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Summary.—A technique for the investigation of wing flutter by means of ground-launched rockets is described. An important feature of the technique is that it can be used at high speeds, including the transonic range.

Model wings are attached to a solid-fuel rocket which has a miniature telemetry set housed in a detachable nose fairing. A vibration pick-up and break wires are fitted in the flutter model and these modulate the output of the telemetry set to transmit flutter information to a ground station. The rocket is fired over an open artillery range and its velocity-time curve is obtained by radio reflection Doppler equipment.

Results are given of tests on flutter models of unswept, untapered wings in the range of Mach number from 0.4 to 1.0.

The effects of longitudinal acceleration on the flutter are shown to be negligible for the range of acceleration and Mach number investigated, and the effect of compressibility is to reduce the margin of the measured speed above the speed calculated on the basis of incompressible flow theory from + 50 per cent at M = 0.4 to - 25 per cent at M = 0.9.

A wing torsional stiffness criterion which includes the Glauert function $\phi(M) = (1 - M^2)^{1/4}$ is shown to give a fair approximation to the test results.

1. Introduction.—Flight methods of flutter testing^{1, 2} provide a means of investigating flutter phenomena in the transonic range—a range which cannot easily be investigated in a wind tunnel. They also provide a useful alternative to tunnel tests for high-subsonic and supersonic conditions. This report describes a flight technique for flutter testing using rocket-propelled flutter models that are launched from the ground, and the technique is applied to flutter tests on unswept, untapered wings in the range of Mach number from 0.4 to 1.0.

The aims of the tests were to determine the effects of rocket acceleration and the effects of compressibility on wing flutter characteristics, and to determine a factor to be applied to the existing criterion for wing torsional stiffness^{6,7} to allow for compressibility effects.

The investigation of acceleration effects shows them to be negligible in the range of acceleration from 19g to 31g, up to a Mach number of 0.65.

The effect of compressibility on flutter speed is determined from a comparison of the measured speed with the speed calculated using two-dimensional incompressible flow theory. This comparison shows that the margin of the measured speed above the theoretical speed is reduced from +50 per cent at M = 0.4 to -25 per cent at M = 0.9. The effect of compressibility on flutter frequency is to reduce it towards the wing bending frequency.

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A comparison of the measured flutter speeds with the speeds derived from the wing torsional stiffness criterion⁷ shows that the Mach function $(1 - M^2)^{1/4}$ forms a reasonable boundary to the results up to M = 0.9. A function to allow for wing density effects is also proposed. In its modified form the criterion is conservative with respect to the test results except in the region of M = 0.8, where it is slightly unconservative.

Further investigations are to be made on wings of different plan-forms, and the technique is to be developed for *ad-hoc* tests on scale models of specific aircraft.

2. Details of the Rocket Model.—A complete rocket model is shown in Figs. 1 and 2. The model was assembled so that the centre of gravity of the complete rocket was at the wing leading edge, to ensure an adequate margin of pitching stability in flight. The average launching weight of a complete model with 3-in. rocket was 55 lb, and at this weight the acceleration of the model during the 1.6 sec burning time was about 24g giving a peak velocity of about 1,200 ft/sec. The ratios of (complete model weight): (wing weight) and (complete pitching inertia): (wing pitching inertia) were not less than 5:1 and 120:1 respectively for any of the models, and were considered adequate to ensure that the flutter obtained approximated to the flutter of a wing fixed at the root. To extend the Mach number range, two of the models were tested using a 5-in. rocket in place of the 3-in. rocket normally used. The launching weight of these models was 140 lb; the acceleration during the burning period of 1.6 sec was about 40g and the peak velocity was about 2,000 ft/sec.

2.1. Telemetry Equipment.—The telemetry equipment was developed by Guided Weapons Department, Royal Aircraft Establishment. The part carried on the rocket (Fig. 3) was a single channel 465 Mc/sec transmitter with its output amplitude modulated at a frequency of 150 ± 15 Kc/sec. The variation of ± 15 Kc/sec corresponded to the range of variation of the inductance of the vibration pick-up, and this variation was recorded as a continuous trace at a ground station. The weight of the unit was $2 \cdot 5$ lb.

2.2. Doppler Equipment.—The velocity of the model was obtained by using a radio reflection Doppler system in which a signal at 212 Mc/sec was transmitted to the model and was reflected back to the transmitting station. The difference in frequency of the transmitted and received signals was a measure of the speed at which the model receded from the transmitter, and this difference was recorded. A frequency difference ranging from 0 to 1000 c/sec corresponded to a velocity of recession ranging from 0 to 2,320 ft/sec. The velocity normally required some correction to allow for trajectory and line of fire of the test vehicle relative to the transmitter, and could be obtained to better than 0.5 per cent accuracy.

