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# The Determination of the Pressure Distribution over an Aerofoil Surface by means of an Electrical Potential Analyser 

By<br>Professor S. C. Redshaw, D.Sc., Ph.D., M.I.C.E., F.R.Ae.S.<br>Reports and Memoranda No. 2915*<br>December, $195^{2}$



Summary.-The potential flow, both with and without circulation, around several thin aeroplane wings has been studied by means of a three-dimensional potential analyser. It is shown that, by using the normal assumptions made in the exercise of the linear perturbation theory, it is possible to obtain the pressure distribution for small angles of incidence, as well as the slope of the lift-incidence curve, easily and rapidly.

Experiments are also described in which it was attempted to remove the effect of boundary restraint in a manner analogous to that used in a flexible-walled wind tunnel.
Suggestions are made for producing a potential analyser of increased scope together with the possibility of extending the work to curved and twisted thin wings.

1. Introduction.-An approximation to the pressure distribution over an aerofoil surface of high aspect ratio, moving at a low subsonic uniform velocity, has been satisfactorily computed by the use of lifting-line theory. After the pressure distribution has been determined the lift, pitching moment and other aerodynamic properties may be easily calculated.

The problem of the determination of the pressure distribution over low aspect ratio swept and unswept plan forms, presents considerable difficulty when using analytical methods based on lifting-surface theory; this report describes the use of a three-dimensional electrical potential analyser for the solution of problems of this nature.

The method consists, basically, of using an electrical analogy whereby the electrical potential, in an appropriately set up pure resistance network, represents the velocity potential of the disturbed flow in the vicinity of the aerofoil surface. Pressure can be calculated from the velocity potentials ; the calculation of other aerodynamic properties follows in the normal manner. It must be emphasised that this method only provides a potential flow solution, although with circulation, for thin plates at small angles of attack.
Two electrical analogies for the pressure distribution on a lifting surface have been discussed by Campbell ${ }^{1}$; the first analogy he mentions is that used in this report, in his proposed second analogy electrical potentials represent the acceleration potentials of the fluid flow. Campbell's proposal is to apply the analogy using a deep electrolytic tank. The acceleration potential analogy is useful for cases in which it is desired to calculate quantities such as control hinge moments.

Malavard and Duquenne ${ }^{2}$, using the velocity potential analogy, have recently studied lifting surfaces by means of an electrolytic tank. They considered that the velocity potential analogy is preferable to the acceleration potential analogy, advocated by Campbell, except for very special cases.

[^0]The electrolytic tank possesses an advantage over an electrical network because it provides a direct analogy to a continuous solution, whereas a network will only provide an analogy to the finite difference form of the appropriate differential equation, it only being possible to take readings at the discrete points provided by the nodal points of the net. Nevertheless, the electrolytic tank presents considerable experimental difficulty; for that reason an electrical network in the form of a three-dimensional potential analyser, which is extremely simple to operate, was chosen for the present investigation.

This report incorporates the work of two previous unpublished reports on the same subject ${ }^{3,4}$.
2. List of Symbols.
$x, y, z \quad$ Cartesian co-ordinates referred to right-hand orthogonal axes. The origin $O$ is taken at the centre of the bottom edge $O x$ of the master tier of the analyser ; the faces $y=0$ and $z=0$ are used as reflecting surfaces, while $y=\hat{y}$ and $x= \pm \hat{x}$ are referred to as walls and $z=\hat{z}$ as the end of the analyser ; $z=0$ is referred to as the master tier
$X, Y, Z \quad$ Cartesian co-ordinates obtained by an affine transformation from co-ordinates $x, y, z$
$x, \omega \quad$ Two orthogonal co-ordinates such that $x$ represents the distance of a point from the plane of a disc and $\omega$ the radial distance of that point from the axis of the disc.
$r, \theta \quad$ Polar co-ordinates
c Radius
$U, V, W \quad$ Velocities along the $x, y$ and $z$-axes respectively
$u, v, w \quad$ Perturbation velocities along the $x, y$ and $z$-axes respectively
$\Phi \quad$ Velocity potential
$\phi \quad$ Perturbation potential, also electrical potential
$\psi \quad$ Stream function
$M \quad$ Mach number
$p \quad$ Fluid pressure
$p_{0} \quad$ Fluid pressure in undisturbed stream
$\Delta p \quad$ Excess pressure defined by $\Delta p=p-p_{0}$
$\rho \quad$ Fluid density, also specific resistivity
$q \quad$ Dynamic pressure
$C_{L} \quad$ Lift coefficient
$\alpha \quad$ Angle of incidence
A Wing area
$I \quad$ Total current from source
$i \quad$ Current density
$L \quad$ Distance between equal source and sink
$D \quad$ Strength of doublet, i.e., limit of $I L$ as $I \rightarrow \infty$ and $L \rightarrow 0$ in such a manner that their product remains finite
$n$ Unit normal vector
ds Element of area
$R \quad$ Electrical resistance
$Q \quad$ A non-dimensional quantity defined in the text.
3. Aerodynamic Theory.-3.1. Two-dimensional Fields of Flow.-The assumptions made with regard to the two-dimensional potential flow of an incompressible fluid are that it possesses no vorticity and no viscosity. As a consequence of the absence of vorticity, the flow may be described by means of a velocity potential function $\phi$ such that the velocity is equal to the gradient of $\phi$. Due to the condition of continuity there also exists a stream function $\psi$ whose gradient gives the mass flow per unit length at right-angles to the flow. Provided the motion is irrotational, a velocity potential exists whether the fluid is compressible or incompressible. The stream function exists only when compressibility is negligible, irrespective of whether the motion is rotational or irrotational.

