R. & M. No. 2557
(11,708, 12,081)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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LONDON: HIS MAJESTY'S STATIONERY OFFICE
1951
PRICE 4*s.* 0*d.* NET

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Communicated by the Principal Director of Scientific Research (Air), Ministry of Supply

Reports and Memoranda No. 2557*

30*th June*, 1948

Summary.—Reasons for Enquiry.—The climb techniques at present used on modern aircraft entail quite high true air speeds and high kinetic energies. It was desired to investigate the effect of kinetic energy variation with height, which is ignored in present methods, on the choice of climb technique.

Scope of Enquiry.—The problem of choosing the best climbing technique is considered and the limitations of the present technique discussed. A new approach is made to the general problem of choosing the best climb technique between any specified end conditions, and with the aid of a geometrical illustration tentative conclusions are deduced concerning the choice of climb technique. These are presented for discussion prior to a fuller investigation.

Conclusions.—It is concluded that the application of present methods of choosing a climb technique to aircraft whose speeds on the climb are high is open to question.

Introduction of 'energy height' as a variable permits a more exact treatment to be attempted and enables a geometrical illustration to be developed of the general problem of optimum climb between specified end conditions. From discussion of this illustration it is tentatively concluded that a revised climb technique, outlined in the Report, will give improved performance by building up a relatively high kinetic energy at low altitudes, where the thrust available is high, for conversion into potential energy (*i.e.* height) at high altitudes.

In a particular example the new technique reduced the times required to climb to 40,000 ft and 45,000 ft by 1·4 minutes (9 per cent) and 2·5 minutes (10 per cent) respectively.

Further Developments.—It is hoped to investigate the proposed technique experimentally with a view to confirming its superiority over present methods.

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1. *Introduction.*—An important feature of the performance characteristics of an aircraft is the minimum time required to bring about any given change in altitude or air speed, particularly the time required to climb to any given altitude from ground level.

The time required to bring about a change in altitude depends on the climb technique adopted, especially the forward speed used and the way in which it is varied with height. The present method of choosing the climb technique assumes that variation in the kinetic energy of the aircraft may be neglected, an assumption which is becoming less tenable as the true speeds used on the climb become higher.

A review of the problem of choosing the best technique of climb was being attempted when a report of some German investigations[1], dated 1944, was encountered. This report suggested a new method of attack which seemed to throw considerable light on the problem and is sketched below. More work will be required before the method of choosing climb technique can be changed;

*A. & A.E.E. Report No. Res. 237—received 14th August, 1948.

it seemed, however, that it would be profitable to bring the outline of the problem to notice without waiting for the results of a long term investigation, in order that any comments and criticisms on matters of principle might be received at an early stage.

2. *Scope of Investigation.*—The problem of proceeding in the quickest way from one combination of height and speed to another is examined, and by use of the concept of 'energy height' a geometrical presentation of the problem is deduced which clarifies the choice of technique. Simple rules by which to choose a climb technique between any end conditions are tentatively proposed. A numerical example is given.

3. *The Information Required Concerning the Climb Performance of an Aircraft.*—Ideally, one should know the shortest time in which a particular aircraft could pass from any one combination of height and speed to any other and what flight technique should be adopted during the transition between the two states. Since, however, such information would probably be too unwieldy for general use one would also like to have general rules from which to derive a sufficiently good approximation to both the time required for, and the technique appropriate to, any particular case.

4. *The Customary Approach to the Problem.*—Up to the present time the climb performance of an aircraft has customarily been considered solely in terms of time to height and rate of change of height—*i.e.* in terms of potential energy only. The long zoom climbs which can be made by recent fighter aircraft vividly illustrate both that the kinetic and the potential energies are to a large extent interchangeable and that the kinetic energy may be very considerable, facts which make a presentation of the climb performance in terms of potential energy (*i.e.* height) alone rather incomplete. However, we will examine this limited approach and consider the choice of the technique of climb for the quickest change of height.

4.1. *The Basic Relationship.*—The basic equation for the longitudinal motion of the aircraft in still air can be written

$$T - D - W \sin \gamma = \frac{W}{g}\frac{dV}{dt}, \qquad (1)$$

where T = thrust,
V = air speed,
D = drag of aircraft,
W = weight of aircraft,
γ = angle of the flight path to the horizontal.