2.3. Vibration Pick-up.—A variable inductance accelerometer-type pick-up was used (Fig. 4). It had a natural frequency of about 100 c/sec and was designed for a range of acceleration of $\pm 20g$. Oil damping was provided to damp the natural frequency oscillations and irregular vibrations experienced in flight.

2.4. Wing Details.—The external dimensions of all the wings tested were as follows:

Length (root to tip) $\dots = 2 \cdot 0$ ft Chord $\dots \dots = 1 \cdot 0$ ft Aspect ratio $\dots = 4 \cdot 4$ (4 $\cdot 6$ for 5-in. rocket models) Thickness/chord $\dots = 0 \cdot 10$ Wing section — RAE 101

Three types of wing structure were used in the course of the tests (*see* Fig. 5). Wings of stressedskin construction (models No. 1101 to 1115) were used for the initial development of the rocket model technique and to investigate the effects of acceleration on flutter characteristics. The construction consisted of a light wooden framework to which the skin was glued. A variation of wing stiffness could be obtained by using skins of plywood, metal and plastic. The construction was found to be generally unsuitable for rocket work since the wing flexural-axis location could not be controlled, models were easily damaged and there were glueing difficulties which resulted in skin failures in flight due to aerodynamic suction.

To overcome these difficulties a solid wing construction was developed. This consisted of a plywood sheet cut to the wing plan-form, carrying a metal spar at 30 per cent of the chord aft of the leading edge, four equally spaced ash ribs and the wing contour filled in with balsa wood. Lead strips glued to the plywood determined the wing inertia-axis position, the location of the spar determined the flexural-axis position and by varying the gauge of the spar material a variation of wing stiffness could be obtained. This construction was used for models 1116 to 1128. Flight tests demonstrated that the flexural axis position was too far aft so that, for models designed to flutter at high speeds, divergence occurred before flutter. There was also a tendency for the nose structure to break away from the spar.

The construction was modified to bring the spar location to 25 per cent chord aft of the leading edge and a plywood skin nose was added forward of the 45 per cent chord line. The divergence characteristics were improved and the nose weakness eliminated. In addition a reduction of wing stiffness could be made after construction by cutting chordwise slots in the stressed-skin nose. Models 1136 to 1143 were of this construction and were used for the high Mach number tests.

3. Test Procedure.—Static measurements of the wing elastic and inertia characteristics were made on all the models. Resonance tests were also made on many of the models to determine the wing natural frequencies.

All models were launched at an elevation of $12\frac{1}{2}$ deg. A continuous photographic record was obtained of the transmitted signals from the vibration pick-up in one wing of each model, and the flight path of the model was followed by radio reflection Doppler equipment and by ciné-cameras.

Wing components were subsequently recovered from the range so that the failures could be examined. Pick-ups recovered with the wings were used in further tests.

3.1. Test Results.—Photographic records were obtained from the telemetry and Doppler stations and from the range ciné-cameras. In general, correlation of the telemetry and Doppler results gave the required flutter information, whilst the ciné-camera record provided a useful check. Results for a typical model are shown in Figs. 6, 7 and 8. Fig. 6 gives three consecutive frames of the rear view camera record and shows the flutter failure. The wing is seen to disintegrate into small fragments. Fig. 7 shows the telemetry record of the wing oscillations in flight. Flutter is presumed to commence when the oscillations are of regular frequency and divergent amplitude. The rapid divergence of the flutter oscillations is shown by the fact that the limit of travel of the pick-up (indicated by the flat-topped curves) is reached after about three cycles and the wing fails after five cycles. However, at this point the speed is considerably greater than at commencement of flutter. Fig. 8 is the velocity-time curve derived from Doppler measurements. Irregularities in the region of flutter failure are probably due to Doppler errors resulting from dispersed reflection from the disintegrated model. From Figs. 7 and 8 the results obtained are:

Flutter frequency	••	= 58 c/sec
Speed of model at beginning of flutter		= 650 ft/sec
Speed of model at wing failure	••	= 760 ft/sec
Acceleration of model during flutter		= 26g.

4. *Theoretical Investigation*.—Theoretical estimates of flutter speed and frequency at ground level were obtained for each model for comparison with the test results. The modes chosen were

the decoupled flexural and torsional fundamental modes of a fixed-root uniform wing, and they were used in conjunction with the static measurements of the wing elastic and inertia characteristics. Calculations were made using two-dimensional vortex-sheet theory derivatives for both incompressible and compressible flow; the former for comparison with the test results to obtain the effect of compressibility, the latter for correlation between compressible flow theory and experiment. No corrections were made for aspect-ratio effects. The derivative values used were obtained from Refs. 3 and 4. Ref. 3 gives derivative values for the range of Mach number from 0 to 0.8 only, but for some of the models values were required within the range of Mach number from 0.8 to 1.0, and these were obtained from Ref. 4. It is probable that the derivative values are considerably in error for Mach numbers approaching unity since in their determination no account has been taken of shock-wave formation. However, the results of calculations made using these derivatives provide an explanation for unique features of the records from models which fluttered at high Mach number (see section 5.3).

