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A Theoretical Calculation of the Reduction in Drag Obtainable by Ejector Action of the Exhaust Gases when Mixed with the Cooling Air-flow of a Typical Air-cooled Engine

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Summary.—The formulae which give the thrust theoretically obtainable in a system where the ejector exhaust is mixed with the engine cooling air are of considerable mathematical complexity, and it is not therefore evident under what conditions the greatest benefit can be obtained from such a system. By considering a special case, it is shown that there is an optimum size of the mixing duct in relation to the exhaust-pipe diameter; and also that the effect of mixing the exhaust gas and cooling air streams is only beneficial when the available total pressure head behind the engine is small.

1. *Introduction.*—It is well known that the reaction thrust obtainable from a jet of gas can be considerably increased by the provision of a mixing tube into which a secondary air stream is induced. With the relatively small amount of momentum available in the exhaust gas from a reciprocating engine, the possible gain by fitting secondary tubes to the exhaust pipes would be inconsiderable, and would not pay for the increased aerodynamic drag.

If the ducted cooling-air of an air-cooled engine is used as the secondary flow, no great additional aerodynamic drag or extra weight need be incurred. A diagrammatic sketch of this arrangement is shown in Fig. 1, and it is with such a scheme in mind that these calculations have been made.

The pulsating character of the exhaust introduces a certain degree of complication. The available mass flow, momentum and energy of the exhaust gasses must be considered separately since no mean value of the exit velocity can be defined which will give both the correct mass flow and the correct momentum. The data from which the values of these quantities are taken is discussed more fully in the next section of this note.

It is assumed that the flow of cooling air is sufficiently steady for mean values of the various pressures to be taken for the purposes of calculation. This could be obtained by grouping the individual pipes in a suitable manner to ensure that the pulsations form a more or less continuous blast, and that the mixing-tube and ducting are fairly long so as to increase the inertia of the flow.

The effect of skin friction has not been considered. There will certainly be increased losses due to the high air velocity through the mixing tube when this latter is used; and in addition, in order to ensure adequate mixing, the ducting may have to be considerably longer than the normal cooling-air annular passage after the engine. The calculated thrusts must therefore be considered as optimistic.

* R.A.E. Technical Note No. Eng. 344 received 26th May, 1945.

The aim of the present note is to evaluate the increased thrust (or decreased drag) obtained by mixing the exhaust gas and cooling air streams under certain given conditions. The results obtained are not sufficiently comprehensive to be used in any way as "design-charts", but the trend of the curves would not be altered by taking different conditions, and important general conclusions may be deduced from them.

It is also believed that the method as used in this note may be of assistance in enabling calculations to be made in any other special case, or for compiling a set of graphs applicable to a range of engine powers, altitudes, cooling air flows, etc.

2. *Exhaust-gas Outlet Conditions.*—2.1. The case under consideration assumes individual exhaust pipes of some length from each cylinder. To get the greatest benefit from the pulsating nature of the exhaust, the capacity of the pipe should be as small as possible. The length is usually determined by installational considerations, and the internal friction drag will be roughly proportional to the inverse fourth power of the diameter. However, the minimum diameter is not so likely to be determined by the optimum thrust condition as by the "negative work" done on the piston during the exhaust stroke, and by the cylinder pressure as the exhaust valve closes (which affects the engine air consumption). An average value for a typical ejector exhaust pipe area is one square inch for every 30 I.H.P. at maximum power.

The dominating factors governing the available thrust from any given system are (i) the engine power and (ii) the back-pressure into which the exhaust-pipe discharges. Tests made on a Sabre single-cylinder unit[1] with typical individual ejector exhaust pipes were examined in order to obtain suitable values for the mass-flow and thrust from a given area.

The condition required was a typical climbing power case (say 75 per cent. of the maximum). The results for 3,700 r.p.m. + 5 lb./sq. in. boost, 12 to 1 mixture strength are given as under

Pipe outlet area	3·45 sq. in.
Total mass flow	650 lb./hr.
Thrust at S/L	8·0 lb.
Thrust at 10,000 ft.	10·2 lb.
Thrust at 30,000 ft.	13·8 lb.

Therefore mass flow/unit area = 0·0523 lb./sq. in/sec.

