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Summary.—This report is an extension of previous theoretical investigations of the elastic buckling in shear of flat plates reinforced by transverse stiffeners. The plates are treated as infinitely long and simply-supported along the long sides. Stiffeners are spaced at regular intervals, dividing the plate into a number of panels of uniform size. The effect of bending and torsional stiffnesses of the stiffeners upon the buckling shear stress is calculated for the complete range of stiffnesses, for panels with ratios of width to stiffener spacing of 1, 2 and 5. The results are presented in tabular and graphical forms.

1. *Introduction*.—Accurate knowledge of the buckling shear stresses of flat plates is necessary for the design of web beams and of panels in fuselage skins and wing surfaces. In this report buckling stresses are obtained for infinitely-long flat plates, simply-supported along the edges and reinforced by equally spaced transverse stiffeners that have both bending and torsional stiffness. But these buckling stresses can be used with little error in calculating the buckling shear stress of finite sheet-stringer combinations if the buckling pattern is repeated several times along the length.

Buckling shear stresses of infinitely-long plates with transverse stiffeners were calculated by Schmieden[1], Seydel[2] and Wang[3] for plates with weak stiffeners, and by Budiansky, Conner and Stein[5] for plates divided into square panels by stiffeners of infinite bending rigidity. Timoshenko[4] found the minimum bending stiffness at which the stiffeners remained undeflected at the buckling stress. More recently Stein and Fralich[6] obtained buckling shear stresses for the complete range of bending stiffness of the stiffeners which were spaced at 1, $\frac{1}{2}$ and $\frac{1}{5}$ the width of the plate.

In all the above investigations no account was taken of the torsional rigidity of the stiffeners. The aim of this report is to calculate the buckling shear stresses of infinitely-long plates for all combinations of the bending and torsional stiffnesses of the stiffeners. The calculations are carried out for different values of a third parameter, the ratio of the width of the panel to the stiffener spacing. The stiffeners are taken as being identical and equally spaced along the length of the plate.

2. *Presentation and Discussion of Results*.—The method adopted for the solution of this problem is an extension of that used by Stein and Fralich. It is an energy method using the Lagrangian Multiplier for the imposition of the stiffener restraints. An assumption is made that the stiffeners act along transverse lines in the central plane of the plate. This assumption incurs little error when the ratio of stiffener flange width to stiffener spacing is small. The effect of offsetting the flexural and torsional axes of the stiffeners from the central plane of the plate is not considered.

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In Appendix I a solution for plates with weak stiffeners is obtained by assuming that the plates are orthotropic. A double Fourier series is used for the deflection function, from which the internal bending energy and external work done are obtained by substitution in energy integrals. These integrals are minimised to yield the buckling parameter.

In Appendix II a solution is obtained for plates reinforced by more rigid stiffeners. By the use of Fourier series for the stiffener deflection and twisting functions, expressions are obtained for the internal bending energy of the plate, for the energy of deformation of the stiffeners, and for the external work done. Taking into account the relations existing between plate and stiffener deformations when minimising these expressions the buckling parameter is again obtained.

The buckling parameters are

$$k_s' = \frac{\tau b^2 t}{\pi^2 D}$$

and

$$k_s = \frac{\tau d^2 t}{\pi^2 D} = \frac{d^2}{b^2} k_s' .$$

These are calculated for various combinations of the bending parameter EI/Dd and the torsional parameter J/Dd for the ratios of b/d equal to 1, 2 and 5. The curves are plotted in Figs. 2, 3 and 4 for the complete range of values of EI/Dd from 0 to infinity.

The results show firstly that for constant torsional stiffness (*i.e.*, J/Dd constant), the buckling shear stress parameter increases rapidly as the bending stiffness EI/Dd increases from zero to some intermediate value depending on the aspect ratio (b/d) of the panels. This part of the curve corresponds to a buckling pattern which extends across a number of stiffeners. As the bending rigidity of the stiffeners increases the buckling pattern becomes more dependent on stiffener spacing and rigidity and suddenly changes to a pattern in which the stiffeners have only relatively small deformations. To this change of pattern there corresponds a point of discontinuity on the curve of k_s against EI/Dd. From this point the buckling shear stress increases less rapidly with stiffener bending stiffness, until finally a further point of discontinuity is reached, after which the buckling shear stress is almost independent of EI/Dd. This final state corresponds to a buckling pattern which is almost entirely confined to the region between the stiffeners.

The effect of increase in the torsional stiffness of the stiffeners is an increase in the buckling shear stress. This increase is small for low values of EI/Dd. Increase of J/Dd also decreases the values of EI/Dd at which the buckling pattern changes. Thus as EI/Dd increases, an increase in J/Dd leads to larger increases in the buckling shear stress. For the region in which the buckling parameter is almost independent of EI/Dd, the effect of increase of J/Dd is an increase in the edge constraint of the panel and hence an increase in the buckling shear stress. In the extreme case of infinite bending and infinite torsional stiffnesses of the stiffeners, the buckling shear stress becomes equal to that of a panel that is simply supported along two sides and clamped along the other two sides.

In most cases the calculations for $J/Dd = 0$ agree with the results obtained by Stein and Fralich[6]. The small differences obtained in a few cases can be attributed to the smaller number of terms in the series for the deflection mode in this report, and usually occurred at large values of EI/Dd.

For the limiting case of EI/Dd and J/Dd both infinite, the use of insufficient terms in the deflection function led to values which were 5 to 10 per cent high. As the inclusion of extra terms in the deflection function would necessitate an excessive amount of computation, more accurate values of the buckling shear stress are taken from the Royal Aeronautical Society Data Sheets[7].

In practice the range of bending stiffnesses encountered is from $EI/Dd = 10$ to 1000. As the ratio of EI/J for a circular section stiffener is $1 + \nu$ the practical range of torsional stiffnesses varies from 0 to 700. For normal top-hat section stringers the practical range of EI/J varies from $1 \cdot 5$ to 10. For this type of stiffener the results show an appreciable increase in the buckling shear stress over the value obtained by neglecting entirely the torsional stiffness of the section.

For open-section stringers the effect of torsional stiffness is much less marked and can be neglected. This assumption is justified because the usual method of attaching the stiffener to the plate, *i.e.*, by one line of riveting, does not ensure integral action between plate and stiffener under a twisting action. It is expected that the use of a good bonded joint between plate and stringer will give slightly higher values for the buckling shear stress of the combination than the use of a riveted joint.

There has been no experimental check of these results but the results of experiments carried out by Stein and Fralich[6] give reasonable agreement with the values calculated for $J = 0$ although some of the experimental values are a little higher than the calculated ones. This difference could be explained by the small effect of torsional stiffness of the stiffeners in their experiments.

