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# The Buckling of Plates Tapered in Thickness

By G. G. POPE, M.Sc. (Eng.)

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*Summary.*

An analysis is given of the buckling of rectangular plates tapered in thickness under uniform end load in the direction of taper, with opposite pairs of edges either clamped or simply-supported.

## 1. *Introduction.*

Although tapered plates are used frequently in aircraft structures, their buckling behaviour appears to have received little attention. In this report an analysis is given of the buckling of rectangular plates tapered in thickness under uniform load in the direction of taper. A linear thickness variation only is considered, but the method used is equally applicable to other thickness variations in which the flexural rigidity can be expressed as a polynomial in the distance along the plate. Results are given graphically for plates in which opposite pairs of edges are either clamped or simply-supported; transverse displacement of the sides is either free or completely prevented.

The analysis is based on the assumption that the buckled shape normal to the direction of taper differs little from the buckled shape across a rectangular plate of constant thickness under end load, with the same boundary conditions along the edges parallel to the loading, but simply-supported at the ends. Assuming this transverse buckled form, a linear differential equation with variable coefficients is obtained for the deflected shape along the plate, using an energy method. A series solution is derived to this equation.

## 2. *Assumptions.*

(1) The plate is perfectly elastic.

(2) The thickness variation is sufficiently gradual for a state of generalised plane stress to be assumed.

(3) The buckling load is not affected significantly by the violation of compatibility by the assumed system of middle-surface forces when transverse displacement is allowed of the sides of the plate. (*See* Section 4.)

(4) The transverse buckled shape is the same as that across a rectangular plate of constant thickness under uniform end load with the same boundary conditions along the sides, but simply-supported at the ends.

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3. *General Analysis.*

The origin of the $(x, y)$ axes is at the centre of a plate of length $a$ and width $b$, as shown in Fig. 1. If the plate, which is tapered in the $x$ direction only, is subjected to a system of middle-surface forces $N_x$, $N_y$, the deflected shape can be expressed approximately (and sometimes exactly) as

$$W = \frac{w}{a} = f(X)\Phi(Y)$$

where

$$X = \frac{x}{a}, \quad Y = \frac{y}{b}$$

and the function $\Phi(Y)$ describes the assumed transverse buckled form. It is shown in the Appendix that if the middle-surface forces are constant over the plate (*see* Section 4), then the governing differential equation for the function $f(X)$ when the plate buckles under this loading is

$$\frac{D}{D_2}(k_0 f'''' + 2k_2 f'' + k_4 f) + \frac{2D'}{D_2}(k_0 f''' + k_2 f') + \\ + \frac{D''}{D_2}(k_0 f'' + \nu k_2 f) - 12\mu^2(1-\nu^2)(k_0 \bar{\sigma}_{x\,2} f'' + k_2 \bar{\sigma}_{y\,2} f) = 0 \tag{1}$$

where a dash denotes differentiation with respect to $X$,

$$\bar{\sigma}_{x\,2} = \frac{a^2 N_x}{12(1-\nu^2)\mu^2 D_2} = \frac{\sigma_{x\,2}}{E}\left(\frac{b}{t_2}\right)^2, \quad \bar{\sigma}_{y\,2} = \frac{\sigma_{y\,2}}{E}\left(\frac{b}{t_2}\right)^2,$$

$$\mu = \frac{a}{b}$$

and the suffix 2 denotes values at $X = \frac{1}{2}$. The coefficients $k_i$ are given by

$$k_i = \mu^i \int_{-1/2}^{+1/2} \Phi \frac{d^i\Phi}{dY^i}\, dY, \quad k_0 = \int_{-1/2}^{+1/2} \Phi^2 dY. \tag{2}$$

If the thickness of the plate varies linearly from $t_1$ at $X = -\frac{1}{2}$ to $t_2$ at $X = +\frac{1}{2}$, the flexural rigidity of the plate is given by

$$\left.\begin{aligned} \frac{D}{D_2} &= m_0 + m_1 X + m_2 X^2 + m_3 X^3 \\ \text{where} \quad m_0 &= 1 + 1{\cdot}5\chi + 0{\cdot}75\chi^2 + 0{\cdot}125\chi^3, \\ m_1 &= -\chi(3 + 3\chi + 0{\cdot}75\chi^2), \\ m_2 &= \chi^2(3 + 1{\cdot}5\chi), \\ m_3 &= -\chi^3 \end{aligned}\right\} \tag{3}$$

and

$$\chi = \frac{t_1}{t_2} - 1.$$

Substituting in equation (1), the following differential equation is obtained for the function $f$:

$$(m_0+m_1X+m_2X^2+m_3X^3)(k_0f''''+2k_2f''+k_4f)+$$
$$+2(m_1+2m_2X+3m_3X^2)(k_0f'''+k_2f')+2(m_2+3m_3X)(k_0f''+\nu k_2f)-$$
$$-12(1-\nu^2)(k_0\bar{\sigma}_{x\,2}f''+k_2\bar{\sigma}_{y\,2}f) = 0. \tag{4}$$

Equation (4) may be solved by substituting

$$f = X^c\sum_{r=0}^{\infty} a_rX^r$$

and equating the coefficients of powers of $X$ to zero. The index $c$ is obtained by equating the coefficient of $X^{c-4}$ to zero, giving the indicial equation

$$c(c-1)(c-2)(c-3) = 0.$$

As equation (4) is linear, the required complete solution is thus given by

$$f = \sum_{r=0}^{\infty} a_rX^r \tag{5}$$

where the coefficients $a_0$, $a_1$, $a_2$ and $a_3$ are arbitrary. In general a coefficient $a_{r+4}$ is obtained by equating the coefficient of $X^r$ to zero, giving

$$k_4m_3a_{r-3}+k_4m_2a_{r-2}+\{k_4m_1+2k_2m_3(r^2-1+3\nu)\}a_{r-1}+$$
$$+\{k_4m_0+2k_2m_2(r^2+r+\nu)-12(1-\nu^2)k_2\bar{\sigma}_{y\,2}\}a_r+$$
$$+(r+1)^2\{2k_2m_1+k_0m_3r(r+2)\}a_{r+1}+$$
$$+(r+1)(r+2)\{2k_2m_0+k_0m_2(r+2)^2-12(1-\nu^2)k_0\bar{\sigma}_{x\,2}\}a_{r+2}+$$
$$+(r+3)(r+2)^3(r+1)k_0m_1a_{r+3}+(r+4)(r+3)(r+2)(r+1)k_0m_0a_{r+4} = 0. \tag{6}$$

Coefficients with negative suffices which occur in this equation when $r < 3$ are, by definition, zero.

