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Note on Profile Drag Calculations for Low-drag Wings with Cusped Trailing Edges

By

R. C. LOCK, M.A.

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Summary.—In R. & M. 1838[1] calculations of profile drag were made based on wing sections of conventional design, and were later extended in an Addendum[2] to " low-drag " wing sections with convex trailing edges. Further calculations were required for low-drag sections of more recent design with cusped trailing edges. Calculations were made on sections of the NACA 65-family of thickness $0 \cdot 12c$ and $0 \cdot 23c$ with maximum thickness at $0 \cdot 4c$ from the leading edge, over a range of Reynolds number and position of the transition points. The results were found to differ considerably from those of Ref. 2 when the transition points were far back from the leading edge, the calculated values of the drag coefficient being in some cases as much as 25 per cent. less than the previous calculations.

The results were in good agreement with wind-tunnel tests made at the National Physical Laboratory and the Royal Aircraft Establishment[6], but showed a large discrepancy with flight tests made at the Royal Aircraft Establishment[7,8]. In these flight tests transition was fixed by means of tapes, but no account was taken of the possibility that transition may have occurred behind the tapes and not necessarily at the tapes themselves. In this way the drag foı a supposed mean transition point may have been underestimated.

1. *Introduction.*—In 1940 Squire and Winterbottom[2] made calculations of the profile drag of wing sections with their maximum-thickness position at about $0 \cdot 5c$ and with convex tails; these sections were representative of low-drag designs at that time. Modern low-drag designs, however, tend to have concave tails, and evidence has recently accumulated which suggest that the calculations of Squire and Winterbottom do not apply to such sections, particularly where the laminar layers are extensive. This is not altogether surprising, for we may expect that with far back transition differences in the shape of the rear of the section and in the trailing-edge angle with the corresponding differences in velocity distribution will result in differences in the profile drag, since the drag is dominated by the contributions of the turbulent parts of the boundary layers.

It was therefore decided to repeat the calculations for low-drag sections more representative of modern practice. The modern trend appears to be towards sections having the maximum thickness in the region of $0 \cdot 4c$, with favourable pressure distributions over the design lift-coefficient range extending back to $0 \cdot 5c$. Further, it is now being recommended, on the basis of German test results, that, on aircraft which are destined to operate under some circumstances above the shock stall, the sections should be symmetrical. The sections chosen for these repeat calculations were therefore symmetrical members of the NACA 65-series.

2. *Method.*—The method of Tetervin[3] and Holt was adopted in place of that of Squire and Young used in Refs. 1 and 2. In this method a similar formula to that of Ref. 4 (which was adopted in place of Polhausen's method in Ref. 2) is used for the laminar boundary layer, but

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for the turbulent boundary layer, instead of the step-by-step method of integrating von Kármán's boundary-layer momentum equation which was originated in Ref. 1, a formula is found to fit the experimental skin-friction relation which makes the momentum equation directly integrable, thus reducing the calculation to a simple integration and considerably lessening the labour involved. As a check, three cases were worked out independently by both methods, and as the difference in no case exceeded 3 per cent. it was concluded that Tetervin's method was of the same order of accuracy as that of Squire and Young, and it was therefore used throughout the subsequent calculations.

3. *Details of Calculations.*—The results for a flat plate used in Refs. 1 and 2 were incorporated without alteration. For the further calculations the aerofoils NACA 65–1, 012 and 65–2, 023 were used, being symmetrical sections of thickness $0 \cdot 12c$ and $0 \cdot 23c$ respectively, each having the maximum-thickness point at $0 \cdot 4c$ and maximum-suction point at $0 \cdot 5c$ over the favourable range of lift coefficient. The velocity distributions for these aerofoils used for the calculations were obtained from Ref. 5, and are shown together with the section profiles in Figs. 1 and 2. Calculations of profile drag were made for Reynolds numbers 10^6, 10^7 and 5×10^7, with the transition points at $0 \cdot 05c$, $0 \cdot 3c$, $0 \cdot 6c$, and $0 \cdot 8c$, at zero lift coefficient. The values obtained for the profile-drag coefficient are given in Table 1, and are plotted against thickness in Figs. 3–5 and against Reynolds number in Figs. 6–9.

In addition, two calculations were made for the section of thickness $0 \cdot 23c$ at a lift coefficient of approximately $0 \cdot 2$. It was found that within the limits of the accuracy of the method, the profile drag was identical with that at zero lift, and it is therefore concluded that the results obtained for zero lift are in fact valid over a small range of lift coefficients.

4. *Comparison with Previous Calculations.*—As was expected, the calculated values of the profile drag for this type of aerofoil are considerably less than the corresponding values of Ref. 2 when the transition points are well back from the leading edge, but agree fairly well when the transition points are near the leading edge. The curves of Ref. 2 are plotted in Figs. 3–5 for comparison.