The flutter equations were solved by the use of an electronic simulator⁵. In general the values of frequency parameter and Mach number assumed for the derivatives were balanced with the values derived from the solution. For the calculations using compressible-flow derivatives the method used to balance the Mach number may be explained by reference to Fig. 11, which shows the results for four of the higher Mach number model wings (1136 to 1139 inclusive). The first diagram of Fig. 11 shows a curve for each model giving the variation of calculated flutter speed with the Mach number assumed in the evaluation of the derivatives. On the same diagram is drawn the straight line giving the relationship between speed and Mach number for actual flight conditions. Intersections of this straight line with the calculated flutter curve then give the theoretical critical flutter conditions in which the Mach number appropriate to the flutter speed is the same as that assumed in the evaluation of the derivatives. In some cases, as for instance the three upper curves of Fig. 11, balance of Mach number could not be achieved because no such intersection occurred. It was then assumed that the theoretical flutter speed was given by the minimum of the flutter curve.

5. Discussion of Results.—5.1. General.—The experimental and theoretical results are given in Table 1. A total of thirty-two models were tested and of these thirty-one were designed for flutter tests and one (model 1126) for divergence tests. Records of flutter oscillations were obtained from twenty-two of the flutter models, and five others probably fluttered though records were not obtained due to telemetry failure; the remaining four failed under aerodynamic suction loads. The elastic and inertia characteristics of the models were varied over a wide range, but an increase in flutter speed was obtained primarily by an increase of wing stiffness. The records of flight characteristics from the wing vibration pick-ups were of five distinct types as follows (see Fig. 9):

- 1. Wing divergence failure.
- 2. Flutter oscillations followed by wing-divergence failure.
- 3. Flutter oscillations leading to wing-flutter failure during the rocket acceleration period.
- 4. Flutter oscillations during the rocket acceleration period dying out as the speed increases, and subsequently reappearing during the deceleration period and leading to wing-flutter failure.
- 5. Flutter oscillations during the acceleration and deceleration periods without wing failure.

In general, the wings were designed to ensure that flutter occurred before divergence, and a record of type 1 was obtained only on model 1126 which was designed to demonstrate divergence failure. Records of type 2 were fairly common, particularly for the models tested at high Mach number, and this despite the fact that for the latter models the wing flexural axis was at about 25 per cent of the wing chord aft of the leading edge. Type 3 records were those most generally

obtained. Records of types 4 and 5 were obtained only on models which fluttered in the range of Mach number from 0.8 to 1.0 and their significance is discussed in section 5.3.

5.2. The Effects of Longitudinal Acceleration.—For this investigation two sets of identical models were constructed and these were tested on rockets ballasted to different launching weights to cover a range of acceleration from 19g to 31g. This range of acceleration was fixed by limitations in range facilities and the total impulse of the rocket. The first set of models (Nos. 1102, 1112, 1113, 1114, 1115) fluttered at a Mach number of about 0.58 and the second set (Nos. 1108, 1109, 1110, 1111) fluttered at a Mach number of about 0.65.

Although the models were designed to be identical, differences in structural characteristics were obtained (*see* Table 1). These differences were levelled out by expressing the experimental results as a ratio to the compressible flow theoretical results, and this ratio is plotted against rocket acceleration in Fig. 10. It can be seen that for the range of acceleration investigated there is no noticeable effect of acceleration on either flutter speed or frequency.

5.3. The Effects of Compressibility.—The records from the wing vibration pick-ups showed that as the Mach number at flutter increased, the flutter frequency was reduced towards the wing fundamental bending frequency, and for the range of Mach number from 0.8 to 1.0 it was in some cases slightly less than the bending frequency. The high Mach number records also indicated that there was a finite speed range for flutter, with no flutter at speeds beyond the upper limit of this range, up to the peak speed of the model.

The results of the theoretical investigation made on models 1136 to 1139 for the range of Mach number from 0 to 1.0 are plotted in Fig. 11. The flutter speed vs. Mach number curves are the flutter boundaries (see section 4) for the various models, and the line through the origin and the point V=1117 ft/sec, M=1.0 represents the rocket flight conditions on a standard day. This line intersects the flutter boundary of model 1139 at the points V=875 ft/sec and V=1000 ft/sec indicating that within this speed range the model will flutter and at higher and lower speeds there will be no flutter. Obviously the acceleration of the model could be such that the flutter region was traversed before the flutter developed to wing failure. The flutter oscillations would then die away to re-appear as the model decelerated through the flutter region and the occurrence of wing failure during deceleration would depend on the rate of deceleration through the flutter region. A pick-up record with the above characteristics is typical of those obtained for the high Mach number tests (records 4 and 5, Fig. 9). There is no intersection of the rocket flight line with the flutter boundaries of models 1136, 1137 and 1138, but in practice model 1136 fluttered during the acceleration and deceleration periods without wing failure, model 1137 fluttered during the acceleration and deceleration periods with wing failure during deceleration, and models 1138 and 1139 fluttered to destruction during the acceleration period. Generally speaking, therefore, the severity of the flutter encountered in the test is reflected in the nearness of the theoretical curve to the rocket line.