The thrusts per unit air consumption are seen to fit closely to the empirical formula

$$\theta = 90 - 3P_a \text{ lb./lb./sec.},$$

where P_a is the atmospheric pressure in lb./sq. in. abs.

Alt. (ft.)	Sabre single cylinder tests	Formula $90-3P_a$
S/L	44·3	45·9
10,000	56·5	59·7
30,000	76·5	76·9

The mass-flow and total temperature of the gases can be considered constant for a given power. In round figures, the following values may be taken for the climbing power case under consideration :

$$M_3/A_3 = 0\cdot050 \text{ lb./sq. in./sec.}$$

$$T_{3t} = 900 \text{ °C. abs.}$$

3. *Solution of Equations.*—In order to obtain satisfactory cooling at a given power, the cooling air mass flow for a given engine should be constant, and will usually vary between five and ten times the engine air consumption. As a typical case, a cooling air flow of eight times the mass of exhaust products is taken, *i.e.* $M_2/M_3 = 8$ with the notation used.

The effect will be considered of varying the mixing-tube area in relation to the exhaust-pipe area, the final exit area always being adjusted to maintain the constant cooling air flow required. The exhaust conditions have been examined and suitable values for the constant found for climbing powers in a previous section. A constant cooling-air temperature is assumed; variations in this will make little difference in the result.

The equations involved are set out in the Appendix, and are reduced to five in number for the purposes of solution.

To summarize, with the notation given at the end of this note the following conditions are taken as constant :—

$$M_2/M_3 = 8 \text{ to } 1$$

$$T_{3t} = 900 \text{ deg. C. abs.}$$

$$T_{1t} = 300 \text{ deg. C. abs.}$$

$$\theta_3 = (90 - 3P_2) \text{ lb./lb./sec.}$$

$$M_3/A_3 = 0\cdot050 \text{ lb./sq. in./sec.}$$

$$P_5 = 10\cdot11 \text{ lb./sq. in. abs.} \; (\equiv 10{,}000 \text{ ft. I.C.A.N.})$$

$$\gamma = 1\cdot400$$

The final thrust θ_5 is required for a range of values of P_{1t}, from a value equal to the static pressure P_5 to about 15-in. water above this.

Referring to the equations of the Appendix, the method of solution used in the present instance is outlined below :—

(i) Take suitable values of P_{1t} and P_2, and substitute in equation (1) to find A_3/A_4.

(ii) Substitute in equation (3) to find $M_4v_4/M_3k\sqrt{T_1}$ in terms of P_4.

(iii) Combine with equation (2) to solve for P_4, as a quadratic.

(iv) Substitute in equation (4) to obtain A_4/A_5.

(v) Finally find θ_5 from equation (5).

Hence simultaneous values of θ_5 and A_3/A_4 are found for the chosen value of P_{1t}, regarding P_2 as a dependent variable.

4. *Presentation of Results.*—Only one specimen set of curves has been calculated, for a cooling-air mass flow of 8 times the mass of engine exhaust products, at an altitude of 10,000 ft. I.C.A.N. This leaves as the only important independent variable the ratio of exhaust-pipe area to mixing-tube area, given by A_3/A_4.

The curves in Fig. 2 are thus drawn to show the variation in total thrust obtainable from the mixed gases against the value of A_3/A_4, for different values of the available total head of cooling air after the engine above atmospheric $(P_{1t} - P_5)$.

It will be realised that the value of this available pressure-head is determined by the amount of ram obtained by flight speed, as diminished by the pressure-drop as the air passes through the baffle system to cool the engine. The net internal drag of the installation is thus proportional to V/g minus the value indicated in Fig. 2, where V is the speed of flight. When comparing the net drag of the mixed flow system with the equivalent quantity for separate cooling and exhaust systems, the difference will always be as shown in Fig. 2, since the intake momentum drag V/g will be the same in each case.

Fig. 2 shows that, for the conditions assumed, there is an optimum area ratio A_3/A_4 of about $0 \cdot 26$ (or mixing-tube diameter twice the exhaust-pipe diameter). It is further evident that no improvement on the unmixed system can be expected in this case for cooling-air total head exit pressures of greater than about $7\frac{1}{2}$-in. water.