5. *Comparison with Experiment.*—5.1. *Wind-tunnel Tests.*—Wind-tunnel tests have been made in the 13 ft. × 9 ft. tunnel at the N.P.L. and in the No. 2, $11\frac{1}{2}$-ft. tunnel at the R.A.E.[6] on the Boulton and Paul low-drag wing, which has the symmetrical section NACA 63–3, 018. The position of the transition points was determined by visual methods, and the profile drag was calculated from pitot-comb measurements in the wake, over a range of Reynolds number from $2 \cdot 5 \times 10^6$ to 13×10^6. The two experimental curves of profile drag coefficient against Reynolds number are shown in Fig. 10, together with the corresponding curves calculated at the N.P.L. by Tetervin's method. (For Reynolds numbers less than 6×10^6 the transition points were in the same position in both experiments; for higher Reynolds numbers they differ in the two cases and the two corresponding theoretical curves have been drawn.) It is seen that there is good agreement between theory and experiment over the greater part of the range of Reynolds number covered, though a divergence begins to appear at low Reynolds numbers, as might be expected. In all the experimental cases transition occurred between $0 \cdot 5c$ and $0 \cdot 6c$, with the exception of two cases at the N.P.L. when the position of the transition point was somewhat uncertain. There is therefore no experimental check on the calculations when the transition point is near the leading edge, but it is probable that in fact they will be of the same order of accuracy.

5.2. *Flight Tests.*—5.2.1. Flight tests have been made at the R.A.E.[7] on the NACA sections 66–2, 116 and 66–2, 216 fitted to the "King-Cobra" F.Z.440. The values of the profile-drag coefficient, determined from pitot-comb measurements in the wake of the wing with the transition points supposedly fixed by tapes, are plotted against the distance of the transition points from the leading edge in Fig. 12, together with the corresponding curves obtained by interpolation from the calculations of the present report and of Ref. 2. It is seen that both sets of theoretical values differ greatly from the experimental values, particularly when the transition points are near the leading edge ; but that the present report shows better agreement than Ref. 2 when the transition points are further back.

Since the discrepancy between theory and experiment was still large, and in the flight tests sections were used with maximum thickness at $0 \cdot 5c$ instead of those with maximum thickness at $0 \cdot 4c$ used for these calculations, two additional calculations were made on the section used for the flight tests, with the Reynolds number of the tests (15×10^6) and with the transition points at $0 \cdot 05c$ and $0 \cdot 6c$. These however showed no appreciable difference from the corresponding values interpolated from the calculations on the 65-family.

There is therefore still a considerable discrepancy to explain. The fact that the theoretical results agree fairly well with the wind-tunnel tests (*see* section 5.1) suggests that the error is probably to be found in the flight tests. The curves of Ref. 7 (*see* Fig. 11) giving the variation in profile-drag increment with thickness of tape, show that in the case when tapes were fitted at $0 \cdot 1c$ the drag increased with tape thickness even at the greatest thickness used; if this rate of increase were due to the drag of the tape, then the tape drag coefficient in terms of frontal area would have to be about $3 \cdot 0$, an inacceptable figure, bearing in mind its fineness ratio.* Little can be deduced from the results with tapes at $0 \cdot 3c$, although the very large scatter of the results with ailerons sealed cannot be overlooked; and with tapes at $0 \cdot 5c$ it appears that the drag increment was practically independent of tape thickness. It is therefore possible that in the first and second cases (tapes at $0 \cdot 1c$ and $0 \cdot 3c$) the transition points had not in fact been fixed at the desired position. This is particularly likely since at these points the pressure gradients are such that a considerable disturbance to the boundary layer would be necessary to produce immediate transition. If in fact transition actually occurred at about $0 \cdot 1c$ behind the tapes, the results would now show fair agreement with theory. When the tapes were fitted at $0 \cdot 5c$ it certainly appears that transition actually occurred in the neighbourhood of the tapes; but in this case the difference between theory and experiment is smaller and probably within the limits of experimental error.

5.2.2. Further flight tests have been made at the R.A.E.[8] on the Hurricane II Z.3687 fitted with low-drag wings of maximum thickness $0 \cdot 17c$ at $0 \cdot 42c$ from the leading edge, designed to give maximum suction at $0 \cdot 5c$. As in the previous tests, the transition points were brought forward by tapes; the results, shown in Figs. 13 and 14, are seen to be of a similar nature to those of the previous tests, and the discrepancy between theory and experiment, which is of the same order as before, may be explained in the same way, viz. the tape height and the drag for which transition occurs at the tapes have been underestimated. Again we may note that if in fact the drag rise shown in Fig. 13 with tape height was due to the drag of the tapes, then the tape drag coefficient in terms of frontal area would have to be about $1 \cdot 7$ with the tapes at $0 \cdot 1c$ and $2 \cdot 4$ with the tapes at $0 \cdot 3c$.

5.2.3. It is suggested in Ref. 7 that these discrepancies between flight tests and theory may be explained on the grounds that the theoretical assumption that transition occurs sufficiently sharply to be located at a point is an over-simplification and that in fact transition occurs over a considerable region. Against this argument we may note that

(*a*) all *direct* measurements of transition made in flight so far have never failed to indicate a sharp transition readily defined for practical purposes by a point

(*b*) tunnel measurements of transition by visual methods at Reynolds numbers sometimes approaching but frequently below those of flight tests have generally shown sharp transition fronts; were transition spread over a large region we should expect to see a blurred indication over this region. Transition regions of appreciable extent have only been noted in wind tunnels at Reynolds numbers below about 10^6

(*c*) the argument of Ref. 7 should apply to all sections, but these flight tests are the first to indicate any serious discrepancy between theory and flight measurements.

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* Ref. 9 shows that a rivet of fineness ratio $0 \cdot 15$ has a drag coefficient in terms of frontal area of $0 \cdot 23$, whilst for a blunt lapjoint the drag coefficient is $0 \cdot 34$.

5.2.4. To clear up this discrepancy between flight and theory it is suggested that in future flight tests some attempt should be made to get a direct indication of transition position, and an investigation of the effect of tape height on transition position would not be unprofitable.

