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The Electronic Simulator for the Solution of Flutter and Vibration Problems

By

F. Smith, M.A.

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SUMMARY

This report describes the basic principles of an electronic simulator to solve the equations for coupled oscillations in several degrees of freedom. The method has direct application to flutter and vibration problems on aircraft. It is suggested that considerable time could be saved by the use of such a machine for obtaining flutter speeds and resonant frequencies, without any limitations in scope or accuracy compared with present methods. In addition, since the simulator gives the whole range of non-critical solutions, other problems can be investigated; in particular, the response of an aircraft to flight resonance excitation can be examined.

The use of electronic amplifiers as adding and integrating units is described and the method of representing a simple harmonic motion is given. Methods of variation of coefficients are developed, and the problem of coupling two or more groups of amplifiers to correspond to two or more degrees of freedom in flutter is discussed.

A prototype simulator for the solution of problems in two degrees of freedom is now being built at the R.A.E. and preliminary work is proceeding on the design of a machine for problems in six degrees of freedom.

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1 Introduction

The prediction of flutter of aircraft usually requires the solution of equations of motion in several degrees of freedom. Since the solution is complicated it is usual to limit the analysis to the determination of critical flutter speeds. The problem then reduces to the solution of a determinant usually complex of the same order as the number of degrees of freedom. Several ingenious mathematical methods have been evolved to reduce the labour of solving the determinant, but the solution is rarely easy and, in addition, the effect of any variation in the coefficients is often only to be found by the solution of the new determinant.

A brief survey of the computational difficulties in the prediction of flutter of complex systems was made by Frazer[1] in 1944. He covered both the purely mathematical considerations and the different approaches in the use of plain and thermionic networks. The Biot Flutter Predictor[3] is in the latter class, but is basically an electrical simulator. Consideration of the limitations of this machine, both in scope and accuracy, led to the lack of interest in this approach. The recent developments in electronic technique have meant that analysers of wide scope and good accuracy are possible and development of equipment for the solution of guided weapon problems is well advanced[3,4].

The electronic simulator described in this report is on the lines of the Guided Weapon Analyser[3], the main differences being the use of push-pull amplifiers, and a layout designed for the solution of a specific type of problem rather than the more flexible layout of the Guided Weapons simulator. With this equipment, coupled mechanical oscillations are represented by corresponding coupled electrical oscillations, the whole motion being slowed down to enable visual observation of the oscillations to be made. The electrical circuits are varied step by step to correspond to a step by step variation in airspeed in the mechanical problem. The flutter solution is determined by the observation of an undamped oscillation in the electrical "degrees of freedom". The method of determining flutter speed in flight by mechanical exciting of the aircraft structure over a suitable range of frequencies and measuring the change of response with flight speeds can also be represented on the simulator ensuring a safer approach to critical conditions.

By the inclusion of a forcing electrical unit the problems of forced mechanical oscillations such as engine-airframe vibrations and normal modes of vibration can be investigated.

2 The flutter problem

The mechanical oscillation of an aerofoil in an airstream in the degree of freedom "x" may be written

$$(A + A') \ddot{X} + (Bv + D) \dot{x} + (Cv^2 + E) x = 0$$

with the usual assumptions that aerodynamic terms of order higher than two may be neglected.

A, A' are the mechanical and aerodynamic inertia coefficients, D, B the corresponding damping coefficients, E, C the stiffness coefficients, and v is proportional to the flight equivalent airspeed.

Since the aerodynamic inertia is, by definition, invariable with flight speed it may for simplicity be included in the mechanical inertia and thus in the remainder of this note "A" will denote the sum of mechanical and aerodynamic coefficients.

Thus the equations of motion in two degrees of freedom are

$$A_{11}\ddot{x}_1 + (B_{11}v + D_{11})\dot{x}_1 + (C_{11}v^2 + E_{11})x_1 = 0$$

$$A_{22}\ddot{x}_2 + (B_{22}v + D_{22})\dot{x}_2 + (C_{22}v^2 + E_{22})x_2 = 0$$

When the cross coupling terms are added they become

$$A_{11}\dot{x}_1+(B_{11}v+D_{11})\dot{x}_1+(C_{11}v^2+E_{11})x_1+A_{12}\ddot{x}_2+(B_{12}v+D_{12})\dot{x}_2+(C_{12}v^2+E_{12})x_2 = 0$$

$$A_{21}x_1+(B_{21}v+D_{21})\dot{x}_1+(C_{21}v^2+E_{21})x_1+A_{22}\ddot{x}_2+(B_{22}v+D_{22})\dot{x}_2+(C_{22}v^2+E_{22})x_2 = 0$$

These equations are more complete than is usually necessary, but in order to make the simulator as general as possible, all possible terms are included. The simulator as envisaged will represent up to six degrees of freedom and all the terms obtained when the above equations in two degrees of freedom are extended to six degrees of freedom will be represented.