3. *Example on the Use of the Results.*—Consider a sheet-stringer combination which is subjected to shearing stresses. The sheet is taken to be simply-supported along the edges normal to the stringer direction and continuous over a number of panels. The stringers are of top-hat section and divide the sheet into panels of uniform size. The dimensions of the combination are taken as

sheet thickness	t	$= 0 \cdot 040$ in.
stiffener spacing	d	$= 8 \cdot 0$ in.
width of sheet	b	$= 40$ in.
thickness of stringer material		$= 0 \cdot 040$ in.
depth of stringer		$= 1$ in.
width of stringer		$= \frac{3}{4}$ in.
width of stringer flanges		$= \frac{3}{8}$ in.
Poisson's ratio	ν	$= 0 \cdot 30$

The stringers are attached to one side of the sheet only and are of the same material as the sheet. It is assumed that the stringers act in the central plane of the sheet.

The moment of inertia of stringer and $1\frac{1}{2}$ in. of sheet is $0 \cdot 0326$ in^4.

Thus

$$\frac{EI}{Dd} = 696.$$

The torsional rigidity

$$J = \frac{4A^2G}{\int \frac{ds}{t}} = 0 \cdot 0132E.$$

Thus

$$\frac{J}{Dd} = 281.$$

Thus from Fig. 4

$$k_s = 9 \cdot 2$$

The value obtained by neglecting the torsional stiffness of the stringer would be

$$k_s = 5 \cdot 6$$

This therefore constitutes an increase of 64 per cent in the buckling shear stress.

4. *Conclusions.*—The buckling shear stresses of simply-supported, infinitely-long plates with transverse stiffeners have been calculated for various values of the bending and torsional stiffnesses of the stiffeners. It has been shown that the torsional stiffness of closed section stringers is sufficient to cause a considerable increase in the buckling shear stress. For open-section stringers the effect of the torsional stiffness on the buckling shear stress is much less marked and can be neglected.

LIST OF SYMBOLS

τ	Buckling shear stress
k_s'	Buckling parameter $= \dfrac{\tau b^2 t}{\pi^2 D} = k_s \dfrac{b^2}{d^2}$
t	Plate thickness
b	Width of plate
d	Spacing of stiffeners
D	The bending rigidity of the plate $= \dfrac{Et^3}{12(1 - \nu^2)}$
E	Modulus of elasticity of material in stringer and plate (assumed equal but not necessary for the purposes of this report)
I	Moment of inertia of stiffener about an axis parallel to the longer side of the plate
J	Torsional rigidity of stiffener
B =	EI/Dd a bending stiffness parameter
Γ =	J/Dd a torsional stiffness parameter
ν	Poisson's ratio for sheet and stiffener material
x, y	Co-ordinate system whose axes are in the longitudinal and transverse directions respectively
w	Deflection of the plate normal to its surface
$(w_s)_i$	Deflection of the ith stiffener normal to the plate surface
$(\theta_s)_i$	Rotation of the ith stiffener = slope of plate in x-direction at stiffener
λ	Half wave-length of buckles in Appendix I
a_n, b_n	Coefficients of deflection function—Appendix I
a_{mn}, b_{mn}	Coefficients of deflection function—Appendix II
Δ_{ni}, Δ_n	Coefficients of stiffener deflection function—Appendix II

V_{ni}', V_n' and V_{ni}, V_n	Coefficients of stiffener rotation function—Appendix II
γ_n, δ_n	Lagrangian multipliers
V	Internal bending strain energy of plate
V_s	Internal bending and torsional strain energy of the stiffeners
T	External work done by applied shear stresses

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APPENDIX I

The Buckling Shear Stress for Low Stiffness Values of the Stiffeners from an Orthotropic Plate Solution

When the transverse stiffeners are weak and unable to prevent buckling taking place across a number of stiffeners, the sheet-stringer combination may be treated as an orthotropic plate, *i.e.*, in this case, a plate which has greater torsional and bending stiffness in the transverse direction than it has in the longitudinal direction. As the stiffeners are increased in rigidity, the buckling pattern becomes more restricted and more dependent on stiffener spacing, until eventually it is confined to areas of the plate between the stiffeners. At certain values of the bending and torsional rigidities, a lower buckling stress is obtained by considering the stiffeners as discrete elements. This part of the solution is given in Appendix II. At the moment, however, the stiffeners are considered as being uniformly distributed along the length of the plate.

The solution is obtained by a strain-energy method.

The internal bending energy of the plate is[4]

$$V = \frac{D}{2} \iint \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \, \partial y} \right)^2 \right] \right\} dx \, dy \quad \ldots \quad (1)$$

integration being carried out over one complete buckling pattern.

The internal bending energy of the stiffeners is

$$V_s = \frac{EI}{2d} \iint \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx \, dy + \frac{J}{2d} \iint \left(\frac{\partial^2 w}{\partial x \, \partial y} \right)^2 dx \, dy \, , \quad \ldots \quad (2)$$

the first term being the internal energy due to bending, and the second, the internal energy due to twisting.

The external work of the shear stresses is

$$T = -\tau t \iint \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \, dx \, dy \, . \quad \ldots \quad (3)$$

It is now assumed that the buckling pattern of the plate can be represented by the following expression for w

$$w = \sin \frac{\pi x}{\lambda} \sum_{n=2,4}^{\infty} a_n \sin \frac{n \pi y}{b} + \cos \frac{\pi x}{\lambda} \sum_{n=1,3}^{\infty} b_n \sin \frac{n \pi y}{b} \quad \ldots \quad (4)$$

where λ is the half-wavelength of the pattern in the longitudinal (x)-direction and b and d the dimensions of the plate as shown in Fig. 1. This expression satisfies the simply-supported boundary condition at the edges $y = 0$ and $y = b$, term by term.

On substituting the expression for w in the energy integrals and integrating over the limits 0 to b and 0 to λ, we obtain,

$$V = \frac{D \lambda \pi^4}{8b^3} \left[\sum_{n=2,4}^{\infty} a_n^2 \left(\frac{b^2}{\lambda^2} + n^2 \right)^2 + \sum_{n=1,3}^{\infty} b_n^2 \left(\frac{b^2}{\lambda^2} + n^2 \right)^2 \right]$$

$$V_s = \frac{EI \pi^4 \lambda}{8b^3 d} \left[\sum_{n=2,4}^{\infty} a_n^2 n^4 + \sum_{n=1,3}^{\infty} b_n^2 n^4 \right] + \frac{J \pi^4}{8 \lambda b d} \left[\sum_{n=2,4}^{\infty} a_n^2 n^2 + \sum_{n=1,3}^{\infty} b_n^2 n^2 \right]$$

$$T = 2\tau t \pi \sum_{n=1,3}^{\infty} \sum_{q=2,4}^{\infty} a_q b_n \frac{nq}{n^2 - q^2} \, .$$

By combining these expressions we obtain the following relation in terms of the potential energy $(V + V_s - T)$

$$\begin{aligned} (V + V_s - T) \frac{8b^3}{D \lambda \pi^4} = & \sum_{n=2,4}^{\infty} a_n^2 \left[\left(\frac{b^2}{\lambda^2} + n^2 \right)^2 + n^4 \frac{EI}{Dd} + n^2 \frac{b^2}{\lambda^2} \frac{J}{Dd} \right] \\ & + \sum_{n=1,3}^{\infty} b_n^2 \left[\left(\frac{b^2}{\lambda^2} + n^2 \right)^2 + n^4 \frac{EI}{Dd} + n^2 \frac{b^2}{\lambda^2} \frac{J}{Dd} \right] \\ & - \frac{16 k_s' b}{\pi \lambda} \sum_{n=1,3}^{\infty} \sum_{q=2,4}^{\infty} a_q b_n \frac{nq}{(n^2 - q^2)} \end{aligned} \quad \ldots \quad (5)$$

where $k_s' = \tau t b^2 / D \pi^2$, the shear buckling parameter.