Figs. 3 and 4 give the values of A_4/A_5 and P_2 corresponding to the curves given in Fig. 2. A_4/A_5 is the amount of convergence or divergence required from the mixing section to exit (*i.e.* the setting of the gills). P_2 is the mean back-pressure as experienced by the engine, which will be benefited by the considerable depressions seen to be available.

Finally in Fig. 5 the optimum case of $A_3/A_4 = 0 \cdot 26$ is taken, and the thrusts as given in Fig. 2 are replotted against the available pressure difference, $P_{1t} - P_5$. The thrust is also given for the case of the normal separate exhaust and cooling-air systems. This confirms that $7\frac{1}{2}$ in. water is the greatest total head of cooling air after the engine for which increased thrust can be expected.

5. *Conclusions.*—It is again emphasised that the values obtained from these calculations refer to one special case only, and that to select the most suitable values for all conditions of flight would require sets of curves for different engine powers and cooling-air mass flows, and different altitudes.

However, two results have a general application :—

(i) For any given altitude and engine power there is an optimum size of mixing tube (in relation to exhaust pipe diameter) which will result in maximum thrust from the system.

(ii) The benefit in thrust obtainable from mixing the exhaust gases with the cooling-air stream is considerable when almost all the ram has been lost in cooling the engine as in a tightly baffled engine or when climbing at low air-speed and at a fairly high engine power. There will be a loss in thrust when the cooling air total head behind the cylinders is large.

List of Symbols Used

P = abs. pressure (lb./sq. in.)
T = abs. temperature (deg. C.)
v = velocity (ft./sec.)
M = mass flow (lb./sec.)
A = area of flow (sq. ft.)
θ = thrust (lb./lb./sec. flowing in mixing tube)

Constants

R = gas constant = 96
$k = \sqrt{(2gJKp)}$ = 147·2
γ = ratio of specific heats = 1·40

Subscripts (*see* Fig. 1)

1 conditions for cooling air after the engine
2 conditions for cooling air at entry to mixing tube
3 conditions for exhaust gas at entry to mixing tube
4 conditions for mixed gases at exit to mixing tube
5 conditions for mixed gases at final exit
t total head conditions, otherwise the static is indicated

REFERENCE

No.	*Author*	*Title, etc.*
1	Hudson, Saunders and Broughton ..	Thrust from Ejector Exhaust, Part III. R.A.E. Report No. Eng. 4114. A.R.C. 7842. May, 1944. (Unpublished.)

APPENDIX

Derivation of Equations

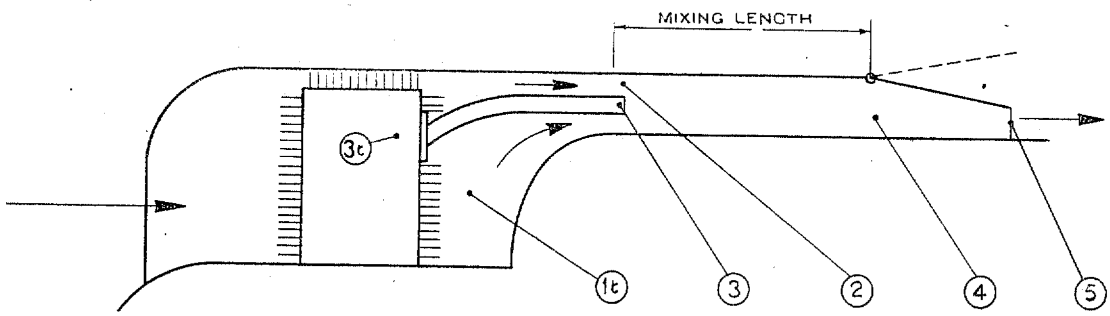


FIG. 1. Diagrammatic Sketch of Mixed Engine Exhaust and Cooling Air Systems.

The notation used in the following is as shown in Fig. 1, and is also summarised in the list of symbols on the previous page.

The mass flow, total energy and momentum per lb. of the exhaust are given by the expressions (M_3/A_3), T_{3t} and θ_3, the values of which have been given in a previous section.

The flow of cooling-air between conditions (1) and (2) is assumed to be adiabatic and frictionless, so that the exponent γ may be used. Adiabatic mixing is then assumed to occur in a parallel tube, and the flow at station (4) is supposed homogeneous. Skin friction drag has not been taken into account, and due to the relatively high velocities in the mixing tube, the thrust values obtained will be optimistic. The final outlet area A_5 is varied in order to regulate the cooling mass-flow to the required value. either by diffusion or throttling. This is also assumed to occur in an ideal manner, with no losses.

