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# ASSESSMENT OF THE POSSIBILITY OF USING SUCTION TO <br> INHIBIT CAVITATION ON CYLINDRICAL SECTIONS 

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## ABSTRACT

Tils report assesses the possibility of using suction to reduce the suction peak in the pressure distribution on a body moving in a fluid, with a view to delaying the onset of cavitation. Two cases are considered:
(1) flow past elliptic cylinders with suction applied over the entire forward half, and
(2) flow past circular cylinders with suction applied over, limited areas.

Results are given of calculations made to detemine the effect of varying amounts of suction as well as, in the case of ( 2 , the added effect of changing the location and extent of the suction area. It is shown that while the onset of cavitacion can be considerably delayed, the amount of suction required to effect such an improvement proves to be excessive. For this reason, it is unilkely that the method can find application; at least, not on non-lifing two-dimensional forms.

## ASSESSMENT OF THE POSSIBILITY OF USING SUCTION TO <br> IGHIBIT CAVITATION ON CYLINDRICAL SECTIONS

## INTRODUCTION

1.1 The inception of cavitation on a body moving in a fluid can be delayed by reducing the suction peak in the pressure distribution. In this report, the possible use of distributed suction is investigated as a means to this end.
1.2 The investigation deals with flow past elliptic cylinders with suction applled over the entire forward half of the cylinder. In addition, and for circular cylinders oniy, a computing programme was undertaken to detemine the effect of suction when applied over limited areas of the surface: in particular, the effect on the suction peak of changing the location and extent of the suction area was considered. Results are given, for both cases, to show the extent by which the free strean speed can be increased before the onset of cavitation for various rates of suction.
1.3 In the assessment it has been assumed that the flow round the cylinder is potential now. This assumption is inevitable as a first approach to the problem. In many practical cases, however, it would be necessary to ensure that it would be approximately fulfilled to avoid cavitation in the wake: this could be achieved by a relatively small amount of suction over the rear half of the cylinder.

## THEORETICAL CONSIDERATIONS

2.1 In this section, theoretical expressions will be derived for two-dimensional potential flow past an ellidtic cylinder with suction distributed over the entire forward half.
2.2 Distributed suction will be represented by a distribution of sinks if a continuous distribution of sinks is taken, then the resulting velocity distribution will also de continuous. A sink distribution of strength prodortional to $\cos \eta$ for $0 \leqslant \eta \leqslant \pi / 2$ and equal to zero for $\pi / 2<|\eta|<\pi$ satisfies this condition, where $\eta$ is the eccentric angle of the ellipse with $\eta=0$ giving the rorward stagnation point.
2.3 The theoretical problem then is to consider two-dimensional potential How past an elliptic cylinder with a sink disuribution over the forward half of strength proportional to cos $\eta$. The analysis will be simplified by considering, in the first place, flow past a circular cylinder and then using the methods of conformal transformation to obtain the corresponding now past an elliptic cylinder.
2.4 Consider a circular cylinder given by $|Z|=a$ in a uniform stream whose velocity at infinity is $U$ in the negative $x$-direction. It is a well-known resuat

that the speed of $a$ at any point $Z=a e^{i \theta}$ on the surface of the cylinder is given by

$$
\begin{equation*}
q=2 U \sin \theta \tag{1}
\end{equation*}
$$

2.5 Consider next the problem or flow past a circuiar cyinder with two sinks of strength $m$ located at points $Z=a e^{l \alpha}$ and $Z=a e^{-l \alpha}$ on the surface. The complex potential of such a flow is

$$
W=U\left[z+\frac{a^{2}}{Z}\right]+2 m \log \left(Z-a e^{i \alpha}\right)+2 m \log \left(Z-a e^{-i \alpha}\right)-2 m \log Z
$$

and the tangential speed $q_{t}$ at any point $Z=a e^{i \theta}$ can be shown to be

$$
\begin{equation*}
a_{t}=2 \sin \theta\left[U+\frac{m}{a} \cdot \frac{1}{\cos \theta-\cos \alpha}\right] \tag{2}
\end{equation*}
$$

