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A Note on Turbulent Boundary-Layer Allowances in Supersonic Nozzle Design

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SUMMARY

Some of the theoretical methods for computing the growth of turbulent boundary layers along both the curved and straight walls of a rectangular supersonic wind tunnel are discussed and the estimated values compared with experiment. Approximate formulae, which may be useful in the initial stages of nozzle design, are suggested for the overall boundary-layer growth along the curved nozzle and flat wall.

1. Introduction

The widespread use of supersonic wind tunnels has led, in the last decade, to a large number of papers dealing with the theoretical design of supersonic nozzles. Such methods, however, can only be used to calculate the nozzle profile in inviscid flow (i.e., the so-called 'potential outline') and for real fluids it is necessary to allow for the growth of the boundary layer along the walls of the tunnel. Usually this is done by displacing the potential outline slightly away from the tunnel centreline, the correction being calculated from the displacement thicknesses of the wall boundary layers. In most cases the boundary layer will be turbulent from some station well upstream of the throat, but in small tunnels (or in tunnels working at low stagnation pressure) some laminar flow may persist into the nozzle proper.

The problem of determining the appropriate boundary-layer growth along the curved nozzle and straight side walls of a rectangular wind tunnel has received less attention than the companion problem of nozzle design for inviscid flow. Admittedly, there are many general papers dealing with boundary layers which could be employed for making the necessary calculations, but this approach is often undesirable; in most cases, rapid approximate methods of estimating the boundary-layer allowances are required.

Several such methods have been proposed, some being satisfactory and others less so; the former do not seem to be as well known perhaps as they deserve and the present brief note is intended to draw attention to them and to compare their predictions with experiment, where this is possible. It should be pointed out perhaps that none of these methods can allow for secondary flow effects.

During the preliminary stages of designing (or adapting) a nozzle profile, an even more approximate estimate of the boundary-layer growth is often needed. The experimental information at present available suggests that suitable approximate formulae for boundary-layer growth can be found which are applicable for nozzle designs of moderate supersonic Mach number.

The present discussion is restricted to tunnels of rectangular cross-section, but some of the theoretical methods (particularly Ref. 6) are applicable to axi-symmetric flow. Heat-transfer effects are not considered; the tunnel walls are assumed to be insulated.

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2. Boundary-Layer Growth Along the Curved Contour

2.1 Theoretical Methods

The need to modify the calculated profile of a supersonic nozzle to allow for the presence of the boundary layer on the nozzle surface was recognized at an early stage[1] in the study of supersonic flows. Early methods of calculating the displacement thickness (δ^*) of a boundary layer in a flow having a considerable longitudinal pressure gradient often involved extensive computation and the need was realised for more approximate and speedy methods, whose use was justified to some extent by the existence of secondary flows within the tunnel which distort the ideal two-dimensional flow.

A rapid and approximate method was given by Puckett[4] in 1946; he assumed, amongst other things, that the boundary-layer velocity profile and the Mach number variation along the nozzle were both linear. This representation is generally agreed to be too crude and a more realistic approach is due to Tucker and appears in two papers[5,6]. The analysis is restricted to turbulent boundary layers and the usual momentum equation is solved in conjunction with Falkner's surface-friction equation[7].

$$\frac{\tau}{\rho U_1^2} = 0.0131 \left(\frac{\mu}{\rho U_1 x} \right)^{1/7} \qquad \ldots (1)$$

The variation of the local surface friction with Mach number can be obtained by choosing the values of μ and ρ in equation (1) at some particular region in the flow. For example, in Ref. 5, Tucker considers the choice of either stream or wall conditions; in the later paper he uses the fluid properties at a temperature which is the arithmetic mean of the wall and stream temperatures. The variation of surface friction with Mach number in this case closely follows the trend of the complicated extension of the Frankl-Voishel[16] analysis and is reasonably well supported by experimental evidence for Mach numbers up to about 3.5 (Fig. 1); it represents a considerable improvement on the use of wall conditions.

The use of equation (1), which is based on flat-plate data, implies of course that the effect of pressure gradient upon skin friction is of secondary importance, and there is some evidence[17] that this is a reasonable assumption for favourable pressure gradients, and possibly even for slight adverse gradients.

Though the use of equation (1) may be criticised in the light of more recent investigations[8,24] it may well be sufficiently accurate for the purpose of calculating nozzle boundary layers. The development of secondary flows within the nozzle and the usual device of compensating for sidewall boundary-layer growth by an additional displacement of the nozzle profile may sometimes nullify the benefits obtained from more accurate calculations. In addition, by using the extensive tabulations of Ref. 6, the boundary-layer growth can be computed very rapidly, and this is often of considerable advantage.

An analysis similar to Tucker's was given by Wilson[9] at about the same time, and uses a different form of skin-friction equation[10] (based on Kármán's mixing-length theory). Sample calculations suggest that for final Mach numbers below about 2.5, there is no great difference between the predictions of this method and Tucker's later theory[6]. Since Ref. 9 lacks suitable tables, it is more laborious to use.

Ruptash/

Ruptash[13,14] has also given a method of calculating the boundary-layer growth in turbulent flow and uses the following surface-friction equation

$$\frac{\tau}{\rho_w U_1^2} = 0.0225 \left(\frac{\mu_w}{\rho_w U \delta} \right)^{1/4} \qquad \ldots (2)$$

where the suffix w relates to wall conditions and δ is the thickness of the boundary layer. The computation is more complicated than that required for Tucker's or Wilson's method and requires two step-by-step integrations.

Tetervin's[2] approximate methods were used by Harrop, the computed boundary-layer growth for $M_d = 2$ being given in Fig. 8 of Ref. 3. If Tucker's second method[6] is used for this case, the boundary layer thickness is in agreement with Harrop's calculation at the throat and for a little way downstream. Further along the curved contour however, Tucker's method predicts a growth some 0.002 in./inch greater. Harrop in fact remarks that the average rate of boundary-layer growth found experimentally was greater than that predicted theoretically by a similar amount.

Mention must also be made of the analysis of Armstrong and Smith[28], and the suggestion of Meyer[29] that since large discrepancies can result from the usual methods of computing boundary-layer growth, use should be made of the Illingworth-Stewartson transformation. So far little use has been made of either of these methods. If a more rigorous analysis is sought, a modification of Ref. 30 would seem to be of considerable value.

Some methods of nozzle design (e.g., Ref. 3) lead to a sudden change of pressure gradient when the design Mach number is achieved, dM/dx being discontinuous at this point. In practice, the local changes in Mach number may well be rapid in this region and the theoretical methods of computing the boundary-layer growth must be made with care.

Whilst the boundary-layer growth should strictly be calculated along the curved surfaces of the nozzle, it is sufficient for moderate design Mach numbers and nozzles of conventional length to work on terms of the longitudinal distance x, since ds/dx is approximately unity. This value of s from throat to run out is only about 2% greater than the corresponding value of x (i.e., ℓ) for design Mach numbers below about 2.5. Even for $M_d = 5$, the ratio s/ℓ is usually about 1.05.

2.2 Comparison with Experiment

Whilst several investigations have been made of the growth of turbulent boundary layers along flat plates, the experimental data for wind-tunnel nozzles are comparatively scanty.

One of the earliest detailed investigations was made by Brinich[11] in 1950 using a nozzle designed for a final Mach number of 2.08. He showed that except in regions of strong longitudinal pressure gradient the boundary-layer profile could be represented with sufficient accuracy by a power-law distribution of velocity of the form

$$\frac{y}{\delta} = \left(\frac{U}{U_1} \right)^{\frac{1}{N}}, \qquad \ldots (3)$$

and/

and that the analysis given by the earlier method of Tucker[5] predicted the boundary-layer growth without serious error. The application of Tucker's later theory[6] to Brinich's results causes some improvement, as is shown in Fig. 2.

Fig. 3 compares the theoretical estimates of Ref. 6 with experiment for a wide range of design Mach numbers; these results are given by Baron in Ref. 12. The boundary-layer growth is underestimated somewhat in every case, particularly at Mach numbers above 2.5, though this discrepancy is reduced when the displacement thickness δ^* is computed. On the other hand, the use of Tucker's second method for computing boundary-layer growth for the nozzle of design Mach number 3 used by Ruptash[13] is in good agreement with experiment (Fig. 4). This figure also shows that the difference in the theoretical estimates of Refs. 6 and 13 is not large for this particular nozzle.