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No.	*Author*	*Title, etc.*
1	Squire and Young	The Calculation of the Profile Drag of Aerofoils. R. & M. 1838. November, 1937.
2	Winterbottom and Squire	Note on Further Wing Profile-drag Calculations. R.A.E. Report No. B.A. 1634. A.R.C. 4871. October, 1940. (Unpublished.)
3	Tetervin	A Method for the Rapid Estimation of Turbulent Boundary Layer Thicknesses for Calculating Profile Drag. N.A.C.A. Advance Confidential Report No. L4G14. July, 1944. A.R.C. 8498.
4	Young and Winterbottom	Note on the Effect of Compressibility on the Profile Drag of Aerofoils at Subsonic Mach Numbers in the Absence of Shock Waves. R. & M. 2400. May, 1940.
5	Jacobs, Abbott and Davidson	Preliminary Low-drag Aerofoil and Flaps Data from Tests at Large Reynolds Numbers and Low Turbulence. N.A.C.A. Advance Confidential Report. May, 1942. A.R.C. 5845.
6	The Staffs of the N.P.L. 13 ft. × 9 ft. Tunnel and the R.A.E. No. 2, $11\frac{1}{2}$ ft. × $8\frac{1}{2}$ ft. Tunnel.	Transition and Drag Measurements on the Boulton Paul Sample of Laminar Flow Wing Construction. November, 1496. R. & M. 2499. (To be published.)
7	Smith and Higton	Flight Tests on "King Cobra" FZ.440 to investigate the Practical Requirements for the Achievement of Low Profile-drag Coefficients on a "Low-drag" Aerofoil. R. & M. 2375. August, 1945.
8	Smith, Higton and Bramwell	Flight Tests on Hurricane II Z.3687 fitted with Special Wings of "Low-drag" Design. R. & M. 2546. September, 1946. (To be published.)
9	Young, Serby and Morris	Flight Tests on the Effect of Surface Finish on Wing Drag. R. & M. 2258. July, 1939.

TABLE 1

Calculations of Profile Drag for Aerofoils of the N.A.C.A. Series 65

Wing $\frac{thickness}{chord}$	Reynolds number	Distance of T.P. behind the L.E.	C_{D0}
0	10^6	0	0·00922
		0·2c	0·00822
		0·4c	0·00712
		0·6c	0·00592
	10^7	0	0·00600
		0·2c	0·00518
		0·4c	0·00422
		0·6c	0·00322
	5×10^7	0	0·00470
		0·2c	0·00394
		0·4c	0·00316
		0·6c	0·00234
0·12	10^6	0·05c	0·01246
		0·3c	0·01054
		0·6c	0·00697
		0·8c	0·00460
	10^7	0·05c	0·00825
		0·3c	0·00654
		0·6c	0·00375
		0·8c	0·00209
	5×10^7	0·05c	0·00639
		0·3c	0·00494
		0·6c	0·00266
		0·8c	0·00135
0·23	10^6	0·05c	0·01652
		0·1c	0·01603
		0·3c	0·01329
		0·6c	0·00782
		0·8c	0·00538
	10^7	0·05c	0·01074
		0·3c	0·00828
		0·6c	0·00394
		0·8c	0·00218
	5×10^7	0·05c	0·00832
		0·3c	0·00629
		0·6c	0·00273
		0·8c	0·00136

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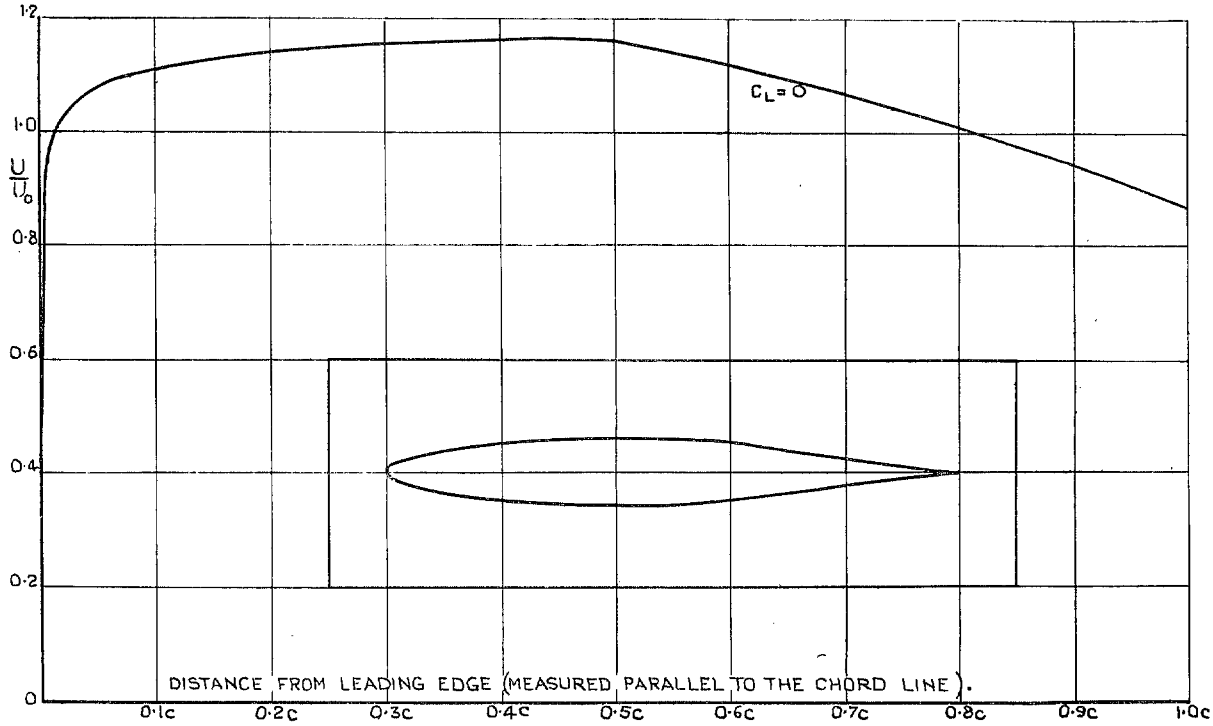


FIG. 1. Profile and velocity distribution for wing section NACA 65,1–012.

