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An Experimental Investigation of the Boundary Layer on a Porous Circular Cylinder

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Summary.—The report describes an experimental investigation of the boundary layer on the surface of a porous circular cylinder at which there is a normal inward velocity. The primary object of the experiments was to test the approximate theory of Ref. 1 for calculating the development of a laminar boundary layer under conditions of continuous suction. The formula given in that reference for calculating the momentum thickness of the layer gave results in accord with the experimental determinations. Owing to practical difficulties in the exploration of the very thin boundary layers and in the determination of the velocity gradient around the surface, other comparisons with the theory (such as the progressive development of the boundary-layer velocity distribution and of the prediction of the separation point) were difficult. Nevertheless reasonable agreement between the theoretical and experimental velocity distributions was obtained particularly for the lower wind speed of the experiment, but no adequate test of the prediction of the separation point was found possible.

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 U_0 Free-stream velocity

 v_0 Normal velocity at the surface of the cylinder.

- ν Kinematic viscosity of air
- ϕ Angular position on the surface of the cylinder measured from the front stagnation point
- *c* Cylinder diameter
- x Distance measured along the surface of the cylinder from the front stagnation point
- *y* Distance normal to the surface
- U Velocity outside boundary layer
- *u x*-component of velocity in boundary layer
- U' Velocity gradient: U' = dU/dx

R Reynolds number:
$$R = U_0 c/v$$

 δ^* Displacement thickness of boundary layer: $\delta^* = \int_0^\infty (1 - u/U) \, dy$

 θ Momentum thickness of boundary layer: $\theta = \int_{a}^{\infty} (u/U)(1 - u/U) dy$

$$V_0 = -(v_0/U_0)\sqrt{F}$$

$$\Delta^* = (\delta^*/c)\sqrt{R}$$

$$\Theta = (\theta/c)\sqrt{R}$$

- *H* Parameter : $H = \delta^*/\theta$
- *l* Parameter : $l = (\theta/U)(\partial u/\partial y)_{y=0}$
- *m* Parameter: $m = (\theta^2/U)(\partial^2 u/\partial y^2)_{y=0}$
- λ Parameter: $\lambda = -v_0\theta/\nu = V_0\theta$
- $\Lambda \qquad \text{Parameter: } \Lambda = -U'\theta^2/\nu = -(U'c/U_0)\Theta^2$

1. Introduction.—A recent trend in aerodynamics has been the search for methods of reducing the drag of a body either by replacing regions of turbulent boundary-layer flow by ones of laminar flow or by delaying or preventing separation. Both theory and experiment show that at high Reynolds numbers a laminar boundary layer is unstable on a solid surface. One way in which large areas of laminar flow may be achieved is to use the stabilising effect on the boundary layer of continuous suction. Continuous suction may also prevent separation. Considerable attention has therefore been given to the theory of the development of a laminar boundary layer under conditions of continuous suction and in Ref. 1 this theory is considered in some detail and an approximate method of solution given. The primary object of the experiments described in this report was to test this theory.

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2. Description of Apparatus and Experimental Method.—(a) General.—A porous circular cylinder 3 in. in diameter and 15 in. long was fitted horizontally between false walls in the National Physical Laboratory 4-ft No. 2 Wind-Tunnel. A photograph of the apparatus is shown in Fig. 1. The model was provided with 36 pressure holes equally spaced around its circumference and staggered in two rows $\frac{1}{2}$ in. on either side of the mid-span. The porous cylinder had an outlet on one side which was joined to a suction pump. The pressure in the interior of the cylinder was measured using an inclined U-tube.

Boundary-layer traverses were made using a small total-head tube which could be moved round the mid-span section of the surface of the cylinder and traversed normal to the surface by a calibrated micrometer screw-head, located outside the tunnel and connected to the total-head tube by a mechanical linkage. A pea-lamp indicated when the total-head tube was in electrical contact with the surface. The tube was joined to a hollow rod spanning the tunnel and connected to one limb of a 26-in. Chattock gauge, the other limb being connected to the surface pressure hole at the total-head tube position.

The apparatus was designed for experiments³ with a Thwaites flap². The small chord of the cylinder and the continuous suction produced very thin boundary layers which were difficult to examine. Thus the apparatus was not ideal for the present experiments.

Throughout the experiment a $\frac{1}{2}$ -in. flap was fitted at the rear of the cylinder ($\phi = 180 \text{ deg}$).