If we further denote the total drag of the aircraft in straight level flight by D_h and the induced drag in this condition by KD_h, then for rectilinear flight at the same speed along a path inclined to the horizontal the induced drag is $KD_h \cos^2 \gamma$, and we may write (*see* Appendix)

$$T - D_h + KD_h \sin^2 \gamma - W \sin \gamma = \frac{W}{g}\frac{dV}{dt},$$

i.e.,

$$T - D_h + KD_h\left(\frac{1}{V}\frac{dH}{dt}\right)^2 - \frac{W}{V}\frac{dH}{dt} = \frac{W}{g}\frac{dV}{dt},$$

where H is the altitude of the aircraft above some arbitrary datum,

i.e.,

$$\frac{W}{V}\left\{\frac{dH}{dt} + \frac{V}{g}\frac{dV}{dt}\right\} = T - D_h + KD_h\left(\frac{1}{V}\frac{dH}{dt}\right)^2, \qquad (2)$$

or

$$\frac{W}{V}\frac{dH}{dt}\left\{1 + \frac{V}{g}\frac{dV}{dH}\right\} = T - D_h + KD_h\left(\frac{1}{V}\frac{dH}{dt}\right)^2. \qquad (2a)$$

4.2. *The Variables Involved.*—Of the variables in the above equations T, D_h and K are functions of air pressure, air temperature, W and V. In a given atmosphere, therefore, they are functions of W, V and H, where H is the altitude of the aircraft relative to some arbitrary datum. Let us now consider how to choose a technique of climb for a given aircraft from given initial conditions.

As the initial weight of the aircraft is then specified, and as the effect of climb technique on weight variation with height would not be expected to affect the choice of the optimum technique, we may now omit W and write

$$\frac{dH}{dt} = f\left(H, V, \frac{dV}{dH}\right) .$$

Any climb technique may be specified by a relation of the form

$$V = \psi\,(H),$$

and we may then write

$$\frac{dV}{dH} = \psi'\,(H) .$$

For a climb following this technique we would then have

$$\begin{aligned}\frac{dH}{dt} &= f\,(H, \psi(H), \psi'\,(H)),\\ &= f_\psi\,(H) \text{ say },\end{aligned}$$

and the time taken to climb from height H_0 to height H_1 would be given by

$$\begin{aligned}{}_0t_1 &= \int_{H_0}^{H_1} \left[f_\psi\,(H) \right]^{-1} dH ,\\ &= F_\psi\,(H_0, H_1) \text{ say} .\end{aligned}$$

Our problem is to determine $\psi(H)$ when some additional condition is imposed—*e.g.* that ${}_0t_1$ shall be a minimum.

It is implied by the above analysis that the climb should strictly be considered as a whole and that the present methods of choosing climb technique, in so far as they involve the consideration of conditions at each altitude independently of the rest of the climb, require justification. For instance, at the start of the climb H_0 may be specified by our end conditions, leaving V_0 and $(dV/dH)_0$ at our disposal. Although the initial value of dH/dt would then be a function of two independent variables we would not be free so to choose these variables as to give, for instance, the highest possible instantaneous value of dH/dt, because this might imply unsuitable values of V at slightly later stages in the climb.

This interdependence of H, V and dV/dH renders visualisation and qualitative treatment of the problem difficult, while analytical treatment (by, for instance the calculus of variations) is hindered by the absence of a suitable analytical expression for the thrust. Hence some simplification is very desirable.

4.3. *Omission of a Variable.*—In the approach to the problem commonly adopted the desired simplification is achieved by neglecting $(V/g)\,(dV/dH)$ in comparison with unity, *i.e.*, by omitting the term containing dV/dH. This simplification has been customary in investigations into climb technique and even, quite often, in performance calculations. Such a simplification requires justification, which is difficult to provide in the absence of a more exact method as a yard stick. To obtain some guidance let us examine the direct effect.

