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By P. BRADSHAW, B.A. and N. GREGORY, M.A.
OF THE AERODYNAMICS DIVISION, N.P.L.

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The Determination of Local Turbulent Skin Friction from Observations in the Viscous Sub-Layer

By P. Bradshaw, B.A. and N. Gregory, M.A.

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Summary—Observations made in the viscous sub-layer of turbulent shear flows with small surface pitot-tubes and hot wires show that such instruments give accurate values of boundary-layer skin friction, at least in zero pressure gradient, but only when calibrated in turbulent flow: calibration in laminar flow gives incorrect results by about 10 per cent with $u_\tau d/\nu = 2$. It is further shown that the region of universality of turbulent velocity profile is confined to the viscous sub-layer, and an upper limit $u_\tau d/\nu = 30$ is suggested for the height d of a flat surface tube if reasonable accuracy is to be obtained in skin-friction measurements.

1. *Introduction.*—In view of the general uncertainty about the relation of skin friction to velocity profile in turbulent shear flow, there is a great need for a reliable method or methods of determining the local skin friction of a boundary layer, particularly in a longitudinal pressure gradient. Some of the existing methods are discussed in Section 7, while the bulk of this report deals with what appears to be one of the most convenient, the determination of skin friction by means of observations, nominally of velocity, in the layer near the surface in which the turbulent shear stress is much less than the direct viscous shear stress, and in which it is therefore to be expected that the mean velocity profile will be determined by the wall shear stress, almost irrespective of the behaviour of the turbulent fluid further from the wall and irrespective of any arguments or hypotheses about the turbulent flow. Most of the experimental investigation reported here concerned the use of small flat pitot-tubes mounted on the surface, but some observations were made with hot wires fixed just above the surface. The first workers to use small pitot-tubes mounted on the surface were Stanton, Marshall and Bryant[1] who found the relation between the pitot-static pressure difference and the velocity profile in laminar flow and used it to deduce the existence of a region of linear variation of velocity close to the wall in turbulent flow. This region was called the laminar sub-layer and the small surface pitot-tubes became known as Stanton tubes. Later, Fage and Falkner[2] used similar tubes to find skin friction in turbulent flow by calibrating them in laminar flow and assuming that the velocity profiles in the flow field of the tube were the same in laminar

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and turbulent flow. Their results for the total skin-friction drag of an aerofoil, when compared with the drag obtained from a momentum balance, indicated that this assumption was justified to within 10 per cent. Fage and Sargent[3] calibrated their surface tubes in a turbulent boundary layer whose skin friction was deduced from a solution of the von Kármán momentum integral equation. Later workers have usually followed Fage and Falkner by calibrating in laminar channel flow.

The present investigation was made in order to decide whether or not the calibrations of velocity-measuring instruments in the viscous sub-layer were the same in laminar and turbulent flow, and whether such instruments could be used to obtain accurate values of boundary-layer skin friction.

2. *The Viscous Sub-layer of Turbulent Shear Flow.*—Following the experiments of Stanton *et al.* referred to above, and on the strength of the 'no-slip' condition at the wall, the concept arose of a 'laminar sub-layer' in which turbulent fluctuations were supposed to be negligible and in which, therefore, the velocity varied linearly with distance from the wall, with a slight curvature of profile due to longitudinal pressure gradient, if any. It was pointed out by Taylor in 1932, and subsequently verified by experiment, for example, that of Laufer,[4] that the ratio of longitudinal turbulent fluctuation velocity to mean velocity at the wall was not zero; in fact, the ratio is of the order of $\sqrt{\{\overline{u'^2}\}}/U = 0{\cdot}3$, though the exact values are still controversial. The name 'laminar sub-layer' is therefore inappropriate except in so far as it implies that the viscous shear stress predominates over the turbulent shear stress, and the name 'linear sub-layer' is to be preferred. For the purposes of this paper the following are defined:

(1) *The 'linear sub-layer'* is the region in which the velocity profile is closely linear with $\partial u/\partial y = \tau_0/\mu$ in the absence of pressure gradient.

(2) *The 'viscous sub-layer'* is the region in which the turbulent shear stress, though appreciable, is considerably less than the viscous shear stress, so that it may be safely assumed that the velocity profile is a unique function of skin friction in all types of turbulent flow.

(3) *The 'universal region'* is the region in which the velocity profile is observed to be a unique function of skin friction in all types of turbulent flow.

(4) *The 'inner law region'* is the region in which the velocity profile is observed to be a unique function of the skin friction, independent of Reynolds number for the flow under consideration.

It is clear that (3) must include (2) and possibly (4) but the latter is a matter of controversy.

A short analysis of the flow in the viscous sub-layer is given here for future reference.

The 'no-slip' condition at the wall, which there is no reason to doubt for ordinary temperatures and pressures, provides that $u = 0$ when $y = 0$; at, and very near the wall, the inertia terms in the Navier-Stokes equations can be neglected, and so

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 U}{\partial y^2}$$

and two similar equations.

Also

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \simeq 0 \quad \text{as} \quad \frac{\partial u}{\partial x} \simeq 0.$$

From these equations and the definition of viscosity,

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_0,$$

we obtain

$$U=\frac{\tau_0}{\mu}y+\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)_0 y^2+\frac{1}{24\nu}\left(\frac{\partial^3 \overline{uv}}{\partial y^3}\right)_0 y^4+0[y^5]$$

$$\sqrt{\{\overline{u'^2}\}}=\sqrt{\left\{\overline{\left(\frac{\partial u}{\partial y}\right)_0^2}\right\}}\,y+0[y^2]$$

$$\sqrt{\{\overline{v'^2}\}}=\frac{1}{2\nu}\sqrt{\left\{\overline{\left(\frac{\partial p}{\partial y}\right)_0^2}\right\}}\,y^2+0[y^3]$$

$$\overline{uv}=\frac{1}{2\rho\nu}\left\{\overline{\left(\frac{\partial u}{\partial y}\frac{\partial p}{\partial y}\right)_0}\right\}y^3+0[y^4]\,.$$

(*See* Ref. 5, p. 218–ff.)