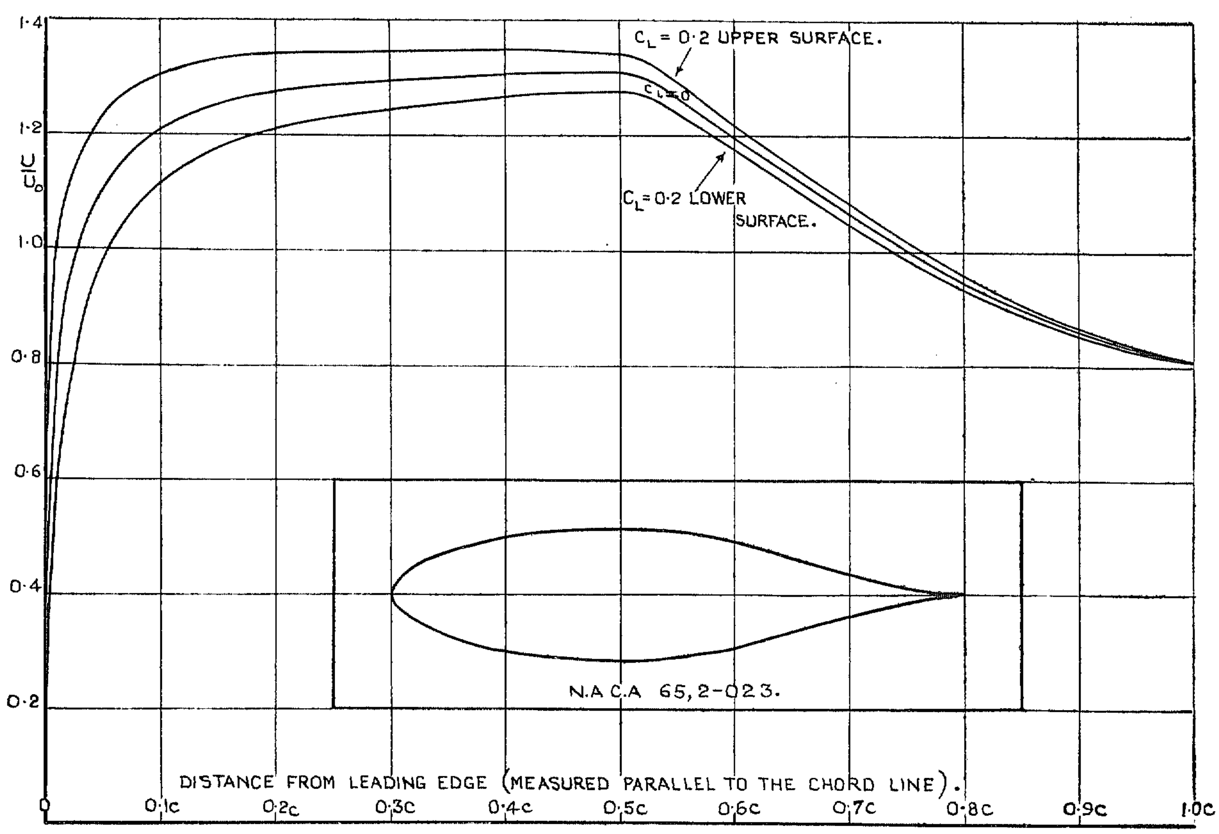


FIG. 2. Profile and velocity distribution for wing section NACA 65,2–023.