In the solution of practical flutter problems in this country it is usual to treat the coefficients B, C as invariable with respect to speed and frequency and this approximation will also be made here. However, as will be seen below, the effect of varying any coefficient is easily found and a more exact solution can be obtained by a method of successive approximations.

The method of solution will normally be by variation of the airspeed coefficient v, although in some cases, notably at speeds near that of sound where the structural stiffness for a given flutter speed may be required, the solution may be by variation of certain coefficients.

3 The electronic units

The principles of the electronic units described here are not new and have been used as the basis of other simulators, in particular in those now being used in Guided Weapons Department R.A.E. In fact, since these principles have been used in other simulators and the reliability of the methods demonstrated, they can be adapted to the solution of the flutter problems with confidence. For clarity, these principles will be outlined here

3.1 Amplifier with resistance feedback

Let us consider the circuit shown in Fig 1 An amplifier of gain "-G" (i.e. the gain is G but the output variation is in the opposite sense to the input variation), where G is large (of the order of 10,000) has a resistance feedback R_1 The input is supplied from a voltage V_2 through resistance R_2. Then, if the impedance from input to earth is infinite, the total current flow from V_1 and V_2 to the amplifier input must be zero, i.e.

$$\frac{V_1 - V_o}{R_1} = - \frac{(V_2 - V_o)}{R_2}$$

and since

$$V_1 = - G\,V_o$$

we have

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_1}{G}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0$$

If G is large we have

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 0$$

which represents the equation

$$A_1 x_1 + A_2 x_2 = 0 .$$

In particular if

$$R_1 = R_2 \quad V_1 = -V_2 .$$

When the input consists of several supplies in parallel (Fig.2) we have

$$\frac{V_1}{R_1} = -\left(\frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4} \quad \ldots\ldots\ldots\right)$$

representing the equation

$$A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4 \ldots\ldots = 0$$

3.2 Amplifier with capacity feedback

If the feedback is through a capacity $\bar{C}$ instead of a resistance, as in Fig.3, we have

$$\bar{C}\frac{d}{dt}(V_1 - V_o) = \frac{-(V_2 - V_o)}{R}$$

or with $V_o = -\frac{V_1}{G}$

$$R\bar{C}\,\dot{V}_1\left(1 + \frac{1}{G}\right) + V_2 + \frac{V_1}{G} = 0 .$$

If V_1, V_2 are of the same order and G is large we have

$$R\bar{C}\,\dot{V}_1 + V_2 = 0$$

or

$$A\,\dot{x}_1 + B\,x_2 = 0$$

Thus V_1 is proportional to the integral of V_2 and the unit acts as an integrator.

3.3 Push-pull amplifier

So far only single sided amplifiers have been described and such amplifiers are known to be liable to drift with time, making the operation of the simulator using such amplifiers more difficult. In addition, since the coefficients of the flutter equations may, in general, be either positive or negative a large number of amplifiers to reverse the sign of a coefficient (see paragraph 3.1) would be needed.

Both these problems are simplified by the use of push-pull amplifiers. Here drift is greatly reduced and the reversal of sign is obtained simply by interchanging the push-pull connecting leads. A push-pull amplifier to be used as an integrating unit is shown in Fig.4. The circuits must, of course, be symmetrical about the earth line.

3.4 The use of potentiometers

In paragraph 3.1 it was shown that

$$A_1 x_1 + A_2 x_2 = 0$$

could be represented by

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 0$$

or in other words

$$A_2 \propto \frac{1}{R_2} .$$

This inverse proportionality makes representation difficult. If, however, only a fraction, S_2, of the voltage V_2 were supplied to R_2 then

$$A_2 \propto \frac{S_2}{R_2}$$

so that, if R_2 is regarded as fixed and S_2 variable, the coefficients A_2, A_3, etc. can be represented directly. Where the feedback from the output voltage V_1 (Fig.1) is through a resistance the same system can be used to represent A_1 directly. Variation of S_1 can be obtained from a tapped potentiometer from V_1 to earth. If the tappings are in tenths of V_1, and three tappings S_a, S_b, S_c, are applied through resistances R_1, $10R_1$, $100R_1$ connected in parallel to the input of the amplifier (Fig.2) we have

$$V_1 \left(\frac{S_{a_1}}{R_1} + \frac{S_{b_1}}{10R_1} + \frac{S_{c_1}}{100R_1}\right) + V_2 \left(\frac{S_{a_2}}{R_2} + \frac{S_{b_2}}{10R_2} + \frac{S_{c_2}}{100R_2}\right) = 0$$

or with $R_1 = R_2$

$$V_1 \left(S_{a_1} + 0.1\ S_{b_1} + 0.01\ S_{c_1}\right) + V_2 \left(S_{a_2} + 0.1\ S_{b_2} + 0.01\ S_{c_2}\right) = 0$$

