the time that the drop is projected into air, is 0.995 second. The period of equilibration is 1.5 per cent of the total.

The period required for a drop to evaporate seems small but in an engine, where the drop may be carried by the air with a velocity of 100 ft/sec or more, evaporation may be inappreciable before the drop has struck some part of the engine. If drops are carried along a pipe, it is instructive to compare the length of pipe necessary for the drops to evaporate with that necessary if the same quantity of liquid is evaporated from the walls of the pipe. The rate of evaporation from the walls can be calculated from the equations given in sections 61 and 89 of Ref. 3. If the velocity is high, it will be found that the length required for evaporation from the walls is a small fraction only of that required for evaporation from the drops.



and a start of the

#### REFERENCES

No.		Name			Title, etc.
1	J. K. Hardy	••	•••	••	Kinetic Temperature of Wet Surfaces etc. NACA, ARR. No. 5G 13. 1945.
<b>2</b>	W. Fuchs	••	••	••	Physical Journal of the Soviet Union. A.R.C. 6,224. 1934.
3	J. K. Hardy	••		•••	Protection of Aircraft against Ice. A.R.C. 10,034. 1946. (Unpublished.)
4	N. Frössling	• •	••	••	Gerlands Beitrage z. Geophysik. A.R.C. 5,270. 1938.
5	McAdams	••	••	••	Heat Transmission, 2nd edn. Page 237. 1942. McGraw-Hill Book Co.

#### APPENDIX

### Evaporation and Heat Transfer When Reynolds number > 1

The rate of evaporation from a wet surface can be calculated with accuracy from data on the transfer of heat. Thus, on the assumption that the drops of liquid are spherical, it is possible to use the data for heat transfer between spheres and air to calculate the rate at which a drop will evaporate. The data, nominally, cover a range of Reynolds number from 20 to 150,000, but are unsatisfactory at values below 500. However, Frössling<sup>4</sup> in a series of careful experiments, has measured the rate of evaporation of drops of liquid covering a range of Reynolds number from 2 to 800. So, the rate of evaporation may be calculated over the entire range which can occur, a limit being set by the instability and disruption of the drops.

Frössling suspended the drops above a small, vertical wind tunnel of the open-jet type. As liquids, he used analine, nitrobenzine and water, and obtained consistent and satisfactory results. He found that rate of evaporation could be expressed by the equation

$$W = 4\pi a D \rho' \left(\frac{e_s - e_0}{P}\right) \frac{M_e}{M_a} \left(1 + 0 \cdot 276 (Tr)^{1/3} \sqrt{R}\right). \quad .. \qquad (18)$$

This equation may be transposed to give the rate of transfer of heat, similar to equation (2), by the use of dimensionless coefficients of heat transfer and evaporation,  $k_h$  and  $k_w$ . This is discussed fully in Ref. 1. These coefficients are defined by the equations

and

$$k_h = rac{H}{4\pi a^2 
ho' V C_p(t_s - t_0)}$$
 $k_w = rac{W}{4\pi a^2 
ho' V \left(rac{e_s - e_0}{P}
ight) rac{M_e}{M_a}}.$ 

By straightforward substitution, it is found that the equation for heat transfer corresponding to (18), in terms of Nusselt number is

$$\frac{hd}{k} = 2 + 0.552 (Pr)^{1/3} \sqrt{R} \qquad \dots \qquad \dots \qquad \dots \qquad (19)$$

the diameter, d, of the drop, being the characteristic length in R.

8

McAdams<sup>5</sup> has published Williams' correlation of the data on heat transfer between spheres and air, and gives the following equation,

The corresponding equation for evaporation, found by reversing the process applied to Frössling's equation, is

For water vapour in air Pr is practically equal to Tr, so equation (21) becomes

The same rate of evaporation is given by equation (21) as is given by Frössling's equation (18) at a Reynolds number slightly below 400, the value of Nusselt number, of course, being also the same (equations (19) and (20)). By analogy with pipes when the flow is transverse, it would be anticipated that Nusselt number should be proportional to R when the flow is laminar or nearly so, and that the index of R should increase as the transfer at the back of the sphere in the region of the wake becomes important. From Frössling's experiments with a sphere of solid napthalene, this occurs, it seems, at about R = 600. At this value of R, rates of heat transfer and of evaporation given by equations (20) and (21) are 8 per cent above those given by equations (18) and (19). On the whole, it seems better to take R = 600 as the limit to which (18) and (19) apply, and to accept the discontinuity. So, (20) and (21) should be used for values of R from 600 upwards.

PRINTED IN GREAT BRITAIN

# Publications of the Aeronautical Research Council

#### ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40s. (40s. 9d.) Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50s. (50s. 10d.)

1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40s. (40s. 10d.) Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60s. (61s.)

1938 Vol. I. Aerodynamics General, Performance, Airscrews. 505. (515.) Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 305. (305. 9d.)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50s. (50s. 11d.) Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63s. (64s. 2d.)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50s. (51s.)

1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control, Structures. 635. (645. 2d.)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75s. (76s. 3d.) Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47s. 6d. (48s. 5d.)

1943 Vol. I. (In the press.) Vol. II. (In the press.)

 ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL

 1933-34
 1s. 6d. (1s. 8d.)
 1937
 2s. (2s. 2d.)

 1934-35
 1s. 6d. (1s. 8d.)
 1938
 1s. 6d. (1s. 8d.)

 April 1, 1935 to Dec. 31, 1936. 4s. (4s. 4d.)
 1939-48
 3s. (3s. 2d.)

INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS AND SEPARATELY-

April, 1950 - - - - R. & M. No. 2600. 2s. 6d. (2s. 71d.)

AUTHOR INDEX TO ALL REPORTS AND MEMORANDA OF THE AERONAUTICAL RESEARCH COUNCIL-

1909-1949 - - - - R. & M. No. 2570. 15s. (15s. 3d.)

INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL-

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	IS. 3d. (IS. 43d.)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1s. (1s. 11d.)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1s. (1s. 1 <sup>1</sup> / <sub>2</sub> d.)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1s. 3d. (1s. 4 <sup>1</sup> / <sub>2</sub> d.)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	IS. 3d. (IS. 41d.)
July, 1951	R. & M. No. 2350.	IS. 9d. (IS. 101d.)

Prices in brackets include postage.

#### Obtainable from

### HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, London, W.C.2; 423 Oxford Street, London, W.I (Post Orders: P.O. Box 569, London, S.E.I); 13a Castle Street, Edinburgh 2; 39 King Street, Manchester 2; 2 Edmund Street, Birmingham 3; I St. Andrew's Crescent, Cardiff; Tower Lane, Bristol 1; 80 Chichester Street, Belfast or through any bookseller.

S.O. Code No. 23-2805