With the notation of Fig. 1, considering flow between (1) and (2) for the cooling air up to the mixing tube,

$$T_2 = T_{1t} - (v_2/k)^2 = A_2 v_2 P_2/RM_2 = T_{1t}\,(P_2/P_{1t})^{(\gamma-1)/\gamma}\,.$$

These equations bring in respectively the conditions of conservation of energy, continuity, and isentropic flow.

In similar manner the flow of the mixed gases between (4) and (5) gives

$$T_4 = T_5 - (v_4/k)^2 + (v_5/k)^2 = A_4 v_4 P_4/RM_4 = T_5\,(P_4/P_5)^{(\gamma-1)/\gamma}$$

and also $\quad T_5 = A_5 v_5 P_5/RM_5\,.$

The following relations must also hold :—

$$M_2 + M_3 = M_4 = M_5$$
$$P_2 = P_3$$
$$A_2 + A_3 = A_4\,.$$

Consideration of the momentum and total energy of the two streams before and after mixing gives the two further equations

$$(P_2A_2 + P_3A_3 - P_4A_4)g = M_4v_4 - M_2v_2 - M_3g\theta_3$$

and

$$M_4\{T_4 + (v_4/k)^2\} = M_2T_{1t} + M_3T_{3t}\,.$$

It now remains to eliminate the unwanted variables, and to present the resulting equations in a convenient form for solution.

From the first set of equations for T_2, we may deduce

$$M_2 = \frac{k}{R}\,\frac{A_2P_2}{\sqrt{T_{1t}}}\,\frac{\sqrt{\{1-(P_2/P_{1t})^{(\gamma-1)/\gamma}\}}}{(P_2/P_{1t})^{(\gamma-1)/\gamma}}$$

and

$$M_2/M_3 = \left(\frac{1-A_3/A_4}{A_3/A_4}\right)\frac{kP_2}{R\sqrt{T_{1t}}\,.\,(M_3/A_3)}\,\frac{\sqrt{\{1-(P_2/P_{1t})^{(\gamma-1)/\gamma}\}}}{(P_2/P_{1t})^{(\gamma-1)/\gamma}} \qquad \ldots \quad \ldots \quad (1)$$

The energy-mixing equation is next considered :—

$$M_4\{M_2T_{1t} + M_3T_{3t}\} = M_4\,[M_4T_4 + M_4\,(v_4/k)^2]\,.$$

Substituting for M_4 and T_4, we get

$$(M_2 + M_3)\,(M_2T_{1t} + M_3T_{3t}) = \frac{A_4P_4}{R}\,(M_4v_4) + \left(\frac{M_4v_4}{k}\right)^2.$$

Therefore $$\left(\frac{M_2}{M_3} + 1\right)\left(\frac{M_2}{M_3} + \frac{T_{3t}}{T_{1t}}\right) = \left(\frac{M_4v_4}{M_3k\sqrt{T_{1t}}}\right)\left[\frac{k}{R\sqrt{T_{1t}}\,.\,(M_3/A_3)\,(A_3/A_4)}\,P_4 + \frac{M_4v_4}{M_3k\sqrt{T_{1t}}}\right] \quad (2)$$

The momentum-mixing equation gives

$$(P_2 - P_4)A_4g = M_4v_4 - M_2v_2 - M_3g\theta_3\,.$$

We have $V_2/k\sqrt{T_{1t}} = \sqrt{\{1 - (P_2/P_{1t})^{(\gamma-1)/\gamma}\}}$.