For example, with $S_{a_1} = 2$, $S_{b_1} = 4$, $S_{c_1} = 7$, and $S_{a_2} = 3$, $S_{b_2} = 8$ $S_{c_2} = 1$, we have

$$0.247\ V_1 + 0.381\ V_2 = 0 .$$

We have thus represented

$$A_1 x_1 + A_2 x_2 = 0$$

with the coefficients A_1 , A_2 represented to three places of decimals, with a maximum value of 1.110.

Such a potentiometer unit is shown in Fig.5 for a push-pull amplifier. Since in practice the resistance $100R_1$, may be difficult to represent a subsidiary resistance potentiometer has been substituted giving the effect of $100R_1$ whilst using no resistance larger than $10R_1$. In addition a reversing switch has been included so that we can represent

$$\pm A_1 x_1 \pm A_2 x_2 = 0$$

3.5 Variable integrating unit

The use of potentiometers is next extended to the integrating unit described in paragraph 3.2. The proposed circuit is shown in Fig.6. We have

$$V_2 \left(\frac{S_a'}{R} + \frac{S_b'}{10R}\right) + \bar{C}\,\dot{V}_1 = 0$$

or

$$V_1 = -\frac{1}{R\bar{C}}\,(S_a' + 0.1\ S_b') \int V_2\,dt\ .$$

Putting $R\bar{C} = \tau$, the time constant of the circuit, we have

$$V_1 = -(S_a' + 0.1\ S_b') \int V_2 \frac{dt}{\tau}$$

Thus we have represented $x_1 = v \int x_2 \frac{dt}{\tau}$, where v is accurate to two places of decimals. Since, as will be shown below, v represents the airspeed we can adjust the airspeed to 1% of the maximum value to be investigated. This accuracy is thought to be sufficient for present needs.

It will further be seen from Fig.6 that the feedback capacity can be selected from the values $0.1\ \bar{C}$, $\bar{C}$ and $10\ \bar{C}$. Thus the time constant τ can be varied from 10 times smaller to 10 times greater than the standard value, having the effect of varying the speed of the integration. In this manner the solution of a problem can be speeded up or slowed down, at the convenience of the operator. Care must be taken, however, to maintain the same time constant throughout the simulator, since, if different parts of the simulator are operating at different speeds, the basic problem is altered.

4 Representation of a one degree of freedom equation

Having considered some of the units of the simulator we can now turn to the representation of the one degree of freedom equation

$$A\,\ddot{x} + (Bv + D)\,\dot{x} + (Cv^2 + E)\,x = 0\ .$$

Let us consider the circuit shown in Fig.7, using the symbol S to represent $S_a + 0.1\ S_b + 0.01\ S_c$ in the summation potentiometer and S'

to represent $S_a' + 0.1\ S_b'$ in the integrating unit. Signs will be omitted since all connections are push-pull and reversing switches are included where necessary.

From Fig.7 and previous equations

$$V_1 = \tau \dot{V}_4 = \tau^2 \ddot{V}_5$$

$$V_1 = \frac{\tau}{S_1'} \dot{V}_2 = \frac{\tau^2}{S_1'\ S_2'} \ddot{V}_3$$

With $\tau = 1$, $S_1' = S_2' = v$ and $V_1 = \ddot{x}_1$, we have

$$V_1 = \ddot{x}_1, \quad V_2 = v\dot{x}_1, \quad V_3 = v^2 x_1, \quad V_4 = \dot{x}_1, \quad V_5 = x_1.$$

Also $\quad \ddot{x}_1 = f(\dot{x}_1,\ x_1,\ v\dot{x}_1,\ v^2 x_1)$

where f is a linear function

Putting $\quad S_1 = A_{11}$, etc. we have

$$A_{11}\ \ddot{x}_1 + (B_{11}\ v + D_{11})\ \dot{x}_1 + (C_{11}\ v^2 + E_{11})\ x_1 = 0$$

Thus the electrical oscillation of the circuit in Fig.7 represents the mechanical oscillation in one degree of freedom and if $S_1 \propto A_{11}$, $S_2 \propto B_{11}$ and so on the oscillations would be identical. However, it is convenient to slow down the mechanical oscillation when transforming into the electrical oscillation, both to make the oscillation easy to follow with the eye and to make the maximum use of the relatively limited range of the potentiometers. If the oscillation is slowed down until the circular frequency is approximately unity then A, C, E will be of the same order.