The direct consequence is overestimation of the instantaneous rate of climb very nearly in the ratio of

$$\left(1 + \frac{V}{g}\frac{dV}{dH}\right), \textit{ i.e. of } \left\{1 + \frac{d}{dH}\left(\frac{1}{2}\frac{V^2}{g}\right)\right\},$$

to unity (equation 2(a)). The amount of this error depends on the way in which the true air speed is varied with altitude, but we may note that

(i) in a climb at constant true air speed the term is zero ;

(ii) in a climb at constant equivalent air speed we have

$$\frac{d}{dH}\left(\frac{1}{2}\frac{V^2}{g}\right) = -\frac{V^2}{2g}\frac{1}{\sigma}\frac{d\sigma}{dH},$$

$$= +0{\cdot}45\left(\frac{V}{100}\right)^2 \text{ per cent at sea level}$$

$$\text{or } 0{\cdot}73\left(\frac{V}{100}\right)^2 \text{ per cent at 40,000 ft,}$$

where V is in ft/sec ;

(iii) present climb techniques, deduced on the basis of the simplification under discussion, usually result in variations of true speed with height lying about midway between the cases (i) and (ii).

As a result of (iii) the value of $\frac{d}{dH}\left(\frac{1}{2}\frac{V^2}{g}\right)$ for jet aircraft using climb techniques deduced by means of the approximation we are discussing will be about half of that for case (ii) ; *i.e.* it will be of the order of $0{\cdot}3\,(V/100)^2$ per cent, and will vary as the square of the true air speed. (iii) above is illustrated by Fig. 1(a), which shows the variation of true air speed (V) and of equivalent air speed ($V\sqrt{\sigma}$) with height for a recent high performance fighter. Fig. 1(b) shows the corresponding variation of $\frac{1}{2}V^2/g$; at 30,000 ft $\frac{d}{dH}\left(\frac{1}{2}\frac{V^2}{g}\right)$ is equal to 0·10—*i.e.* 10 per cent on rate of climb. The effect of the term omitted is further illustrated in Figs. 2(a) and 2(b), which show the variation of instantaneous rate of climb with air speed, at each of two heights, for a hypothetical jet fighter in cases (i) and (ii), *i.e.*

(i) when climbing at constant true speed, and

(ii) when climbing at constant equivalent speed.

It will be seen that the two cases differ considerably. The former is the rate of climb which would be estimated using the approximation under discussion $\left(\frac{d}{dH}\left(\frac{1}{2}\frac{V^2}{g}\right) \text{ assumed zero}\right)$ and the latter that which would be observed in partial climbs at constant equivalent air speed, customarily made in determining the optimum speed for climb. The instantaneous rate of climb on a continuous climb according to present techniques would lie midway between these cases.

Thus the direct effect of the term omitted on estimates of the rate of climb on a continuous (as distinct from a 'partial') climb may be considerable (10 per cent or so). Its effect on the choice of climb technique is, as was remarked above, difficult to assess in the absence of a more precise method to provide a standard, but since the term is proportional to speed squared its effect will be less at low than at high climbing speeds, and one would expect the approximation to be satisfactory at low speeds and unsatisfactory at high speeds, the definitions of 'low' and 'high' depending on, among other things, the accuracy required. It certainly cannot be assumed without investigation that so large a term can be neglected for present high performance aircraft, which tend to climb at forward speeds much higher than those used until recently.

5. *The Proposed Alternative Approach.*—We have seen that in considering the climb performance of an aircraft three independent variables apart from the aircraft weight should, strictly, be considered and that if the fundamental variables H and V are taken as two of these the influence of the third must, for high performance aircraft, be considered. On the other hand the elimination of one variable is highly desirable to simplify the problem. Let us, therefore, now consider an alternative approach.

