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By

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Cambridge University Engineering Department
Communicated by Prof. W. A. Mair

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SUMMARY

A new type of auxiliary equation has been evolved for calculating the development of the form-parameter H $(= \delta^*/\theta)$ in turbulent boundary layers with adverse pressure gradients. This has been achieved by considerably extending part of a previous analysis by Spence. The chief advantage of this new method lies in the rapidity and ease with which the growth of H may be calculated, while results compare favourably with those obtained using the methods of other investigators. Several results of the calculations are compared with experiment. Finally, attention is drawn to the similarity between the form of the new equation and that of Schuh's auxiliary equation.

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1./

1. Introduction

Whereas the growth of momentum thickness in the turbulent boundary layer can now be calculated with reasonable speed and accuracy by such methods as those of Ref's. 1 and 2, which involve only simple quadrature, the prediction of the form parameter remains a somewhat tedious and lengthy task, while results obtained using the various methods available do not always justify the complexity of the calculations involved.

In view of this, and prompted by the personal need for a means of performing rapidly a large number of calculations of this nature, the author felt it worth while to attempt the construction of a new method for determining the growth of the form parameter H, concentrating on speed and ease of application, and accepting the need for certain approximations in achieving this.

Consideration of the various parameters which influence the growth of H shows that the external velocity distribution is by far the most important. This then suggests an auxiliary equation in which H is primarily a function of U, with a second-order dependence upon θ and R_θ. In order to formulate an auxiliary equation of this form, part of an analysis by Spence[3] has been introduced, and this is developed in the following section to give an integral form of the auxiliary equation.[+]

2. List of Symbols

x, y	co-ordinates parallel and normal to surface
u, v	mean velocity components in x, y directions
U	free stream velocity at edge of boundary layer
τ, τ_w	shear stress, shear stress at wall
$u_\tau = \left(\frac{\tau_w}{\rho}\right)^{\frac{1}{2}}$	friction velocity
$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$	local skin friction coefficient
δ	thickness of boundary layer
$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$	displacement thickness of boundary layer
$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$	momentum thickness of boundary layer
$\epsilon = \int_0^\delta \frac{u}{U}\left[1 - \left(\frac{u}{U}\right)^2\right] dy$	energy thickness of boundary layer

R_θ/

+The author's attention has been drawn to an unpublished communication in which D. A. Spence proposes the use of an integral form of the auxiliary equation. This equation requires numerical or graphical integration except in the particular case of a linear adverse velocity gradient, where it reduces to a form similar to that of the author's auxiliary equation.

$R_\theta = \dfrac{\theta U}{\nu}$	Reynolds number based on momentum thickness
$H = \dfrac{\delta^*}{\theta}$	form parameters for velocity profile
$\gamma = \left(\dfrac{u}{U}\right)_{y=\theta}$	form parameters for velocity profile
A, B	constants in universal logarithmic velocity profile
k	constant defined in equation (9)
α	constant defined in equation (10)
$n = 2(1 - \alpha)\sqrt{1 + k^2}$	
$K = \dfrac{An}{1 - \alpha}$	
Z	function defined in equation (12)
G	function defined in Ref. 4, assumed constant (= 2.8) in present paper
$s = x_2 - x_1$	length of step in calculation procedure
$\eta = \dfrac{x - x_1}{x_2 - x_1}$	non-dimensional distance in x-direction
$\beta = \dfrac{s}{\theta_1 R_{\theta_1}^{0.402}}$	
ϕ	function defined in equation (26)
$y^* = \dfrac{y\, u_\tau}{\nu}$	
$\theta^* = \dfrac{\theta\, u_\tau}{\nu}$	

Additional Symbols Used in Describing Schuh's Method

I_1	integral defined by equation (33)
$\Xi = \theta R_\theta^n$	
m, a, b, h, g, p	constants

λ/

$$\lambda = \frac{U_2}{U_1}$$

$$\mathcal{K} = \frac{1 - \lambda}{x_2 - x_1}$$

3. General Equations

3.1 Derivation of Spence's form-parameter equation

Spence utilised the equation for the universal logarithmic velocity profile, as confirmed by the measurements of Ludwieg and Tillmann[4].

$$u = u_\tau f(y^*) \qquad \ldots(1)$$

where

$$f(y^*) = A.\log_e y^* + B ,$$

$$y^* = \frac{u_\tau y}{\nu} ,$$

$$A = 2.5$$

and

$$B = 5.5 .$$

Using equation (1), together with the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \qquad \ldots(2)$$

the equation of motion

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \frac{1}{\rho} \frac{\partial \tau}{\partial y} \qquad \ldots(3)$$

may be formulated at the height $y = \theta$ (for which equation (1) is still generally valid), thus giving:-

$$u_\tau u_\tau' f^2(\theta^*) = U \frac{dU}{dx} + \frac{1}{\rho} \left(\frac{\partial \tau}{\partial y} \right)_{y=\theta} , \qquad \ldots(4)$$

where $\theta^* = \dfrac{\theta u_\tau}{\nu}$ and the prime denotes differentiation with respect to x.

Introducing the form parameter $\gamma = \left(\dfrac{u}{U} \right)_{y=\theta}$, $f(\theta^*)$ is then found by putting $y = \theta$ in equation (1), to obtain

$$\gamma = \frac{u_\tau}{U} f(\theta^*) . \qquad \ldots(5)$$

$\dfrac{u_\tau'}{u_\tau}$ /

$\frac{u_\tau'}{u_\tau}$ is found by differentiating equation (5) with respect to x to obtain

$$\left(1 + \frac{Au_\tau}{y\,U}\right)\frac{u_\tau'}{u_\tau} = \frac{y'}{y} + \frac{U'}{U} - \frac{Au_\tau}{U}\cdot\frac{\theta'}{\theta} \ . \qquad \ldots(6)$$

Spence neglects the term $\frac{Au_\tau}{y\,U}$ in the brackets on the left-hand side, on the grounds that it is much less than unity in practice. The term $\frac{\theta'}{\theta}$ is given by the momentum equation, i.e.,

$$\frac{\theta'}{\theta} = \frac{1}{\theta}\left(\frac{u_\tau}{U}\right)^2 - (H + 2)\frac{U'}{U} \ . \qquad \ldots(7)$$

With the aid of equations (5), (6) and (7), equation (4) may be written

$$yy' - \frac{Ay}{\theta}\left(\frac{u_\tau}{U}\right)^3 = \frac{U'}{U}\left[1 - (H + 2)\frac{Au_\tau}{U}\cdot y - y^2\right]$$

$$+ \frac{\partial}{\partial y}\left(\frac{\tau}{\rho U^2}\right)_{y=0} \ . \qquad \ldots(8)$$