If the stress coefficient $\bar{\sigma}_{y\,2}$ is assumed to be proportional to $\bar{\sigma}_{x\,2}$, the latter can be used as the buckling coefficient. This is evaluated using a digital computer. Assuming first a value of $\bar{\sigma}_{x\,2}$ known to be numerically less than the correct solution, the coefficients of the series are calculated in terms of the arbitrary coefficients $a_0$, $a_1$, $a_2$ and $a_3$ using equation (6). Four linear simultaneous equations are obtained for these coefficients from the boundary conditions along the edges $X = \pm\frac{1}{2}$. The buckling condition is satisfied only if the determinant of the coefficients of these equations is zero. This determinant is evaluated for the assumed value of $\bar{\sigma}_{x\,2}$, which is then adjusted until the determinant changes sign. Subsequent approximations to $\bar{\sigma}_{x\,2}$ are made by successively interpolating and re-evaluating the determinant until the required accuracy is reached.

4. *Middle-Surface Forces in Plate.*

Mansfield[3] has shown that the middle-surface forces in a plate of variable thickness satisfy the equation

$$\nabla^2\left(\frac{1}{2}\nabla^2\Psi\right)-(1+\nu)\Diamond^4\left(\frac{1}{t},\Psi\right) = 0 \tag{7}$$

where

$$\Diamond^4(\alpha,\beta) = \frac{\partial^2\alpha}{\partial x^2}\frac{\partial^2\beta}{\partial y^2}-2\frac{\partial^2\alpha}{\partial x\,\partial y}\frac{\partial^2\beta}{\partial x\,\partial y}+\frac{\partial^2\alpha}{\partial y^2}\frac{\partial^2\beta}{\partial x^2} \tag{8}$$

($\Phi$ is pronounced 'die') and $\Psi$ is a force function such that

$$N_x = \frac{\partial^2 \Psi}{\partial y^2} \text{ etc.}$$

In the present example $N_x$ and $N_y$ are assumed constant over the plate and $N_{xy}$ is zero, so that

$$\Psi = \frac{N_x}{2}(y^2 + \lambda x^2) \tag{9}$$

where

$$\lambda = \frac{N_y}{N_x}.$$

Now $t$ varies in the $x$ direction only, so that equation (7) becomes

$$N_x(1+\lambda)\frac{\partial^2}{\partial x^2}\left(\frac{1}{t}\right) - N_x(1+\nu)\frac{\partial^2}{\partial x^2}\left(\frac{1}{t}\right) = 0,$$

from which it is seen that, except for the special case when $(\partial^2/\partial x^2)(1/t) = 0$ (i.e., $1/t$ is a linear function of $x$), a compatible stress system is only obtained if

$$\lambda = \nu.$$

This is the solution when transverse displacement of the sides of the plate is completely prevented. Now, in the examples that follow the stress system (9) is also used when the plate, which is loaded in the $x$ direction, is free to deform in the $y$ direction (i.e., $\lambda = 0$). The buckling load should not be seriously affected by this approximation.

## 5. *Applications.*

The analysis of the preceding sections is now applied to the buckling of a plate under uniform load in the direction of taper (parallel to $OX$) with opposite pairs of edges either clamped or simply-supported; transverse displacement of the sides $Y = \pm \frac{1}{2}$ is either unrestrained or completely prevented. Expressions are given in Sections 5.1 and 5.2 for the constants required in equation (1) under these conditions and the results plotted are listed in Section 5.3.

### 5.1. *Plate Simply-Supported along Sides.*

The assumed transverse buckled shape is here given by

$$\Phi = \cos \pi Y$$

so that

$$k_0 = -\frac{k_2}{\mu^2 \pi^2} = \frac{k_4}{\mu^4 \pi^4}.$$

In this particular case the assumed transverse deflected shape satisfies the buckling differential equation. Furthermore, if transverse movement of the edges $Y = \pm \frac{1}{2}$ in the plane of the plate is completely prevented, the assumed middle-surface force distribution is also rigorously correct and this method gives an exact (series) solution to a physically admissible problem.

5.2. *Plate Clamped along Sides.*

The assumed transverse buckled shape is here given by

$$\Phi = \cosh pY - q\cos pY$$

where $p$ is the first positive root ($4\cdot73004$) of the equation

$$\sinh\frac{p}{2}\cos\frac{p}{2} + \cosh\frac{p}{2}\sin\frac{p}{2} = 0 \tag{10}$$

and

$$\left.\begin{aligned} q &= \frac{\cosh\frac{p}{2}}{\cos\frac{p}{2}} \\ &= (\sec p)^{1/2}. \end{aligned}\right\} \tag{11}$$

Hence

$$k_0 = \tfrac{1}{2}(q^2+1),$$

$$\frac{k_2}{p^2\mu^2} = -\tfrac{1}{2}(q^2-1) - \frac{1}{p}(q^4-1)^{1/2},$$

and

$$\frac{k_4}{p^4\mu^4} = k_0.$$

5.3. *Results.*

The variation of the buckling coefficient $\bar{\sigma}_{x2}$ with $a/b$ has been plotted for a series of values of $t_1/t_2$ with various combinations of boundary conditions, as listed in the following table.

| Ends $X = \pm\frac{1}{2}$ | Sides $Y = \pm\frac{1}{2}$ | Transverse displacement of sides $Y = \pm\frac{1}{2}$ | Fig. |
|---|---|---|---|
| Simply-supported | Simply-supported | Free | 2 |
| Clamped | Simply-supported | Free | 3 |
| Simply-supported | Clamped | Free | 4 |
| Clamped | Clamped | Free | 5 |
| Simply-supported | Simply-supported | Completely prevented | 6 |
| Clamped | Simply-supported | Completely prevented | 7 |
| Simply-supported | Clamped | Completely prevented | 8 |
| Clamped | Clamped | Completely prevented | 9 |

Specimen buckled shapes are shown in Figs. 10 and 11.