Thus in general

$$S_1 = kA_{11},\ S_2 = knB_{11},\ S_3 = kn^2C_{11},\ S_4 = knD_{11},\ S_5 = kn^2E_{11},$$

where k and n are chosen to make the maximum use of the accuracy of the simulator.

5 Extension to six degrees of freedom

We have now seen how the simulator can represent an oscillation in one degree of freedom:-

$$A_{11}\ \ddot{x}_1 + (B_{11}\ v + D_{11})\ \dot{x}_1 + (C_{11}\ v^2 + E_{11})\ x_1 = 0$$

With a second unit we can represent the independent oscillation

$$A_{22}\ddot{x}_2 + (B_{22}v + D_{22})\dot{x}_2 + (C_{22}v^2 + E_{22})x_2 = 0$$

In order to represent the flutter equations we must now add the cross coupling terms discussed in paragraph 2. To do this we must add extra electrical couplings and potentiometer units so that each amplifier output in unit 1 is coupled to the input of unit 2, in addition to being coupled to the input of 1. Similarly for the output of the amplifiers of unit 2.

When the simulator is extended to cover six degrees of freedom, the output of every amplifier is coupled to the input of each degree of freedom unit making a total of 180 coupling leads. Such a circuit is difficult to present in diagrammatic form, so in Fig.8 a representative circuit is shown. Here the couplings of one typical amplifier in each degree of freedom to the inputs of each degree of freedom unit are shown. The diagram is arranged in determinant form to indicate the analogy between the electrical couplings and the mechanical coupling terms they represent.

6 Accuracy

Only the general principles of the simulator have been described in this note, and considerable care must be taken at every stage in the development to ensure the accuracy of the representation of the problem. It is a little difficult to determine the effect of variation of components on the overall accuracy of the complete simulator, since the calculations involved are lengthy. However, a series of calculations in two degrees of freedom has been made, varying the coupling terms slightly one by one. The largest inaccuracy in airspeed due to the limitations of the simulator was $\frac{1}{4}$% and the largest inaccuracy due to the estimated maximum errors in components about $\frac{1}{3}$%.

From these figures it is deduced that errors may amount to an inaccuracy in airspeed of 2% in six degrees of freedom (about 20 ft/sec in typical problems), whilst the repeatability should be appreciably better. Flutter frequency should be obtained with considerably better accuracy.

7 Conclusions

It is considered that a flutter simulator of the type described here will possess adequate scope and accuracy for the solution of most flutter and vibration problems at present in hand and likely to arise in the near future and at the same time enable solutions to be obtained much more quickly.

Preliminary work on the design of a simulator with six degrees of freedom is now proceeding at the R.A.E. and it is hoped that the machine can be completed by the end of 1950. In the meantime a prototype with 2-degrees of freedom is being made.

Acknowledgements

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Wt.2078.CP26.43. Printed in Great Britain.

FIG. 1,2,3 & 4.

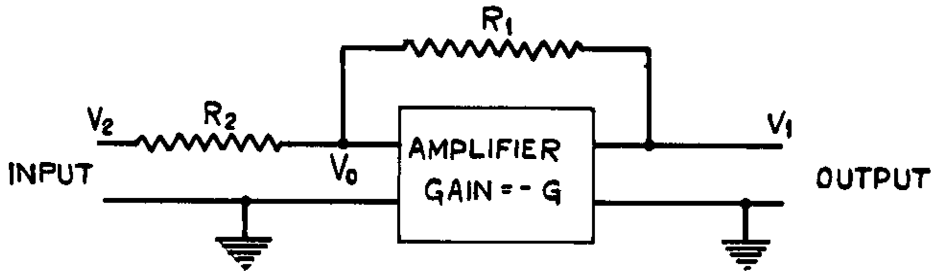


FIG. 1 AMPLIFIER WITH RESISTANCE FEEDBACK.