(b) Total-head Tube.—Because the boundary layers being investigated were very thin (having displacement thicknesses of the order 0.005 in. to 0.010 in.) it was important that the total-head tube used should be as small as possible. Various sizes and shapes were tried and it was found that a tube roughly rectangular in cross-section and of internal dimensions 0.005 in. by 0.025 in. (with the smaller dimension normal to the surface) was the most satisfactory. With any smaller tube the time lag was excessive.

(c) Range of Tests.—At nominal wind speeds of 25 and 69 ft/sec ($R = 4.20 \times 10^4$ and 1.011×10^5) boundary-layer traverses were made at various positions on the surface of the cylinder for different suction quantities (including no suction) and the surface pressure distributions were recorded. With no suction, separation occurred near 80 deg (measured from the front stagnation point) indicating that the boundary layer was laminar.

The suction quantities were all chosen fairly small (the largest velocity into the surface being 0.4 ft/sec) so that the boundary layers did not become so thin that satisfactory measurements could not be made, and also that measurements could be taken of boundary layers about to separate.

After a certain time the porosity of the cylinder suddenly decreased and prevented the completion of the original experimental programme in its entirety.

At the conclusion of the experiment the true wind speeds between the false walls were measured with the model removed.

(d) Analysis of Results.—The normal velocity, v_0 , at the surface of the cylinder was obtained from the pressure drop across it using previously measured values of the porosity of the material. Variation of the static pressure around the surface of the cylinder results, of course, in a variation of v_0 for constant pressure inside the cylinder, full allowance for which was made in the comparison with the theory.

The displacement and momentum thicknesses of the boundary-layer (δ^* and θ respectively) were calculated from the readings of the total-head tube assuming that the tube registered the total-head at its centre.

One source of error in these values of δ^* and θ arises from the zero error, ε , in the distance of the tube from the surface. If it is assumed that the velocity varies linearly from the surface up to the position of the first reading, u_1 , these errors are easily calculated in terms of ε and u_1/U .

Thus the error in δ^* is $\varepsilon(1 - \frac{1}{2}u_1/U)$ and the error in θ is $\varepsilon[\frac{1}{2}(u_1/U) - \frac{1}{3}(u_1/U)^2]$. It is estimated that -0.0005 in. $< \varepsilon < 0.0005$ in. and the error in θ has been evaluated and indicated in the figures.

Errors also arise from the finite size of the total-head tube, from interference effects when the tube is near the surface, from the effect of fluctuations in the boundary layer and from variations in static pressure through the boundary layer. Examination of the velocity profiles shows that near the separation point the measured values of u/U near the surface are consistently too large. (See for example curves 1 and 5 of Fig. 9, and Fig. 12). It is thought that these increased readings may be due to the fluctuations, u', of the x-component of velocity in the boundary layer. Such fluctuations uncorrelated with the v-component are possible in a laminar layer: Schubauer⁴ measured such fluctuations on an elliptic cylinder and obtained values of u'/U_0 as large as 0.3 near the surface which certainly are large enough to account for the increased readings.

3. Summary of Approximate Theory.—The basic assumption of the theory of Ref. 1 is that the boundary-layer characteristics when put in suitable non-dimensional form are functions only of λ and Λ where $\lambda = -v_0\theta/\nu$ and $\Lambda = -U'\theta^2/\nu$.

The momentum thickness, θ , itself is expressible as a function of x/c, U/U_0 , λ and Λ . The momentum equation can be written in the form

It was shown in Ref. 1 that for all known solutions of boundary-layer flow the right-hand side of equation (1) can be well represented by a single function of λ and Λ , and the equation then becomes

$$\frac{U}{\nu}\frac{d\theta^2}{dx} = \frac{U}{U_0}\frac{d\Theta^2}{d(x/c)} = 0.45 - 1.28\lambda + 0.76\lambda^2 + 6\Lambda \qquad \dots \qquad (2)$$

where U_0 and c are a representative velocity and length respectively and $\Theta = (\theta/c)\sqrt{R}$, R being the Reynolds number U_0c/ν . In any problem, v_0 and U are known functions of x/c: thus if $V_0 = -(v_0/U)\sqrt{R}$, λ and Λ are both known functions of Θ since they are given respectively by $\lambda = V_0\Theta$ and $\Lambda = -(U'c/U_0)\Theta^2$. Equation (2) can therefore be solved for Θ as a function of x/c. H, $l = (\theta/U)(\partial u/\partial y)_{y=0}$ and $m = (\theta^2/U)(\partial^2 u/\partial y^2)_{y=0}$ then follow, being known functions of λ and Λ (see Ref. 1).

