R. & M. No. 2973
(14,353)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Estimation of the Change in Performance Characteristics of a Turbine resulting from Changes in the Gas Thermodynamic Properties

By

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LONDON : HER MAJESTY'S STATIONERY OFFICE
1956
FOUR SHILLINGS NET

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Communicated by the Principal Director of Scientific Research (Air), Ministry of Supply

Reports and Memoranda No. 2973*

June, 1951

Summary.—Frequently turbines are tested in a laboratory with cool air which has thermodynamic properties different from those of the hot gas for which the turbine is designed. This note presents methods for correcting such tests to allow for a difference in gas thermodynamic properties.

1. *Introduction*.—To obtain experimentally the performance characteristics of an axial flow turbine it is common practice to test with a turbine inlet temperature much below the designed operating temperature. Such a practice leads to simplicity in testing since rotational speeds may be reduced, output power is reduced, the lower temperature enables cheap materials to be used for the test rig, and the problem of providing instruments to withstand high temperatures is avoided. The practice, however, introduces two problems of its own : (*a*) the thermodynamic properties of cold air differ slightly from those of hot air and (*b*) the geometric dimensions of a turbine tested with cold air will differ slightly from those appertaining to the heated (and consequently expanded) condition ; this applies essentially to blade tip or shroud clearances and to annulus area.

The object of this note is to suggest an approximate but simple method of correcting turbine characteristics for changes in the gas thermodynamic properties (geometric dimensions being unaltered) and in addition to indicate the order of magnitude of the changes in the characteristics which will result from variations in annulus area and blade tip or shroud clearances.

A further application of the suggested procedure appertains to performance calculation. To calculate the characteristics of a turbine it is usually necessary to have at hand a table or chart of standard flow functions of the gas. It is not convenient to have a set of tables or charts covering a wide range of thermodynamic constants and it is therefore useful to be able to calculate the turbine performance for some standard set of gas properties and finally to correct the performance characteristics to suit the gas properties actually required.

It is assumed throughout that the gas approximates to a perfect gas as it expands through the turbine and consequently that the thermodynamic properties are determined by the values of K_p and γ, these values remaining unchanged during the actual expansion process.

Thermodynamic properties of air and combustion products of hydrocarbon fuels over a wide range of temperatures may conveniently be determined from Ref. 3.

2. *Outline of Method*.—If geometrically identical turbine stages are operated with different gases, then if gas incidences on to the blades, gas Mach numbers, and gas Reynolds numbers are identical in each unit, so also will the stage efficiencies and gas outlet angles from the blade rows be similar. This is the basic assumption behind the advocated method of correction.

*N.G.T.E. Memo. M.118, received 1st November, 1951.

Such an assumption is well substantiated for incompressible fluids, where similarity in Reynolds number has long been proved to be the essential criterion for similarity in the aerodynamic characteristics of geometrically similar bodies in different fluids. The assumption is not so well established for compressible gases since little or no controlled tests have yet been made to compare the performance of geometrically similar bodies in different gases at supersonic or high subsonic speeds. In the U.S.A. some work has been carried out using Freon-12 as a substitute for air and the relatively crude comparisons that are available suggest that at similar incidences, Mach numbers, and Reynolds numbers the lift and drag coefficients of bodies such as cascades of blades are not very markedly affected by the large change in γ ($1 \cdot 4$ for air ; $1 \cdot 125$ for Freon-12) for Mach numbers up to as high as $1 \cdot 5$. In the present instance, where smaller changes in γ are envisaged (say $1 \cdot 4$ to $1 \cdot 3$) and lower Mach numbers (not greater than $1 \cdot 1$) there appears to be no reason to regard the basic assumption as invalid.

The assumption cannot be applied exactly, however, since it is impossible to achieve precise similarity between gas incidences and gas Mach numbers throughout the entire stage of a single-stage turbine when comparing identical turbines operated by different gases. The small but unavoidable discrepancy in flow conditions between different gases will become greater as the expansion ratio is increased. On a single-stage turbine the discrepancy is not likely to prove significant but it must be given careful consideration on multi-stage turbines, particularly when the expansion ratio is high. In a multi-stage turbine it is only possible, therefore, to approximate to exact similarity of flow between different gases. Two approximations are possible : either

(i) The average incidence on all the stages can be made identical together with inlet gas Mach number. This leads to local discrepancies in incidences (between different gases) on the first and last stages. This discrepancy in incidence can be more conveniently expressed as a difference (between different gases) in overall gas density ratios.

or

(ii) Incidences through stages can be made identical together with overall gas density ratio. This leads to almost exact similarity between different gases in the velocity triangles throughout the turbine but there will be a small discrepancy in Mach number.

Since experience indicates that turbine efficiency is not generally susceptible to small changes in Mach number the second type of approximation to flow similarity can be claimed to result in equality of stage efficiency. The same claim can be made for the first type of approximation to flow similarity providing the discrepancy in gas density ratio (or incidences) between the different gases is small. A method of correcting the relationship between efficiency, temperature ratio ($\Delta T/T_1$), speed ($N/\sqrt{T_1}$), and pressure ratio for a change in gas properties can be framed around either of the above assumptions for flow similarity. A method framed around the first type of approximation (similar mean incidence and inlet Mach number) is preferred when possible due to its comparative simplicity computationally. If, however, the system leads to discrepancies in gas density ratio between the original and corrected gases which are greater than about 8 per cent, then the second method should certainly be employed. Larger discrepancies in gas density ratio may lead to discrepancies in incidence angle on first and last stages greater than about 5 deg.

The problem of correcting the relationship between the flow function ($W\sqrt{T_1}/P_1$), pressure ratio, and $N/\sqrt{T_1}$ has to be tackled separately. Providing the change to be considered in the thermodynamic properties of the gas is not large ($\Delta K_p/K_p < 25$ per cent, $\Delta(\gamma - 1)/(\gamma - 1) < 25$ per cent) a satisfactory correction can be made by comparing the turbine with a simple nozzle. The change of the characteristic of flow function *vs.* pressure ratio (for a simple nozzle) caused by a change of the thermodynamic gas constants can be computed fairly easily and the corresponding change of flow function with pressure ratio at constant $N/\sqrt{T_1}$ in a turbine will approximately be proportionately similar.

