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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
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Summary.—This report develops an approximate theory of longitudinal response which applies to the slow mode of motion after a disturbance. This theory is complementary to that for the quick-period motion given by Gates and Lyon[5] (1944). It predicts the slow motions which occur after the quick-period motions have died out.

The approximate equations given are of second order only and can therefore be solved algebraically. This has been done and the general solutions are given in Tables 1 and 2.

Some numerical examples have been computed to indicate the accuracy which can be expected. The agreement with the first approximation is quite good and in some components it can be improved by the use of a further correction term.

1. *Introduction*.—Recent flight and wind-tunnel tests have shown that aircraft flying in the transonic range are likely to have manoeuvre and static margins well outside the ranges which have been usual in the past and in particular that large negative static margins may occur, accompanied by moderately large positive manoeuvre margins. This condition must lead to instability of the long-period motion and it is desirable that some method should be available which enables the seriousness of this effect on the aircraft response to control movements and outside disturbances to be assessed easily.

The motion of an aeroplane after a longitudinal disturbance is a mixture of two modes of motion. Usually these two modes are both damped oscillations, one with a period of the order of one second (the short-period oscillation) and one with a period of the order of 100 seconds (the long-period oscillation or phugoid). Occasionally however, either of these modes can consist of a pair of exponential motions in which the time of decay corresponds roughly to the period of the oscillation. The period or time of decay is called the time constant of the motion. Because the time constants of the two types of motion differ so much it is possible to find approximate methods to treat each separately. A theory has already been developed which enables the rapid response to be calculated and this report aims at producing a comparable theory for the slow response.

2. *Complete Linear Theory*.—The equations of motion of an aircraft for small symmetrical displacements from a steady flight condition are well known and have been given by Bryant and Gates[3] (1937) and in a modified form by Whatham and Priestley[4] (1946). They may be written:

* R.A.E. Report Aero. 2438, received 29th March, 1952.

$$\left.\begin{aligned}
&\left(\frac{d}{d\tau} - x_u\right)\hat{u} \quad - x_w\hat{w} \quad + k\theta = 0 \\
&- z_u\hat{u} + \left(\frac{d}{d\tau} - z_w\right)\hat{w} \quad - \hat{q} \quad = 0 \\
&\varkappa\hat{u} + \left(\chi\frac{d}{d\tau} + \omega\right)\hat{w} + \left(\frac{d}{d\tau} + \nu\right)\hat{q} \quad = \mathscr{C}_m \\
&- \hat{q} + \frac{d\theta}{d\tau} = 0
\end{aligned}\right\} \quad \ldots \quad (1)$$

In these equations τ is the measure of time in dimensionless units, the unit of time being $\hat{t} = m/\rho SV$ and

$$\left.\begin{aligned}
&\hat{u} = \frac{u}{V} \qquad \hat{w} = \frac{w}{V} \qquad \hat{q} = \frac{\mu q l}{V} = q\hat{t} \\
&\varkappa = -\frac{\mu m_u}{i_B} \qquad \chi = -\frac{\mu m_{\dot{w}}}{i_B} \\
&\omega = -\frac{\mu m_w}{i_B} \qquad \nu = -\frac{m_q}{i_B} \qquad \mathscr{C}_m = \frac{\mu C_m}{i_B}
\end{aligned}\right\} \quad \ldots \quad (2)$$

The remaining symbols are the standard ones of R. & M. 1801[3] (1937).

These differential equations are of fourth order so that the characteristic equation is of fourth degree. The roots of this equation usually occur as one large pair and one small pair which may be real or complex. These correspond to motions with time constants of the order of 1 second and 100 seconds respectively.

The difference between these time constants suggests that the motion for a few seconds after a disturbance will be governed almost entirely by the large roots and the motion after a long time by the small roots. We should therefore expect that approximate theories for these two conditions should be possible. The approximate theory of manoeuvrability[5] which neglects the effect of changes of speed provides this for the few seconds after a disturbance. The present aim is to produce a corresponding theory which will predict the motion a long time after the disturbance.

3. *Previous Approximations.*—As early as 1908 a theory of the slow-period motions was given by Lanchester[1]. In this theory he did not assume that the displacement of the aircraft from the equilibrium condition was small, so that the theory does not compare with any of our usual theories. Jones[2] (1936) has however reduced this theory to one corresponding to small disturbances.

The assumptions of this theory may be written:

(*a*) The inertia in pitch is negligible, *i.e.*, we may neglect the term in $d\hat{q}/d\tau$ in the third of equations (1).

(*b*) The aircraft is statically stable, i.e., $\omega > 0$.

(*c*) There is no damping in pitch and no damping due to downwash delay, *i.e.*, the terms $\nu\hat{q}$ and $\chi(d\hat{w}/d\tau)$ may be omitted from the pitching moment equation.

(*d*) The variation of pitching moment with speed may be neglected, *i.e.*, the term $\varkappa\hat{u}$ may be omitted.

(*e*) The thrust and drag are equal at all times, i.e., $x_u = 0$.

It can be seen that the effect of these assumptions is to reduce the pitching moment equation to

$$\omega\hat{w} = \mathscr{C}_m$$

so that $\hat{w}$ is simply proportional to $\mathscr{C}_m$ and in the absence of a pitching moment $\hat{w}$ is zero. We may therefore ignore $\hat{w}$ in the other equations and these then become

$$\left.\begin{aligned} \frac{d\hat{u}}{d\tau} \qquad\qquad + k\theta &= 0 \\ - z_u\hat{u} - \hat{q} \qquad\qquad &= 0 \\ - \hat{q} + \frac{d\theta}{d\tau} &= 0 \end{aligned}\right\} \qquad (3)$$

These reduce to

$$\frac{d_2\theta}{d\tau^2} - kz_u\theta = 0. \qquad (4)$$

Now since z_u is negative, this is the equation of a steady oscillation. The complete theory predicts a damped oscillation but in this approximation all the damping terms such as ν have been omitted. Also assumption (*d*) that there is no pitching moment due to change of speed makes the theory inapplicable at high subsonic speeds where this term may be an important one.

4. *The Present Theory.*—For the purpose for which this theory is needed we can no longer omit the damping terms and we must also include the change of pitching moment with speed. The omission of the damping term and the term in m_u from the pitching-moment equation led in Lanchester's theory to the result that the incidence was constant. Instead of making the same assumptions as Lanchester we shall assume that the incidence is not constant but varies only slowly with time so that its time differential may be neglected. Our assumptions may be summarized as follows:

(*a*) The inertia in pitch may be neglected.

(b) Terms in rate of change of incidence may be neglected.

With these assumptions the equations of motion become:

$$\left.\begin{aligned} \left(\frac{d}{d\tau} - x_u\right)\hat{u} - x_w\hat{w} \qquad\qquad + k\theta &= 0 \\ - z_u\hat{u} - z_w\hat{w} - \hat{q} \qquad\qquad &= 0 \\ \varkappa\hat{u} + \omega\hat{w} + \nu\hat{q} \qquad\qquad &= \mathscr{C}_m \\ - \hat{q} \qquad + \frac{d\theta}{d\tau} &= 0 \end{aligned}\right\} \qquad (6)$$

These equations are of second order and the characteristic equation is

$$\Delta \equiv (\omega - z_w\nu)\lambda^2 + \{ - x_u(\omega - z_w\nu) + x_w(\varkappa - z_u\nu)\} \lambda + k(z_w\varkappa - z_u\omega) = 0. \qquad (7)$$

We have now to find the initial conditions to be applied to these equations and in order to do this we must consider the general nature of the complete motion. We know that this motion consists of a rapid motion which, if it is stable, decays quickly, leaving much slower motions to persist. If our equations are to represent just the slower motions, our assumptions must be equivalent to assuming that the rapid motion occurs infinitely quickly. It has been shown that these rapid motions are represented with good accuracy by the equations:

$$\left.\begin{aligned} \left(\frac{d}{d\tau} - z_w\right)\hat{w} \qquad\qquad - \hat{q} &= 0 \\ \left(\chi\frac{d}{d\tau} + \omega\right)\hat{w} + \left(\frac{d}{d\tau} + \nu\right)\hat{q} &= \mathscr{C}_m \end{aligned}\right\} \qquad (8)$$

These equations are derived on the assumption that the forward speed is unchanged during the rapid motion and in writing them it has been assumed that there is no disturbance in forward speed. If there were a disturbance in $\hat{u}$ these equations would become

$$\left.\begin{aligned} -z_u\hat{u} + \left(\frac{d}{d\tau} - z_w\right)\hat{w} \qquad\qquad - \hat{q} &= 0 \\ \varkappa\hat{u} + \left(\chi\frac{d}{d\tau} + \omega\right)\hat{w} + \left(\frac{d}{d\tau} + \nu\right)\hat{q} &= \mathscr{C}_m \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

$\hat{u}$ here being assumed constant for the rapid motion.

Now let us suppose that the given initial conditions are

$$\hat{u} = u_0, \quad \hat{w} = w_0, \quad \hat{q} = q_0, \quad \theta = \theta_0 \quad \text{at} \quad \tau = 0$$

and that the intial conditions we have to impose on our approximate equations are

$$\hat{u} = u_1, \quad \hat{w} = w_1, \quad \hat{q} = q_1, \quad \theta = \theta_1 \quad \text{at} \quad \tau = 0.$$

Since we assume that $\hat{u}$ is constant in the rapid motion it follows that $u_1 = u_0$. Since $\hat{w}$ and $\hat{q}$ are the variables involved in the rapid motion $w_1 \neq w_0$ and $q_1 \neq q_0$ but since θ does not occur explicitly in these equations but only its differential $\hat{q}$ and the time for which the rapid motion persists is small we may assume as a first approximation that $\theta_1 = \theta_0$.

If we assume the initial value u_1 then w_1 and q_1 must satisfy the second and third of equations (6), namely:

$$\left.\begin{aligned} -z_u\hat{u} - z_w\hat{w} - \hat{q} &= 0 \\ \varkappa\hat{u} + \omega\hat{w} + \nu\hat{q} &= \mathscr{C}_m \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (10)$$

where $\mathscr{C}_m$ has its value at $\tau = 0$.

Now if we assume that the short-period motion is stable then the motion will die out so that the differential terms disappear, and the final conditions are given by the equations (9) with the differential terms deleted. It can be seen that these are precisely the same as equation (10). This means that the initial conditions we assume for $\hat{w}$ and $\hat{q}$ in the long period motion are the final conditions for the short period motion.

We can extend this principle to obtain equivalent initial conditions for the long-period motion corresponding to disturbances in $\hat{w}$ and $\hat{q}$, as follows. We may solve equations (10) for w_1 and q_1 giving

$$\left.\begin{aligned} w_1 &= \frac{1}{(\omega - z_w\nu)}\left\{\mathscr{C}_m - (\varkappa - z_u\nu)u_1\right\} \\ q_1 &= \frac{1}{(\omega - z_w\nu)}\left\{-z_w\mathscr{C}_m + (z_w\varkappa - z_u\omega)u_1\right\} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \qquad (11)$$

These equations apply at the end of the rapid motion. Now during the rapid motion we have from (9)

$$\left.\begin{aligned} -z_u\hat{u} + \left(\frac{d}{d\tau} - z_w\right)\hat{w} \qquad\qquad - \hat{q} &= 0 \\ \varkappa\,\hat{u} + \left(\chi\frac{d}{d\tau} + \omega\right)\hat{w} + \left(\frac{d}{d\tau} + \nu\right)\hat{q} &= \mathscr{C}_m \end{aligned}\right\}.$$

Whence, eliminating $\hat{w}$

$$(z_w\varkappa - z_u)\hat{u} + (\omega + \chi z_w)\frac{d\hat{w}}{d\tau} + z_w\frac{d\hat{q}}{d\tau} - (\omega - z_w\nu)\hat{q} = z_w\mathscr{C}_m. \quad \ldots \quad \ldots \quad \ldots \qquad (12)$$

Now integrating with respect to τ,

$$(z_w\varkappa - z_u)\int\hat{u}\,d\tau + (\omega + \chi z_w)\hat{w} + z_w\hat{q} - (\omega - z_w\nu)\theta$$
$$= z_w\int\mathscr{C}_m\,d\tau + (\omega + \chi z_w)w_0 + z_w q_0 - (\omega - z_w\nu)\theta_0. \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Now we may substitute for w_1 and q_1 in terms of u_1 from (11) and obtain

$$(z_w\varkappa - z_u)\int\hat{u}\,d\tau + \left(\frac{\omega + \chi z_w}{\omega - z_w\nu}\right)\left\{\mathscr{C}_m - (\chi - z_u\nu)u_1\right\} + \frac{z_w}{(\omega - z_w\nu)}\left\{-z_w\mathscr{C}_m + (z_w\varkappa - z_u\omega)u_1\right\}$$
$$- (\omega - z_w\nu)\theta_1 = z_w\int\mathscr{C}_m\,d\tau + (\omega + \chi z_w)w_0 + z_w q_0 - (\omega - z_w\nu)\theta_0. \quad \ldots \quad \ldots \quad (14)$$

Now if we write $u_1 = u_0$ we may solve this equation for θ_1 and obtain

$$\theta_1 = \theta_0 - \left(\frac{\omega + \chi z_w}{\omega - z_w\nu}\right)w_0 - \frac{z_w}{(\omega - z_w\nu)}q_0 + \frac{(\omega + z_w\chi - z_w^2)}{(\omega - z_w\nu)^2}\mathscr{C}_m$$
$$+ \frac{\{z_w(z_w\varkappa - z_u\omega) - (\omega + \chi z_w)(\varkappa - z_u\nu)\}}{(\omega - z_w\nu)^2}u_0 - \frac{z_w}{(\omega - z_w\nu)}\int\mathscr{C}_m\,d\tau + \frac{(z_w\varkappa - z_u)}{(\omega - z_w\nu)}\int u_0\,d\tau. \quad (15)$$

Now θ_1 is the equivalent initial value of θ for the long-period motion and since we are assuming that the rapid motion takes place infinitely quickly we may take the upper limit of the last two integrals zero so that they vanish.