If the sink strength is $m \cos \alpha$, the tangential speed at $Z=a e^{i \theta}$ is given by

$$
\begin{equation*}
a_{t}=2 \sin \theta\left[U+\frac{m}{a} \frac{\cos \alpha}{\cos \theta-\cos \alpha}\right\} \tag{3}
\end{equation*}
$$

2.6 Extending the problem to one of uniform flow past a circular cylinder with a distribution of sinks over the forward hal f of strength $m \cos \alpha$ per unit arc length, the tangential speed $a_{t}$ induced at any point $P\left(Z=a e^{i \theta}\right)$, is obtained by integrating the expression for the speed induced at $P$ by two symmetrically placed elementary sinks each of length $a d \alpha$. It follows from equation (3), that the tangential speed is

$$
2 \sin \theta\left[U+m \frac{\cos \alpha d u}{\cos \theta-\cos \alpha}\right]
$$

On integrating,

$$
\begin{equation*}
q_{t}=2 \sin \theta(U+m I) \tag{4}
\end{equation*}
$$

where

$$
I=\int_{0}^{\pi / 2} \frac{\cos \alpha}{\cos \theta-\cos \alpha} d \alpha=-\frac{\pi}{2}+\int_{0}^{\pi / 2} \frac{\cos \theta}{\cos \theta-\cos \alpha} d \alpha
$$

This integral can be evaluated to give

$$
I=-\frac{\pi}{2}-\left\{\begin{array}{l}
2 \cot \theta \operatorname{coth}^{-1} \cot \frac{\theta}{2} \text { for } 0<\theta<\frac{\pi}{2} \\
2 \cot \theta \tanh ^{-1} \cot \frac{\theta}{2} \text { for } \frac{\pi}{2}<\theta<\pi
\end{array}\right.
$$

In addition there is a normal velocity given by

$$
a_{n}=\left\{\begin{array}{cc}
\pi \pi \cos \theta & \operatorname{tor} 0<|\theta|<\frac{\pi}{2}  \tag{5}\\
0 & \frac{\pi}{2} \leqslant|\theta| \leqslant \pi
\end{array} \quad\right. \text { (see Re f. 1). }
$$

The resultant speed $q$ at $P$ can now be obtained from equations (4) and (5) and is given by

$$
\begin{equation*}
q=\sqrt{q_{t}^{2}+a_{n}^{2}} \tag{e}
\end{equation*}
$$

2.7 The solution of the problem of flow past an elliptic cyilnder can now be deduced from the solution already obtained for a circie. Consider the conformal transformation

$$
\begin{equation*}
Z=\frac{1}{2}\left(z+\sqrt{z^{2}-c^{2}}\right) \text { with } c^{2}=a^{2}-b^{2} \tag{7}
\end{equation*}
$$

This transformation mans the region outside the ellipse of semi-axes $a, b$ in the $z$ piane into the region outside tie circle of radius $\frac{1}{2}(a+b)$ in the $Z$ plane. Furthermore, any point $P$ on the ellipse can be expressed in terms of the eccentric angle $\eta$ as $z=a$ cos $\eta+i b \sin \eta$, and corresponds to a point $P^{\prime}$ on the circle given by $Z=\frac{1}{2}(a+b) e^{i \eta}$. Again, if $d s$ and $d s^{\prime}$ are corresponding elements of length containing $P$ and $P^{\prime}$ respectively, then

$$
\frac{d s}{d s^{\prime}}=\left|\frac{d z}{d z}\right|=\left|\frac{-a \sin \eta+i b \cos \eta}{\frac{1}{2}(a+b) e^{i \eta}}\right|=\frac{\sqrt{a^{2} \sin ^{2} \eta+b^{2} \cos ^{2} \eta}}{\frac{1}{2}(a+b)}
$$

Hence, a sink distribution of strength $m$ cos $\eta$ per unit length on the ellipse correspands to a sink distribution of strength

$$
\frac{m \cos \eta \sqrt{a^{2} \sin ^{2} \eta+b^{2} \cos ^{2} \eta}}{\frac{1}{2}(a+b)} \text { per unit length on the circle. }
$$