Two electrical analogies, devised by Taylor and Sharman ${ }^{5}$, exist for the solution of problems concerning two-dimensional fields of flow.
3.2. Three-dimensional Fields of Flow.-For three-dimensional fields of flow the velocity potential $\phi$ still satisfies Laplace's equation and is analogous to the electrical potential for the steady flow of electricity through a block of conducting material. The stream function $\psi$ at a given point is no longer defined except in the special case of axisymmetrical flow. The reason for this being that a point specifies a certain streamline ; in the two-dimensional case this line is sufficient to divide the flow into regions, whereas in the three-dimensional case a surface is required. For axially symmetrical flow the surface formed by the revolution of the streamline about the axis of symmetry can be used ; as might be expected, even here $\psi$ is no longer a solution of Laplace's equation and although streamlines are still normal to equipotential surfaces there is no form of analogy corresponding to the second analogy of Taylor and Sharman.
3.3. Linear Perturbation Theory.-With certain limitations the linear perturbation theory considerably simplifies the analysis of certain cases of potential flow.

If we place an aerofoil in a uniform stream moving with a velocity $U$ parallel to the $x$-axis, say, the motion will be perturbed and the velocity potential will become

$$
\begin{equation*}
\Phi=U x+\phi(x, y, z) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{1}
\end{equation*}
$$

so that $\phi$ is the perturbation potential.
The velocity at any point which was formerly $(-U, 0,0)$, now becomes $(-U+u, v, w)$. The perturbation is said to be linear if $u / U, v / U$ and $w / U$ are small quantities of the first order whose squares and products are negligible. No limitation is imposed on $U$ which may be either large or small. The approximation fails near a stagnation point where $-U+u=0$ so that $u / U=1$.

Therefore the assumptions made in the exercise of the linearised theory are :-
(a) The aerofoil is represented by an infinitely thin plate
(b) The camber of the aerofoil must be small
(c) Only small angles of incidence may be considered
(d) The vortex lines at the rear of the aerofoil surface remain in the same plane as the aerofoil surface and run immediately aft.

An aerofoil defined by these limiting assumptions is suitable for high speeds and therefore in this case the linear theory will give satisfactory results.

The Prandtl-Glauert equation, satisfied by the perturbation potential is

$$
\begin{equation*}
\left(1-M^{2}\right) \frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}=0 \quad \text {.. .. .. .. .. .. } \tag{2}
\end{equation*}
$$

where $M$ is the Mach number of the undisturbed flow.
Glauert ${ }^{6}$ and Prandt $1^{7}$ have shown that, at subsonic speeds, a distribution of potential satisfying Laplace's equation will also satisfy the linearised compressible-flow equation (equation (2)) if the distribution $\phi(x, y, z)$ is foreshortened along the direction of motion by the affine transformation

$$
\begin{equation*}
X=\frac{x}{\sqrt{ }\left(1-M^{2}\right)}, \bar{Y}=y, Z=z . \quad . \quad . \quad . \quad . \quad . \quad . \tag{3}
\end{equation*}
$$

Using (3) equation (2) transforms to

$$
\begin{equation*}
\frac{\partial^{2} \phi}{\partial X^{2}}+\frac{\partial^{2} \phi}{\partial Y^{2}}+\frac{\partial^{2} \phi}{\partial Z^{2}}=0 \quad \text {. } \quad . \quad \text {.. .. .. .. .. } \tag{4}
\end{equation*}
$$

which"is Laplace's equation.
Thus, if we start with a fictitious aerofoil longer in the $x$-direction than the true one and calculate the potential distribution for this aerofoil by methods applicable to incompressible flow, the correct dimensions and correct distribution of $\phi$ are obtained when the affine transformation is applied ; the transformation is not conformal. As will be seen later, the PrandtlGlauert method can be applied directly with great advantage to the experimental results obtained from the potential analyser.

As the linearised theory only permits small angles of incidence to be considered and, for small angles of yaw, we may ignore the free-stream component velocities $V$ and $W$ in the $y$ and $z$-axes respectively in comparison with the velocity $U$ along the $x$-axis. Then if $p$ is the pressure at any point and $p_{0}$ is the pressure in the unperturbed stream we have, from Bernoulli's theorem

$$
\begin{equation*}
p+\frac{1}{2} \rho\left[(-U+u)^{2}+v^{2}+w^{2}\right]=p_{0}+\frac{1}{2} \rho U^{2} \quad . . \quad . . \quad . . \quad . \tag{5}
\end{equation*}
$$

and on the assumption that $u, v$ and $w$ are so small that their squares and products may be neglected, and defining the excess pressure as

$$
\Delta p=p-p_{0}
$$

and

$$
\begin{equation*}
q=\frac{1}{2} \rho U^{2} \tag{6}
\end{equation*}
$$

we have from (5)

$$
\begin{equation*}
\Delta p=\frac{2 u}{U} q . \quad . \quad . . \quad . \quad . . \quad . . \quad . . \quad . \quad . . \tag{7}
\end{equation*}
$$

The assumption that as the perturbation velocities and the angle of incidence are small, so that their squares and products may be neglected, fails at a stagnation point, for there $u=U$. This entails infinite pressure at the leading edge except for the ideal angle of attack ${ }^{8}$. Campbell points out that there is no trouble in the absence of discontinuity in the streamline direction at the leading edge, for in this case $u=0$. Rounding the leading edge removes the infinite velocity except, of course, at the stagnation point. At a finite lift the stagnation point moves to the lower surface but if the thickness, curvature and angle of incidence are diminished, the domain in which the error is considerable shortens to the immediate neighbourhood of the leading edge ${ }^{9}$.

The Joukowski condition that the air speed at the trailing edge must be finite, has to be satisfied in order that the flow of an ideal fluid may approximate to that of a real fluid. This postulates that the streamlines at the trailing edge of a thin aerofoil must be continuous in direction with no infinite velocities.