This equation achieves what we set out to do in that we have obtained equivalent initial conditions θ_1 for a disturbance w_0 or q_0. The physical interpretation is that during the short-period motion the disturbances w_0 and q_0 die away and during this motion an angle of pitch θ_1 is developed which excites the long-period motion.

In equation (15) however we have also terms in $\mathscr{C}_m$ and u_0. These terms mean that u_0 and $\mathscr{C}_m$ disturbances produce an angle of pitch during the short-period motion even if we assume it takes place very quickly, so that even if at the beginning of a disturbance only u is non-zero when we come to the beginning of the long-period motion θ has assumed a finite value. It will be shown later that these terms are usually a small correction to the calculated response.

5. *Damping and Frequency.*—If the characteristic equation (7) has complex roots $r \pm is$ the motion is oscillatory and its frequency is

$\frac{s}{2\pi}$ cycles per unit of aerodynamic time.

The damping coefficient, the logarithm of the factor by which the amplitude is multiplied in one unit of time, is r. From equation (7) we have

$$\left.\begin{aligned} 2r &= -x_u + \frac{x_w(\varkappa - z_u\nu)}{(\omega - z_w\nu)} \\ r^2 + s^2 &= \frac{k(z_w\varkappa - z_u\omega)}{(\omega - z_w\nu)} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

Approximate formulae have been given previously by Lyon and others[6] (1942) for these quantities. These are:

$$\left.\begin{aligned} 2r &= \left(\frac{D}{C} - \frac{BE}{C^2}\right) \\ r^2 + s^2 &= \frac{E}{C} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

where
$$\left.\begin{aligned} B &= N + \nu + \chi \\ C &= N\nu + \omega \\ D &= Q\omega + P\nu + R\chi - S\varkappa \\ E &= R\omega - T\varkappa. \end{aligned}\right\} \quad \ldots \qquad (18)$$

Here
$$\left.\begin{aligned} N &= -x_u - z_w \doteqdot -z_w \\ P &= x_u z_w - x_w z_u \\ Q &= -x_u \\ R &= -kz_u \\ S &= k - z_w \\ T &= -kz_w. \end{aligned}\right\} \quad \ldots \qquad (19)$$

If we consider only the first term of the damping we have

$$2r = \frac{D}{C}$$

$$= \frac{1}{(\omega - z_w\nu)}\left\{ -x_u(\omega - z_w\nu) + x_w(\varkappa - z_u\nu) - k(z_u\chi + \varkappa)\right\} \quad \ldots \qquad (20)$$

so that the present theory (16) produces the first two of the three terms in the bracket and neglects the third. This may lead to some error in the damping but since the damping is usually small the effect on the response of using a slightly inaccurate value of this small quantity should not be serious.

The expression for the constant term of the quadratic is, from (17),

$$r^2 + s^2 = \frac{E}{C} = \frac{k(z_w\varkappa - z_u\omega)}{(\omega - z_w\nu)} \quad \ldots \qquad (21)$$

which agrees with the value predicted by the present theory (16).

Since the expression obtained by this method for the damping is not very accurate it is not recommended for use if only the damping and not the response is required. If a good approximation to the damping is needed the formula of R. & M. 2075[6] (1942) should be used. The values of the stability roots calculated for four examples are given in the following table. The values of the derivatives assumed are given in Table 3.

Comparison of Exact and Approximate Stability Roots

Example	Exact	Method of Ref. 6	Method of this theory
1	$-0\cdot00702 \pm 0\cdot1843i$	$-0\cdot00702 \pm 0\cdot1842i$	$-0\cdot00760 \pm 0\cdot1846i$
2	$+0\cdot1745,\ -0\cdot1740$	$+0\cdot1746,\ -0\cdot1741$	$+0\cdot1734,\ -0\cdot1753$
3	$-0\cdot0358\ \pm 0\cdot1301i$	$-0\cdot0346\ \pm 0\cdot1276i$	$-0\cdot0322\ \pm 0\cdot1292i$
4	$-0\cdot0250\ \pm 0\cdot5408i$	$-0\cdot0247\ \pm 0\cdot5395i$	$-0\cdot0656\ \pm 0\cdot5424i$

6. *Solution of Equations of Motion.*—The approximate equations of motion are from (6)

$$\left.\begin{array}{lr} \left(\dfrac{d}{d\tau} - x_u\right)\hat{u} - x_w\hat{w} & + k\theta = 0 \\ - z_u\hat{u} - z_w\hat{w} - \hat{q} & = 0 \\ \varkappa\hat{u} + \omega\hat{w} + \nu\hat{q} & = \mathscr{C}_m \\ - \hat{q} & + \dfrac{d\theta}{d\tau} = 0 \end{array}\right\} \quad \ldots \quad \ldots \quad \ldots \qquad (22)$$

Assuming $\hat{u} = u_0$, $\theta = \theta_0$ at $t = 0$ and applying the Laplace transform these equations become

$$\left.\begin{array}{lr} (p - x_u)\bar{u} - x_w\bar{w} & + k\bar{\theta} = u_0 \\ - z_u\bar{u} - z_w\bar{w} - \bar{q} & = 0 \\ \varkappa\bar{u} + \omega\bar{w} + \nu\bar{q} & = \overline{\mathscr{C}}_m \\ - \bar{q} & + p\bar{\theta} = \theta_0 \end{array}\right\} \quad \ldots \quad \ldots \quad \ldots \qquad (23)$$

where $\bar{u}$, $\bar{w}$, $\bar{q}$, $\bar{\theta}$ and $\overline{\mathscr{C}}_m$ denote the Laplace transforms of $\hat{u}$, $\hat{w}$, $\hat{q}$, θ and $\mathscr{C}_m$ respectively. The solution of these equations is

$$\bar{u} = \begin{vmatrix} u_0 & -x_w & 0 & k \\ 0 & -z_w & -1 & 0 \\ \overline{\mathscr{C}}_m & \omega & \nu & 0 \\ \theta_0 & 0 & -1 & p \end{vmatrix} \div \Delta \quad \ldots \quad \ldots \quad \ldots \qquad (24)$$

$$\bar{w} = \begin{vmatrix} p - x_u & u_0 & 0 & k \\ -z_u & 0 & -1 & 0 \\ \varkappa & \overline{\mathscr{C}}_m & \nu & 0 \\ 0 & \theta_0 & -1 & p \end{vmatrix} \div \Delta \quad \ldots \quad \ldots \quad \ldots \qquad (25)$$

$$\bar{q} = \begin{vmatrix} p - x_u & -x_w & u_0 & k \\ -z_u & -z_w & 0 & 0 \\ \varkappa & \omega & \overline{\mathscr{C}}_m & 0 \\ 0 & 0 & \theta_0 & p \end{vmatrix} \div \Delta \quad \ldots \quad \ldots \quad \ldots \qquad (26)$$

$$\bar{\theta} = \begin{vmatrix} p - x_u & -x_w & 0 & u_0 \\ -z_u & -z_w & -1 & 0 \\ \varkappa & \omega & \nu & \overline{\mathscr{C}}_m \\ 0 & 0 & -1 & \theta_0 \end{vmatrix} \div \Delta \quad \ldots \quad \ldots \quad \ldots \qquad (27)$$

where

$$\Delta = \begin{vmatrix} p - x_u & -x_w & 0 & k \\ -z_u & -z_w & -1 & 0 \\ \varkappa & \omega & \nu & 0 \\ 0 & 0 & -1 & p \end{vmatrix}$$

$$= (\omega - z_w\nu)p^2 + \{ - x_u(\omega - z_w\nu) + x_w(\varkappa - z_u\nu)\}p + k(z_w\varkappa - z_u\omega) \,. \quad \ldots \quad \ldots \qquad (7)$$

We shall write these expressions in the form

$$\left.\begin{aligned}\bar{u} &= \{\, F_{uu}\, u_0 + F_{um}\, \overline{\mathscr{C}}_m + F_{u0}\, \theta_0 \,\} \div \Delta \\ \bar{w} &= \{\, F_{wu}\, u_0 + F_{wm}\, \overline{\mathscr{C}}_m + F_{w0}\, \theta_0 \,\} \div \Delta\end{aligned}\right\} \qquad \ldots \quad \ldots \qquad (28)$$

and similarly for $\bar{q}$ and $\bar{\theta}$.

The expressions F_{uu}, F_{um}, F_{wu}, etc., are polynomials in p and a complete list of them is given in Appendix I.

The complete solutions for all components have been written down in this analysis but since the second and third equations are purely algebraic they can be solved for $\hat{w}$ and $\hat{q}$ in terms of $\hat{u}$ as follows:

$$\left.\begin{aligned}\hat{w} &= \frac{1}{(\omega - z_w \nu)}\left\{\mathscr{C}_m - (\varkappa - z_u \nu)\hat{u}\right\} \\ \hat{q} &= \frac{1}{(\omega - z_w \nu)}\left\{- z_w \mathscr{C}_m + (z_w \varkappa - z_u \omega)\hat{u}\right\}\end{aligned}\right\}. \qquad \ldots \quad \ldots \quad \ldots \qquad (29)$$

It may be more convenient in computation to use these relations than to use the expressions for $\hat{w}$ and $\hat{q}$ in terms of the stability roots.

We can consider any component of the motion as the sum of three parts due to the initial speed error, the initial disturbance in pitch and the applied pitching moment. These parts may be evaluated by inverting the Laplace transform by the method of partial fractions. The analysis differs according as the characteristic equation (7) has real or complex roots.

For example we have for an initial forward speed error u_0

$$\frac{\bar{u}}{u_0} = \frac{F_{uu}}{\Delta} = \frac{\Omega p}{\Omega p^2 + \{-x_u \Omega + x_w Y\}p + kZ} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (30)$$

where

$$\left.\begin{aligned}\Omega &= (\omega - z_w \nu) \\ Y &= (\varkappa - \nu z_u) \\ Z &= (\varkappa z_w - \omega z_u)\end{aligned}\right\}. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (31)$$

If $\Delta = 0$ has complex roots $r \pm is$ we may write

$$\Delta \equiv \Omega p^2 + \{-x_u \Omega + x_w Y\}p + kZ \equiv \Omega\{p^2 - 2rp + (r^2 + s^2)\}$$

so that
$$\frac{\bar{u}}{u_0} = \frac{p}{(p - r)^2 + s^2} = \frac{p - r}{(p - r)^2 + s^2} + \frac{r}{s}\cdot\frac{s}{(p - r)^2 + s^2}$$

and hence
$$\frac{\hat{u}}{u_0} = e^{r\tau}\left\{\cos s\tau + \frac{r}{s}\sin s\tau\right\}.$$

If $\Delta = 0$ has two real roots λ_1 and λ_2 we may write

$$\Delta \equiv \Omega(p - \lambda_1)(p - \lambda_2)$$

then
$$\frac{\bar{u}}{u_0} = \frac{p}{(p - \lambda_1)(p - \lambda_2)} = \frac{1}{\lambda_1 - \lambda_2}\left(\frac{\lambda_1}{p - \lambda_1} - \frac{\lambda_2}{p - \lambda_2}\right)$$

and therefore
$$\frac{\hat{u}}{u_0} = \frac{1}{\lambda_1 - \lambda_2}\left(\lambda_1 e^{\lambda_1 \tau} - \lambda_2 e^{\lambda_2 \tau}\right)$$

Similarly we may obtain the responses in all components to initial disturbances in speed or angle of pitch.