3. *Correction for Change of K_p and γ of the Characteristics Relating Efficiency, Rotational Speed, and Pressure Ratio.*—Consider the velocity triangles of a single stage of a turbine.

The work done per pound of gas per second is given by:

$$\text{Work done} = K_p.\Delta T_s = UV_{a1} (\tan \alpha_1 - \Gamma \tan \alpha_2) \qquad (1)$$

$$\text{where} \quad \Gamma = V_{a2}/V_{a1} \qquad (2)$$

Hence, since $U/V_{a1} = \tan \alpha_0 + \tan \alpha_1$

$$K_p \Delta T_s/U^2 = (\tan \alpha_1 - \Gamma \tan \alpha_2)/(\tan \alpha_0 + \tan \alpha_1) \qquad (3)$$

Equation (3) demonstrates that if in geometrically similar stages working with different gases the velocity triangles are similar then so also will the value of $K_p \Delta T_s/U^2$ be similar.

With certain limitations it may be shown that the converse is true. Experience has shown that the outlet flow angle from a row of blades is little influenced by changes in the nature of the gas or fluid providing Reynolds number, Mach number and incidence are roughly similar. Thus α_0 and α_2 should not be sensibly affected by changes in K_p and γ. Hence if in geometrically similar stages the value of $K_p \Delta T_s/U^2$ and Γ are equal then the stage velocity triangles (or gas incidence on to blading) will be identical. Changes in the value of Γ are proportional to changes in the stage density ratio and, as pointed out in section 2, it is not possible to achieve exact similarity between all the parameters density ratio, $K_p \Delta T_s/U^2$, Mach number, and Reynolds number for different gases in geometrically similar stages. Two methods of correction are outlined below. Method 1 is based on the assumption that stage efficiency is unaltered when $K_p \Delta T_s/U^2$ and inlet Mach number is similar and method 2 is based on the assumption that stage efficiency is unaltered when $K_p \Delta T_s/U^2$ and density ratio is similar.

Method 1.—From the known turbine characteristics the polytropic efficiency is related to $K_p \Delta T/N^2$ for various values of inlet Mach number. The Mach number selected is the ratio of blade speed to the velocity of sound in the operating gas at inlet stagnation temperature, and may be adequately expressed by the parameter $N/\sqrt{\{K_p(\gamma - 1)T_1\}}$. Such a set of curves has been derived from the characteristics shown in Fig. 2 and is plotted in Fig. 3(a). This family of curves may be regarded as valid for all values of K_p and γ. Selecting new values of K_p and γ, reading off values of η_{pol} for selected values of $K_p \Delta T/N^2$ and $N/\sqrt{\{K_p(\gamma - 1)T_1\}}$ from Fig. 3(a), and employing the relationships:

$$\frac{\Delta T}{T_1} = \frac{K_p.\Delta T}{N^2}\left(\frac{N}{\sqrt{T_1}}\right)^2 \frac{1}{K_p} \qquad (4)$$

$$\frac{P_2}{P_1} = \left(1 - \frac{\Delta T}{T_1}\right)^{\frac{n}{n-1}} \qquad (5)$$

$$\frac{n}{n-1} = \frac{\gamma}{\gamma - 1} \frac{1}{\eta_{pol}} \qquad (6)$$

it is simple to reconstruct the characteristics relating η_{isen}, $\Delta T/T_1$, P_2/P_1 and $N/\sqrt{T_1}$ for the new values of K_p and γ. Such a reconstruction is shown in Fig. 4.

On Fig. 4 it will be observed that as drawn the curve of efficiency against pressure ratio for each value of $N/\sqrt{T}$ is identical for both gases. In fact the efficiencies are not exactly identical but the difference between the two gases is too small to represent graphically on the scale shown and for normal purposes of assessing gas-turbine engine performance the difference may be neglected.

Method 2.—From the known turbine characteristics the polytropic efficiency is related to $K_p \Delta T/N^2$ for various values of overall gas density ratio (*e.g.*, as shown in Fig. 3(b)).

Selecting values of $K_p \Delta T/N^2$ and $N/\sqrt{T_1}$ and employing the relationship

$$\frac{\rho_2}{\rho_1} = \left[1 - \left(\frac{\Delta T}{T_1}\right)\right]^{\frac{1}{n-1}} \qquad (7)$$

in addition to equations (4), (5) and (6), it is possible to find the magnitude of η_{pol} from Fig. 3(b) for each selected pair of values of $K_p \Delta T/N^2$ and $N/\sqrt{T_1}$. Thence curves connecting η_{isen}, $\Delta T/T_1$, P_2/P_1, and $N/\sqrt{T_1}$ may be constructed for new values of K_p and γ.

In practice it will usually be found both easier and quicker to employ method (1). On the other hand method (2) is probably the more accurate and should certainly be used if application of method (1) leads to discrepancies in the value of density ratio between the original and new values of K_p and γ greater than about 8 per cent.

4. *Correction for Change in K_p and γ of the Characteristics Relating Flow Function, Pressure Ratio, and Rotational Speed.*—A typical example is illustrated in Fig. 5 of the relationship between $W\sqrt{T_1}/P_1$ and P_2/P_1 for a fixed value of $N/\sqrt{T_1}$ and two pairs of values of K_p and γ. Except in the region of choked and nearly choked flow the change of flow function with K_p and γ is very small.