The theoretical effect of compressibility on flutter frequency is to reduce this frequency towards the wing bending frequency with increasing Mach number. The trend is less pronounced than is obtained experimentally (*see* Table 1). However, close agreement of theory with experiment is not to be expected, particularly in view of the uncertainties attached to the derivative values at high Mach number.

The experimental results are expressed as ratios of the theoretical results in Table 1, and these ratios are plotted against Mach number at flutter in Fig. 12. The comparison of experiment with incompressible-flow theory shows that the margin of the experimental speed above the theoretical speed is reduced from about +50 per cent at M=0.4 to -25 per cent, at M=0.9; a similar comparison with compressible flow theory gives margins of about +50 per cent at M=0.4 reducing to -12 per cent at M=0.9. In both cases the frequency ratio shows a marked reduction with increasing Mach number.

5.4. Wing Torsional Stiffness Criterion.—A criterion for wing torsional stiffness has been proposed^{6,7} of the form:—

The experimental data of the present report form a basis for the extension of the above criterion to include the effects of compressibility.

The flutter speeds determined on the basis of the criterion are given in Table 2, and the ratio of experimental flutter speed to criterion flutter speed is plotted against Mach number at flutter in Fig. 13. The curve of the Glauert function $\phi(M) = (1 - M^2)^{1/4}$ is also shown, and it can be seen that this curve forms a reasonable boundary to the results, with the following limitations:

$$egin{aligned} \phi(M) &= (1-M^2)^{1/4}, \, 0 < M < 0.9 \ &= 0.66, \qquad 0.9 < M < 1.0 \ . \end{aligned}$$

The discrepancy between this boundary and the experimental results is greatest for the results at the lower Mach numbers. However, these results are mainly for wings of low relative density $(\sigma_w < 10)$, and there is no allowance for variation of wing density in the criterion. An investigation of the effect of wing relative density on flutter speed has been made for one of these wings⁵, and from these results a relative density factor to be applied to the criterion has been derived. This factor is of the form:

$$\phi(\sigma_w) = \left(0.95 + rac{1\cdot 3}{\sigma_w}
ight).$$

By including this factor the agreement of the results with the boundary formed by $\phi(M) = (1 - M^2)^{1/4}$ is improved.

5.4.1. Suggested modified criterion.—On the basis of the above results a modified criterion is suggested of the form:

$$\begin{split} \frac{1}{V} \Big(\frac{m_0}{\rho_0 dc_m^2} \Big)^{1/2} &= \frac{0 \cdot 9(g - 0 \cdot 1)(1 \cdot 3 - h)}{(0 \cdot 9 - 0 \cdot 33k)(1 - 0 \cdot 1r) \Big(0 \cdot 95 + \frac{1 \cdot 3}{\sigma_w} \Big) \phi(M) \, \sec^{3/2} \Big(\beta - \frac{\pi}{16} \Big) \\ \phi(M) &= (1 - M^2)^{1/4}, \, 0 < M < 0 \cdot 9 \\ &= 0 \cdot 66 \,, \qquad 0 \cdot 9 < M < 1 \cdot 0 \,. \end{split}$$

It should be noted that the suggested modifications are based on results for unswept wings of one particular plan-form. Further experimental and theoretical verification is required for their acceptance.

6. Conclusions.—The technique of using ground-launched rockets provides a means for flutter investigation at high-subsonic and supersonic speeds, including transonic conditions. The technique can be applied to the empirical determination of the effects of relevant parameters on the flutter characteristics and to the development of more accurate theoretical prediction of flutter characteristics.

The general conclusions from tests on unswept, untapered wings are as follows:

(a) The effects of acceleration on flutter characteristics are negligible for these models, within the range of acceleration from 19g to 31g.

(b) The effect of compressibility on the fixed-root flutter speed is to reduce the margin of the true flutter speed above the speed calculated using two-dimensional incompressible flow theory from about + 50 per cent at M = 0.4 to - 25 per cent at M = 0.9.

(c) The effect of compressibility on the flutter frequency is to reduce this frequency towards the wing fundamental bending frequency.

(d) Modifications to the wing torsional stiffness criterion are suggested to allow for the effects of compressibility and wing relative density on flutter speed. The suggested compressibility function is:

$$egin{aligned} \phi(M) &= (1 - M^{\, extsf{2}})^{1/4}, \, 0 < M < 0.9 \ &= 0.66 \;, \qquad 0.9 < M < 1.0 \;. \end{aligned}$$

The suggested wing relative density function is:

$$\phi(\sigma_w) = \left(0.95 + \frac{1.3}{\sigma_w}\right)$$

7. Further Developments.—Similar investigations to the above are to be made on wings of different plan-forms. Tests are in progress on untapered wings swept back to an angle of 40 deg. The technique is also being developed for *ad hoc* flutter tests on scale models of specific aircraft.