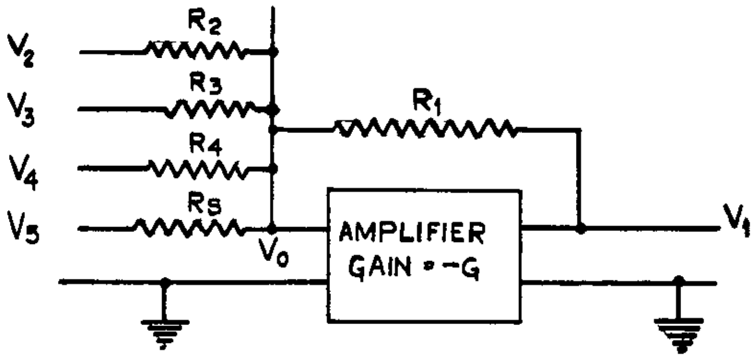


FIG. 2. SUMMING UNIT.

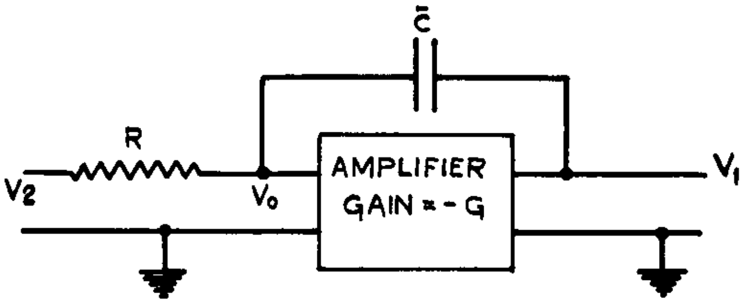


FIG. 3. INTEGRATING UNIT.

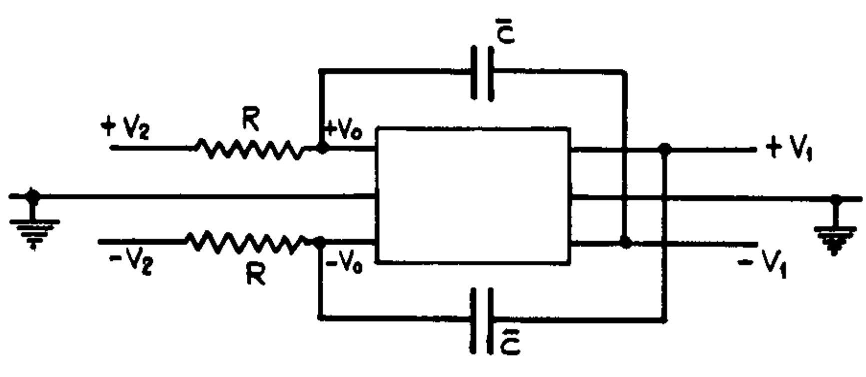


FIG. 4 PUSH-PULL INTEGRATOR.

FIG. 5.