The incidence of the aerofoil is given by the ratio of the vertical velocity, in a region not affected by the wing, and the horizontal velocity thus

$$
\begin{equation*}
\alpha=\frac{W}{U} \cdot \quad . \quad \quad . \quad . . \quad . \quad . . \quad . \quad . \quad . \quad . \quad . \tag{8}
\end{equation*}
$$

The pressure increment per radian of incidence is, from equations (7) and (8)

$$
\begin{equation*}
\frac{\Delta p}{\alpha}=\frac{2 u}{W} q . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{9}
\end{equation*}
$$

Hence

$$
\begin{equation*}
\frac{\Delta p}{q \alpha}=\frac{2 u}{W} \tag{10}
\end{equation*}
$$

Noting that the pressure on the lower surface of the wing is equal and opposite to the pressure on the corresponding point of the upper surface, we have for the lift coefficient

$$
\begin{equation*}
C_{L}=\frac{1}{A} \int_{-s}^{s} d y \int_{0}^{c} \frac{2 \Delta p}{q} d x \quad . . \quad . . \quad . . \quad . . \quad . \quad . . \tag{11}
\end{equation*}
$$

from which

$$
\begin{equation*}
\frac{\partial C_{L}}{\partial \alpha}=\frac{\partial}{\partial \alpha} \frac{1}{A} \int_{-s}^{s} d y \int_{0}^{c} \frac{2 \Delta p}{q} d x . \quad \ldots \quad . \quad . \quad \ldots \quad . . \tag{12}
\end{equation*}
$$

4. The Electrical Analogy.-If we identify fluid flow with electrical flow we have a direct analogy in which the velocity potential is represented by an electrical potential.

$$
\text { Now as } \quad u=-\frac{\partial \phi}{\partial x} \quad .
$$

and, at a point remote from the aerofoil

$$
\begin{equation*}
W=-\frac{\partial \phi}{\partial z}, \quad . \quad . . \quad . \quad . \quad . \quad . . \quad . \quad . \quad . \quad . \tag{14}
\end{equation*}
$$

we may rewrite equation (10) as

$$
\begin{equation*}
\frac{\Delta p}{q \alpha}=2 \frac{\partial \phi / \partial x}{\partial \phi / \partial z} \tag{15}
\end{equation*}
$$

where $\phi$ denotes either velocity or electrical potential. From equation (12), by the use of equations (13), (14) and (15) we have

$$
\begin{align*}
& \frac{\partial C_{L}}{\partial \alpha}=\frac{4}{A \partial \phi / \partial z} \int_{-s}^{s} d y \int_{0}^{c} \frac{\partial \phi}{\partial x} d x \\
& \frac{\partial C_{L}}{\partial \alpha}=\frac{4}{A \partial \phi / \partial z} \int_{-s}^{s}(\phi)_{x=c} d y \ldots  \tag{16}\\
& \ldots \\
& \ldots
\end{align*} \quad \ldots \quad \ldots \quad . \quad . \quad .
$$

since $\phi$ is zero at the leading edge.

The potential analyser will be fully described in the next section but, for the purpose of illustrating the analogy, it may be described here as consisting essentially of a cubical mesh of resistances so arranged that there are ten square tiers placed one above the other, each tier having a mesh separation of $1 / 24$ the side length. The analogy, can then be realised by :-
(a) Slitting the bottom tier in its own plane over an area corresponding in shape to the aerofoil considered
(b) Short-circuiting the area surrounding the model and short-circuiting the whole of the top tier
(c) Applying a known electrical potential between the short-circuited top tier and the shortcircuited margin of the bottom tier.
In practice the bottom tier is not actually slit because as the flow is applied normal to the model this tier may be considered as a plane of symmetry. It is, therefore, only necessary to double the value of the resistances over the area which represents the model. It will be observed that this analogy is a three-dimensional form of the direct analogy of Taylor and Sharman.

An alternative method of representing the velocity potential analogy when using an electrolytic tank has been proposed by Campbell, who arranged that constant currents should be fed into a number of electrodes representing the aerofoil surface, the short-circuited margin surrounding the aerofoil being set at zero potential. Thus a distribution of source-link doublets or dipoles is used to produce the perturbation velocity.

Apart from being an easier method to apply when an electrolytic tank is used, this method possesses the advantage that twisted and cambered wings may be represented and 'flexible walls' also may be more easily represented at the boundaries of the instrument.

Although this alternative method was not used in this investigation, there is no inherent limitation in the design of the potential analyser which prevents this method being applied.

If experiments concerning potential flow without circulation are made, boundary conditions present no difficulty but special consideration must be given to practical cases which require flow with circulation. When flow with circulation is being investigated the Joukowski condition at the trailing edge has to be satisfied ; this may be realised by ensuring that points immediately behind the trailing edge shall be at the same potential as corresponding points along the trailing edge. On the potential analyser this is effected by short-circuiting lines along the $x$-axis, and raising the potential of each individual line so that its potential is the same as at the corresponding point on the trailing edge, one mesh separation away from it. We thus have in the wake, as $\phi$ is independent of $x$.
and

$$
\phi_{\mathrm{W}}=\phi_{\mathrm{T} . \mathrm{E}}
$$

$$
\begin{equation*}
\left(\frac{\partial \phi}{\partial x}\right)_{\mathrm{w}}=0 . \quad . . \quad . . \quad . . \quad . \quad . \quad . \quad . \quad . \quad . \tag{17}
\end{equation*}
$$

The second of these conditions applying since the pressure, which is proportional to $\partial \phi / \partial x$, vanishes on either side of the wake and is, of course, zero at the trailing edge. $\phi$ must be continuous at the trailing edge as any discontinuity would cause an infinite velocity there.
5. Description of Potential Analyser.-The three-dimensional potential analyser, which was used for the experiments, has been previously fully described ${ }^{10}$. The instrument consists essentially of a cubical mesh of resistors arranged in nine main tiers with an auxiliary tier, each tier consisting of a square array of $25 \times 25$ points interconnected with resistors to form a square mesh. The nodal points on each tier were connected to the adjacent tiers through intertier resistors of the same value as the tier resistors. The individual mesh resistances were provided by special precision woven resistors having a tolerance of $\pm 1$ per cent on a nominal resistance of 200 ohms.