Therefore $$\frac{M_4v_4}{M_3k\sqrt{T_{1t}}} = \frac{(P_2 - P_4)A_4g}{M_3k\sqrt{T_{1t}}} + \frac{M_2}{M_3}\sqrt{\left\{1 - (P_2/P_{1t})^{(\gamma-1)/\gamma}\right\}} + \frac{g\theta_3}{k\sqrt{T_{1t}}}\,.$$

Therefore $$\frac{M_4v_4}{M_3k\sqrt{T_{1t}}} = \left[\frac{g}{k\sqrt{T_{1t}}\,.\,(M_3/A_3)\,(A_3/A_4)}(P_2 - P_4) + \frac{M_2}{M_3}\sqrt{\left\{1 - (P_2/P_{1t})^{(\gamma-1)/\gamma}\right\}} + \frac{g\theta_3}{k\sqrt{T_{1t}}}\right] \quad (3)$$

We also have

$$\frac{A_5v_5P_5}{M_5R} - (v_4/k)^2 + (v_5/k)^2 = \frac{A_4v_4P_4}{M_4R} = \frac{A_5v_5P_5}{M_5R}\left(\frac{P_4}{P_5}\right)^{(\gamma-1)/\gamma}.$$

Therefore $$A_5P_5\,(M_5v_5) - \frac{R}{k^2}\,(M_4v_4)^2 + \frac{R}{k^2}\,(M_5v_5)^2 = A_4P_4\,(M_4v_4)$$

$$= A_5P_5\,(M_5v_5)\left(\frac{P_4}{P_5}\right)^{(\gamma-1)/\gamma}.$$

Eliminating M_5v_5, we get

$$A_4P_4\,(M_4v_4)\left(\frac{P_5}{P_4}\right)^{(\gamma-1)/\gamma} - \frac{R}{k^2}\,(M_4v_4)^2 + \frac{R}{k^2}\,(M_4v_4)^2\left(\frac{A_4}{A_5}\,\frac{P_4}{P_5}\right)^2\left(\frac{P_5}{P_4}\right)^{2\,.\,(\gamma-1)/\gamma}$$
$$= A_4P_4M_4v_4\,.$$

Therefore $$A_4P_4\left\{\left(\frac{P_5}{P_4}\right)^{(\gamma-1)/\gamma} - 1\right\} = M_4v_4\,\frac{R}{k^2}\left\{1 - \left(\frac{A_4}{A_5}\right)^2\left(\frac{P_4}{P_5}\right)^{2/\gamma}\right\}.$$

Therefore $$\frac{kP_4}{R\sqrt{T_{1t}}\,(M_3/A_3)\,(A_3/A_4)}\left\{\left(\frac{P_5}{P_4}\right)^{(\gamma-1)/\gamma} - 1\right\} = \frac{M_4v_4}{M_3k\sqrt{T_{1t}}}\left\{1 - \left(\frac{A_4}{A_5}\right)^2\left(\frac{P_4}{P_5}\right)^{2/\gamma}\right\}. \quad (4)$$

The final thrust is given by

$$\theta_5 = \frac{v_5}{g} = \frac{v_4}{g}\cdot\frac{A_4}{A_5}\cdot(P_4/P_5)^{1/\gamma}\,.$$

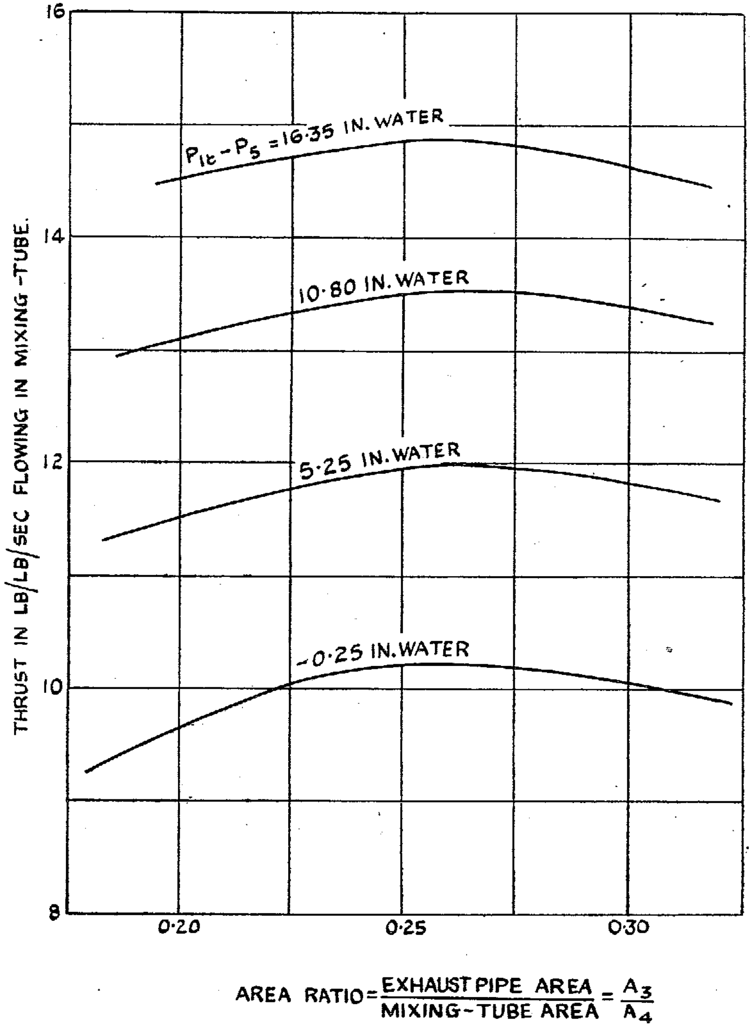
Therefore $$\theta_5 = \frac{k\sqrt{T_{1t}}}{g}\left(\frac{M_4v_4}{M_3k\sqrt{T_{1t}}}\right)\frac{1}{(M_2/M_3 + 1)}\,(A_4/A_5)\,(P_4/P_5)^{1/\gamma}\ldots \quad (5)$$