Minimising the potential energy with respect to a_n and b_n we obtain the following set of linear equations,

$$a_n \left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4 \frac{EI}{Dd} + \frac{n^2 b^2}{\lambda^2} \frac{J}{Dd}\right] - \frac{8b}{\pi\lambda} k_s' \sum_{q=1,3}^{\infty} b_q \frac{nq}{(q^2 - n^2)} = 0$$

$$(n = 2, 4, \ldots\ldots.) \qquad (6)$$

$$b_n\left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4 \frac{EI}{Dd} + n^2 \frac{b^2}{\lambda^2} \frac{J}{Dd}\right] - \frac{8b}{\pi\lambda} k_s' \sum_{q=2,4}^{\infty} a_q \frac{nq}{(n^2 - q^2)} = 0$$

$$(n = 1, 3, \ldots\ldots.) \qquad (7)$$

Substituting for a_n from equation (6) in equation (7), there results

$$\begin{bmatrix} C_{11} & C_{13} & C_{15} \ldots\ldots\ldots \\ C_{31} & C_{33} & C_{35} \ldots\ldots\ldots \\ C_{51} & C_{53} & C_{55} \ldots\ldots\ldots \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} b_1 \\ b_3 \\ b^5 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = 0 \qquad (8)$$

in which

$$C_{nn} = \left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4 \frac{EI}{Dd} + \frac{n^2 b^2}{\lambda^2} \frac{J}{Dd}\right]$$

$$- \left(\frac{8bk_s'}{\pi\lambda}\right)^2 \sum_{q=2,4}^{\infty} \frac{n^2 q^2}{(n^2 - q^2)\left[\left(\frac{b^2}{\lambda^2} + q^2\right)^2 + q^4 \frac{EI}{Dd} + q^2 \frac{b^2}{\lambda^2} \frac{J}{Dd}\right]}$$

$$C_{rn} = C_{nr} = -\left(\frac{8k_s' b}{\pi\lambda}\right)^2 \sum_{q=2,4}^{\infty} \frac{rnq^2}{(n^2 - q^2)(r^2 - q^2)\left[\left(\frac{b^2}{\lambda^2} + q^2\right)^2 + q^4 \frac{EI}{Dd} + \frac{q^2 b^2}{\lambda^2} \frac{J}{Dd}\right]}$$

for $n \neq r$.

A solution exists if a value of k_s' can be found which makes the determinant of the matrix C vanish.

By considering only the term C_{11} we can obtain a first approximation to the solution. This is seen to be equivalent to assuming a buckling pattern which is represented by the b_1 and all the a_n terms. A second approximation which is equivalent to assuming a buckling pattern represented by the b_1, b_3 and the a_n terms is obtained by equating

$$C_{11}C_{33} - C_{13}^2 = 0.$$

This equation gives k_s' as a function of b/λ, EI/Dd and J/Dd. On minimising k_s' with respect to b/λ, the buckling parameter is obtained as a function of EI/Dd and J/Dd.

Further approximations can be obtained from the matrix C but the small increase in accuracy gained is insufficient to justify the increase in computation required.

APPENDIX II

The Buckling Shear Stress for Higher Values of the Bending and Torsional Stiffnesses of the Transverse Stiffeners

As the stiffnesses of the stiffeners increase, the buckling pattern of the plate becomes much more dependent on stiffener spacing than it is for the low stiffnesses in Appendix I. For this reason the buckling configuration is now assumed to be periodic over an integral number of panels and that it is one of two types, symmetric or antisymmetric about the midpoint of each bay. The stiffeners are to be considered as discrete elements and not uniformly distributed over the whole plate. It is necessary to ensure that their deflections and angles of twist are identical with the plate deflections and angles of twist along the lines of attachment. This is done by use of the Lagrangian Multiplier.

Although the following five buckling configurations were investigated, *i.e.*,

(i) symmetric buckling periodic over each panel

(ii) antisymmetric buckling periodic over each panel

(iii) symmetric buckling periodic over two panels

(iv) antisymmetric buckling periodic over two panels

(v) symmetric buckling one panel, antisymmetric buckling next panel, periodic over four panels,

only the last two are governing cases for the ratios of width of plate to stiffener spacing considered.

The following series are assumed for the buckling patterns (iv) and (v).

Antisymmetric buckling, periodic over two panels,

$$w = \sum_{m=1,3}^{\infty} \sum_{n=2,4}^{\infty} a_{mn} \sin \frac{m\pi x}{d} \sin \frac{n\pi y}{b} + \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} b_{mn} \cos \frac{m\pi x}{d} \sin \frac{n\pi y}{b}. \quad .. \quad (1)$$

Symmetric buckling one panel, antisymmetric buckling next panel, periodic over four panels,

$$w = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} a_{mn} \left[\sin \frac{m\pi x}{2d} + (-1)^{\frac{1}{2}(m-1)} \cos \frac{m\pi x}{2d} \right] \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=1,3}^{\infty} \sum_{n=2,4}^{\infty} b_{mn} \left[\sin \frac{m\pi x}{2d} - (-1)^{\frac{1}{2}(m-1)} \cos \frac{m\pi x}{2d} \right] \sin \frac{n\pi y}{b}. \quad .. \quad (2)$$

The co-ordinate system used in these deflection functions is shown in Fig. 1. Consideration of these expressions shows that the simply-supported boundary conditions at the edges $y = 0$ and $y = b$ are satisfied, together with the conditions for plate continuity at the stiffeners $x = 0$, d, $2d$, etc.