Resistance elements were not assembled on the master tier but Post Office type terminals were used instead, these terminals being connected to the inter-tier resistors in the normal manner. Resistors, depending on the nature of the particular experiment in hand, were attached to the terminals of the master tier as required. The resistors on the right-hand side and bottom edges of each tier were doubled in value, while the inter-tier resistors connecting the bottom right-hand corners of the tiers were quadrupled in value, thus providing ' selvedges' for problems involving a field of single or double symmetry. The reason for using the doubled resistors is that the net can be considered to be split at the plane of symmetry. When resistance elements are added to the master tier, which is a plane of symmetry, they are also doubled in value. Figs. 1 and 2 show front and rear views of the instrument. The analyser is capable of solving the threedimensional finite difference form of Laplace's equation.
6. Experimental Procedure.-6.1. Flow Normal to the Surface.-It was decided that the first series of experiments should be concerned with the combined undisturbed and perturbed potential flows, without circulation, normal to the following four plan forms :-
(a) A circular lamina
(b) A rectangular wing, constant chord, aspect ratio 6
(c) A delta wing, 45-deg sweep, aspect ratio 4
(d) A swept-back wing, 45-deg sweep, constant chord, aspect ratio 3.

The setting-up procedure, alike for the four experiments, consisted of representing the planform of the model by a resistance mesh on the master tier, which normally had no tier resistors. As this tier coincides with a plane of symmetry, the resistors used were double the value of the standard mesh resistors. The remaining area of the tier was short-circuited with heavy copper wire.

As the circular lamina and the rectangular wing each possess two axes of symmetry in planform, it was possible to set these forms up on the master tier so that the axes of symmetry coincided with the mutually perpendicular selvedges of the tier.

With the other two forms only one axis of symmetry existed and therefore one selvedge only could be used. All the nodes on the auxiliary tier (tier 9), were short-circuited and a potential of one volt was maintained between this tier and the short-circuited portion of the master tier.

A certain amount of importance is attached to the model size, the optimum size is the one having the largest number of points available for measurement while remaining unaffected by channel restraint due to the proximity of the boundary. The actual model sizes which were used are shown in Fig. 3.

A potential of one volt was applied to the analyser and the tiers were scanned by plugging in a probe at each socket in turn, the potentials being measured by means of a Muirhead Type D. 72 A potentiometer.' All readings were taken to the nearest millivolt, this being the limit of accuracy of the potentiometer. The electrical circuit which was used for the experiments is shown in Fig. 4.

In the case of the circular lamina the boundary of the model cut through the square mesh and therefore resistors having a resistance value proportional to the reduced mesh arm were mounted on the master tier at the boundary of the model. This method of using a reduced resistance is analogous to the use of an irregular star in the relaxation process ${ }^{11}$.

A brief description of each experiment is given here in order to illustrate the technique employed.
6.1.1. The Circular lamina.-Two sizes of model were used having mesh separations of $\frac{1}{6}$ th and $\frac{1}{12}$ th of the diameter of the circle. This experiment provides the only possible example of axial symmetry and a theoretical solution to the problem exists.

Fig. 5 shows the results for the coarse and fine nets respectively, this figure represents the variation of velocity potential with radius for the master tier and tiers one and two.

The theoretical curves have been drawn by using the equation
where

$$
\begin{array}{rlllll}
\frac{\pi \phi}{2 W} & =Q+x \tan ^{-1} \frac{x}{Q} \quad \ldots & \ldots & \ldots & \ldots & \ldots \\
& \ldots & \ldots  \tag{19}\\
Q & =\sqrt{\left\{\frac{\left[\left(x^{2}+\omega^{2}-c^{2}\right)^{2}+4 c^{2} x^{2}\right]^{1 / 2}-\left[x^{2}+\omega^{2}-c^{2}\right]}{2}\right\}} & \ldots & \ldots
\end{array}
$$

$x$ and $\omega$ being two orthogonal co-ordinates such that $x$ represents the distance of a point from the plane of the disc and $\omega$ the radial distance of this point from the axis of the disc, $\phi$ is the potential at point $(x, \omega)$ for a flow velocity $W$ normal to a disc of radius $c$.

This expression has been derived directly from Lamb's theoretical work ${ }^{12}$.
It will be seen that a closer approximation to the theoretical value is obtained when a finer mesh is used. Additional curves illustrate the improvement to the results which can be obtained by using Richardson's correction, which gives results approximating to the use of a still finer mesh ${ }^{13}$.

Using the relaxation technique the nodal residuals for the experiments were calculated; the results show that the discrepancy between the experimental and theoretical curves cannot be attributed to experimental errors, as essentially the same result would have been obtained from a relaxation calculation. The differences can probably be attributed to mesh size, the boundary of the model not coinciding with the net nodes as assumed. A discussion on the effective model size will be given in a later section.

The times taken to set up the apparatus and perform the various experiments are given in Table 1. One semi-skilled person was employed throughout, with the assistance of an unskilled person during the reading and recording stage.