From these results we note that

$$\left(\frac{\sqrt{\{\overline{u'^2}\}}}{U}\right)_0=\frac{\mu\sqrt{\left\{\overline{\left(\frac{\partial u}{\partial y}\right)_0^2}\right\}}}{\tau_0}\neq 0$$

in general, that the Reynolds shear stress $-\rho\overline{u'v'}$ varies as the cube of the distance from the wall, and that the mean velocity profile in zero pressure gradient has no quadratic or cubic terms, indicating that it remains effectively linear for some distance from the wall until the effect of the quartic term due to the Reynolds shear stress becomes apparent.

We therefore see that if an instrument with linear response to velocity fluctuations could be introduced into the effectively linear part of the profile, it could be calibrated in laminar flow and used to find the skin friction in turbulent flow. Unfortunately there is no simple instrument with linear response and it appears to be very difficult to make an instrument which is entirely immersed in the linear sub-layer. It is therefore necessary to ensure that the flow in which the instrument is calibrated corresponds as nearly as possible with the flow to be measured with respect to mean velocity profile and turbulent intensities in the region occupied by the instrument. In particular, the practice of calibration in laminar flow and use in turbulent flow is suspect *a priori*. Moreover, if the instrument records a pressure difference it may also be influenced by the fluctuations in static pressure at the wall, which Willmarth[6] and others have shown to be of the same order as the wall shear stress.

3. *Outline of Present Investigation.*—In view of the above considerations, some experiments have been made to investigate the hypothetical differences in calibration between laminar and turbulent flow and to provide some evidence concerning the assumption of complete universality of turbulent velocity profile in some region near the wall. The work arose from the investigation of the 'inner law' region reported in Ref. 7. It was not intended to carry out an extensive investigation of the properties of the viscous sub-layer, for which the apparatus was not particularly suitable, but only to obtain information about the behaviour of the surface tubes.

The procedure was to calibrate a number of small flat surface pitot-tubes in laminar and turbulent duct flow, and to deduce from the calibrations values of the skin friction of a turbulent boundary layer on a wind-tunnel wall. A series of larger tubes was tested in the turbulent flows only. Observations were also made with hot wires used instead of pitot-tubes, so as to compare results obtained with two instruments with different responses to turbulent fluctuations.

4. *Apparatus and Procedure.*—The measurements were made in turbulent flow in the 12-in. × 2-in. rectangular duct already described in Ref. 7, in a 3·75 in. × 0·040 in. × 21 in. duct in which the flow remained laminar up to a Reynolds number of 2,000 based on maximum velocity and duct height, and in the turbulent boundary layer on the wall of a 13 in. × 8 in. open-circuit wind tunnel. The flow in the wind-tunnel was rather unsteady, and although considerable improvements to it were made during the experiment, the results given here are somewhat scattered.

The instruments were mounted on the $3\frac{1}{2}$-in. metal discs used in the experiment of Ref. 7, so that they could be transferred between holes in the walls of the tunnel and ducts.

4.1. *Stanton Tubes.*—The flat surface pitot-tubes were made in a number of different ways. At first machined half-pitots were screwed to the surface, then razor blades were soldered in position, and finally pieces of 0·002-in. steel skin with one edge chamfered were attached with cellulose adhesive or solder with the lip immediately above the front edge of a square pressure tapping, as shown in Fig. 1. This last method was found to be most satisfactory, producing a tube of very small height, acceptably short lag, and reasonable stability of calibration. The susceptibility of the tubes to dust was perhaps exaggerated in Ref. 7 or possibly dust was liberated from the 12-in. × 2-in. wooden duct when new: at any rate, comparatively little dust trouble has been experienced recently.

A static pressure hole of 0·040 in. diameter was located with its centre accurately abreast of the pitot-tube mouth and usually about 1 in. to the side. The pitot-static pressure differences were read on Chattock gauges and inclined alcohol manometers. Although it is believed that the pressure-measuring systems normally used in experimental aerodynamics are effectively linear in their response to pressure fluctuations, so that a manometer will indicate a mean value of the pressure presented to it, the simple precaution was taken of keeping the whole system of surface tube, connecting tubes and manometer, intact for a given set of calibration and measurement runs, so that the dynamic response should be the same for all runs.

4.2. *Hot Wires.*—Wires of 0·0002-in. diameter platinum-rhodium alloy were used. The probe is shown in Fig. 1. The height of the wires above the surface was nominally 0·002 in. but the wires were not straight and no attempt was made to measure their distance from the surface accurately. The wire length was usually about 2 mm giving a cold resistance of 15 to 25 ohms. A constant current, usually 40 mA, was used, and the maximum ratio of hot resistance to cold resistance, R/R_A, was about 1·4, indicating a temperature some 200 deg C above ambient.

4.3. *Reduction of Observations.*—The wall shear stress in the two ducts was calculated from the measured pressure gradient. In the case of the 6 : 1 aspect ratio turbulent flow duct, an allowance had to be made for the non-uniformity of skin friction round the perimeter. In order to do this, it was temporarily assumed that the distribution was independent of Reynolds number in the range tested: this enabled the exponent n of the Stanton-tube calibration,

$$S-p \propto \tau_0{}^n,$$

to be obtained from readings of $(S-p)$ and of the pressure gradient. Then the distribution of the ratio $(S-p)/(S-p)_{\text{centre line}}$ was found by placing the Stanton tube at various points on the floor and wall of the duct; and finally $(\tau_0)_{\text{average}}/(\tau_0)_{\text{c.l.}}$ followed from the graph of

$$\tau_0/(\tau_0)_{\text{c.l.}} = \{(S-p)/(S-p)_{\text{c.l.}}\}^{1/n}.$$

It was found that $(\tau_0)_{\text{average}}/(\tau_0)_{\text{c.l.}}$ was indeed independent of Reynolds number and equal to $0{\cdot}87_5$.

The laminar flow channel had an aspect ratio of nearly 100 and no attempt was made to check the skin-friction distribution round the perimeter.

The Stanton-tube pressure differences were correlated in the manner suggested by Preston.[8] He showed that if $u/u_\tau = f_1(u_\tau y/\nu)$, then a series of geometrically similar surface tubes should have a single calibration curve

$$\frac{(P-p)\,d^2}{4\rho\nu^2} = f_2\left(\frac{\tau_0 d^2}{4\rho\nu^2}\right).$$

This method of plotting was used for the series of, nominally, similar flat surface tubes, but for routine skin-friction measurements $\log\{(P-p)/\Lambda\}$ was plotted against $\log(\tau_0/\Lambda)$ (where $\Lambda = \rho\nu^2/(\rho\nu^2)_{\text{ref}}$). It was found that straight lines fitted the calibrations adequately in the range of skin friction covered, both in laminar and in turbulent flow, indicating a power-law variation of the tube dynamic pressure with shear stress.