Wilson has compared his own boundary-layer thickness estimates with experiment for nozzles having design Mach numbers of 2.0 and 5.0, and obtains satisfactory agreement in both cases. Tucker's method has also been applied to these profiles and agrees well at the lower Mach number; at the high Mach number, the growth is slightly underestimated.

It may be worth noting that in the cases where Tucker's method underestimated the boundary-layer growth at high design Mach numbers the nozzles were comparatively short. For example, the nozzle used by Baron[12] with $M_d = 3.0$ had an ℓ/h ratio of 4.02 (almost the minimum-length value[27]). Good agreement between Tucker's theory and experiment was obtained at $M_d = 3.0$ in Ref. 13 with a liner in which ℓ/h was 6.5.

This suggests the possibility that Tucker's method over-estimates the effect of the longitudinal pressure gradient in reducing the boundary-layer growth, resulting in a boundary-layer growth which is too small when the pressure gradient is more marked. More information is required, however, before this can be proved.

All the experimental results quoted in this section were obtained in tunnels with a working-section width (b) greater than one-third of the full tunnel height $(2h)$; for most results the ratio $2h/b$ was much nearer unity. In tunnels having a narrow width compared with the height, the effects of the secondary flows and the sidewall boundary-layer growth become of more importance and may modify the boundary-layer growth on the curved walls.

2.3 Mean Rate of Growth

In many cases the measured boundary-layer growth is approximately linear with distance along the nozzle surface and this linearity is often obtained in the theoretical estimates too (see Figs. 2, 4 and 8). The mean rate of growth $(\Delta\delta/\ell)$ from the nozzle throat to the run-out position may well be useful in the initial stages of nozzle design, when no great accuracy is required. The available experimental evidence known to the authors has been plotted in Fig. 5 in terms of the wall Reynolds number at the run-out position (i.e., based on design M and ℓ). Wilson's estimates[9] for three Foelsh-type[18] nozzles and the results of applying Tucker's later theory to the liner used in Ref. 11 for a range of Reynolds number have also been added.

The experimental points for design Mach numbers below 2.5 are grouped together and seem to be represented approximately by the curve

$$\frac{\Delta\delta}{\ell} = \frac{0.29}{R_R^{1/5}} \qquad \ldots (4)$$

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This is of similar form to the approximate equation for the growth of a turbulent boundary layer along a flat plate in incompressible flow[19],

$$\frac{\delta}{x} = \frac{0.37}{R_x^{1/5}} , \qquad \dots (5)$$

which assumes a friction relation similar to equation (2) and a boundary-layer velocity profile of the form

$$\frac{U}{U_1} = \left(\frac{y}{\delta}\right)^{1/7}$$

For a tunnel with a working section about 2 feet square, the value of $\Delta\delta/\ell$ for atmospheric stagnation pressure at a Mach number of 2 is about 0.010. This represents a mean rate of growth of the displacement thickness of about 0.002 in./inch, a value often quoted as being suitable for the boundary layer allowance for one wall of moderately large supersonic wind tunnels.

For design Mach numbers above 2.5 there are insufficient experimental data from which to deduce approximate formulae for the overall growth. Nozzles designed for the higher supersonic Mach numbers tend to vary more widely in length for a given M_d, according to the method of design employed, and this may become of significance when considering the overall growth of the boundary layer. It may thus be more satisfactory to perform a complete calculation to estimate the boundary layer rather than rely on an equation similar to (4) above, bearing in mind that as mentioned above, the method of Ref. 6 may somewhat underestimate boundary-layer growth for short nozzles.

2.4 Boundary-Layer Growth Upstream of the Throat

In nozzle design the assumption is often made that the boundary-layer thickness is zero at the throat due to the large favourable pressure gradient upstream of this position. Whilst it is true that the pressure gradient restricts the growth of the boundary layer, the small amount of experimental evidence available suggests that its thickness is only effectively zero when the design Mach number is above about 3. Fig. 6, for example, shows the thickness of the boundary layer at the throat of the family of nozzles tested by Baron[12].

Though most of the theoretical methods can be employed to calculate the growth of the boundary layer along the contraction surface upstream of the throat, it is often difficult to decide where to begin the calculation and what thickness to assign the boundary layer at this station. The theoretical values shown in Fig. 6 were based on a simple approximate method described in Ref. 12, which depends on a knowledge of the measured boundary-layer thickness at the throat of one of the nozzle family.

In the absence of specific information, however, an estimate must be made of the boundary-layer thickness, preferably at some station well upstream of the throat. The effect of the contraction on the boundary-layer growth can then be computed and the boundary-layer thickness at the throat found. The calculation can subsequently be extended along the curved profile of the nozzle.

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The assumption of zero thickness at the throat leads to a slightly higher rate of growth on the surface downstream because of the smaller Reynolds number associated with the boundary-layer development. In addition, the effective throat width will be over-estimated. These effects combine to cause the effective area ratio of the nozzle to be overestimated; hence the actual Mach number obtained in the working section is a little higher than that designed for. Fortunately the throat boundary layer is likely to be thickest at the lower nozzle design Mach numbers where the throat is large. Thus errors in estimating the value of the boundary-layer thickness at the throat are smaller in proportion to the throat width.

With very small tunnels or those working at low stagnation pressures, laminar boundary layers may exist on the tunnel walls in the throat region. This is discussed briefly in section 6 below.

3. Boundary-Layer Growth Along the Flat Sidewalls

The boundary layer on the flat sidewalls downstream from the throat is subject not only to a pressure gradient along the length of the tunnel, but also to a lateral pressure gradient caused by the Mach number variation across the tunnel in the region where the flow is being accelerated to the design Mach number. Near the throat, due to the curvature of the lines of equal Mach number caused by the radial-like nature of the flow, the highest pressure on the sidewall is at the centre line (for a double-sided nozzle). Further downstream this pressure gradient is reversed, since the design Mach number is achieved nearest the throat on the sidewall centre line. This transverse pressure gradient will usually be more severe than that experienced by the flow near the throat and will cause the sidewall boundary layer to thicken near the centre line (Fig. 7)[9], the effect being more pronounced at higher design Mach numbers where the transverse pressure gradient is stronger. Additional experimental evidence is given in Refs. 11, 12, and 21. In Ref. 21 it is shown that shaped fences placed on the sidewall retard this secondary flow and make the boundary layer thickness more uniform. This is of particular importance at high supersonic Mach numbers where the boundary layer may well occupy an appreciable fraction of the tunnel width at the sidewall centreline.

The theoretical methods which might normally be used for predicting the boundary-layer growth along the sidewalls assume two-dimensional flow and do not, of course, allow for secondary-flow effects; it is thus probable that at the higher design Mach numbers, large discrepancies will occur. At present, the available evidence suggests that for nozzles of conventional length and with design Mach number below about 2.5, the accumulation of boundary-layer fluid at the side-wall centre line is not large and the theoretical estimate of boundary-layer growth (assuming two-dimensional flow under the design longitudinal pressure gradient) is in reasonable accordance with experiment (Fig. 8). It is possible that some improvement might result in applying the three-dimensional boundary-layer theory for displacement effects given in Ref. 22, but as yet no suitable method of doing this has been evolved.

It is perhaps worth pointing out that downstream of the run-out position, in a region nominally free from pressure gradients, the transverse flow of the boundary layer may continue because of the secondary flow effects peculiar to square ducts (see Fig. 105 of Ref. 23).

Fig. 8 also shows the theoretical estimates[6] on boundary-layer displacement thickness for the curved contours of the nozzles. The difference in boundary-layer growth between the curved and flat walls is negligible at $M_d = 1.71$ and even at $M_d = 3.5$, the average rate

of/

of growth from throat to run-out position on the two walls is not greatly different. This evidence supports the usual practice of compensating for the boundary-layer growth on the sidewalls by a proportionate increase in the boundary-layer allowance placed on the curved nozzle contour, the magnitude of the increase depending only on the tunnel width and height. Thus for a tunnel of width w and local height $2h$, having curved nozzles on the lower and upper walls, the effective displacement thickness applied to each nozzle would be

$$\delta^*_{EFF} = \frac{2h + w}{w} \delta^*_{contour}$$

The mean boundary-layer growth along the tunnel walls may also be found from pressure measurements made along the axis of symmetry of the nozzle, the presence of a steady pressure gradient downstream of the apex of the test rhombus usually indicating an incorrect boundary-layer allowance. When the nozzle blocks can be tilted slightly, this gradient can often be removed, the magnitude of the tilt applied being equivalent to the deficiency in the estimated effective boundary-layer displacement thickness slope.