TABLE 1

| Experiment | Setting-up <br> time <br> hours | Reading and recording |  |
| :--- | :---: | :---: | :---: |
|  | Semi-skilled <br> hours | Unskilled <br> hours |  |
| Circle (3 unit radius) | 4 | 20 | 20 |
| Circle (6 unit radius) | 4 | 12 | 12. |
| Rectangular wing | .. | 4 | 24 |
| Delta wing ... | .. | 8 | 32 |
| Swept-back wing | .. | 8 | 28 |

6.1.2. Rectangular wing, constant chord, aspect ratio 6.-Owing to its double axis of symmetry it was only necessary to set up a quarter of the rectangular wing, thus allowing a model of enhanced size to be used. The results call for no particular comment.
6.1.3. Delta wing, 45-deg sweep, aspect ratio 4.-In the case of the delta wing only one axis of symmetry exists and only one selvedge of the board could be used. The model which was used for this experiment was about the largest which could have been selected, as an examination of the results showed that there was a slight amount of channel restraint as evidenced by the readings in the region of the tier edges.
6.1.4. Swept-back wing, 45-deg sweep, constant chord, aspect ratio 3.-The swept-back wing and delta experiments were very similar ; only one axis of symmetry could be used and the model size was approximately the same. The accuracy of the readings was comparable with those obtained from the other experiments.
6.2. Flow Normal to the Surface using the Flexible-Wall Analogy.-In the experiments described in the foregoing section, the model size for the analyser was influenced by two opposing considerations; the model had to be sufficiently large to enable a reasonably fine mesh to be used, and yet not so large as to give rise to undue channel interference.

The use of a graded net with a mesh which is fine near the model growing coarser further away, would be one method by which this problem could be attacked ${ }^{14,16}$.

An alternative method would be to start with a very small model, observe the boundary potentials and use these for a larger model. This method would not be very reliable as, with the coarse mesh necessitated by a small model, the model size becomes uncertain and the model boundary might not be correctly interpreted as ending at the net nodal points.

In the tests which are described in this report, and which have previously been reported by Bruce ${ }^{4}$, a correction was introduced to offset the effect of what might be termed channel interference. This correction was not obtained by coarsening the mesh at a point remote from the model, but by calculating the boundary conditions for a finite net which give the same distribution of potential for a particular model, a doublet, as if it were immersed in an infinite net; the correction, which is reasonably independent of small changes in the model, was suggested by Mr. K. V. Diprose of the Royal Aircraft Establishment, and is analogous to the use of a slotted wind tunnel ${ }^{16}$.

This section of the report is not concerned with circulation and therefore the analogy between the electrical and fluid flow problems is complete. It has been found convenient to treat the present problem in electrical terms.
6.2.1. Theory of analogy to flexible-walled tunnel.-The case of a doublet immersed in a uniform infinite conducting medium is considered ; the current flow normal to, and the potential distribution over the walls of a cube containing the model being calculated. The conditions within the cube are unaltered if boundary resistors are used instead of the further medium, providing that these resistors give the right ratio of potential to normal current.

The potential distribution at a distance $r$ from an isolated source is found as follows :-

$$
\begin{align*}
& \rho i=-\operatorname{grad} \phi \\
& \operatorname{div} i=0, \\
& \int_{s}^{i} d s=I, \quad \text {.. .. .. .. .. .. .. .. .. }  \tag{20}\\
& \phi=\frac{\rho I}{4 \pi r} . \quad \text {. . . . .. .. .. .. .. .. } \tag{21}
\end{align*}
$$

hence $\quad i=\frac{I}{4 \pi \gamma^{2}}$
and

The potential distribution due to a doublet strength $D$, see Fig. 6, is thus

$$
\begin{equation*}
\phi=\frac{\rho D \cos \theta}{4 \pi r^{2}} . \quad . . \quad . \quad . \quad . . \quad . . \quad . \quad . . \quad . \tag{22}
\end{equation*}
$$

The value of the wall correction boundary resistors may be found from the ratio of the potential to the normal potential gradient at the appropriate point, the potential gradient representing the density of the outward flowing current. Consider a rectangular wind tunnel with a flat plate normal to the current flow and take the origin $O$ in the plate with right-hand orthogonal axes $O x, O y, O z$, such that $O z$ is the axis of the doublet, that is to say $O z$ is normal to the plate. Let the radius vector from the origin $O$ to some point on the tunnel wall be $r$ and let the unit vector normal to the wall at this point be $n$, then the density of current flow normal to the wall is

$$
\begin{equation*}
\frac{n \operatorname{grad} \phi}{\rho} \tag{23}
\end{equation*}
$$

where the components of grad $\phi$ are

$$
\begin{equation*}
\frac{\partial \phi}{\partial x}=\frac{\partial \phi}{\partial r} \cdot \frac{\partial r}{\partial x}+\frac{\partial \phi}{\partial \theta} \cdot \frac{\partial \theta}{\partial x} \text { etc. .. .. .. ... .. ... .. } \tag{24}
\end{equation*}
$$

Now,

$$
\begin{equation*}
\phi=\frac{\rho D \cos \theta}{4 \pi r^{2}} . \tag{25}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
\frac{\partial \phi}{\partial r}=-\frac{\rho D \cos \theta}{2 \pi r^{3}} \tag{26}
\end{equation*}
$$

and

$$
\frac{\partial \phi}{\partial \theta}=-\frac{\rho D \sin \theta}{4 \pi r^{2}}
$$

while

$$
\begin{equation*}
r^{2}=x^{2}+y^{2}+z^{2} \tag{27}
\end{equation*}
$$

and $\quad \tan ^{2} \theta=\frac{x^{2}+y^{2}}{z^{2}}$.
Hence

$$
\begin{align*}
\frac{\partial \phi}{\partial x} & =\frac{-\rho D}{2 \pi}\left[\frac{\cos \theta}{r^{3}} \cdot \frac{x}{r}+\frac{\sin \theta}{2 r^{2}} \cdot \frac{x}{z^{2} \tan \theta} \overline{\sec ^{2} \theta}\right] \\
& =\frac{-3 \rho D x \cos \theta}{4 \pi r^{4}} . \quad \ldots \tag{28}
\end{align*} \ldots \quad . \quad .
$$