1. *Antisymmetric Buckling, Periodic over Two Panels.*—The deflection of the plate is given by equation (1) of this Appendix. It is assumed that the deflection function of the ith stiffener is

$$(w_s)_i = \sum_{n=1,3}^{\infty} \Delta_{ni} \sin \frac{n\pi y}{b} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3)$$

and that the angular rotation of the ith stiffener is

$$(\theta_s)_i = \sum_{n=2,4}^{\infty} V_{ni}' \sin \frac{n\pi y}{b}. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (4)$$

As the buckling pattern is periodic over two panels, there are two stiffeners included in this interval, *i.e.*, $i = 1$ and 2. To satisfy the boundary conditions, stiffener deflection equals plate deflection, and stiffener rotation equals plate slope along the line of attachment, we must have,

$$\left.\begin{aligned} w(id, y) - (w_s)_i &= 0 \\ \text{and} \qquad \frac{\partial w(id, y)}{\partial x} - (\theta_s)_i &= 0 \end{aligned}\right\} (i = 1, 2)$$

which give upon substitution from (1), (3) and (4),

$$\left.\begin{aligned} \sum_{m=1,3}^{\infty} b_{mn} + \Delta_{n1} &= 0 \\ \sum_{m=1,3}^{\infty} b_{mn} - \Delta_{n2} &= 0 \end{aligned}\right\} (n = 1, 3 \ldots)$$

and

$$\left.\begin{aligned} \sum_{m=1,3}^{\infty} \frac{m\pi}{d} a_{mn} + \nabla_{n1}' &= 0 \\ \sum_{m=1,3}^{\infty} \frac{m\pi}{d} a_{mn} - \nabla_{n2}' &= 0 \end{aligned}\right\} (n = 2, 4 \ldots) .$$

From these equations it is seen that $\Delta_{n1} = -\Delta_{n2}$ and $\nabla_{n1}' = -\Delta_{n2}'$. Letting $\Delta_n = \Delta_{n1} = -\Delta_{n2}$ and $\nabla_n' = \nabla_{n1}' = -\nabla_{n2}' = (\pi/d)\nabla_n$ the boundary conditions simplify to

$$\left.\begin{aligned} \sum_{m=1,3}^{\infty} b_{mn} + \Delta_n &= 0 \quad (n = 1, 3, 5 \ldots) \\ \text{and} \qquad \sum_{m=1,3}^{\infty} m a_{mn} + \nabla_n &= 0 \quad (n = 2, 4, 6 \ldots) \end{aligned}\right\} \qquad (5)$$

The energy expressions V, V_s and T for the plate and stiffener internal energy and the external work done are,

$$\left.\begin{aligned} V &= \frac{D}{2}\int_0^b\int_0^{2d}\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right\} dx\,dy \\ V_s &= \sum_{i=1}^{2}\frac{EI}{2}\int_0^b\left[\frac{\partial^2 (w_s)_i}{\partial y^2}\right]^2 dy + \sum_{i=1}^{2}\frac{J}{2}\int_0^b\left[\frac{\partial(\theta_s)_i}{\partial y}\right]^2 dy \\ T &= -\tau t\int_0^b\int_0^{2d}\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\,dx\,dy \end{aligned}\right\} \qquad (6)$$

Substitution of the functions for w, w_s and θ_s gives the following expressions

$$\left.\begin{aligned} V &= \frac{Dd\pi^4}{4b^3}\left[\sum_{m=1,3}^{\infty}\sum_{n=2,4}^{\infty} a_{mn}^2\left(m^2\frac{b^2}{d^2} + n^2\right)^2 + \sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty} b_{mn}^2\left(m^2\frac{b^2}{d^2} + n^2\right)^2\right] \\ V_s &= \frac{EI\pi^4}{2b^3}\sum_{n=1,3}^{\infty}\Delta_n^2 n^4 + \frac{J\pi^4}{2bd^2}\sum_{n=2,4}^{\infty}\nabla_n^2 n^2 \\ T &= 4\tau t\pi\sum_{m=1,3}^{\infty}\sum_{n=2,4}^{\infty}\sum_{q=1,3}^{\infty} a_{mn}\,b_{mq}\frac{mnq}{(q^2 - n^2)} \end{aligned}\right\} \qquad (7)$$

As a first approximation to this set of equations, the b_{m1}'s and the a_{m2}'s are considered. Thus

$$A_{m1}b_{m1} + \tfrac{2}{3}mSa_{m2} = 2B\Delta_1$$

$$\tfrac{2}{3}mSb_{m1} + A_{m2}a_{m2} = 8m\Gamma' V_2 .$$

On solving for b_{m1} and a_{m2} and substituting in equations (5), we obtain

$$\begin{bmatrix} \sum\limits_{m=1,3}^{\infty} \dfrac{A_{m2}}{X_m} + \dfrac{1}{2B} , & -\sum\limits_{m=1,3}^{\infty} \dfrac{2}{3}\dfrac{m^2S}{X_m} \\ -\sum\limits_{1,3}^{\infty} \dfrac{2}{3}\dfrac{m^2S}{X_m} , & \sum\limits_{1,3}^{\infty} \dfrac{m^2 A_{m1}}{X_m} + \dfrac{1}{8\Gamma'} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ V_2 \end{bmatrix} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

where

$$X_m = A_{m1}A_{m2} - \tfrac{4}{9}m^2S^2 .$$

For a non-trivial solution of this equation, the determinant of the matrix must be set equal to zero. The resulting equation then permits solution for the buckling parameter k_s'.

As a second approximation the equations in the b_{m1}, a_{m2}, b_{m3} terms are considered. Carrying out the same procedure as above, we arrive at the determinantal equation

$$\begin{vmatrix} \sum \dfrac{A_{m2}A_{m3} - \frac{36}{25} m^2S^2}{X_m'} + \dfrac{1}{2B} , & -\sum \dfrac{\frac{2}{3} m^2SA_{m3}}{X_m'} , & -\sum \dfrac{\frac{4}{5} m^2S^2}{X_m'} \\ \sum -\dfrac{\frac{2}{3} m^2SA_{m3}}{X_m'} , & \sum \dfrac{m^2A_{m1}A_{m3}}{X_m'} + \dfrac{1}{8\Gamma'} , & \sum \dfrac{\frac{6}{5} m^2SA_{m1}}{X_m'} \\ \sum -\dfrac{\frac{4}{5} m^2S^2}{X_m'} , & \sum \dfrac{\frac{6}{5} m^2SA_{m1}}{X_m'} , & \sum \dfrac{A_{m1}A_{m2} - \frac{4}{9} m^2S^2}{X_m'} + \dfrac{1}{162B} \end{vmatrix} = 0 \ldots (12)$$

in which

$$X_m' = A_{m1}A_{m2}A_{m3} - \tfrac{36}{25}m^2S^2A_{m1} - \tfrac{4}{9}m^2S^2A_{m3} .$$

This equation permits of a more accurate evaluation of k_s'. Comparison of these values with those obtained from a third approximation, *i.e.*, by considering all the b_{m1}, a_{m2}, b_{m3}, and a_{m4} terms, shows that the second approximation gives results which are sufficiently accurate for the purposes of this report.