4. Boundary-Layer Growth in the Absence of a Pressure Gradient

Downstream of the run-out position on the curved wall, and the upstream inclined characteristic from that point on the sidewall, the flow is theoretically free from any pressure gradients, apart from those caused by secondary flows. Thus the boundary-layer growth is similar in many respects to that on a flat plate in a uniform stream at the nozzle design Mach number, and if desired, the boundary-layer growth could be calculated with considerable accuracy[24]. However, in most cases the methods discussed in Section 2 are of sufficient accuracy and the absence of the pressure gradient simplifies still further the computation.

For example, if the method of Ref. 6 is used for this type of flow, the following equation can be deduced, using the basic assumptions of the method and assuming $\gamma = 1.40$,

$$\delta = \frac{0.0153x}{\Lambda f R_x^{1/7}}, \qquad \dots (6a)$$

where $\Lambda = (1 + 0.1M^2)^{5/7}$, $f = \theta/\delta$ (and can be obtained as a function of M and power-law index N in the report), x is the distance along the equivalent flat plate from the beginning of the boundary-layer growth and R_x is the Reynolds number based on free-stream velocity and kinematic viscosity, and distance x. The appropriate flat-plate x at the beginning of the uniform-flow region in the tunnel can be computed from a knowledge of the boundary-layer thickness at that position.

Equation (6a) can be simplified by noting that when $N = 7$, the product Λf is nearly 0.1 for Mach numbers between 1 and 4. Thus

$$\delta \simeq \frac{0.153x}{R_x^{1/7}} \qquad \dots (6b)$$

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It is interesting to note that this equation gives values of δ/x which are similar to those obtained from equation (5):-

$R_x \times 10^{-6}$	1	3	5	7	10	15	25
$\dfrac{\delta}{x} \times 10^{3}$ (eqn. (6b))	21.3	18.2	16.9	16.2	15.3	14.3	13.5
$\dfrac{\delta}{x} \times 10^{3}$ (eqn. (5))	23.4	18.8	17.0	15.8	14.8	13.6	12.3

Fig. 9 suggests that there is some experimental support for equation (6b) for moderate values of M_d; alternatively the approximate equation for the average boundary-layer growth along the curved part of the nozzle contour

$$\frac{\Delta\delta}{\ell} = \frac{0.29}{R_x^{1/5}} \qquad \ldots (4)$$

can be modified for use in uniform flow, by writing

$$\frac{\Delta\delta}{\Delta x} \simeq \frac{d\delta}{dx} \simeq \frac{0.29}{R_x^{1/5}} \qquad \ldots (4a)$$

This form may sometimes be convenient in preliminary design work by enabling a single equation to be used for boundary-layer development along the complete nozzle block. Fig. 9 shows that equation (4a) does not give markedly different results from equation (6a) above.

Alternatively use may be made of the fact that for a fixed velocity distribution within the boundary layer

$$C_F = 2\left(\frac{\theta}{x}\right) = 2\left(\frac{\delta}{x}\right)\left(\frac{\theta}{\delta}\right),$$

$$\text{and} \quad C_f = 2\left(\frac{d\theta}{dx}\right) = 2\left(\frac{d\delta}{dx}\right)\left(\frac{\theta}{\delta}\right).$$

Thus curves of C_F and C_f against Reynolds number, for various stream Mach numbers (as in Ref. 32) can be used directly to determine the momentum thickness or its rate of growth; by means of suitable tables or graphs, the value of δ or $d\delta/dx$ may be found. The rate of growth of the boundary layer as computed from Cope's[32] curves of the local surface-friction coefficient (C_f) for $M = 2$ is shown in Fig. 9, this curve being in reasonable agreement with the others.

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5. Calculation of Displacement Thickness

The foregoing discussion has been mainly concerned with the calculation of the boundary-layer thickness; to correct the potential outline of the nozzle profile, the displacement thickness of the boundary layer must be used, where

$$\delta^* = g\delta$$

The value g has been given in numerous reports in either tabular or graphical form; the most comprehensive tables are probably those of Ref. 6. For convenience, values of g are shown in Fig. 10 of the present note.

The choice of a value for N appropriate to the conditions being considered is often difficult. Tucker[6], for example, suggests

$$N = 2.2\, R_x^{1/14}\, (1 + 0.1M^2)^{1/7} \qquad \ldots (7)$$

whilst Ruptash[14] and Baron[12] neglect the factor containing M; the former also uses a numerical coefficient of 2.6. Wilson[9] recommends that N should be put equal to 7 for all Reynolds numbers.

Experimentally the value of N is not easy to determine accurately from the boundary-layer traverse data[11]. Baron[12] found for values of M_d between 1.5 and 3.5 that on the curved nozzle contour, a power-law velocity distribution with $N = 7$ fitted most of the measurements made on the curved sections of nozzles; on the sidewall, the experimental values of N were around 8 away from the tunnel centre-line, and somewhat higher on the centreline, particularly in regions of large pressure gradient.

Brinich[11] discusses this matter at some length and concludes that the power-law velocity profile is approximated to most closely where the pressure gradient is either very small or zero. His results for an $M_d = 2.08$ nozzle suggest that a value of N of about 8 would be most satisfactory along the curved wall and 7 along the flat side walls (Fig. 11). Also shown on this Figure is Tucker's semi-empirical relationship for N. Whether the increase in N down the tunnel is due to the increase in R_x or to less distortion of the boundary-layer velocity profile by the longitudinal pressure gradient is uncertain. Ruptash[13] found a similar increase in the best value of N with x but in Baron's[12] results the effect is less evident.

In the absence of more conclusive evidence a value of 7 for N would seem to be reasonable for both sidewall and curved contour, at least in the preliminary design stage.

6. Boundary-Layer Transition

In the foregoing it has been assumed that the boundary layer is turbulent from some station well upstream of the throat, either as a result of natural or forced transition in the wall boundary layer. It may be however that in some small tunnels, the boundary layer is laminar near the throat. For example, Brinich[11] found that transition occurred on the side wall of a 10 in. × 3.84 in. tunnel at Reynolds numbers (based on boundary-layer displacement thickness and stream kinematic viscosity) between 913 and 1982. No laminar layers were observed on the bottom

wall,/

wall, where the local Reynolds number was higher. These Reynolds numbers are approximately in accordance with those quoted in Ref. 15 for the occurrence of transition on a flat plate and presumably can be used as a rough guide to the likelihood of laminar flow persisting into the throat region of the nozzle. An accurate prediction of boundary-layer transition is a difficult matter however. As Dryden[31] has pointed out, the transition is affected by Mach number, free-stream turbulence and local-heat transfer.

There seems to be little evidence for the assertion sometimes made that transition occurs on the curved contour at the point of inflection downstream of the throat.

Since the transition position will fluctuate with tunnel stagnation pressure and because the boundary layer growth near the transition region is not easy to calculate, it may be desirable to provoke transition artificially (by means of a wire or surface roughness, for example) well upstream of the throat.

7. Concluding Remarks

It is hoped that this note may be of use in indicating means of calculating turbulent boundary-layer growth along the walls of supersonic wind tunnels, particularly when great accuracy is not required or desired.

8. Acknowledgement

The authors wish to acknowledge the help given to them during discussions on the development of turbulent boundary layer by Dr. G. E. Gadd of the Aerodynamics Division, N.P.L.

