

## Swept Wings in Supersonic Flight

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR), MINISTRY OF SUPPLY

## Reports and Memoranda No. 2818

December, 1946

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Summary.—Opinion seems still unsettled on the aerodynamic merit of swept wings in supersonic flight. To elucidate this, Ackeret's theory of two-dimensional wave reaction is here extended to include sweep. The formulae so derived are used to compare the performance of a straight wing with one swept through 45 deg, making some allowance for frictional drag.

As the wave form-drag varies as  $(\text{thickness})^2$  it is this part of the drag which causes most trouble. A straight wing of given thickness/chord ratio can be swept through an angle  $\psi$  either by yawing it or by shearing it. In both cases the critical M is increased from 1 to sec  $\psi$  and the favourable lift/incidence effects above the critical M are the same. But the form-drag of the yawed wing begins to be less than that of the straight wing soon after  $M = \sec \psi$  and is reduced in the ratio  $\cos^2 \psi : 1$  at large M; while the form-drag of the sheared wing always exceeds that of the straight wing. Thus to make the best use of sweep in a supersonic speed range beginning at  $M = \sec \psi$  the straight wing thickness which must be tolerated should be yawed through an angle  $\psi$ .

1. Introduction.—In A.R.C. 8806<sup>1</sup> McKinnon Wood goes a long way to remove some of the confusions which exist in assessing the aerodynamic merit of swept wings. Yet this paper still seems to leave open the argument between those who think of the wing section along the direction of flight and those who think of it as perpendicular to the leading edge. Nor does it fully rebut the not uncommon opinion that as sweep merely delays the critical Mach number its usefulness at supersonic speeds is doubtful. As Ackeret's theory of wave reaction at supersonic speeds can be simply extended to include sweep on the lines suggested by McKinnon Wood, I have worked this out in the hope of throwing further light on the subject. The calculation given below follows Taylor's outline of Ackeret's theory (R. & M. 1467<sup>2</sup>) with some changes of notation.

2. Ackeret's Theory Extended to Include Sweep.—Consider an infinite unswept wing of unit chord moving at speed V (Mach number M > 1) and incidence  $\alpha$  referred to the chord joining the sharp leading and trailing edges. If the wing is now yawed through an angle  $\psi$  in the plane of its edges without altering its motion, its speed and incidence in a plane perpendicular to its edges are respectively  $V \cos \psi$  and  $\alpha \sec \psi$  (Fig. 1). It is the motion in this plane which produces the plane sound waves springing from the surface, and so we calculate the wave reaction by using the appropriate Mach angle  $\mu$  in this plane and substituting  $V \cos \psi$  for V and  $\alpha \sec \psi$  for  $\alpha$  in Taylor's equations. The geometry is sketched in Fig. 2.

The Mach angle  $\mu$  is given by

(60497)

At the surface the component velocity of the air normal to the surface is equal to the velocity of the surface normal to itself. The surface condition is therefore

 $V \cos \psi \cdot \beta = u \cos (\mu - \beta)$ 

if u is the wave velocity and  $\beta$  the inclination of the tangent of the surface to the undisturbed flow.  $\beta$  being small compared with  $\mu$  we have from this

But the wave pressure p is given by

$$\begin{split} p &= \rho a u \\ &= \rho a V \beta \cos \psi \sec \mu \qquad \text{from (2)} \\ &= \frac{\rho V^2 \cos^2 \psi}{\sqrt{(M^2 \cos^2 \psi - 1)}} \beta \qquad \text{from (1)} \dots \dots \dots \dots \dots \dots \dots \dots \dots (3) \end{split}$$

If  $\beta_1$ ,  $\beta_2$  are respectively the slopes of the upper and lower surfaces referred to the chord,  $\beta$  is  $(-\alpha \sec \psi + \beta_1)$  along the upper surface and  $(\alpha \sec \psi + \beta_2)$  along the lower surface.

