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The Flow Through Short Straight Pipes in a Compressible Viscous Stream

By

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The flow through short straight pipes in a compressible viscous stream

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SUMMARY

In the design of small models of aircraft or missiles for testing in supersonic tunnels, it may be desired to represent jet engine nacelles by means of simple hollow pipes. The note sets out the principal characteristics of compressible flow in such pipes at zero yaw and gives a theory for calculating the effect of the boundary layer. This is checked against the results of tests with a series of pipes of varying size, at Mach numbers from 1.34 to 2.41.

Curves are presented for determining the maximum length/radius ratio of a parallel pipe which will permit supersonic internal flow, in terms of the Mach number of the stream and Reynolds number of the pipe: the curves are given for both laminar and turbulent internal boundary layers.

The effect of inclination of the pipe to the stream is discussed briefly, on the basis of results at one Mach number (1.86).

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1 Introduction

In the design of small scale models of aircraft or missiles for testing in supersonic tunnels, it is sometimes desired to represent jet engine nacelles by means of simple hollow pipes allowing a free flow through the inside. Such a flow may be either subsonic or supersonic, depending on the Mach and Reynolds numbers and the internal taper of the pipe. It is desirable to be able to predict which type of flow will be obtained, primarily in order to determine whether or not the front external shock wave will be attached to the lip of the pipe. The nature and position of this shock will, in general, have some effect on the pressure distribution of the wing or other surfaces in proximity to the nacelle.

The principal characteristics of compressible flow through a short straight pipe immersed in a stream at zero angle of yaw are set out in the present note. The effect of an internal boundary layer is calculated on the assumption that the radius of the pipe at any point is reduced by an amount equal to the displacement thickness. With this assumption a relationship is derived giving the free stream Mach number at which the flow in the pipe just becomes supersonic, in terms of the Reynolds number and dimensions of the pipe. The relationship is given for both laminar and turbulent internal layers. The theory is supported by the results of a series of simple tests on parallel pipes of various diameters and length/radius ratios, at Mach numbers from 1.34 to 2.41.

The effect of inclination of the pipe to the stream direction is discussed briefly, on the basis of results of a few tests at one Mach number (1.86).

2 Theory

Some characteristics of the flow of a non-viscous, compressible fluid through a straight tapered pipe at zero yaw are set out in Appendix I. This shows the variation of mass flow coefficient with Mach number and the relationship between free stream tube area of the through-flow, A_1, and the entry and exit areas of the pipe, A_2 and A_3 respectively. The Mach number ranges in which the internal flow is respectively subsonic and supersonic are defined.

It is shown that, in the case of a contracting pipe ($A_3 < A_2$), the minimum value of area ratio $\psi (= A_3/A_2)$, which will allow supersonic flow to be established through the pipe, is given in terms of the free stream Mach number $M_1 (> 1)$ by the relationship:-

$$\psi = \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)\, M_1^2}\right]^{\frac{1}{2}} \left[\frac{2\gamma}{\gamma+1} - \frac{\gamma-1}{(\gamma+1)\, M_1^2}\right]^{1/(\gamma-1)} \qquad (1)$$

The relationship is determined by the condition that when a normal shock stands across the entry to the pipe, the flow at exit is just sonic. For all exit areas greater than the limiting value so defined, the normal shock is capable of passing through the pipe if a slightly greater pressure drop is applied, as for example by a small increase in free stream Mach number. The internal flow is then supersonic throughout. It is important to note that the condition necessary for the establishment of supersonic internal flow actually relates to the state of subsonic internal flow behind a normal shock.

Equation (1) was given previously by Lukasiewicz[1] (in algebraically different form) in a discussion of the flow in supersonic diffusers. The relationship is plotted in Fig.1.

The effect of viscosity is to increase the apparent contraction of the pipe - i.e. to decrease the effective exit area - because of a deficiency of mass flow in the boundary layer. It is assumed that the radius of the pipe is effectively reduced by an amount equal to the boundary layer displacement thickness. Thus if r_3 is the geometric exit radius and δ^* the displacement thickness at exit, the effective exit area is

$$(A_3)_{eff.} = A_3 \left(1 - \frac{\delta^*}{r_3}\right)^2 \qquad (2)$$

The effective area ratio is

$$\psi_{eff.} = \frac{(A_3)_{eff.}}{A_2} = \frac{A_3}{A_2}\left(1 - \frac{\delta^*}{r_3}\right)^2 \qquad (3)$$

It is further assumed that axial pressure gradients resulting from the rotational symmetry and effective taper of the pipe have a negligible effect on the boundary layer characteristics; so that δ^*/r_3 may be evaluated as for uniform flow over a flat plate.

We now consider the cases of laminar and turbulent boundary layers in turn.

2.1 Laminar layer

For a laminar layer, the velocity profile in incompressible flow is assumed to take the form:-

$$\frac{u}{U} = \sin \frac{\pi y}{2\delta_i} \qquad (4)$$

The thickness ratio (Goldstein, ref.2) is then

$$\frac{\delta_i}{\ell} = 4.8\, R^{-\frac{1}{2}} \qquad (5)$$

and the displacement thickness ratio is

$$\frac{\delta_i^*}{\delta_i} = 0.363 \qquad (6)$$

For the variation of displacement thickness with Mach number, we use Howarth's result[3], which is

$$\frac{\delta^*}{\delta_i^*} = 1 + 0.227\, M^2 \qquad (7)$$

[This results from an increase in thickness by a factor $1 + 0.08\,M^2$ coupled with a change in profile in the sense of becoming more nearly linear as the Mach number is increased]. Writing

$$\frac{\delta^*}{r_3} = \frac{\delta^*}{\delta_i^*}\cdot\frac{\delta_i^*}{\delta_i}\cdot\frac{\delta_i}{\ell}\cdot\frac{\ell}{r_3} \tag{8}$$

equation (3) becomes, for the laminar layer,

$$\psi_{\text{eff.}} = \frac{A_3}{A_2}\left[1 - 1.74\,(1 + 0.227\,M^2)\,R^{-\frac{1}{2}}\cdot\frac{\ell}{r_3}\right]^2 \tag{9}$$

where ℓ is the length of the pipe.

To determine the critical area ratio given by equation (1), it is necessary to insert into equation (9) values of M and R appropriate to the state of subsonic internal flow at the critical point. A close approximation is obtained by using mean values of M and R between those at the entry, where the conditions are those behind a normal shock at free stream Mach number, and those at the exit, where the Mach number is unity. The relationships between these mean values $\bar{M}$ and $\bar{R}$ and the free stream values are shown plotted in Fig.2. We write

$$\left(\frac{\bar{R}}{R_1}\right)^{\frac{1}{2}} = f_1(M_1) \tag{10}$$

This function has been calculated for a stagnation temperature of 20°C, and is strictly unique only on the assumption that viscosity is directly proportional to temperature. This assumption has already been invoked in using equation (7) above and in the present context is certainly adequate in view of the use of mean Reynolds numbers in a field of varying velocity.

The effective area ratio of the pipe may now be written in the form

$$\psi_{\text{eff.}} = \frac{A_3}{A_2}\left[1 - \frac{1.74}{f_1(M_1)}\,(1 + 0.227\,\bar{M}^2)\,R_1^{-\frac{1}{2}}\cdot\frac{\ell}{r_3}\right]^2 \tag{11}$$

The critical free stream Mach number, or minimum Mach number for supersonic internal flow, is obtained by equating the right-hand sides of equations (1) and (11). It is seen that the critical Mach number is a function of the geometric area ratio of the pipe, the length/radius ratio and the Reynolds number based on pipe length and free stream velocity. Otherwise put, with a completely laminar internal boundary layer, the condition for supersonic internal flow is:-

$$R_1^{-\frac{1}{2}}\cdot\frac{\ell}{r_3} < \frac{0.575\,f_1(M_1)}{(1+0.227\,\bar{M}^2)}\left[1 - \left(\frac{A_2}{A_3}\right)^{\frac{1}{2}}\left\{\frac{(\gamma-1)}{(\gamma+1)} + \frac{2}{(\gamma+1)\,M_1^2}\right\}^{\frac{1}{4}} \times \left\{\frac{2\gamma}{(\gamma+1)} - \frac{(\gamma-1)}{(\gamma+1)\,M_1^2}\right\}^{\frac{1}{2(\gamma-1)}}\right] \tag{12}$$

2.2 Turbulent layer

For the case in which the internal layer is fully turbulent, it is assumed that the velocity profile in incompressible flow takes the form

$$\frac{u}{U} = \left(\frac{y}{\delta_i}\right)^{1/7} \tag{13}$$

and that in the range $0 \leqslant M \leqslant 1.0$ there is no change of thickness or of profile with Mach number. The displacement thickness variation is then as given by Cope[4] and may be approximated by the formula:-

$$\frac{\delta^*}{\delta_i} = 0.128 \left(1 + 0.219 M^2\right) \tag{14}$$

Using for the thickness ratio in incompressible flow the formula

$$\frac{\delta_i}{\ell} = 0.37 R^{-1/5} \tag{15}$$

equation (3) becomes, in this case,

$$\psi_{eff.} = \frac{A_3}{A_2}\left[1 - 0.0474 \left(1 + 0.219 \bar{M}^2\right) \bar{R}^{-1/5} \frac{\ell}{r_3}\right]^2 \tag{16}$$

The mean internal Mach number and Reynolds number are the same as for the laminar case. Writing

$$\left(\frac{\bar{R}}{R_1}\right)^{1/5} = f_2(M_1) \tag{17}$$

(this function is plotted in Fig.2), we have

$$\psi_{eff.} = \frac{A_3}{A_2}\left[1 - \frac{0.0474}{f_2(M_1)} \left(1 + 0.219 \bar{M}^2\right) . R_1^{-1/5} . \frac{\ell}{r_3}\right]^2 \tag{18}$$

The condition for supersonic internal flow is therefore

$$R_1^{-1/5} . \frac{\ell}{r_3} < \frac{21.1\ f_2(M_1)}{(1+0.219\ \bar{M}^2)}\left[1 - \left(\frac{A_2}{A_3}\right)^{\frac{1}{2}} \left\{\frac{(\gamma-1)}{(\gamma+1)} + \frac{2}{(\gamma+1)\ M_1^2}\right\}^{\frac{1}{4}} \times \left\{\frac{2\gamma}{(\gamma+1)} - \frac{(\gamma-1)}{(\gamma+1)\ M_1^2}\right\}^{\frac{1}{2(\gamma-1)}}\right] \tag{19}$$

The relationships (12) and (19) may be used to determine the maximum length/radius ratio for supersonic internal flow in terms of the free stream Mach and Reynolds numbers. Results of such calculations for the particular case of a parallel pipe ($A_3 = A_2$) are given in Fig.16 (see Appendix II).

3 Comparison with experiment

3.1 Details of tests

In connection with a proposal to represent engine nacelles on a small scale model of a supersonic aircraft by means of straight parallel pipes, a brief series of tests was made to explore the lower limits of size which would allow supersonic internal flow at various free stream Mach numbers. The results are compared with the theory of the preceding section.

Straight parallel pipes ($A_3/A_2 = 1.0$) of various sizes were mounted in succession on a strut in a small supersonic tunnel ($5\frac{1}{2}$" × $5\frac{1}{2}$") and the nature of the internal flow (i.e. subsonic or supersonic) was determined both by schlieren observation of the external shock pattern and also - a more conclusive check - by measurement of the internal static pressure at a single point halfway along the length of the tube.

The pipes were made of thin-walled, drawn, brass or steel tubing. The leading edge of each pipe was sharpened on the outside to a 3° wedge. The inside surface was cylindrical throughout and was given a reasonably smooth finish by polishing.

Fourteen pipes were tested in all. The lengths ranged from 0.41 in. to 3.75 in. and the internal radii from 0.043 in. to 0.375 in. The dimensions are tabulated in Fig.3, which shows the relative placing of the pipes on scales of the appropriate parameters for laminar and turbulent flow.