When a turbine is choked it may be assumed that the flow changes by an amount which is proportionally similar to the change which would occur to the choked flow in a simple nozzle with isentropic expansion. Now the choking flow in a simple nozzle is given by[2]

$$\frac{W\sqrt{T_1}}{AP_1} = \left(\frac{\gamma}{R}\right)^{1/2} \cdot \left[\frac{2}{\gamma + 1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} . \qquad (8)$$

Thus the ratio of choking flows in two geometrically identical turbines using different gases in which the two pairs of values of K_p and γ are designated by the suffices a and b respectively is

$$\left(\frac{Q_a}{Q_b}\right)_{\text{choked}} \simeq \frac{[\gamma_a^2/K_{pa}(\gamma_a - 1)]^{1/2}[2/(\gamma_a + 1)]^{(\gamma_a+1)/2(\gamma_a-1)}}{[\gamma_b^2/K_{pb}(\gamma_b - 1)]^{1/2}[2/(\gamma_b + 1)]^{(\gamma_b+1)/2(\gamma_b-1)}} \qquad (9)$$

(for convenience the parameter $W\sqrt{T_1}/P_1$ is represented by the symbol Q). Similarly it is assumed that in the nearly choked region of flow the change in flow function at a given value of pressure ratio and $N/\sqrt{T_1}$ will be roughly proportional to the change in flow function through an efficient nozzle working at the same ratio of Q/Q_{choking}. In Fig. 5b is shown a curve connecting relative change in flow (change in flow/change in choking flow) with the flow ratio Q/Q_c estimated for a simple convergent nozzle with isentropic flow. Thus if ΔQ_c is the change of Q resulting from alterations in K_p and γ when the turbine is fully choked [*see* equation (9) and Fig. 5a] then it is assumed that the change ΔQ at other flows may be approximately represented by the curve of $\Delta Q/\Delta Q_c$ plotted against Q/Q_c shown in Fig. 5b.

No allowance is made for the fact that the choking pressure ratio alters slightly with change in the gas properties. It is argued that this is admissible since the flow quantity becomes increasingly insensitive to pressure ratio as the choking pressure ratio is approached and exceeded.

This correction for mass flow is only approximate but since the maximum change in flow function $\Delta Q_c/Q_c$ is generally small and in the region of 1 or 2 per cent then a considerable error in the increment ΔQ can be tolerated without involving an appreciable error in the total flow.

To demonstrate the methods of correction outlined in sections 3 and 4 an example is worked out in detail in Appendix II.

5. *Correction for Small Changes in Annulus Dimensions and Blade-Tip Clearances.*—The corrections to turbine performance that have been suggested for a change in the gas constants are reasonable providing the turbine remains geometrically unchanged. If, for example, cold-air turbine tests are to be corrected to represent the performance of the turbine working with hot gas then the characteristics as adjusted in accordance with sections 3 and 4 will only be correct if the hot dimensions and blade clearances are identical to those appertaining to the cold-air test. Generally this will not be quite true and a further correction will be required.

Roughly, providing the difference in annulus, or blade throat, areas is small (of the order of 1 per cent) and the differences in blade tip or shroud clearances are small (of the order of 1 per cent of the blade height) then the change in flow can be considered as roughly proportional to the change in mean annulus area and the change in efficiency and temperature drop can be considered as proportional to the change in blade tip or shroud clearance.

The effect of tip clearance was discussed in Ref. 1. There it was indicated that the change in efficiency resulting from changes in clearance in rotor and stator were very approximately expressible as

$$\Delta\eta = -[\Delta(k/h)_{\text{rotor}} + \Delta(k/h)_{\text{stator}}]2B \qquad \qquad (10)$$

where $B = 0\cdot5$ for simple radial tip clearance

$B = 0\cdot25$ for simple shrouding.

A slightly better but more complex expression can be used if required, namely:

$$\Delta\eta = -\frac{\eta^2.B}{(2.K_p.\Delta T_s/V_a^2)}\cdot\left\{\underbrace{\sec^3\alpha_m.C_L/(s/c)^2.\Delta(k/h)}_{\text{stator}} + \underbrace{\sec^3\alpha_m.C_L/(s/c)^2.\Delta(k/h)}_{\text{rotor}}\right\} \qquad (11)$$

If the problem is approached in detail (Ref. 1) it will be found that the flow function is dependent upon efficiency and blade clearances as well as annulus or blade throat areas. However, the introduction of these variables into the flow function correction would involve a much greater amount of computation and at the present stage of knowledge it is doubtful if the accuracy of the corrections would be appreciably enhanced. Providing the changes in clearance and annulus area are very small the corrections outlined above should be sufficiently accurate for normal purposes of engine performance prediction and comparison.

6. *Discussion.*—In the example shown in Fig. 4 the relationship between η_{isen}, P_2/P_1, and $N/\sqrt{T_1}$ remained substantially unchanged when the gas constants were varied. It is obviously of interest to determine the necessary conditions for such an occurrence.

If a turbine stage is operated with two different gases '*a*' and '*b*' and if the efficiency is unaltered at any fixed value of P_2/P_1 and $N/\sqrt{T_1}$ then from section 3 it may be implied that

$$K_{pa}\,\Delta T_a/N_a^2 \simeq K_{pb}\Delta T_b/N_b^2 \qquad \qquad (12)$$

Now,

$$\frac{K_p\Delta T}{N^2} = \eta_{isen}\left(R\frac{\gamma}{\gamma-1}\right)\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]\left[\frac{\sqrt{T_1}}{N}\right]^2 \qquad \qquad (13)$$

From equation (13) it may be seen that if equation (12) is to hold at a given value of P_2/P_1 and $N/\sqrt{T_1}$ then

$$R_a\left(\frac{\gamma_a}{\gamma_a-1}\right)\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma_a-1}{\gamma_a}}\right] \simeq R_b\left(\frac{\gamma_b}{\gamma_b-1}\right)\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma_b-1}{\gamma_b}}\right] \qquad (14)$$

When $R_a = R_b$ and $1\cdot3 < \gamma < 1\cdot4$ it is found that equation (14) approximately holds. The range of γ specified is that covered by air and combustion products of 'standard' fuel (Ref. 3) in conventional gas-turbine engines.

Under such conditions therefore a very simple correction for change in gas properties becomes possible. The relationship between η_{isen}, P_2/P_1, and $N/\sqrt{T_1}$ may be assumed to be independent of K_p and γ and the relationship between $\Delta T/T_1$ and P_2/P_1 at any given value of $N/\sqrt{T_1}$ and any value of γ in the range $1\cdot3 < \gamma < 1\cdot4$ immediately follows from

$$\frac{\Delta T}{T_1} = \eta_{isen}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad \qquad (15)$$

7. *Conclusions.*—Approximate methods for correcting overall axial-turbine characteristics for a change in operating gas are presented. Such a correction is useful when the performance of a turbine is determined either experimentally or theoretically for one type of gas and it is desired to predict the characteristics of the same turbine when operated with a different type of gas.