Acknowledgment.—Acknowledgments are due to the staff of Guided Weapons Department, Trials Division, for assistance given in the calibration and testing of these models.

Experimental results										<u> </u>	Theoret	retical results Batio: Experiment						nt																
										Incompressible Compressible								TT M fc TT					T					ble	Theory					
Model No.	lφ	m_{θ}	h	g		w	n _B	'nT	Vc	ⁿ c	ωC		G	<i>V</i> _{<i>F</i>}	V_1	<i>n</i> ₁	ω1	V_2	n ₂	ω2	V_C/V_1	n_C/n_1	V_C/V_2	n_C/n_2										
$ 1101 \\ 1102 \\ 1103 \\ 1104 $	1290 1420 3580 3180	680 533 2180 1890	$0.25 \\ 0.29 \\ 0 \\ 0 \\ 0$	$0.47 \\ 0.47 \\ 0.47 \\ 0.47 \\ 0.47$	$ \begin{array}{c} 0.30 \\ 0.29 \\ 0.29 \\ 0.29 \\ 0.29 \end{array} $	$1 \cdot 02 \\ 0 \cdot 55 \\ 0 \cdot 68 \\ 0 \cdot 61$	22 30 39 38	88 96 159 148	740 650	$\begin{bmatrix} 43\\58\\ \end{bmatrix}$ W	0.36 0.56 ing sk	0.66 0.58 in failu	26 26 ires amic	835 760 940 960	537 496	47 59	$0.55 \\ 0.74$	540 508	40 53	$\begin{array}{c} 0\cdot47\\ 0\cdot66 \end{array}$	$1.38 \\ 1.31$	$\begin{array}{c} 0.91 \\ 0.98 \end{array}$	$1 \cdot 37 \\ 1 \cdot 28$	1.08 1.09										
1105 1106 1108 1109 1110	3050 3050 1930 1830	1940 1880 707 732 702	$ \begin{array}{c} 0 \\ 0 \\ 0 \cdot 34 \\ 0 \cdot 26 \\ 0 \cdot 24 \end{array} $	$ \begin{array}{c} 0 \cdot 47 \\ 0 \cdot 47 \\ \end{array} $	0.29 0.29 0.29 0.29 0.29 0.29	$0.66 \\ 0.59 \\ 0.71 \\ 0.71 \\ 0.71$	35 35	134	740 720	f su 60 Teler 60	ction 0.51 metry 0.52	loads 0.66 failure 0.64	28	950 950 755 840 780	549 534 520	61 61 59	$0.69 \\ 0.72 \\ 0.72$	578 566 547	55 54 54	$0.59 \\ 0.60 \\ 0.62$	1·35 1·38	$\begin{array}{c} 0.98\\ 1.02 \end{array}$	$1 \cdot 28$ $1 \cdot 31$	1.09 1.10										
1111 1112 1113 1114	1650 1650 1690 1630	712 546 532 583	$0 \cdot 24 \\ 0 \cdot 24 \\ 0 \cdot 22 \\ 0 \cdot 20 \\ 0 \cdot 23$	$ \begin{array}{c} 0 \cdot 47 \\ 0 \cdot 47 \\ 0 \cdot 47 \\ 0 \cdot 47 \\ 0 \cdot 47 \end{array} $	$ \begin{array}{c} 0 \cdot 29 \\ \end{array} $	0.71 0.55 0.55 0.55	32 33 33	96 96 98	715 640 655 640	59 60 58 58	$ \begin{array}{c} 0 \cdot 52 \\ 0 \cdot 58 \\ 0 \cdot 56 \\ 0 \cdot 57 \\ \end{array} $	$ \begin{array}{c} 0.64 \\ 0.57 \\ 0.59 \\ 0.57 \end{array} $	22 19 23 23	790 710 760 760	528 474 462 509	60 57 61 60	$0.71 \\ 0.75 \\ 0.83 \\ 0.74 \\ 0.72$	558 495 470 516	54 53 54 56	0.61. 0.67 0.73 0.68	$1 \cdot 35$ $1 \cdot 35$ $1 \cdot 42$ $1 \cdot 26$ $1 \cdot 97$	$ \begin{array}{c} 0.98 \\ 1.05 \\ 0.95 \\ 0.97 \\ 0.08 \end{array} $	$1 \cdot 28 \\ 1 \cdot 29 \\ 1 \cdot 39 \\ 1 \cdot 24 \\ 1 \cdot 25$	$1 \cdot 09$ $1 \cdot 13$ $1 \cdot 07$ $1 \cdot 04$ $1 \cdot 09$										
$ 1115 \\ 1116 \\ 1117 \\ 1118 \\ 1119 \\ 1119 $	$ 1500 \\ 1650 \\ 1450 \\ 1600 \\ 2180 $	568 351 315 394 382	$ \begin{array}{c} 0 \cdot 26 \\ 0 \cdot 30 \\ 0 \cdot 32 \\ 0 \cdot 31 \\ 0 \cdot 30 \end{array} $	$ \begin{array}{c} 0 \cdot 47 \\ 0 \cdot 33 \\ 0 \cdot 47 \\ 0 \cdot 39 \\ 0 \cdot 47 \end{array} $	$ \begin{array}{c c} 0 \cdot 29 \\ 0 \cdot 29 \\ 0 \cdot 28 \\ 0 \cdot 31 \\ 0 \cdot 28 \\ \end{array} $	$ \begin{array}{c} 0 \cdot 55 \\ 1 \cdot 12 \\ 1 \cdot 00 \\ 1 \cdot 40 \\ 1 \cdot 16 \end{array} $	31 23	97 61	640 495 590	60 Tele: 34 25 Tele:	0.59 metry 0.31 0.26 metrv	10.57 failure 0.44 0.53 failure	31 23 27	765 740 565 730 649	505 581 346 452 371	34 37 31 39	$ \begin{array}{c} 0.76 \\ 0.36 \\ 0.68 \\ 0.43 \\ 0.65 \end{array} $	592 340 440 360	55 32 32 29 34	$ \begin{array}{c} 0.87 \\ 0.34 \\ 0.59 \\ 0.42 \\ 0.58 \end{array} $	$1 \cdot 27$ $1 \cdot 43$ $1 \cdot 31$	0.93 0.92 0.81	$1 \cdot 25$ $1 \cdot 46$ $1 \cdot 34$	1.06 0.86										