To simplify this equation empirically, Spence replaced two slowly varying quantities by constants to be fitted from experimental results. First, since $(H + 2)$ increases while $\frac{u_\tau}{U}$ decreases, the approximate relation

$$(H + 2)\,A\,\frac{u_\tau}{U} = 2\,k \qquad \ldots(9)$$

may be accepted, k being an appropriate constant. Secondly, examination of the shear stress profiles given by Schubauer and Klebanoff[5] shows that $\frac{\partial}{\partial y}\left(\frac{\tau}{\rho U^2}\right)_{y=0}$ becomes a progressively larger fraction of $-\frac{1}{U}\frac{dU}{dx}$ (which is the normal gradient of $\frac{\tau}{\rho U^2}$ at $y = 0$) as x increases, and therefore as H and the quantity $(1 - 2ky - y^2)$ increase. This suggests the approximation

$$\frac{\partial}{\partial y}\left(\frac{\tau}{\rho U^2}\right)_{y=0} = -\,\alpha(1 - 2\,ky - y^2)\,\frac{U'}{U} \ , \qquad \ldots(10)$$

where α is a constant to be determined from experiment.

With these approximations, equation (8) becomes

$$(1 - \alpha)\frac{U'}{U} - \frac{yy'}{1 - 2yk - y^2} = -\frac{y}{1 - 2ky - y^2}\,\frac{A}{\theta}\cdot\left(\frac{u_\tau}{U}\right)^3 \qquad \ldots(11)$$

Solution/

Solution of (11) involves the function

$$Z = [\sqrt{1+k^2} + (k+y)]^{\sqrt{1+k^2}+k} . [\sqrt{1+k^2} - (k+y)]^{\sqrt{1+k^2}-k} \qquad ...(12)$$

Differentiating logarithmically,

$$\frac{1}{Z}\frac{dZ}{dy} = -2\sqrt{1+k^2} . \frac{y}{1-2ky-y^2} . \qquad ...(13)$$

Thus (11) may be written,

$$\frac{1}{2\sqrt{1+k^2}} . \frac{1}{Z}\frac{dZ}{dx} + (1-\alpha)\frac{1}{U}\frac{dU}{dx}$$

$$= -\frac{y}{1-2ky-y^2} . \frac{A}{\theta}\left(\frac{u_\tau}{U}\right)^3 , \qquad ...(14)$$

which constitutes the initial equation of the present analysis.

3.2 Derivation of present form-parameter equation

Making the substitutions

$$n = 2(1-\alpha)\sqrt{1+k^2}$$

and

$$K = \frac{An}{1-\alpha} ,$$

equation (14) may be integrated to give

$$\log_e \frac{Z_2}{Z_1} + n \log_e \frac{U_2}{U_1} = -K\int_{x_1}^{x_2} \frac{y\left(\frac{c_f}{2}\right)^{3/2}}{\theta(1-2ky-y^2)} dx . \qquad ...(15)$$

The problem is now to express the right-hand side of equation (15) in an integrable form. The procedure adopted here may be outlined as follows.

First, c_f is expressed in terms of H and R_θ using the skin friction relation of Ludwieg and Tillmann[4].

Second, $\frac{\theta}{\theta_1}$ is expressed in terms of $\frac{U}{U_1}$ using the approximate empirical relation given by Ross and Robertson[6]. The integral is now obtained in terms of $\frac{U}{U_1}$, H, and y.

Third,/

Third, since the right-hand side of equation (15) is normally very small compared with either of the terms on the left-hand side, it may be neglected to obtain a first approximation for $\frac{U}{U_1}$ in terms of Z (which is simply a function of γ and the constant k). The variables in the integral are now reduced to γ and H.

Fourth, the relation between γ and H given by Spence[3] is introduced to eliminate γ. The integral now assumes the form

$$\int_{x_1}^{x_2} F(H)\, dx.,$$

with some functions of the initial values outside the integral sign.

The procedure outlined above is described in detail in the following paragraphs and in Section 3.3, and approximations to the simplified integral are given, thus enabling the form-parameter equation to be used in the integral form.

Introducing Ludwieg and Tillmann's skin friction equation

$$\frac{c_f}{2} = 0.123\ 10^{-0.678H}\ R_\theta^{-0.268}, \qquad \ldots(16)$$

equation (15) may be written

$$\log_e \frac{Z_2}{Z_1} + n \log_e \frac{U_2}{U_1} = -\ 0.0431\ K \int_{x_1}^{x_2} \left\{ \frac{\gamma}{\theta(1 - 2k\gamma - \gamma^2)}\ 10^{-1.017H}\ R_\theta^{-0.402} \right\} dx\ . \qquad \ldots(17)$$

Ross and Robertson[6] gave, as an approximate expression for the growth of momentum thickness in an adverse pressure gradient, the relation

$$\frac{\theta}{\theta_1} = \left(\frac{U_1}{U}\right)^{2+G}, \qquad \ldots(18)$$

where the subscript 1 refers to some initial point, and G is a function which in general depends upon Reynolds number, but which may be considered constant without great loss in accuracy. In this analysis G will therefore be assumed to have the constant value 2.8 as Ross and Robertson suggest.

Equation (18) then enables (17) to be written

$$\log_e \frac{Z_2}{Z_1} + n \log_e \frac{U_2}{U_1} = -\ \frac{0.0431\ K}{\theta_1 R_{\theta_1}^{0.402}} \int_{x_1}^{x_2} \left\{ \frac{\gamma}{(1 - 2k\gamma - \gamma^2)}\ 10^{-1.017H} \left(\frac{U}{U_1}\right)^{1.402(2+G)\ -\ 0.402} \right\} dx\ . \qquad \ldots(19)$$

In/

In adverse pressure gradients, to which application of this method is limited, the term on the right-hand side of (19) is small compared with either of those on the left-hand side, and the following relation is approximately true;

$$\frac{U}{U_1} = \frac{Z_1}{Z}^{\frac{1}{n}} \quad . \qquad \ldots(20)$$

It seems reasonable, therefore, to use equation (20) in order to simplify further the right-hand side of (19).