If $q, q^{\prime}$ be the speeds at $P$ and $P^{\prime}$ respectively, then

$$
\begin{equation*}
q=a^{\prime}\left|\frac{d Z}{d z}\right|=q^{\prime} \frac{\frac{1}{2}(a+b)}{\sqrt{a^{2} \sin ^{2} \eta+b^{2} \cos ^{2}} \eta} \tag{8}
\end{equation*}
$$

2.8 It follows from equationa (4), (5) and (8), that the speed $q^{\prime}$ at $P^{\prime}$ is given by

$$
q^{\prime}=\sqrt{q^{\prime} t^{2}+q^{\prime} n^{2}}
$$

where

$$
q^{\prime} t=2 \sin \eta(U+m J) \text { with } J=\int_{0}^{\pi / 2} \frac{\cos \alpha \sqrt{a^{2} \sin ^{2} \alpha+b^{2} \cos ^{2} \alpha}}{\frac{1}{2(a+b)}} \cdot \frac{d \alpha}{\cos \eta-\cos x}
$$

and

$$
q_{n}^{\prime}=\left\{\begin{array}{cc}
\frac{m \pi \cos \eta \sqrt{a^{2} \sin ^{2} \eta+b^{2} \cos ^{2} \eta}}{\frac{1}{2}(a+b)} & \text { for }-\frac{\pi}{2}<\eta<\frac{\pi}{2} \\
0 & \text { for } \\
\frac{\pi}{2} \leqslant \eta \leqslant \frac{3 \pi}{2}
\end{array}\right.
$$

Hence, since $q$ and $q^{\prime}$ are related by the expression given in (8), the speed $q$ at any point of the ellipse (given in terms of its eccentric angle $\eta$ ) is

$$
\begin{equation*}
q=\sqrt{q_{t}^{2}+q_{n}^{2}} \tag{9}
\end{equation*}
$$

where $\quad \sigma_{t}=2 \sin \eta(D+m J) \frac{\frac{1}{2}(a+b)}{\sqrt{a^{2} \sin ^{2} \eta+b^{2} \cos ^{2} \eta}}$
and $\quad \eta_{n}=\left\{\begin{aligned} m \pi \cos \eta & \text { for }-\frac{\pi}{2}<\eta<\frac{\pi}{2} \\ 0 & \text { for } \frac{\pi}{2}<\eta \leqslant \frac{3 \pi}{2}\end{aligned}\right.$
The integral $J$ can be solved in tems of complete elliptic integrals: the solution is $\left(\frac{a+b}{2 \pi}\right) J=K k^{2} \cos ^{2} \eta+k \cos \eta \ln \left(\frac{k^{\prime}}{1-k}\right)+\cot \eta \sqrt{1-k^{2} \cos ^{2} \eta}\left[\ln \left|\frac{\sqrt{1-k^{2}} \cos \eta}{\sin \eta+\sqrt{1-k^{2}} \cos ^{2} \eta}\right|-K R\left(\frac{\pi}{2}-\eta, k\right]-E\right.$
where $K=\int_{0}^{\pi / 2} \frac{d \alpha}{\sqrt{1-{h^{2}}^{2} \sin ^{2} \alpha}}$ is the complete ellidtic integral of the first kind,

$$
E=\int_{0}^{\pi / 2} \sqrt{1-k^{2} \sin ^{2} \alpha d \alpha} \text { is the complete elliptic integral of the second kind, }
$$

and $Z(\beta, k)$, the Jacobian zeta function, is defined as

$$
Z(\beta, k)=E(\beta, k)-\frac{E}{K} F(\beta, k)
$$

2.9 In the absence of suction, it follows from equation (i) using the transformation in (7), that the speed at any point on the elliptic cylinder defined by its eccentric angle $\eta$ is

$$
\begin{equation*}
q=20 \sin \eta \frac{\frac{1}{2}(a+b)}{\left(a^{2} \sin ^{2} \eta+b^{2} \cos ^{2} \eta\right)^{\frac{1}{2}}} \tag{10}
\end{equation*}
$$