The term $KD_h \left(\frac{1}{V}\frac{dH}{dt}\right)^2$, *i.e.* $KD_h \sin^2\gamma$, in equation (2) represents the effect of the reduction in the induced drag due to inclination of the flight path to the horizontal. This term is commonly ignored, and it is shown in the Appendix that this is permissible, but it has been retained in the present discussion up to now because, with the previous choice of variables, omission of it did not help. If we now delete it we can re-write equation (2) in the form

$$\frac{W}{V}\frac{d}{dt}\left(H + \tfrac{1}{2}\frac{V^2}{g}\right) = T - D_h ,$$

and if we now write

$$H_e = H + \tfrac{1}{2}\frac{V^2}{g} , \qquad \qquad (3)$$

the equation above reduces to the very simple form

$$\frac{dH_e}{dt} = \frac{V}{W}(T - D_h) . \qquad \qquad (4)$$

We have already remarked that T and D_h are, at given W, functions of H and V; they are therefore, from equation (3), functions of H_e and V, and we may write

$$\frac{dH_e}{dt} = f(H_e, V) . \qquad \qquad (5)$$

Since we may calculate the time to height by integrating $1/(dH_e/dt)$ with respect to H_e between the appropriate limits it will be appreciated that we have, subject to the validity of our omission of the term in $\sin^2 \gamma$, reduced the climb performance to a matter of two variables apart from aircraft weight.

It may be objected that H_e is a difficult conception to handle, but the elimination of a variable by some device is almost essential to a grasp of the problem*. Also, it can be argued that H_e, which represents the total energy of the aircraft, is, in view of the ready interchangeability of kinetic and potential energy already referred to, of greater tactical significance than H. Furthermore, adoption of H_e and V as variables permits an attack to be made on the general problem as outlined in section 3, instead of on the limited aspect only of the time required for a change of height (section 4). No approach which involves neglecting the term in dV/dH can be expected to deal adequately with the general problem.

6. *The Concept of 'Energy Height'.*—H_e, which we may call the energy height, is the height at which the potential energy of the aircraft would be equal to the sum of its potential and kinetic energy at height H and speed V. The application of this concept to the climb problem was first brought to the attention of the author by Reference 1, in which it was adopted in considering the particular case of the Me. 262. Energy height is plotted against true speed for various true

* It seems probable that means could be derived of indicating H_e instrumentally by reference to pitot and static air pressures.

heights in Fig. 3. It will be seen that $(H_e - H)$ may amount to several thousand feet at high speeds ; it is, for instance, about 4,000 ft at 500 ft/sec.

Some care is needed in using the concept. We have, for instance shown that dH_e/dt may be considered independent of the flight path angle, being nearly equal to $(V/W)\ (T - D_h)$ (equation (4)), which at given W is a function V and H only. Hence H_e will increase as long as T is greater than D_h, *i.e.* as long as the speed does not exceed that for steady level flight at the engine setting being considered, whether the aircraft is climbing or descending. In the latter case the increase in kinetic energy outweighs the decrease in potential energy.

7. *A Geometrical Illustration of the Climb Characteristics of the Aircraft.*—If ${}_1t_2$ is the time required to change from H_{e1} to H_{e2} according to a particular technique, then

$$
\begin{aligned}
{}_1t_2 &= \int_{H_{e1}}^{H_{e2}} \left\{ \frac{1}{\dfrac{dH_e}{dt}} \right\} dH_e , \\
&= \int_{H_{e1}}^{H_{e2}} \phi\ (H_e,\ V)\ dH_e \text{ say} .
\end{aligned}
$$

It is now convenient to adopt a geometrical illustration of our problem, to clarify ideas. The curve of $\phi\ (H_e,\ V)$ against V at given H_e and engine settings will be of the form sketched in Fig. 4 ; it will have a single minimum, at the speed at which the excess thrust power is greatest, and will be asymptotic to the vertical at the maximum and minimum speeds at which level flight can be maintained at the given engine settings. Outside this region $\phi\ (H_e,\ V)$ is negative ; it is asymptotic to zero as V tends to infinity. The value of $\phi\ (H_e,\ V)$ at the minimum will in general increase with increase of altitude owing to decrease in the thrust available.