The values of Reynolds number, based on pipe length, were in most cases below one million. Thus it was reasonable to expect, with the smooth pipes, a completely laminar internal boundary layer, except for the longest pipes at the lowest Mach numbers. For some of the tests, a turbulent layer was obtained by applying a narrow transition strip of thin tape round the inside circumference a short distance in from the entry. It was possible to use this technique only with the pipes of larger diameter but the number of tests made in this way was sufficient to give additional support to the theory (see Section 3.2).

The tests were made at Mach numbers 1.34, 1.53, 1.86 and 2.41. In addition, a few tests were made at M = 1.86 with the pipes inclined at various angles to the flow (Section 4).

3.2 Results at zero yaw

Results for the smooth pipes at zero yaw are plotted in Figs.4,5. In the upper half of Fig.4, the internal pressure ratio p_1/p_{01} is plotted against the parameter $R_1^{-\frac{1}{2}} \times \ell/r$ for each of the test Mach numbers. Each curve shows a jump where the internal flow changes from supersonic to subsonic ($R_1^{-\frac{1}{2}} \times \ell/r$ increasing). The lower half of the diagram shows the corresponding internal Mach numbers. These are calculated on the assumption that before the jump occurs the total pressure in the tube (outside the boundary layer) is equal to that in the free stream, while after the jump the total pressure is that behind a normal shock at the free stream Mach number.

Fig. 5 shows for each test Mach number the range of values of $R_1^{-\frac{1}{2}} \times \ell/r$ covered by the pipes tested. The nature of the symbol indicates whether the internal flow was subsonic or supersonic (as deduced from the pressure measurements). The curve from equation (12), defining the theoretical boundary for wholly laminar flow is plotted and, on the whole the experimental results conform well to this boundary. At the two lowest Mach numbers, some cases of subsonic flow occurred at values of $R_1^{-\frac{1}{2}} \times \ell/r$ below the theoretical critical value. These results were obtained with some of the largest pipes, having a Reynolds number greater than 10^6, and the probable explanation is that in these cases the boundary layer became turbulent before the exit.

It is concluded that the results support the theory of section 2.

As a further point of interest, it may be shown from equation (11) that after the normal shock is swallowed, changing the internal flow from subsonic to supersonic, the laminar boundary layer actually thickens because the effect of increase of Mach number outweighs that of the Reynolds number change. The supersonic flow therefore sustains a degree of contraction greater than that which just allows the normal shock to pass. This is confirmed by the pressure readings, from which it can be shown that even halfway along the tube the Mach number (Fig.4) in supersonic flow near the critical point is lower than that corresponding to the critical area ratio.

Figs. 6, 7 show the results obtained with transition strips inside the tubes. The basis of plotting is the parameter for wholly turbulent flow, $R_1^{-1/5} \times \ell/r$. The number of experimental points is small owing to the difficulty of applying the turbulence strip technique to the tubes of smaller diameter but on the whole the results give further support to the theory. There is a suggestion that the experimental boundary is displaced from the theoretical one in a direction restricting the development of supersonic flow. This may be either because the transition strips gave the effect of turbulent layers of length somewhat greater than the actual pipe lengths or because theory underestimates the increase of displacement thickness with Mach number in the case of the turbulent layer. The former explanation seems the more likely.

In one case at $M_1 = 1.34$, both the supersonic and subsonic internal flow states were observed at different times during the run. It is seen that this case lies close to the theoretical boundary line. Care is clearly necessary in using the theoretical curve outside the range in which it is supported by the practical results, i.e. beyond $M = 2$.

In Figs. 8-11 schlieren pictures are presented showing the two types of flow, with laminar boundary layer, at $M = 1.34$, 1.53 and 1.86 respectively, and also two cases where the addition of a transition strip caused a change from supersonic to subsonic flow.

The difference in external shock pattern, according as the internal flow is supersonic or subsonic, becomes less obvious as the free stream Mach number is increased.

4 Flow in inclined pipes

At one Mach number ($M_1 = 1.86$), a few tests were made to determine the effect of inclining the pipe at an angle to the stream. In Figs.12(a) and (b) the internal Mach number, calculated as before from the single static pressure measurement midway along the pipe, is plotted as a function of angle of inclination, or yaw, β. Fig.12(a) applies to pipes in which the boundary layer at zero yaw is laminar, Fig.12(b) to those in which it is turbulent. Two features of the curves are to be noted:-

(1) In each set of tests the order in which the jumps occur with increase of β corresponds to the order of placing on a scale of the turbulent flow parameter $R_1^{-1/5} \times \ell/r$, as shown in Fig.3. From this it is inferred that when, at zero yaw, the internal boundary layer is laminar and the internal flow supersonic, the effect of the first few degrees of yaw is to bring transition forward up the pipe, as a result of disturbances just inside the entry. Thereafter the yaw effect is qualitatively similar to that for a pipe starting with a turbulent layer at zero yaw.

(2) The velocity at the specified point inside the pipe is not necessarily subsonic after the jump, but becomes increasingly supersonic with increase of β. It is presumed that the flow separates from the sharp leading edge of the inclined pipe, forming a throat further downstream inside the pipe. The velocity at this throat becomes sonic and the throat is then followed by a supersonic expansion extending some further distance down the pipe. Hence the measured internal pressure may correspond to a supersonic velocity even though a detached shock is present at the entry. Further increase of the angle of inclination would cause the sonic throat to contract progressively until, at or near 90 degrees yaw, the mass flow in the pipe became zero.

Two comparisons of external flow patterns on opposite sides of the jump are shown in Fig.11. In the first comparison, pipe No. 4 is shown at 0° and $12\frac{1}{2}^{\circ}$ yaw. The difference in external shock formation at the two angles can be detected but is fairly small. It should be noted that the plane of yaw is at right angles to the plane of the photograph; bigger differences than those shown may exist in the plane of yaw. The plane of the photograph is however the more appropriate for indicating the degree of interference of, say, a nacelle at pitching incidence on a wing.

The second comparison shows two shock formations obtained under nominally identical conditions with pipe No. 7. At the critical angle of 6° it was observed that over a period of the order of a minute the flow starting from the supersonic configuration, grew slowly more subsonic (internal pressure rising, external bow wave widening) and then, having reached a limit went quickly supersonic again. It appears, therefore, that periodic fluctuations are liable to occur near the changeover condition.

From inspection of the results, it is deduced that for pipes starting with a completely laminar layer, about 7° of yaw is required to make the layer fully turbulent. If the value of $R_1^{-1/5} \cdot \ell/r$ is greater than the critical, the entry flow will by this time have become subsonic. If the value of $R_1^{-1/5} \cdot \ell/r$ is less than the critical, the change of flow will occur at some higher angle, determined by the value of this turbulent flow parameter rather than that of the parameter for laminar flow, $R_1^{-\frac{1}{2}} \cdot \ell/r$. Thus, as a first approximation, a single plotting of all the results in terms of the turbulent flow parameter is possible. This is shown in Fig. 13. For each test made, the value of $R_1^{-1/5} \cdot \ell/r$ of the pipe is plotted against $(\beta - \beta_t)$, where β_t is defined as the angle for which the internal boundary layer first becomes completely turbulent. β_t is zero for the pipes with transition strip and is taken to be 7° for the smooth pipes. A single boundary can be defined between the cases giving supersonic internal flow and those for which the flow at entry is subsonic.

As a suggestion for the form of the boundary curve, if it is assumed that the curve is symmetrical about the axis $(\beta - \beta_t) = 0$, and that when ℓ/r is zero (i.e. when the pipe becomes a thin ring) the critical angle is $\pi/2$*, then an appropriate formal relationship is:-

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* Strictly, when $\beta_t = 7^{\circ}$, the critical angle for $\ell/r = 0$ is $\beta - \beta_t = \pi/2 - 7^{\circ}$ but this correction is unimportant in relation to the moderate angles of yaw under consideration.

$$Z = Z_{max} \, e^{-k \tan^2(\beta - \beta_t)} \qquad (20)$$

where Z is written for the parameter $R_1^{-1/5} \cdot \ell/r$ and Z_{max} is the critical value for turbulent flow at $\beta = 0$ and the particular Mach number, given by equation (19). A curve with $k = 7$ provides a good fit to the few results obtained.

5 Conclusions

Straight open pipes may be used to provide simple representation of jet engine nacelles on supersonic wind tunnel models. The conditions that the flow through should be unchoked have been determined theoretically and checked experimentally, with good general agreement.

The effect of yaw has been considered briefly.

List of symbols

p	static pressure
T	temperature
M	Mach number
$\overline{M}$	mean Mach number of subsonic internal flow
a	sonic velocity
A	cross-sectional area
m	rate of mass flow
ℓ	length of pipe
R	Reynolds number based on ℓ
$\overline{R}$	mean Reynolds number of subsonic internal flow
r	radius of pipe
y	normal distance from surface
δ	thickness of boundary layer
δ^*	displacement thickness of boundary layer
u	local velocity in boundary layer
U	local velocity just outside boundary layer
ψ	pipe area ratio, exit area ÷ entry area
t	T/T_o
γ	ratio of specific heats
$f_1(M_1)$	$(\overline{R}/R_1)^{\frac{1}{2}}$
$f_2(M_1)$	$(\overline{R}/R_1)^{1/5}$

List of symbols (cont'd.)

β angle of inclination (incidence or yaw) of pipe

β_t minimum value of β for which internal boundary layer is fully turbulent

Z critical value of $R_1^{-1/5} \cdot \ell/r$

Z_{max} critical value of $R_1^{-1/5} \cdot \ell/r$ at zero yaw

k empirical constant in yaw relationship - equation (20)

λ critical value of ℓ/r

Suffixes (except in f_1 and f_2) :-

o stagnation condition

1 in free stream

2 at pipe entry (internal)

3 at pipe exit (internal)

ℓ pertaining to laminar flow

t pertaining to turbulent flow

i pertaining to incompressible flow (Section 2) or to conditions at the representative point inside the pipe (Sections 3,4)

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APPENDIX I

Note on the characteristics of non-viscous, compressible flow in straight pipes of monotonic taper

1 Notation

p = pressure

T = temperature

M = Mach number

a = sonic velocity

A = cross-sectional area

m = rate of mass flow

γ = ratio of specific heats

t = T/T_o

ψ = A_3/A_2

$(\ )_o$ refers to stagnation conditions

$(\ )_1$ " " conditions in the free stream

$(\ )_2$ " " " at the pipe entry

$(\ )_3$ " " " " " " exit

$(\ )'$ " " " behind a normal shock

2 Parallel pipe

In a straight parallel pipe, with sharp leading edge, the internal Mach number is always equal to the free stream Mach number*. The mass flow enclosed by the pipe is given by the equation:-

$$\frac{m\ a_o}{A_2\ p_o} = \gamma\ M_1\ t_1^{(\gamma+1)/2(\gamma-1)} \tag{21}$$

This is plotted for air ($\gamma = 1.4$) in Fig. 14 (curve (1)). The mass flow is a maximum at $M_1 = 1.0$, when

$$\frac{m\ a_o}{A_2\ p_o} = \gamma \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} = 0.810 \tag{22}$$

---

* Clearly, if the wall is infinitely thin, the presence of a parallel pipe creates no disturbance in the flow, either internally or externally.

The free stream tube area, A_1, is equal to the entry area, A_2, at all Mach numbers. At supersonic speeds, the condition $A_1 = A_2$ is usually termed "full mass flow".