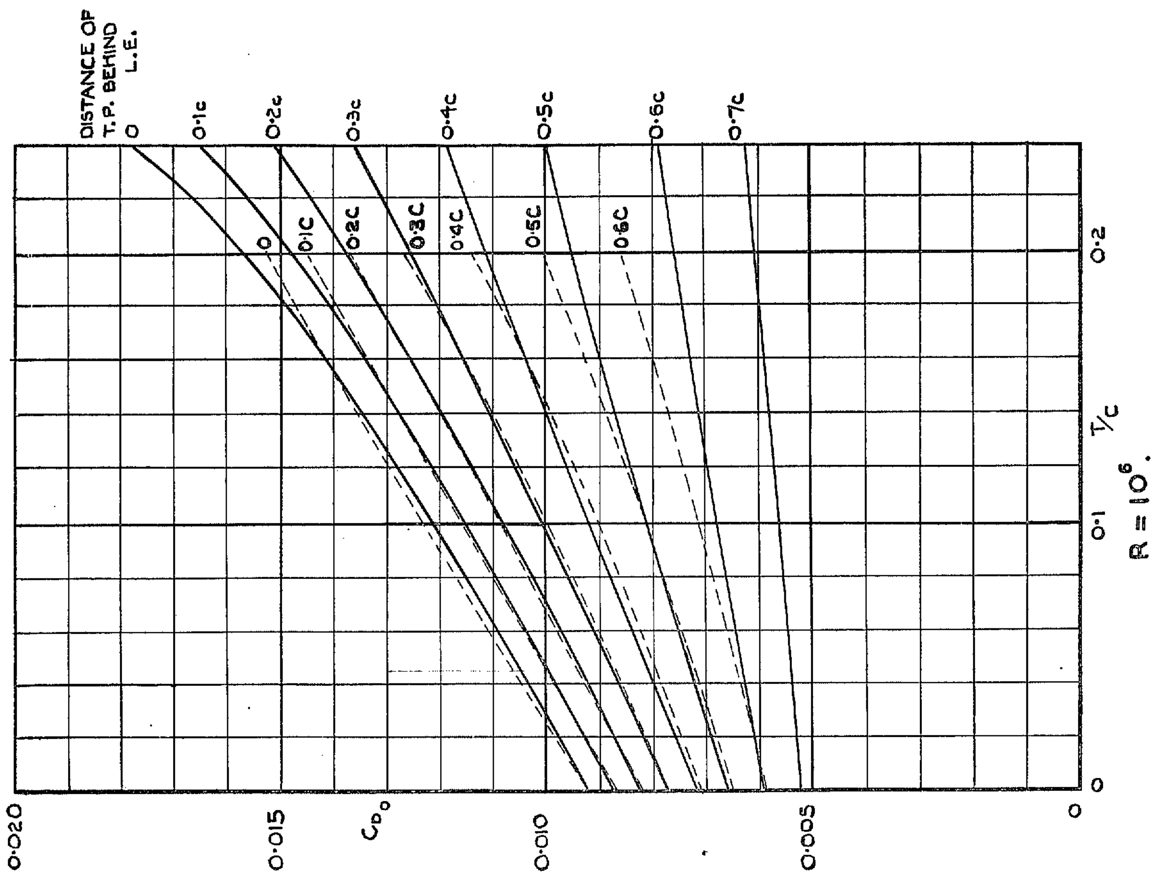


FIG. 3. Variation of calculated profile-drag coefficient with wing thickness and position of transition point.

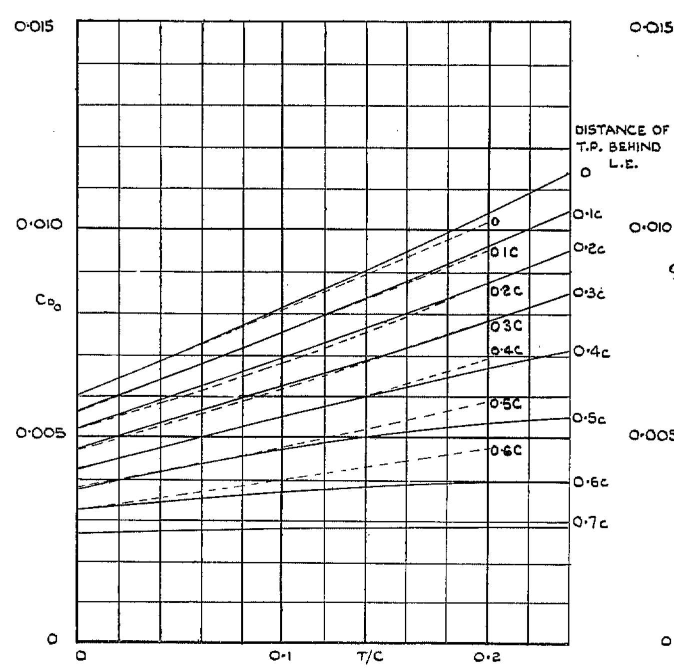


FIG. 4. $R = 10^7$.