As only two independent variables are involved the relation can be presented in a three-dimensional diagram. An attempt to sketch such a diagram is shown in Fig. 5. The surface $A_1B_1C_1CC_2B_2A_2A_1$, which represents $\phi\ (H_e,\ V)$ has a valley, the trough of which gives the variation of V with H_e for which $\phi\ (H_e,\ V)$ is always a minimum. The variation of V and $\phi\ (H_e,\ V)$ with energy height H_e when a particular climb technique is used is represented by a curve, such as B_1BB_2, on this surface. The variation of $\phi\ (H_e,\ V)$ with H_e with this climb technique is given by the projection P_1PP_2 of B_1BB_2 on the plane $V = 0$, and the time required for a given change of energy height is the area under the corresponding part of this curve.

8. *The Climb Technique Requiring the Minimum Time for a Specified Change of Height and Air Speed.*—The above geometrical representation gives a qualitative picture of the effect of climb technique on the time required to bring about a specified change of height and speed. Consider, for instance, a few specific problems.

8.1. *The Most Rapid Increase of Energy Height with Free Choice of the Aircraft Speed Throughout.*—If there is complete freedom of choice of the air speed throughout, the climb technique which will enable a given increase in energy height (*i.e.* of energy) to be achieved in the minimum time—*i.e.* with the minimum area under the corresponding curve P_1PP_2—is that for which the variation of speed V with height corresponds to the bottom of the valley in the surface $A_1B_1C_1CC_2B_2A_2A_1$. That is, the variation of V with H_e is that given by the projection on the horizontal plane $\phi\ (V,\ H_e) = 0$ of the line along the bottom of the valley.

8.2. *The Most Rapid Transition from a Given Combination of Height and Speed to Another.*—As height, speed and energy height are inter-related, any combination of height and speed can be represented on the diagram in Fig. 5 by an energy height and an air speed. Thus the problem can be represented as the determination of the most suitable path by which to pass over the surface $A_1B_1C_1CC_2B_2A_2A_1$ from, for example, B_1 to B_2. In general, of course, the end points chosen will not be at the bottom of the valley—*i.e.* the starting and finishing speeds will not be those which give the most rapid increase of energy at the respective energy heights ; when they are the technique of section 8.1 would be used.

The most suitable path over the surface is that which descends into the valley as rapidly as possible at the beginning of the climb and climbs out of it as rapidly as possible at the end, since the proportion of the path which is high upon the 'hill-side' should be as small as possible to make the area under P_1PP_2 the minimum possible. The rapidity of descent into or ascent from the valley is, however, limited ; the velocity of the representative point P over the surface has components $dH_e/dt = \{\phi\ (V,\ H_e)\}^{-1}$ parallel to the H_e axis and dV/dt parallel to the V axis, and hence the rate at which the representative point P can descend into or ascend from the valley is limited, at any one position, by the limitations on the range of dV/dt. This must lie between $g\left(\frac{T-D}{W}+1\right)$ and $g\left(\frac{T-D}{W}-1\right)$, which correspond to vertical descent and vertical ascent respectively.

Thus, if the specified starting speed were below the speed corresponding to the bottom of the valley of Fig. 5 at the relevant energy height and the specified final speed above, the best climb technique would be (according to this simplified consideration) to start with a vertical descent (unless the climb is started at ground level) in order to achieve the optimum speed as quickly as possible, climb at the speed corresponding to the valley up to a point near the desired end point and then descend vertically again until the desired speed and height had been attained.

These conclusions are, however, based on approximations and ignore, for instance, the effect of the considerable changes in normal accelerations which will occur during any violent changes in flight-path angle. The technique outlined is probably a close approximation to the real optimum, but it is thought that moderate departures from it would entail little loss of time.

8.3. *The Most Rapid Transition from one Height to Another with Free Choice of Air Speed Throughout.*—Representation of this problem on Fig. 5 necessitates the introduction of lines of constant true height (H) on the representative surface. To avoid confusion these are sketched in a new figure (Fig. 6) ; they are the family of lines whose projections on the plane $f\ (V,\ H_e) = 0$ are given by the equation

$$H_e = H + \frac{1}{2}\frac{V^2}{g},$$

where H is a parameter.

The best climb technique is represented by the curve on the representative surface which joins the relevant pair of these lines in such a way as to make the area under the corresponding projected curve P_1PP_2 as small as possible. In this case the problem is complicated by the fact that the initial and final values of H_e may be varied, but it seems fairly clear that the greater part of a long climb would be best made according to the technique corresponding to the bottom of the valley. The choice of technique at the start and finish of the climb requires a more detailed examination than is proposed in the present note.