Hence the upper surface pressure  $p_1$  is given by

and for the lower surface

We can now integrate along the unit chord to get the section coefficients as follows

where

 $C_{M}$  about leading edge

$$= \frac{1}{\frac{1}{2}\rho V^{2}} \int_{0}^{1} (p_{1} - p_{2})x \, dx$$

$$= \frac{1}{\sqrt{(M^{2} - \sec^{2} \psi)}} \left\{ -2\alpha + 2\int_{0}^{1} (\beta_{1} - \beta_{2})x \, dx \right\}$$

$$= -\frac{1}{2}C_{L} + \frac{\cos \psi}{\sqrt{(M^{2} - \sec^{2} \psi)}} \left\{ 2\int_{0}^{1} (\beta_{1} - \beta_{2})x \, dx - \int_{0}^{1} (\beta_{1} - \beta_{2}) \, dx \right\}. \quad ... \quad (9)$$

Thus the aerodynamic centre is at half the chord, and

$$C_{M0} = \frac{\cos \psi}{\sqrt{(M^2 - \sec^2 \psi)}} \left\{ 2 \int_0^1 (\beta_1 - \beta_2) x \, dx - \int_0^1 (\beta_1 - \beta_2) \, dx \right\} \, . \qquad (10)$$

In the above  $C_D$  and  $C_{M0}$  are referred to the yawed chord and the yawed span. We are more interested in their components  $C_D \cos \psi$ ,  $C_{M0} \cos \psi$  referred to the direction of flight, and denoting these by the suffix F we have

$$C_{M0F} = \frac{\cos^2 \psi}{\sqrt{(M^2 - \sec^2 \psi)}} \left\{ 2 \int_0^1 (\beta_1 - \beta_2) x \, dx - \int_0^1 (\beta_1 - \beta_2) \, dx \right\} \qquad \dots \qquad (10')$$

3. Biconvex Section.—Equations (6) to (10') give the wave reaction produced by an infinite yawed wing in the general case. To discuss the effects of the sweep  $\psi$  in more detail consider a biconvex section in which the edge values of  $\beta_1$ ,  $\beta_2$  are  $\beta_{m1}$ ,  $\beta_{m2}$  so that

$$\frac{\beta_1}{\beta_{m1}} = \frac{\beta_2}{\beta_{m2}} = 1 - 2x \,.$$

If the thickness/chord ratio is t and the camber of the centre-line is  $\tau$  we have

$$t = \frac{1}{4}(\beta_{m1} + \beta_{m2})$$
  
$$\tau = \frac{1}{8}(\beta_{m1} - \beta_{m2})$$

It follows that

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$$\int_{0}^{1} (\beta_{1} - \beta_{2}) dx = 0$$
  
$$\int_{0}^{1} (\beta_{1}^{2} + \beta_{2}^{2}) dx = \frac{1}{3} (\beta_{m1}^{2} + \beta_{m2}^{2})$$
  
$$= \frac{8}{3} (t^{2} + 4\tau^{2})$$
  
$$\int_{0}^{1} (\beta_{1} - \beta_{2}) x dx = -\frac{1}{6} (\beta_{m1} - \beta_{m2})$$
  
$$= -\frac{4}{3}\tau.$$

The results for a biconvex section are therefore

$$C_{L} = \frac{4\alpha}{\sqrt{(M^{2} - \sec^{2}\psi)}}$$

$$C_{DF} = \frac{1}{\sqrt{(M^{2} - \sec^{2}\psi)}} \left\{ 4\alpha^{2} + \frac{1.6}{3}(t^{2} + 4\tau^{2})\cos^{2}\psi \right\}$$

$$\frac{L}{\overline{D}} = \frac{\alpha}{\alpha^{2} + \frac{4}{3}(t^{2} + 4\tau^{2})\cos^{2}\psi}$$

$$C_{M0F} = -\frac{8}{3} \frac{\tau \cos^{2}\psi}{\sqrt{(M^{2} - \sec^{2}\psi)}}$$

$$3$$

$$(11)$$

4. Symmetrical Biconvex Wing with Frictional Drag.—The analysis can be made rather more realistic by applying it to the case of finite swept wings, and making some allowance for the skin friction of the wings and auxiliary surfaces, while still neglecting the aspect-ratio effects and any other wave drag or interference drag which may be present. This amounts to increasing  $C_{DF}$  by  $KC_f$  where  $C_f$  is the skin-friction coefficient (assumed to be independent of sweep) and K is the ratio of total wetted area to wing area.