2. *Symmetric Buckling One Panel, Antisymmetric Next Panel, Periodic over Four Panels.*—The series for w giving this type of buckling is given by equation (2) of this Appendix, and is

$$w = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} a_{mn} \left[\sin \frac{m\pi x}{2d} + (-1)^{\frac{1}{2}(m-1)} \cos \frac{m\pi x}{2d}\right] \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=1,3}^{\infty} \sum_{n=2,4}^{\infty} b_{mn} \left[\sin \frac{m\pi x}{2d} - (-1)^{\frac{1}{2}(m-1)} \cos \frac{m\pi x}{2d}\right] \sin \frac{n\pi y}{b} \ldots \quad \ldots \quad (2)$$

As before we assume the series for the deflection of the ith stiffener

$$(w_s)_i = \sum_{n=1,3}^{\infty} \Delta_{ni} \sin \frac{n\pi y}{b} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

and for the angular rotation of the ith stiffener

$$(\theta_s)_i = \sum_{n=2,4}^{\infty} V_{ni}' \sin \frac{n\pi y}{b} = \sum_{n=2,4}^{\infty} V_{ni} \frac{\pi}{2d} \sin \frac{n\pi y}{b} . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

This buckling pattern is periodic over four bays. It is therefore necessary to consider four stiffeners, *i.e.*, $i = 1$ to 4 in our analysis. The conditions

$$w(id, y) - (w_s)_i = 0$$

and

$$\frac{\partial w(id, y)}{\partial x} - (\theta_s)_i = 0$$

give upon substituting (2) (13) and (14), the relations

$$\left. \begin{array}{l} \Delta_n = -\Delta_{n1} = \Delta_{n2} = \Delta_{n3} = -\Delta_{n4} \\ \nabla_n = \nabla_{n1} = \nabla_{n2} = -\nabla_{n3} = -\nabla_{n4} \end{array} \right\} \quad n = 1, 3 \ldots$$

$$\left. \begin{array}{l} \Delta_n = -\Delta_{n1} = -\Delta_{n2} = \Delta_{n3} = \Delta_{n4} \\ \nabla_n = -\nabla_{n1} = \nabla_{n2} = \nabla_{n3} = -\nabla_{n4} \end{array} \right\} \quad n = 2, 4 \ldots$$

which this give the boundary conditions

$$\left. \begin{array}{l} \sum\limits_{m=1,3}^{\infty} a_{mn} (-1)^{\frac{1}{2}(m-1)} + \Delta_n = 0 \ (n = 1, 3, 5 \ldots) \\ \sum\limits_{m=1,3}^{\infty} b_{mn} (-1)^{\frac{1}{2}(m-1)} + \Delta_n = 0 \ (n = 2, 4, 6 \ldots) \\ \sum\limits_{m=1,3}^{\infty} m a_{mn} + \nabla_n = 0 \ (n = 1, 3, 5 \ldots) \\ \sum\limits_{m=1,3}^{\infty} m b_{mn} + \nabla_n = 0 \ (n = 2, 4, 6 \ldots) \end{array} \right\} \ldots \quad .. \quad .. \qquad (15)$$

On substituting the deflection function (2) in the energy integrals (6), we obtain,

$$V = \frac{D d \pi^4}{b^3} \left[\sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} a_{mn}^2 \left(\frac{m^2 b^2}{4d^2} + n^2 \right)^2 + \sum_{m=1,3}^{\infty} \sum_{n=2,4}^{\infty} b_{mn}^2 \left(\frac{m^2 b^2}{4d^2} + n^2 \right)^2 \right]$$

$$V_s = \frac{EI\pi^4}{b^3} \left[\sum_{n=1,2}^{\infty} \Delta_n^2 n^4 \right] + \frac{J\pi^4}{bd^2} \sum_{n=1,2}^{\infty} \nabla_n n^2$$

$$T = 8\tau t \pi \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \sum_{q=2,4}^{\infty} a_{mn} b_{mq} (-1)^{\frac{1}{2}(m-1)} \frac{mnq}{(n^2 - q^2)} .$$

Again using the Lagrangian Multiplier method we have to minimise the following function,

$$F = \frac{V + V_s - T}{\frac{\pi^4 D d}{b^3}} + \sum_{n=1,3}^{\infty} \gamma_n \left[\sum_{m=1,3}^{\infty} a_{mn} (-1)^{\frac{1}{2}(m-1)} + \Delta_n \right] + \sum_{n=2,4}^{\infty} \gamma_n \left[\sum_{m=1,3}^{\infty} b_{mn} (-1)^{\frac{1}{2}(m-1)} + \Delta_n \right]$$

$$+ \sum_{n=1,3}^{\infty} \delta_n \left[\sum_{m=1,3}^{\infty} m a_{mn} + \nabla_n \right] + \sum_{n=2,4}^{\infty} \delta_n \left[\sum_{m=1,3}^{\infty} m b_{mn} + \nabla_n \right] \quad .. \quad .. \quad .. \quad .. \qquad (16)$$

in which the γ's and δ's are Lagrangian Multipliers.

Minimising with respect to the a_{mn}, b_{mn}, ∇_n and Δ_n terms and substituting for γ_n and δ_n as in equations (9) and (10), we obtain the relations,

$$
\begin{bmatrix}
A_{m1} & \frac{2}{3}mS(-1)^{\frac{1}{2}(m-1)} & 0 & \frac{4}{15}mS(-1)^{\frac{1}{2}(m-1)} & 0 & \frac{6}{35}mS(-1)^{\frac{1}{2}(m-1)} & \cdots \\
\frac{2}{3}mS(-1)^{\frac{1}{2}(m-1)} & A_{m2} & -\frac{6}{5}mS(-1)^{\frac{1}{2}(m-1)} & 0 & -\frac{10}{21}mS(-1)^{\frac{1}{2}(m-1)} & 0 & \cdots \\
0 & -\frac{6}{5}mS(-1)^{\frac{1}{2}(m-1)} & A_{m3} & \frac{12}{7}mS(-1)^{\frac{1}{2}(m-1)} & 0 & \frac{2}{3}mS(-1)^{\frac{1}{2}(m-1)} & \cdots \\
\frac{4}{15}mS(-1)^{\frac{1}{2}(m-1)} & 0 & \frac{12}{7}mS(-1)^{\frac{1}{2}(m-1)} & A_{m4} & -\frac{20}{9}mS(-1)^{\frac{1}{2}(m-1)} & 0 & \cdots \\
0 & -\frac{10}{21}mS(-1)^{\frac{1}{2}(m-1)} & 0 & -\frac{20}{9}mS(-1)^{\frac{1}{2}(m-1)} & A_{m5} & \frac{30}{11}mS(-1)^{\frac{1}{2}(m-1)} & \cdots \\
\frac{6}{35}mS(-1)^{\frac{1}{2}(m-1)} & 0 & \frac{2}{3}mS(-1)^{\frac{1}{2}(m-1)} & 0 & \frac{30}{11}mS(-1)^{\frac{1}{2}(m-1)} & A_{m6} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{bmatrix}
\begin{bmatrix}
a_{m1} \\ b_{m2} \\ a_{m3} \\ b_{m4} \\ a_{m5} \\ b_{m6} \\ \vdots
\end{bmatrix}
$$

$$
= \begin{bmatrix}
(-1)^{\frac{1}{2}(m-1)} & 2B\Delta_1 + 2n\Gamma'\nabla_1 \\
(-1)^{\frac{1}{2}(m-1)} & 32B\Delta_2 + 8m\Gamma'\nabla_2 \\
(-1)^{\frac{1}{2}(m-1)} & 162B\Delta_3 + 18m\Gamma'\nabla_3 \\
(-1)^{\frac{1}{2}(m-1)} & 512B\Delta_4 + 32m\Gamma'\nabla_4 \\
(-1)^{\frac{1}{2}(m-1)} & 1250B\Delta_5 + 50m\Gamma'\nabla_5 \\
(-1)^{\frac{1}{2}(m-1)} & 2592B\Delta_6 + 72m\Gamma'\nabla_6 \\
\vdots & \vdots
\end{bmatrix} \quad \ldots \quad (17)
$$

where
$$A_{nm} = 2\left(\frac{m^2b^2}{4d^2} + n^2\right)^2$$

and S, B and Γ' are as in (10).