# NOTATION

Suffices 1 and 2 on stress, middle-surface force and length symbols indicate values at $x = -a/2$ and $x = a/2$ respectively.

| | | |
|---|---|---|
| $a$, $b$ | | Length and width of plate |
| $t$ | | Plate thickness |
| $x$, $y$ | | Cartesian co-ordinates, $x$ lies along the plate |
| $w$ | | Deflection |
| $X$, $Y$ | = | $\dfrac{x}{a}, \dfrac{y}{b}$ |
| $W$ | = | $\dfrac{w}{a}$ |
| $\mu$ | = | $\dfrac{a}{b}$ |
| $\chi$ | = | $\dfrac{t_1}{t_2} - 1$ |
| $\nu$ | | Poisson's ratio (taken as $0 \cdot 3$ for computational purposes) |
| $E$ | | Young's modulus |
| $D$ | | Flexural rigidity $= Et^3/12(1-\nu^2)$ |
| $N_x$, $N_y$ | | Middle-surface forces |
| $\sigma_x$, $\sigma_y$ | | Middle-surface stresses |
| $\bar{\sigma}_x$, $\bar{\sigma}_y$ | = | $\dfrac{\sigma_x}{E}\left(\dfrac{b}{t_2}\right)^2, \quad \dfrac{\sigma_y}{E}\left(\dfrac{b}{t_2}\right)^2$ |
| $\Psi$ | | Middle-surface force function such that $N_x = \dfrac{\partial^2 \Psi}{\partial y^2}$ etc. |
| $\Phi$ | | Assumed transverse buckled shape |
| $\nabla^2$ | | Laplacian differential operator |
| $\Diamond^4$ | | Differential operator defined by equation (8) |
| $f$ | | Function of $X$ |
| $\lambda$ | = | $\dfrac{N_y}{N_x}$ |
| $p$ | | Coefficient defined by equation (10) |
| $q$ | | Coefficient defined by equation (11) |
| $a_i$ | | Coefficients defined by equation (5) |
| $k_i$ | | Coefficients defined by equations (2) |
| $m_i$ | | Coefficients defined by equations (3) |
| $T$ | | Work done by middle-surface forces |
| $U$ | | Strain energy |

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## APPENDIX

*Derivation of Differential Equation*

The basic method used here, which is due to Ritz, has been applied extensively by Kantorovich. In classical small-deflection theory[2], the strain energy of bending of a plate is given by

$$U = \iint \frac{D}{2}\{(\nabla^2 w)^2 - (1-\nu)\Diamond^4(w, w)\}\, dx\, dy \tag{12}$$

where the operator $\Diamond^4$ is defined by equation (8). The work done on the plate by the middle-surface forces $N_x$ and $N_y$, which are assumed constant (*see* Section 4), is given by

$$T = -\tfrac{1}{2}\iint \left\{N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2\right\} dx\, dy. \tag{13}$$

If the deflection of the plate can be expressed as

$$W = f(X)\Phi(Y)$$

where $W = w/a$, $X = x/a$, $Y = y/b$, expressions (12) and (13) may be rewritten as

$$U = \frac{1}{2\mu}\iint D\{(f''\Phi + \mu^2 f\Phi'')^2 - 2\mu^2(1-\nu)(ff''\Phi\Phi'' + f'^2\Phi'^2)\}\, dX\, dY, \tag{14}$$

$$T = -\frac{a^2}{2\mu}\iint (N_x f'^2\Phi^2 + \mu^2 N_y f^2\Phi'^2) dX\, dY \tag{15}$$

where

$$\mu = \frac{a}{b} \text{ and } f' = \frac{df}{dX},\ \Phi' = \frac{d\Phi}{dY} \text{ etc.}$$

To obtain an approximate solution, a known function $\Phi$ is assumed for the transverse deflected shape. The corresponding differential equation for $f$ is found by considering an infinitesimal virtual variation $\delta f$ which satisfies the boundary conditions of the plate. The resulting increments of the strain energy of and the work done on the plate are

$$\delta U = \frac{1}{\mu}\iint D\{(f''\Phi + \mu^2 f\Phi'')(\delta f''\Phi + \mu^2\delta f\Phi'') -$$

$$- (1-\nu)\mu^2[(f\delta f'' + \delta f f'')\Phi\Phi'' + 2f'\delta f'\Phi'^2]\}\, dX\, dY, \tag{16}$$

$$\delta T = -\frac{a^2}{\mu}\iint (N_x f'\delta f'\Phi^2 + \mu^2 N_y f\delta f\Phi'^2) dX\, dY. \tag{17}$$

Now, integrating by parts, it is seen that

$$\left.\begin{aligned} &\int_{-1/2}^{+1/2} \Phi''^2 dY = \Big[\Phi'\Phi''\Big]_{-1/2}^{+1/2} - \Big[\Phi\Phi'''\Big]_{-1/2}^{+1/2} + \int_{-1/2}^{+1/2} \Phi\Phi''''dY \\ \text{and}\quad &\int_{-1/2}^{+1/2} \Phi'^2 dY = \Big[\Phi\Phi'\Big]_{-1/2}^{+1/2} - \int_{-1/2}^{+1/2} \Phi\Phi'' dY. \end{aligned}\right\} \tag{18}$$

The terms in the square brackets in expressions (18) vanish, because the functions $\Phi$ are chosen to satisfy the boundary conditions along the sides of the plate, which are here either simply-supported or clamped. Thus,

$$\int_{-1/2}^{+1/2} \Phi''^2 dY = \int_{-1/2}^{+1/2} \Phi\Phi'''' dY \text{ and } \int_{-1/2}^{+1/2} \Phi'^2 dY = -\int_{-1/2}^{+1/2} \Phi\Phi'' dY.$$

Hence, if $D$ is a function of $X$ only, the increments of strain energy and work done due to the virtual variation $\delta f$ can be integrated by parts with respect to $Y$, giving

$$\delta U = \frac{1}{\mu}\int_{-1/2}^{+1/2} D\{\delta f''(k_0 f'' + \nu k_2 f) + 2(1-\nu)k_2 f'\delta f' + \\ + \delta f(k_4 f + \nu k_2 f'')\}\, dX \tag{19}$$

and

$$\delta T = -\frac{a^2}{\mu}\int_{-1/2}^{+1/2} (N_x k_0 f'\delta f' - N_y k_2 f\delta f) dX \tag{20}$$

where

$$k_i = \mu^i \int_{-1/2}^{+1/2} \Phi \frac{d^i\Phi}{dY^i} dY \text{ and } k_0 = \int_{-1/2}^{+1/2} \Phi^2 dY.$$

If expressions (19) and (20) are now integrated by parts with respect to $X$, the following expressions are obtained.