If it is desired to correct the characteristics of a turbine obtained on a test rig using cold air to those which would appertain to the same turbine using hot air and combustion products it is demonstrated that a very simple correction becomes possible. In such an instance dimensional alterations resulting from thermal expansion may occur and should be allowed for. Approximate expressions for change of efficiency and temperature drop with blade-tip clearance are quoted.

The general accuracy of the corrections should be well within the limitations of $\pm$ 2 per cent in flow and efficiency which frequently appertain to detail experimental or predicted overall performance at the present time.

A representative example of the application of the method is worked out in detail in Appendix II.

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APPENDIX I

NOTATION

The velocity triangles and convention of signs adopted in a turbine stage are illustrated in Fig. 1.

List of Symbols

A	Flow area
R	Gas constant
h	Blade height
k	Minimum radial clearance between blade tip and adjacent wall or minimum clearance between blade shroud and adjacent casing
K_p	Specific heat at constant pressure
N	r.p.m.
n	Index identifying the nature of the gas expansion through the turbine, *viz.*, P/ρ^n = constant
P_1	Turbine inlet total pressure
P_2	Turbine outlet total pressure
Q	Symbol designating the parameter $W\sqrt{T_1}/P_1$
T_1	Turbine inlet total temperature

NOTATION—*continued*

ΔT	Overall total temperature drop across turbine
ΔT_s	Average temperature drop per stage
U	Blade peripheral speed
V_{a1}	Gas axial velocity at inlet to a rotor blade
V_{a2}	Gas axial velocity at outlet from a rotor blade
W	Gas mass flow per second through turbine
γ	Ratio of specific heats
ρ_1	Gas density at inlet to turbine
ρ_2	Gas density at outlet from turbine
Γ	Ratio V_{a2}/V_{a1}
η_{pol}	Turbine polytropic expansion efficiency
η_{isen}	Overall isentropic efficiency of turbine

APPENDIX II

Application of Method of Correction

Problem.—It will be assumed that the characteristics shown in Fig. 2 have been obtained from a turbine tested with cold air in which

$$K_{pa} = 10{,}870 \text{ ft/pdl/lb/deg C}$$
$$\gamma_a = 1 \cdot 3963.$$

Stator clearances are zero.

Rotor radial tip clearances are $1 \cdot 5$ per cent of the blade height.

It will further be supposed that it is required to find the characteristics of the same turbine at the same values of $N/\sqrt{T_1}$ when using hot air in which

$$K_{pb} = 12{,}300 \text{ ft/pdl/lb/deg C}$$
$$\gamma_b = 1 \cdot 333$$

Stator clearances are zero.

Rotor radial tip clearances are $2 \cdot 0$ per cent of the blade height.

Hot annulus areas are $0 \cdot 8$ per cent greater than in cold tests.

Procedure (*Method* 1).—(*a*) Express the rotor speeds as a Mach number, using the parameter

$N/\sqrt{\{K_{pa}(\gamma_a - 1)T_1\}}$

$N/\sqrt{T_1}$	250	200	150	100
$N/\sqrt{\{K_{pa}(\gamma_a - 1)T_1\}}$ (cold air)	3·82	3·05	2·29	1·53
$N/\sqrt{\{K_{pb}(\gamma_b - 1)T_1\}}$ (hot air)	3·91	3·13	2·34	1·56

(*b*) From the test results and for the values of $N\sqrt{\{K_{pa}(\gamma_a - 1)T_1\}}$ tested plot curves relating η_{pol} with $K_p.\Delta T/N^2$

N.B.

$$\eta_{pol} = \frac{\gamma}{\gamma - 1} \frac{\log (T_2/T_1)}{\log (P_2/P_1)}$$

and

$$\frac{K_p \Delta T}{N^2} = \frac{\Delta T}{T_1} \cdot \frac{1}{(N/\sqrt{T_1})^2} \cdot K_{pa} .$$

Such a family of curves is plotted in Fig. 3a. This family is now regarded as valid for all values of K_p and γ.

(*c*) Select a number of values of $K_{pb}\Delta T/N^2$ and for the required values of $N/\sqrt{\{K_{pb}(\gamma_b - 1)T_1\}}$ read off from Fig. 3a the corresponding values of η_{pol}.

(*d*) For each point selected in (*c*) find the values of η_{isen}, $\Delta T/T_1$, and P_2/P_1 for the new values of K_p and γ (i.e., K_{pb} and γ_b).

N.B.

$$\frac{\Delta T}{T_1} = \frac{K_p.\Delta T}{N^2} \left(\frac{N}{\sqrt{T_1}}\right)^2 \cdot \frac{1}{K_{pb}}$$

$$\frac{P_2}{P_1} = \left(1 - \frac{\Delta T}{T_1}\right)^{\frac{n}{n-1}}$$

$$\frac{n}{n-1} = \frac{\gamma_b}{\gamma_b - 1} \cdot \frac{1}{\eta_{pol}}$$

$$\eta_{isen} = \frac{\Delta T/T_1}{1 - [P_2/P_1]^{(\gamma_b - 1)/\gamma_b}} .$$

(*e*) For each $N/\sqrt{T_1}$ line select values of P_2/P_1 and the corresponding test result values of $W\sqrt{T_1}/P_1$. Also determine the value of $(W\sqrt{T_1}/P_1)/(W\sqrt{T_1}/P_1)$ choking.