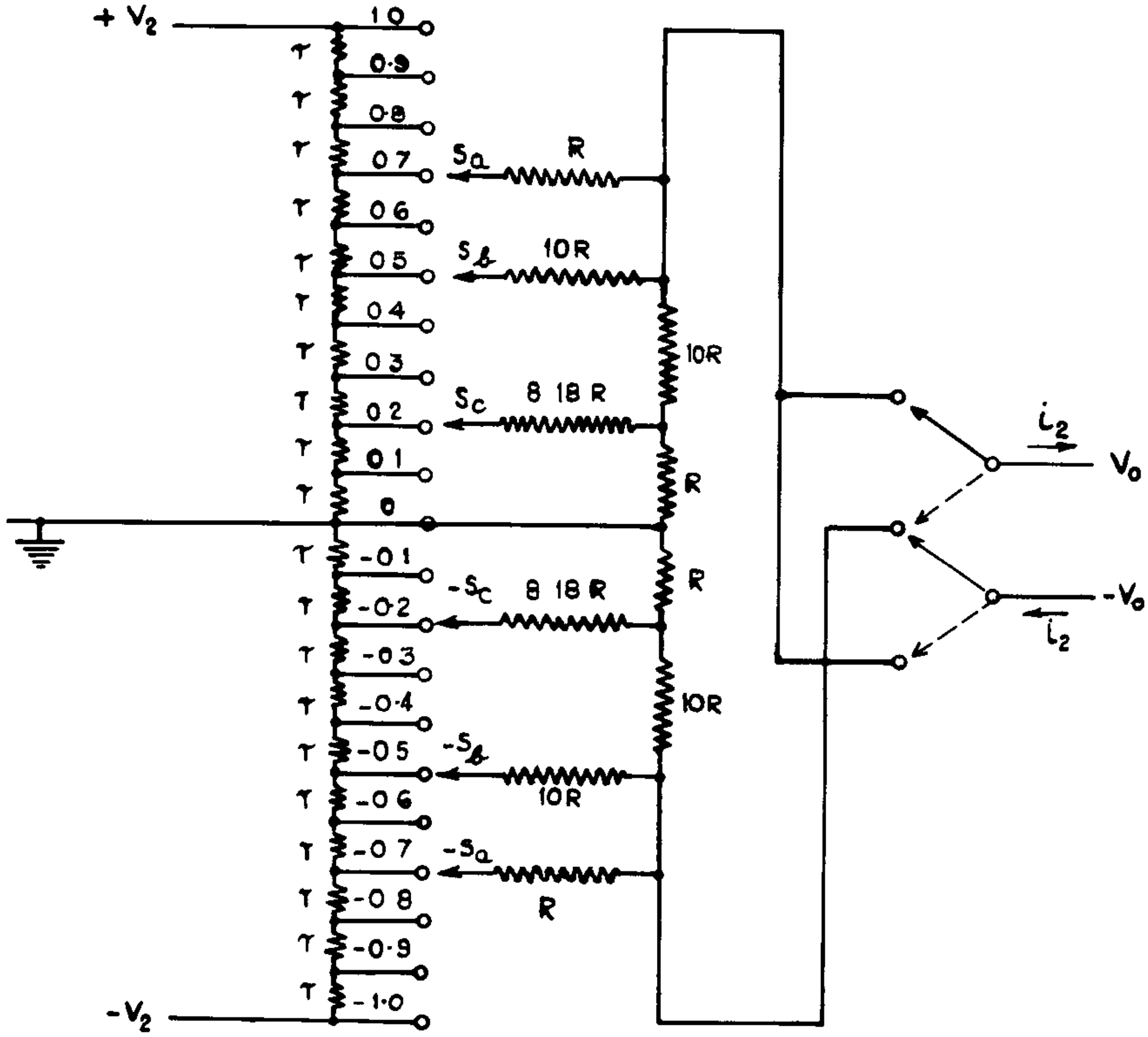


FIG. 5. POTENTIOMETER AND REVERSING UNIT

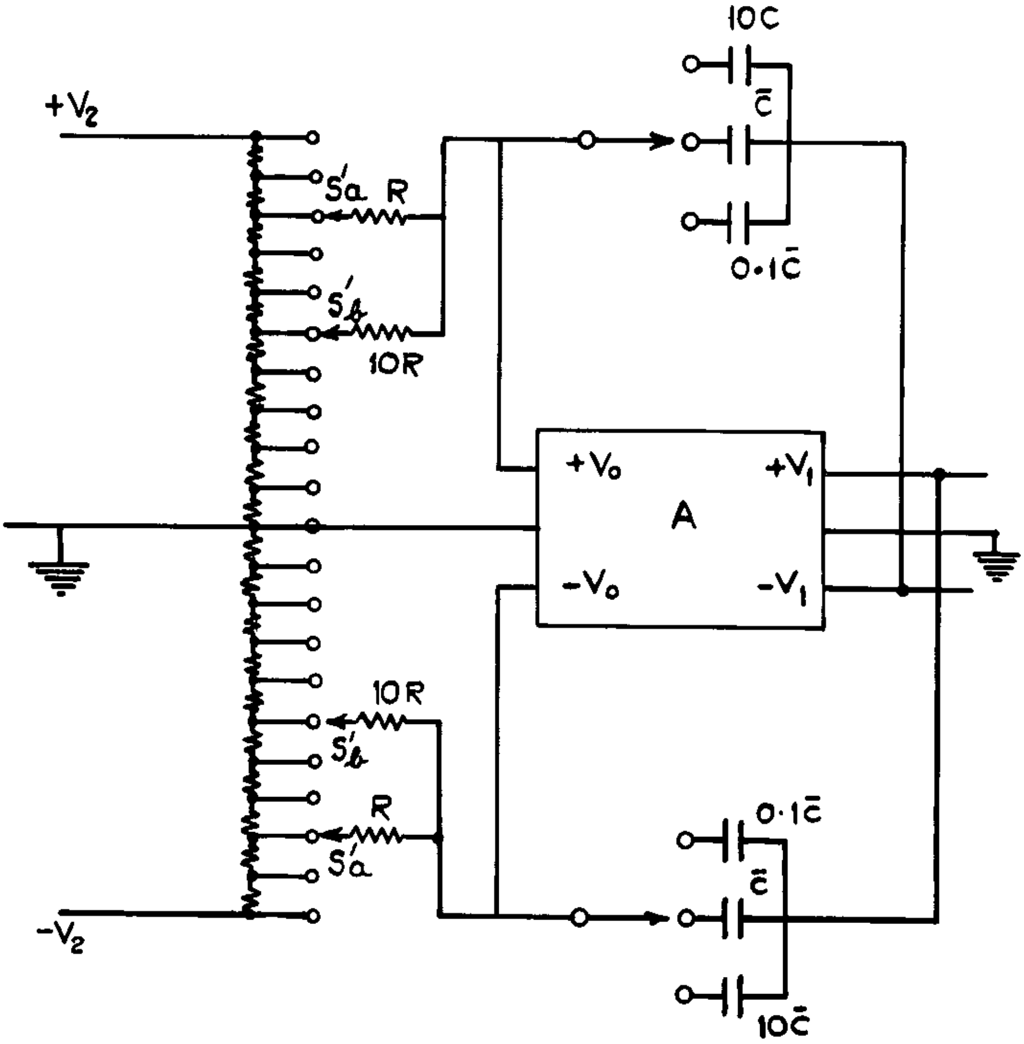


FIG. 6. VARIABLE INTEGRATING UNIT.