3.1 In this section, equations will be derived for flow past a circular cylinder with suction applied through two areas which are located symmetrically with respect to the fore-and-aft plane of symetry but which do not in general cover the entire front half of the cylinder.
3.2 Let the extremities of one such area be given by angular co-ordinates $\theta_{1}$ and $\theta_{2^{\prime}}$

then the other suction area is given by co-ordinates $-\theta_{1}$ and $-\theta_{2} \quad\left(\theta_{2}-\theta_{1}\right)$ will then be a measure of the width of the suction area If

$$
\bar{\theta}=\frac{\theta_{1}+\theta_{2}}{2}
$$

$\bar{\theta}$ gives its location.
3.3 Consider flow past a circular cyilnder with a distribution of sinks on the surface from $\alpha=\theta_{1}$ to $\alpha=\theta_{2}$ and from $\alpha=-\theta_{1}$ to $\alpha=-\theta_{2}$ of strength

$$
m \cos \left[\left(\frac{|\alpha|-\bar{\theta}}{\theta_{2}-\theta_{1}}\right\} \pi\right]
$$

Der unit arc length. This suction distribution falls to zero at its edges and thus the possibility of an infinite suction peak at these points is avolded.
3.4 From 2.8(4), it follows that
$\frac{q_{t}}{U}=2 \sin \theta\left(1+\frac{m}{U} I\right\}$ where $I=\int_{\theta_{1}}^{\theta_{2} \frac{\cos \left[\frac{\alpha-\bar{\theta}}{\theta_{2}-\theta_{1}} \pi\right] d \alpha}{\cos \theta-\cos \alpha}}$
and $\frac{q_{n}}{\partial}=\left\{\begin{array}{c}\frac{m}{U} \pi \cos \left[\frac{|\theta|-\bar{\theta}}{\theta_{2}-\theta_{1}}\right] \text { for } \theta_{1} \leqslant|\theta| \leqslant \theta_{2} \\ 0 \quad \text { for all other } \theta\end{array}\right.$

Let $\left(\theta_{2}-\theta_{1}\right)$ be chosen such that $\frac{\pi}{\theta_{2}-\theta_{1}}=$ an integer $n$.
Then

$$
\begin{align*}
I \equiv I_{n} & =\int_{\theta_{1}}^{\theta_{2} \cos n(\alpha-\bar{\theta})} \frac{\cos \theta-\cos \alpha}{} d \alpha=\cos \bar{\theta} \int_{\theta_{1}}^{\theta_{2}} \frac{\cos n \alpha}{\cos \theta-\cos \alpha} d \alpha+\sin \bar{\theta} \int_{\theta_{1}}^{\theta_{2}} \frac{\sin n \alpha}{\cos \theta-\cos \alpha} d \alpha \\
& =\cos n \bar{\theta} I_{C_{n}}+\sin n \bar{\theta} I_{S_{n}} \tag{12}
\end{align*}
$$

where

$$
I_{c_{n}}=\int_{\theta_{1}}^{\theta_{2}} \frac{\cos n \alpha}{\cos \theta-\cos \alpha} d \alpha \text { and } I_{S_{n}}=\int_{\theta_{1}}^{\theta_{2}} \frac{\sin n \alpha}{\cos \theta-\cos \alpha} d \alpha
$$

It can be shown that $I_{C}$ and $I_{S}$ satisfy the following recurrence formulat:-

$$
\begin{align*}
& I_{c_{n}}+I_{c_{n-2}}=2 \cos \theta I_{c_{n-1}}-\frac{4}{n-1} \sin (n-1) \frac{\theta_{2}-\theta_{1}}{2} \cos (n-1) \bar{\theta}  \tag{13}\\
& I_{s_{n}}+I_{s_{n-2}}=2 \cos \theta I_{s_{n-1}}-\frac{4}{n-1} \sin (n-1) \frac{\theta_{2}-\theta_{1}}{2} \sin (n-1) \bar{\theta} \tag{14}
\end{align*}
$$

By successive reduction of these formulae, expressions can be obtained giving $I_{c_{n}}$ in terms of $I_{C_{0}}$ and $I_{S_{n}}$ in terms of $I_{S_{1}} ; \quad I_{C_{O}}$ and $I_{S_{1}}$ can be evaluated to give
$\left.I_{C_{0}}=-\frac{1}{\sin \theta} \ln \left\lvert\, \frac{\cos \bar{\theta}-\cos \left[\theta+\theta_{2}-\theta_{1}\right.}{\cos \bar{\theta}-\cos \left(\theta-\theta_{2}-\theta_{1}\right.}\right.\right\} \mid$ and $I_{s_{1}}=\ln \left|\frac{\cos \theta-\cos \theta_{2}}{\cos \theta-\cos \theta_{1}}\right|$