Taking as a first approximation, all the a_{m1} and b_{m2} terms and proceeding as in section 1 of this Appendix the following determinantal equation is obtained.

$$\begin{vmatrix} \sum\limits_{1,3}^{\infty} \frac{A_{m2}}{X_m} + \frac{1}{2B} & \sum\limits_{1,3}^{\infty} \frac{mA_{m2}(-1)^{\frac{1}{2}(m-1)}}{X_m} & \sum\limits_{1,3}^{\infty} -\frac{2}{3}\frac{mS(-1)^{\frac{1}{2}(m-1)}}{X_m} & \sum\limits_{1,3}^{\infty} -\frac{2}{3}\frac{m^2S}{X_m} \\ \sum\limits_{1,3}^{\infty} \frac{mA_{m2}}{X_m}(-1)^{\frac{1}{2}(m-1)} & \sum\limits_{1,3}^{\infty} \frac{m^2A_{m2}}{X_m} + \frac{1}{2\Gamma'} & \sum\limits_{1,3}^{\infty} -\frac{2}{3}\frac{m^2S}{X_m} & \sum\limits_{1,3}^{\infty} -\frac{2}{3}\frac{m^3S(-1)^{\frac{1}{2}(m-1)}}{X_m} \\ \sum\limits_{1,3}^{\infty} -\frac{2}{3}\frac{mS(-1)^{\frac{1}{2}(m-1)}}{X_m} & \sum\limits_{1,3}^{\infty} -\frac{2}{3}\frac{m^2S}{X_m} & \sum\limits_{1,3}^{\infty} \frac{A_{m1}}{X_m} + \frac{1}{32B} & \sum\limits_{1,3}^{\infty} \frac{mA_{m1}}{X_m}(-1)^{\frac{1}{2}(m-1)} \\ \sum\limits_{1,3}^{\infty} -\frac{2}{3}\frac{m^2S}{X_m} & \sum\limits_{1,3}^{\infty} -\frac{2}{3}\frac{m^3S(-1)^{\frac{1}{2}(m-1)}}{X_m} & \sum\limits_{1,3}^{\infty} \frac{mA_{m1}}{X_m}(-1)^{\frac{1}{2}(m-1)} & \sum\limits_{1,3}^{\infty} \frac{m^2A_{m1}}{X_m} + \frac{1}{8\Gamma'} \end{vmatrix} = 0 \quad (18)$$

in which
$$X_m = A_{m1}A_{m2} - \tfrac{4}{9}m^2S^2.$$

A second approximation is obtained by taking all the a_{m1}, b_{m2} and a_{m3} terms. This leads to a sixth-order determinantal equation, which in most cases gives results which are sufficiently accurate. In some cases a higher approximation is required, but as it proves to be too involved for computational purposes, the results for $B = \Gamma = \infty$ are taken from the values given in the Royal Aeronautical Society Data Sheets[7].

For the case of the aspect ratio equal to 5, lower values of k_s' are obtained for large values of EI/Dd and J/Dd by using all the a_{m4}, b_{m5} and a_{m6} terms.

The results of the calculations are given in Table 1 and in Figs. 2, 3 and 4. The figures are plotted with k_s against $B(EI/Dd)$ for various values of $\Gamma(J/Dd)$.

TABLE 1

Values of the Buckling Shear Stress Parameter

For Various Aspect Ratios and Stiffness Parameters

Torsional Stiffness	Orthotropic plate solution		Aspect ratio	Antisymmetric Buckling over two panels		Buckling over four panels				
						a_{m1}, b_{m2}, a_{m3} terms (also b_{m2}, a_{m3}, b_{m4})		b_{m4}, a_{m5}, b_{m6} terms		
Γ	B	k_s'	b/d	B	k_s'	B	k_s'	B	k_s'	
0	0 2 5 20 50 100 200	5·34 10·34 16·07 37·14 68·99 112·2 184·6	1·0	0 2·91 7·78 22·29 ∞	5·53 7·85 9·82 11·78 13·86	0 7·09 19·03 ∞	6·08 10·0 10·5 10·86			From Stein and Fralich N.A.C.A. T.N.1851
			2·0	0 3·35 14·50 22·99 33·11 45·77 61·97 82·92 112·3 605 ∞	9·65 12·0 16·0 18·0 20·0 22·0 24·0 26·0 28·0 35·0 37·05	0 5·48 11·93 26·37 36·29 68·92 145·4 625 ∞	5·54 15·0 20·0 23·5 24·5 26·0 27·0 28·0 28·2			
			5·0	0 18·02 90·99 176·8 444·7 704·4	42·5 70·0 90·0 100 120 140	0 49·19 112·8	13·31 60·0 100	220 ∞	140 143	
0			2·0	4·0 25·0 60·0 500	12·27 18·4 23·65 34·5	1 10 12·5 40 ∞	7·78 18·85 20·75 24·9 28·3			Additional Values Calculated
			5·0	138·3 631·3 1877 8883	96·3 136·0 192·4 248·4	34·7 75·0 130·7 860	49·6 78·54 111·1 192·1	960	142·8	
0·25			1·0	0 1·0 4·0	5·58 6·60 8·60	5·0 10·0 30·0 ∞	10·37 10·80 11·35 11·52			
			2·0	0 4·0 25·0 60·0 500 ∞	11·31 12·7 18·85 24·4 35·15 37·8	0 1·0 6·0 10·0 40·0 100	6·40 8·96 18·45 22·5 29·95 33·15			