Thus,

$$\log_e \frac{Z_2}{Z_1} + n \log_e \frac{U_2}{U_1} = - \frac{0.0431\ K}{\theta_1 R_{\theta_1}^{0.402}} \int_{x_1}^{x_2} \left\{ \frac{\gamma}{(1 - 2k\gamma - \gamma^2)} \; 10^{-1.017H} \; \frac{Z_1}{Z}^{\frac{1.402(2+G)-0.402}{n}} \right\} dx \; . \qquad \ldots(21)$$

Inserting the values:

$$k = 0.2 \; ,$$

$$\alpha = -\,0.18 \; ,$$

$$n = 2.4 \; ,$$

as found experimentally by Spence and

$$G = 2.8$$

as suggested by Ross and Robertson, and introducing the non-dimensional distance

$$\eta = \frac{x - x_1}{x_2 - x_1} = \frac{x - x_1}{s} \; ,$$

equation (21) may be written in the form

$$\frac{U_2}{U_1} = \frac{Z_1}{Z_2}^{\frac{1}{2.4}} e^{\left[-0.0915 Z_1^{2.633} \cdot \frac{s}{\theta_1 R_{\theta_1}^{0.402}} \cdot \int_0^{1.0} \left\{ \frac{\gamma}{(1-0.4\gamma-\gamma^2)} \cdot \frac{10^{-1.017H}}{Z^{2.633}} \right\} d\eta . \right]} \qquad \ldots(22)$$

Accepting the power-law approximation to the velocity profiles, γ may be expressed in terms of H as

$$\gamma = \left[\frac{H - 1}{H(H + 1)} \right]^{\frac{1}{2}(H-1)} \quad . \qquad \ldots(23)$$

Using/

Using equations (12) and (23), the integral may now be written in the form

$$\int_0^{1.0} F(H)\, d\eta \ ,$$

and equation (22) becomes

$$\frac{U_2}{U_1} = \left(\frac{Z_1}{Z_2}\right)^{\frac{1}{2.4}} e^{\left[-0.0915 Z_1^{2.633} \cdot \frac{s}{\theta_1 R_{\theta_1}^{0.402}} \cdot \int_0^{1.0} F(H)\, d\eta\right]} . \qquad \ldots(24)$$

Z and y are plotted against H in Figs. 1 and 2, while in Fig. 3

$$\left(\frac{Z_1}{Z_2}\right)^{\frac{1}{2.4}}$$

is plotted as a function of H_1 and H_2.

3.3 Determination of $\int_0^{1.0} F(H)\, d\eta$

It is now necessary to express the integral $\int_0^{1.0} F(H)\, d\eta$ in terms of H_1 and H_2, the values of H corresponding to the lower and upper limits of integration.

The function $F(H)$ is plotted in Fig. 4. Now, because of the characteristic variation of H with η in adverse pressure gradients, (which may be inferred from Figs. 5 to 7) it would not be unreasonable to expect that a plot of $F(H)$ against η, instead of against H should tend much more closely to linearity. Investigation of some experimental results[5,7,8] confirms this, thereby suggesting the simplification

$$\int_0^{1.0} F(H)\, d\eta = \frac{F(H_1) + F(H_2)}{2} . \qquad \ldots(25)$$

By defining the functions β and φ as

$$\beta = \frac{s}{\theta_1 R_{\theta_1}^{0.402}}$$

and

$$\varphi = 0.0915 Z_1^{2.633} \int_0^{1.0} F(H)\, d\eta$$

$$= 0.04575 Z_1^{2.633} \left[F(H_1) + F(H_2)\right] \qquad \ldots(26)$$

Equation/

Equation (24) may be written as

$$\frac{U_2}{U_1} = \left(\frac{Z_1}{Z_2}\right)^{\frac{1}{2.4}} e^{-\beta\phi}$$

or, since $\beta\phi << 1.0$,

$$\frac{U_2}{U_1} \doteq \left(\frac{Z_1}{Z_2}\right)^{\frac{1}{2.4}} (1 - \beta\phi) \qquad \ldots(27)$$

with the aid of Figs. 1 and 4, ϕ may be determined for any given values of H_1 and H_2.

4. Application of Theory

4.1 Method of solution

It is assumed that the velocity distribution and initial value of H are known, and that the development of θ has been calculated independently of H by a method of quadrature such as has been evolved by Truckenbrodt[1] or Maskell[2]. Equation (27) can then be solved in the following manner.

Commencing with H_1, the initial value of H, a value of H_2 is selected (say $H_1 + 0.1$). From Fig. 3, the corresponding value of $\dfrac{U_2/U_1}{1 - \beta\phi}$ is found, while ϕ is evaluated from Figs. 1 and 4 as described in 3.3. A guess must now be made as to the distance s required for H to increase from H_1 to H_2, and hence a first approximation to β may be found.

U_2/U_1 can now be calculated and hence x read off from the velocity distribution. The new value of s $(= x_2 - x_1)$ is fed back to yield a more accurate value of β and a second value of x_2 is determined. In general, the solution converges rapidly and further iteration should be unnecessary. The procedure is then repeated in step-by-step fashion from H_2 to H_3, and so on.

It will be found, however, that the value of $\beta\phi$ rapidly becomes negligible after the first few steps. Thereafter any step length may be used without loss of accuracy and the calculation is continued using Fig. 3 only, in conjunction with the velocity distribution.

It has been found that acceptable accuracy is obtained by using steps of 0.1 in H until the term $\beta\phi$ becomes negligible.

4.2 Comparison with experimental results

The following three sets of experiments have been used to test the accuracy of the present method:

Author	Experiment	Fig.
Schubauer and Klebanoff[5]	N.B.S. Turbulent Boundary Layer.	5
von Doenhoff and Tetervin[7]	Aerofoil NACA 65(216)-222 approx. $R = 2.64 \times 10^6$, $\alpha = 10.1°$.	6
Sandborn[8]	N.B.S. Turbulent Boundary Layer.	7

In each case the experimental values of momentum thickness have been used in calculating the growth of H by the present method, but this does not invalidate comparison with other methods which have used calculated values of momentum thickness. This is because the value of θ is only of importance during the initial stages of the present calculation, in which range any divergence between the experimental and calculated momentum thicknesses must still be small.

In comparing the results with those of Spence, it should also be borne in mind that, whereas the present analysis involves the Ludwieg-Tillmann skin friction law

$$c_f = 0.246. \quad 10^{-0.678H} R_\theta^{-0.268} , \qquad \ldots(28)$$

Spence adopted Squire and Young's[9] law:

$$c_f = 2(3.59 + 2.56 \log_e R_\theta)^{-2} \qquad \ldots(29)$$

for the reason that, through over-estimating the value of skin friction near separation, it partly compensated for the omission of the transverse pressure gradient and Reynolds stress terms which are no longer negligible at that stage.