1121 1122 1123 1126	2110 2500 1850 3860	395 384 417 4050	$0.29 \\ 0.30 \\ 0.29 \\ 0.36$	$ \begin{array}{c} 0 \cdot 34 \\ 0 \cdot 47 \\ 0 \cdot 40 \\ 0 \cdot 47 \end{array} $	$ \begin{array}{c c} 0 & 20 \\ 0 & 29 \\ 0 & 28 \\ 0 & 31 \\ 0 & 28 \end{array} $	$1 \cdot 25$ $1 \cdot 40$ $1 \cdot 24$ $2 \cdot 15$			655 520 610	32 35 32 Diverg	$\begin{vmatrix} 0.33 \\ 0.40 \\ 0.33 \\ ence t \end{vmatrix}$	$\begin{vmatrix} 0.59 \\ 0.47 \\ 0.55 \\ est mo$	25 25 22 del	800 650 740 1130	560 362 473	35 35 36	$ \begin{array}{c} 0.39 \\ 0.60 \\ 0.48 \end{array} $	546 354 463	33 32 33	$0.38 \\ 0.57 \\ 0.45$	$1 \cdot 17$ $1 \cdot 44$ $1 \cdot 29$	$ \begin{array}{c} 0.91 \\ 1.00 \\ 0.89 \end{array} $	$1 \cdot 20$ $1 \cdot 47$ $1 \cdot 32$	$ \begin{array}{c} 0.97 \\ 1.09 \\ 0.97 \\ 0.97 \end{array} $										
1127 1128 1136 1137	4370 3130 5090 4700	906 1180 2290 1950	$ \begin{array}{c} 0 \cdot 32 \\ 0 \cdot 30 \\ 0 \cdot 26 \\ 0 \cdot 25 \\ 0 \cdot 25 \\ \end{array} $	$ \begin{array}{c} 0.38 \\ 0.40 \\ 0.37 \\ 0.38 \\ 0.38 \end{array} $	$ \begin{array}{c} 0 \cdot 22 \\ 0 \cdot 29 \\ 0 \cdot 25 \\ 0 \cdot $	1.78 2.29 1.96 1.89	31 25 27 31	95 94 130 120	845 1090 1010	38 Tele 27 28	0.28 metry 0.16 0.17	0.79 failure 0.98 0.90 0.85	25	$ \begin{array}{r} 1010 \\ 1005 \\ 1000 \\ 1020 \end{array} $	832 821 1340 1160	47 39 65 59 61	$ \begin{array}{c} 0.35 \\ 0.30 \\ 0.30 \\ 0.32 \\ 0.31 \end{array} $	811 810 1180 1000 1060	41 36 51 47 49	$ \begin{array}{c} 0.32 \\ 0.28 \\ 0.27 \\ 0.29 \\ 0.29 \\ 0.29 \end{array} $	1.02 0.81 0.87 0.78	$ \begin{array}{c c} 0.81 \\ 0.42 \\ 0.47 \\ 0.52 \end{array} $	1.04 0.92 1.01 0.89	0.93 0.53 0.60 0.65										
1138 1139 1140 1141 1142	$ \begin{array}{r} 4000 \\ 3350 \\ 2210 \\ 5590 \\ 4850 \end{array} $	$ \begin{array}{c c} 1940 \\ 1530 \\ 1270 \\ 1420 \\ 1550 \\ \end{array} $	$ \begin{array}{c c} 0 \cdot 26 \\ 0 \cdot 25 \\ 0 \cdot 24 \\ 0 \cdot 22 \\ 0 \cdot 23 \\ \end{array} $	0.38 0.39 0.36 0.38 0.37	$ \begin{array}{c c} 0.25 \\ 0.27 \\ 0.24 \\ 0.25 \\ 0.25 \end{array} $	$1 \cdot 78$ $1 \cdot 67$ $2 \cdot 31$ $2 \cdot 00$ $2 \cdot 13$	33 31 24 27 28	119 112 99 107 104	943 856 875 945	$\begin{vmatrix} 32\\ 34\\ 24\\ Te\\ 40 \end{vmatrix}$	0 · 25 0 · 17 lemetr 0 · 27	0.77 0.78 0.78 0.85	25 26 re 43	985 1040 900 1050	993 912 832 891	55 43 54 51	$ \begin{array}{c} 0.35 \\ 0.29 \\ 0.41 \\ 0.36 \end{array} $	875 766 700 773	43 34 45 41	$ \begin{array}{c} 0.31 \\ 0.28 \\ 0.40 \\ 0.33 \end{array} $	$ \begin{array}{r} 0 \cdot 86 \\ 0 \cdot 96 \\ 1 \cdot 06 \end{array} $	$ \begin{array}{c c} 0.62 \\ 0.56 \\ 0.78 \\ \end{array} $	$0.98 \\ 1.14 \\ 1.22$	0.79 0.71 0.97										
1143	3590	1450	0.22	0.38	0.26	1.83	27	114	835	28	0.20	0.75	42	990	850	54	0.40	731	41	0.35	0.98	0.52	1.14	0.08										