The variables used in the analysis of the hot-wire calibration were based on King's law for the heat transfer which states that

$$Nu = A + B\sqrt{(Re)},$$

where

$$Nu \text{ (Nusselt number)} = \frac{I^2 R}{k(T-T_A)\,l}$$

and

$$Re \text{ (Reynolds number)} = \frac{Ud}{\nu}.$$

For simplicity, we may omit all the factors which are constant for a given set of tests, and make the assumption of linear variation of resistance with temperature. Noting that $\bar{u}$ is a unique function of τ_0, we arrive at the arguments

$$\frac{R}{R-R_A} \quad \text{and} \quad \sqrt{\left(\frac{\tau_0}{\Lambda}\right)}$$

in terms of which the hot-wire calibration should be a straight line if the sub-layer velocity profile is itself linear.

5. *Results.*—The calibrations of a hot-wire probe and of one of the smallest surface tubes are shown, for both laminar and turbulent channel flow, in Figs. 2 and 3. The differences between the laminar and turbulent calibrations are in each case considerable. Moreover, the discrepancies are of totally different character for the two instruments.

In the case of the Stanton tube, the calibrations diverge considerably at higher values of wall shear stress, and the best-straight-lines on the logarithmic plot seem to converge near

$$\log\{(\tau_0 d^2)/(4\rho\nu^2)\} \simeq 1{\cdot}5 \quad \text{or} \quad u_\tau d/\nu \simeq 1.$$

Attempts to explore this region were not very successful. With $d = 0{\cdot}002$ in., $u_\tau d/\nu = 1$ implies that $u_\tau \simeq 1$ ft/sec, and as the Stanton-tube static-pressure difference in this region is of order τ_0 or $\frac{1}{2}\rho u_\tau^2$, accurate readings were difficult to obtain. Even at the lowest shear stress at which readings were taken, however, there still seemed to be an appreciable difference between the calibrations, in the sense to be expected if the local velocity in turbulent flow were less than that in laminar flow. The general behaviour of instruments in the sub-layer will be more fully discussed in Section 6: at this stage it is sufficient to draw the important conclusion that the calibrations do differ considerably even for the smallest size of Stanton tube.

The hot-wire calibrations, on the other hand, diverge at the lower end of the shear-stress scale, with the turbulent-flow calibration indicating a higher velocity than the laminar-flow one. The calibrations were not carried to such low values of wall shear stress as in the case of the Stanton tube because of the likely effect of free convection at very low air speeds. (A check on free convection effects, at one of the smallest shear stresses actually used, was made by reading the hot-wire resistance with the laminar-flow duct in various different attitudes: no measurable effect was observed.) Again, it is clear that the differences in calibration would result in very different predictions of skin friction.

In Fig. 4 are shown the skin-friction coefficients for the turbulent boundary layer obtained from the turbulent duct calibrations for both Stanton tube and hot wire. It is seen that there is no consistent difference between the values obtained from the two instruments, at least within the scatter of about ± 3 per cent. A reasonable estimate for the maximum probable error would be ± 1 per cent on skin friction. The skin-friction graphs obtained by using the laminar-flow calibrations for each instrument clearly differ from each other and from the turbulent-flow calibration values by far more than this experimental scatter.

Fig. 5 shows the calibrations in the turbulent-flow duct of a series of flat surface pitot-tubes of increasing size, and which were intended to be geometrically similar, compared with Preston's calibration of a series of similar round pitot-tubes. The fact that the calibrations do not lie exactly on one smooth curve is merely an indication that geometrical similarity was not accurately attained, for Preston's reasoning should apply to flat pitot-tubes as well as round ones providing that they remain immersed in the inner-law layer. The calibrations are seen to run fairly smoothly from the 0·002-in. tube, for which the laminar and turbulent calibrations are nearly parallel, to the 0·058-in. and 0·104-in. tubes whose calibrations are nearly parallel to Preston's round-tube results, though it will be seen later that this latter result is not necessarily to be expected.

The same tubes were tested in the wind tunnel. On making Preston's assumption that the calibrations of a large pitot-tube are the same in all turbulent flows (in this case, rectangular-duct flow and boundary-layer flow), values were deduced for the skin friction on the tunnel wall. Taking the values given by the smallest tube as the correct ones a discrepancy appears at about

$$u_\tau d/\nu = 30\{\log(u_\tau d/\nu) \simeq 1{\cdot}5\}$$

and increases to about 10 per cent at $u_\tau d/\nu = 300$. At values of $u_\tau d/\nu$ less than 30 there seems to be no systematic difference between the calibrations. The reason for these results will be demonstrated below by a comparison between the velocity profiles: the point of the present data is that the values of skin friction given by the surface tubes are the same to within about ± 1 per cent, for all $u_\tau d/\nu < 30$.

The behaviour of the effective centres of the series of tubes is shown in Fig. 6. The effective centre is defined as the position at which the dynamic pressure of the (undisturbed) flow is equal to the pressure difference actually observed in the tube. It is clear that especially in a viscous flow the effective centre should not be confused with the stagnation point or line and, particularly in the case of a flat tube resting on the surface where the front face of the tube is in a separated region, it has not much relation to the streamline pattern. The effective centre positions for the flat tubes are seen to vary over a very wide range, those for the smallest sizes being well above the top of the tube, while the effective centres for the largest tubes are at only 0·1 to 0·2 of the tube height from the surface, compared with a constant height of $0{\cdot}62 \times$ diameter for Preston's large round surface tubes. It may be noted that this curve may also be interpreted as one of displacement effect for flat traverse pitots when actually touching the wall, so that only when $u_\tau d/\nu$ is about $3\{\log(u_\tau d/\nu) \simeq 0{\cdot}4\}$ will the tube read the dynamic pressure at its geometrical centre.

A sample velocity profile (calculated) for the laminar-flow duct is also shown. It should be noted that this parabolic profile does not plot as a universal function in Fig. 7, as $u/u_\tau = (u_\tau y/\nu)(1 - y/2a)$.