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List of Symbols

b	tunnel breadth
f	θ/δ
g	δ^*/δ
h	tunnel half-height
ℓ	distance from throat to run-out position on nozzle
M	fluid Mach number
M_d	nozzle design Mach number
N	inverse power index in velocity profile (equation (3))
R	Reynolds number
R_R	Reynolds number at run-out position
s	distance along curved surface of nozzle
u	fluid velocity
w	tunnel width
x	distance along longitudinal axis of tunnel
y	distance normal to wall
C_F	mean surface-friction coefficient (friction force per unit wetted area divided by free-stream dynamic pressure)
C_f	local surface-friction coefficient
δ	boundary-layer thickness
$\Delta\delta$	boundary-layer growth from nozzle throat to run-out position
δ^*	boundary-layer displacement thickness
γ	ratio of specific heats of gas
θ	boundary-layer momentum thickness
μ	fluid viscosity
ρ	fluid density
τ	shear stress at wall
A	$(1 + 0.1M^2)^{8/7}$

Suffices

1	conditions at the edge of the boundary layer
w	conditions at the wall
x	conditions at position x
i	value in incompressible flow

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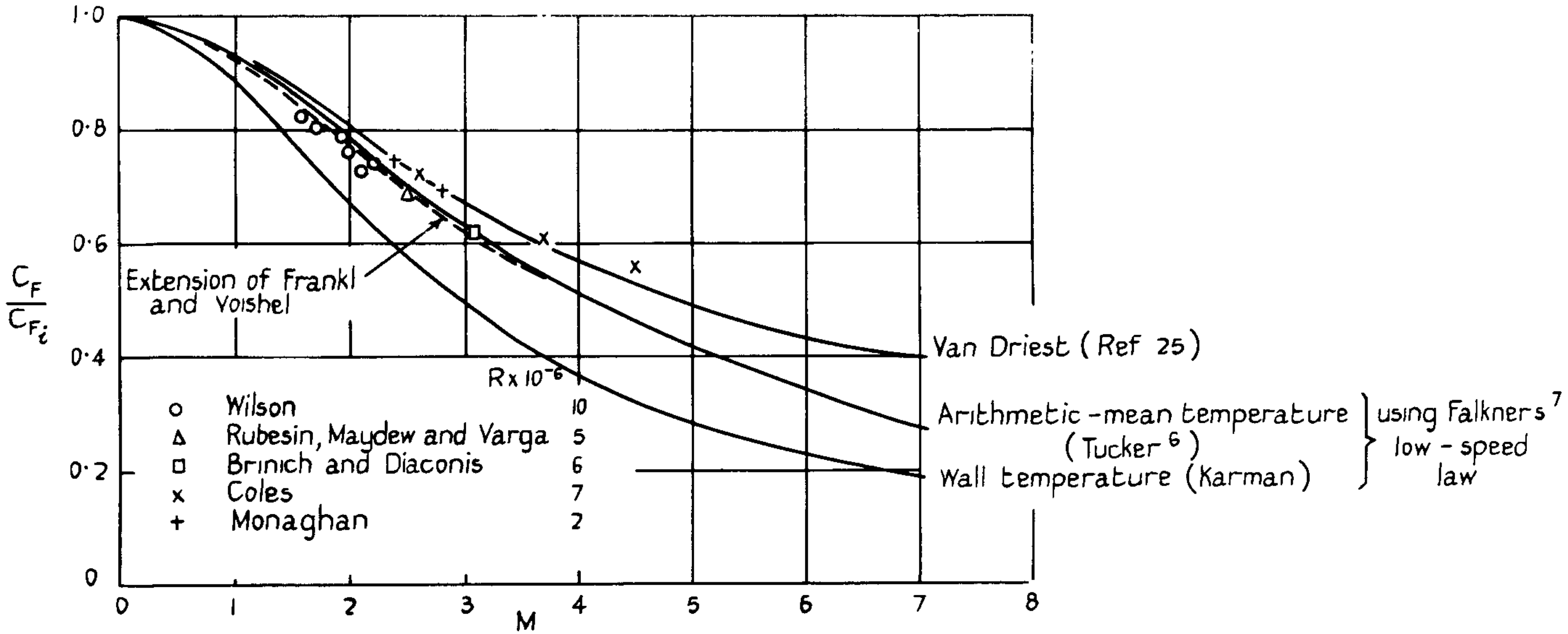
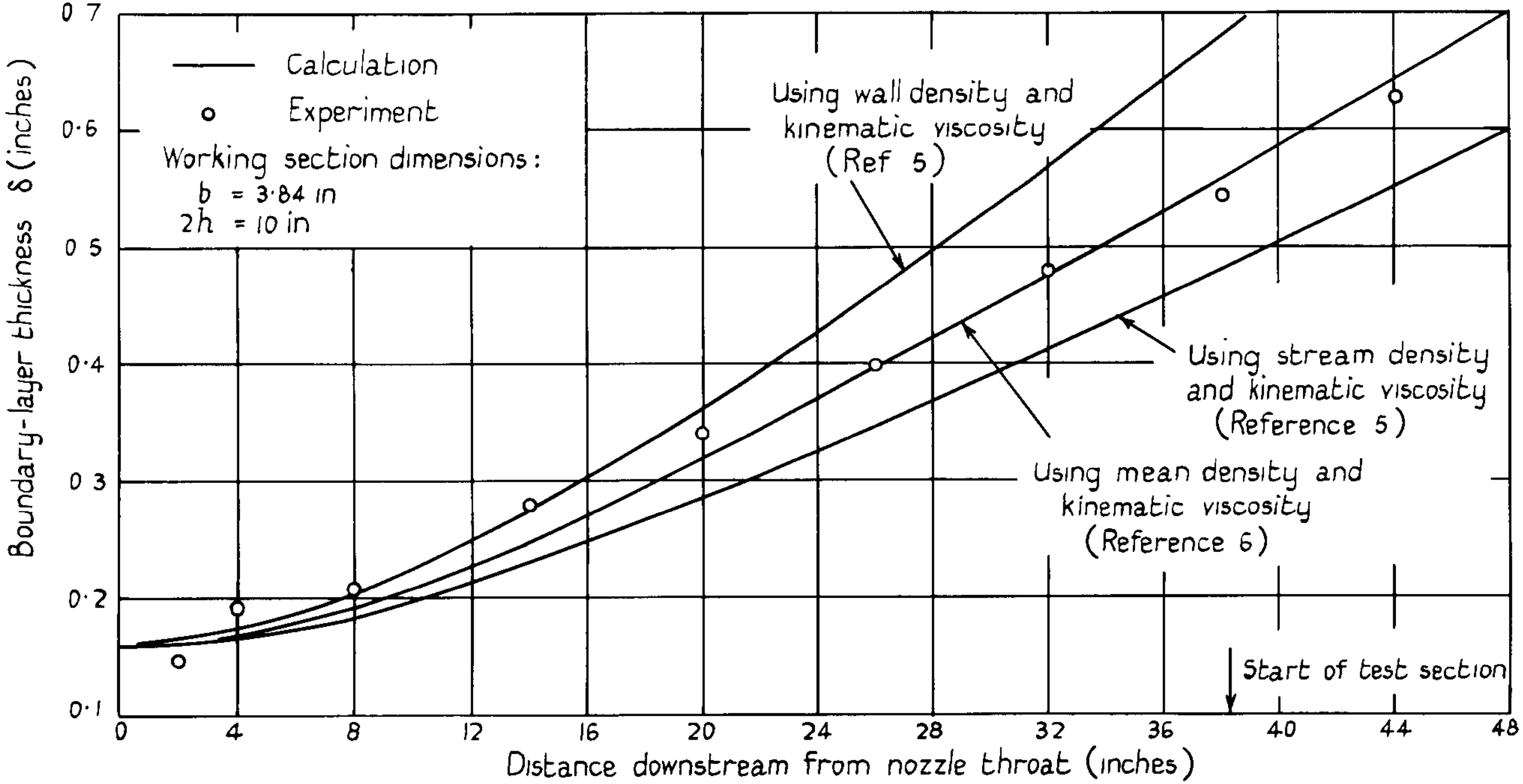
30	D. A. Spence	The Growth of Turbulent Boundary Layers. J. Ae. Sc. Vol. 23, p. 3. (1956).
31	H. Dryden	Recent Investigations of the problem of transition. Zeitschrift für Flugwissenschaften (4) p. 89 (1956).
32	W. F. Cope	Notes and graphs for boundary layer calculations in compressible flow. C.P. 89. August, 1951.

FIG. 1.