3 Expanding pipe

Corresponding characteristics of the flow through an expanding pipe are shown by curve (2) of Fig.14. At low subsonic speeds the flow is governed by the condition that the internal static pressure attains the free stream value at the exit. Hence the exit Mach number is equal to M_1, the free stream tube area is equal to A_3, and the mass flow coefficient is greater than that for the parallel pipe in the ratio A_3/A_2. The value of mass flow coefficient given by equation (22) above is consequently reached at some value of M_1 below unity. Here the entry chokes ($M_2 = 1.0$) and between this Mach number and unity the mass flow remains constant while the free stream tube area decreases from the value A_3 to the value A_2.

Above the choking Mach number, a normal shock travels along the pipe as M_1 is increased, and at some higher value, which may be either subsonic or supersonic, the shock reaches the exit. From this point onwards the internal flow is completely supersonic and the pressure difference at exit is resolved through a train of shock waves and expansions, beginning with shocks.

These points are illustrated further in Fig.15, where theoretical curves are shown for the choking boundary and the Mach number at which the internal flow becomes completely supersonic.

Above $M_1 = 1.0$ the entry is at "full mass flow" ($A_1 = A_2$) and this mass flow is the same as for a parallel pipe having the same entry area.

4 Contracting pipe

Curve (3) of Fig.14 illustrates the flow through a contracting pipe. At subsonic speeds, the condition that the static pressure at exit has the free stream value implies that the exit Mach number is equal to M_1, the free stream tube area is equal to A_3 and therefore the mass flow coefficient is lower than that for the parallel pipe in the ratio A_3/A_2. Consequently in this case the value of mass flow given by equation (22), corresponding to choking of the entry is never attained. At $M_1 = 1.0$ however, the exit chokes ($M_3 = 1.0$).

This condition then extends into the supersonic range. Since the exit is choked, the flow remains subsonic throughout the pipe: hence a normal shock is formed in the free stream ahead of the entry. The mass flow is determined by the conditions at the exit, namely, that $M_3 = 1.0$ and the total pressure is that behind a normal shock at the free stream Mach number. We may therefore formulate an expression for mass flow similar to that of equation (22) for the entry choke. We have, then

$$\frac{m\ a_o}{A_3\ p_o'} = \gamma \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} \tag{23}$$

In terms of the entry area and free stream total pressure, this becomes

$$\frac{m\ a_o}{A_2\ p_o} = \gamma \cdot \psi\ \frac{p_o'}{p_o} \cdot \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)} \tag{24}$$

where $\psi = A_3/A_2$ and p_o'/p_o is the total pressure ratio across a normal shock at Mach number M_1, given by

$$\frac{p_o'}{p_o} = \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)\,M_1^2}\right]^{-\gamma/(\gamma-1)} \times \left[\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right]^{-1/\gamma-1} \qquad (25)$$

This mass flow coefficient is plotted in Fig. 14 for the Mach number range corresponding to exit choke. In the same range the free stream tube area, which at $M_1 = 1.0$ is equal to A_3, increases with increase of M_1 towards the value A_2. The normal shock moves downstream towards the entry.

A value of M_1 is reached at which the mass flow coefficient given by equation (24) is equal to that of curve (1) for supersonic internal flow. At this point, the free stream tube area is equal to A_2 and the normal shock lies across the plane of the entry. Above this point, the condition of choked exit and subsonic internal flow would give a greater mass flow than is obtained with supersonic internal flow. The former condition would require the free stream tube area to be greater than A_2, with a detached normal shock followed by subsonic acceleration into the entry. This is not a stable solution. Instead, at the critical Mach number, the shock is swallowed and the condition of supersonic internal flow is obtained as with the parallel and expanding pipes. The entry is at "full mass flow" ($A_1 = A_2$), the mass flow is given by equation (21), and a residual pressure difference at exit is resolved by means of a shock-expansion train beginning with expansions.

The critical Mach number, corresponding to swallowing of the shock, is obtained in terms of the area ratio of the pipe by equating the two expressions for mass flow given by equations (21) and (24). This leads to the relationship:-

$$\psi = \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)\,M_1^2}\right]^{\frac{1}{2}} \left[\frac{2\gamma}{\gamma+1} - \frac{\gamma-1}{(\gamma+1)\,M_1^2}\right]^{1/(\gamma-1)}$$

which is equation (1) of the present report.

APPENDIX II

Critical length/radius ratio for parallel pipes in viscous flow

Equations (12) and (19) define the relationships between the Mach number at which the internal flow becomes supersonic and the parameters $R_1^{-\frac{1}{2}} \cdot \ell/r$ and $R_1^{-1/5} \cdot \ell/r$ for laminar and turbulent boundary layers respectively. For a parallel pipe ($A_3 = A_2$), the relationships become:-

$$R_1^{-\frac{1}{2}} \cdot \ell/r = \frac{0.575\ f_1(M_1)}{(1 + 0.227\ \bar{M}^{-2})} (1 - \sqrt{\psi}) \tag{12a}$$

for a laminar layer, and

$$R_1^{-1/5} \cdot \ell/r = \frac{21.1\ f_2(M_1)}{(1 + 0.219\ \bar{M}^2)} \ (1 - \sqrt{\psi}) \tag{19a}$$

for a turbulent layer, where ψ is defined by equation (1).

From these equations, the critical length/radius ratio of a parallel pipe has been calculated for various Mach numbers from 1.0 to 3.0 and various Reynolds numbers between 10^4 and 10^7. The results are plotted in Figs.16(a) and (b) - for laminar and turbulent layers respectively - in the form of curves of ℓ/r against $\log R_1$ at constant M_1.

At $R_1 = 10^4$ the critical ratio is somewhat greater for a turbulent layer than for a laminar layer. At $R_1 = 10^5$ the values are much the same in the two cases. At $R_1 = 10^6$ the values for the laminar layer are about 50% greater than those for the turbulent layer.

We note that since $\bar{M} \leq 1.0$, the denominators of the right hand sides of equations (12a) and (19a) are the same to within 1%. Equating these expressions leads to the following approximate result for the ratio of critical values. Writing λ for the critical value of ℓ/r, with appropriate suffix for laminar or turbulent flow, we have

$$\frac{\lambda_\ell}{\lambda_t} \doteqdot \frac{0.575\ f_1(M_1).R_1^{\frac{1}{2}}}{21.1\ f_2(M_1)\ R_1^{1/5}} = \frac{0.575\ (\bar{R})^{\frac{1}{2}}}{21.1\ (\bar{R})^{1/5}} = 0.02725\ (\bar{R})^{0.3} \tag{26}$$

when $\bar{R} = 10^6$ this has the value 1.72. In general therefore, the ratio is

$$\frac{\lambda_\ell}{\lambda_t} = 1.72 \left[10^{(\log \bar{R} - 6)}\right]^{0.3}$$

or, since $10^{0.3} \doteqdot 2.0$,

$$\frac{\lambda_\ell}{\lambda_t} \doteqdot 1.72 \times 2^{(\log \bar{R} - 6)} \qquad (27)$$

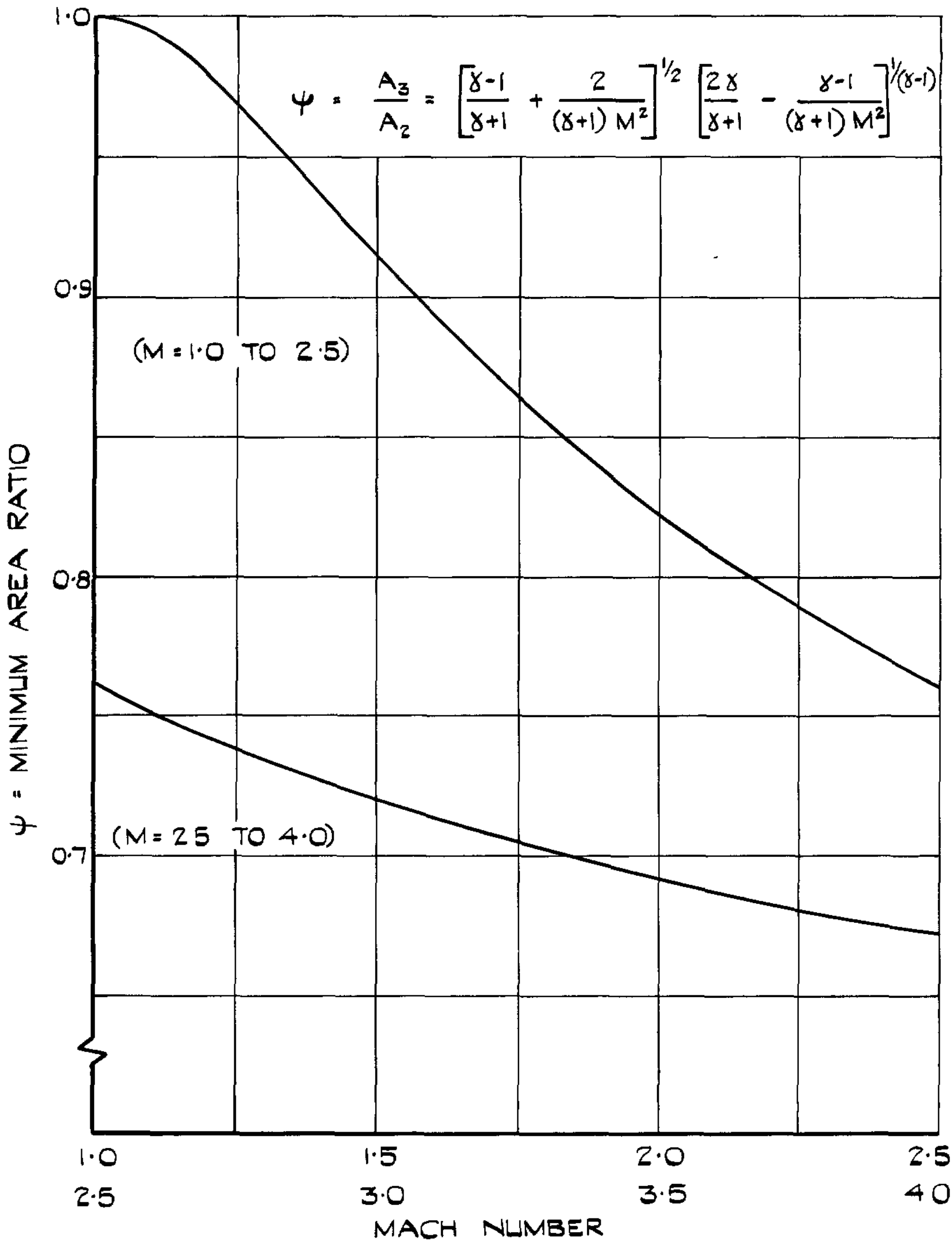


FIG. 1. MINIMUM AREA RATIO OF A PIPE FOR SUPERSONIC INTERNAL FLOW.