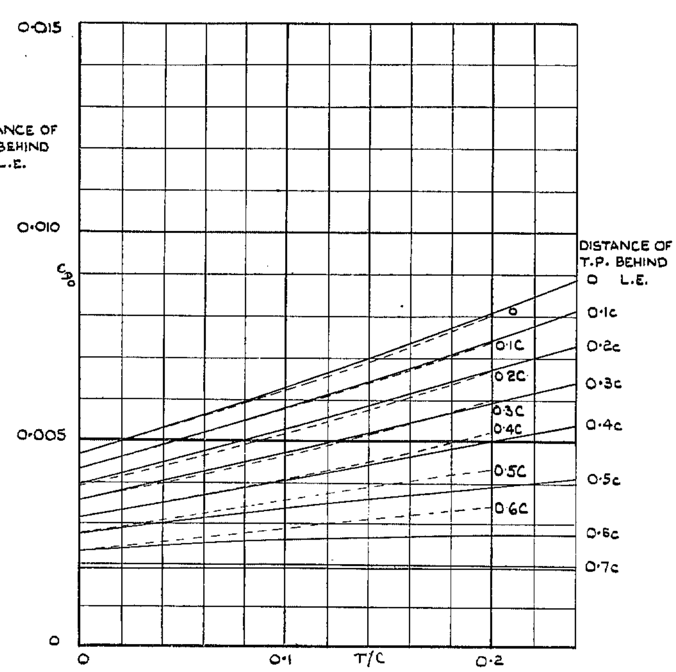
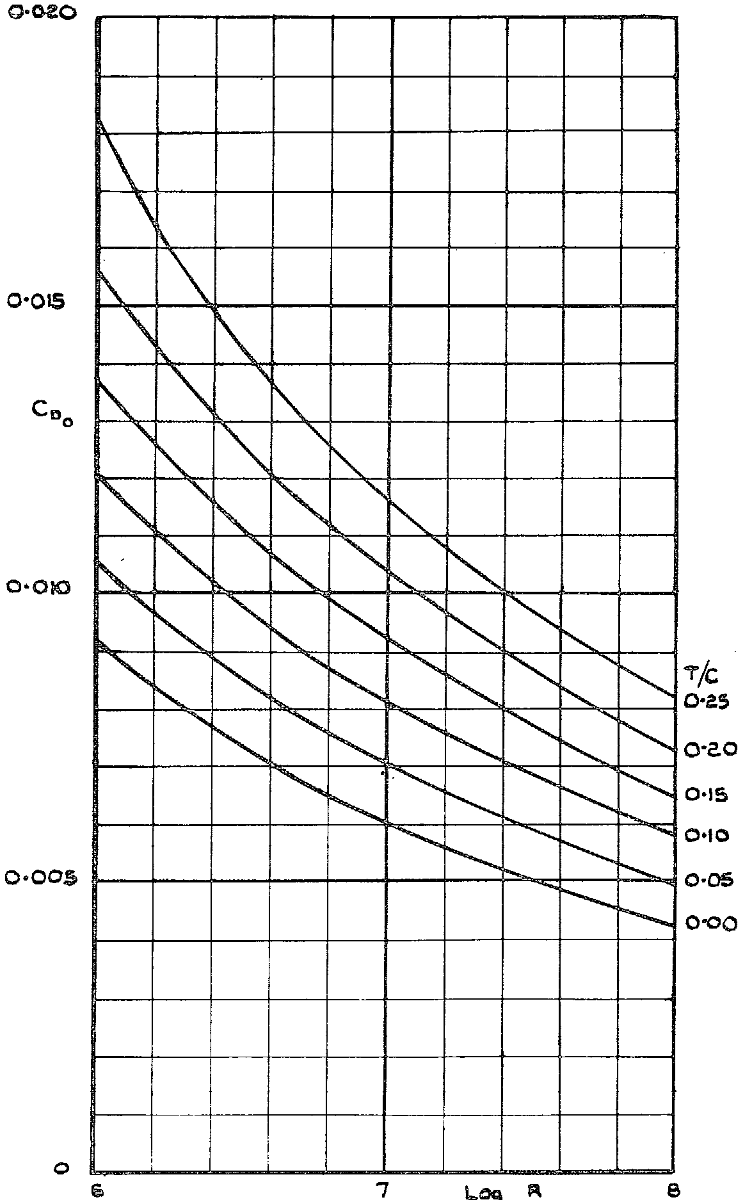


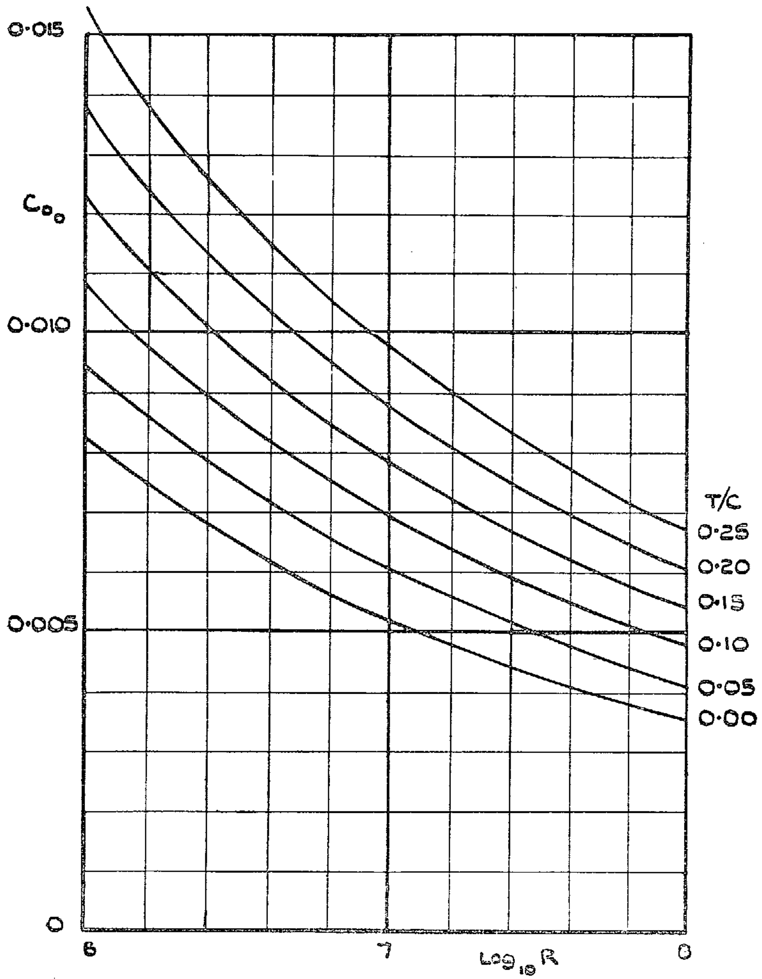
FIG. 5. $R = 5 \times 10^7$.

Variation of calculated profile-drag coefficient with wing thickness and position of transition point.