The calculation of response to an arbitrary pitching moment $\mathscr{C}_m = \mathscr{C}_m(\tau)$ is more complicated and will depend on the actual form of the function $\mathscr{C}_m(\tau)$. We shall take as a simple example the function

$$\left.\begin{array}{ll} \mathscr{C}_m(\tau) = 0 & \tau < 0 \\ \mathscr{C}_m(\tau) = 1 & \tau > 0 \end{array}\right\} .$$

This represents a sudden displacement of the elevator at time $\tau = 0$.

We have then $\bar{\mathscr{C}}_m(p) = 1/p$.

The Laplace transform can be inverted by the use of partial fractions as before.

We have for the response in forward speed

$$\bar{u} = F_{um}\bar{\mathscr{C}}_m/\Delta = (x_w p + z_w k)/p\Delta \ .$$

If the roots of $\Delta = 0$ are complex and equal to $r \pm is$ we have as before

$$\bar{u} = \frac{x_w p + z_w k}{\Omega p\{(p - r)^2 + s^2\}}$$

$$= \frac{z_w}{Z} \cdot \frac{1}{p} - \frac{z_w}{Z}\frac{(p - r)}{(p - r)^2 + s^2} + \frac{x_w(r^2 + s^2) + kz_w r}{kZs} \cdot \frac{s}{(p - r)^2 + s^2}$$

$$\hat{u} = \frac{z_w}{Z}(1 - e^{r\tau}\cos s\tau) + \frac{x_w(r^2 + s^2) + kz_w r}{kZs} e^{r\tau}\sin s\tau .$$

A similar method may be used when the roots are real.

Solutions may be obtained in the same way when $\mathscr{C}_m(\tau)$ is a polynomial, exponential or sinusoidal function of τ, or any combination of these.

Full solutions in terms of initial disturbances u_0 and θ_0 and of instantaneously applied pitching moment $\mathscr{C}_m$ are given in Tables 1 and 2.

Solutions for initial disturbances w_0 and q_0 are obtained by using the equivalent angle of pitch θ_1 given by equation (15) with the integral terms omitted. Thus if K is any component of the motion

$$\frac{K}{w_0} = \left(\frac{\omega + \chi z_w}{\omega - z_w\nu}\right)\frac{K}{\theta_0}$$

and

$$\frac{K}{q_0} = \frac{-z_w}{(\omega - z_w\nu)}\frac{K}{\theta_0}$$

K/θ_0 being given in the tables.

Equation (15) also gives the correction terms for the u_0 and $\mathscr{C}_m$ solutions as

$$\Delta\left(\frac{K}{u_0}\right) = \frac{z_w(z_w\varkappa - z_u\omega) - (\omega + \chi z_w)(\varkappa - z_u\nu)}{(\omega - z_w\nu)^2}\frac{K}{\theta_0}$$

$$\Delta\left(\frac{K}{\mathscr{C}_m}\right) = \frac{\omega + z_w\chi - z_w^2}{(\omega - z_w\nu)^2}\frac{K}{\theta_0} .$$

7. *Numerical Examples.*—To obtain some idea of the magnitude of the errors involved in this approximate theory the response of four aeroplanes, calculated by the exact theory as described by Whatham and Priestley[4] (1946), has been compared with the response calculated by the method of this report. The examples have been taken from calculations performed for other reasons, so that calculations of the same components were not available in all the examples.

The aeroplane of the first example has a large static margin and a large manoeuvre margin. In the second example the manoeuvre margin is the same but the static margin is large and negative so that the motion we are considering is unstable. In the third and fourth examples both stability margins are small. The third and fourth examples are taken from Refs. 4 and 7.

The solutions for examples 1, 3 and 4 are all oscillatory. The oscillatory nature can be seen clearly in example 4 but in examples 1 and 3 only about one quarter of a cycle has been plotted.

The agreement is fairly good for the curves calculated of response to initial speed error, instantaneously applied pitching moment and initial angle of pitch. The larger errors occur in the curves of angle of pitch. In an attempt to improve the accuracy the equivalent initial angle of pitch as derived in section 4 was calculated and the response to this term included. The correction term obtained from this is small in all cases but produces a substantially better agreement in the values of θ and also in the other components which arise from the application of pitching moment. The values of $\hat{w}$ and $\hat{u}$ arising from an initial error in $\hat{u}$ do not seem to be improved by this correction term. In example 1 the correction term is negligible and in examples 2 and 3 the agreement between the approximate and the exact theory is worse when this extra term is included. No explanation has been found for this anomaly.

It has been mentioned in section 5 that the value given by this method for the damping of the oscillation may not be very accurate. The effect of an error in obtaining the damping is shown in example 4. The agreement over the early part of the oscillation is quite good but deteriorates as the time increases.

The curves for change of incidence (Figs. 2, 5, 8, 11, 14, 17a and 20b) show clearly the short-period motion which has been neglected. The values obtained by the present theory differ considerably from the exact values over the first second but after that the difference becomes quite small.

8. *Discussion and Conclusions.*—The method described seems to predict the long-period longitudinal motion with an accuracy which should be sufficient for most purposes.

When taken in conjunction with the approximate theory of the short-period motion it gives a simple representation of the response to elevator application. From the theory of the quick oscillation we see that an application of a positive pitching moment (up elevator) causes a rapid increase of incidence and a positive rate of pitch which are usually accompanied by considerable oscillation. During this stage of the motion the forward speed and the attitude change very little. This theory shows that in the subsequent motion (if it is stable) the speed falls slowly and the aircraft pitches nose-up while the incidence remains almost constant.

LIST OF SYMBOLS

a		Coefficient in the analytical formulae for response (Tables 1 and 2)
B		Approximation to the coefficient of the cubic term in the stability quartic (section 5)
b		Coefficient in the analytical formulae for response (Tables 1 and 2)
C		Approximation to the coefficient of the quadratic term in the stability quartic (section 5)
C_m		Pitching-moment coefficient due to disturbing moment
$\mathscr{C}_m$	=	$\mu C_m / i_B$
c		Coefficient in the analytical formulae for response (Tables 1 and 2)
D		Approximation to the coefficient of the linear term in the stability quartic (section 5)
E		Constant term in the stability quartic (section 5)
F_{uu} F_{um} $F_{u\theta}$ F_{wu} F_{wm} $F_{w\theta}$ F_{qu} F_{qm} $F_{q\theta}$ $F_{\theta u}$ $F_{\theta m}$ $F_{\theta\theta}$		Co-factors of the determinant Δ (section 6)
i_B		Inertia coefficient of the aircraft in pitch
k		$\frac{1}{2}C_L$
l		Reference length for stability derivatives
m_u		Dimensionless derivative of pitching moment with respect to forward velocity
m_w		Dimensionless derivative of pitching moment with respect to normal velocity
$m_{\dot{w}}$		Dimensionless derivative of pitching moment with respect to rate of change of normal velocity
m_q		Dimensionless derivative of pitching moment with respect to rate of pitch
N	=	$-x_u - z_w$
P	=	$x_u z_w - x_w z_u$
p		The variable of the Laplace transform
Q	=	$-x_u$
q		Rate of pitch (radn/sec)
$\hat{q}$		Dimensionless rate of pitch $\mu ql/V$
q_0		Initial value of $\hat{q}$
$\bar{q}$		The Laplace transform of $\hat{q}$
R	=	$-kz_u$
r		Real part of a complex root of the stability equation
S	=	$k - z_w$
s		Imaginary part of a complex root of the stability equation
T	=	$-kz_w$
u		Change in forward velocity of aircraft (ft/sec)

$\hat{u}$		Dimensionless change in forward velocity u/V
u_0		Initial value of $\hat{u}$
$\bar{u}$		Laplace transform of $\hat{u}$
V		Forward velocity of aircraft (ft/sec)
w		Normal velocity of aircraft
$\hat{w}$		Dimensionless normal velocity w/V
w_0		Initial value of $\hat{w}$
$\bar{w}$		Laplace transform of $\hat{w}$
x_u		Dimensionless derivative of forward force with respect to forward velocity
x_w		Dimensionless derivative of forward force with respect to normal velocity
Y	$=$	$\varkappa - \nu z_u$
Z	$=$	$\varkappa z_w - \omega z_u$
Δ		The characteristic equation of the approximate equations of motion (section 4)
θ		Angle of pitch of aircraft
θ_0		Initial value of θ
$\bar{\theta}$		Laplace transform of θ
θ_1		Equivalent initial value of θ (section 4)
$\varkappa$	$=$	$-\mu m_u/i_B$
μ		Relative density parameter of the aircraft
ν	$=$	$-m_q/i_B$
τ		Dimensionless measure of time
χ	$=$	$-\mu m_{\dot{w}}/i_B$
Ω	$=$	$\omega - z_w \nu$
ω	$=$	$-\mu m_w/i_B$

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APPENDIX

List of Formulae for Co-factors

$$F_{uu} = (\omega - z_w\nu)p \quad \text{(A.1)}$$
$$F_{um} = x_wp + z_wk \quad \text{(A.2)}$$
$$F_{u0} = -k(\omega - z_w\nu) \quad \text{(A.3)}$$

$$F_{wu} = -(\varkappa - \nu z_u)p \quad \text{(A.4)}$$
$$F_{wm} = p^2 - x_up - kz_u = \frac{1}{(\omega - z_w\nu)}\left\{\Delta - (\varkappa - z_u\nu)(x_wp + z_wk)\right\} \quad \text{(A.5)}$$
$$F_{w0} = k(\varkappa - z_u\nu) \quad \text{(A.6)}$$

$$F_{qu} = (\varkappa z_w - \omega z_u)p \quad \text{(A.7)}$$
$$F_{qm} = -z_wp^2 + (x_uz_w - x_wz_u)p \quad \text{(A.8)}$$
$$F_{q0} = -k(z_w\varkappa - z_u\omega) \quad \text{(A.9)}$$

$$F_{0u} = (\varkappa\, z_w - \omega z_u) \quad \text{(A.10)}$$
$$F_{0m} = -z_wp^e + (x_uz_w - x_wz_u) \quad \text{(A.11)}$$
$$F_{00} = (\omega - z_w\nu)p + \left\{-x_u(\omega - z_w\nu) + x_w(\varkappa - z_u\nu)\right\} = \frac{1}{p}\left\{\Delta - k(z_w\varkappa - z_u\omega)\right\}. \quad \text{(A.12)}$$

These expressions may be simplified by writing

$$\omega - z_w\nu = \Omega$$
$$\varkappa z_w - \omega z_u = Z$$
$$x_uz_w - x_wz_u = P$$
$$\varkappa - \nu z_u = Y.$$

Then

$$F_{uu} = \Omega p$$
$$F_{um} = x_wp + z_wk$$
$$F_{u0} = -k\Omega$$

$$F_{wu} = -Yp$$
$$F_{wm} = \mathrm{p}^2 - x_up - kz_u = \frac{1}{\Omega}\left\{\Delta - Yx_wp - Yz_wk\right\}$$
$$F_{w0} = kY$$

$$F_{qu} = Zp$$
$$F_{qm} = -z_wp^2 + Pp$$
$$F_{q0} = -kZ$$
$$F_{0u} = Z$$
$$F_{0m} = -z_wp + P$$
$$F_{00} = \Omega p + (x_wY - x_u\Omega) = \frac{1}{p}\{\Delta - kZ\}.$$

With the same notation

$$\Delta = \Omega p^2 + \{-x_u\Omega + x_wY\}p + kZ.$$

TABLE 1

Solutions of the Equations of Motion when the Stability Equation has Real Roots λ_1, λ_2

The solution is for each component of the form

$$a + b\,e^{\lambda_1 \tau} + c\,e^{\lambda_2 \tau}$$

where a, b and c are given in the following table.