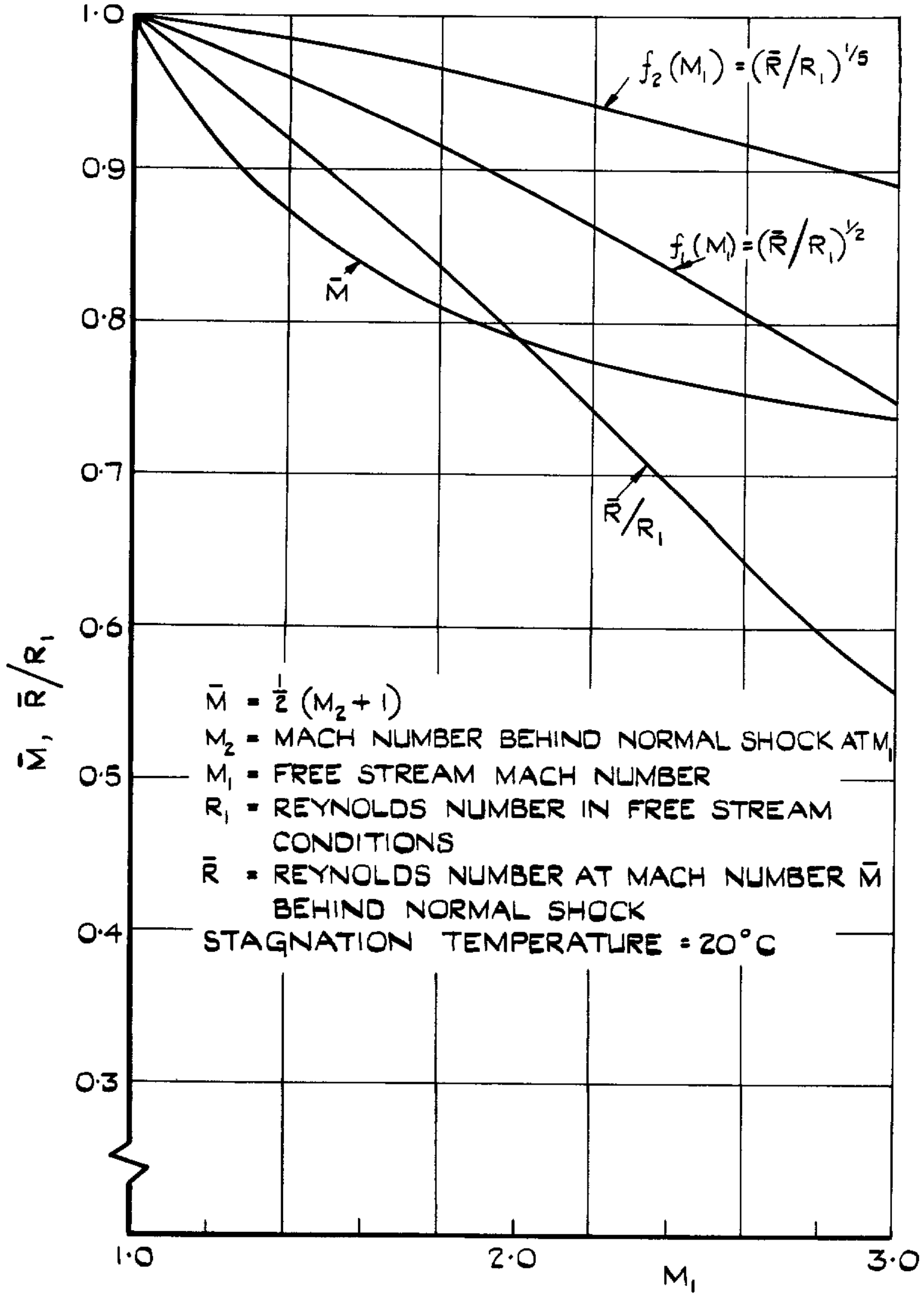
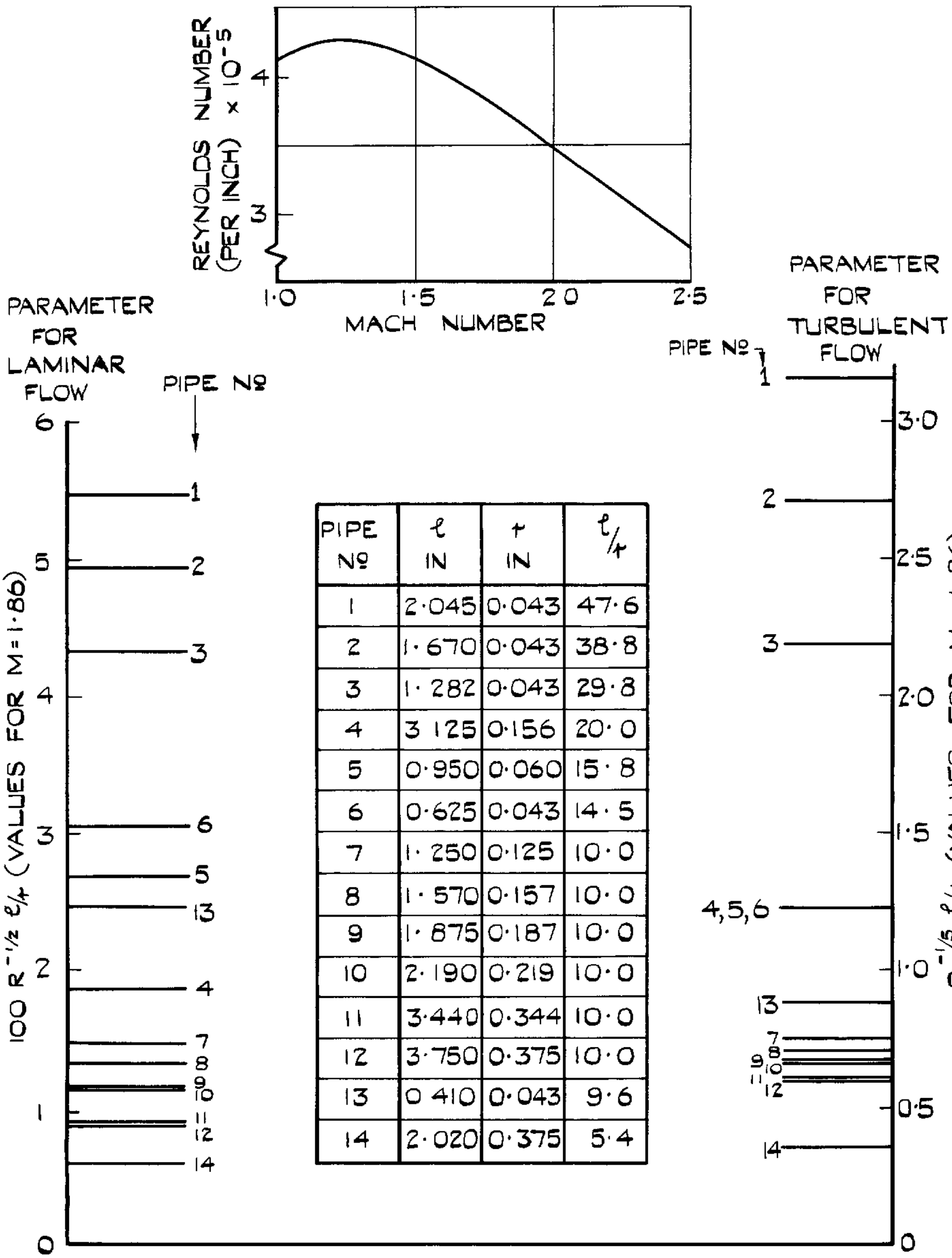


FIG. 2. INTERNAL MACH AND REYNOLDS NUMBERS AS FUNCTIONS OF FREE STREAM M_1.



PIPE Nº	ℓ IN	t IN	ℓ/t
1	2·045	0·043	47·6
2	1·670	0·043	38·8
3	1·282	0·043	29·8
4	3 125	0·156	20·0
5	0·950	0·060	15·8
6	0·625	0·043	14·5
7	1·250	0·125	10·0
8	1·570	0·157	10·0
9	1·875	0·187	10·0
10	2·190	0·219	10·0
11	3·440	0·344	10·0
12	3·750	0·375	10·0
13	0 410	0·043	9·6
14	2·020	0·375	5·4

FIG. 3. DETAILS OF MODEL PIPES SHOWING ORDER OF PLACING ON SCALES OF PARAMETERS FOR LAMINAR & TURBULENT FLOW.

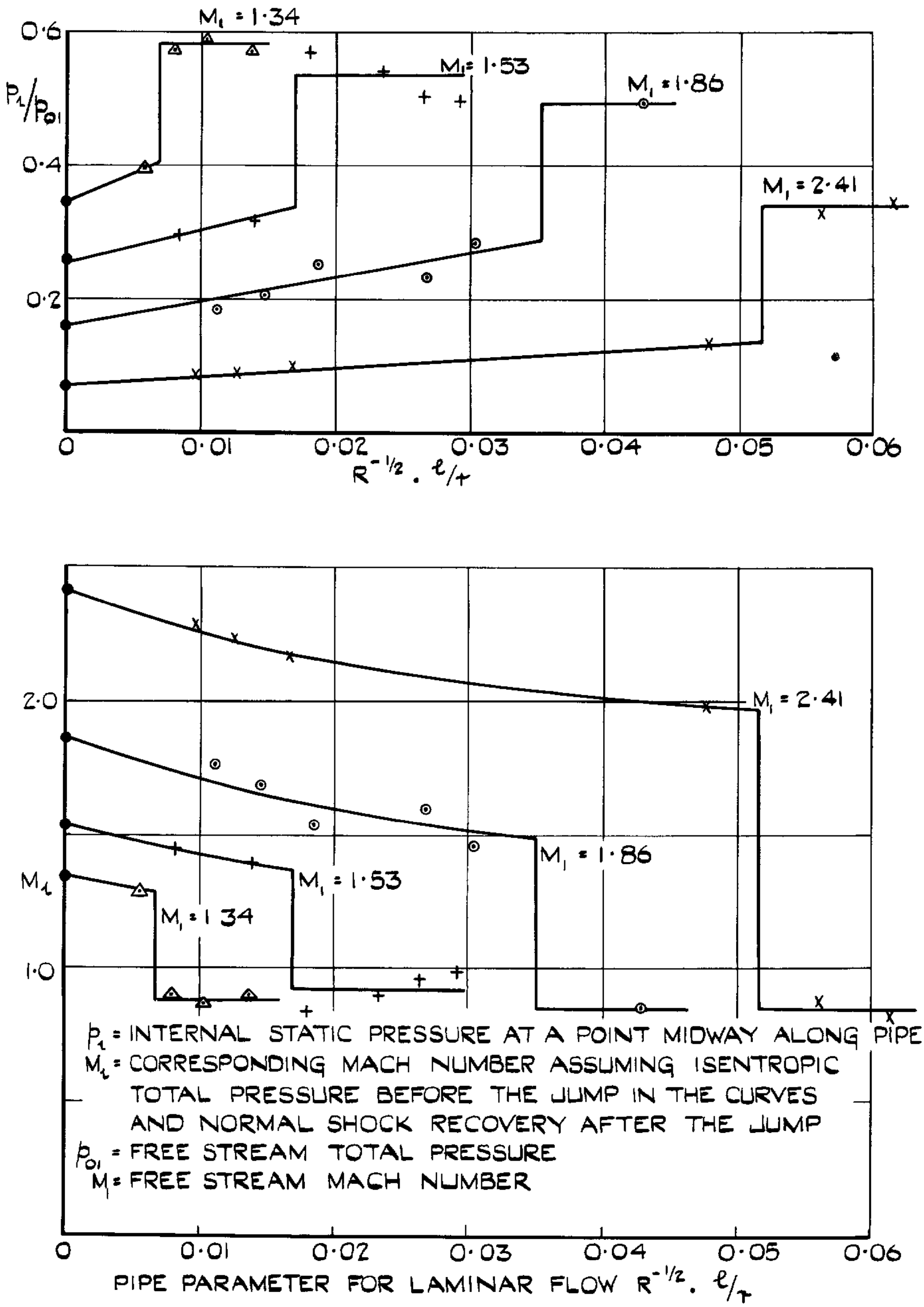


FIG. 4. STATIC PRESSURE AND MACH NUMBER IN SMOOTH PARALLEL PIPES.