TABLE 1—*continued*

Torsional stiffness	Orthotropic plate solution		Aspect ratio	Antisymmetric Buckling over two panels		Buckling over four panels			
						a_{m1}, b_{m2}, a_{m3} terms (also b_{m2}, a_{m3}, b_{m4})		b_{m4}, a_{m5}, b_{m6} terms	
Γ	B	k_s'	b/d	B	k_s'	B	k_s'	B	k_s'
0·25			2·0			∞	34·95		
			5·0					∞	175·2
1·0	0 10·0 50·0	6·50 28·9 80·3	1·0	0 1·0 4·0 10·0 20·0	5·80 6·84 8·78 10·52 11·75	5·0 10·0 30·0 ∞	10·88 11·75 12·08 12·18		
			5·0	70·7 438·4 730·8 1453 4394	96·2 136·0 157·1 192·4 248·4	17·3 56·8 103 299 746	49·54 78·54 111·1 192·1 248·4	30·3 133 ∞	198·6 222·1 231·4
2·5			2·0	0 4·0 25·0 60 500 ∞	14·16 17·0 22·4 28·9 37·25 39·6	0 1·0 6·0 10·0 40·0	8·62 11·9 19·5 23·8 35·5		
10·0	0 10·0 30·0 50·0 100	15·3 40·2 74·4 99·8 144·2	5·0	68·36 173·9 450·3 1196	136·0 157·1 192·4 248·4	3·50 23·2 55·1 193·2	49·6 78·54 111·1 192·1	12·0 45·8 ∞	198·6 222·1 232·0
25·0			2·0	0 4·0 25·0 60·0 500 ∞	15·15 18·78 25·1 30·3 38·7 40·4				
∞			1·0	0 1·0 4·0 10·0 20·0 100	6·44 7·17 8·95 10·61 11·92 13·07	5·0 10·0 100 ∞	11·28 11·94 12·34 12·40		
			2·0	0 4·0 12·5 25·0 60·0 500 ∞	15·3 19·1 22·35 25·7 30·9 39·1 40·8	0 1·0 6·0 10·0 40·0	10·1 12·6 19·7 24·1 36·2		
			5·0	12·34 74·8 271·5 352·1	136·0 192·4 235·6 248·4	0·72 13·2 36·0 148·5 296	49·6 78·54 111·1 192·1 248·4	11·7 44·2 ∞	198·6 222·1 232·0

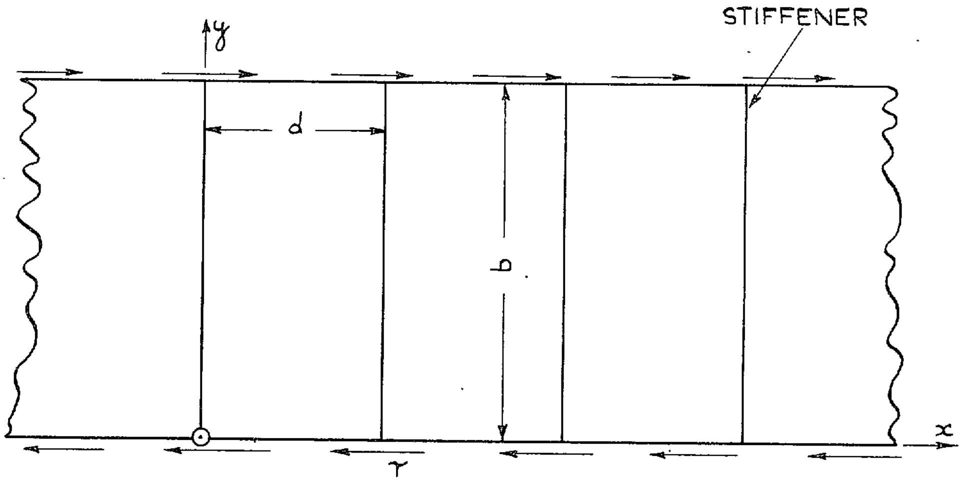

FIG. 1. Infinitely-long simply-supported plate with transverse stiffeners.

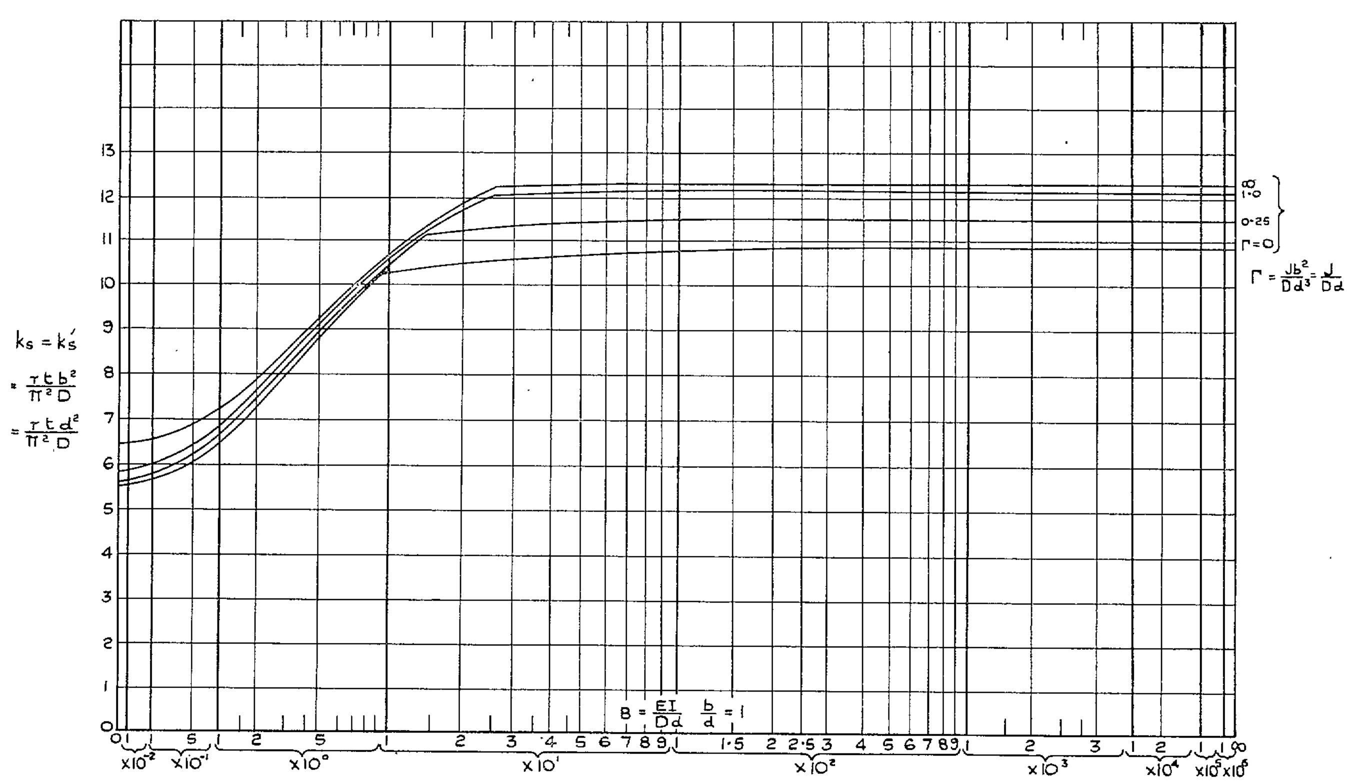

FIG. 2. Buckling shear stress for aspect ratio of panel = 1·0.