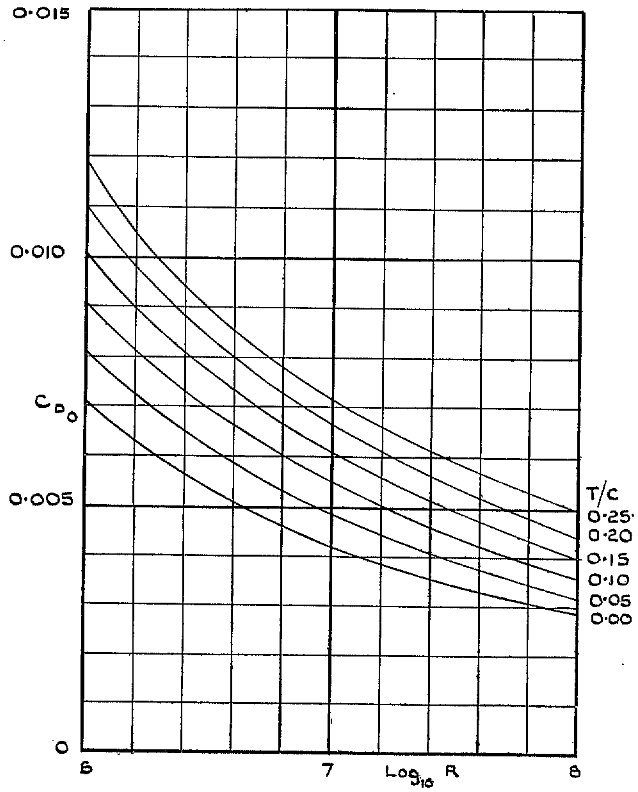
TRANSITION POINT AT L.E.

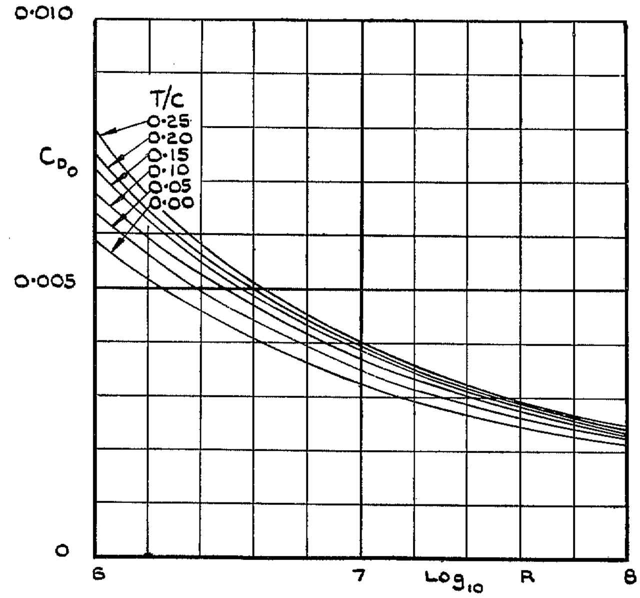
FIG. 6. Variation of profile drag with Reynolds number and wing thickness.



TRANSITION POINT AT 0·2c.

FIG. 7. Variation of profile drag with Reynolds number and wing thickness.





FIGS. 8 & 9. Variation of profile drag with Reynolds number and wing thickness.

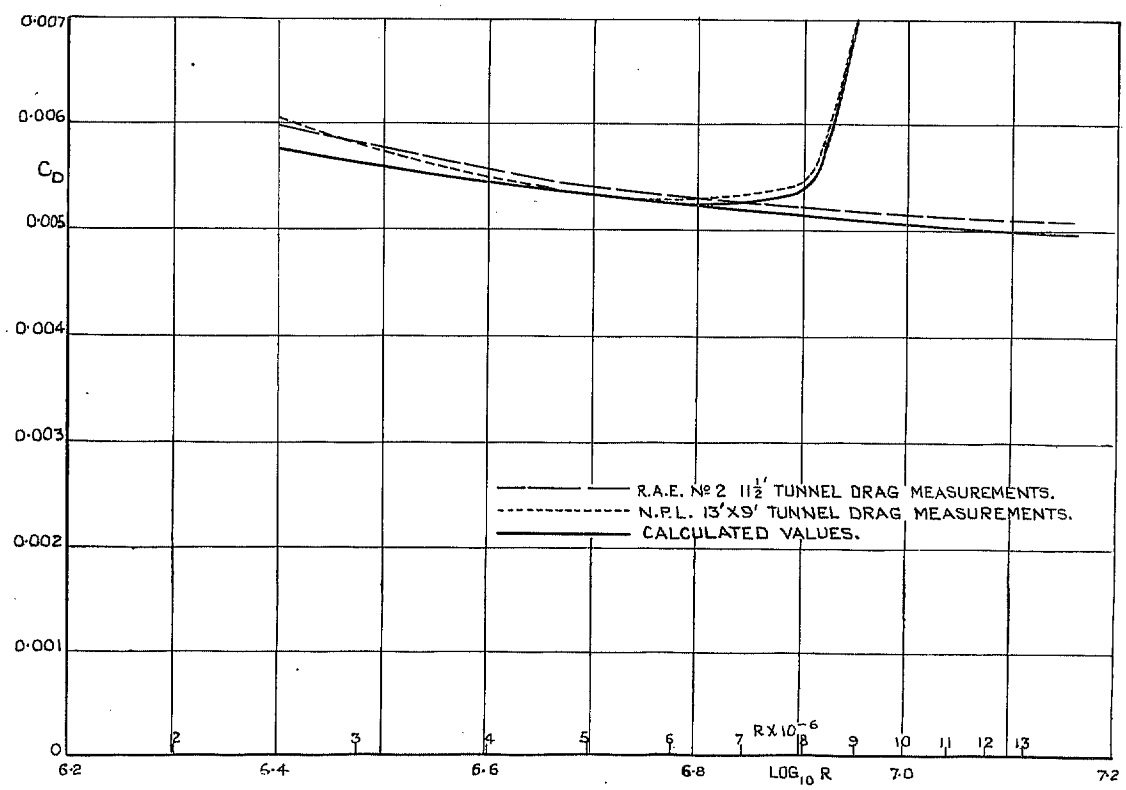


FIG. 10. Comparison with wind-tunnel tests on the Boulton Paul low-drag wing NACA 653-018.
(*see* Ret. 6)