Where possible, curves calculated according to the methods of Maskell[2], Spence[3], Schuh[10] and Truckenbrodt[1] have been included. It will be seen that the present method compares favourably with those of the other investigators.

It should be noted that, in the case of the Schubauer-Klebanoff experiment, Maskell commenced his calculation at the 0.5 feet position, whereas in the other methods illustrated, including the present one, the beginning of the test section at 17.5 feet was selected as the initial point; moreover his initial values of H were obtained from the approximate empirical relationship

$$H_0 = 1.630 - 0.0775 \log_{10} R_\theta , \qquad \ldots(30)$$

whereas the present calculations are based on an initial experimental point.

5./

5. Similarity Between the Form of the Present Auxiliary Equation and that of Schuh+

Using the energy equation

$$\frac{d}{dx}(U^3\epsilon) = \frac{2}{\rho}\int_0^\infty \tau \frac{\partial u}{\partial y}\,dy , \qquad \ldots(31)$$

Schuh[10] obtained the following auxiliary equation for the growth of H:

$$H_2 = -\frac{1}{g}\log_{10}\left\{10^{-gH_1} + 2.303.\ (10)^{-p}g\left(I_1 + 2.303\log_{10}\left(\frac{U_2}{U_1}\right)\right)\right\}$$

$$\text{for } H \geqslant 1.30, \qquad \ldots(32)$$

where

$$I_1 = h\int_{x_1}^{x_2}\frac{dx}{\mathfrak{Z}} = 2.303\frac{h}{a}\log_{10}\left\{1 + \frac{a}{\mathfrak{Z}_1}\int_{x_1}^{x_2}\frac{U_2}{U_1}dx\right\} , \ldots(33)$$

$$\mathfrak{Z} = \theta R_0^m ,$$

$a = 0.0185$, $b = 4.27$, $m = 0.268$, $h = 1 \times 10^{-3}$, $g = 1.535$

and $$p = 2.17 .$$

In equation (32), I_1 is small compared with the second term in the inner brackets, and Schuh suggests the assumption of a linear relation for $\frac{U_2}{U_1}$. Equation (33) then becomes

$$I_1 = 0.121\log_{10}\left(1 + \frac{1 - \lambda^{5\cdot 3}}{280\ \mathfrak{Z}_1 \mathcal{K}}\right) , \qquad \ldots(34)$$

where $$\lambda = \frac{U_2}{U_1}$$

and $$\mathcal{K} = \frac{1 - \lambda}{x_2 - x_1} .$$

As/

+To avoid confusion, several of Schuh's symbols have been altered.

As I_1 has little influence in equation (32), H is primarily a function of $\frac{U_2}{U_1}$, with only a second order dependence on Z_1 and $\mathcal{H}$. The similarity between this form of equation and that used in the present analysis is quite clear, since $(Z_1 \mathcal{H})^{-1}$ corresponds closely to the term $\beta\phi$ in equation (24), as shown below.

Schuh's Analysis	Present Analysis
$Z_1 = \theta_1 R_\theta^{0.268}$	$\beta = \frac{s}{\theta_1 R_{\theta_1}^{0.402}}$
$\mathcal{H} = \frac{1 - \frac{U_2}{U_1}}{s}$	$\phi = f(H_1, H_2,)$
$\frac{1}{Z_1 \mathcal{H}} = \frac{S}{\theta_1 R_{\theta_1}^{0.268}} \left(\frac{U_1}{U_1 - U_2} \right)$	$\beta\phi = \frac{S}{\theta_1 R_{\theta_1}^{0.402}} f(H_1, H_2,)$

The terms $\left(\frac{U_1}{U_1 - U_2} \right)$ and $f(H_1, H_2)$ may be considered equivalent, since both the present analysis and Schuh's suggest that H is primarily a function of $\underset{\sim}{U}$. The parameters $\frac{1}{Z_1 \mathcal{H}}$ and $\beta\phi$ are therefore of similar form and are also of comparable order of magnitude.

By assuming a linear relation for U_2/U_1, Schuh was able to plot U_2/U_1 as a function of H_1 for various values of H_2 and $\mathcal{H} Z_1$. The case where $\mathcal{H} Z_1 = \infty$ corresponds to the appropriate H_2 curves in Fig. 3 of the present report, which may be considered as a plot of U_2/U_1 against H_1 for $\beta\phi = 0$.

Schuh's curves for $H_2 = 2.0$ and ∞ are drawn as broken lines in Fig. 3. Agreement between the two curves for $H_2 = 2.0$ is fairly good. For $H_2 = \infty$, however, Schuh's curve predicts that this value is reached at higher values of U_2/U_1, and consequently further upstream. The difference is also illustrated in Figs. 5 and 7, where Schuh's values of H rise more rapidly towards the end of the calculation.

6. Conclusions

A method has been evolved for calculating the growth of the form parameter H in turbulent boundary layers with adverse pressure gradients. The auxiliary equation used has been derived by extending an analysis by Spence and introducing the findings of Ross and of Ludwieg and Tillmann. The equation is different in form from those used by previous investigators, although it has been shown to correspond closely to a simplified form of Schuh's auxiliary equation. The new method is easily and rapidly applied and results compare favourably with those obtained using other methods.

7. Acknowledgements

The author is indebted to Prof. W. A. Mair and Dr. M. R. Head for their assistance in the preparation of this report.