It will be useful to compare the performance of the swept with the unswept wing over a range of Mach number, wing loading, and altitude. The wing loading w is introduced by the relation

$$C_L = \frac{2w/r}{\gamma p_0 M^2}$$

where  $p_0$  is sea-level pressure (2110 lb/ft<sup>2</sup>), r is relative pressure  $p/p_0$ , and  $\gamma$  is the ratio of the specific heats of air, taken constant at 1.4. The relative pressure r is shown as a function of altitude in Fig. 3. The equations for a symmetrical biconvex section are on these assumptions :—

$$C_{DF} = \frac{4}{\sqrt{(M^2 - \sec^2 \psi)}} \left( \alpha^2 + \frac{4}{3} t^2 \cos^2 \psi \right) + K C_f \qquad \dots \qquad \dots \qquad \dots \qquad (13)$$

The most useful basis of drag comparison appears to be what may be called the specific drag D/Sr, i.e., the drag per unit area per relative pressure. This is obtained from (12) and (13), by eliminating  $\alpha$ , in the form

5. *Performance Comparisons.*—The characteristics given by (12) to (15) are surveyed in Figs. 4 to 9 where the following numerical values and ranges have been taken

M =Critical to 4.

ψ

- $KC_f = 0.01.$  This can be only a very rough typical value, as K depends on the proportion of wing to body and  $C_f$  depends on the Reynolds number, which varies between 10<sup>7</sup> and 10<sup>8</sup> in the range considered. In this range  $C_f$  is of the order 0.0020 to 0.0025, and so the value of  $KC_f$  chosen represents a design in which wetted area is four or five times wing area.
  - w/r w may vary from 20 to over 100, and r is less than 0.1 above 50,000 ft. A range of w/r from 100 to 1000 is covered.
  - t 0 and  $0 \cdot 1$ .

0 and 45 deg.

Lift and Incidence (Eqns. 12).— $C_L$ ,  $dC_L/d\alpha$  and  $\alpha$ , which are independent of thickness, are plotted against M in Figs. 4 to 6. These diagrams demonstrate the low values of  $C_L$  and  $\alpha$  which suffice for supersonic flight in the stratosphere at wing loadings less than 100. Fig. 6 shows in

particular the maximum in incidence which is characteristic of supersonic flight at any given w/r. This is found from equation (12) as

$$\alpha_{\max} = 0.0048 \frac{\omega}{r} \cos \psi \quad \text{degrees} \quad \dots \quad (16)$$

and occurs at  $M = \sqrt{2}$  sec  $\psi$ .

An indication of the  $C_L$ ,  $\alpha$ -régime is given by the following table which shows the values required to produce 1g and 5g when W/S = 50:—

		CL	<i>C</i> <sub><i>L</i></sub>		$\alpha_{\rm max}$ (deg)	
		M = 1.5	2.5	$\psi = 0$	45 deg	
ground	1g 5g	$\begin{array}{c} 0\cdot 008\\ 0\cdot 040\end{array}$	$\begin{array}{c} 0\cdot 003\\ 0\cdot 015\end{array}$	$ \begin{array}{c} 0\cdot 24 \\ 1\cdot 20 \end{array} $	0·17 0·85	
52,000 ft.	1g 5g	$\begin{array}{c} 0 \cdot 08 \\ 0 \cdot 40 \end{array}$	$0.029 \\ 0.145$	$ \begin{array}{r} 2\cdot 4 \\ 12\cdot 0 \end{array} $	1.7 8.5	

The advantage of sweep in reducing the incidence necessary for any flight conditions is obvious.

Lift/Drag.—In Fig. 7 L/D is plotted against  $\alpha$  for several values of M for  $\psi = 0$  and 45 deg, at thickness  $t = 0 \cdot 1$ . This shows clearly the merit of the yawed wing, but is chiefly of interest when studied in relation to the incidence survey of Fig. 6. At a given value of w/r the  $(L/D)_{max}$  available can only be utilised if the incidence range in which this occurs is exceeded by the  $\alpha_{max}$  given by Fig. 6 or equation (16). If w/r is of the order 100, supersonic flight is confined to incidences of less than  $0 \cdot 01$  radians and L/D's less than 1. Even at w/r = 1000 the L/D reached is considerably less than that available, and it is only when w/r exceeds 1500 that  $(L/D)_{max}$  is reached in some part of the M range. The wedge-shaped curves sketched in the figure show the operating conditions for several values of w/r. Thus in the case considered maximum efficiency can only be realised in the troposphere at enormous wing loadings; with loadings of less than 100 it is necessary for maximum efficiency to fly high in the stratosphere. Conditions become easier with thinner wings than the 10 per cent illustrated, since  $\alpha$  for  $(L/D)_{max}$  decreases with thickness. It is nevertheless generally true that efficient supersonic flight puts a premium on high altitude.