Finally, by using (12), the following general fomula emerges for $I_{n}$ $I_{n}=\cos \overline{n \theta} \cos n \theta I_{c_{O}}+\sin n \bar{\theta} \frac{\sin n \theta}{\sin \theta} I_{S_{1}}-\cos \overline{n \theta} \frac{\sin n \theta}{\sin \theta}\left(\theta_{2}-\theta_{1}\right)$

$$
\begin{equation*}
-\sum_{m=1}^{n-1} \frac{\sin m \theta}{\sin \theta} \cdot \frac{4}{n-m} \sin (n-m)\left[\frac{\theta_{2}-\theta_{1}}{2}\right] \cos \bar{m} \bar{\theta} \tag{15}
\end{equation*}
$$

3.5 From equations (11) and (15) the velocity distribution around the cylinder corresponding to chosen values of $\bar{\theta}$ and $\left(\theta_{2}-\theta_{1}\right)$ can be computed.

## NON-DIMENSIONAL COEFFICIENTS

### 4.1 SUCTION WJANTITY COEFFICIENT

A suction quantity coefficient - which is a non-dimensional messure of the quantity of suction applied - can be defined as
$S=\frac{\text { Quantity of fluid sucked through unit length of surface in unit time }}{20 b}=\frac{v}{20 b}$
where $U=$ free stream velocity and $b=$ semi-minor axis of ellipse.
Now $\quad V=2 \pi n \int_{0}^{\pi / 2} \cos \eta\left(a^{2} \sin ^{2} \eta+b^{2} \cos ^{2} \eta\right)^{\frac{1}{2}} d \eta$
which on integrating, becomes

$$
\begin{align*}
& 2 \pi m a\left[0.5+\frac{1-k^{2}}{2 k} \ln \sqrt{\frac{1+k}{1-k}}\right] \text { where } k^{2}=1-\frac{b^{2}}{a^{2}}  \tag{16}\\
\therefore & S=\pi \cdot \frac{m}{U} \cdot \frac{a}{b}\left[0.5+\frac{1-k^{2}}{2 k} \ln \sqrt{\frac{1+k}{1-k}}\right]
\end{align*}
$$

### 4.2 CONDITIONS FOR INCIPIEITT CAVITATION

The pressure and velocity at any point on the ellipse will be governed by the free stream conditions and the shape of the ellipse. Let the free stream pressure be denoted by $P_{\text {. }}$ at the point of minimum pressure on the ellipse, (where, it follows, the velocity will be maximum) let the pressure and velocity be denoted by $p_{c}$ and $q_{c}$ respectively. Cavitation will occur first at this point and will commence when $p_{C}=$ vapour pressure of the Inuld. These parameters are related by Bernoulli's theorem thus:-

$$
\begin{equation*}
P+\frac{1}{2} \rho U^{2}=p_{c}+\frac{1}{2} \rho a_{c}^{2} \tag{17}
\end{equation*}
$$

where $\rho$ is the density of the fluid.
Furthemore, to describe the conditions under which cavitation occurs use will be made of the cavitation number $Q$, which is defined as

$$
\begin{equation*}
Q=\frac{P-p_{c}}{\frac{1}{2} \rho U^{2}} \tag{18}
\end{equation*}
$$

Combining (17) and (18) gives

$$
\begin{equation*}
Q=\left[\frac{\pi}{d}\right]^{2}-1 \tag{19}
\end{equation*}
$$

By putting $\eta_{C}=$ vapour pressure of the fluld, free stream conditions corresponding to incipience of cavitation can be deduced from the value of $\ell$.