9. *Proposed New Rules by which to Choose Climb Techniques.*—As was remarked in section 3, it is desirable to have general rules by which one may choose a technique sufficiently near the optimum in any given case.

The kinetic and the potential energy of the aircraft are readily interchangeable and hence their relative proportions are not in themselves important except near the beginning and the end of the climb, where they are dictated by the required end conditions. Over the remainder of the climb attention may be concentrated on achieving the most rapid increase in total energy, *i.e.* one may use the technique corresponding to the valley of the representative surface. It is therefore suggested that this be adopted as a basic technique, and that such deviations be added at the beginning and end of the climb as will give a convenient and rapid transition between the basic technique and the end conditions. It is thought that provided the transition is rapid the details are of secondary importance.

This approach is analagous to that adopted by Prandtl for viscous flow, the solution being divided into three regions, one near each of the boundaries and a middle region. Over the middle region the optimum technique is sensibly that given by $\partial\phi/\partial V = 0$; it is this which we have called the 'basic' technique. Within the boundary layers the optimum technique is, of course, determined by the boundary conditions, and will consist of reducing ϕ from its boundary value as rapidly as possible. If, for instance, the initial speed were below that given by the basic technique at the boundary value of H_e the aircraft should, according to the simplified theory given above, be dived vertically, as this would give the most rapid acceleration and hence the most rapid reduction of ϕ.

It is clear that, apart from questions of practicability, the theory given above is inadequate for a precise consideration of the technique within the ' boundary layers ' but the time spent in them is a small proportion of any long climb, and it is thought that any technique reasonably favourable to a rapid transition through them should give a time close to the optimum.

10. *Comparison with Present Practice.*—Present practice is to base the air speed used on the climb on ' partial climbs ' made at constant equivalent air speed $V\sqrt{\sigma}$, and mean height. At each height on the climb the air speed is such that

$$\frac{\partial}{\partial V}\left(\frac{dH}{dt}\right)\bigg|_{H\ \text{constant}} = 0 ,$$

where dH/dt is the rate of climb in a 'partial' climb, at constant equivalent air speed, through a small range of altitude about the height in question. The partial differentiation is made with H constant.

In the technique corresponding to the valley of the representative diagram the air speed at each height on the climb is such that

$$\frac{\partial}{\partial V}\left(\frac{dH_e}{dt}\right)\bigg|_{H_e\ \text{constant}} = 0 ,$$

i.e. a partial differentiation with regard to air speed is made of the rate of change of energy height (*i.e.* of energy), with the mean energy height H_e (*not* H) kept constant. dH_e/dt is equal to the rate of climb which would be obtained in a partial climb made at constant true air speed, V. Thus the technique might be derived from partial climbs made at constant true air speed with the mean energy height kept constant.

To fix ideas a numerical example has been taken. Times from sea level to 40,000 ft and 45,000 ft have been calculated for a hypothetical aircraft similar to the Meteor 4 at climb rating,

(a) climbing according to the customary technique, using the initial and final speeds appropriate to that technique,

(b) climbing according to the new technique, but using the same initial and final speeds as in (a). The climb was started by accelerating in horizontal flight (at sea level) to the speed demanded by the new basic technique, and ended with a 20 deg. zoom—as giving a rapid but convenient transition to the desired end conditions.

Fig. 7(a) shows the two techniques* and Fig. 7(b) the change in the time to height. About 9 seconds is required for the acceleration at sea level, and a rather greater time is spent at the lower altitudes. The excess speed acquired at the price of this can, however, be quickly converted into height if desired, as is illustrated by the two cases shown, and results in a net saving in time to height. When climbing to 40,000 ft the last 1,500 ft is covered in about 6 seconds at

* The kinks in the curves at the tropopause are due to the more rapid fall in engine performance with further increase of height.

the expense of a loss of about 60 ft/sec in speed, whereas in the 45,000 ft case the last 500 ft is covered in about 2 seconds for a loss in speed of about 20 ft/sec. The time gained by the new technique is 1·4 minutes to 40,000 ft (in 16 minutes) and 2·5 minutes to 45,000 ft (in 24 minutes), *i.e.* 9 per cent and 10 per cent respectively. It is considered that the brevity of the transition phases confirms the view expressed above (section 9) that the technique employed in these is of secondary importance.