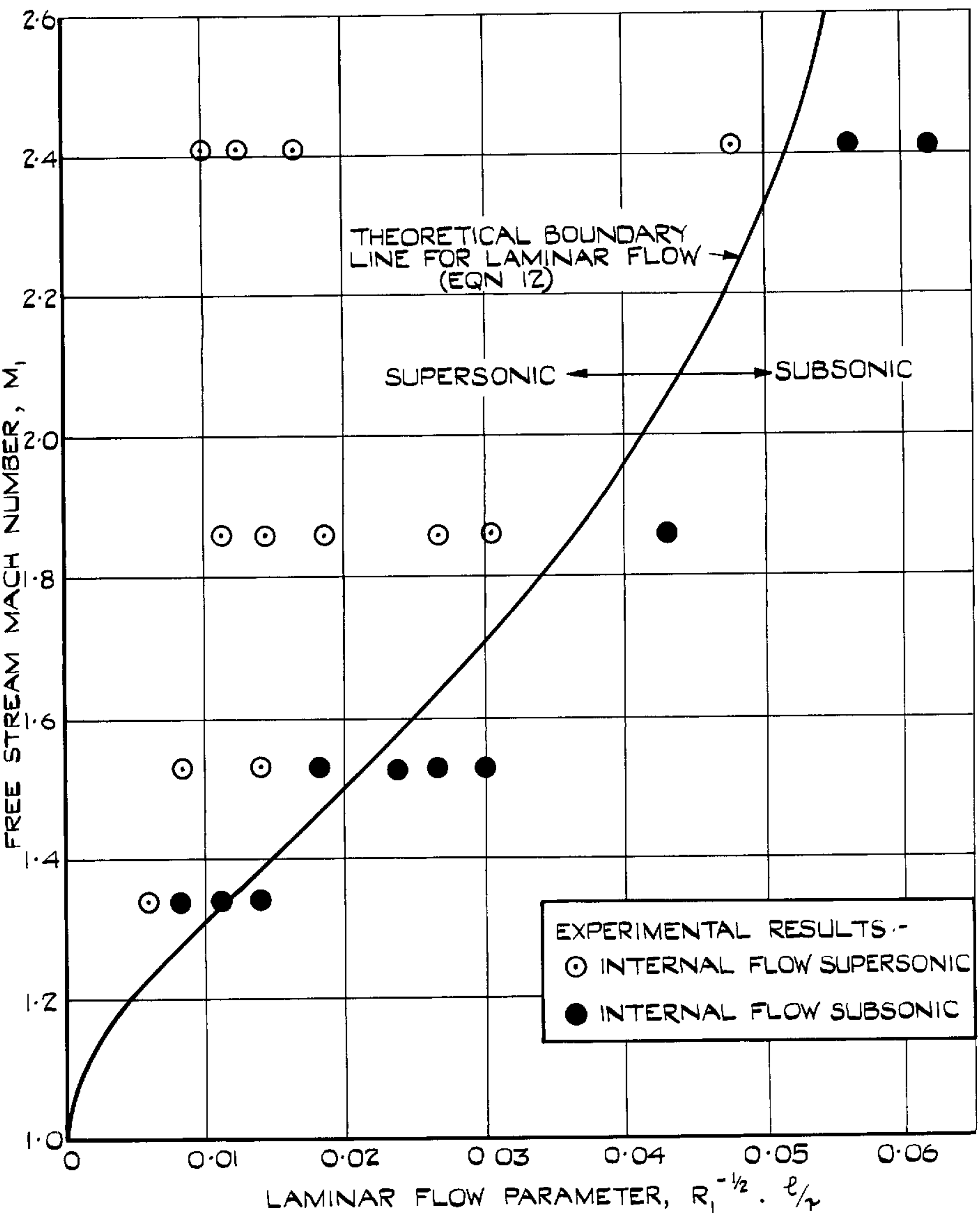


FIG. 5. CHARACTER OF FLOW IN SMOOTH PARALLEL PIPES.

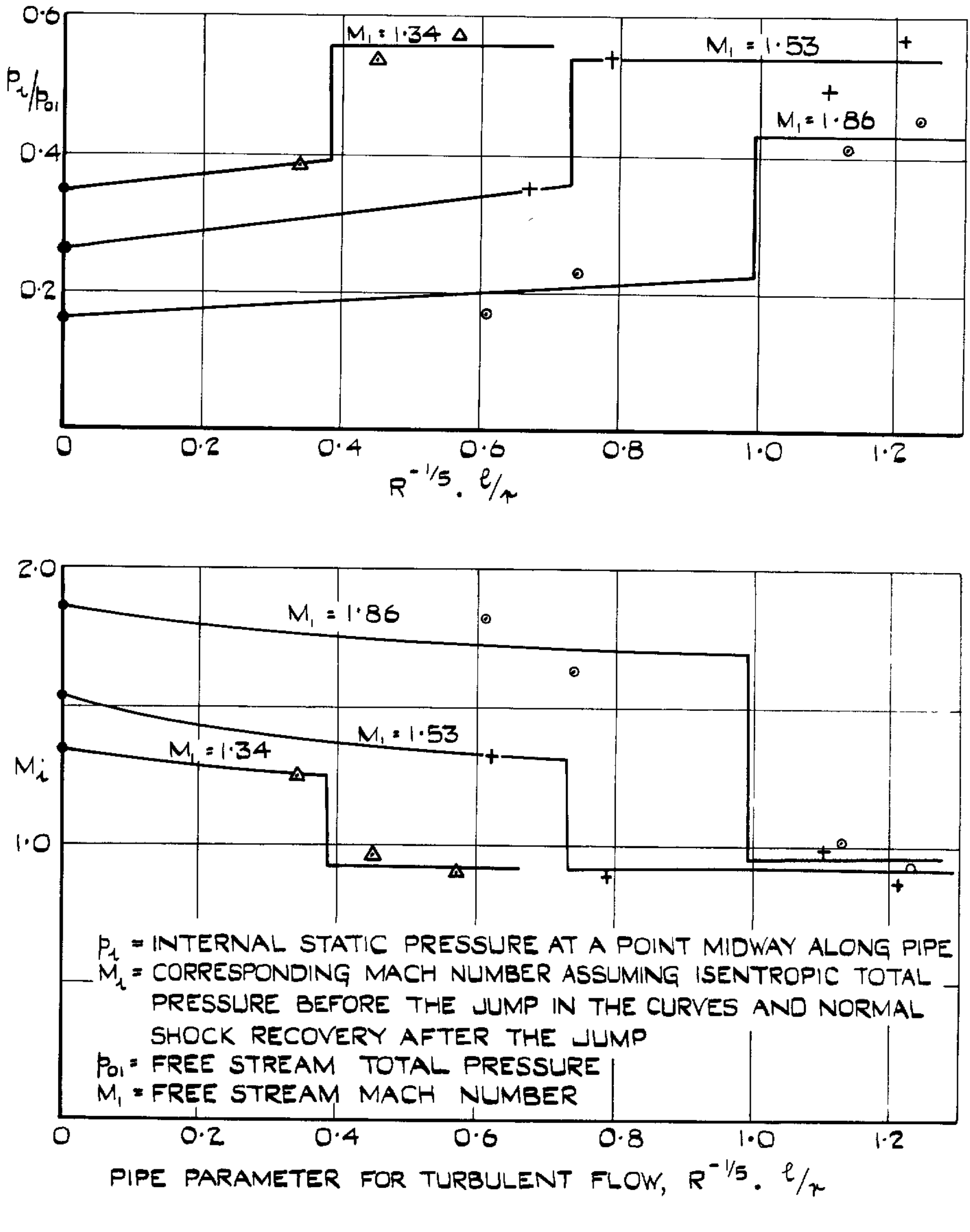


FIG. 6. STATIC PRESSURE AND MACH NUMBER IN PARALLEL PIPES WITH TRANSITION STRIP.

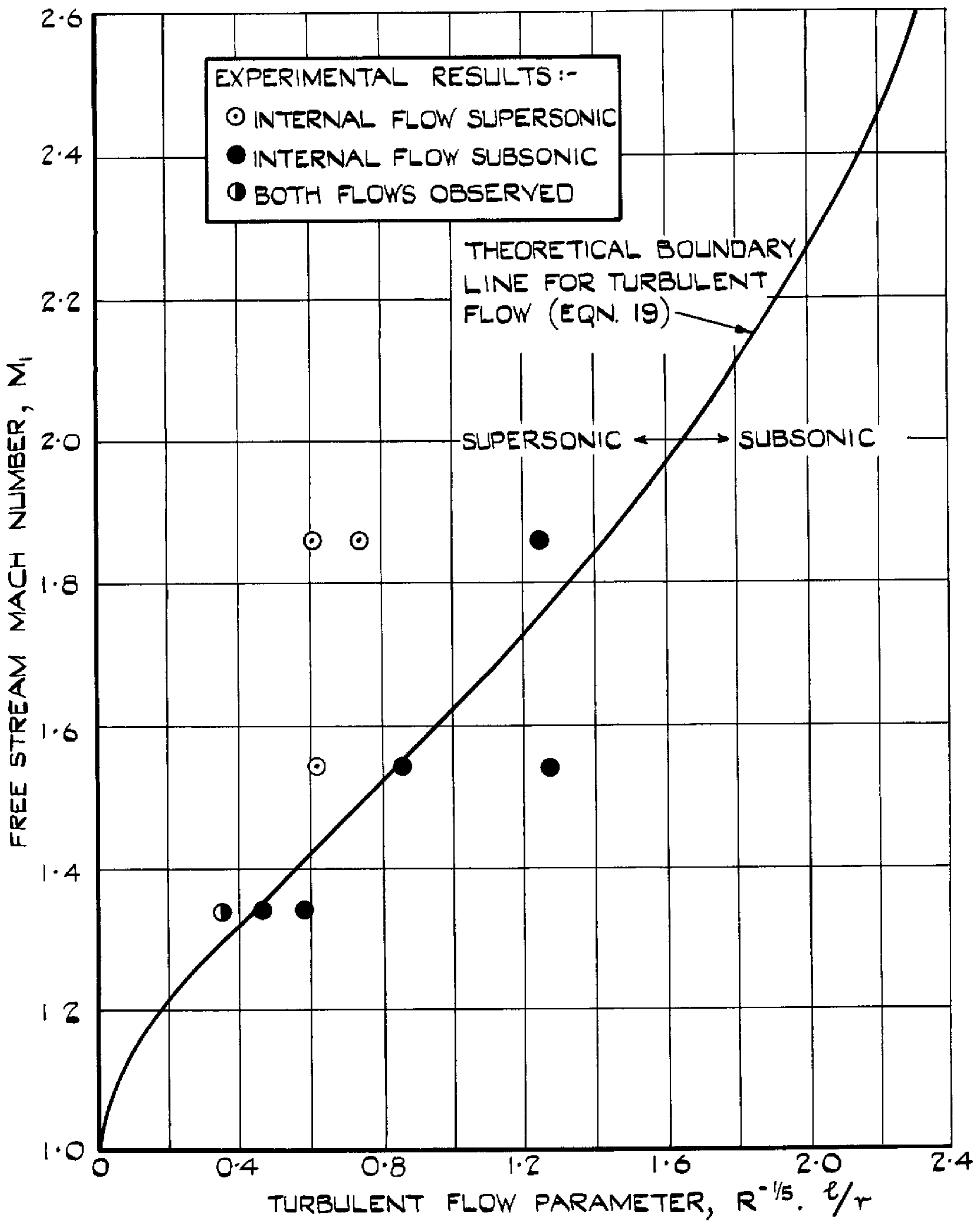
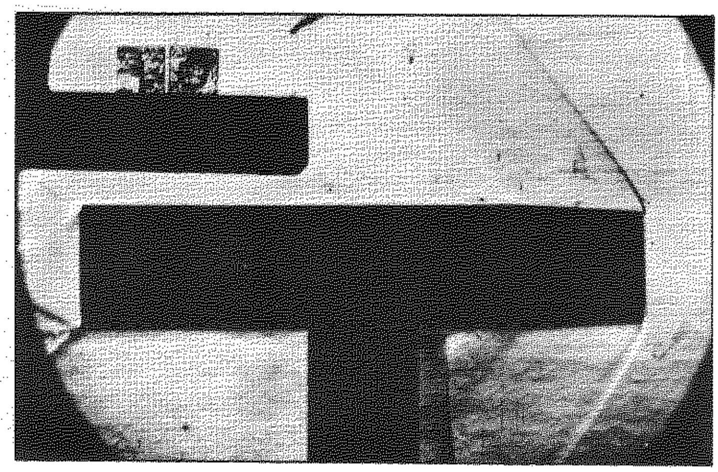
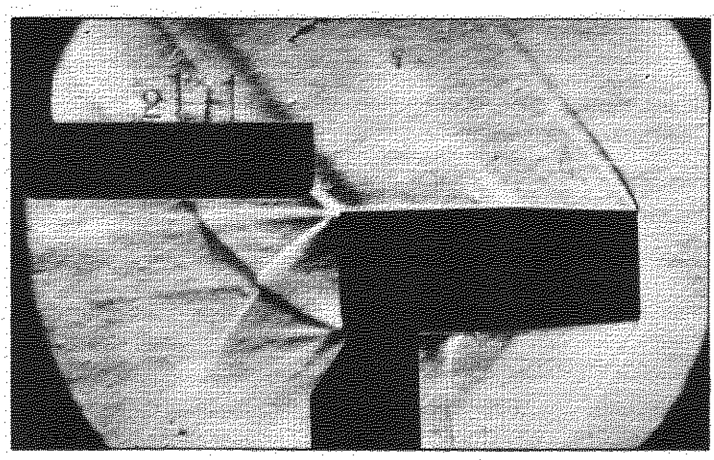