Disturbance	Component	a	b	c
Initial speed error (u_0)	$\dfrac{\hat{u}}{u_0}$	—	$\dfrac{\lambda_1}{\lambda_1 - \lambda_2}$	$\dfrac{\lambda_2}{\lambda_2 - \lambda_1}$
,, ,, ,, ,,	$\dfrac{\hat{w}}{u_0}$	—	$-\dfrac{Y}{\Omega}\dfrac{\lambda_1}{\lambda_1 - \lambda_2}$	$-\dfrac{Y}{\Omega}\dfrac{\lambda_2}{\lambda_2 - \lambda_1}$
,, ,, ,, ,,	$\dfrac{\hat{q}}{u_0}$	—	$\dfrac{Z}{\Omega}\dfrac{\lambda_1}{\lambda_1 - \lambda_2}$	$\dfrac{Z}{\Omega}\dfrac{\lambda_2}{\lambda_2 - \lambda_1}$
,, ,, ,, ,,	$\dfrac{\theta}{u_0}$	—	$\dfrac{Z}{\Omega}\dfrac{1}{\lambda_1 - \lambda_2}$	$\dfrac{Z}{\Omega}\dfrac{1}{\lambda_2 - \lambda_1}$
Initial pitch error (θ_0)	$\dfrac{\hat{u}}{\theta_0}$	—	$-\dfrac{k}{\lambda_1 - \lambda_2}$	$-\dfrac{k}{\lambda_2 - \lambda_1}$
,, ,, ,, ,,	$\dfrac{\hat{w}}{\theta_0}$	—	$\dfrac{Y}{\Omega}\dfrac{k}{\lambda_1 - \lambda_2}$	$\dfrac{Y}{\Omega}\dfrac{k}{\lambda_2 - \lambda_1}$
,, ,, ,, ,,	$\dfrac{\hat{q}}{\theta_0}$	—	$-\dfrac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$	$-\dfrac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$
,, ,, ,, ,,	$\dfrac{\theta}{\theta_0}$	—	$-\dfrac{\lambda_2}{\lambda_1 - \lambda_2}$	$-\dfrac{\lambda_1}{\lambda_2 - \lambda_1}$
Instantaneous pitching moment ($\mathscr{C}_m$)	$\dfrac{\hat{u}}{\mathscr{C}_m}$	$\dfrac{z_w}{Z}$	$\dfrac{x_w\lambda_1 + kz_w}{\Omega\lambda_1(\lambda_1 - \lambda_2)}$	$\dfrac{x_w\lambda_2 + kz_w}{\Omega\lambda_2(\lambda_2 - \lambda_1)}$
,, ,, ,, ,,	$\dfrac{\hat{w}}{\mathscr{C}_m}$	$\dfrac{1}{\Omega} - \dfrac{Yz_w}{\Omega Z}$	$-\dfrac{Y}{\Omega^2}\dfrac{x_w\lambda_1 + kz_w}{\lambda_1(\lambda_1 - \lambda_2)}$	$-\dfrac{Y}{\Omega^2}\dfrac{x_w\lambda_2 + kz_w}{\lambda_2(\lambda_2 - \lambda_1)}$
,, ,, ,, ,,	$\dfrac{\hat{q}}{\mathscr{C}_m}$	—	$\dfrac{Z(x_w\lambda_1 + kz_w)}{\Omega^2\lambda_1(\lambda_1 - \lambda_2)}$	$\dfrac{Z(x_w\lambda_2 + kz_w)}{\Omega^2\lambda_2(\lambda_2 - \lambda_1)}$
,, ,, ,, ,,	$\dfrac{\theta}{\mathscr{C}_m}$	$\dfrac{P}{kZ}$	$\dfrac{-z_w\lambda_1 + P}{\Omega\lambda_1(\lambda_1 - \lambda_2)}$	$\dfrac{-z_w\lambda_2 + P}{\Omega\lambda_2(\lambda_2 - \lambda_1)}$

Note: Although the complete expressions for $\hat{w}$ and $\hat{q}$ are given it may be more convenient to calculate them from the formulae

$$\hat{w} = -\frac{Y}{\Omega}\hat{u} + \frac{1}{\Omega}\mathscr{C}_m$$

$$\hat{q} = \frac{Z}{\Omega}\hat{u} - \frac{z_w}{\Omega}\mathscr{C}_m$$

TABLE 2

Solutions of the Equations of Motion when the Stability Equation has Complex Roots $(r \pm is)$

The solution is for each component of the form

$$a + b\,e^{r\tau}\cos s\tau + c\,e^{r\tau}\sin s\tau$$

where a, b and c are given in the following table.

Disturbance	Component	a	b	c
Initial speed error (u_0)	$\frac{\hat{u}}{u_0}$	—	1	$\frac{r}{s}$
,, ,, ,, ,,	$\frac{\hat{w}}{u_0}$	—	$-\frac{Y}{\Omega}$	$-\frac{Yr}{\Omega s}$
,, ,, ,, ,,	$\frac{\hat{q}}{u_0}$	—	$\frac{Z}{\Omega}$	$\frac{Zr}{\Omega s}$
,, ,, ,, ,,	$\frac{\theta}{u_0}$	—	—	$\frac{Z}{\Omega s}$
Initial pitch error (θ_0)	$\frac{\hat{u}}{\theta_0}$	—	—	$-\frac{k}{s}$
,, ,, ,, ,,	$\frac{\hat{w}}{\theta_0}$	—	—	$\frac{Yk}{\Omega s}$
,, ,, ,, ,,	$\frac{\hat{q}}{\theta_0}$	—	—	$-\frac{Zk}{\Omega s}$
,, ,, ,, ,,	$\frac{\theta}{\theta_0}$	—	1	$-\frac{r}{s}$
Instantaneously applied pitching moment $(\mathscr{C}_m)$	$\frac{\hat{u}}{\mathscr{C}_m}$	$\frac{z_w}{Z}$	$-\frac{z_w}{Z}$	$\frac{x_w(r^2+s^2)+kz_wr}{kZs}$
,, ,, ,, ,, ,,	$\frac{\hat{w}}{\mathscr{C}_m}$	$\frac{1}{\Omega}-\frac{Yz_w}{\Omega Z}$	$\frac{Yz_w}{\Omega Z}$	$-\frac{Y}{\Omega}\,\frac{x_w(r^2+s^2)+kz_wr}{kZs}$
,, ,, ,, ,, ,,	$\frac{\hat{q}}{\mathscr{C}_m}$	—	$-\frac{z_w}{\Omega}$	$\frac{x_w(r^2+s^2)+kz_wr}{k\Omega s}$
,, ,, ,, ,, ,,	$\frac{\theta}{\mathscr{C}_m}$	$\frac{P}{kZ}$	$-\frac{P}{kZ}$	$\frac{-z_w(r^2+s^2)+Pr}{kZs}$

Note: Although the complete expressions for $\hat{w}$ and $\hat{q}$ are given it may be more convenient to calculate them from the formulae

$$\hat{w} = -\frac{Y}{\Omega}\hat{u} + \frac{1}{\Omega}\mathscr{C}_m$$

$$\hat{q} = \frac{Z}{\Omega}\hat{u} - \frac{z_w}{\Omega}\mathscr{C}_m$$

TABLE 3

Values used in Numerical Examples

	Example 1	Example 2	Example 3	Example 4
C_L	$0 \cdot 3$	$0 \cdot 3$	$0 \cdot 5$	$1 \cdot 0$
x_u	$-0 \cdot 015$	$-0 \cdot 015$	$-0 \cdot 0325$	$-0 \cdot 09$
z_u	$-0 \cdot 24$	$-0 \cdot 24$	$-0 \cdot 5$	$-1 \cdot 0$
x_w	$0 \cdot 065$	$0 \cdot 065$	$0 \cdot 15$	$0 \cdot 23$
z_w	$-2 \cdot 2$	$-2 \cdot 2$	$-2 \cdot 016$	$-2 \cdot 25$
$\varkappa$	0	$28 \cdot 5$	0	0
ω	138	138	1	10
χ	$1 \cdot 0$	$1 \cdot 0$	$1 \cdot 2$	$1 \cdot 0$
ν	$3 \cdot 68$	$3 \cdot 68$	3	3

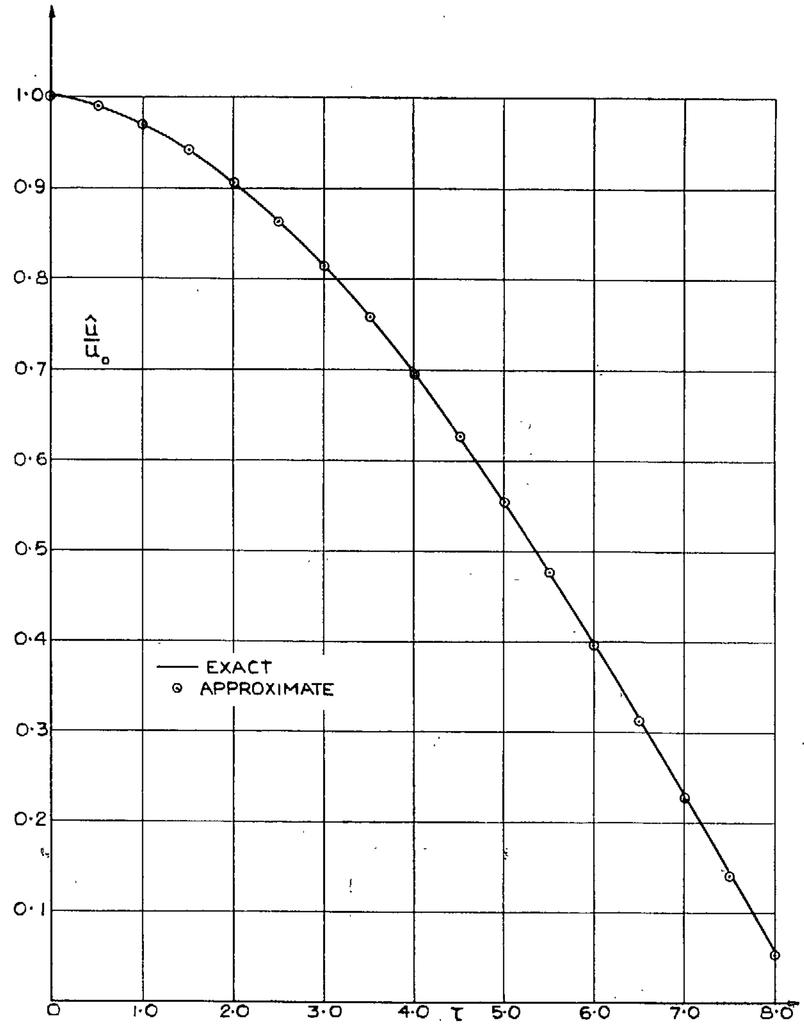

FIG. 1. Response to initial speed error.
Change of speed. Example 1.

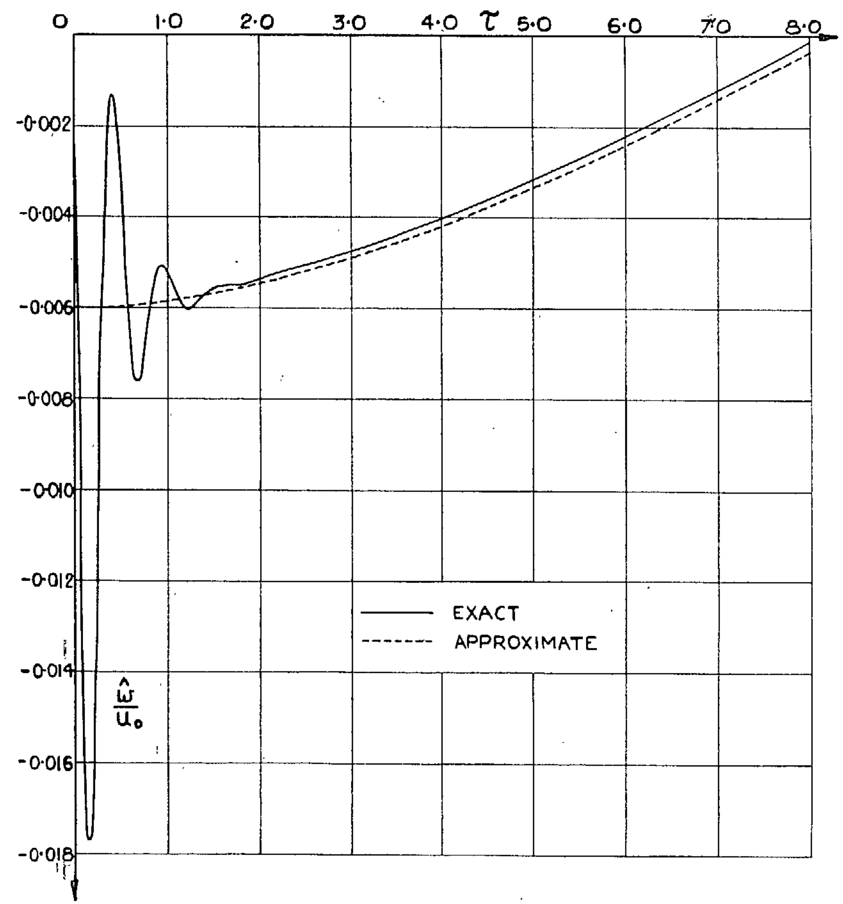

FIG. 2. Response to initial speed error.
Change of incidence. Example 1.