6. *Discussion.*—In view of the steady divergence of the Stanton-tube calibrations in laminar and turbulent flow, the chief reason for the difference is probably the failure to make a tube small enough for its field of influence to be confined within the linear sub-layer. The Stanton tube may be regarded as producing local separation of the sub-layer ahead of itself, with a bubble of almost constant pressure between the separation point and the face of the tube. If separation is crudely assumed to occur at a value of adverse pressure gradient depending only on the sub-layer parameters and independent of the size of the tube, the separation point will be only a few tube heights upstream of the tube face for a small tube, but many tube heights upstream for a large tube because the large tube will produce a larger adverse pressure gradient than the small tube. Therefore the vertical extent of the disturbance caused by the tube will be proportionally greater for the smaller tube because of the larger upwash angle, so that its reading will be influenced more by the velocity profile well above the tube (this is quite a different argument from that based on 'effective centres' which have little physical significance). This approach also shows that the pressure in the separation bubble is not likely to vary very much with the size of the tube, and in fact the present results show that the pressure varies only as $d^{1\cdot2}$ for the smallest tubes at a given shear stress instead of varying as d^2, as might be expected if the flow patterns were similar and the flow essentially inviscid. The concept of a constant pressure bubble ahead of the tube has been explored quantitatively by Gadd.[9] For the present purpose one need only note that the smallest tubes have proportionally the largest vertical field of influence, and that the divergence of the turbulent-flow profile from linearity appears to take place well within this field of influence.

The turbulent velocity fluctuations in the linear sub-layer may also have some effect, but at first sight one would expect that their effect would be to increase the pressure difference in turbulent flow above that for laminar flow whereas the discrepancy is in the opposite sense. However, one must always beware of the application of inviscid-flow concepts, like constancy of dynamic pressure along streamlines, to what is in fact a flow at very low local Reynolds number.

Willmarth[6] has shown that the r.m.s. fluctuating static pressure at the wall below a turbulent boundary layer is about $0{\cdot}003 \times \frac{1}{2}\rho U_1^2$, that is, of the same order as the wall shear stress τ_0. For the smallest tubes, therefore, the static-pressure fluctuations are of the same order as the tube pressure difference, as seen in Fig. 5 and it is possible that these, like the velocity fluctuations, may also influence the results.

The reference pressure which was subtracted from the tube total-pressure reading was that in a 0·040-in. static-pressure tapping. Recent work by Shaw[10] has shown that the effect of finite hole size on the recorded pressure is considerable, but Preston[8] has pointed out that this effect can be correlated in just the same way as the disturbance produced by surface tubes (which are just another form of imperfection in the surface), so that one would not expect it to account for the laminar-turbulent calibration differences except that the turbulent fluctuations may affect the static hole as well as the pitot-tube. Some unreported experiments in the turbulent duct by Mr. M. T. Gee of Aerodynamics Division, N.P.L., have shown that the difference between the static pressure recorded by a 0·040-in. hole and that obtained by extrapolating to zero hole diameter is about $4\tau_0^{1\cdot6}$ lb/ft² which is only about 10 per cent of the reading of the smallest Stanton tube. Presumably the difference between the errors in laminar flow and in turbulent flow would be an order of magnitude smaller and therefore only 1 per cent or so of the Stanton-tube reading.

The hot-wire calibration differences are more difficult to explain. At least at the lowest speeds, the wires should have been entirely within the linear sub-layer, so that an explanation must be sought in the fluctuating properties of the turbulent flow or in the experimental conditions. The discrepancy amounts to about 20 per cent on speed at the lowest wall shear stress, and it is unlikely that the skin friction in either duct was this much in error. The curvature of the laminar-flow profile due to the pressure gradient results in an error, but only of about 5 per cent at $y = 0{\cdot}002$ in., the nominal height of the wire.

The blockage caused by the probe arms may have been responsible, though some care was taken to make the probe as small as possible, and an alternative design with the wire mounted directly on top of the vertical steel wires was also tried on one occasion and gave similar results, though the wire was rather further from the surface and the calibrations did not quite cross at their lower ends. Turbulent velocity fluctuations are expected to decrease the velocity recorded by the wire for a given mean shear stress, provided that the responses of the electrical circuit components are linear. This latter point was checked by shunting the wire with a 2200 μF capacitor (impedance 10 Ω at 10 c.p.s.). The violence of the oscillations of the galvanometer needle was much reduced but the balance point was unaffected.

It is possible that the heat loss to the wall varies between laminar and turbulent flow, for Cox[11] has stated that a wire ten diameters from a perfectly conducting wall loses 30 per cent of the heat supplied to it by conduction to the wall. Cox's conclusions are controversial, and much more work, not strictly relevant to the development of methods of skin-friction measurement, would be needed to establish the exact behaviour of hot wires near a wall. The chief conclusion to be drawn is that the behaviour of hot wires, like that of pitot-tubes, seems to differ greatly between laminar and turbulent flow.

In view of these differences in calibration, and of the satisfactory agreement of the skin-friction measurements based on the turbulent-flow calibrations, it is certain that the assumption of the applicability of the laminar-flow calibration to turbulent flow gives incorrect answers for skin friction. The hypothesis that the turbulent-flow calibration gives the right answer is supported by the good agreement between hot wires and surface tubes, by the coincidence of the duct and boundary-layer velocity profiles in the viscous sub-layer and the resulting agreement of skin-friction values obtained from the smaller surface tubes, and by the agreement of the results of Ref. 7, obtained with surface tubes, with those of Ref. 12, obtained by wake-traverse and differentiation, and those of Ref. 13, obtained from floating surface element readings. The experiments reported in these References were all carried out in boundary layers at zero pressure gradient, so that they only support the use of surface tubes in this case. It remains to be seen whether the results for boundary layers in pressure gradients are consistent, but the grounds for using the Stanton-tube or hot-wire methods are at least as strong as those in favour of other methods.

7. *Comparison of Skin-Friction Measurement Methods.*—The best-known methods of local skin-friction measurement are as follows:

(1) Momentum balance

(2) Floating surface element

(3) Heat-transfer surface element

(4) Surface pitot-tubes

(5) Extrapolation of shear-stress profile from hot-wire measurements.

The floating element cannot easily be used in pressure gradients, which tends to limit its use to that of providing standards against which to calibrate substandard instruments. It is possible that some way may be found of overcoming the difficulties caused by horizontal buoyancy effects and finite gap width, but this does not seem to be in sight at present, and the instrument needs a large flat area for its mounting.