Effect of Mach number on the mean friction-drag on flat plates

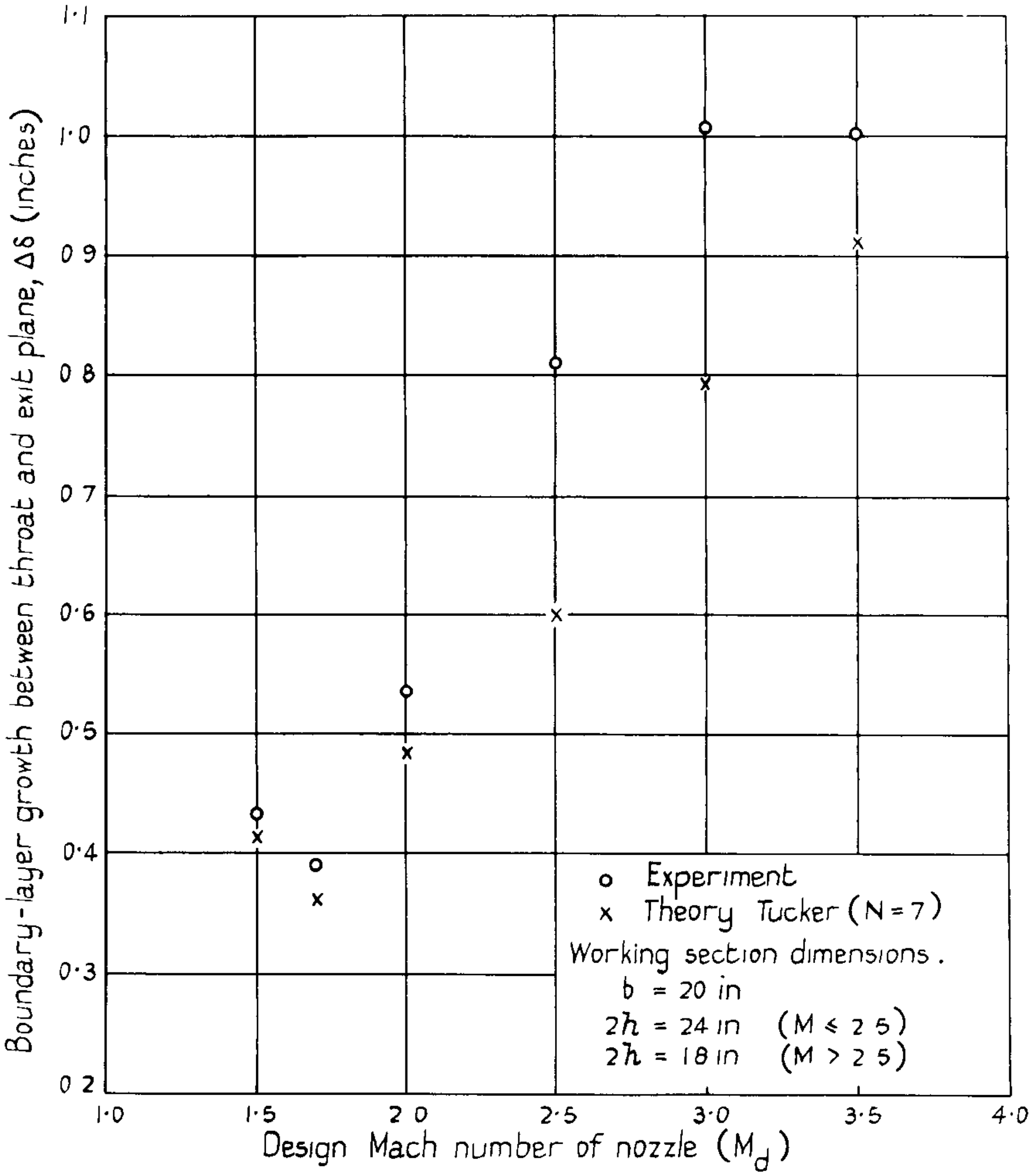
(Theoretical curves are for $R = 7 \times 10^6$)



Comparison of calculated and experimental boundary-layer growth for $M_d = 2{\cdot}08$ (Reference 11)

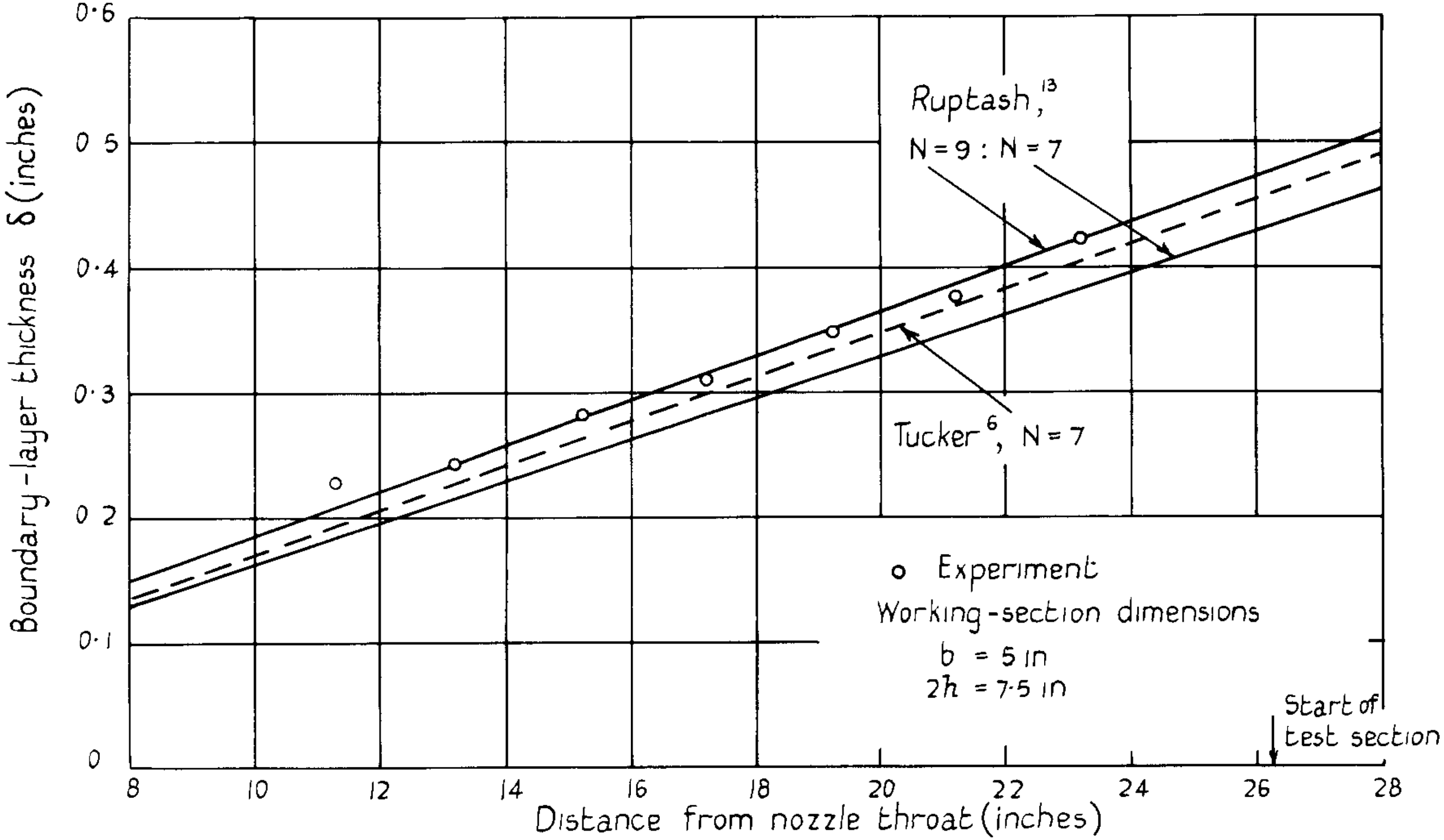
FIG. 2.

FIG 3.