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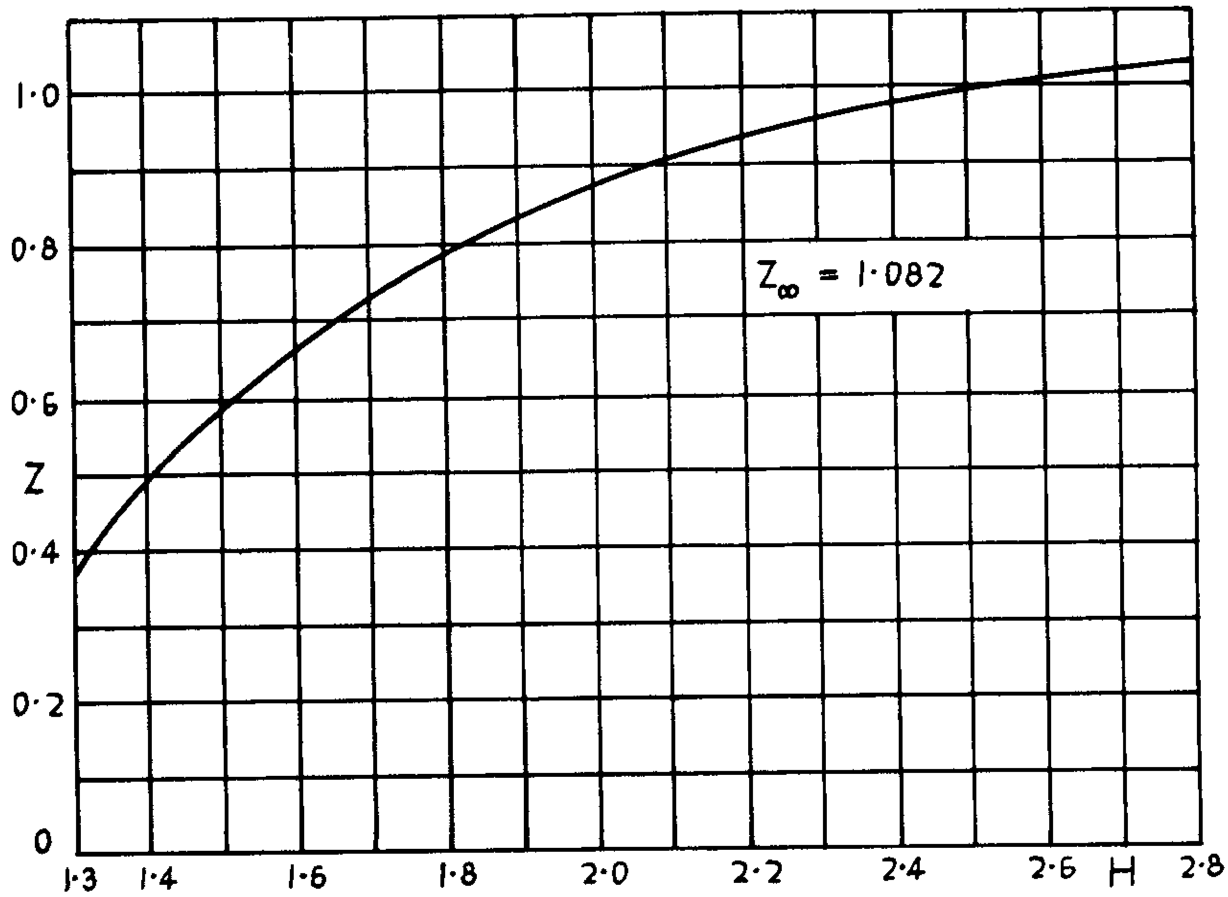
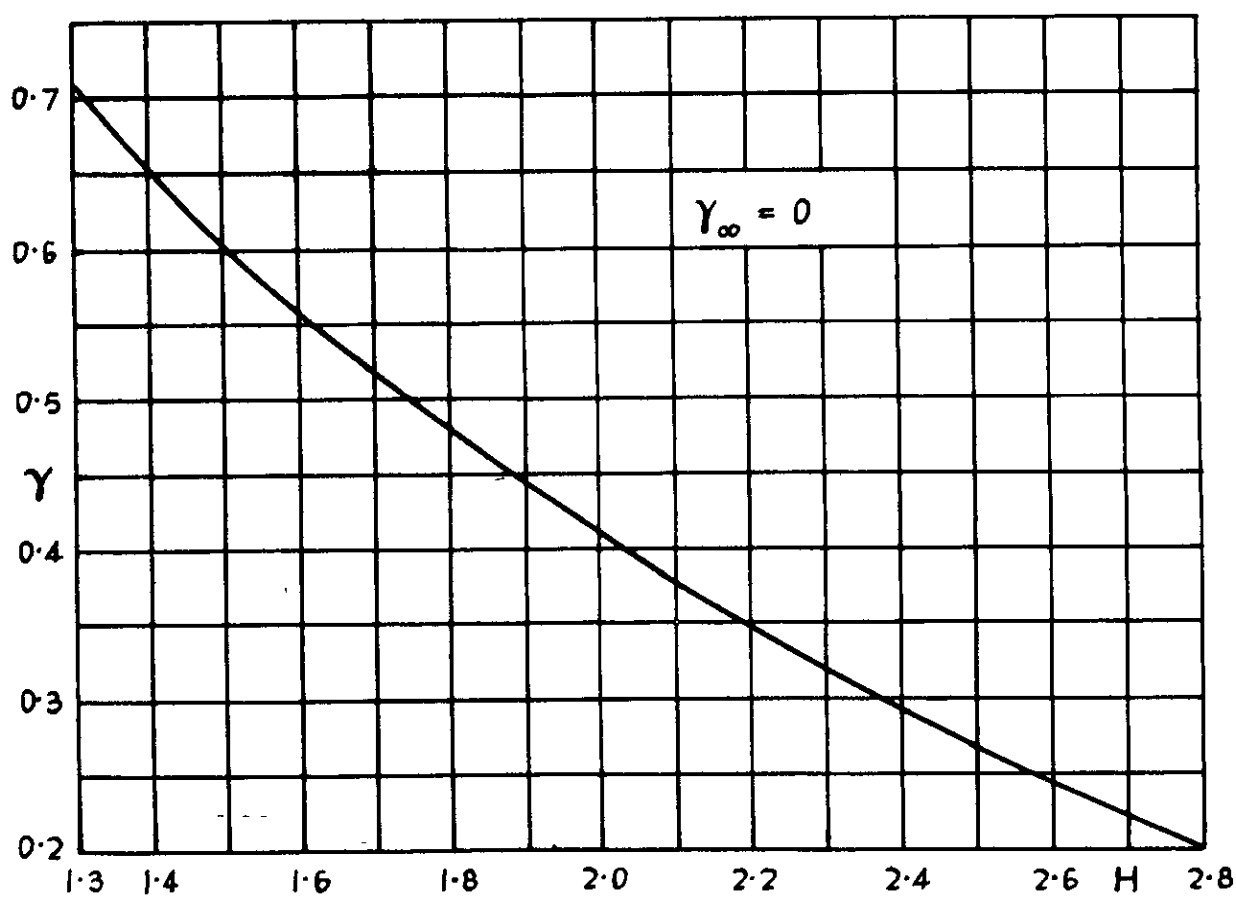
FIGS. 1 & 2.