Of the methods available for use in pressure gradients, the extrapolation of the shear-stress profile is inconvenient unless the profile is actually required as one of the results of the investigation. Schubauer and Klebanoff[14] obtained quite consistent though not necessarily accurate results by this method, but it is necessary that the boundary layer should be sufficiently thick to accommodate the hot wires. One advantage is that a special surface is not needed.

The surface-tube and heat-transfer methods are substandard methods, requiring calibration, and make some sort of assumption about universality of velocity profile, be it in the linear sub-layer or in the whole inner-law region. The Stanton-tube and hot-wire methods described in this report involve the least restrictive assumptions and are probably the easiest to apply.

The history of the use of the von Kármán momentum integral equation for skin-friction determination is not a happy one. Slight departures from two-dimensional or axisymmetric flow may produce highly inaccurate estimates of skin friction, particularly in adverse pressure gradients where the skin-friction coefficient is the small difference of two large terms in the equation, and it is also necessary to take account of the turbulent fluctuations, both in their contribution to momentum flux and in their effect on the pitot-tube or hot wire used to measure the velocity profiles. The most that can be said is that the momentum equation and independently derived skin-friction results should be used to check each other.

It therefore seems that the Stanton-tube method is the most acceptable one for finding skin friction in a completely unknown flow. Where the inner law has been established by previous experiment, Preston's method of using larger surface tubes is preferable because large tubes are more robust, easier to mount, and give larger pressure differences.

8. *Conclusions.*—Surface pitot-tubes of non-dimensional height $u_\tau d/\nu < 30$, calibrated in turbulent channel flow, give values of skin friction in a wind-tunnel wall boundary layer which agree among the different sizes of tube and also agree with the skin friction indicated by a hot wire, mounted very near the surface and similarly calibrated. Skin-friction curves in turbulent flow obtained from calibrations of the two sorts of instrument in laminar channel flow disagree with each other; they also disagree with the values obtained from turbulent-flow calibration, so that the past practice of calibration in laminar flow for use in turbulent flow leads to inaccurate results. In view of the internal consistency of the turbulent-flow results and their agreement with results obtained by other methods in other experiments, it is concluded that the calibration of Stanton surface pitot-tubes in the viscous sub-layer of turbulent duct flow gives accurate values of turbulent boundary-layer skin friction, at least in zero pressure gradient, and that it is one of the most convenient methods of skin-friction measurement.

9. *Acknowledgements.*—The authors wish to acknowledge the benefit of several discussions with Dr. G. E. Gadd. They are also grateful to Dr. D. L. Schultz for the loan of electrical equipment and much helpful advice.

NOTATION

a	Half height of laminar-flow duct = 0·020 in.
$c_f = \dfrac{\tau_0}{\frac{1}{2}\rho U_1^2}$	Local coefficient of skin friction
d	Diameter of round surface tube, overall height of flat surface tube, diameter of hot wire
f	A function
I	Wire heating current
k	Thermal conductivity of air
l	Length of hot wire
p	Static pressure or pressure in static-pressure hole
p'	r.m.s. fluctuating wall static pressure
P	Stagnation pressure in round surface tube
R	Hot-wire resistance
R_A	Wire resistance at ambient temperature
$R_x = \dfrac{Ux}{\nu}$	Reynolds number. $x = 2{\cdot}5$ ft for observation in tunnel
S	Stagnation pressure in flat surface tube
T	Temperature
$u_\tau = \sqrt{\left(\dfrac{\tau_0}{\rho}\right)}$	
U, V, W	Mean velocities
u, v, w	Instantaneous velocities
u', v', w'	r.m.s. fluctuating velocities
x, y, z	Perpendicular axes
$\Lambda = \dfrac{\rho\nu^2}{\rho_0 \nu_0^2}$	Suffix $_0$ implying 15 deg C and 760 mm Hg
μ	Viscosity of air
ν	Kinematic viscosity of air = μ/ρ
ρ	Density of air
τ_0	Wall shear stress

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12	E. F. Relf, R. C. Pankhurst and W. S. Walker ..	The use of pitot-tubes to measure skin friction on a flat plate. R. & M. 3185. August, 1954.
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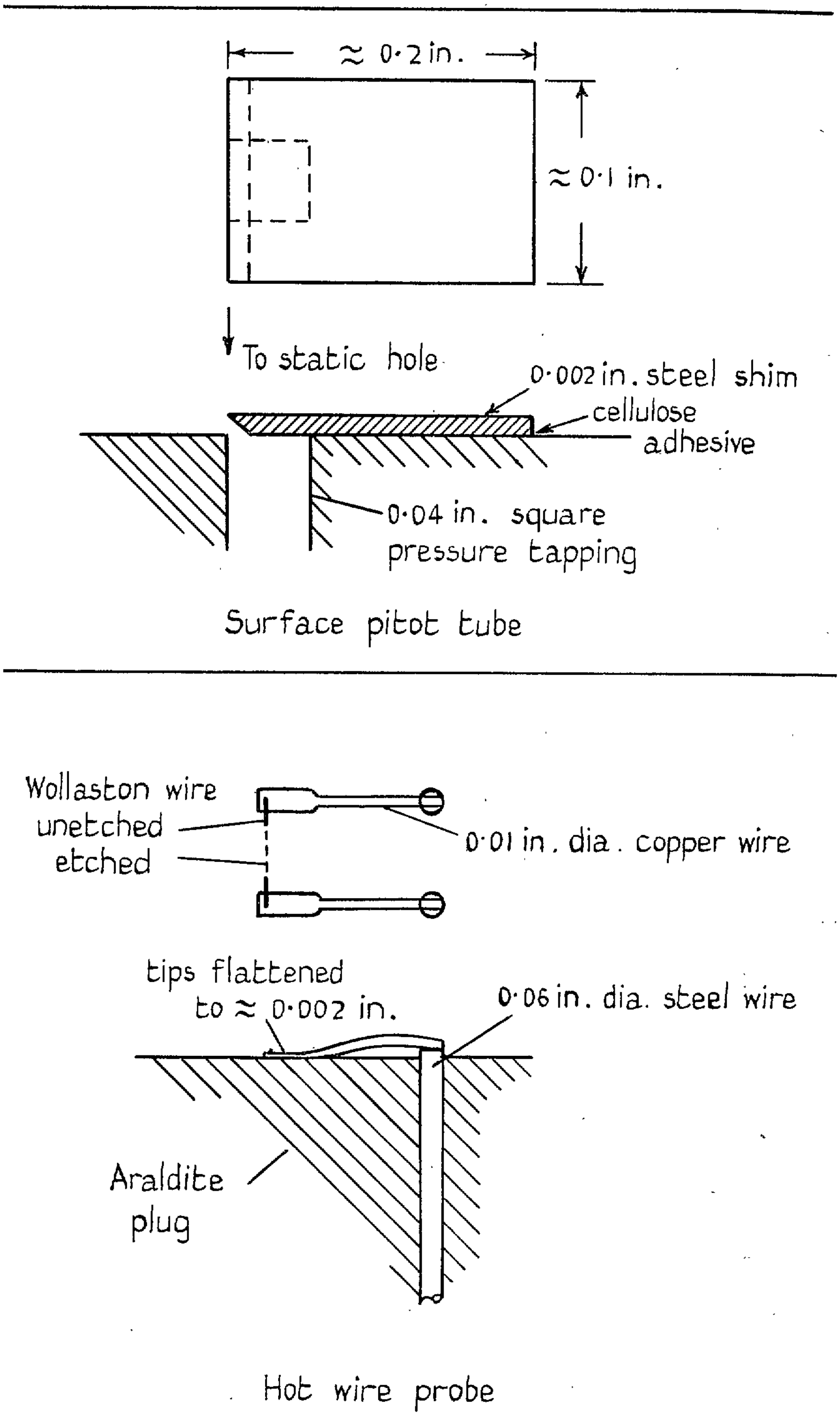