$$\mu\delta U = \Big[D(k_0 f'' + \nu k_2 f)\Big]_{X=-1/2}^{X=+1/2} \delta f' - \Big\{D[k_0 f''' + (2-\nu)k_2 f'] + \\ + D'(k_0 f'' + \nu k_2 f)\Big\}_{X=-1/2}^{X=+1/2} \delta f + \\ + \delta f \int_{-1/2}^{+1/2} \{D(k_0 f'''' + 2k_2 f'' + k_4 f) + 2D'(k_0 f''' + k_2 f') + \\ + D''(k_0 f'' + \nu k_2 f)\} dX, \tag{21}$$

$$\mu\delta T = -a^2\Big[N_x k_0 f' + 2N_{xy} k_1 f\Big]_{X=-1/2}^{X=+1/2} \delta f + a^2 \delta f \int_{-1/2}^{+1/2} (N_x k_0 f'' + N_y k_2 f) dX. \tag{22}$$

Now if the edges at $X = \pm\frac{1}{2}$ are simply-supported

$$[\delta f]_{X=\pm 1/2} = [f]_{X=\pm 1/2} = [f'']_{X=\pm 1/2} = 0,$$

and if they are clamped

$$[\delta f]_{X=\pm 1/2} = [\delta f']_{X=\pm 1/2} = 0.$$

Hence the unevaluated integrals are the only terms in expressions (21) and (22) which do not vanish.

By the principle of virtual displacements

$$\delta U = \delta T$$

and, because the variation of $\delta f$ is arbitrary everywhere except along the edges of the plate, the term under the integral sign in expressions (21) and (22) may be equated, giving the following differential equation for $f$.

$$D(k_0 f'''' + 2k_2 f'' + k_4 f) + 2D'(k_0 f''' + k_2 f') + \\ + D''(k_0 f'' + \nu k_2 f) - a^2(N_x k_0 f'' + N_y k_2 f) = 0.$$

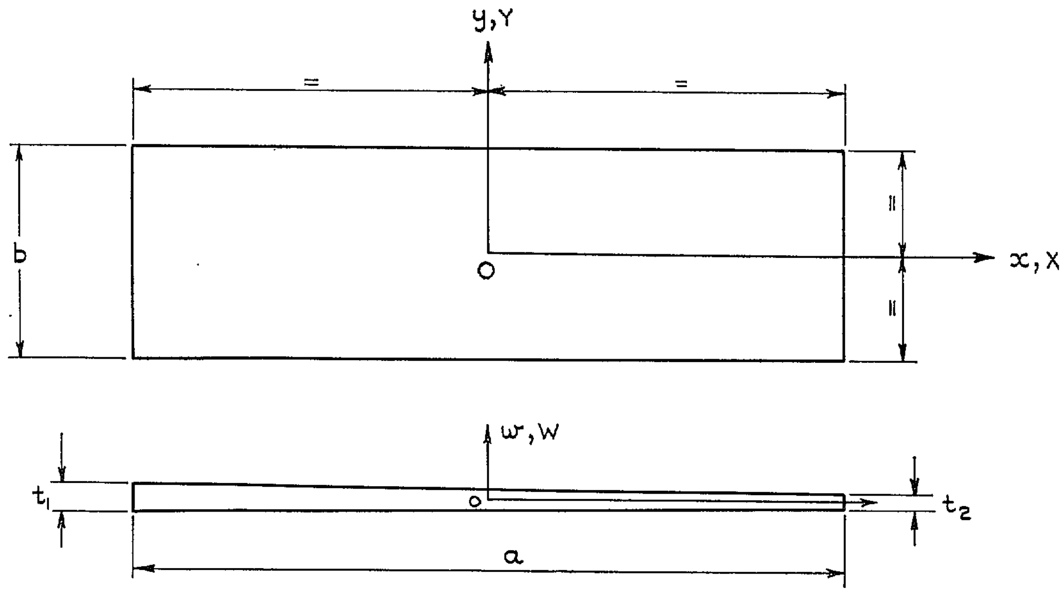


FIG. 1. Axes and notation.

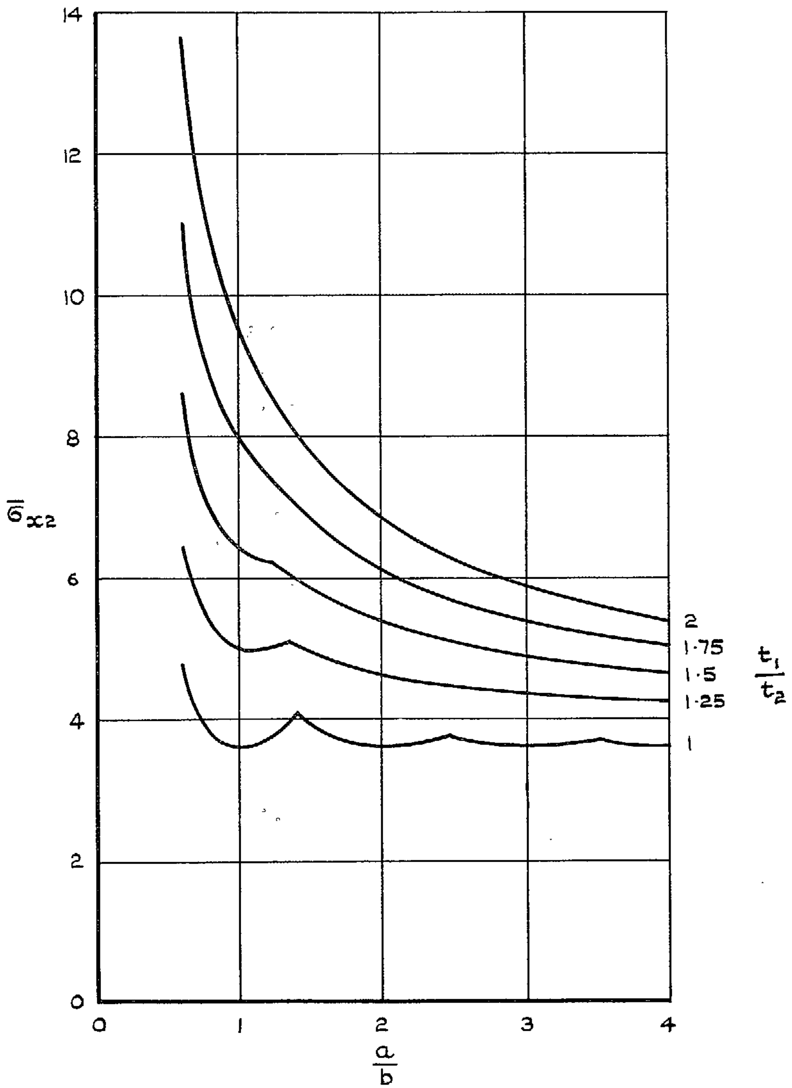


FIG. 2. Buckling stress diagram. Sides and ends simply-supported. Free transverse displacement of sides.