TABLE 1Experimental and Theoretical Results

Notation

 l_{ϕ} Wing bending stiffness measured at 0.7s, lb ft/radn

 m_{θ}^{τ} Wing twisting stiffness measured at 0.7s, lb ft/radn

- \check{h} Distance of wing flexural axis aft of leading edge \div wing chord
- g Distance of wing mertra axis aft of leading edge \div wing chord
- K_G Radius of gyration of wing section about inertia axis \div wing chord
- w Wing weight per foot run, lb/ft
- n_B Wing fundamental flexure frequency, cycles/sec
- $\tilde{n_T}$ Wing fundamental torsion frequency, cycles/sec
- G Gravitational acceleration, ft/sec²

- V_C Critical flutter speed, ft/sec
- n_{C} Flutter frequency, cycles/sec
- ω_C Flutter frequency parameter
- $\dot{M_C}$ Mach number at critical flutter speed
- $f_{\rm C}$ Rocket acceleration at critical flutter speed, ft/sec²
- V_F Speed at wing failure, ft/sec
- $V_{1,2}$ Calculated flutter speeds, ft/sec
- $n_{1,2}$ Calculated flutter frequencies, cycles/sec
- $\omega_{1,2}$ Calculated flutter frequency parameters

TABLE 2

Model No.	l_{ϕ}	m_{θ}	r	g	h	$ ho_w G$ (lb/cu ft)	V_A	V_B	Vc	M_{C}	$\frac{V_C}{V_A}$	$\frac{V_C}{V_B}$
1101	1290	680	0.59	0.47	0.25	1.02	631	662	740	0.66	1.17	1.12
1102	1420	533	0.84	0.47	0.29	$\hat{0}\cdot 55$	563	638	650	0.58	1.15	$1 \cdot 02$
1103	3580	2180	0.51	0.47	0	0.68	920	1010	000			1 . 02
1104	3180	1890	0.52	0.47	Ŏ	0.61	855	953				
1105	3040	1940	0.49	0.47	Ō	0.66	869	960				
1106	3050	1880	0.50	0.47	Ō	0.59	856	958				
1108	1930	• 707	0.85	0.47	0.34	0.71	684	747	740	0.66	1.08	0.99
1109	1830	732	0.77	0.47	0.26	0.71	648	707				
1110	1900	702	0.84	0.47	0.24	0.71	618	675	720	0.64	1.16	1.07
1111 ′	1660	712	0.72	0.47	0.24	0.71	628	686	715	0.64	1.14	1.04
1112	1560	546	0.88	0.47	0.22	0.55	534	605	640	0.57	$1 \cdot 20$	1.06
1113	1690	532	0.98	0.47	0.20	0.55	511	.578	655	0.59	1.28	1.13
1114	1630	583	0.86	0.47	0.23	0.55	556	630	640	0.57	1.15	1.02
1115	1500	568	0.81	0.47	0.26	0.55	567	642	640	0.57	1.13	1.00
1116	1650	351	1.45	0.33	0.30	$1 \cdot 12$	694	721				
1117	1450	315	$1 \cdot 42$	0.47	0.32	$1 \cdot 00$	418	440	495	0.44	$1 \cdot 18$	1.12
1118	1600	394	$1 \cdot 26$	0.39	0.31	$1 \cdot 40$	603	617	590	0.53	0.98	0.96
1119	2180	382	1.76	0.47	0.30	1.16	434	450				
1121	2110	395	1.65	0.34	0.29	$1 \cdot 25$	684	705	655	0.59	0.96	0.93
1122	2500	384	$2 \cdot 01$	0.47	0.30	$1 \cdot 40$	423	433	520	0.47	1.23	$1 \cdot 20$
1123	1850	417	1.37	0.40	0.29	$1 \cdot 24$	581	599	610	0.55	$1 \cdot 05$	$1 \cdot 02$
1126	3860	4050	0.29	0.47	0.36	2.15	1780	1770				
1127	4370	906	$1 \cdot 48$	0.38	0.32	1.78	932	937	845	0.79	0.91	0.90
1128	3130	1180	0.82	0.40	0.30	$2 \cdot 29$	1050	1040				
1136	5090	2290	0.69	0.37	0.26	1.96	1590	1590	1090	0.98	0.69	0.69