## 5. DESCRIPTION OF COMPUTATIONS

## A. FLOW PAST AN ELLIPTIC CYINDER WITH SUCTION APPLIED <br> OVER THE ENTIRE FORNARD HALF

5.1 Using equation (9), velocity distributions were computed corresponding to varying rates of suction for the following elliptic cylinders:-

$$
\frac{b}{a}=1,0.866,0.707,0.5,0.332,0.2
$$

The resuits of these computations for a typical case $(b / a=0.5)$ are presented graphically in Flgures 1, 2 and 3. It will be observed (Figure 1) that flows with suction possess two suction peaks - one on the forward half and the other on the rear half of the cylincer. These peaks are plotted separately against $S$ giving the two curves shown in Figure a their intersection gives tre optimum suction peak. In Figure 3 , a comparison is given of flow with optimum suction and flow in the absence of suction.

## 5. 2 Similar curves were drawn for the other five elliptic cylinders; these results

 are summarlsed in Figure 4 where deak velocity is plotted against $b / a_{0}$5.3 From (19) and the values of $q_{C} / V$ evaluated for each ellipse, the corresponding values of $Q$ can also be calculated. Rewriting (18) in the form

$$
U^{R}=\frac{P-p_{C}}{\frac{1}{2} \rho Q}
$$

enables values of $U$ corresponding to any value of $P$ to be calculated for conditions of inclpient cavitation. The results of such calculations with water at $70^{\circ} \mathrm{F}$ are presented in Figure 5. The three cases dealt with indicate the extent by which the free stream velocity can be increased with optimum suction. This point is further illustrated in Flgure 6 where $U$ is plotted against $b / a$ for a free stream pressure of $15 \mathrm{ibs} / \mathrm{sq}$. in. With $b / a>0.3$, optimum suction considerably delays the onset of cavitation: for example, with $b / a=0.5$, the free stream speed can be more than doubled before the inception of cavitation.
B. FLOW PAST A CIRCULAR CYLINDER WITH SUCTION OVER LIIITED AREAS
5.4 In this case, the computing progranme undertaken was designed to determine the effect on the suction peak of suction area location and extent at various rates of suction. Cal culations were made for the following values of $\left(\theta_{2}-\theta_{1}\right)$ and $\theta$ :-

| $\left(\theta_{2}-e_{1}\right)$ | 90 | 60 | 30 | 20 | 15 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{\theta}$ | 45 | $30,45,60$ | $15,30,45,60,75$ | $15,30,45,60,75$ | $15,30,45,60,75$ | $15,30,45,60,75$ |

5. 5 The results of these calculations are presented in Figures 7 and 8 . In Figure 7. peak velocity is plotted against $\vec{\theta}$ for each value of $\left(\theta_{2}-\theta_{1}\right)$ and curves are drawn relatin these parameters at constant values of $S$ furthermore, for each value of $\left(\theta_{2}-\theta_{1}\right)$ a curve relating optimum peak velocity and $\theta$ is also given.
5.6 Flgure 7 shows that, for any $\left(\theta_{2}-\theta_{1}\right)$, a curve of deak velocity against $\bar{\theta}$ for constant $S$ either
(1) falls with increasing $\ddot{\theta}$ to a minimum where it meets the optimum curve, or
(2) remains above the ontimum curve over the entire range of $\vec{\theta}$.

In the latter case it will be seen that the lovest value of the peak velocity occurs at the upper limit of $\vec{\theta}$, at which point $\theta_{2}=90^{\circ}$. Study of cases obeying (1), indicates that, with increasing $\left(\theta_{2}-\theta_{1}\right)$ and for any particular value of $S$, whe minimum peak velocity decreases while the corresponding value of $\bar{\theta}$ increases. In the 11 mit , the
value of $\left(\theta_{2}-\theta_{1}\right)$ will be such that the $S$ curve and the optimum curve meet at the upper 11mit of $\bar{\theta}$; that 1 s , where $\theta_{2}=90^{\circ}$. Hence, for any $S$ the optimum values of ( $\theta_{2}-\theta_{1}$ ) and $\vec{\theta}$ always satisty the condition $\theta_{2}=90^{\circ}$. In order to determine the optimum combination of $\left(\theta_{2}-\theta_{1}\right)$ and $\bar{\theta}$ for any $S$, values of the peak velocity on the forward and rear hali of the cylinder are plotted against values of $\bar{\theta}$ for which $\theta_{2}=80^{\circ}$ in Figure 8 . A pair of curves can then be drawn for any value of $\$$ one giving the peak velocity on the forward half and the other the peak velocity on the rear half. The point of intersection of any such pair gives the optimum values of $\bar{\theta}$ and the peak velocity, whereas the corresponding value of $\left(\theta_{2}-\theta_{1}\right)$ is given by $\left(\theta_{2}-\theta_{1}\right)=2(90-\bar{\theta})$.