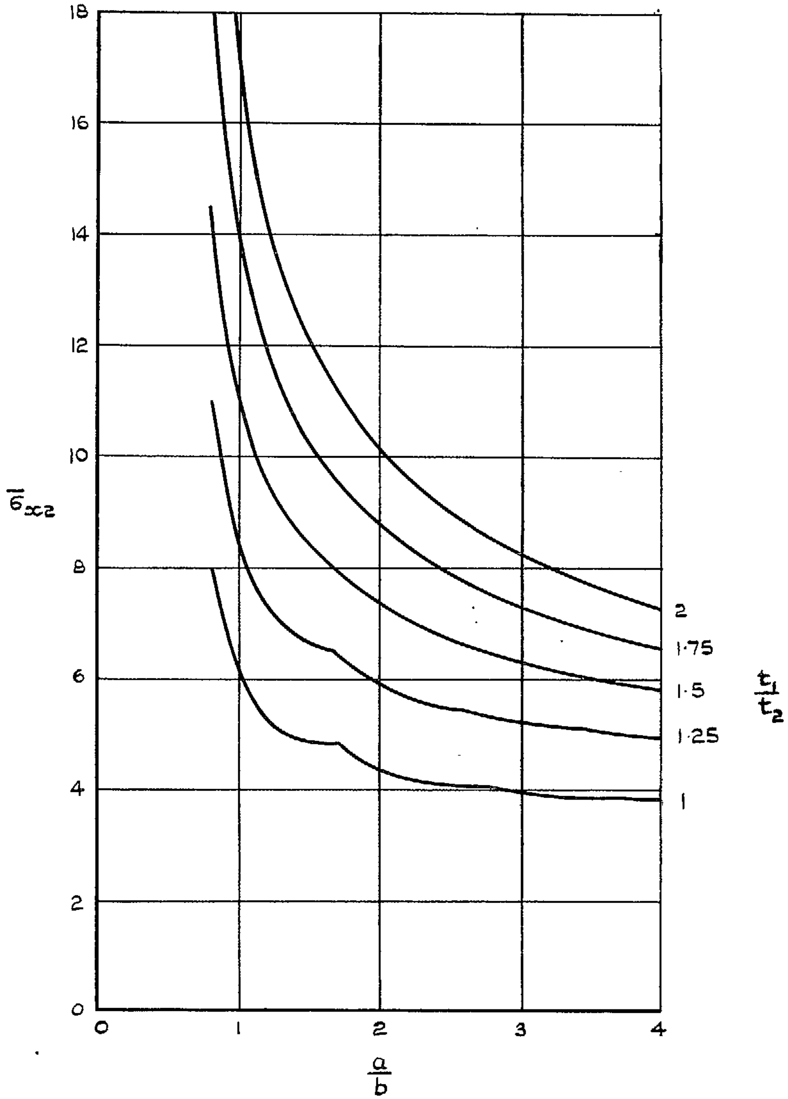


FIG. 3. Buckling stress diagram. Sides simply-supported, ends clamped. Free transverse displacement of sides.

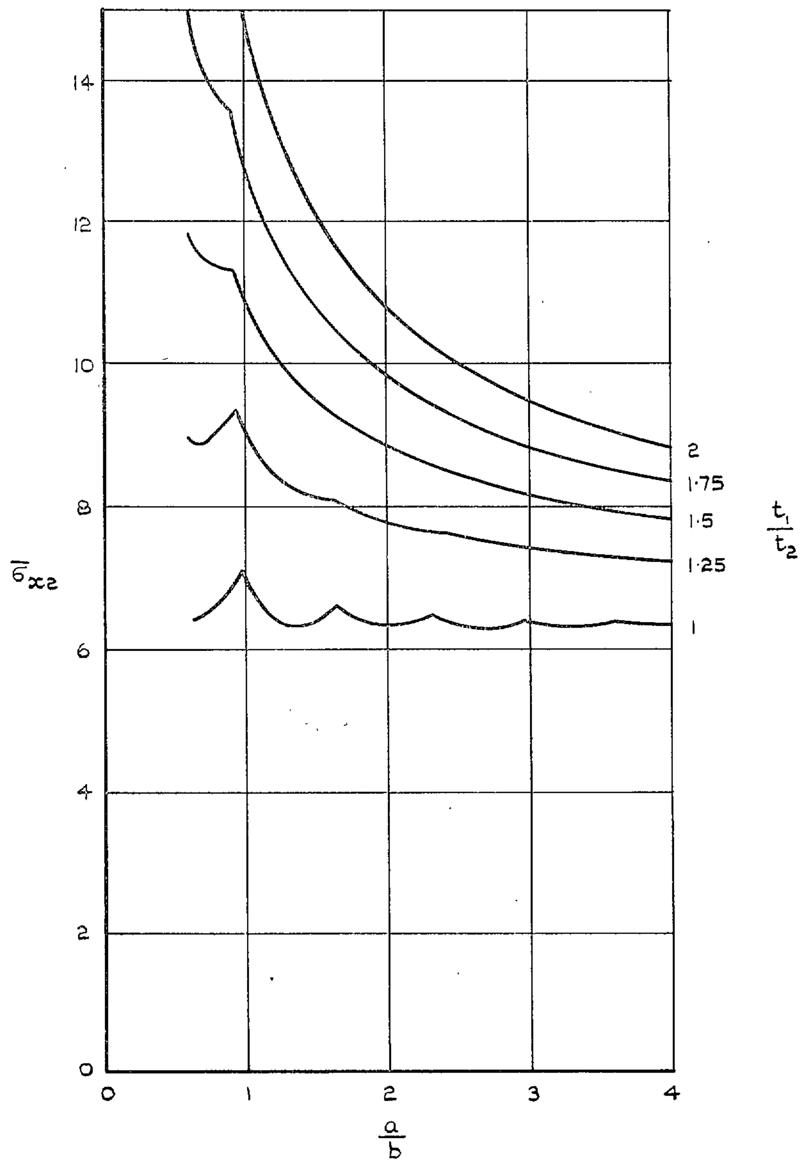


FIG. 4. Buckling stress diagram. Sides clamped, ends simply-supported. Free transverse displacement of sides.