(*f*) Find change in Q_c [N.B., $Q = W\sqrt{T_1}/P_1$; $Q_c = (W\sqrt{T_1}/P_1)$ choking] resulting from change in γ and K_p

Thus

$$(Q_c)_b = (Q_c)_a \frac{[\gamma_b^2/K_{pb}(\gamma_b - 1)]^{1/2}[2/(\gamma_b + 1)]^{(\gamma_b + 1)/2(\gamma_b - 1)}}{[\gamma_a^2/K_{pa}(\gamma_a - 1)]^{1/2}[2/(\gamma_a + 1)]^{(\gamma_a + 1)/2(\gamma_a - 1)}}$$

(Equation 9)

and

$$\Delta Q_c = (Q_c)_b - (Q_c)_a .$$

(*g*) From Fig. 5b find $\Delta Q/\Delta Q_c$, and hence ΔQ, for each of the values of Q/Q_c tabulated under (*e*). Thus find new value of $W\sqrt{T_1}/P_1$ (viz., $Q + \Delta Q$). These new values of Q assume same annulus areas as for cold-air tests. Since the annulus area will be 0·8 per cent greater when running with hot air these new values of Q must be multiplied by 1·008.

(*h*) The values of η_{isen} and $\Delta T/T_1$ found in (*d*) correspond to tip clearances identical to those in the cold-air tests. Since under hot conditions the rotor blade-tip clearance will be increased by an amount equal to (0·005 × blade height) the efficiency will be reduced approximately by (*see* equation 10)

$$\Delta\eta = -(0 + 0{\cdot}005) \times 1{\cdot}0 = -0{\cdot}5 \text{ per cent.}$$

As a result of this change the values of η_{isen} found in (*d*) must be reduced by 0·5 per cent and $\Delta T/T_1$ must be reduced by $(0{\cdot}005/\eta_{isen})(\Delta T/T_1)$.

The following table illustrates numerically the correction outlined in steps (*c*) to (*g*) for the characteristics shown in fig. 2 when $N/\sqrt{T_1} = 150$.

TABLE 1

Line	Step							Notes
1	(*c*)	$100\, K_p \Delta T/N^2$	1·2	2·4	3·6	5·2	8·0	Selected arbitrarily
2	(*c*)	η_{pol}	0·58	0·731	0·790	0·802	0·767	From Fig. 3(a), for $N/\sqrt{\{K_p(\gamma - 1)T_1\}} = 2 \cdot 34$
3	(*d*)	$\Delta T/T_1$	0·0219	0·0439	0·0659	0·0950	0·1463	Equation (4)
4	(*d*)	$n/(n - 1)$	6·90	5·48	5·065	4·99	5·215	Equation (6)
5	(*d*)	P_2/P_1	0·8580	0·783	0·7085	0·6075	0·4375	Equation (5)
6	(*d*)	$1 - (P_2/P_1)^{(\gamma_b - 1)/\gamma_b}$	0·0375	0·0594	0·0825	0·117	0·1867	This is equal to $\Delta T_{isen}/T_1$
7	(*d*)	η_{isen}	0·585	0·739	0·799	0·812	0·784	$\eta_{isen} = \Delta T/\Delta T_{isen}$
8	(*e*)	$W\sqrt{T_1}/P_1$ (or Q)	15·15	18·48	20·9	23·0	24·45	From Fig. 2, corresponding to P_2/P_1 in line 5
9	(*e*)	Q/Q_c	0·62	0·755	0·855	0·94	1·0	From Fig. 2, $Q_c = 24 \cdot 45$
10	(*g*)	$\Delta Q/\Delta Q_c$	0·05	0·13	0·28	0·5	1·0	From Fig. 5(b)
11	(*g*)	ΔQ	—0·015	—0·04	—0·086	—0·154	—0·308	Since ΔQ_c, calculated from equation (9), is $-0 \cdot 308$
12	(*g*)	Q (corrected)	15·13	18·44	20·81	22·85	24·14	Q (line 8) $+ \Delta Q$

N.B.—No correction is included in the above table for variations in tip clearance or annulus area.

Procedure (Method 2).—(*a*) From the test results plot curves relating η_{pol} with $K_p\Delta T/N^2$ for various constant values of density ratio

N.B. $\rho_2/\rho_1 = (P_2/P_1)/(1 - \Delta T/T_1)$.

Such curves are shown in Fig. 3(b). This family of curves is now regarded as valid for all values of K_p and γ.

(*b*) Select arbitrarily a number of values of $K_p\Delta T/N^2$ and for each value of $N/\sqrt{T_1}$ find the corresponding value of $\Delta T/T_1$ for the new value of K_p

i.e.,
$$\frac{\Delta T}{T_1} = \frac{K_p\Delta T}{N^2} \cdot \left(\frac{N}{\sqrt{T_1}}\right)^2 \cdot \frac{1}{K_{pb}} .$$

(*c*) For each value of $K_p\Delta T/N^2$ listed under (*b*) find the corresponding density ratio (ρ_2/ρ_1), and read off the value of η_{pol} from Fig. 3b.

N.B.
$$(\rho_2/\rho_1) = \left(1 - \frac{\Delta T}{T_1}\right)^{\frac{1}{n-1}}$$

and
$$\frac{n}{n - 1} = \frac{\gamma_b}{\gamma_b - 1} \frac{1}{\eta_{pol}} .$$

It will be observed that a small amount of trial and error may be necessary here ; but the variation of η_{pol} with ρ_2/ρ_1 at any value of $K_p \Delta T/N^2$ is sufficiently small to make either the first or second approximation to η_{pol} sufficiently accurate.

(*d*) For each point listed under (*b*) and (*c*) find corresponding value of P_2/P_1 and η_{isen} for the new values of K_p and γ