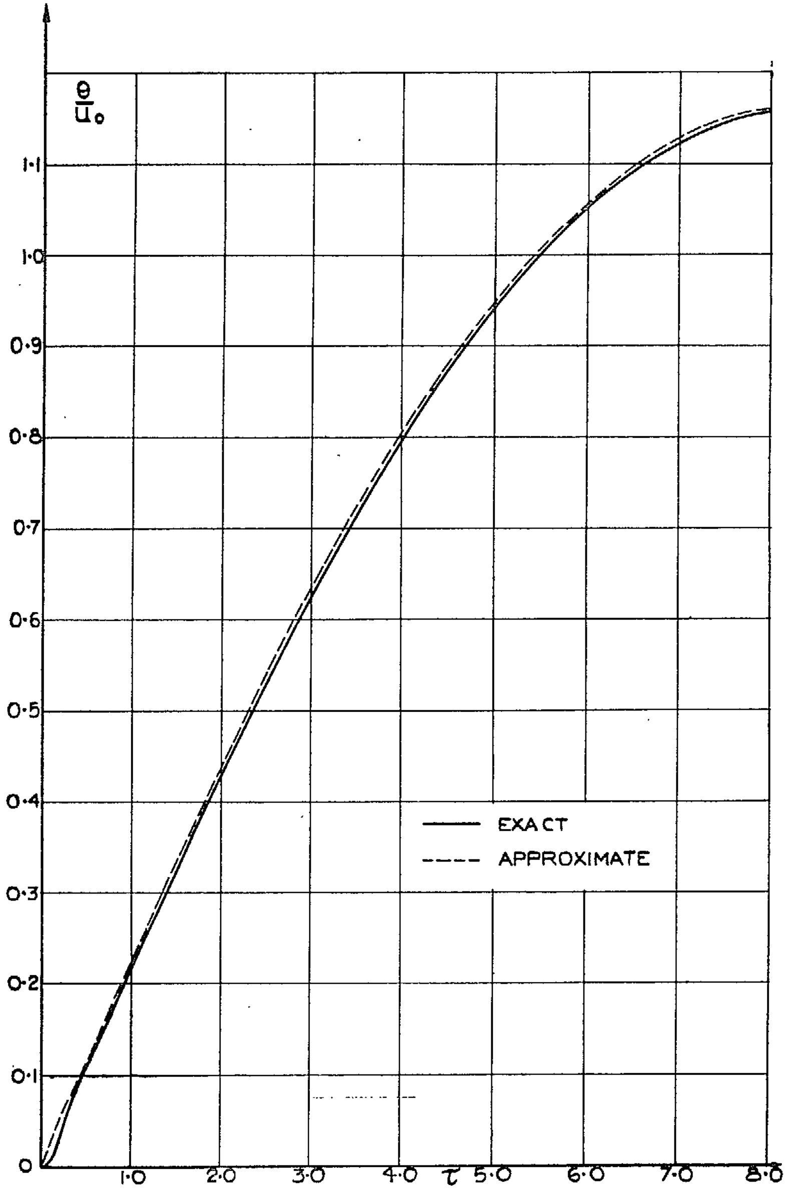

FIG. 3. Response to initial speed error. Angle of pitch. Example 1.

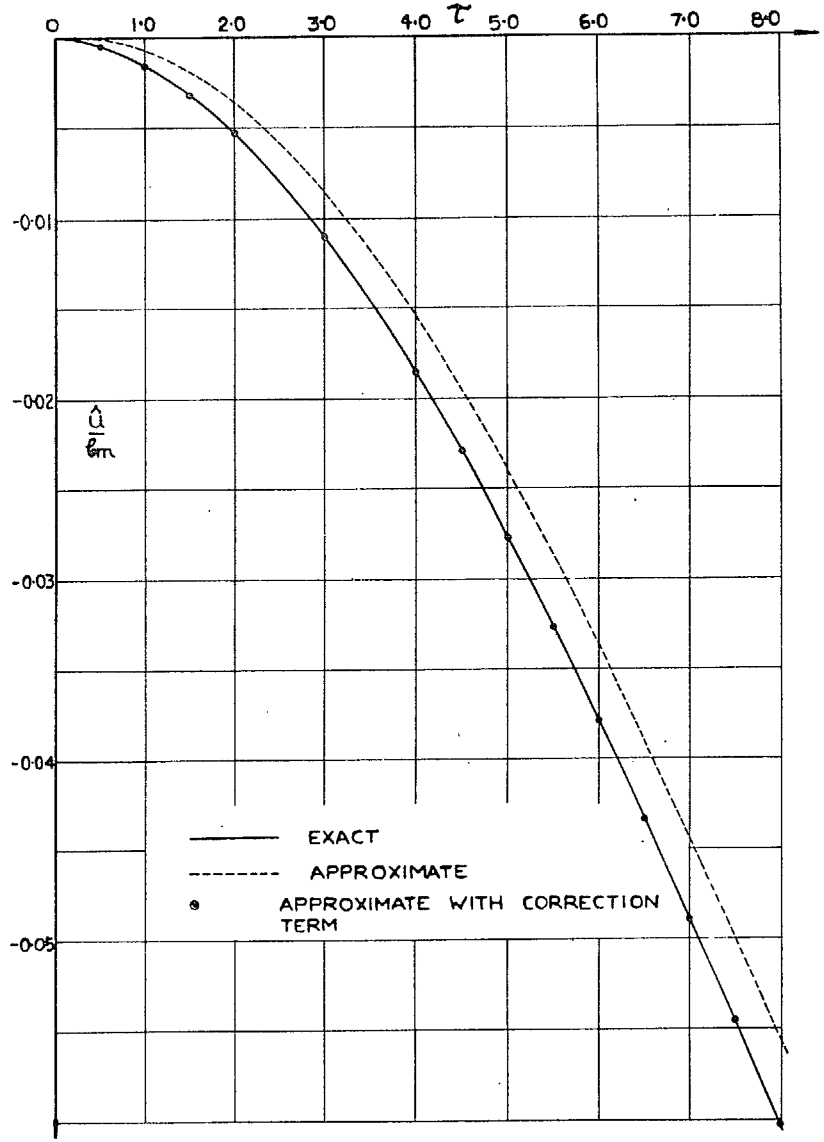

FIG. 4. Response to instantaneously applied pitching moment. Change of speed. Example 1.

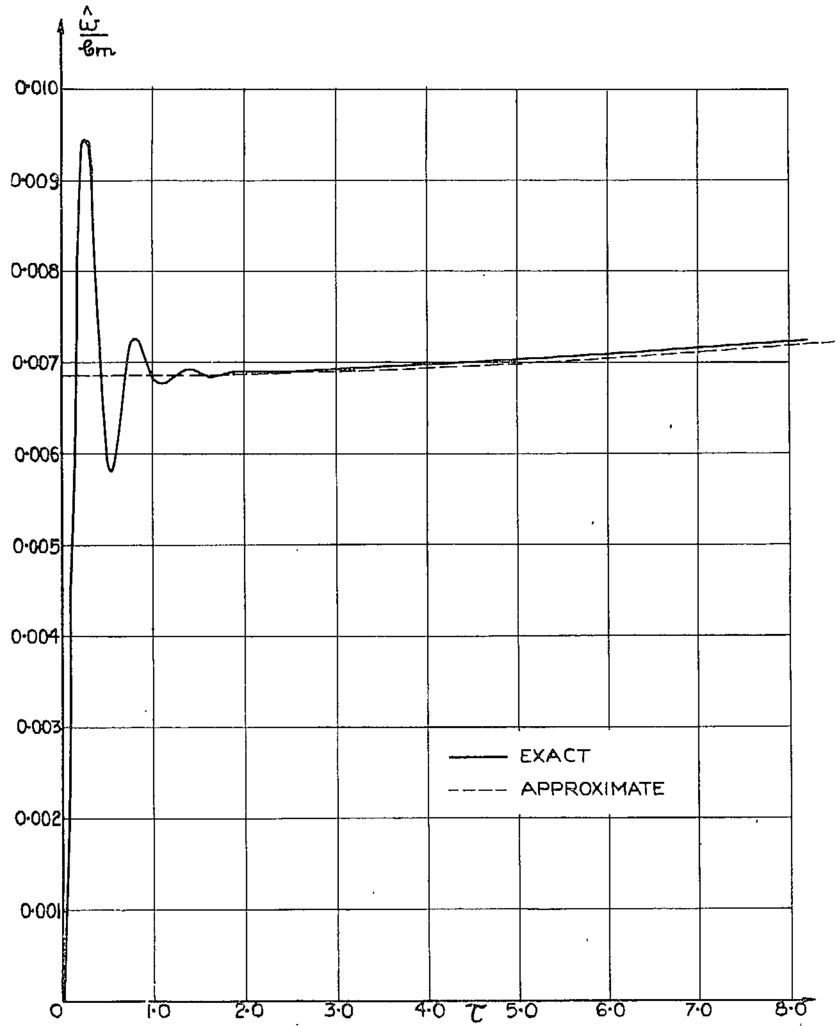

FIG. 5. Response to instantaneously applied pitching moment. Change of incidence. Example 1.

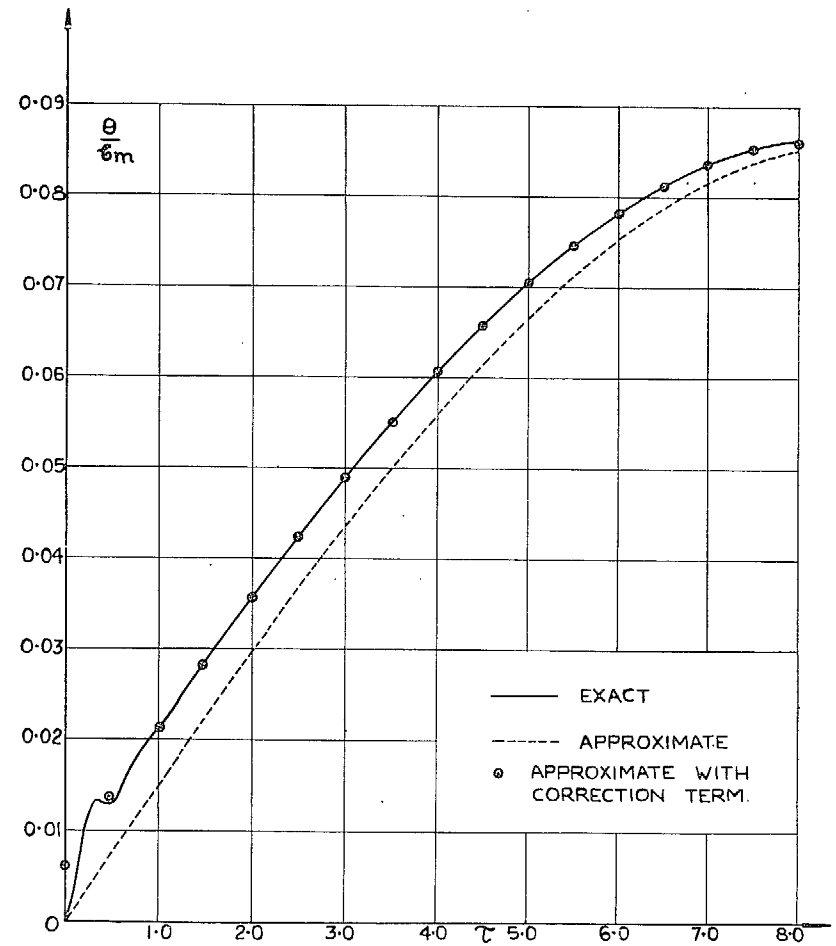

FIG. 6. Response to instantaneously applied pitching moment. Angle of pitch. Example 1.