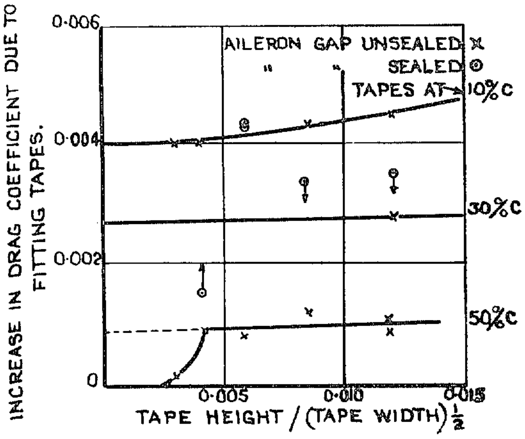


FIG. 11. Increase in drag coefficient with tape height (*see* Ref. 7).

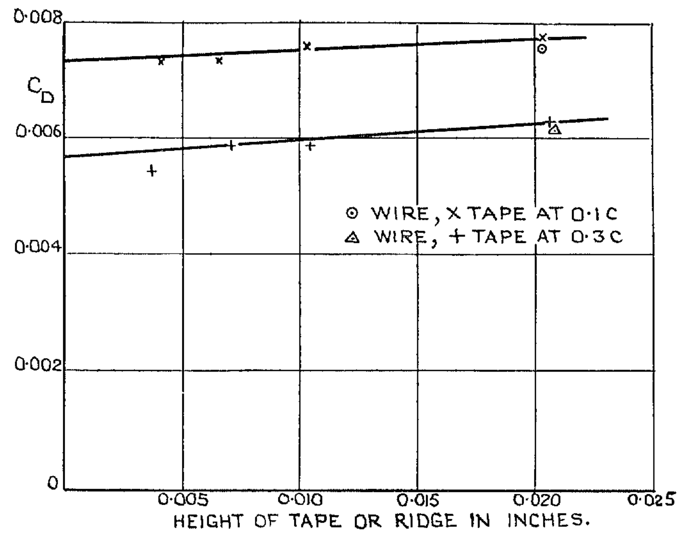


FIG. 13. Increase in drag coefficient with tape height (*see* Ref. 8).

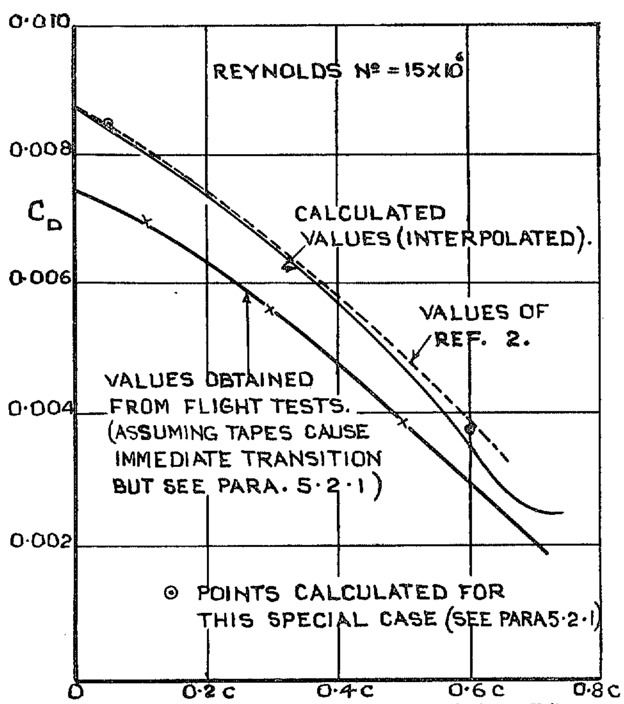


FIG. 12. Comparison with flight tests on the "King Cobra" FZ.440 (*see* Ref. 7).

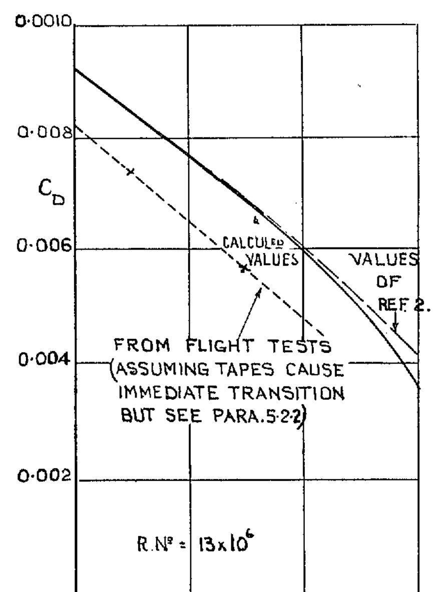


FIG. 14. Comparison with flight tests on Hurricane Z.3687 (*see* Ref. 8).

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