FIG. 1. Details of surface tubes and hot wires.

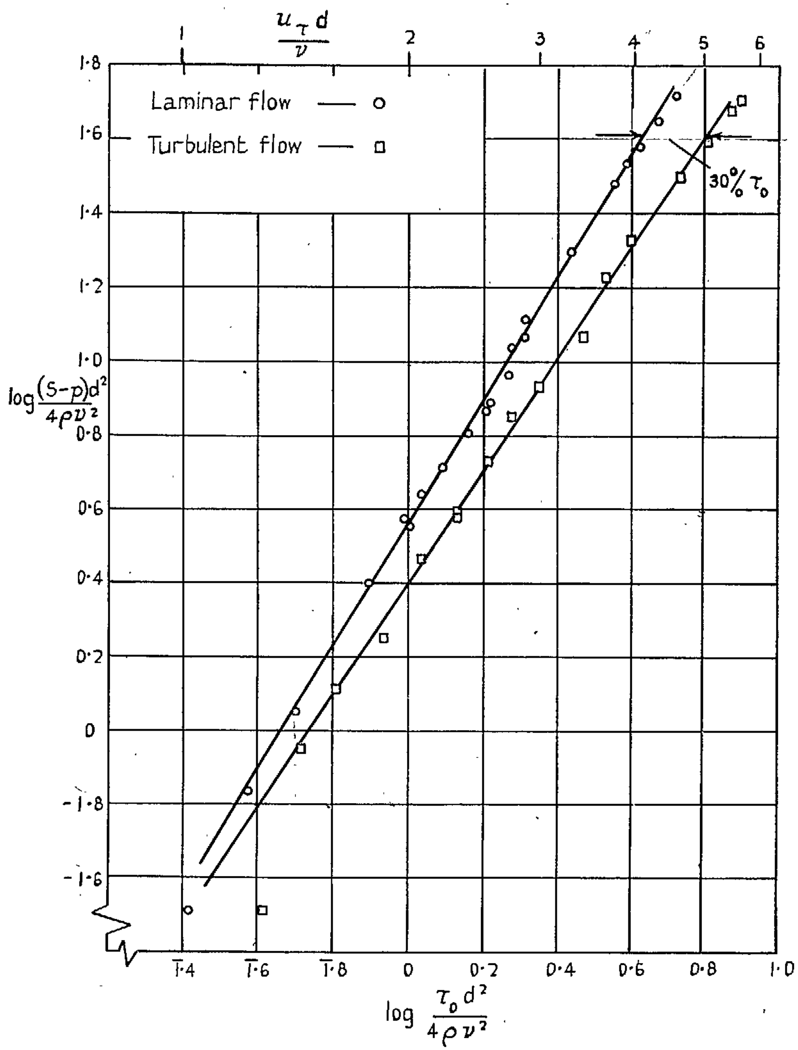

Fig. 2. 0·002-in. surface-tube calibrations.

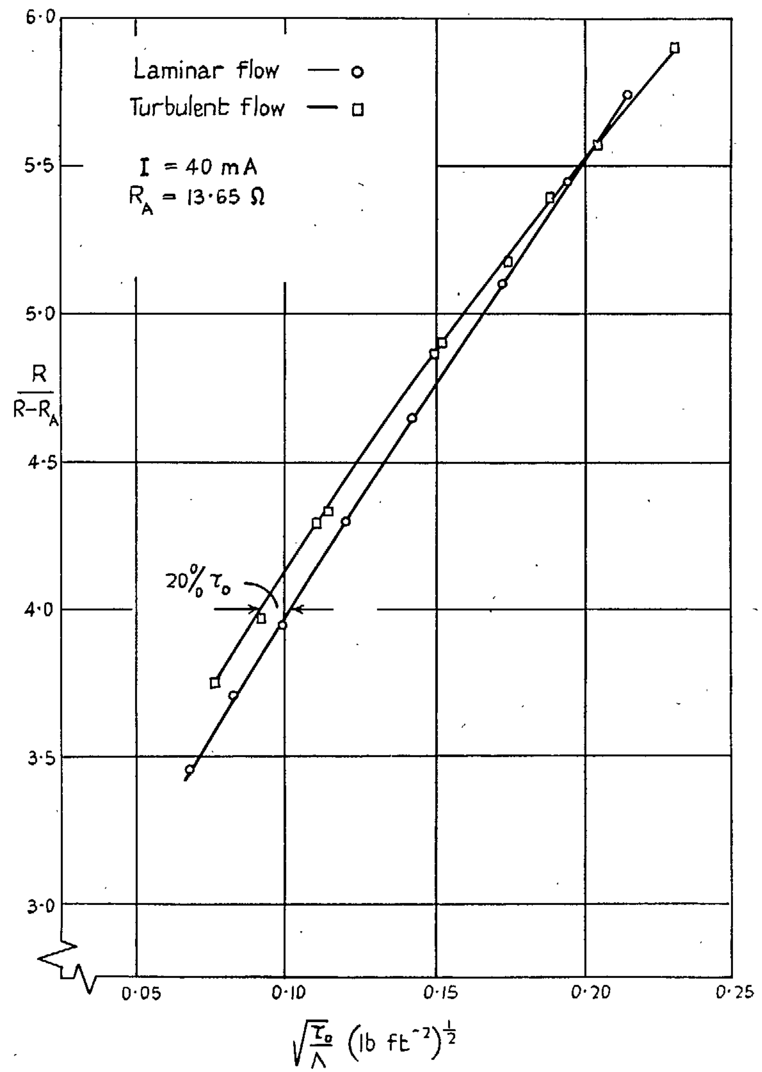

Fig. 3. Hot-wire calibrations.