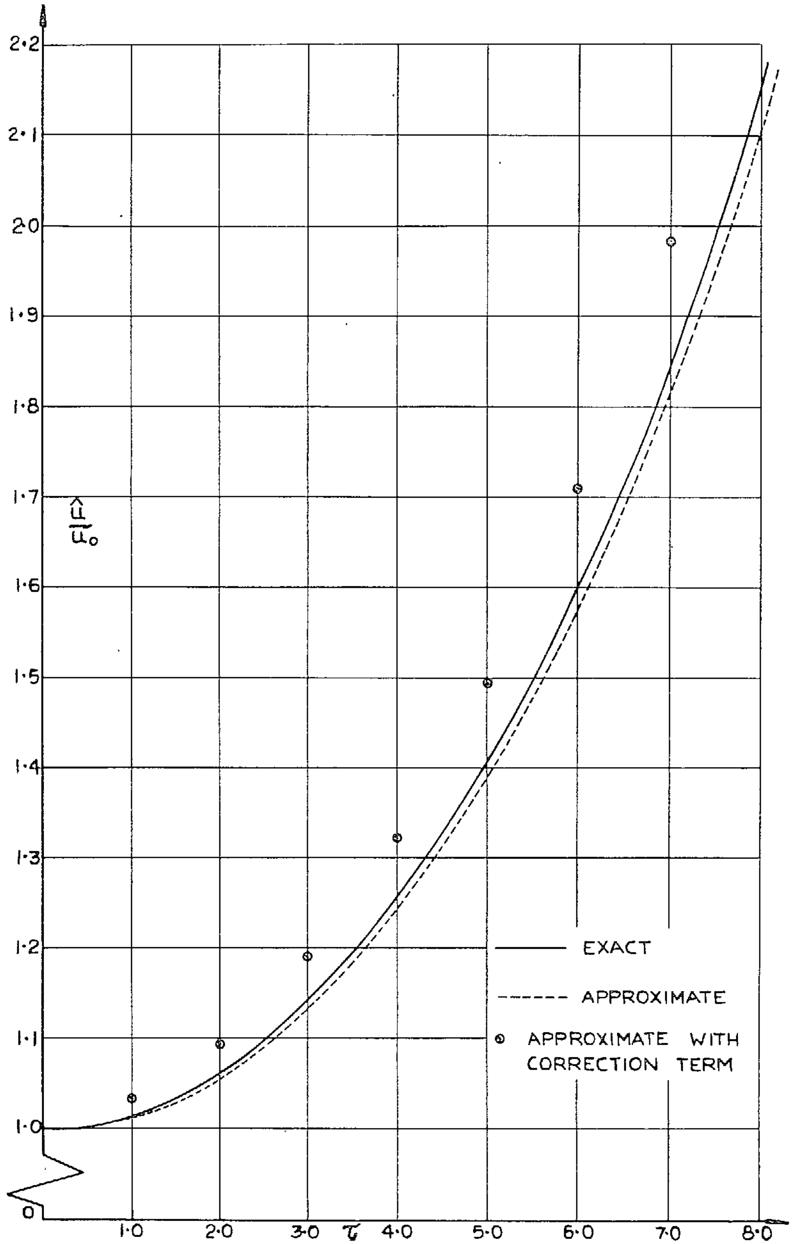

FIG. 7. Response to initial speed error. Change of speed. Example 2.

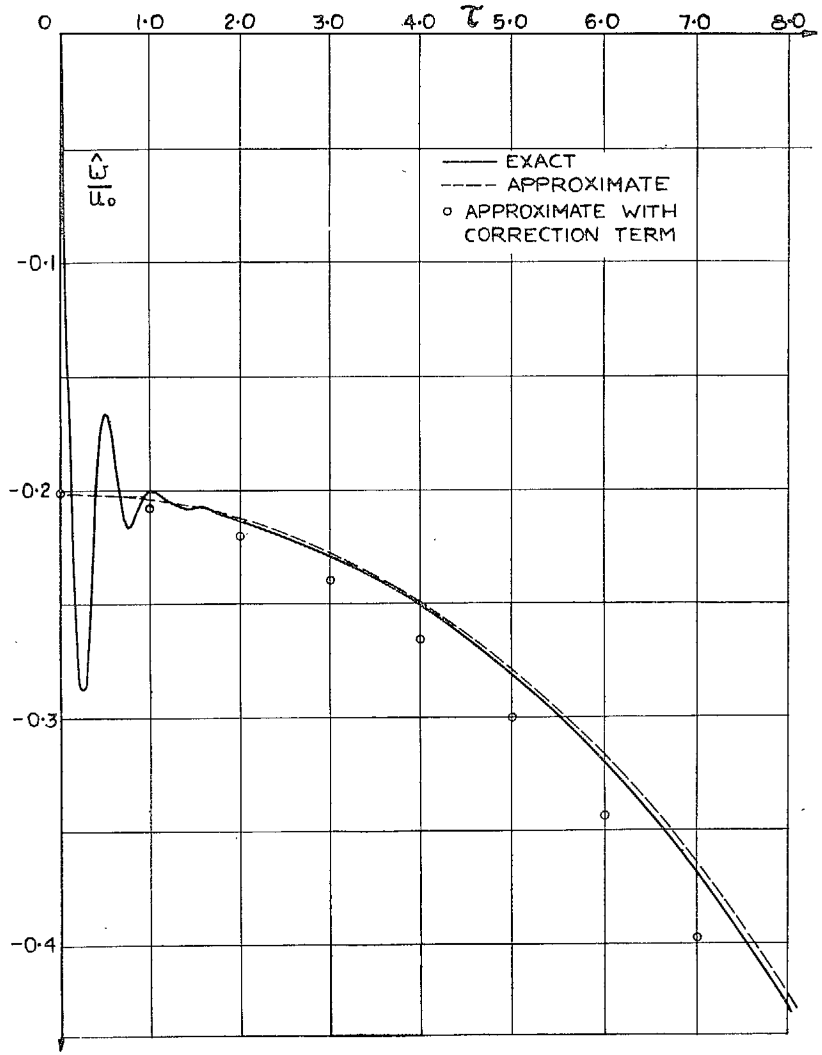

FIG. 8. Response to initial speed error. Change of incidence. Example 2.

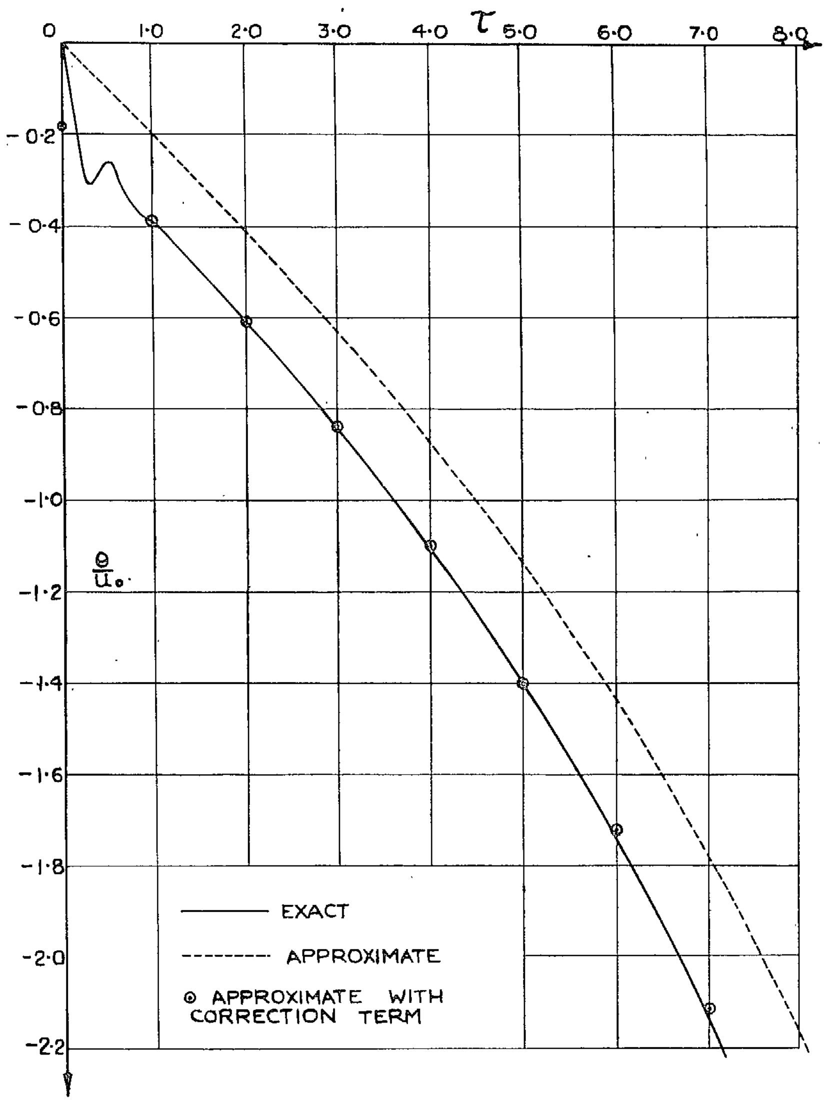

FIG. 9. Response to initial speed error. Angle of pitch. Example 2.

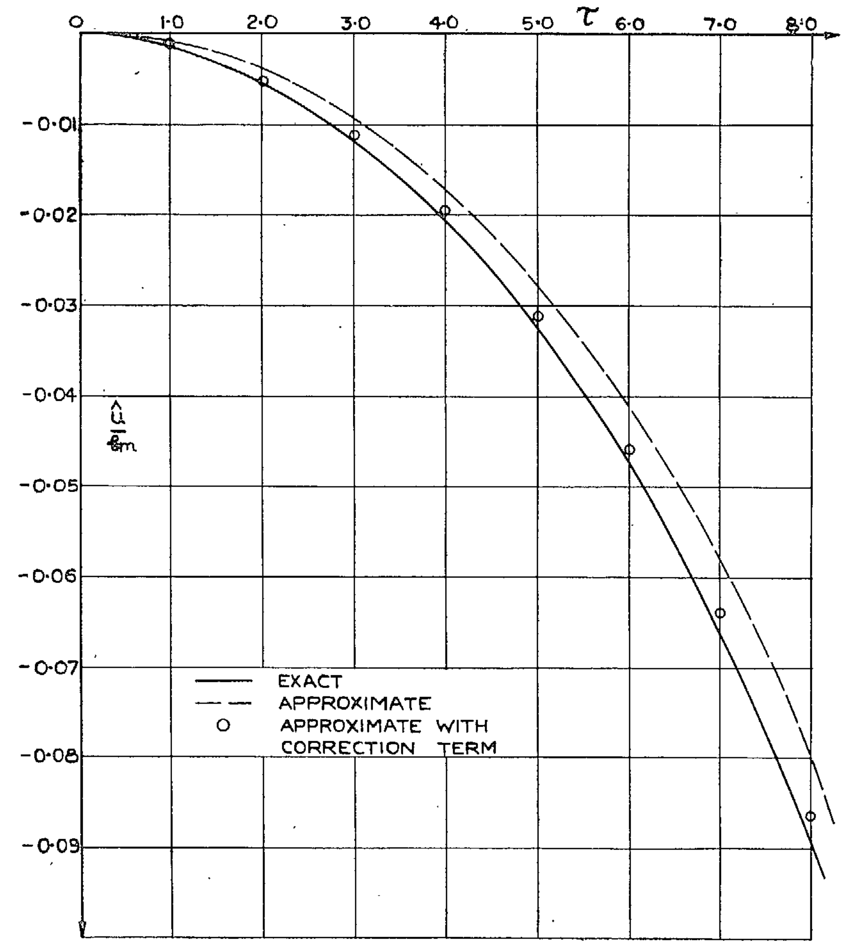

FIG. 10. Response to instantaneously applied pitching moment. Change of speed. Example 2.