In obtaining the thrust available from the separate exhaust and cooling air systems, it is assumed that the exit pressure is P_5 (*i.e.* static atmosphere pressure) in each case.

For the exhaust, the thrust per lb./sec., by the given formula is $90 - 3P_5$.

For the cooling air, it would be $\frac{k\sqrt{T_{1t}}}{g}\sqrt{\left\{1 - (P_5/P_{1t})^{(\gamma-1)/\gamma}\right\}}$ on the same assumptions as in those used above. For the combined flows, the thrust per lb. of total flow, comparable with the figure for the mixed system, is given by

$$\theta = \left(\frac{1}{1 + M_2/M_3}\right)(90 - 3P_5) + \left(\frac{M_2/M_3}{1 + M_2/M_3}\right)\frac{k\sqrt{T_{1t}}}{g}\sqrt{\left\{1 - (P_5/P_{1t})^{(\gamma-1)/\gamma}\right\}}\,. \quad (6)$$



$(P_{1t} - P_5)$ = Total head of cooling air after engine above atmosphere. 10,000 ft. altitude.

Cooling air mass flow = 8 × engine air cons.

FIG. 2. Variation of Thrust with Mixing-Tube Area.

$(P_{1t} - P_5)$ = total head of cooling air after engine, above atmosphere.