The above improvement is considerable, and it is considered that the proposed new rules promise to provide a satisfactory basis on which to choose climb techniques for high performance aircraft.

11. *Conclusions.*—In present methods of choosing a climb technique one of the four independent variables involved is neglected to make the problem tractable. This approximation is strictly applicable only when the air speeds used on the climb are low, and its use in connection with recent jet aircraft, which climb at quite high air speeds, is open to question.

By introducing the concept of energy height it has been found possible so to choose the independent variables that one can legitimately be neglected. With this simplification a geometrical representation of the climb performance has been developed to facilitate qualitative grasp of the problem of choosing the best climb technique, including the best method of passing from any one specified combination of height and speed to another.

Brief examination of this problem with the aid of the geometrical representation leads to tentative proposals for a climb technique for general use which promises to be superior to that deduced by present methods. The technique would result in relatively high kinetic energy being acquired at low altitudes, where the thrust available is high, for conversion into potential energy (*i.e.* height) at high altitudes.

In a particular example the proposed technique reduced the time required on the climb by 1·4 minutes (9 per cent) for a climb to 40,000 ft and by 2·5 minutes (10 per cent) on a climb to 45,000 ft.

12. *Further Developments.*—It is hoped to investigate the proposed technique experimentally with a view to confirming its superiority over present methods.

REFERENCE

No.	*Author*	*Title, etc.*
1	Kaiser	The Climb of Jet Propelled Aircraft—Part I. Speed along the Path in Optimum Climb. RTP/TIB Translation GDC/15/148T. (April, 1944).

APPENDIX

The Term in the Climb Equation Containing $KD_h \sin^2\gamma$

If we denote the total drag in straight level flight by D_h and the induced drag in this condition by KD_h (section 4·1) then for straight flight along a path inclined to the horizontal at an angle γ the total drag is

$$(1 - K)\, D_h + KD_h \cos^2\gamma = (1 - K)\, D_h + KD_h\,(1 - \sin^2\gamma) = D_h - KD_h \sin^2\gamma.$$

The second term of the last expression is normally small compared with D_h and may, as shown below, be ignored if required. It represents the reduction of the induced drag which results from inclination of the flight path to the horizontal.

The exact form of equation (4) would be

$$\frac{dH_e}{dt} = \frac{V}{W}\,(T - D_h) + K\,\frac{V}{W}\,D_h \sin^2\gamma\ .$$

We are unlikely to want to deal with air speeds below that for minimum drag (which for turbine jet aircraft is approximately the speed for best angle of steady climb) so

$$\frac{KD_h}{W} < \frac{1}{2}\,\frac{1}{(L/D)_{\text{max.}}}\ ,$$

where $(L/D)_{\text{max}}$ is the maximum ratio of lift to drag for the particular aircraft, and will normally be in the range 10 to 15.

For quasi-steady conditions such as may obtain on a long climb $\frac{d}{dt}\left(\frac{1}{2}\,\frac{V^2}{g}\right)$ is not large compared with dH_e/dt and to assess the order of our error we may take

$$\sin\gamma = \frac{1}{V}\,\frac{dH_e}{dt} = \frac{T - D_h}{W} \text{ approximately.}$$

Then
$$\frac{\triangle\left(\frac{dH_e}{dt}\right)}{\frac{dH_e}{dt}} < \frac{1}{2}\,\frac{1}{(L/D)_{\text{max.}}}\,\frac{1}{V}\,\frac{dH_e}{dt}\ ,$$

$$= \frac{1}{2}\,\frac{1}{(L/D)_{\text{max.}}}\,\frac{T - D_h}{W}\ ,$$

$$< \frac{1}{2}\,\frac{1}{(L/D)^2_{\text{max.}}}\left\{\frac{T}{D_{\text{min.}}} - 1\right\}.$$

where $D_{\text{min.}}$ is the minimum value of D_h and hence $D_h/D_{\text{min.}} \geqslant 1$.