 \varTheta is most conveniently found from equation (2) by using a step-by-step process. Its integral form

$$\Theta^{2} = \left(\frac{U_{0}}{U}\right)^{6} \int_{0}^{\frac{x}{c}} \left(0 \cdot 45 - 1 \cdot 28\lambda + 0 \cdot 76\lambda^{2}\right) \left(\frac{U}{U_{0}}\right)^{5} d\left(\frac{x}{c}\right) \quad \dots \quad \dots \quad (3)$$

is useful as a check on the accuracy of the calculation.

At a stagnation point, the right-hand side of (2) must vanish for a regular solution. Thus

$$0.45 - 1.28\lambda + 0.76\lambda^2 + 6A = 0$$

$$0.45 - 1.28\lambda + \left(0.76 - \frac{6U'c}{U_0} \cdot \frac{1}{V_0^2}\right)\lambda^2 = 0 \quad .. \qquad (4)$$

which is a quadratic equation giving the value of λ at a stagnation point. For normal values of V_0 there is only one positive root.

Separation occurs, according to this theory, when $f(\lambda, \Lambda) = 0$, the function f being given in Ref.1

4. Velocity Distribution outside Boundary Layer.—(a) In laminar boundary-layer theory scale effect may automatically be taken into account by the use of suitable non-dimensional variables. Four such variables are U/U_0 , $\Theta = (\theta/c)\sqrt{R}$, $\Delta^* = (\delta^*/c)\sqrt{R}$ and $V_0 = (v_0/U_0)\sqrt{R}$. The five

suction quantities used in the experiment were chosen to give four values of V_0 common to the two wind speeds of the experiment (25 ft/sec and 60 ft/sec nominal). It was thus hoped to obtain experimental verification of the validity of the use of these particular non-dimensional variables. Such a verification requires the same values of U/U_0 at the various Reynolds numbers. However, in cases where separation occurs, in which the dependence of the wake on Reynolds number is of much greater complexity, the same values of U/U_0 for given V_0 and different R cannot be expected and in fact were not found.

In Fig. 2, U/U_0 is given for the two wind speeds and five suction quantities (including $V_0 = 0$). The values of V_0 shown on the graphs are the values at the front stagnation point: V_0 varies round the surface due to the varying external pressure. (This variation is greatest for the higher wind speed and smallest suction velocity and then amounts to about 30 per cent.) It is clear that V_0 is not the only determining parameter.

It was found that for some V_0 and U_0 the position around the surface of the boundary-layer total-head tube had a considerable effect on the velocity distribution. The readings for different positions of the tube in the two cases when this was most noticeable ($U_0 = 26.25$ ft/sec, $V_0 = 2.05$ and 3.08) are shown in Fig. 3. The scatter of the points is seen to be large in the region of decreasing velocity particularly for the larger suction velocity when the point of separation had been delayed by the suction about 30 deg. A general waviness of the curves is also noticeable; this may be due to the two spanwise positions of the pressure holes.

(b) Smoothed Values of U and U'.—To calculate the momentum thickness, θ , of the boundary layer from equation (2) of section 3 smoothed values of U and U' are needed at closely spaced points around the surface of the cylinder. These values were obtained at intervals of $2\frac{1}{2}$ deg from the experimental velocity distributions by graphical methods. The determination of U was straightforward but the uncertainty in the values of U' was large near the separation point. This was because the spacing of the pressure holes was large (10 deg) and because U' changes rapidly near the separation point. It is possible that these uncertainties in U' do not introduce large errors in the calculated values of θ , but large errors will result in $\Lambda = -(U'/\nu)\theta^2$ and hence in the calculated values of the other boundary-layer characteristics which according to the theory are given by the values of λ and Λ .

(c) Effect of Flap on Velocity Distribution when $V_0 = 0$.—The comparison (Fig. 4) between the velocity distributions without a flap and with a $\frac{1}{2}$ -in. flap fitted at the rear of the cylinder when $V_0 = 0$ is interesting. It shows that the flap reduces the velocity over the rear of the cylinder and that the reduction extends as far forward as the 40 deg station. This reduced velocity (and increased pressure) will result in an appreciable reduction in the form drag of the cylinder.

A similar effect has been observed previously⁵ but under somewhat different conditions. Figs. 5 and 6 (reproduced from Ref. 5) show schlieren and interferometer photographs of the flow about a circular cylinder at a Mach number of 0.3 with and without a long obstruction in the wake. The photographs show that the obstruction has a marked stabilising effect on the wake and the evaluation of the interferometer photographs shows that the form drag of the cylinder is reduced appreciably.

It is intended to investigate this matter further by means of flow visualization experiments in a water tunnel.