Specific Drag D/Sr.—It is clear from equation (15) that  $\psi$  affects both the induced drag (arising through w/r) and the form drag (arising through thickness t). To separate these effects, the specific drag is plotted in Fig. 8 for the ideal case of zero thickness, and also for t = 0.1\*.

The effect of 45 deg yaw on the induced drag is shown in the lower group of curves; it is favourable but small except near the critical  $M = \sqrt{2}$ .

The yaw effect on form drag is much more important, as is seen by noting the difference between a curve in the upper group and its corresponding one in the lower group. For instance at w/r = 500 and M = 2 the form drag of the unyawed wing is AC; this is reduced to BC by yawing through 45 deg. The yaw effect increases with M, and at large M the form drag is approximately halved by yawing through 45 deg.

It should perhaps be noted that L/D values can be quickly obtained from a specific drag curve; we have only to divide w/r by its ordinates. If the diagram is examined in this way it will be found to conform to the L/D discussion already given.

<sup>\*</sup> A rather extreme thickness has been chosen for illustration. At the more usual thicknesses of 7 per cent and 5 per cent the form drags shown would be respectively halved and quartered.

6. Yawed and Sheared Wings.—The discussion so far has compared a straight wing with one of the same thickness which is yawed. It is evident, however, that the drag equations (13) and (15) admit another interpretation, for t cos  $\psi$  is merely  $t_F$  the thickness/chord ratio of the yawed wing measured in the direction of flight. If we use  $t_F$  instead of  $t \cos \psi$  in (15) we are clearly comparing the drag of a straight wing of thickness/chord ratio  $t_F$  with that of the wing sheared through an angle  $\psi$ . The result of this at  $t_F = 0.1$  is shown in Fig. 9, to be compared with Fig. 8. The sheared wing has more drag than the straight wing above the critical M because its thickness/chord ratio in a section perpendicular to its edges has been increased, and the comparison with the yawed wing is very striking. The lift and incidence comparisons of Figs. 5 and 6 remain unaltered.

This distinction between the yawed and the sheared wing seems very pertinent to supersonic design. We have seen that supersonic form drag is particularly serious because it increases as  $t^2$ . The minimum thickness which can be tolerated is usually dictated by non-aerodynamic consideration, and should be settled in relation to the straight wing. The critical M can now be increased from 1 to sec  $\psi$  by sweeping the wing through  $\psi$ , however this is effected. If the wing is yawed the

form drag is multiplied, relative to the straight wing, by the factor  $\cos^2 \psi \left(\frac{M^2 - 1}{M^2 - \sec^2 \psi}\right)^{1/2}$  which

becomes unity shortly after  $M = \sec \psi$  and  $\rightarrow \cos^2 \psi$  as  $M \rightarrow \infty$ . If the wing is sheared the corresponding factor is  $\left(\frac{M^2-1}{M^2-\sec^2 \psi}\right)^{1/2}$  which is always > 1 and  $\rightarrow 1$  as  $M \rightarrow \infty$ .

It seems then that in designing for a range of supersonic speeds beginning at  $M = \sec \psi$  the correct course is to turn the thickness which has to be tolerated through an angle  $\psi$  away from the direction of flight by yawing the wing through  $\psi$ . The application of this simple rule to plan forms of small aspect ratio and high taper, such as the delta wing, is of course very doubtful. But the above analysis seems to resolve the argument between the 'fore-and-aft' and the ' normal to leading edge ' schools of thought, in so far as effects of sweep can be isolated from the other parameters of a finite plan form.

## REFERENCES

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No.	Author			Title, etc.
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Vcos ψ











FIG. 7. Comparison of Yawed and Unyawed Biconvex Wings; 10 per cent thickness.



**Ehickne** 

% 0

**Ehickness** 

Zero

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S.O. Code No. 23-2818

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