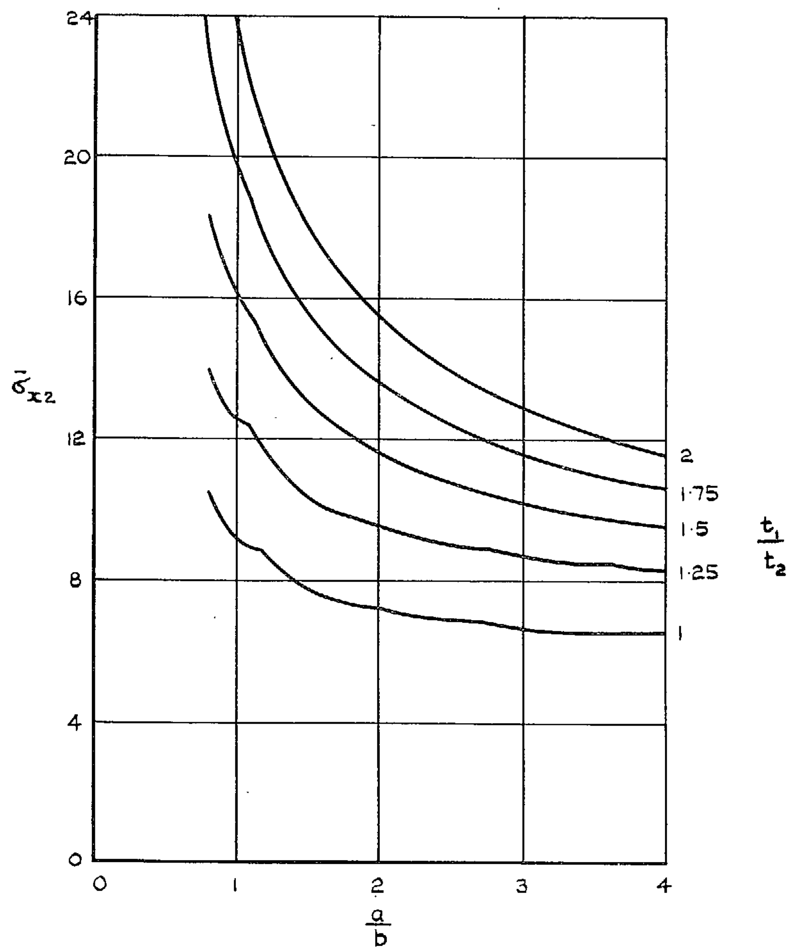


FIG. 5. Buckling stress diagram. Sides and ends clamped. Free transverse displacement of sides.

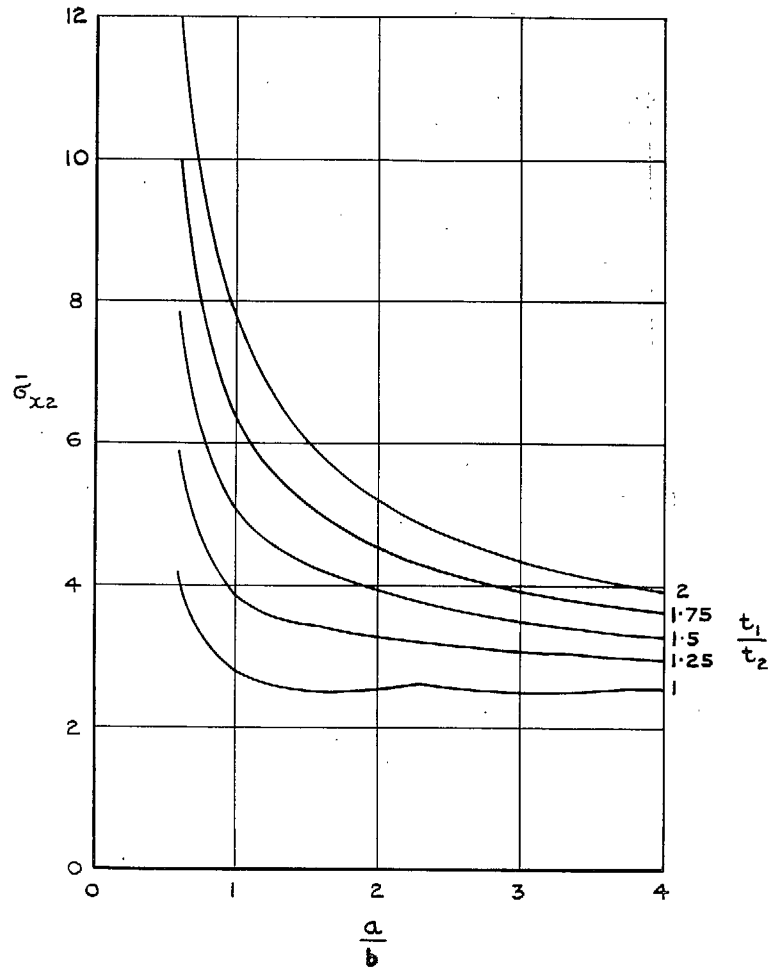


FIG. 6. Buckling stress diagram. Ends and sides simply-supported. No transverse displacement of sides.