5. Description of Experimental Results and Comparison with Theory.—(a) Momentum Thickness of Boundary Layer.—For each of the two wind speeds and five suction quantities the momentum thickness was calculated using equation (2) with steps of $2\frac{1}{2}$ deg. The accuracy of the step-by-step method was checked in one case by the integral equation (3). The results of the calculations are compared with the experimental values in Figs. 7 and 8. The range of values indicated about each experimental point shows the error, described in section 2 (d), arising from the inaccuracy in the determination of the distance from the surface of the total-head tube. It is seen that the agreement is good over the whole range of observation. For the higher Reynolds number especially, any divergence between theory and experiment could be explained by known errors in the experimental technique. At the lower Reynolds number there is a certain scatter in the experimental points: a characteristic of the lower speed was a general unsteadiness of the flow and consequent uncertainty of many of the readings. The only discrepancy that cannot be explained in these latter results is that the momentum thickness measured at 47 deg has too great a value for each suction quantity.

These results give confidence that formula (2) is a good approximation even close to the separation point.

(b) Velocity Distributions in the Boundary Layer.—Comparison can also be made between theoretical and experimental velocity distributions. According to the theory the velocity distribution expressed as a function of y/θ depends only on the values of λ and Λ , and comparison is therefore made between experimental and theoretical distributions having the same values of these parameters. Exact solutions of boundary-layer flow with suction appear in Ref. 1 and in Iglisch's flat-plate constant-suction solution (Ref. 6). To compare with the former, one must choose one of the given solutions whose values of λ and Λ occur together in the above calculations (of θ) based on the experimental velocity distribution. In the Iglisch solution, $\Lambda = 0$ and it is therefore possible to compare only at points of maximum velocity (U' = 0) on the cylinder. Except for the largest value of V_0 at the lower Reynolds number this point is in each case close to 67 deg, thus the experimental distributions at this point are compared with those of Iglisch's solution with the same value of λ .

The comparisons of these two kinds are shown in Figs. 9 and 10. Although the agreement is reasonable there appear to be systematic differences between the theoretical and experimental curves particularly for the higher wind speed. The experimental curves are seen to lie above the theoretical ones over the inner portion of the boundary layer. To determine whether this disagreement is due to the approximate nature of the theory or to errors in the experimental method exact solutions of the boundary-layer equations for the experimental velocity distribution are needed. For the case of zero suction such solutions can be calculated by the method of series provided that the velocity distribution can be expressed as a polynomial in the distance around the surface from the front stagnation point. For the case $U_0 = 63 \cdot 13$ ft/sec, $V_0 = 0$ it was found that the experimental velocity distribution was closely represented by the expression

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 $U/U_{\rm 0} = 0.0284\phi - 1.09 \times 10^{-6}\phi^{\rm 3} - 1.36 \times 10^{-10}\phi^{\rm 5}$

where ϕ is the angular position in degrees measured from the front stagnation point.

Using this expression for U/U_0 the boundary-layer profile at 67 deg was calculated and found to be almost identical with the corresponding approximate theoretical distribution shown as curve 6 in Fig. 10. It is concluded therefore that for $V_0 = 0$ the disagreement in Figs. 9 and 10 is due to experimental errors and not to the approximate nature of the theory. Since the disagreement in the general case when $V_0 \neq 0$ is similar to that when $V_0 = 0$ it seems reasonable to conclude that it too is due mainly to errors in the experimental method.

The nature of the disagreement and the fact that it is most pronounced at the higher wind speed and lowest suction quantities suggest that it may be due to the start of gradual transition.

(c) The Separation Point.—According to the theory separation occurs at the point on the surface at which Λ equals $\Lambda_0(\lambda)$, the function $\Lambda_0(\lambda)$ being defined in Ref. 1. In Fig. 11 $\Lambda/\Lambda_0(\lambda)$ is plotted against ϕ for each of the two wind speeds and five suction quantities of the experiment; the theoretical separation points are therefore given by the points of intersection of the curves and the line $\Lambda/\Lambda_0(\lambda) = 1$.

The sudden decrease in the porosity of the cylinder referred to in section 2 (c) forestalled an accurate experimental determination of the positions of separation. However, from the boundary-layer traverses which were made at intervals of 10 deg around the surface until beyond separation, the separation points are known to within 5 deg and are tabulated in Fig. 11.

A critical comparison between the theoretical and experimental separation points is not possible and that for two reasons. Firstly, as remarked above, the experimental separation points are not accurately known and secondly the errors in $\Lambda = -U'\theta^2/\nu$ are large near separation due to inaccuracies in the determination of U' (see section 4 (b)). All that can be said of the comparison of Fig. 11 is that the theory correctly exhibits the delay of separation by suction and that the separation points given by it are of the right order of magnitude.