Similarly,

$$
\frac{\partial \phi}{\partial y}=\frac{3 \rho D y \cos \theta}{4 \pi r^{4}}
$$

and

$$
\begin{align*}
\frac{\partial \phi}{\partial z} & =-\frac{\rho D}{2 \pi r^{4}}\left[z \cos \theta-\frac{\left(x^{2}+y^{2}\right)}{r}\right] \\
& =-\frac{\rho D}{2 \pi r^{5}}\left[z^{2}-x^{2}-y^{2}\right] . \quad \ldots  \tag{29}\\
\cdots & \ldots
\end{align*} . \quad \ldots \quad \ldots
$$

For a tunnel wall parallel to $O y z$ the components of $n$ are $(1,0,0)$ hence

$$
\rho i=\frac{3 \rho D x \cos \theta}{4 \pi r^{4}}
$$

and

$$
\begin{equation*}
\frac{\phi}{i}=\rho \frac{r^{2}}{3 x} \tag{30}
\end{equation*}
$$

For a resistance network this becomes :-

$$
\begin{equation*}
R_{\text {termination }}=R_{\text {net }}\left(\frac{x^{2}+y^{2}+z^{2}}{3 x}\right) \quad . \quad . \quad \quad . \quad . . \quad . . \quad . \quad . \tag{31}
\end{equation*}
$$

$x, y$ and $z$ now being measured in mesh units.
Similarly for a wall parallel to $O x z$ :-

$$
\begin{equation*}
\dot{R}_{\text {termination }}=R_{\text {net }}\left(\frac{x^{2}+y^{2}+z^{2}}{3 y}\right) \quad . \quad . \quad . . \quad . . \quad . . \quad . . \quad . \tag{32}
\end{equation*}
$$

while for a tunnel end parallel to $O x y$

$$
\begin{align*}
\frac{\phi}{i} & =\frac{r^{3} \cos \theta}{2\left(z^{2}-x^{2}-y^{2}\right)} \\
& =\frac{r^{2} z}{2\left(z^{2}-x^{2}-y^{2}\right)} . \tag{33}
\end{align*}
$$

or for the network

$$
\begin{equation*}
R_{\text {termination }}=R_{\text {net }} \frac{z}{2} \frac{\left(x^{2}+y^{2}+z^{2}\right)}{\left(z^{2}-x^{2}-y^{2}\right)} \cdot \ldots \quad . \quad . . \quad . \quad . \quad . \quad . \tag{34}
\end{equation*}
$$

It will be observed that this result postulates negative values of $R$ when the value of $z$, for the furthest tier, is less than the greatest radial distance of any point on the tiers from the $O z$-axis.

$$
\begin{equation*}
\text { i.e., } z_{\max }<\left[\sqrt{ }\left(x^{2}+y^{2}\right)\right]_{\max } . \quad \ldots \tag{35}
\end{equation*}
$$

6.2.2. Application of the theory.-The investigation in the previous section considered termination resistances for a model which could be represented sufficiently accurately by a single doublet. In consequence, this case covered only perturbation velocity potentials for flow without circulation; under these conditions, the model was represented by a set of current sources, the current at each point being proportional to the slope of the centre of the aerofoil at that point. Nodes outside, but in the plane of the model, were held at a uniform zero potential and the ends of all terminating resistances were also taken to zero potential.
In the actual experiments which were undertaken, the model was a flat plate and the velocity potentials which were observed were the total velocity potentials for flow normal to the plate ; that is to say, the sum of the perturbation velocity potentials and the undisturbed flow potentials. The alteration to the boundary conditions was such that no connection was made to the nodes on the model, while the nodes over the most remote tier were raised to a uniform potential ; under these conditions the channel and terminating resistors had to be omitted, while the wall resistors were not connected to zero potential but to the potential at infinity, appropriate to their own particular tiers. For the actual analyser which was used the end resistors could not, in any case, have been used, for many of them would have to be of negative value.

Where circulation was represented on the analyser, the appropriate terminating resistances could not be found for the surface through which the trailing-edge vortex sheet passed.
The models considered had an axis of symmetry and this axis was represented as the axis $O x$ and, therefore, the surface $y=0$ was a reflecting surface and not a channel wall ; that is, it needed no terminating resistors.

Fig. 7 shows, diagrammatically, the electrical circuit used for the channel wall resistors. A view of the apparatus set up for an experiment is shown in Fig. 8.

The resistance values of the wall resistors are tabulated in Appendix I, to this report.
6.2.3. Experiments using flexible-wall analogy.-Experiments were carried out to determine the velocity potentials for flow normal to flat plates of the following wing shapes:-
(a) Rectangular
(b) Delta
(c) Swept-back.

These were represented on the master tier, as before, by leaving the corresponding mesh nodes disconnected while shorting the remaining points on the master tier to zero potential. In case (a), the rectangular plate had to be moved to the centre of the master tier as the terminating resistors had been calculated on the assumption of a doublet at this point.

The flow normal to the plates was again represented by raising the auxiliary tier, $z=9$, to a potential of 1,000 units, approximately 1 volt, and the experiments then consisted of measuring the potentials at other nodes. Terminating resistors on tier 1 had a potential of $111 \pm 5$ units at their far end and $222 \pm 5$ units on tier 2, etc., these potentials being derived from a 5 -ohm potential divider with 100 tapping points.
6.2.4. Results and comments.-The experiment which was made to the delta model was the only one which showed any significant difference from the experiments which have been described in section 6.1. In the flexible-wall experiment the potentials over the centre of the model increased by about 2 units, approximately 1 per cent change. A visual inspection of the various model shapes and sizes, compared with the tier size, showed that the delta model would certainly be one which would be most affected by channel constraint.