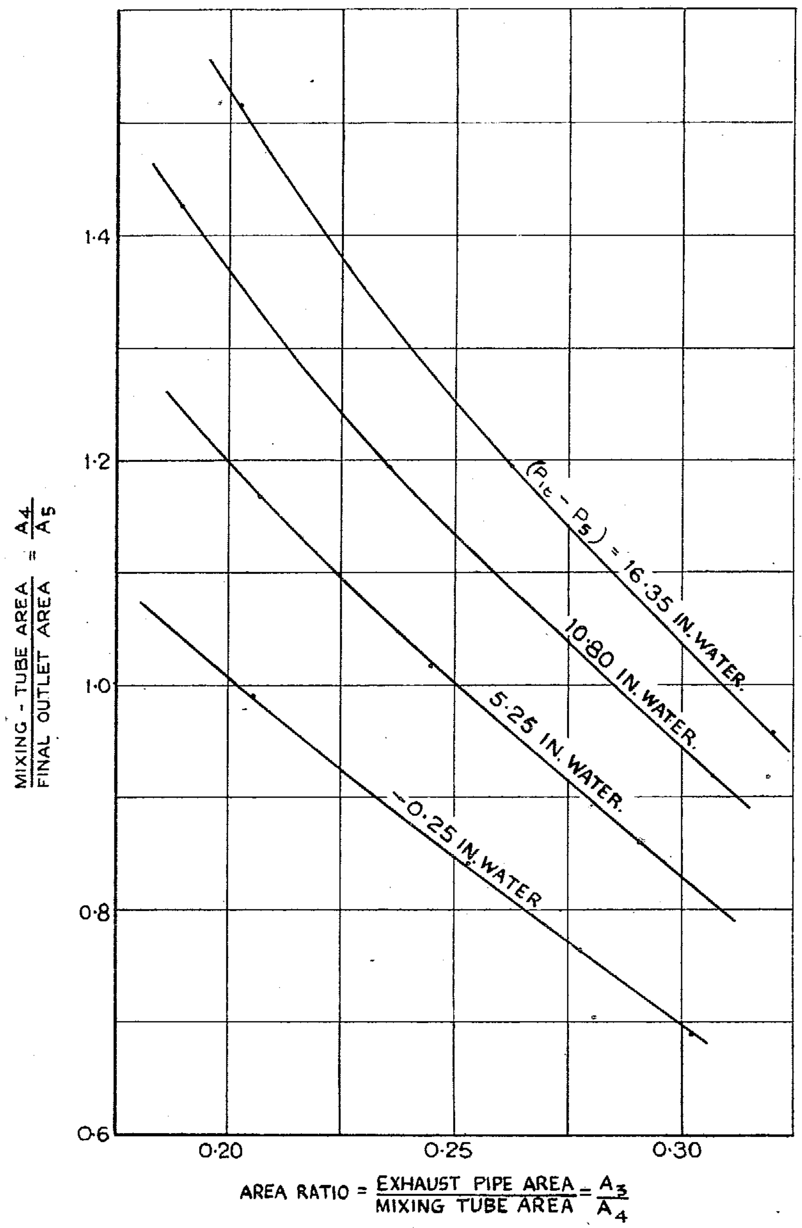


FIG. 3. Final Exit Areas Required to Give Conditions of Fig. 2.

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P_{1t} = total head of cooling air after engine.

P_2 = static pressure at exhaust exits.

P_5 = atmospheric pressure.