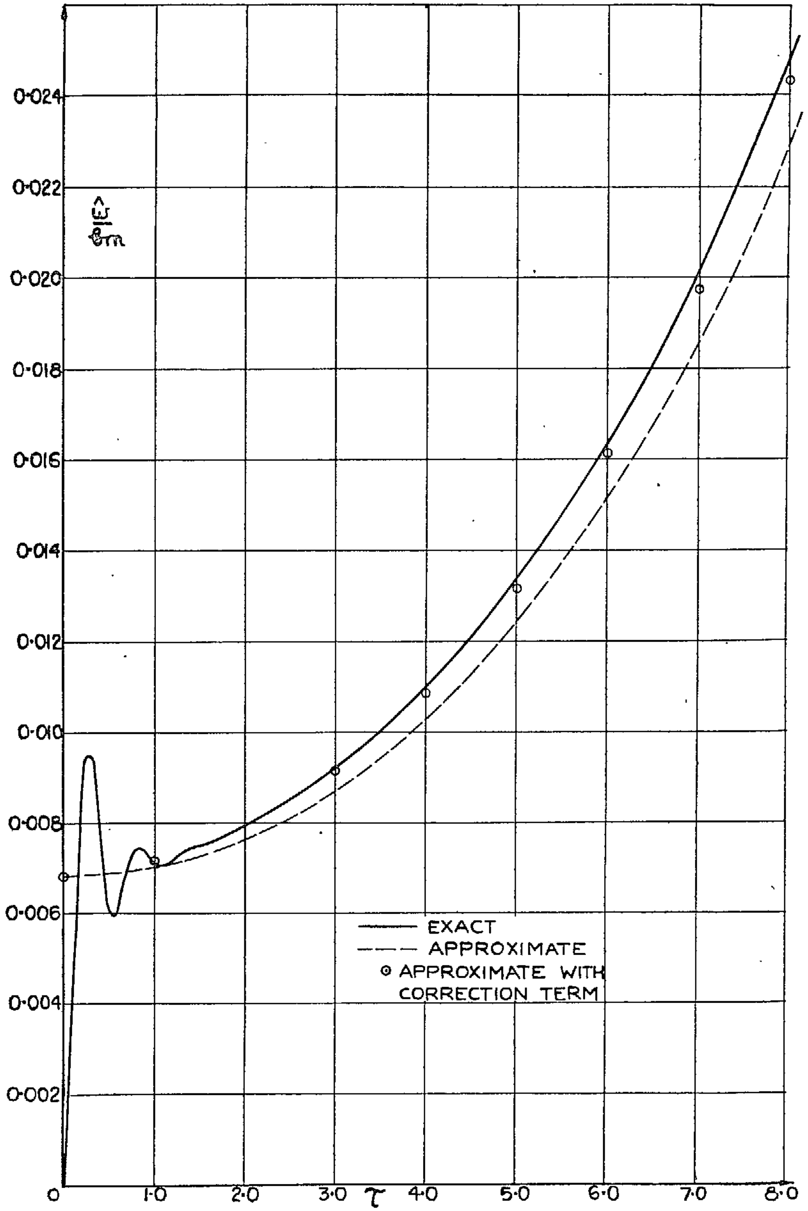

FIG. 11. Response to instantaneously applied pitching moment. Change of incidence. Example 2.

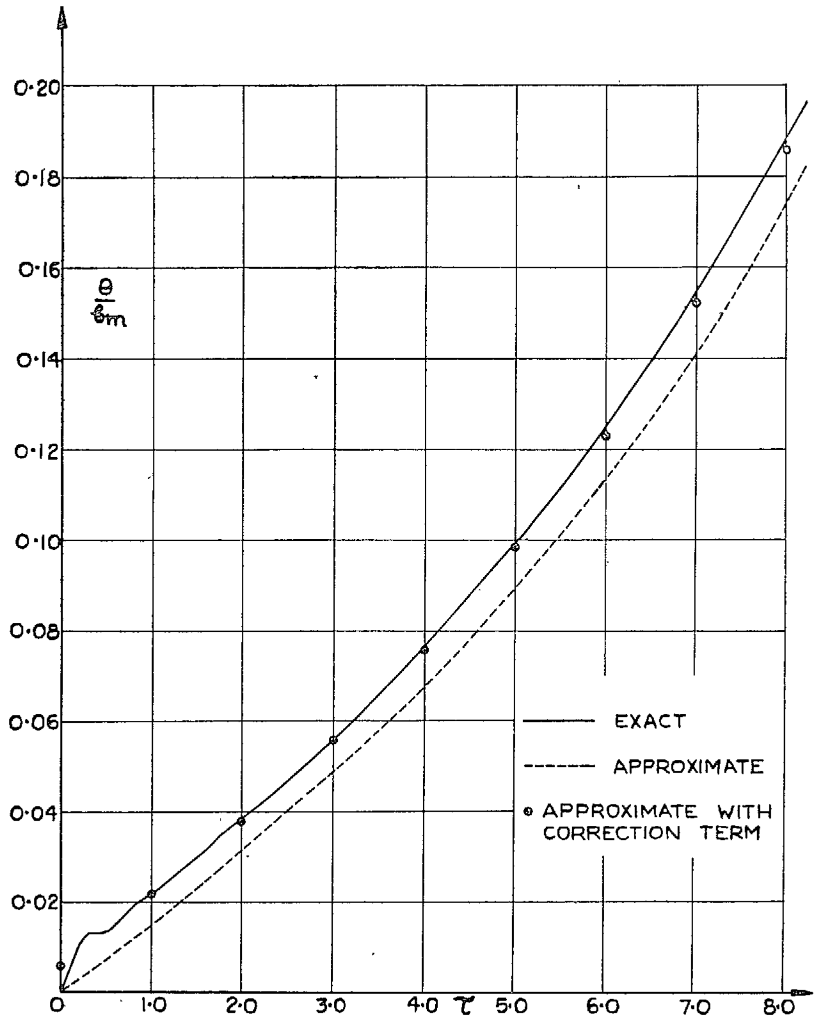

FIG. 12. Response to instantaneously applied pitching moment. Angle of pitch. Example 2.

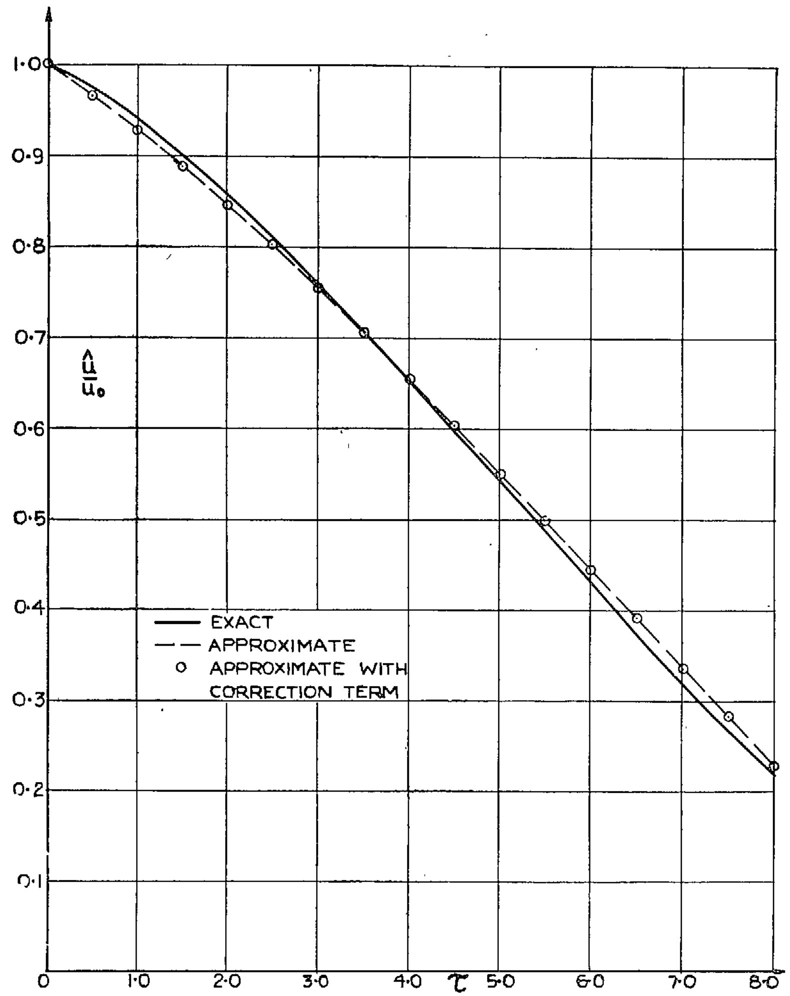

FIG. 13. Response to initial speed error.
Change of speed. Example 3.

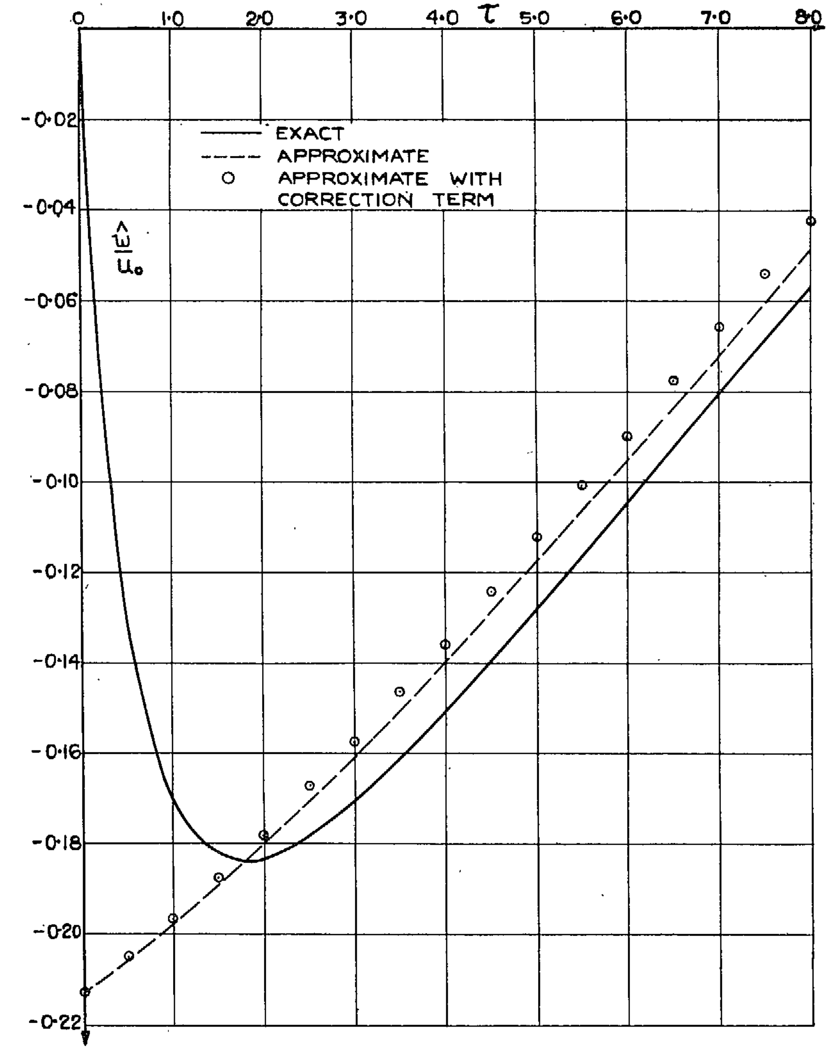

FIG. 14. Response to initial speed error.
Change of incidence. Example 3.

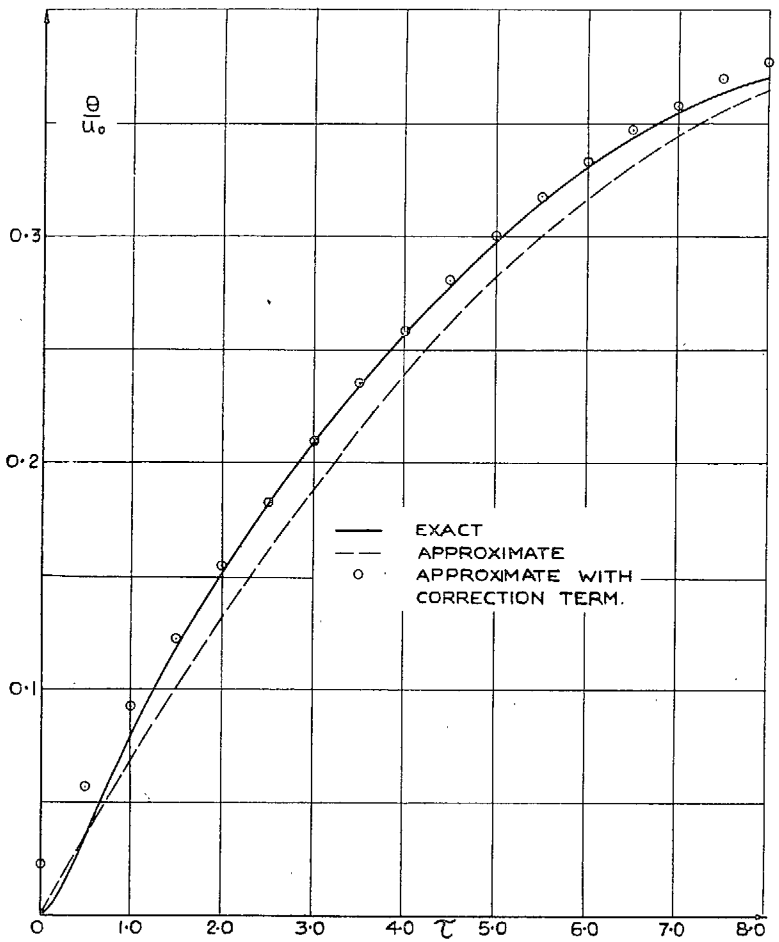

FIG. 15. Response to initial speed error.
Angle of pitch. Example 3.