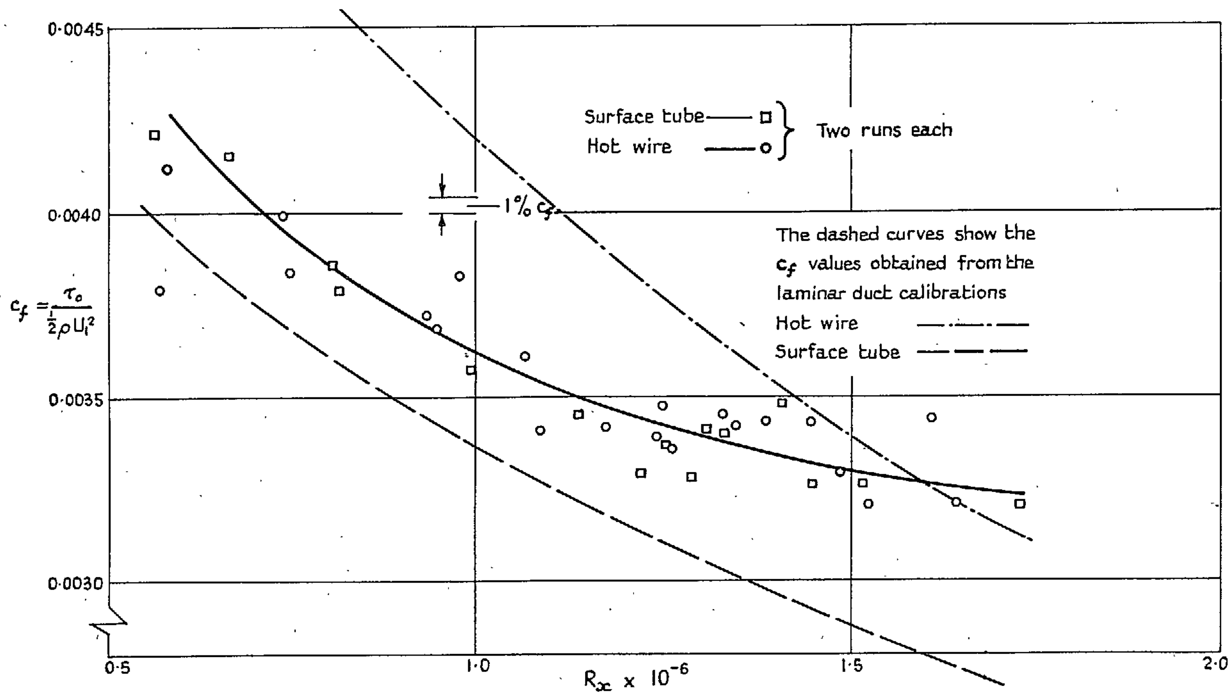

FIG. 4. Skin-friction measurements on tunnel wall. Comparison of surface-tube and hot-wire results.

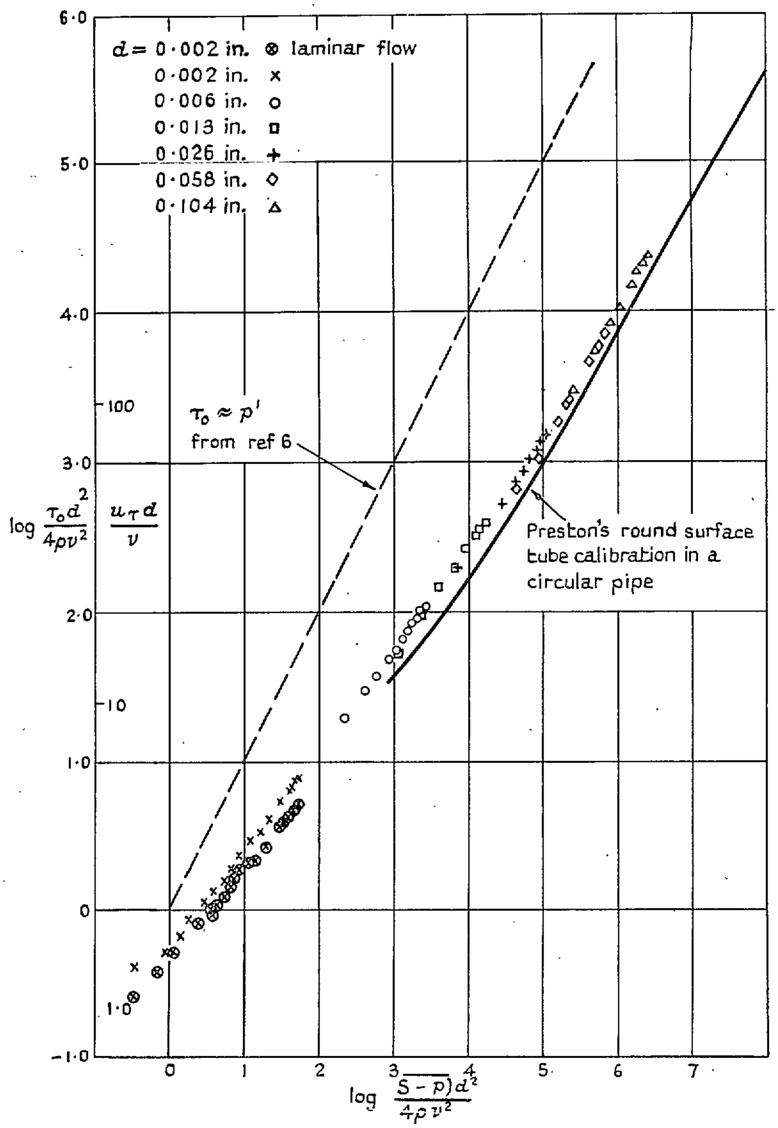

FIG. 5. Flat surface-tube calibration in channel flow.