It seems likely that the end effect will swamp the wall effect, for the analyser has only 9 tiers, on either side of the master tier, although it has an equivalent cross-section of $24 \times 48$ units. It is impossible, however, to add correcting resistors to the top tier ; this is due both to the method of application of the flow conditions and to the fact that even if this were rectified by representing perturbation flow only, as suggested in this report, then negative resistors would be required. The solution of this difficulty would appear to lie in the addition of a net of increasing coarseness.
6.3. Flow Normal to the Surface with Circulation.-It has already been pointed out that it would have been possible to have made tests to determine the perturbation flow without circulation, together with the circulation originating in a trailing vortex sheet, but without the undisturbed flow normal to the aerofoil surface. However, for the purpose of this investigation it was decided to combine the three flows; therefore the results of this section, when combined with the desired amount of undisturbed flow along the aerofoil surface, give the complete solution to the problem under consideration without recourse to the results of the previous experiments.

In this series of experiments it was not possible to use the flexible-walled terminal analogy because the appropriate terminating resistances could not be found for the surface through which the trailing-edge vortex sheet passed.

With the exception of the circular lamina all the planforms previously investigated were re-investigated to determine the effect of circulation.

A diagrammatic arrangement of the set-up for the master tier is shown in Fig. 9. Reference to this illustration shows that the model was set out on the tier in the normal way and that the portion of the tier unoccupied either by the model, or the vortex sheet immediately behind it, was short-circuited and set at zero potential. Commencing with nodal points, such as $\mathrm{A}^{\prime}, \mathrm{B}^{\prime}$, $C^{\prime}$, etc., which are one mesh unit to the rear of corresponding points $A, B, C$, etc., successive nodal points were shorted to the boundary of the tier. Thus the area from one unit behind the trailing edge to the boundary was shorted by bars running in one direction only. Each shorting bar was connected through a rheostat to a potential divider. The condition for
circulation is that the potential at a point immediately behind the surface shall be at the same potential as the corresponding point on the surface trailing edge. The experimental procedure was to apply a uniform unit voltage at the auxiliary tier and then by means of the potential divider and rheostats, to adjust the potential on the shorting bars so that the potentials at $\mathrm{A}^{\prime}, \mathrm{B}^{\prime}, \mathrm{C}^{\prime}$, etc., coincided with the potentials at the corresponding points $\mathrm{A}, \mathrm{B}, \mathrm{C}$, etc. It will be appreciated that adjustment to the potential on any bar upset the values of the potentials on neighbouring bars and the setting up developed into what might be termed an electrical iteration process. It was found, however, that with a little practice the setting up of a model was quite rapid. After the set up had been completed the method of scanning was identical to that carried out for the previous experiments.

The results of these experiments are given in Appendix II to this report.
6.3.1. Results and comments.-From the foregoing experiments the velocity potential $\phi$ was plotted for chordwise sections of the aerofoil, these curves being graphically differentiated to obtain values of $\partial \phi / \partial x$. Then, by the use of equation (15), values of $\Delta p / q \alpha$ were computed and are pictorially represented in Figs. 10, 11 and 12 for the three experiments. These figures call for no special comment although it will be observed that in each case the load distribution curves appear to be reasonable.

Values of $\partial C_{L} / \partial \alpha$ for each aerofoil were calculated from equation (16) and are compared with theoretical values, supplied by Mr. W. P. Jones of the National. Physical Laboratory, in Table 2.

TABLE 2

| Experiment | $\frac{\partial C_{L}}{\partial \alpha}$ |  |
| :---: | :---: | :---: |
|  | Theoretical | Experimental |
| Rectangular wing <br> Aspect ratio $=6$.. | $4 \cdot 26$ | $4 \cdot 80$ |
| Delta wing <br> Aspect ratio $=4$ | $3 \cdot 47$ | $3 \cdot 22$ |
| Swept-back wing <br> Aspect ratio $=3$.. | $2 \cdot 75$ | $2 \cdot 75$ |

When making these calculations it was difficult to assess the true area of the aerofoil, a difficulty which has been previously mentioned in section 6.1.1. The difficulty arises because the net used was too coarse for the true boundary position to be precisely located. The actual locus of the boundary must lie between lines joining the nodal points on the assumed boundary, which was short-circuited on the model set up on the analyser, and lines joining the nodal points one mesh distance within the model from the short-circuited boundary. For the purpose of the present investigation the true boundary was assumed to lie along lines one half mesh distance inside the model, from the short-circuited boundary. This difficulty could have been obviated if the tier size had been larger, this would have allowed a larger model to be used with, of course, a finer mesh separation.
7. Conclusions.-The potential analyser proved to be easy and quick to operate, even by unskilled personnel. The apparatus was designed for the general solution of the three-dimensional form of Laplace's equation and this, to a certain extent, proved to be a handicap when it was used for the particular flow problems described in this report.

For problems concerning potential flow around thin plates, under conditions in which the linear-perturbation theory applies, it is only necessary to measure the electrical potentials over the master tier. It would thus be possible to have a very large master tier, thus permitting a small mesh separation, so avoiding the difficulty regarding the indeterminancy of the model boundary. The effect of additional tiers would have to be produced but, as the tiers would not need to be accessible, a considerable simplification in the construction would be possible. Furthermore, a graded resistance net could be used, thus increasing the effective size of the network.

In addition to the study of problems of the type described in this report, curved and twisted aerofoils could also be studied and in this connection there are many practical advantages to be gained in using an electrical network as opposed to an electrolytic tank.

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## APPENDIX I

## Correction Resistors

The arrangement of the wall correction resistors for the tiers 1 to 8 inclusive is shown diagrammatically below.