1137	4700	1950	0.75	0.38	0.25	1.86	1380	1383	1010	0.90	0.73	0.73
1138	4000	1940	0.64	0.38	0.26	1.78	1410	1417	945	0.85	0.67	0.67
1139	3350	1530	0.67	0.39	0.25	1.67	1200	1212	856	0.77	0.70	0.71
1140	2210. 5500	1270	0.54	0.36	0.24	2.31	1220	1210	875	0.78	0.72	0.72
1141	3390	1420	1.22	0.38 0.27	0.22	2.00	1110	1110	0.45	0.05	0 77	0 77
1144	4800	1000	0.97	0.32	0.23	2.13	1230	1230	945	0.85	0.77	0.77
1140	3390	1400	0.11	0.30	0.77	1.02	1100	1100	892	0.12	0.72	0.12

Comparison of Experimental and Criterion Flutter Speeds

LIST OF SYMBOLS

$$\begin{split} V_{A} &= \left(\frac{m_{\theta}}{\rho_{\theta}dc_{m}^{2}}\right)^{1/2} \frac{(0\cdot9-0\cdot33k)(1-0\cdot1r)\sec^{3/2}\left(\beta-\frac{\pi}{16}\right)}{0\cdot9(g-0\cdot1)(1\cdot3-k)} \\ V_{B} & \left(\frac{m_{\theta}}{\rho_{\theta}dc_{m}^{2}}\right)^{1/2} \frac{(0\cdot9-0\cdot33k)(1-0\cdot1r)\left(0\cdot95+\frac{1\cdot3}{\sigma_{w}}\right)\sec^{3/2}\left(\beta'-\frac{\pi}{16}\right)}{0\cdot9(g-0\cdot1)(1\cdot3-k)} \\ V_{C} & \text{Critical flutter speed, ft/sec} \\ M_{C} & \text{Mach number at critical flutter speed} \\ g & \text{Gravitational acceleration, ft/sec}^{2} \\ c_{m} & \text{Wing mean chord, ft} \\ d & \text{Wing length to equivalent tip } (=0\cdot9s) \\ g & \text{Distance of inertia axis aft of leading edge ÷ wing chord} \\ k & \text{Wing taper ratio} \left(=\frac{\text{Tip chord}}{\text{Root chord}}\right) \\ h & \text{Distance of flexural axis aft of leading edge ÷ wing chord} \\ l_{4} & \text{Wing flexural stiffness measured at } 0\cdot7s, \text{ lb ft/radn} \\ m_{\theta} & \text{Wing torsional stiffness measurement at } 0\cdot7s, \text{ lb ft/radn} \\ r & \text{Stiffness ratio} \left(=\frac{l_{\theta}}{d^{3}}/\frac{m_{\theta}}{dc_{m}^{2}}\right) \\ s & \text{Wing density at sea-level, slugs/cu ft} \\ \rho_{w} & \text{Wing relative density} \left(=\frac{\text{Density of wing}}{\text{Density of surrounding air}}\right) \end{split}$$

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FIG. 1. General arrangement of rocket model. Third angle projection.

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FIG. 2. Rocket model on launcher.















FLUTTER FREQUENCY 58 CYCLES/SEC.

FIG. 7. Telemeter record of wing oscillations.



FIG. 8. Velocity-time curve from Doppler measurement.



FIG. 9. Typical records of flight characteristics.



FIG. 10. Flutter speed and frequency ratios vs. acceleration at flutter.



FIG. 11. Effect of compressibility on calculated flutter values (models 1136-1139)



FIG. 12. Flutter speed and frequency ratios vs. Mach number at flutter





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