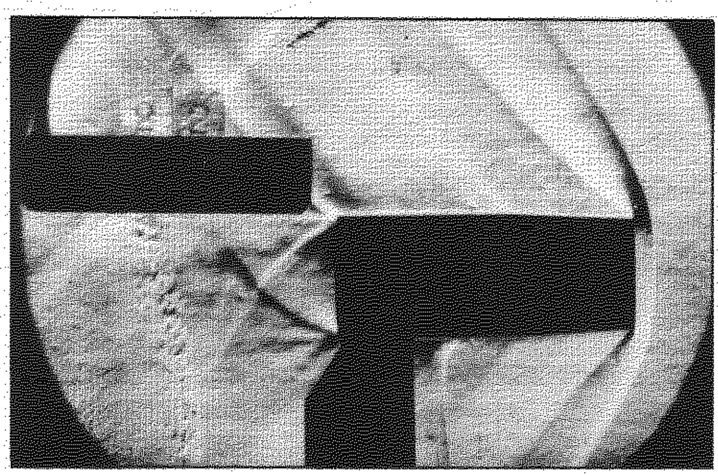
FIG. 7. CHARACTER OF FLOW IN PARALLEL PIPES WITH TRANSITION STRIP.



TUBE NO.12 (SMOOTH)
$\ell/r = 10.0$; $R = 1.58 \times 10^6$
INTERNAL FLOW SUBSONIC.

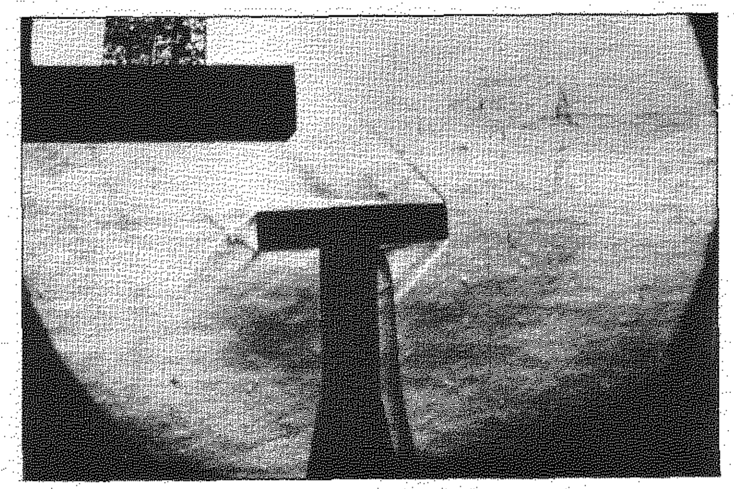


TUBE NO.14 (SMOOTH)
$\ell/r = 5.39$; $R = 8.44 \times 10^5$
INTERNAL FLOW SUPERSONIC.

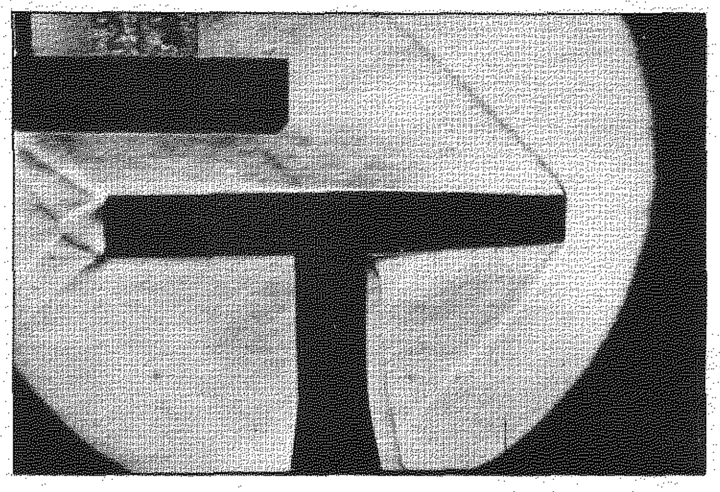


TUBE NO.14 WITH
TURBULENCE STRIP.
$\ell/r = 5.39$; $R = 8.44 \times 10^5$
INTERNAL FLOW SUBSONIC.

FIG.8. EXTERNAL SHOCK FORMATIONS AT M = 1.3
- LAMINAR AND TURBULENT LAYERS

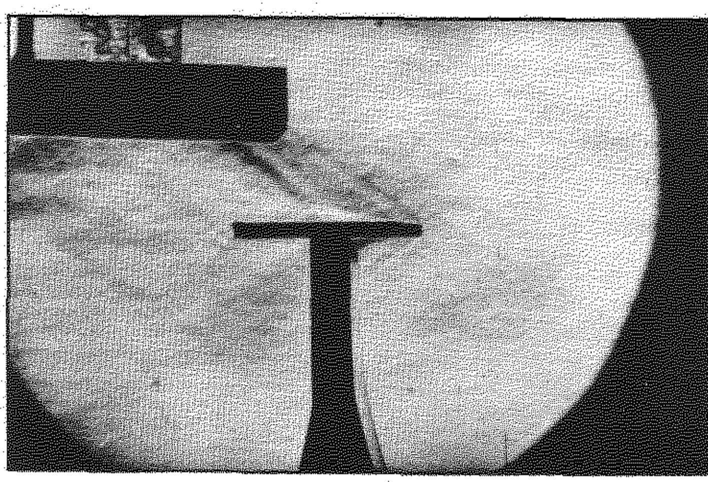


TUBE NO.7 (SMOOTH)
$\ell/r = 10.0$; $R = 5.02 \times 10^5$
INTERNAL FLOW SUPERSONIC

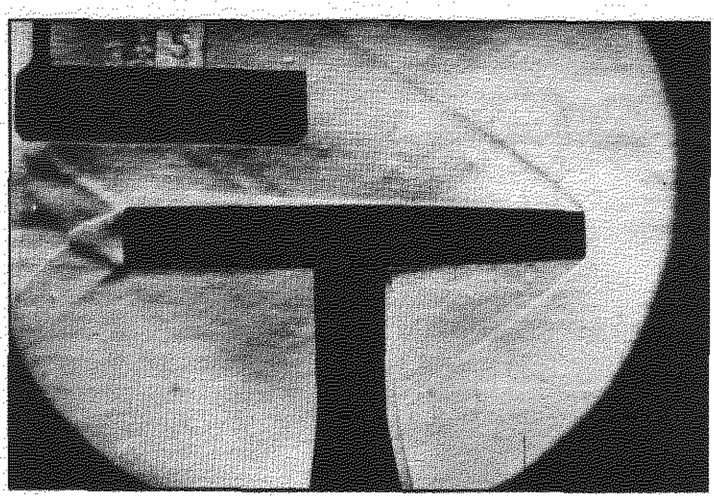


TUBE NO.4 (SMOOTH)
$\ell/r = 20.0$; $R = 1.26 \times 10^6$
INTERNAL FLOW SUBSONIC

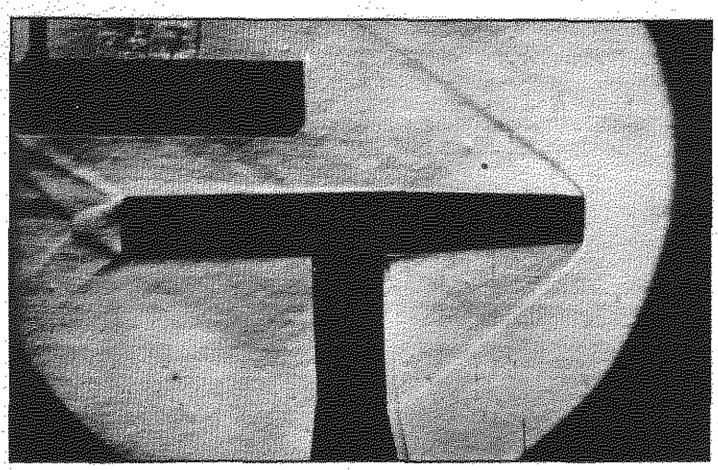
FIG.9. EXTERNAL SHOCK FORMATIONS AT M = 1.53 - LAMINAR LAYERS



TUBE NO.3 (SMOOTH)
$\ell/r = 29.8$; $R = 4.71 \times 10^5$
INTERNAL FLOW SUBSONIC.

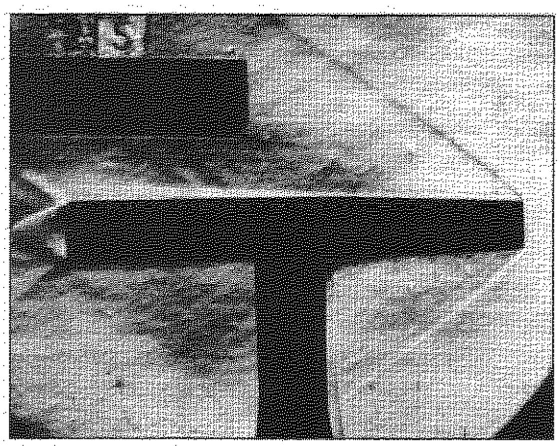


TUBE NO.4 (SMOOTH)
$\ell/r = 20.0$; $R = 1.15 \times 10^6$
INTERNAL FLOW SUPERSONIC.

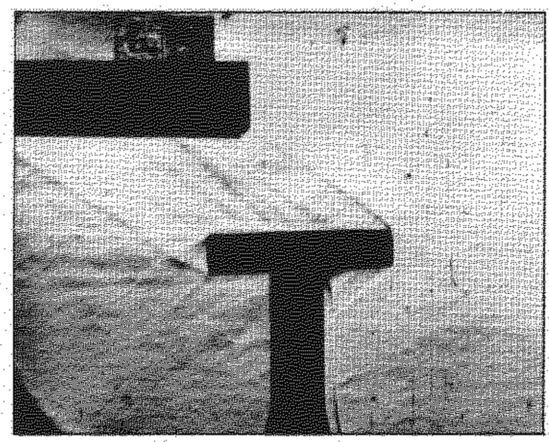


TUBE NO.4 WITH
TURBULENCE STRIP
INTERNAL FLOW SUBSONIC.