The following table gives the values of each resistor for each tier.

|  | BOUNDARY POINT RESISTOR (Ohms) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 | 2200 | 2200 |
| 2 | 820 | 820 | 820 | 820 | 1000 | 1000 | 1000 | 1200 |
| 3 | 820 | 820 | 820 | 1000 | 1000 | 1000 | 1000 | 1200 |
| 4 | 820 | 820 | 820 | 1000 | 1000 | 1000 | 1200 | 1200 |
| 5 | 820 | 1000 | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 |
| 6 | 1000 | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 |
| 7 | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 1500 |
| 8 | 1000 | 1000 | 1200 | 1200 | 1200 | 1200 | 1500 | 1500 |
| 9 | 1200 | 1200 | 1200 | 1200 | 1200 | 1500 | 1500 | 1500 |
| 10 | 1200 | 1200 | 1500 | 1500 | 1500 | 1500 | 1500 | 1800 |
| 11 | 1500 | 1500 | 1500 | 1500 | 1500 | 1500 | 1800 | 1800 |
| 12 | 1500 | 1500 | 1500 | 1500 | 1500 | 1800 | 1800 | 1800 |
| 13 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 |
| 14 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 | 2200 |
| 15 | 1800 | 1800 | 1800 | 1800 | 2200 | 2200 | 2200 | 2200 |
| 16 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2700 |
| 17 | 2200 | 2200 | 2200 | 2200 | 2200 | 2700 | 2700 | 2700 |
| 18 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 |
| 19 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 |
| 20 | 2700 | 2700 | 2700 | 2700 | 2700 | 3300 | 3300 | 3300 |
| 21 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 |
| 22 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 | 3900 |
| 23 | 3300 | 3300 | 3300 | 3300 | 3900 | 3900 | 3900 | 3900 |
| 24 | 3900 | 3900 | 3900 | 3900 | 3900 | 3900. | 3900 | 3900 |
| 25 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 |
| 26 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 |
| 27 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 |
| 28 | 1800 | 1800 | 1800 | 2200 | 2200 | 2200 | 2200 | 2200 |
| 29 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 | 2200 | 2200 |
| 30 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 | 2200 |
| 31 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 |
| 32-42 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 |
| 43 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 |
| 44 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 | 2200 |
| 45 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 | 2200 | 2200 |
| 46 | 1800 | 1800 | 1800 | 2200 | 2200 | 2200 | 2200 | 2200 |
| 47 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 |
| 48 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 |
| 49 | 1500 | 1500 | 1500 | 1500 | 1500 | 1500 | 1500 | 1500 |
| 50 | 3900 | 3900 | 3900 | 3900 | 3900 | 3900 | 3900 | 3900 |
| 51 | 3300 | 3300 | 3300 | 3900 | 3900 | 3900 | 3900 | 3900 |
| 52 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 | 3900 |
| 53 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 | 3300 |
| 54 | 2700 | 2700 | 2700 | 2700 | 2700 | 3300 | 3300 | 3300 |
| 55 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 | 3300 |
| 56 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 | 2700 |
| 57 | 2200 | 2200 | 2200 | 2200 | 2700 | 2700 | 2700 | 2700 |
| 58 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2700 | 2700 |
| 59 | 1800 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 |
| 60 | 1800 | 1800 | 1800 | 1800 | 2200 | 2200 | 2200 | 2200 |
| 61 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 2200 |
| 62 | 1500 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 |
| 63 | 1500 | 1500 | 1500 | 1500 | 1500 | 1800 | 1800 | 1800 |
| 64 | 1500 | 1500 | 1500 | 1500 | 1500. | 1500 | 1800 | 1800 |
| 65 | 1200 | 1200 | 1500 | 1500 | 1500 | 1500 | 1500 | 1800 |
| 66 | 1200 | 1200 | 1200 | 1200 | 1500 | 1500. | 1500 | 1500. |
| 67 | 1200 | 1200 | 1200 | 1200 | 1200 | 1500 | 1500 | 1500 |
| 68 | 1000 | 1000 | 1200 | 1200 | 1200 | 1200 | 1500 | 1500 |
| 69 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 1200 | 1500 |
| 70 | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 | 1500 |
| 71 | 1000 | 1000 | 1000 | 1.000 | 1000 | $\lambda 200$ | 1200 | 1200 |
| 72 | 1000 | 1000 | 1000 | 1000 | 1000 | 1200 | 1200 | 1200 |
| 73 | 2800 | 1800 | 1800 | 2200 | 2200 | 2200 | 2200 | 2700 |

## APPENDIX II

Tabulated Results of Experiments;
Flow with Circulation



- THREE DIMENSIONAL POTENTIAL ANALYSER -













## -THREE DIMENSIONAL POTENTIAL ANALYSER -







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FIg. 1. General view of potential analyser.


FIG. 2. Rear view of potential analyser.


Fig. 3. Model sizes.


Fig. 4. Electrical circuit for analyser experiments.


Frg. 5. Results of circular-disc experiments.


StrengTh Of DOublet, $D_{1}=I L$

POTENTIAL DISTRIBUTION, $\phi_{1}=\frac{P D \cos \theta}{4 \pi r^{2}}$

Fig. 6. Potential distribution due to doublet strength.

Tier 9 (AUXILIARY TIER)

TiER 8.


InTERMEDATE Tiers.

TIER I.


CORrECTION RESISTORS.

Fig. 7. Electrical circuit for channel wall resistors.


Fig. 8. Potential analyser set up for an experiment.


FIG. 9. Electrical circuit for flow with circulation.


Fig. 10. Pressure distribution over rectangular wing.


Fig. 11. Pressure distribution over delta wing.


Fig. 12. Pressure distribution over swept-back wing.

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