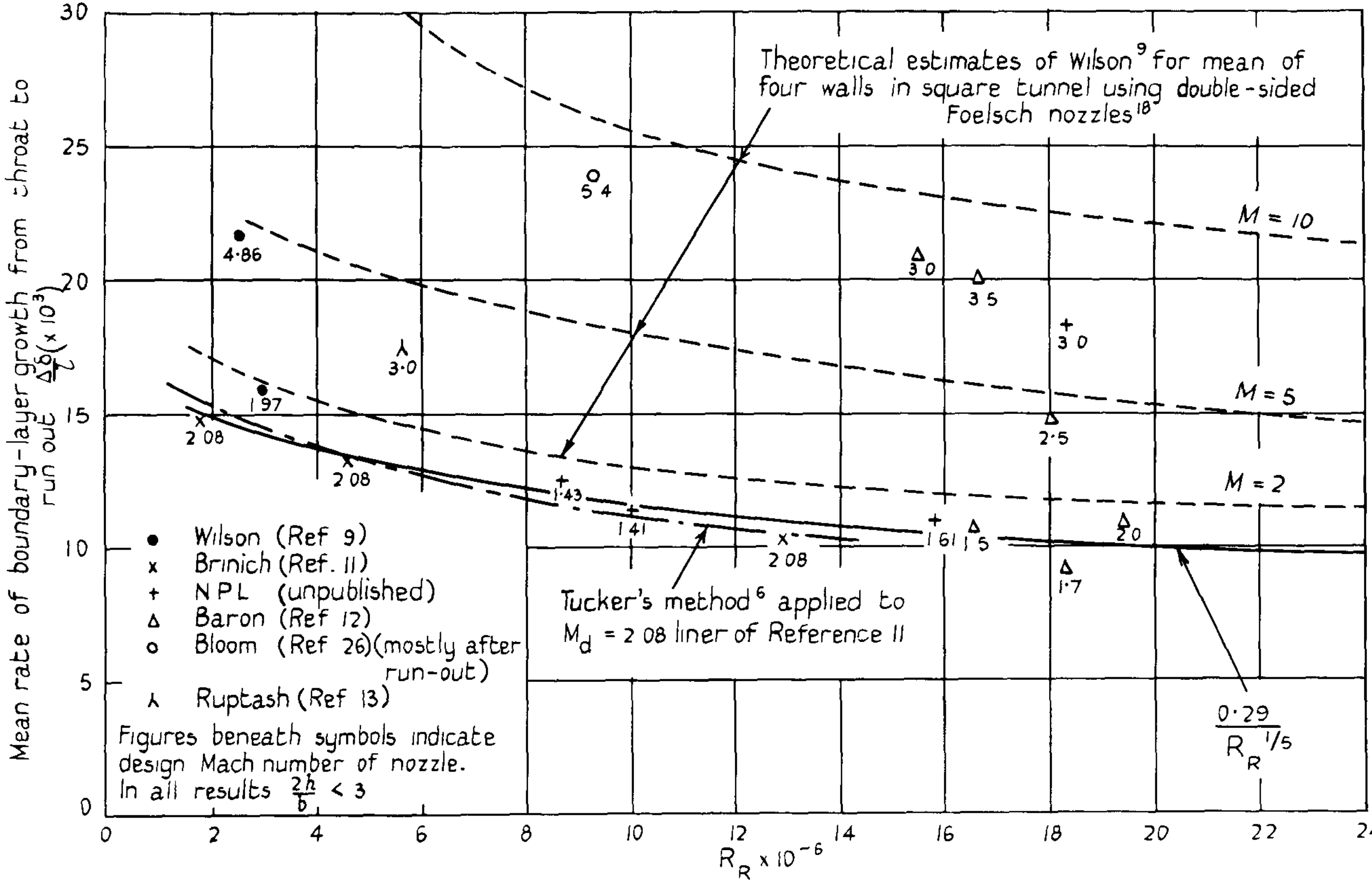


Comparison of theoretical and experimental boundary-layer growth (based on results given in Ref 12)

FIG. 4



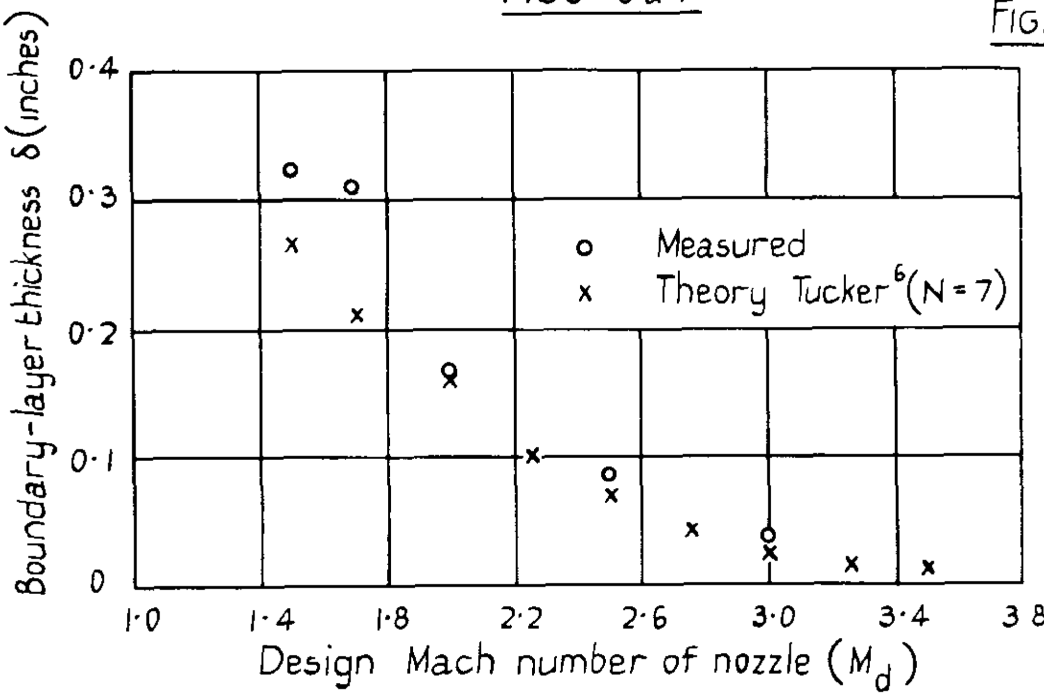
Boundary-layer growth along curved contour of $M_d = 3$ nozzle