N.B. $$P_2/P_1 = \left(1 - \frac{\Delta T}{T_1}\right)^{\frac{n}{n-1}}$$

$$\frac{n}{n-1} = \frac{\gamma_b}{\gamma_b - 1}\frac{1}{\eta_{pol}}$$

$$\eta_{isen} = \frac{\Delta T/T_1}{1 - [P_2/P_1]^{(\gamma_b - 1)/\gamma_b}} .$$

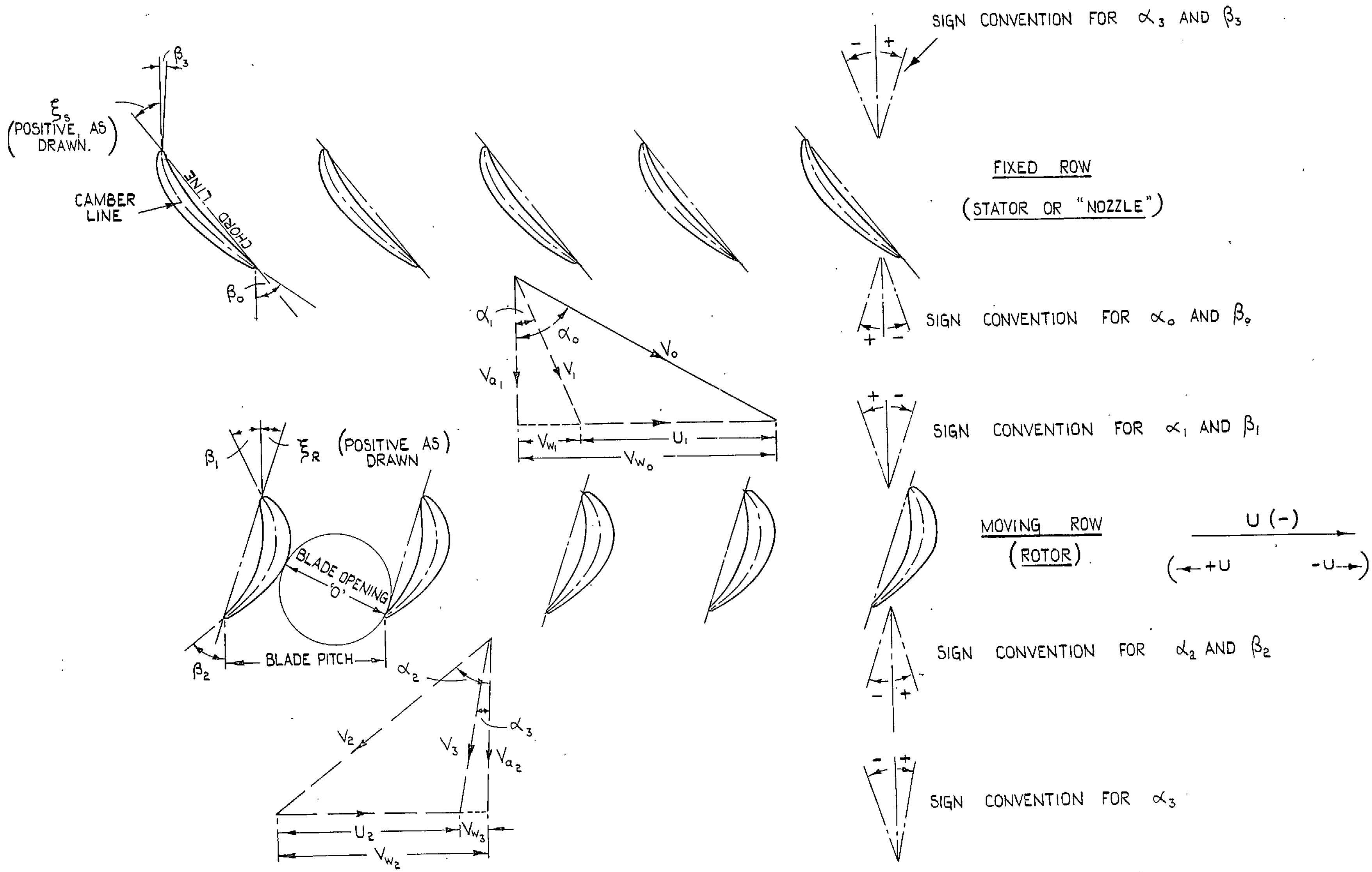
The corrections to the flow function, Q, and the corrections for increase in annulus area and blade-tip clearances are identical to those enumerated in Method 1 and need not be repeated here.

The following table illustrates numerically the corrections outlined in steps (*b*) to (*d*) for the characteristics shown in Fig. 2 when $N/\sqrt{T_1} = 150$.

TABLE 2

Line	Step							Notes
1	(*b*)	100 $K_p \Delta T/N^2$	1·2	2·4	3·6	5·2	8·0	Selected arbitrarily
2	(*b*)	$\Delta T/T_1$	0·0219	0·0439	0·0659	0·0950	0·1463	Equation (4)
3	(*c*)	η_{pol} (approx.)	0·60	0·75	0·81	0·815	0·77	Approximately from Fig. 3b ; true value of ρ_2/ρ_1 is yet unknown
4	(*c*)	$1/(n - 1)$	5·66	4·34	3·94	3·91	4·2	$1/(n - 1) = (n/n - 1) - 1)$; *see* equation (6)
5	(*c*)	ρ_2/ρ_1	0·882	0·823	0·765	0·676	0·515	$\rho_2/\rho_1 = \left(1 - \frac{\Delta T}{T_1}\right)^{1/(n-1)}$
6	(*c*)	η_{pol}	0·58	0·731	0·795	0·805	0·772	From Fig. 3b and corresponding to value of ρ_2/ρ_1 in line 5. This is second approximation and accurate for correction purposes
7	(*d*)	$n/(n - 1)$	6·90	5·48	5·04	4·98	5·19	Corresponding to η_{pol} in line 6
8	(*d*)	P_2/P_1	0·858	0·782	0·709	0·608	0·4405	Equation (5)
9	(*d*)	$1 - (P_2/P_1)^{(\gamma_b - 1)/\gamma_b}$	0·0375	0·0594	0·0826	0·1167	0·185	equal to $\Delta T_{isen}/T_1$
10	(*d*)	η_{isen}	0·585	0·740	0·798	0·815	0·791	equal to $\Delta T/\Delta T_{isen}$

The values in lines 2, 8, and 10 in the above table may be compared with the corresponding values deduced by Method 1 in lines 3, 5 and 7 of Table 1. It will be observed that the differences are very small and that each of the two methods of correction give almost identical answers.



NOTE: WHEN THE STAGGER ANGLE IS POSITIVE IN SIGN THEN THE ANGLE DEFINED BY $\cos^{-1} o/s$ MUST BE ASSIGNED NEGATIVE.

FIG. 1. Notation and sign convention for velocity triangles on an axial-flow turbine stage. (Positive stagger on blades.)