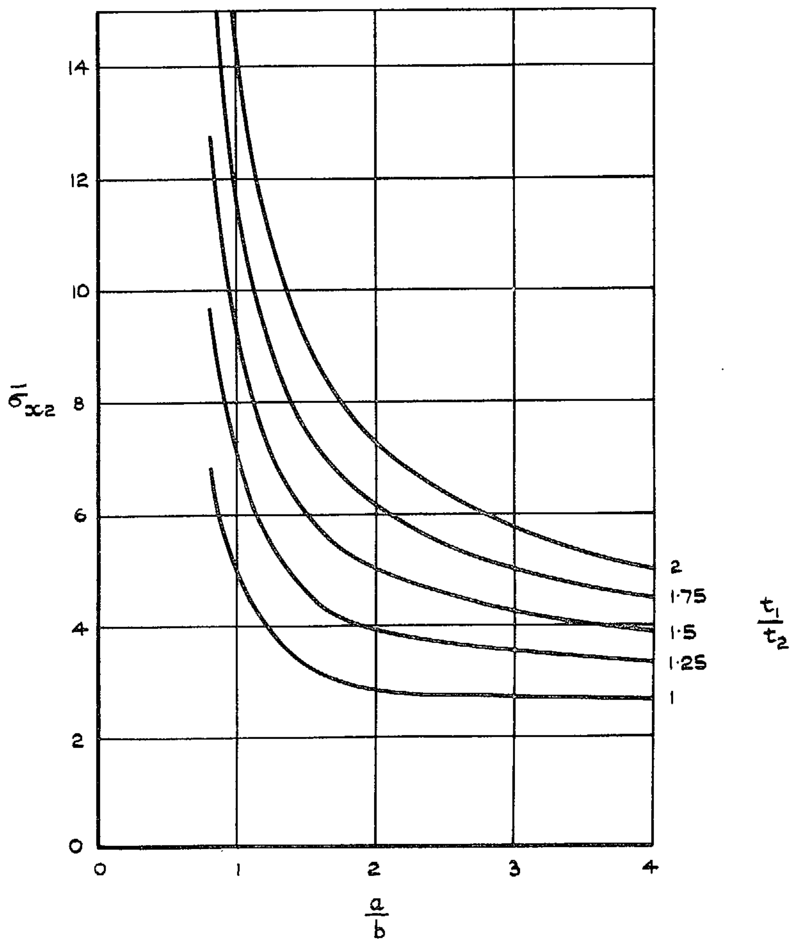


FIG. 7. Buckling stress diagram. Ends clamped, sides simply-supported. No transverse displacement of sides.

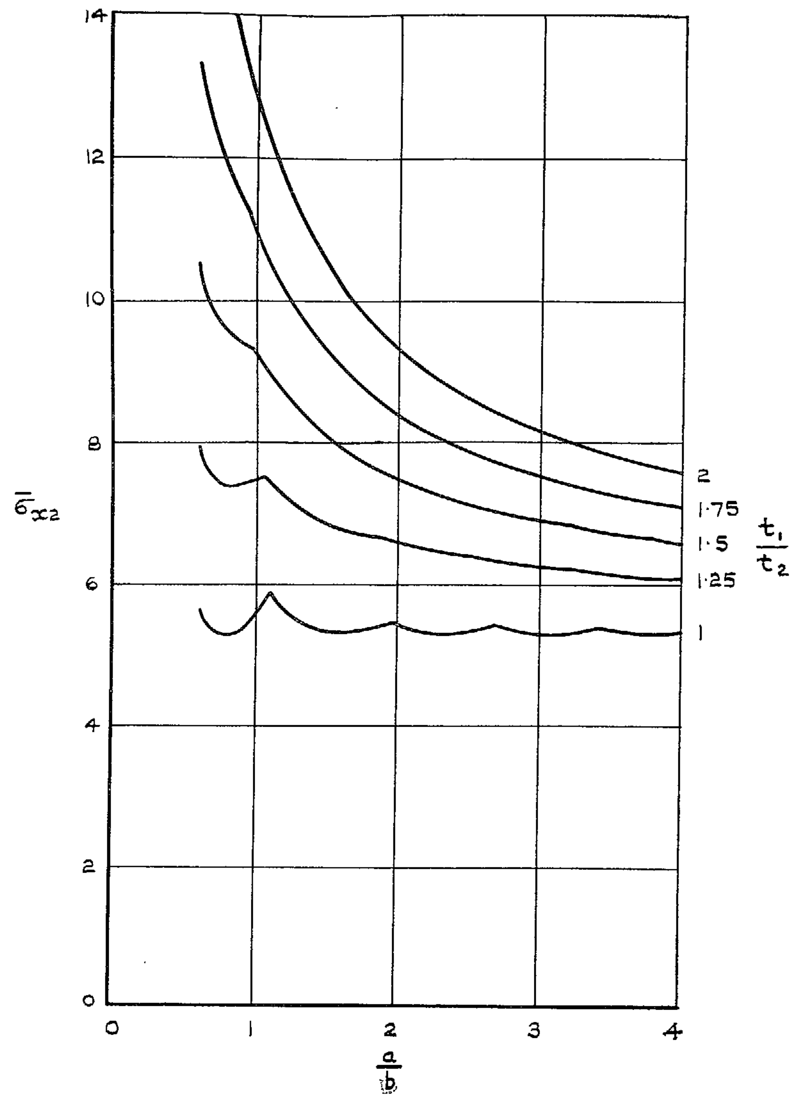


FIG. 8. Buckling stress diagram. Sides clamped, ends simply-supported. No transverse displacement of sides.