Overall rate of boundary-layer growth along curved surface of supersonic nozzle

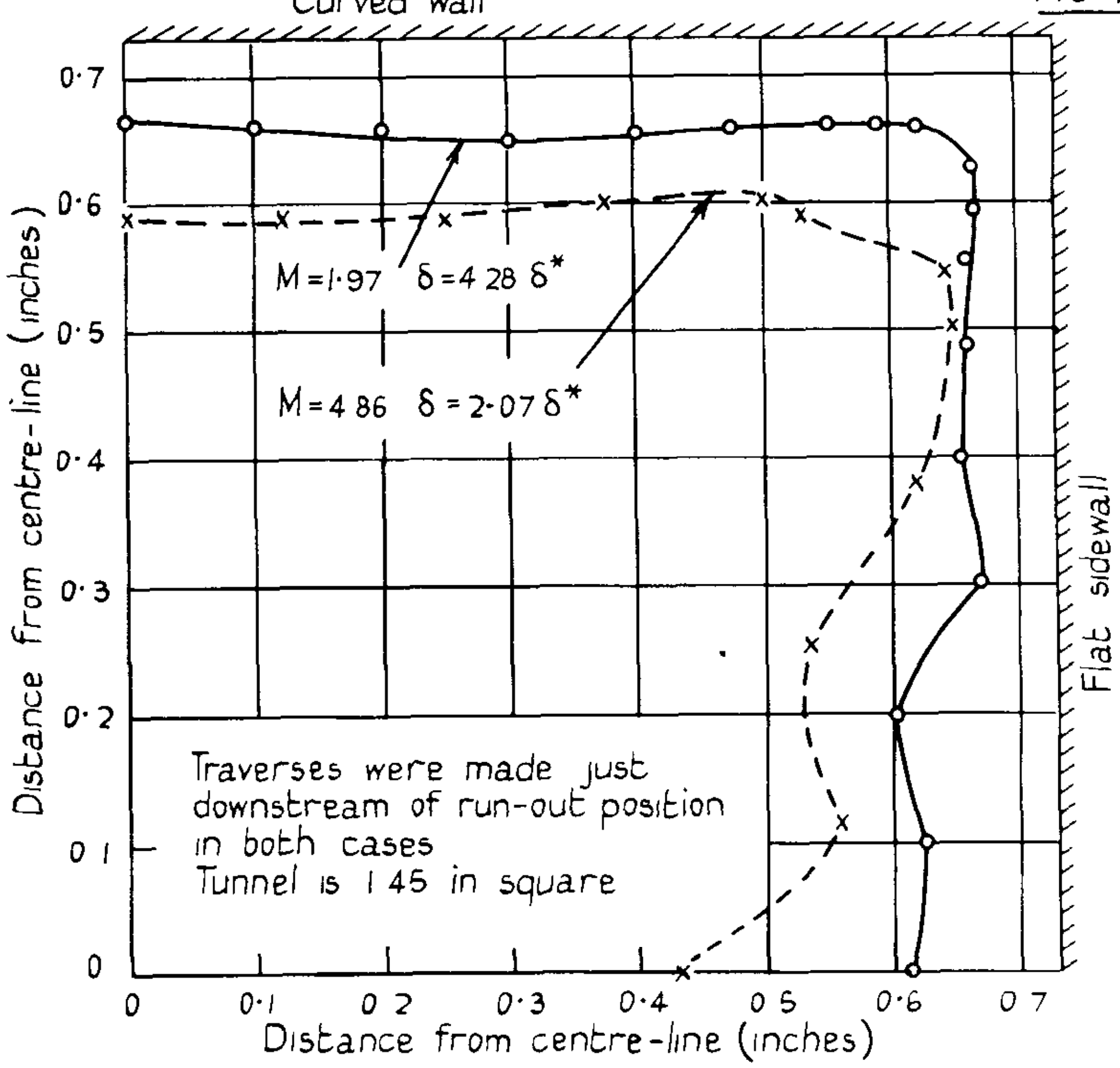
FIG. 5

FIG. 6



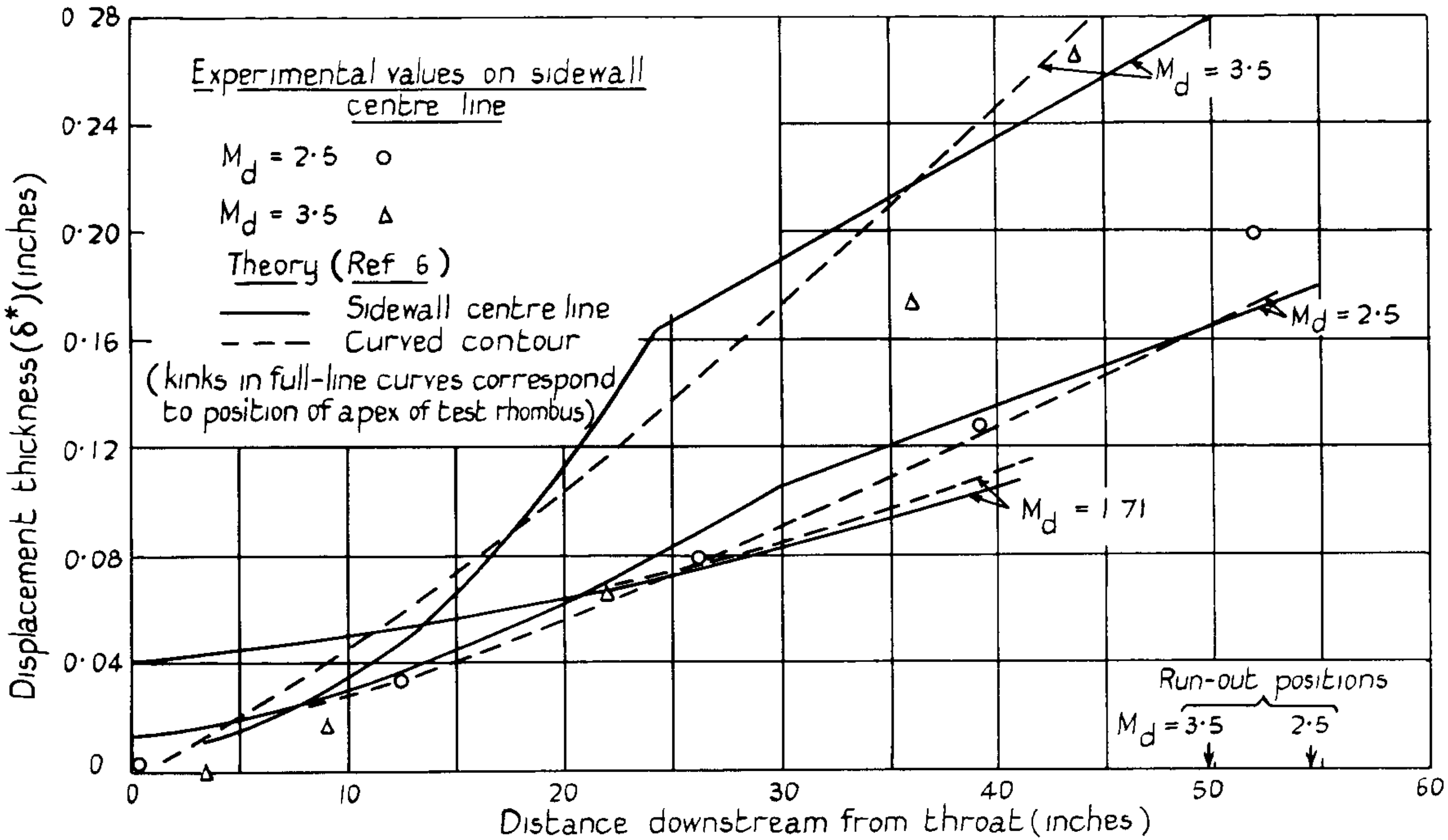
Comparison of theoretical and experimental boundary-layer thickness at throat (based on results given in Ref. 12).

FIG 7



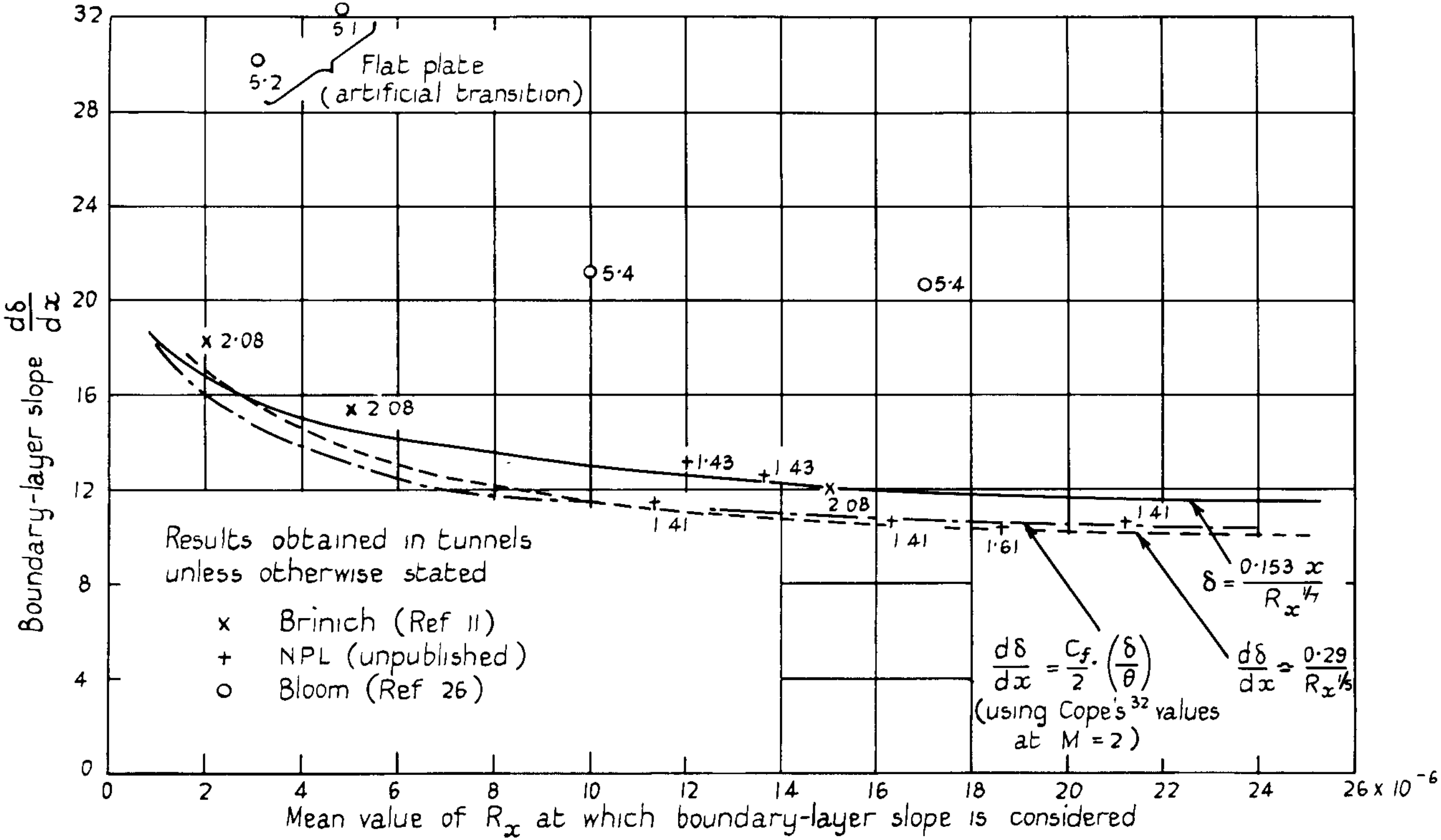
Distribution of boundary-layer thickness δ in one quarter of tunnel (from Ref 9)

FIG. 8.