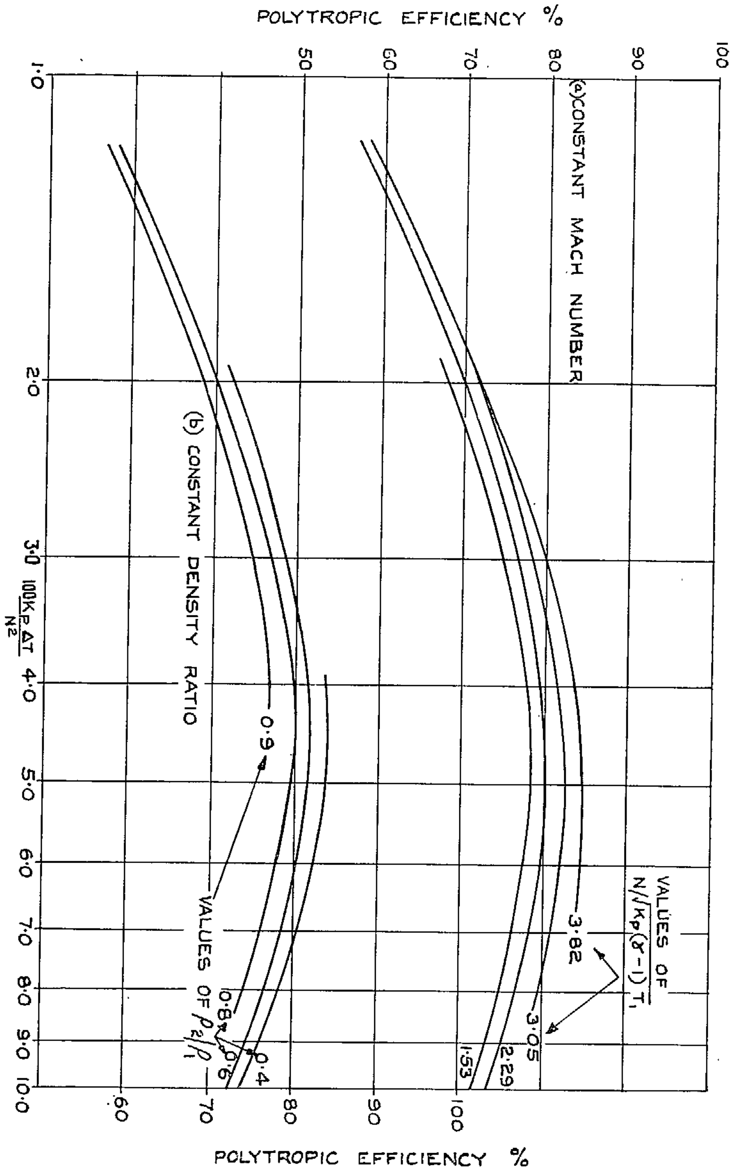


FIG. 3. Relationship between polytropic efficiency and $K_p \Delta T/N^2$.

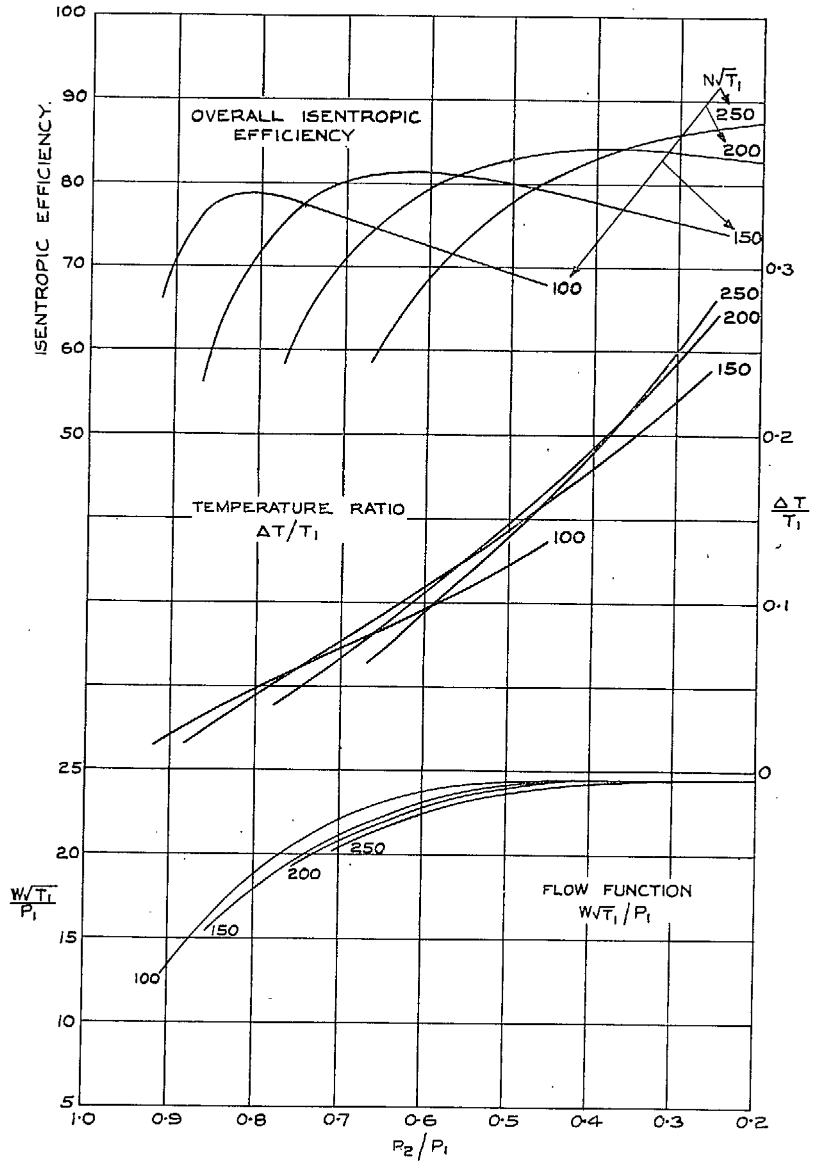


FIG. 2. Typical turbine characteristics.