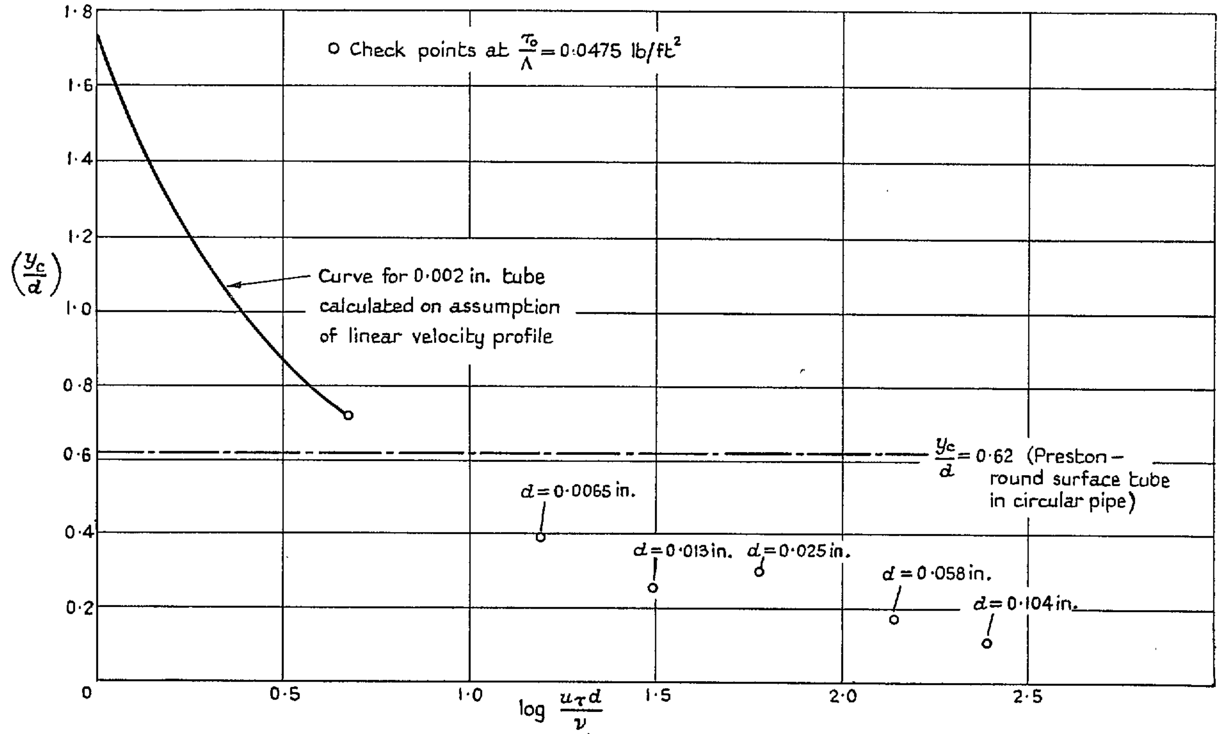

FIG. 6. Positions of effective centres of flat surface tubes.

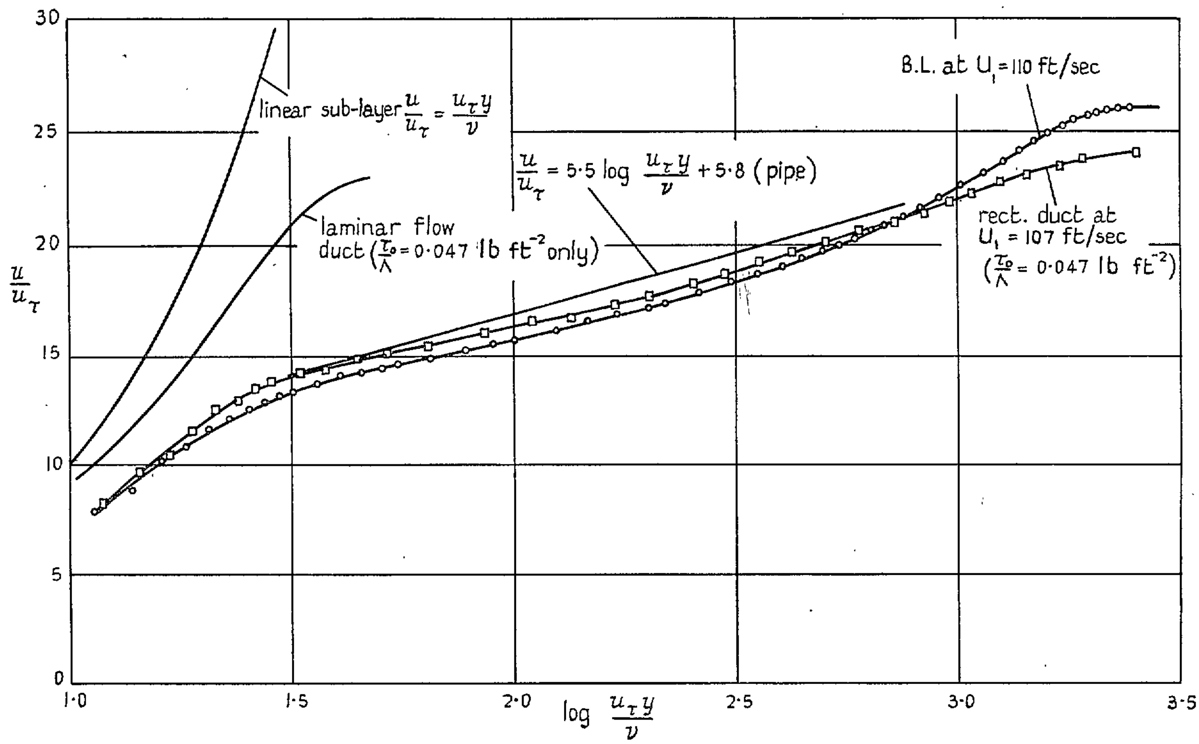

FIG. 7. Velocity profiles in duct and boundary layer.

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