B

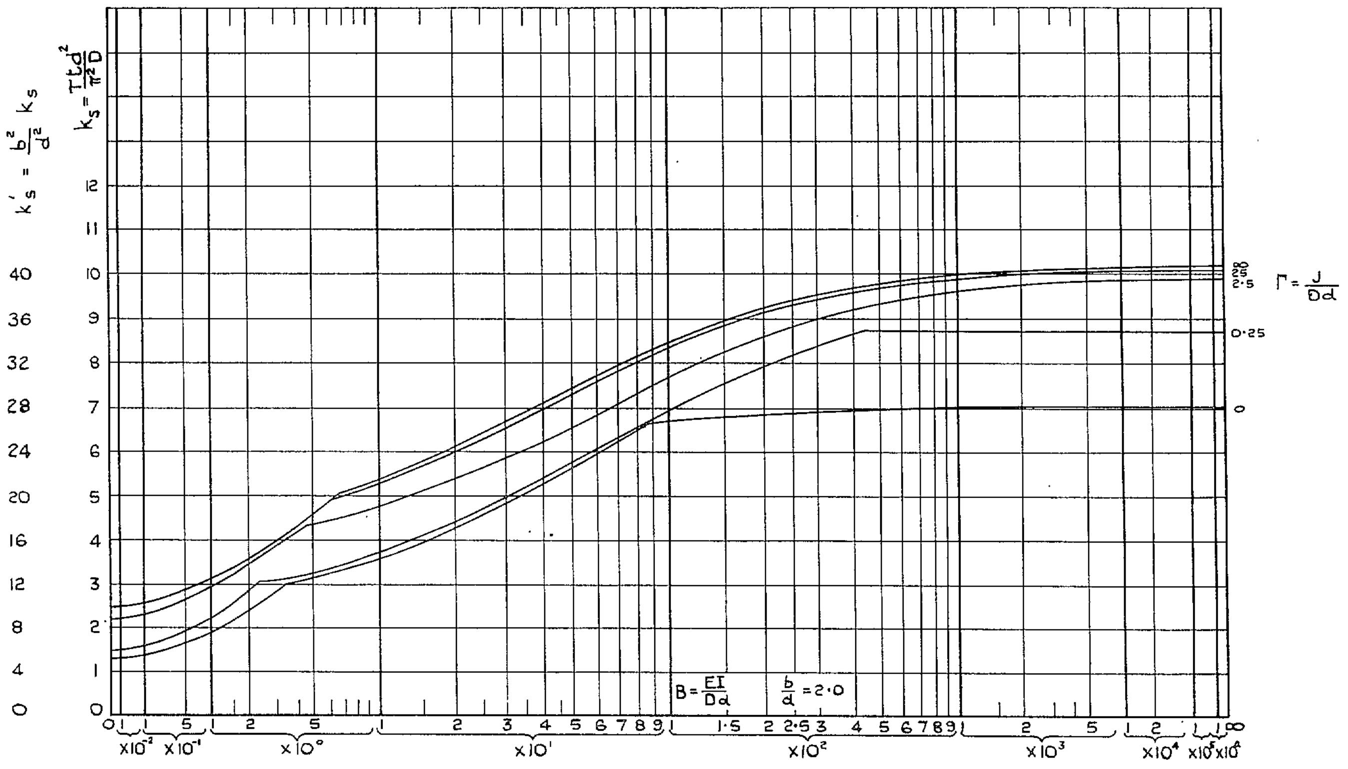

FIG. 3. Buckling shear stress for aspect ratio of panel $= 2 \cdot 0$.

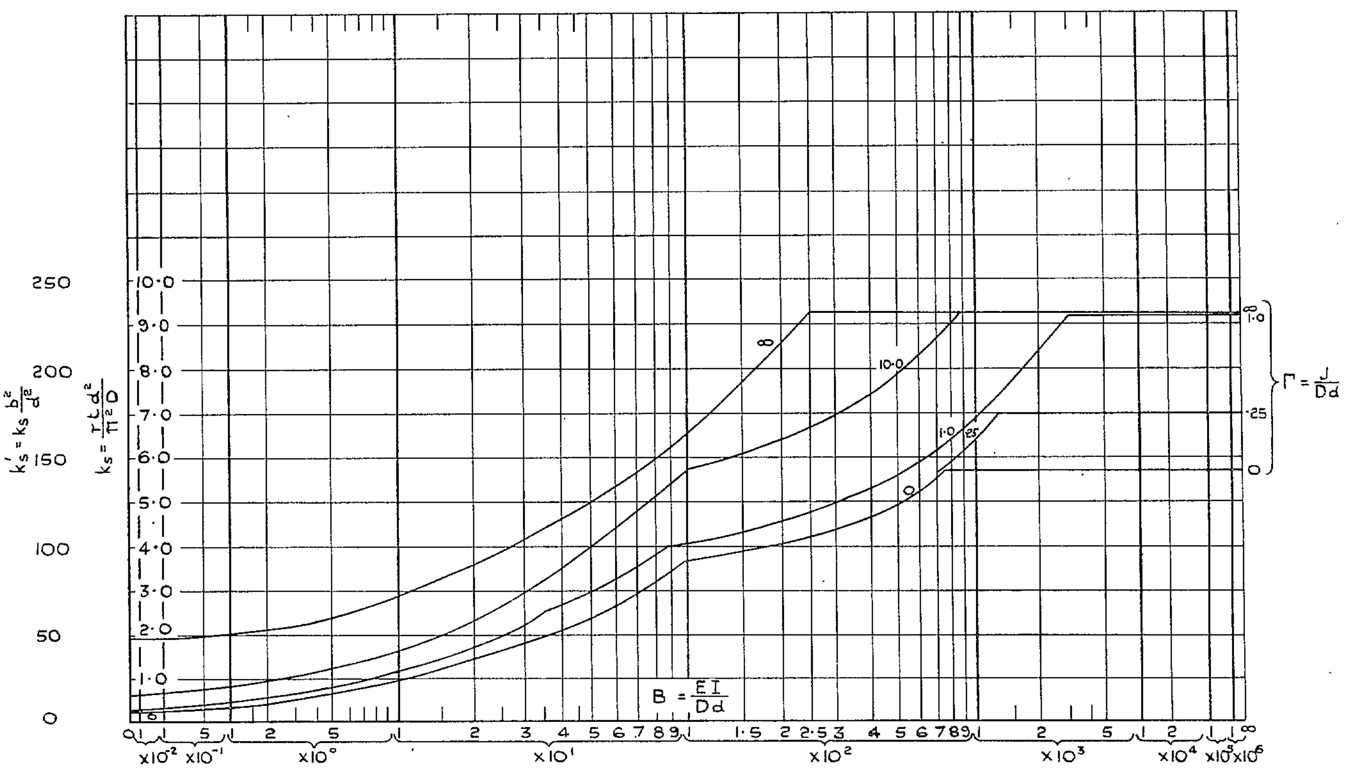

FIG. 4. Buckling shear stress for aspect ratio of panel $= 5 \cdot 0$.

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