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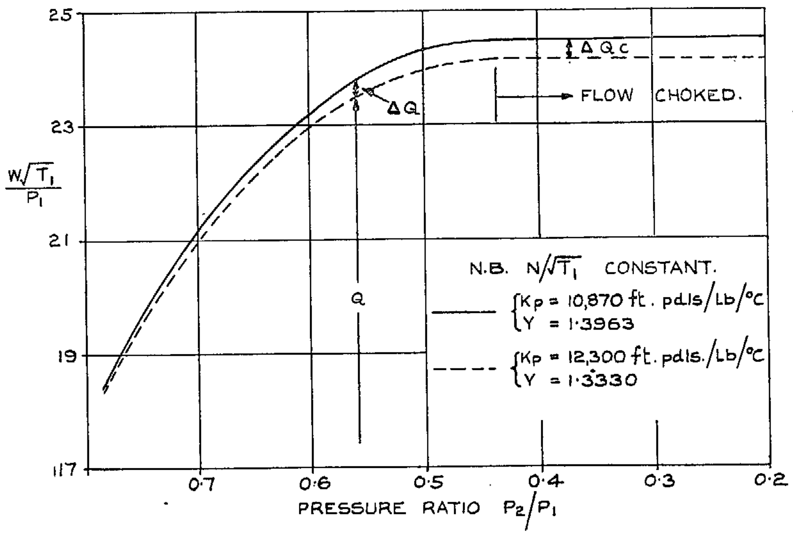

FIG. 5a.

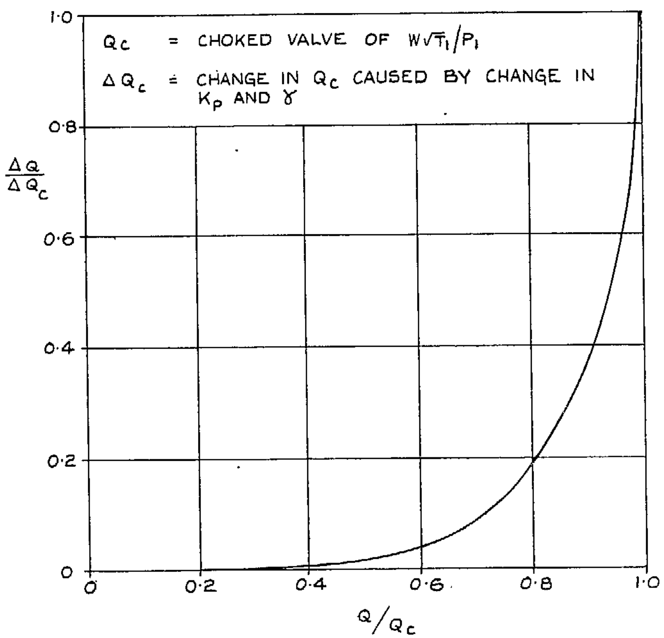

FIG. 5b.

FIG. 5. Variation of flow function with change of K_p and γ.

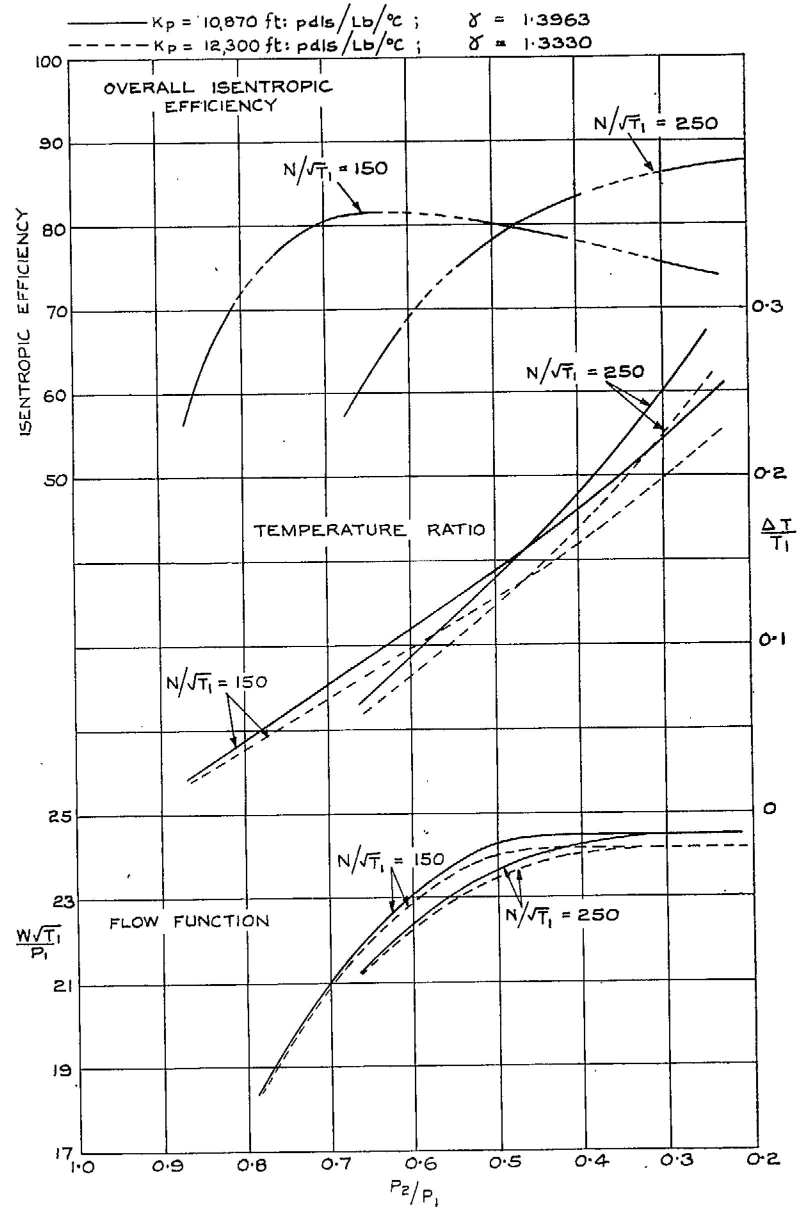

FIG. 4. Correction of turbine characteristics for change of K_p and γ.
(*N.B.* No variation of turbine dimensions or clearances.)

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B

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