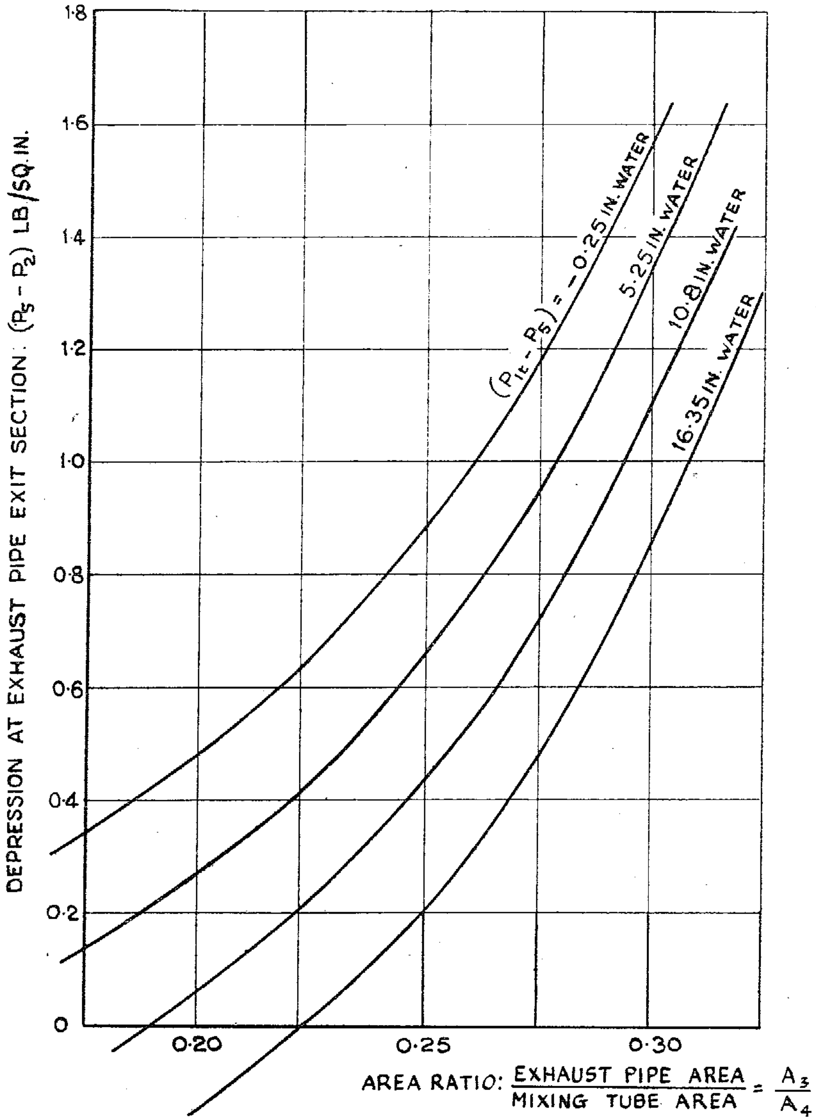
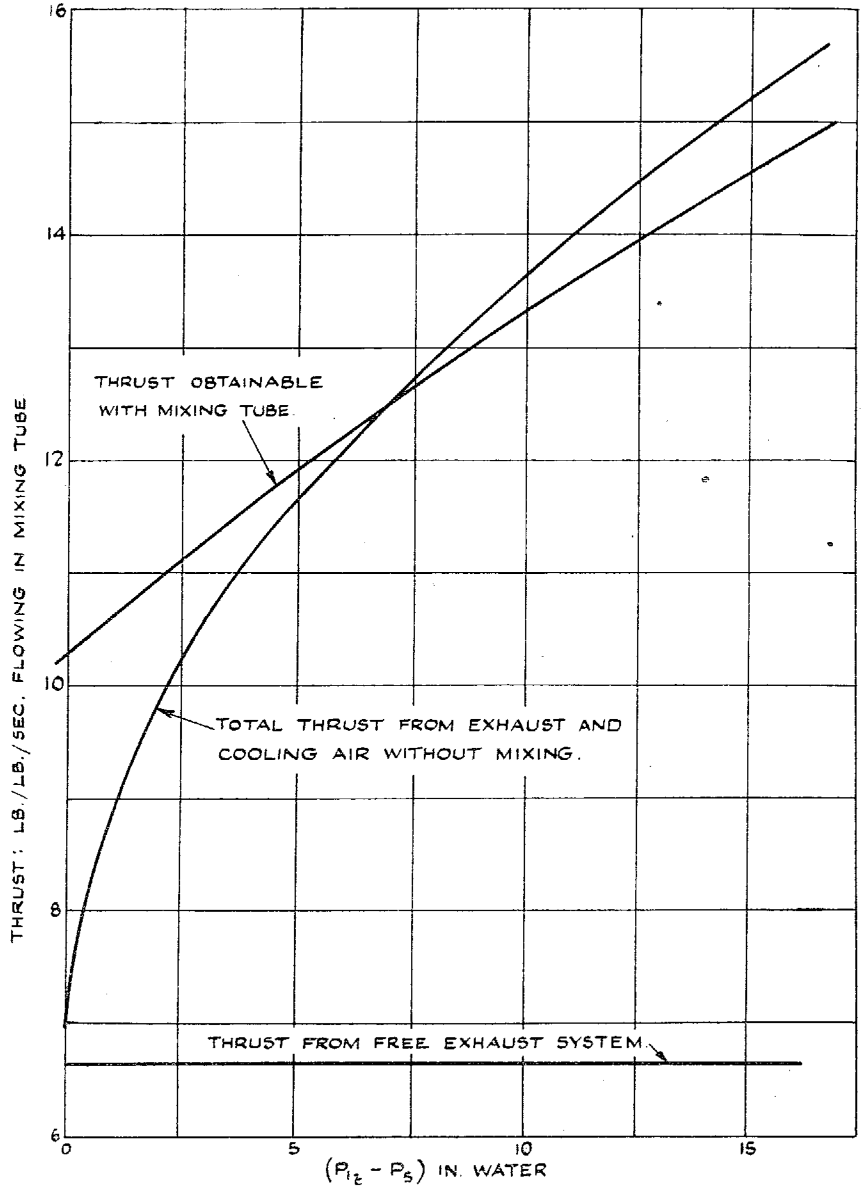


FIG. 4. Depression at Exhaust Outlet for Points Plotted in Fig. 2.



$(P_{1t} - P_5)$ = Total head after engine above atmosphere.

$$\frac{\text{Exhaust pipe area}}{\text{Mixing-tube area}} = 0 \cdot 26$$

10,000 ft. altitude

8 to 1 cooling air

FIG. 5. Variation of Thrust with Total Pressure Head Available.

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