FIG. 1



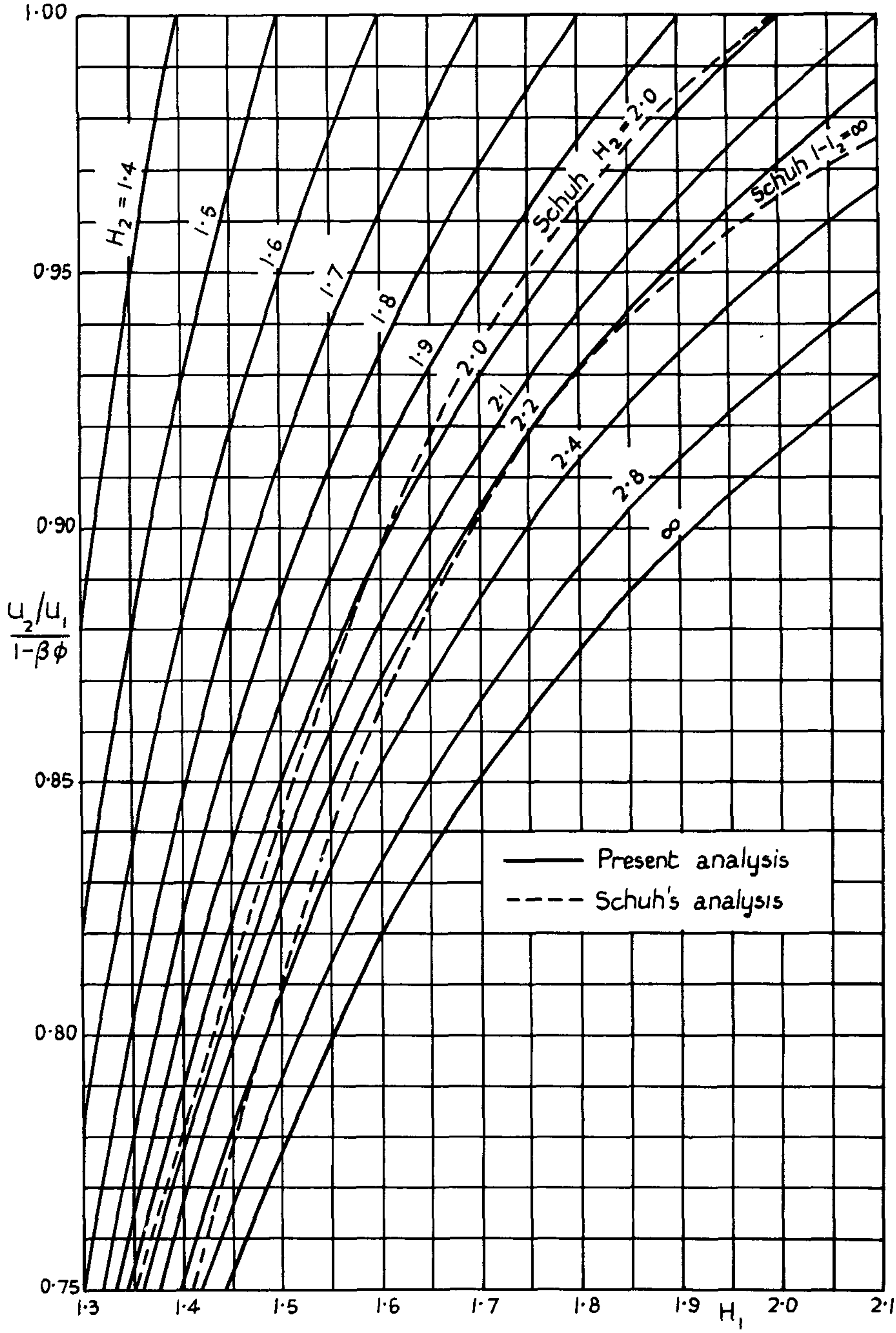
Z (eqn. 12) as a function of H.

FIG. 2.



γ (eqn 23) as a function of H.

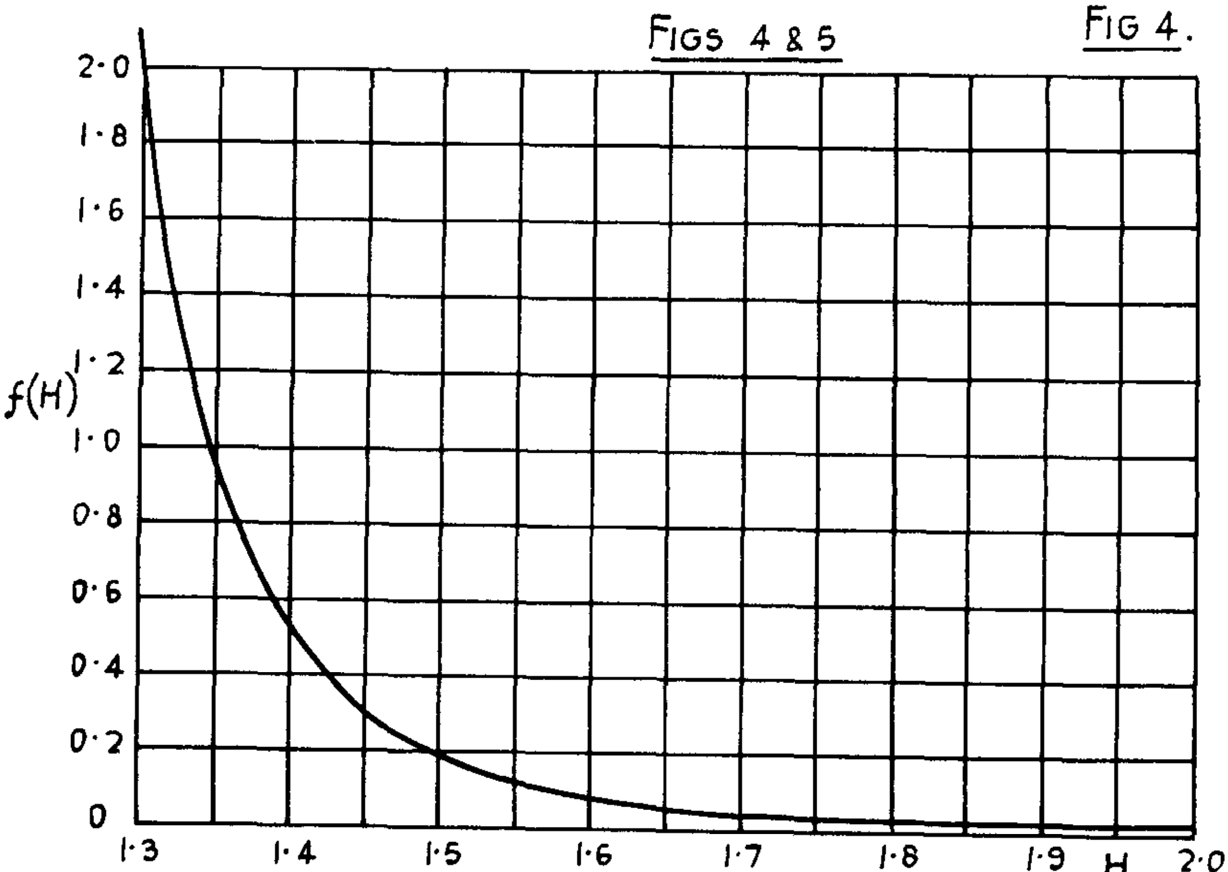
FIG. 3.