## NUMERICAL RESULTS

5.7 While it has been shown that distributed suction can considerably delay the onset of cavitation, the practicability of the method must obviously depend on the rate of suction required. The following tables give the permissible free stream speeds at conditions of inclpient cavitation corresponding to various suction rates expressed in galions per minute.

A - KLOW PAST ELLIPTIC CYLINDERS WITH A DISTRIBUTION OF STNKS
OVER THE ENTIRE FORWARD HALF OF STRENGTH PROPORTIONAL TO $\cos \eta$
TABLE 1: $\frac{b}{a}=1$ and $P=15 \mathrm{lbs} / \mathrm{sq} . \mathrm{in}$.

| $S$ | $\frac{\eta_{C}}{U}$ | $Q$ | $U$ | Suction rate per foot <br> length of cyl inder or <br> diame ter 2" |
| :--- | :---: | :---: | :---: | :---: |
| 0.0 | 2 | 3 | 26.9 | 0 |
| 0.05 | 1.960 | 2.842 | 27.6 |  |
| 0.10 | 1.923 | 2.698 | 28.4 | 86 |
| 0.20 | 1.849 | 2.419 | 30.0 | 178 |
| 0.60 | 1.630 | 1.657 | 36.2 | 376 |
| 1.0 | 1.320 | 0.742 | 54.1 | 1130 |
| 1.21 (opt) | 1.209 | 0.482 | 68.5 | 3380 |
|  |  |  | 5180 |  |

TARLE 2: $-\frac{b}{a}=0.5$ and $P=15 \mathrm{dbs} / \mathrm{Sq} . \mathrm{in}$.

| $S$ | $\frac{q_{C}}{U}$ | $Q$ | $U$ | Suction rate per foot <br> length of cylinder <br> with $b$ = 1" |
| :--- | :--- | :--- | :--- | :--- |
| - gallons per mimute |  |  |  |  |$|$

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TABLE 3: $\quad \frac{b}{a}=0.2$ and $P=15 \mathrm{lbs} / \mathrm{se} . \mathrm{in}$.

| $S$ | $\frac{q_{C}}{U}$ | $Q$ | $U$ | Suction rate per foot <br> length of cylinder <br> with $b=$ 1" |
| :--- | :--- | :---: | :---: | :---: |
| 0.0 | 1.20 | 0.44 | 70.3 | 0 |
| 0.05 | 1.190 | 0.416 | 72.2 | 226 |
| 0.10 | 1.180 | 0.392 | 74.4 | 485 |
| 0.20 | 1.188 | 0.360 | 77.7 | 970 |
| $0.30(\mathrm{opt})$ | 1.160 | 0.346 | 79.2 | 1485 |

b - FLOW PAST a circular cilinder with a distribution of sinks over LIMITED AREAS OF STRINATH PROPORTICNAL TO cos $\left.\left[\frac{|\alpha|-\bar{\theta}}{\theta_{2}-\theta_{1}}\right\} \pi\right]$ _-

TABLE 4: $\quad P=15 \mathrm{lbs} / \mathrm{sq} . \mathrm{in}$.