(d) Progressive Development of the Boundary Layer.—The progressive development of the boundary-layer velocity distributions round the surface for two suction velocities ($V_0 = 0$ and $3 \cdot 08$) at the lower wind speed is shown in Fig. 12. For no suction, the rapid growth of the boundary layer is clearly seen between 47 deg and 77 deg, and separation is about to occur at the latter point. For $V_0 = 3 \cdot 08$ a stable velocity distribution is obtained at least until 97 deg (the surface pressure distribution is, of course, now more favourable—see Fig. 2), and at 107 deg separation is imminent. The effect of the suction in reducing the thickness of the boundary layer in comparison with $V_0 = 0$ is also clearly shown.

(e) Other Comparisons with Theory.—The theory enables the displacement thickness δ^* to be calculated from values of θ by giving $H = \delta^*/\theta$ as a function of λ and Λ . It is mentioned, however, in Ref. 1 that the values of $H(\lambda, \Lambda)$ would be greatly improved by a knowledge of more exact solutions of boundary-layer flow and that the values derived from the 'similar solutions' are probably not good. No comparison therefore is made between the experimental and theoretical values of δ^* in this report. For a similar reason, calculated values of $l = (\theta/U)(\partial u/\partial y)_{y=0}$ and $m = (\theta^2/U)(\partial^2 u/\partial y^2)_{y=0}$ are not compared with experimental values, nor are the velocity profiles calculated (as suggested in Ref. 1), by using values of l, m and H.

6. Suggestions for the Design of Future Experiments.—The work described in this report has emphasized the importance of the following features of the design of experiments investigating laminar boundary layers, particularly with continuous suction.

(a) Provision should be made for the measurement of pressure at more closely spaced points round the surface, so that the pressure gradient or U' can be more accurately determined.

(b) In order to obtain thick boundary layers at a given Reynolds number, the chord should be as large as possible and the wind speed as low as is consistent with accurate measurement of pressure.

(c) In conditions of continuous suction the flow appears to be especially sensitive to disturbances introduced by the measuring apparatus, even when these are very small.

(d) The addition of another variable quantity, the suction velocity, has greatly increased the number of observations to be taken and the subsequent computation. Serious attention should be paid to the possibility of automatic recording and computation by means of standardised equipment which can be applied to all aerofoil experiments.

7. Conclusions.—(a) The experimental values of Θ compare well with those calculated by the formula

$$\frac{U}{U_0}\frac{d\Theta^2}{dx} = 0.45 - 1.28\lambda + 0.76\lambda^2 + 6\Lambda$$

where $\lambda = -V_0 \Theta$ and $\Lambda = -(U'c/U_0)\Theta^2$.

This gives confidence that this formula for calculating the momentum thickness of a laminar boundary layer under conditions of continuous suction is reasonably accurate until close to the separation point. (b) Owing to practical difficulties in the exploration of the very thin boundary layers and in the determination of the velocity gradient round the surface comparisons with the theory of the progressive development of the boundary-layer velocity distribution and of the prediction of the separation point were difficult. Nevertheless reasonable agreement between the theoretical and experimental velocity distributions were obtained particularly for the lower wind speed of the experiment, but no adequate test of the prediction of the separation point was found possible.

(c) A flap fitted at the rear of a solid cylinder increases the pressure in the wake and so decreases the form drag of the cylinder. Other observations (Ref. 5) show that such a flap may also have the effect of stabilising the wake.

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FIG. 1. Photograph of Apparatus.



FIG. 2. Velocity distributions for different wind speeds and suction quantities.

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FIG. 3. The velocity distribution for different positions of the total-head tube. $U_0 = 26.25$ ft/sec.





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FIG. 5. Schlieren and interferometer photographs of the flow about a cylinder at M = 0.3. (Ref. 5.)

FIG. 6. Schlieren and interferometer photographs of the flow about a cylinder with a long obstruction in the wake at M = 0.3. (Ref. 5.)



FIG. 7. The momentum thickness of the boundary layer for different suction velocities. $U_0 = 63 \cdot 13$ ft/sec.



FIG. 8. The momentum thickness of the boundary layer for different suction velocities. $U_0 = 26 \cdot 25$ ft/sec.



FIG. 9. Boundary-layer velocity distributions.



FIG. 10. Boundary-layer velocity distributions.

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FIG. 12. Development of the boundary-layer velocity distributions with and without suction.

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