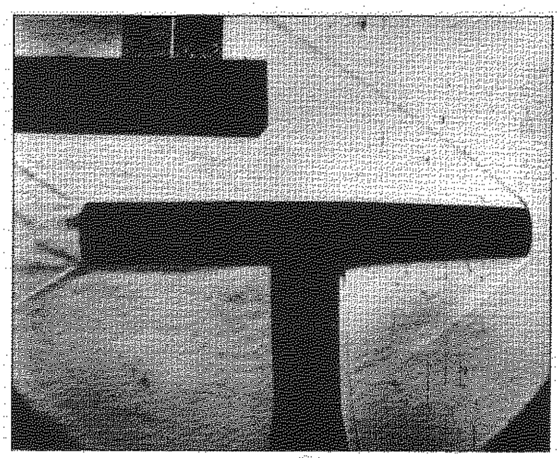
FIG.10. EXTERNAL SHOCK FORMATIONS AT M = 1.86
- LAMINAR AND TURBULENT LAYERS



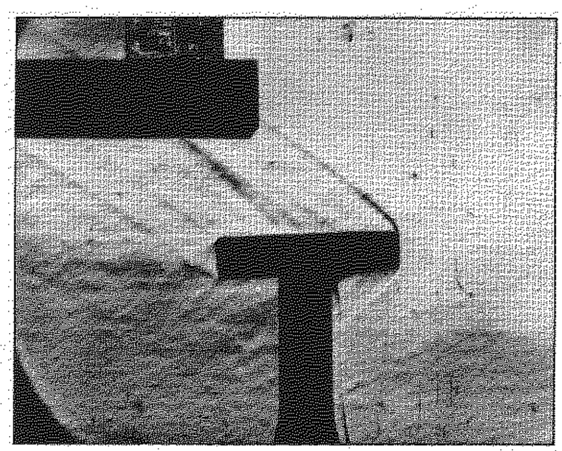
TUBE NO.4 (SMOOTH)
ℓ/r = 20.0; R = 1.15 x 10^6
ANGLE OF YAW, β = 0°
ENTRY FLOW SUPERSONIC.



TUBE NO.7 WITH
TURBULENCE STRIP (ℓ/r = 10.0;
R = 4.59 x 10^5) β = 6°
ENTRY FLOW SUBSONIC.

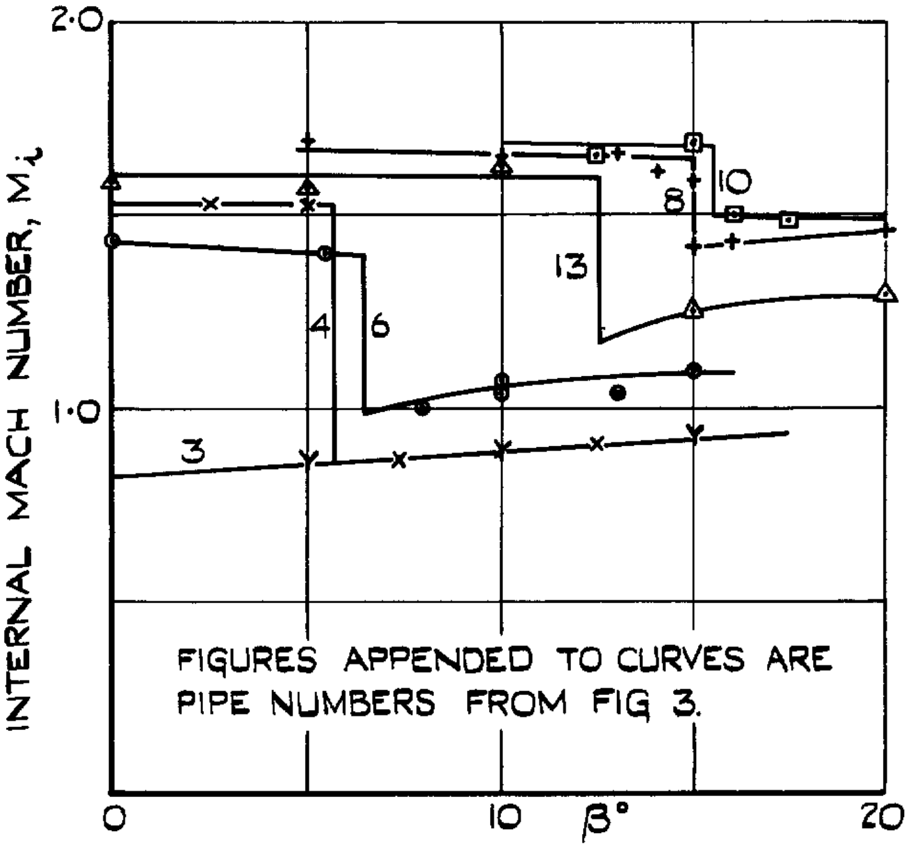


TUBE NO.4 (SMOOTH)
ℓ/r = 20.0; R = 1.15 x 10^6
β = 12.5°
ENTRY FLOW SUBSONIC.

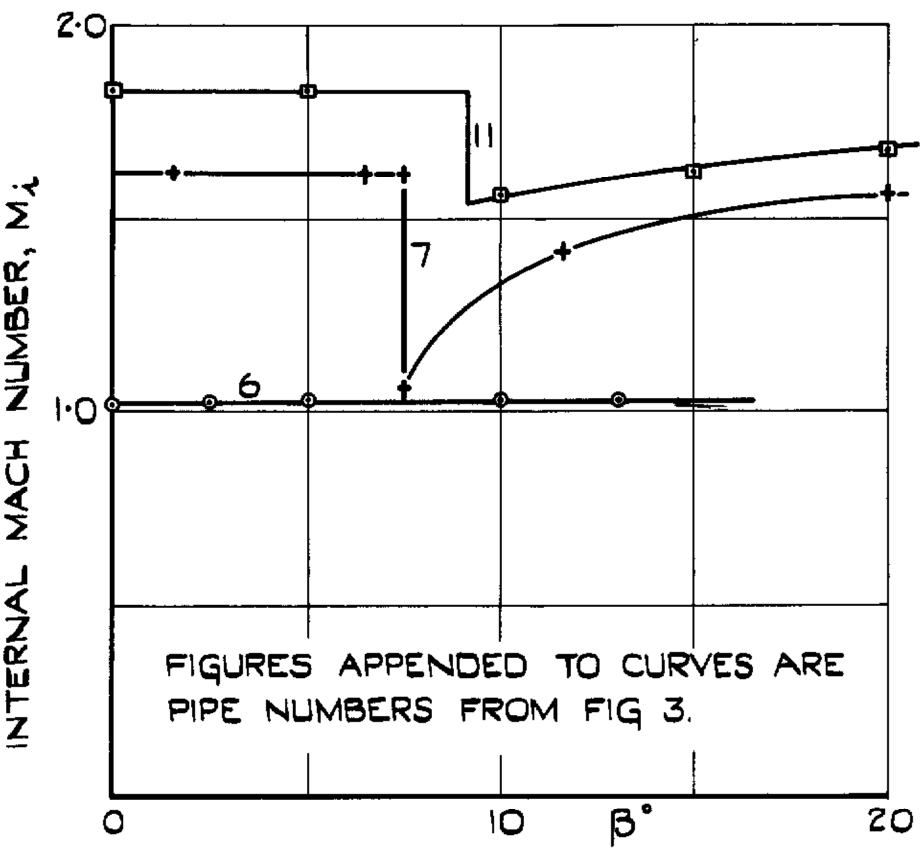


TUBE NO.7 WITH
TURBULENCE STRIP (ℓ/r = 10.0;
R = 4.59 x 10^5) β = 6°
ENTRY FLOW SUPERSONIC

FIG.11. EFFECT OF YAW ON EXTERNAL SHOCK FORMATIONS AT M = 1.86



(a) PIPES WITH LAMINAR FLOW AT ZERO YAW.



(b) PIPES WITH TURBULENT FLOW AT ZERO YAW.

FIG. 12 (a&b) INTERNAL MACH NUMBER OF INCLINED PIPES AT $M_1 = 1 \cdot 86$.

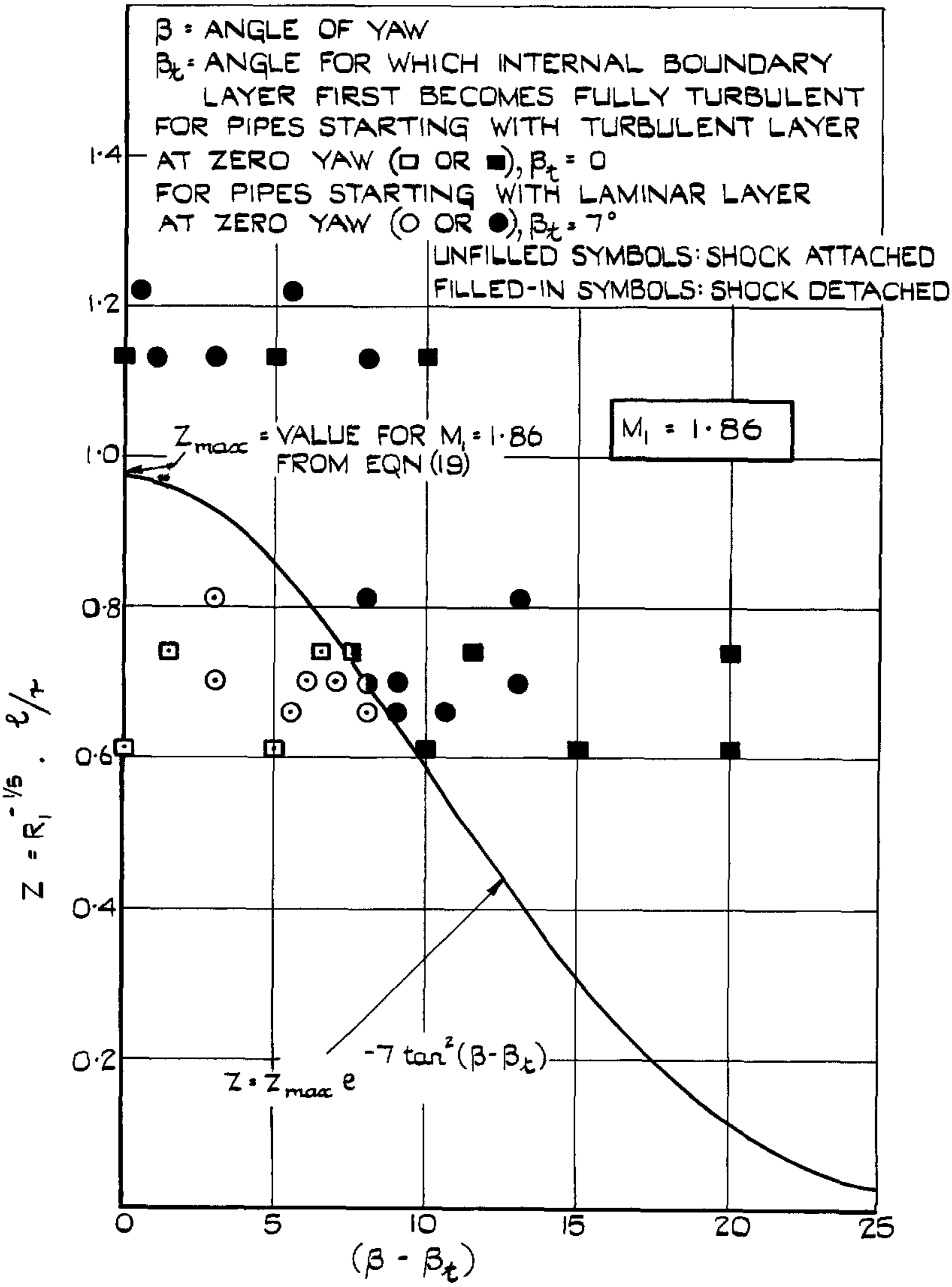


FIG. 13. VARIATION OF CRITICAL VALUE OF $R_1^{-1/5} . \ell/r$ WITH ANGLE OF YAW.