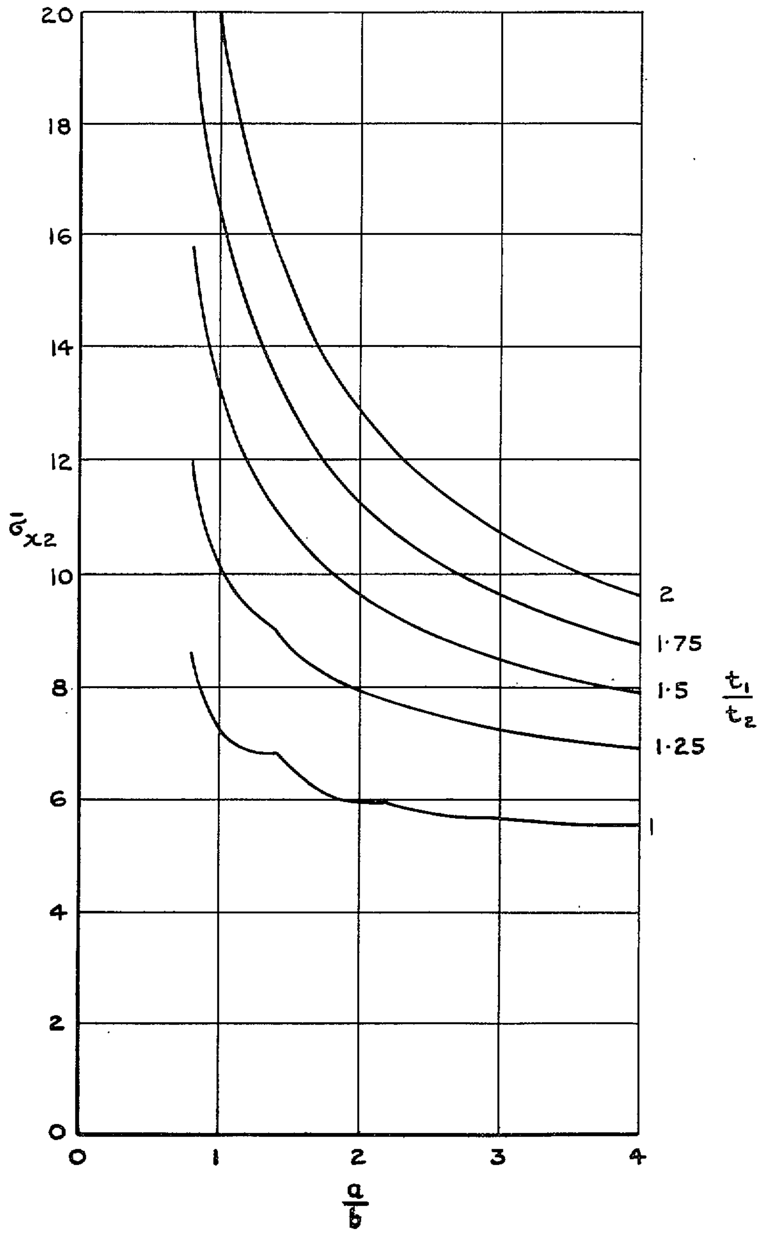


FIG. 9. Buckling stress diagram. Sides and ends clamped. No transverse displacement of sides.

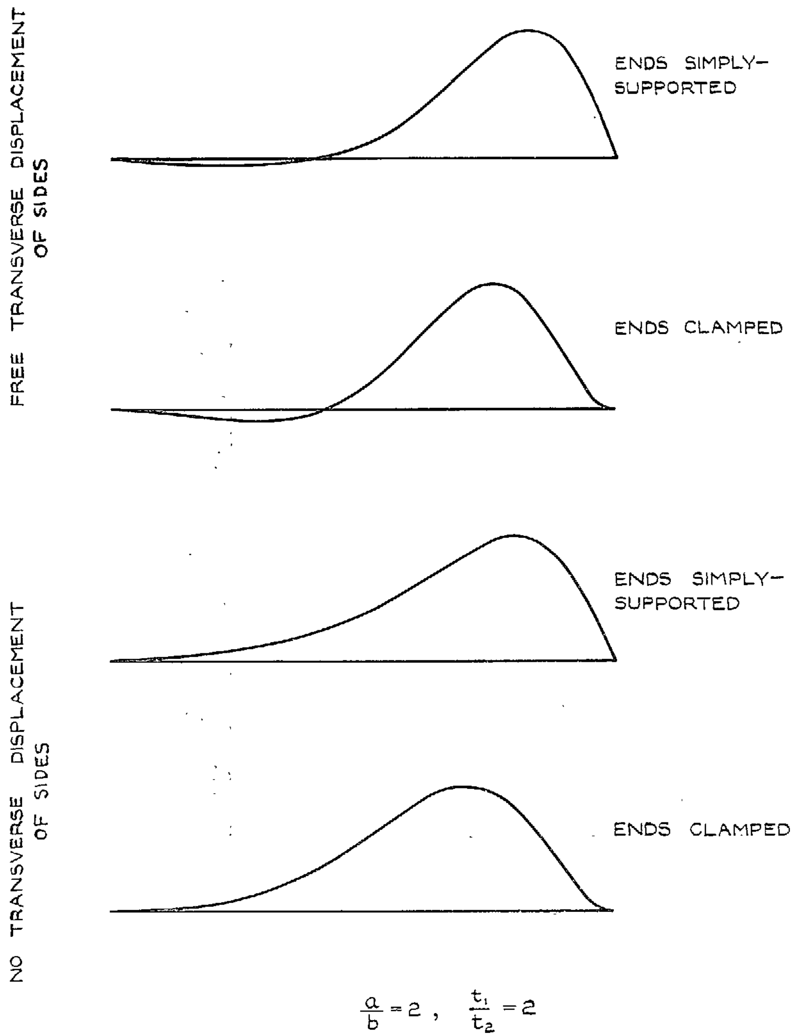


FIG. 10. Specimen buckled shapes, sides simply-supported.

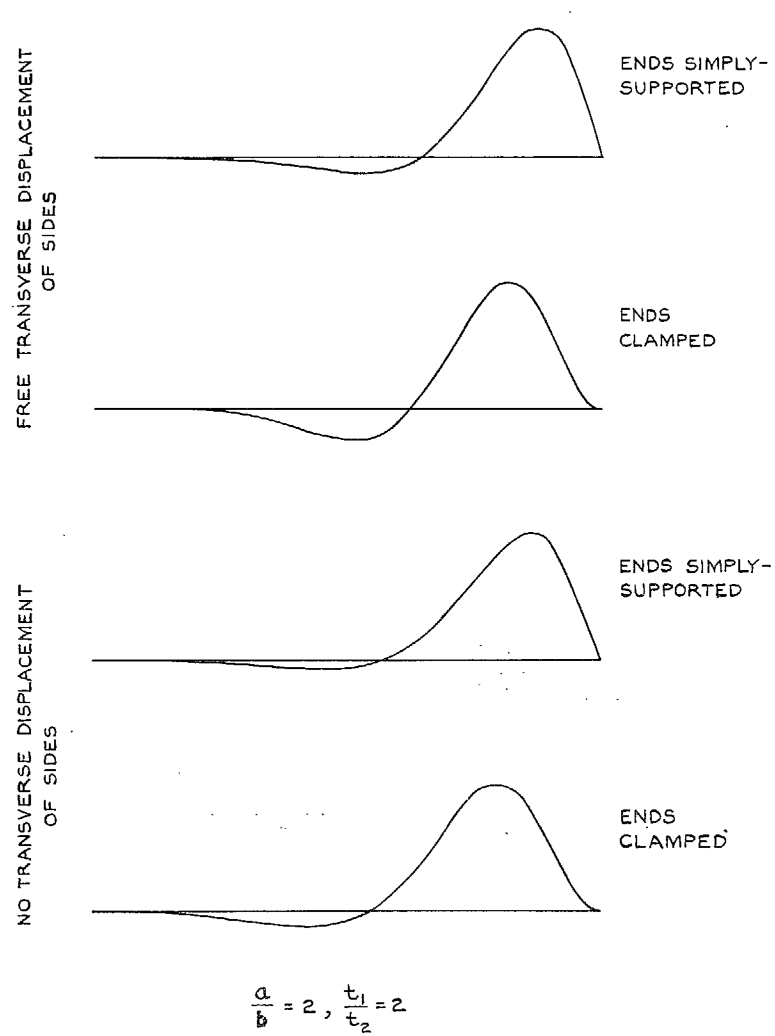


FIG. 11. Specimen buckled shapes, sides clamped.



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