$\left(\frac{Z_1}{Z_2}\right)^{\frac{1}{2 \cdot 4}} \left[= \frac{u_2/u_1}{1-\beta\phi} \right]$ as a function of H_1 and H_2

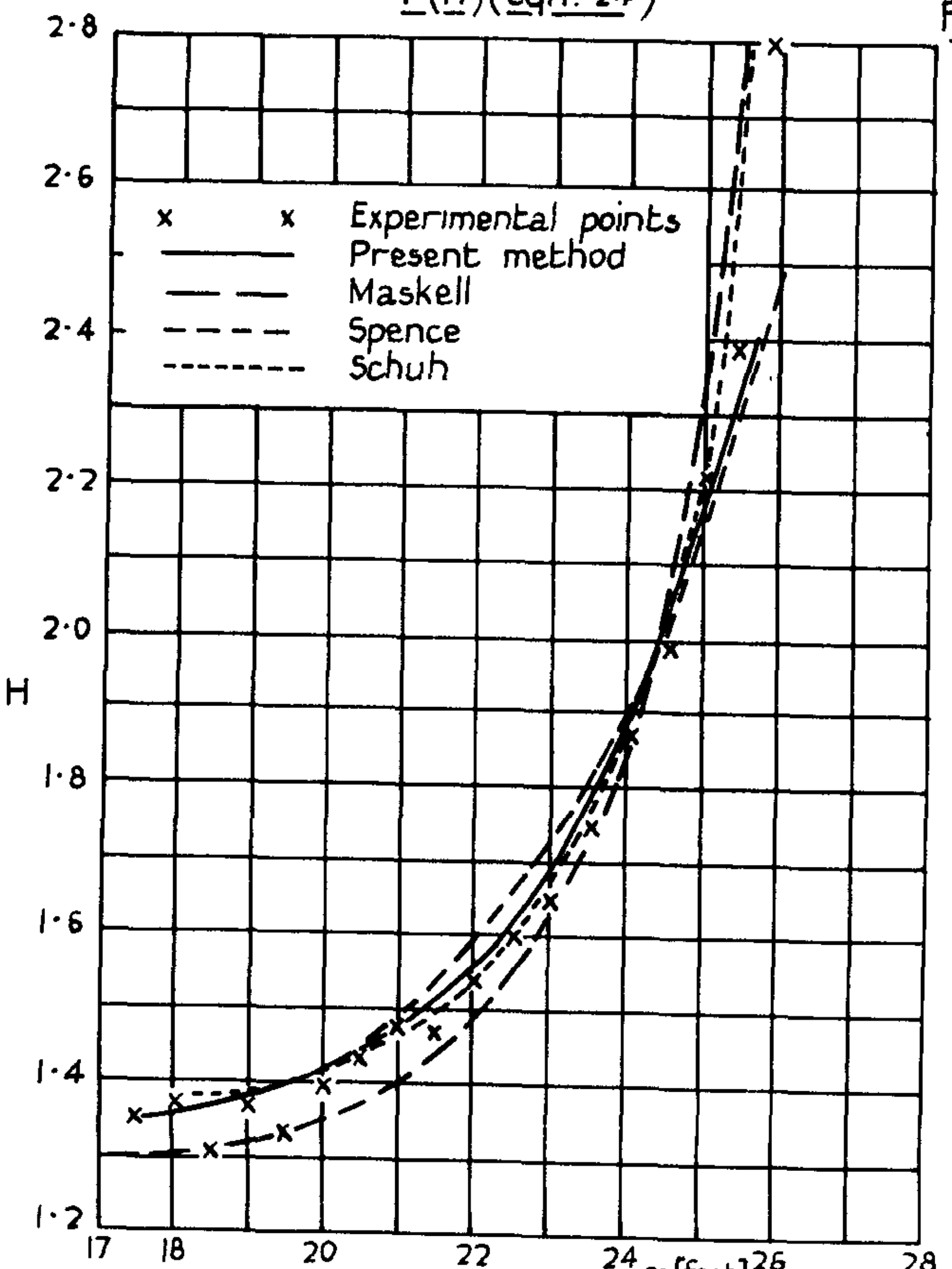
FIGS 4 & 5

FIG 4.