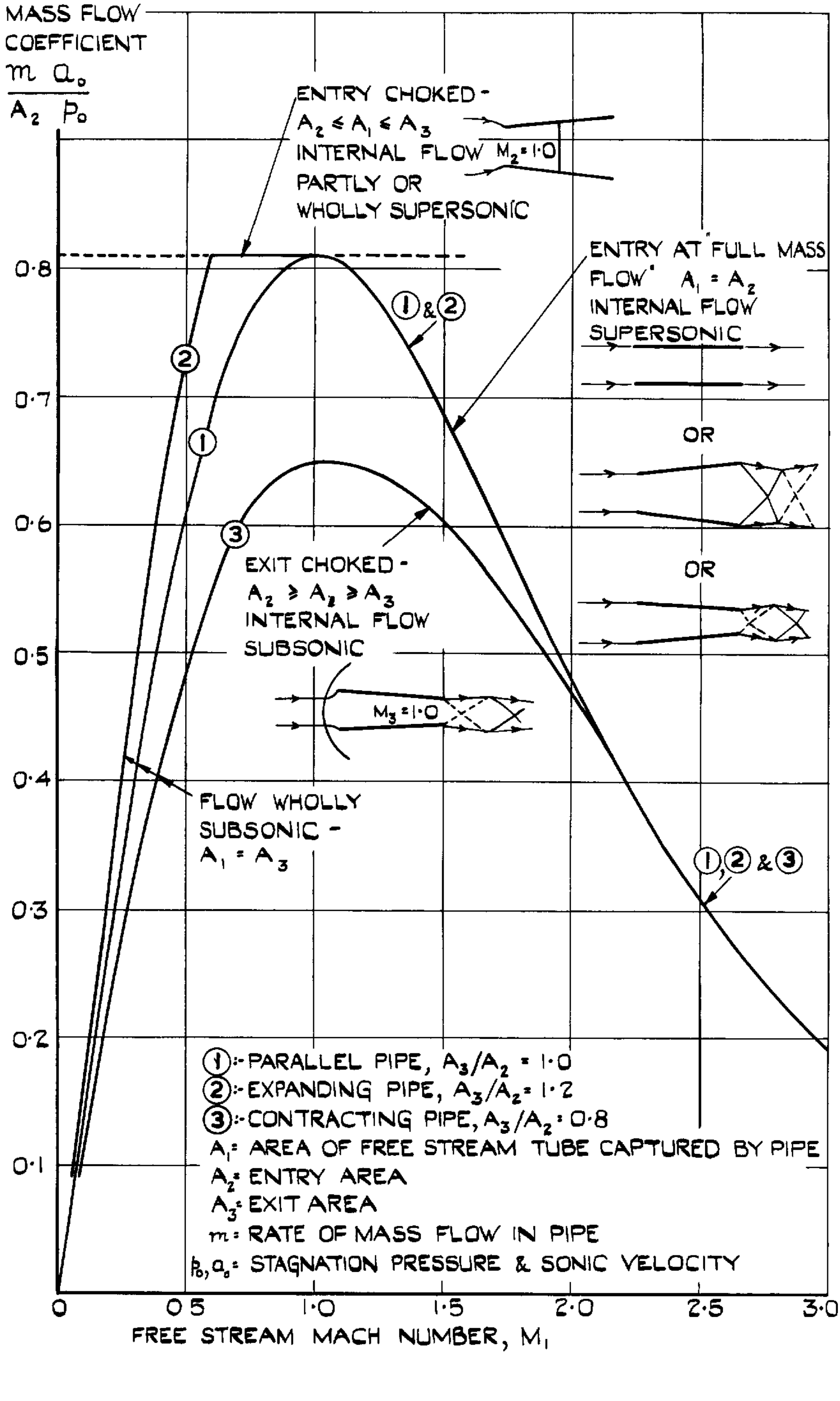


FIG. 14. COMPRESSIBLE FLOW IN FRICTIONLESS PIPES WITH MONOTONIC TAPER.

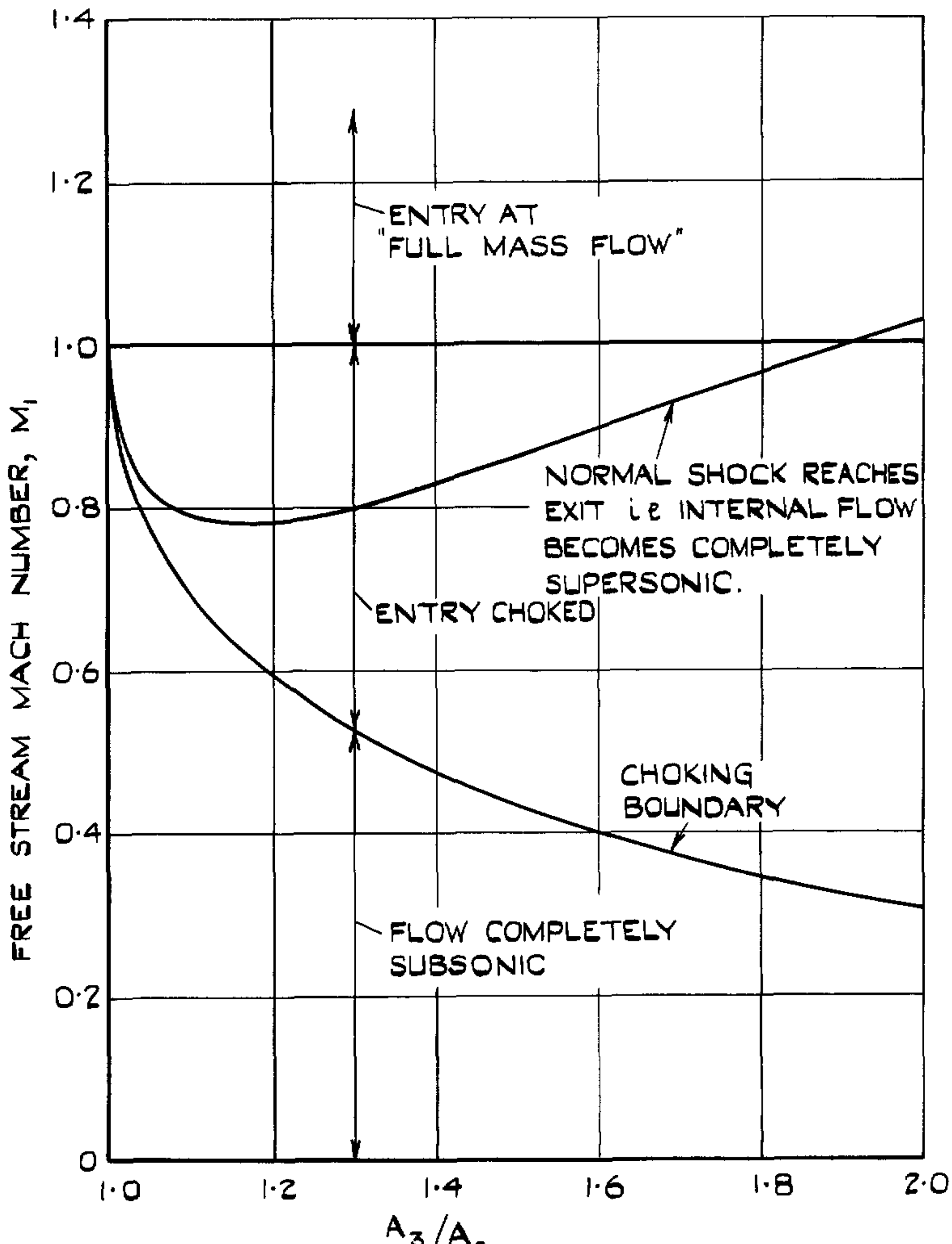


FIG. 15. COMPRESSIBLE FLOW IN EXPANDING PIPE: TRANSITION FROM SUBSONIC TO SUPERSONIC INTERNAL FLOW.

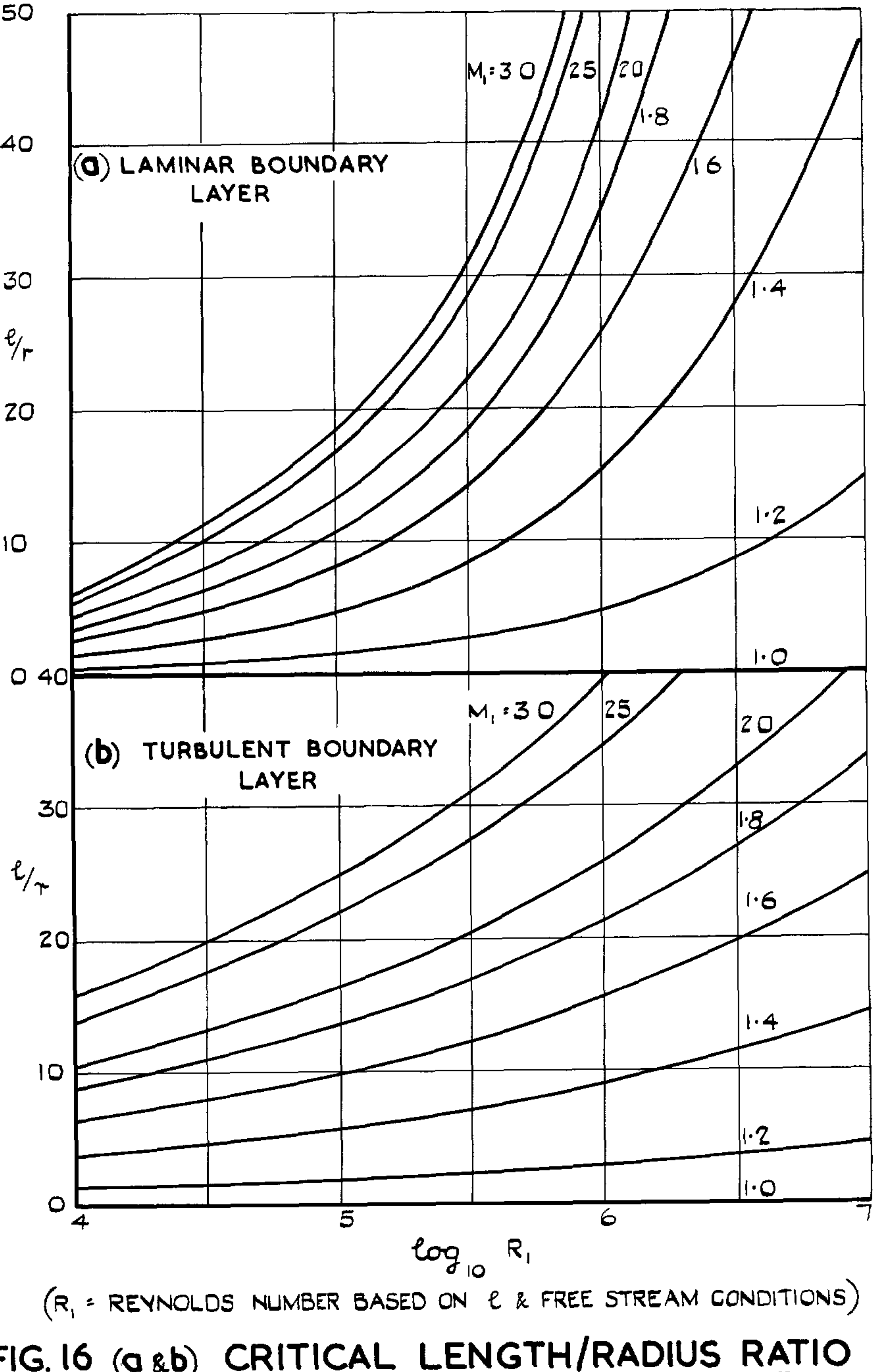


FIG. 16 (a & b) CRITICAL LENGTH/RADIUS RATIO FOR PARALLEL PIPES WITH LAMINAR OR TURBULENT BOUNDARY LAYERS AS FUNCTION OF REYNOLDS AND MACH NUMBERS.

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