| $S$ | Odtimun values |  |  | 2 | U | Suction rate per foot length of cylinder of diameter $2^{\prime \prime}$ <br> - galions per minute |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\bar{\theta}$ | $\left(\theta_{2}-\theta_{1}\right)$ | $\frac{q_{c}}{0}$ |  |  |  |
| 0.0 | - | - | 2 | 3 | 26.9 | 0 |
| 0.03 | $70^{\circ}$ | $40^{\circ}$ | 1.945 | 2.783 | 27.9 | 52 |
| 0.06 | $65{ }^{\circ}{ }^{\circ}$ | $49^{\circ}$ | 1.905 | 2.629 | 28.7 | 108 |
| 0.10 | $63 \frac{1}{2}^{\circ}$, | $53^{\circ}$ | 1.830 | 2.460 | 29.7 | 188 |
| 0.15 | $62^{\circ}$ | $56^{\circ}$ | 1.810 | 2.276 | 30.9 | 290 |
| 0.80 | $60^{\circ}$ | $60^{\circ}$ | 1.770 | 2.133 | 31.9 | 389 |
| 0.25 | $78^{\circ}$ | $64^{\circ}$ | 1.725 | 1.976 | 33.2 | 519 |

5.8 It is clear from the tables of 5.7, that in order to effect an appreciable delay in the onset of cavitation, an excessive rate of suction is required. For this reason alone, it is unlikely that the method of distributed suction can be usefully employed in practice, at least on non-lifting two-dimensional forms.

## 6. SLUMMARY AND CONCLUSIONS

Consideration is given to the possibility of using distributed suction to reduce the suction peak lit the pressure distribution on a body in two-dimensional flow and so delay the onset of cavitation. Flow equations are derlved for two cases:
(a) flow past elliptic cylinders with suction applied over the entire forward half, and
(b) flow past circuiar cylinders with suction applied over 1 imited areas.

Calculations have been made to determine the effect of varying amounts of suction and, in the case of $(0)$, the added effect of changing the location and extent of the suction area. These results are presented graphically. While it has been shown that the onset of cavitation can be considerably delayed (for example, a circular cylinder with cotirnum suction cavitates for the same free stream conditions as an elliptic cylinder of fineness ratio 5:1 in the absence of suction) the rate of suction required to effect such an improvement is excessive: even a small improvenent requires a considerable rate of suction. For thls reason, it is umluely that the method can be use fully employed in practice, at least on nuir-11fting two-dimensional forms.

## ACFINTLEDGTMTMTN

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## RTHRMCUS

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$\mathbb{N}(2), S=1 \cdot 30 ;$
$\mathbb{N}(3), S=043$.


TWO-DIMENSIONAL FLOW PAST AN ELLIPTIC CYLINDER $(b / a=0.5)$ AT ZERO INCIDENCE

FIG. I.


TWO-DIMENSIONAL FLOW PAST AN ELLIPTIC CYLINDER ( $\mathrm{b} / a=0.5$ ) AT ZERO INCIDENCE

FIG. 2.
-- WITHOLT SUCTION
— WITH OPTIMUM SUCTION.


TWO-DIMENSIONAL FLOW PAST AN ELLIPTIC CYLINDER ( $b / a=0.5$ ) AT ZERO INCIDENCE

FIG.3.


TWO-DIMENSIONAL FLOW PAST ELLIPTIC CYLINDERS AT ZERO INCIDENCE.

FIG. 4.
-- WITHOUT SUCTION
—WITH OPTIMUM SUCTION
CLRVES (I) REFER TO ELLIPSE WITH $b / a=1$
CURVES (2) REFER TO ELLIPSE WITH b/a=0.5 CURVES (3) REFER TO ELLIPSE WITH b/a=0.2


FREE STREAM CONDITIONS CORRESPONDING TO INCIPIENCE OF CAVITATION 'FOR TWO-DIMENSIONAL FLOW PAST ELLIPTIC CYLINDERS AT ZERO INCIDENCE.

FIG. 5.


INCIPIENT CAVITATION SPEEDS AT FREE STREAM PRESSURE OF 15 LBS./SQ.IN. FOR TWO-DIMENSIONAL FLOW PAST ELLIPTIC CYLINDERS AT ZERO INCIDENCE.

FIG. 6.

- CLRVES OF CONSTANT SLCTION.
-一一 CURVE OF OPTIMUM SUCTION.

two-dimensional flow past a circular cylinder With suction applied over limited areas

FIG.7.


DETERMINATION OF OPTIMUM LOCATION OF SUCTION AREA CORRESPONDING TO DIFFERENT VALUES OF $S$ FOR TWO-DIMENSIONAL FLOW PAST A CIRCULAR CYLINDER.

FIG. 8.

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