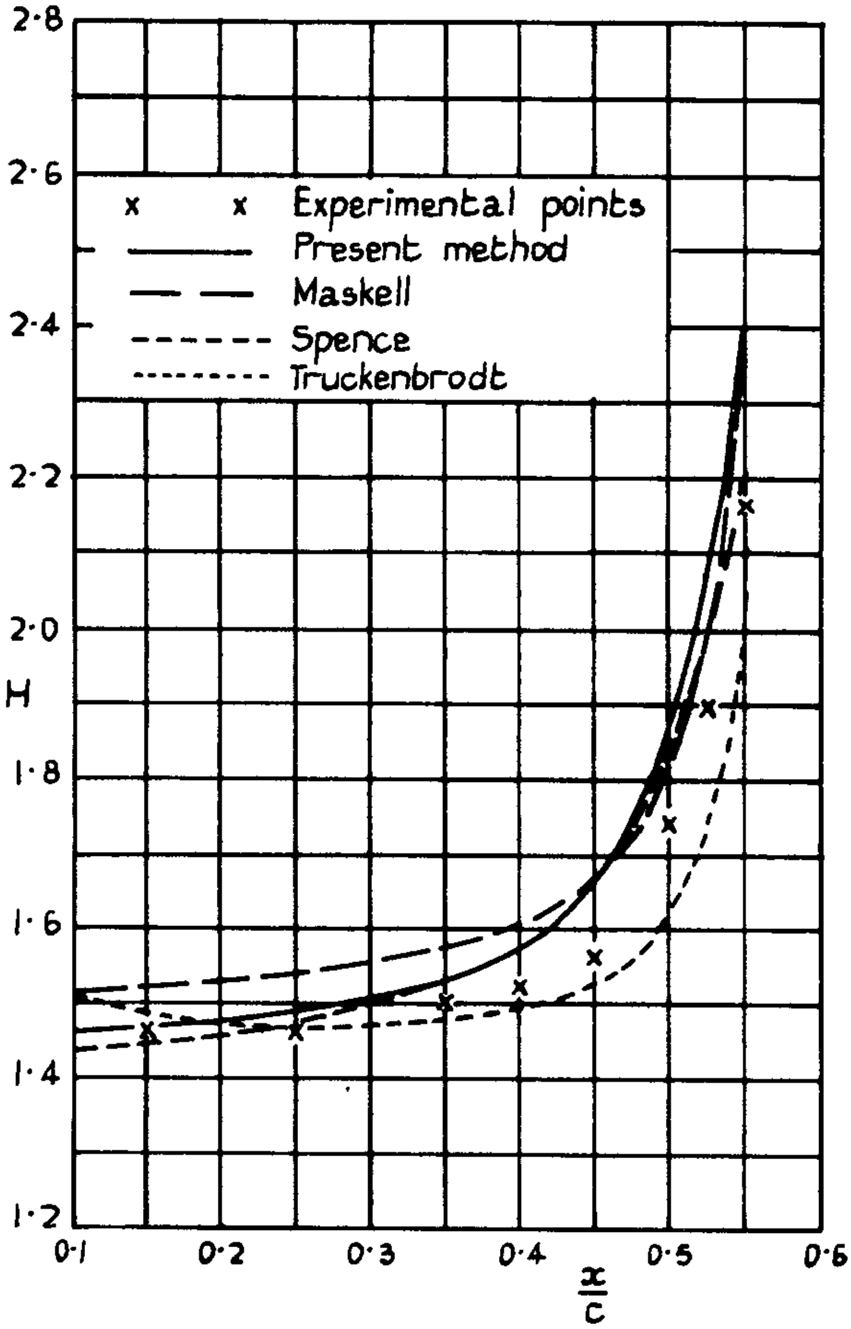
F(H)(eqn. 24)

FIG 5



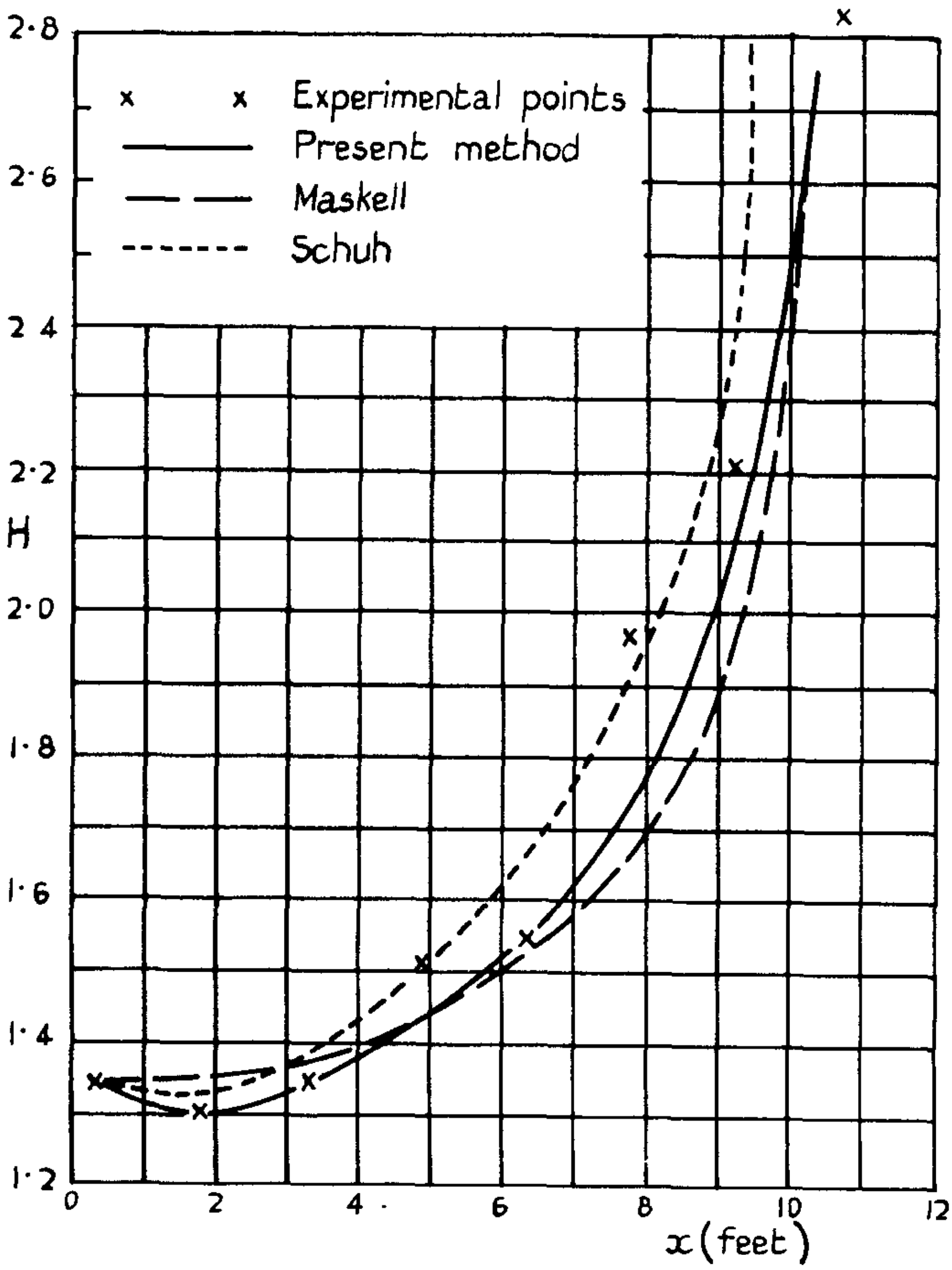
Comparison with measurements of Schubauer and Klebanoff [5]

FIG. 6.



Comparison with measurements made on N.A.C.A. 65 (216) 222 approx $R = 2{\cdot}64 \times 10^6$
$\alpha = 10{\cdot}1^\circ$

FIG. 7



Comparison with measurements made by Sandborn (8) with highest rate of suction.

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