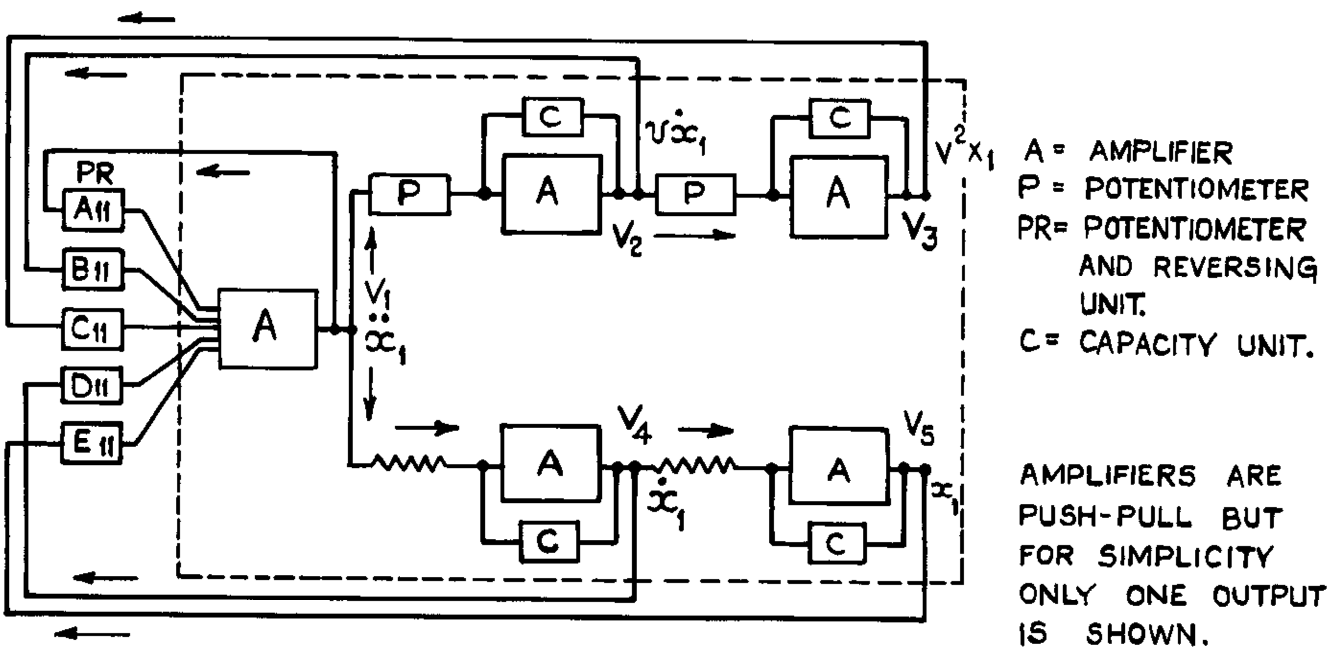
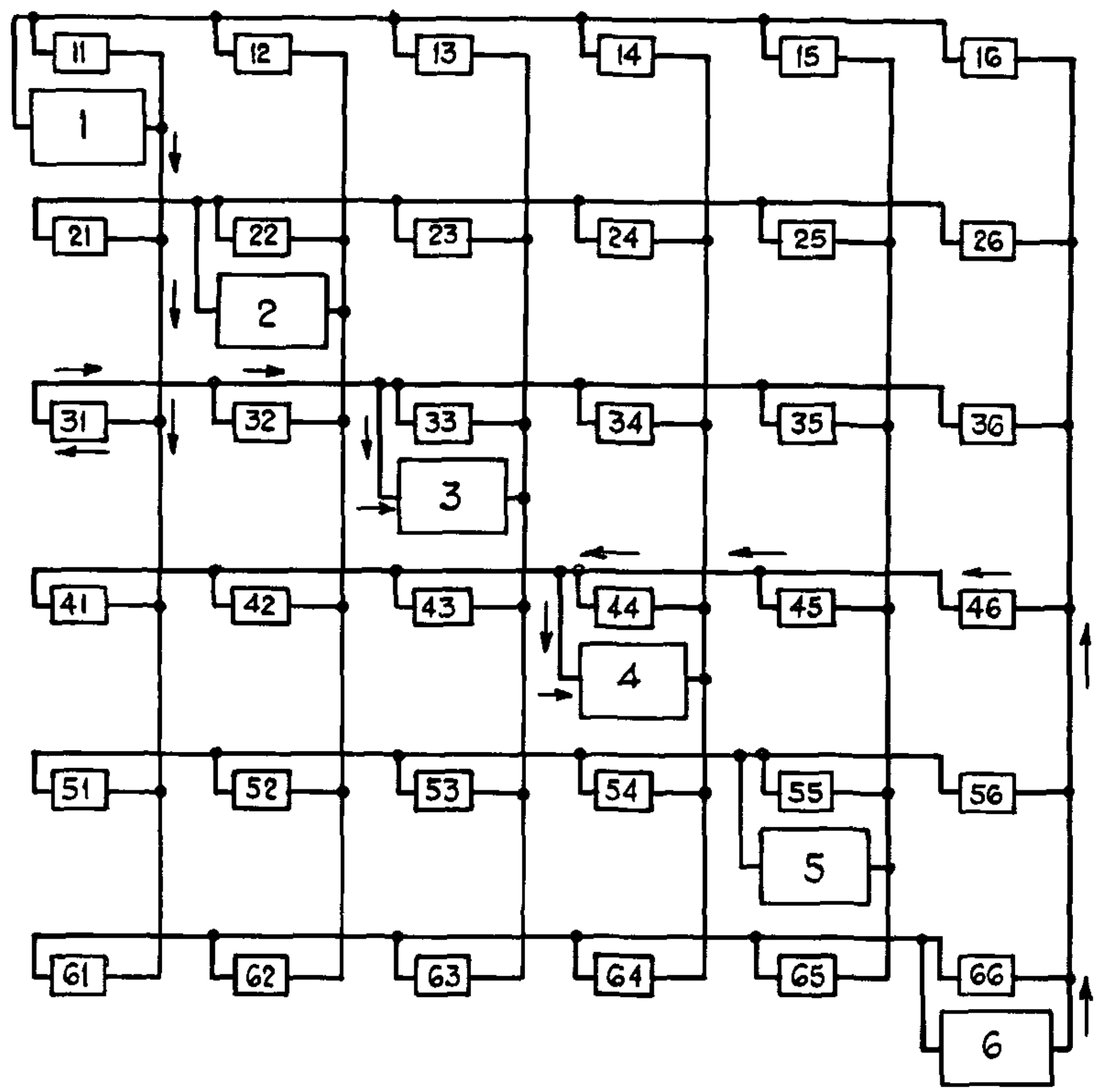


FIG. 7. CIRCUIT FOR ONE DEGREE OF FREEDOM.

FIG. 8.



= ONE DEGREE OF FREEDOM UNIT CONTAINED WITHIN DOTTED LINE OF FIG 7

pq = POTENTIOMETER REVERSING UNIT CONNECTING OUTPUT OF UNIT q TO INPUT OF UNIT p

ARROWS INDICATE REPRESENTATIVE FLOW OF CURRENT FROM OUTPUT OF UNIT 1 TO INPUT OF UNIT 3 AND FROM OUTPUT OF UNIT 6 TO INPUT OF UNIT 4

ONLY ONE OUTPUT FROM EACH UNIT SHOWN FOR SIMPLICITY. THERE ARE ACTUALLY FIVE PUSH-PULL OUTPUTS FROM EACH UNIT.

FIG. 8. BASIC CIRCUIT FOR 6 DEGREES OF FREEDOM.

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