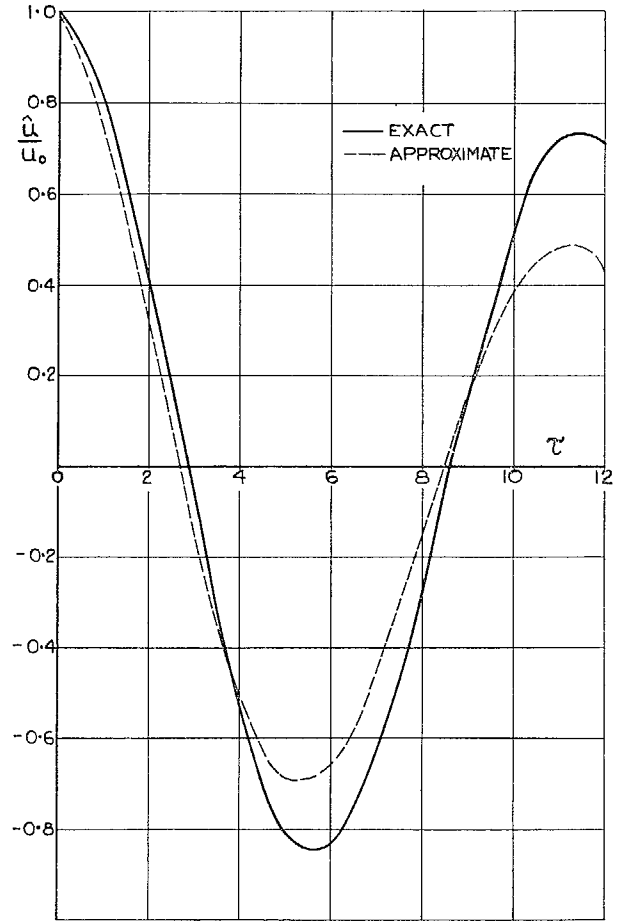

FIG. 16. Response to initial speed error.
Change of speed. Example 4.

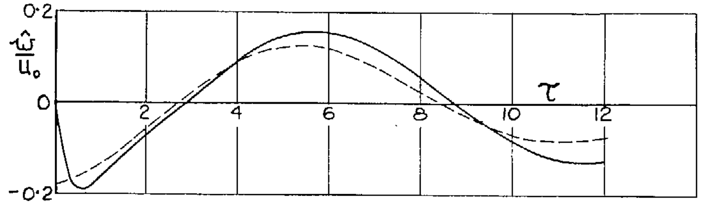

FIG. 17a. Change of incidence.

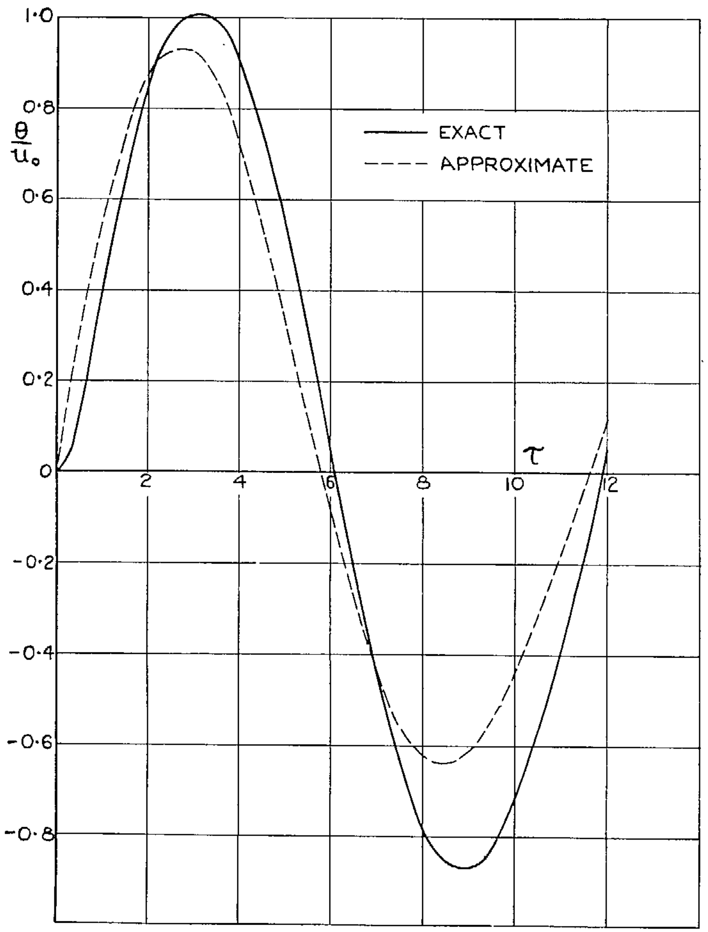

FIG. 17b. Angle of pitch.

FIGS. 17a and 17b. Response to initial speed error. Example 4.

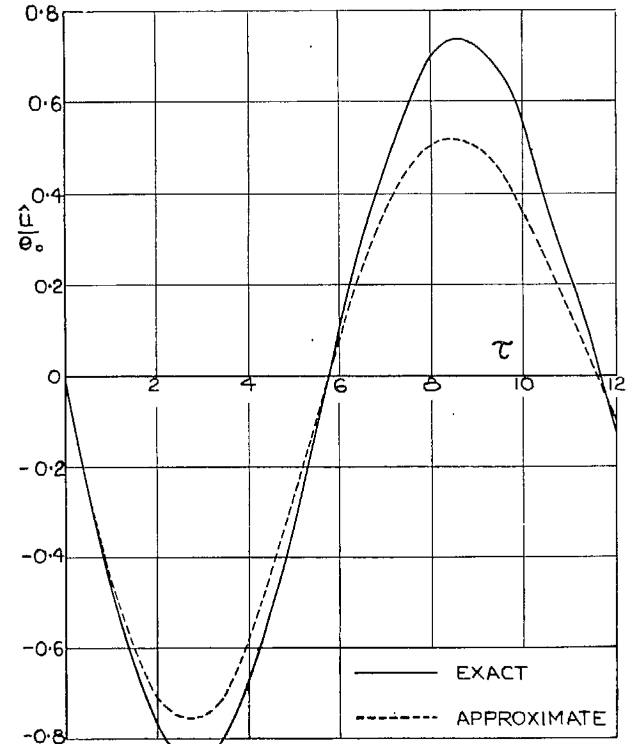

FIG. 18a. Change of speed.

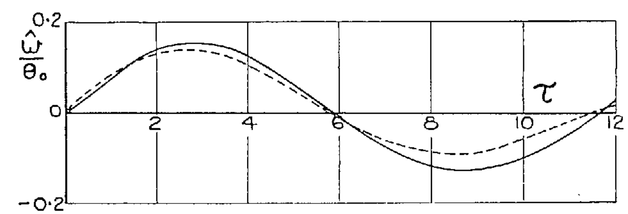

FIG. 18b. Change of incidence.

FIGS. 18a and 18b. Response to initial angle of pitch. Example 4.

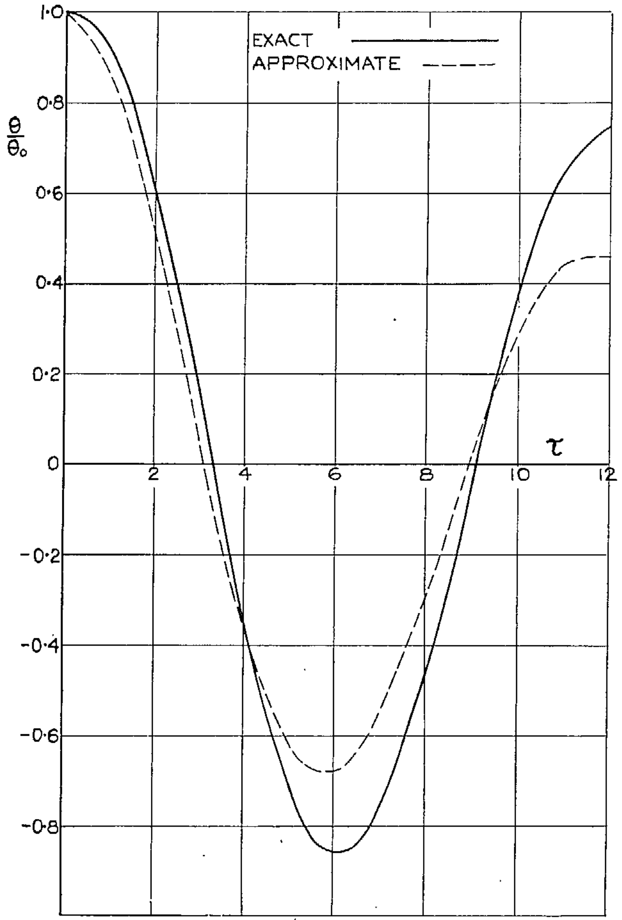

FIG. 19. Response to initial angle of pitch. Angle of pitch. Example 4.

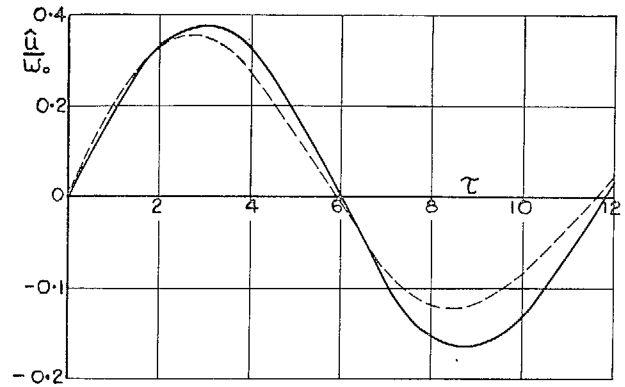

FIG. 20a. Change of speed.

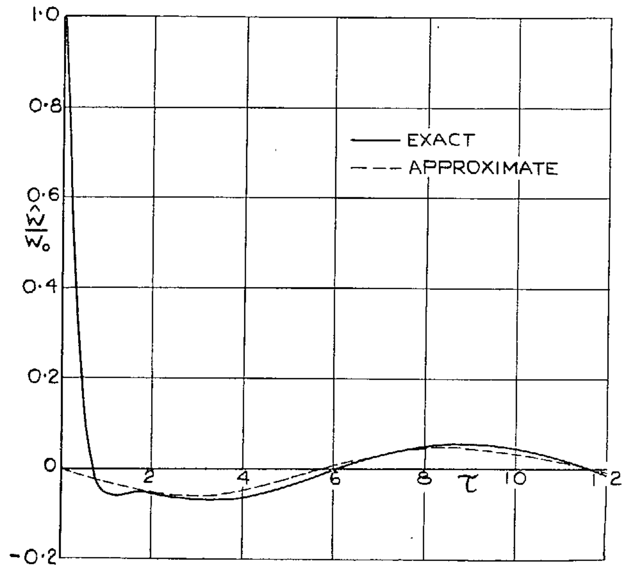

FIG. 20b. Change of incidence.

FIGS. 20a and 20b. Response to initial incidence error. Example 4.

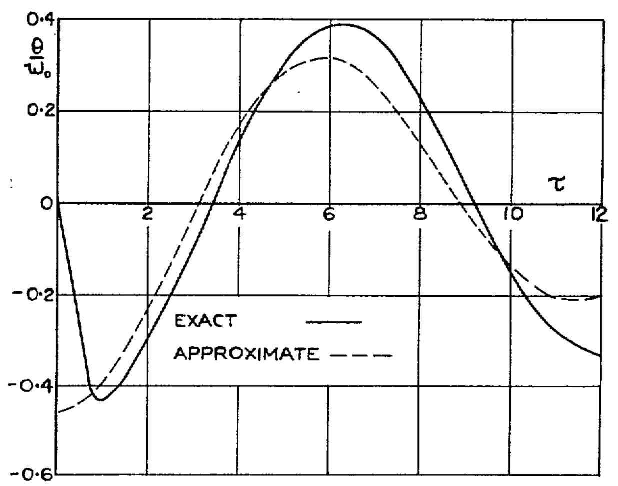

FIG. 21. Response to initial incidence error. Angle of pitch. Example 4.

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