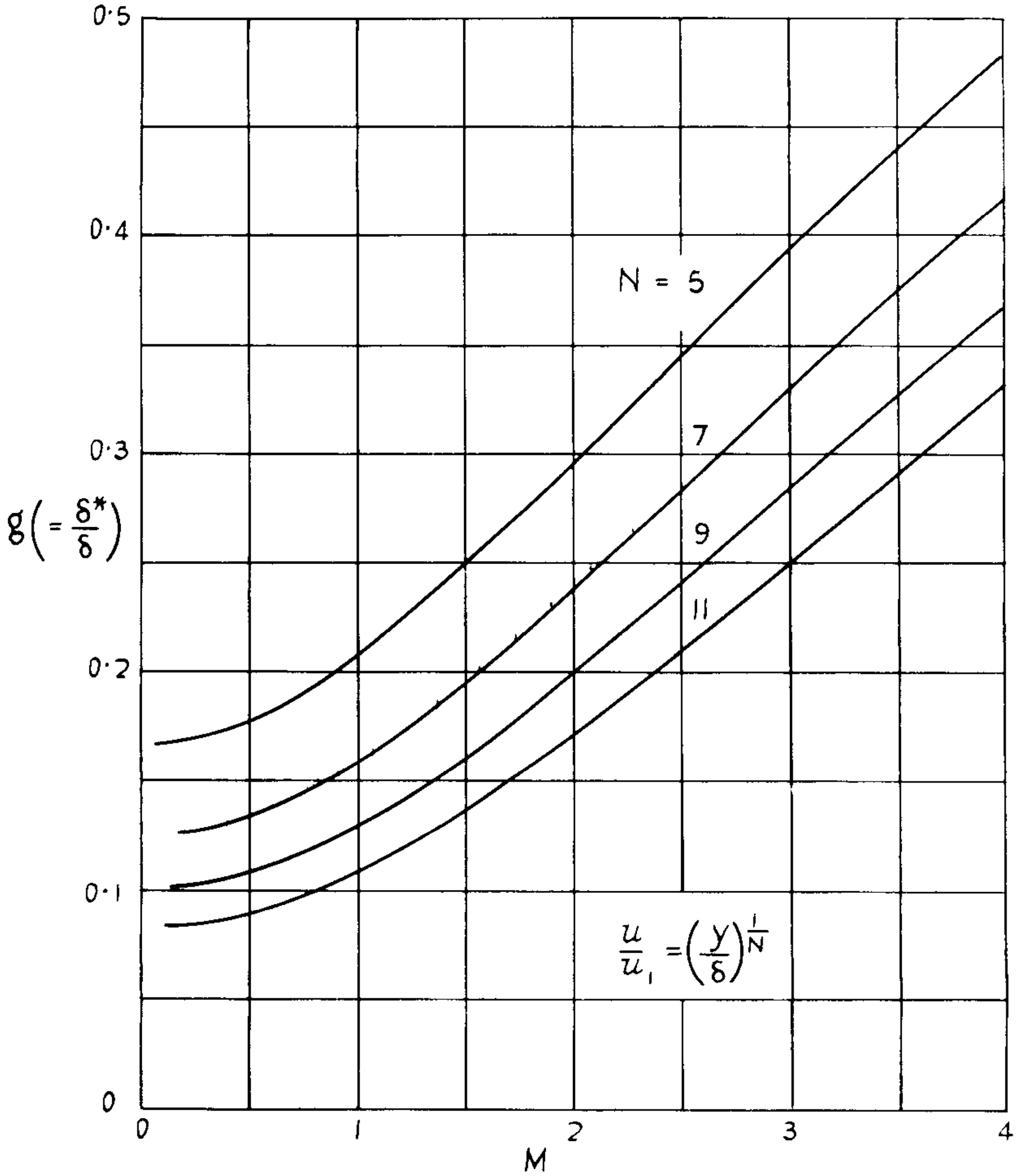
Comparison of estimated and measured boundary-layer displacement thickness of sidewall centreline (from data given in Ref 12)

FIG 9.



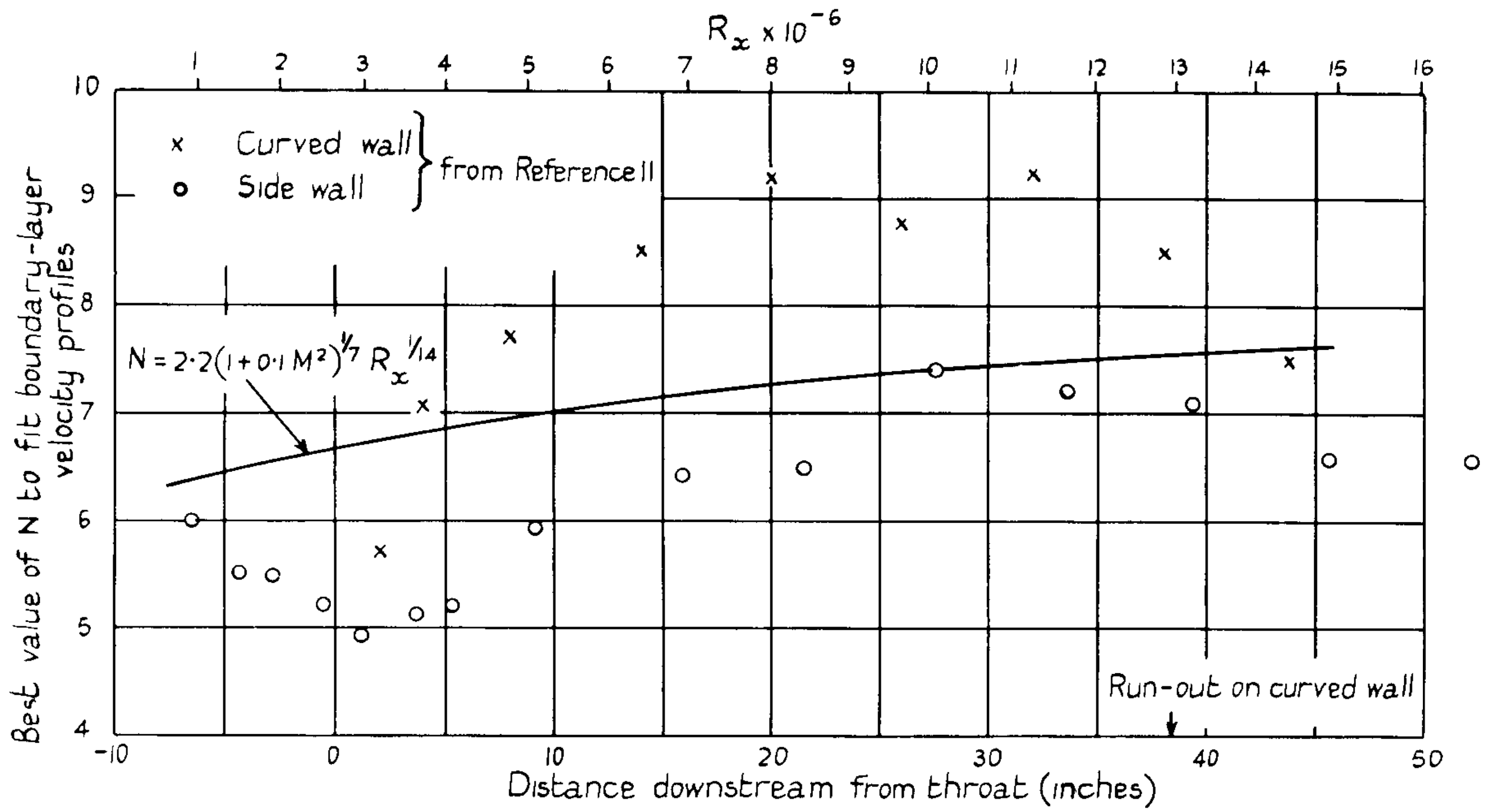
Boundary growth on nozzle profiles downstream of run-out position

FIG 10.



Variation of thickness parameter g with Mach number and N

FIG. 11



Experimental best values of N at $M_d = 2{\cdot}08$

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