If, for instance, $(L/D)_{\text{max.}} \geqslant 10$ and $T/D \leqslant 5$ the error on the quasi-steady climb will be less than 2 per cent.

Thus the error introduced by neglecting the term in $\sin^2\gamma$ is small except perhaps in steep zooms or steep dives, which will be transient conditions. This approximation seems, therefore, to be justified, particularly as a means of visualising the problem of choosing the optimum climb technique ; it should not distort the general view of the problem.

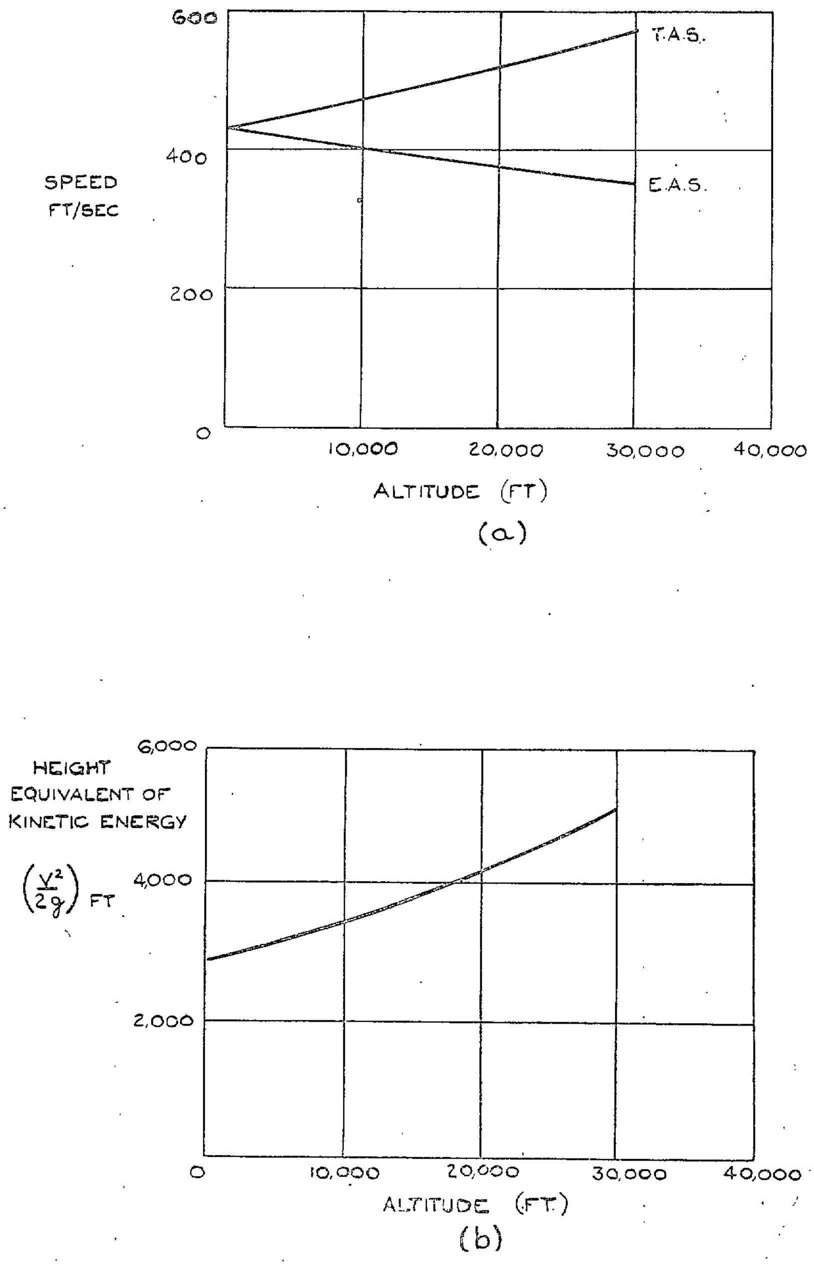
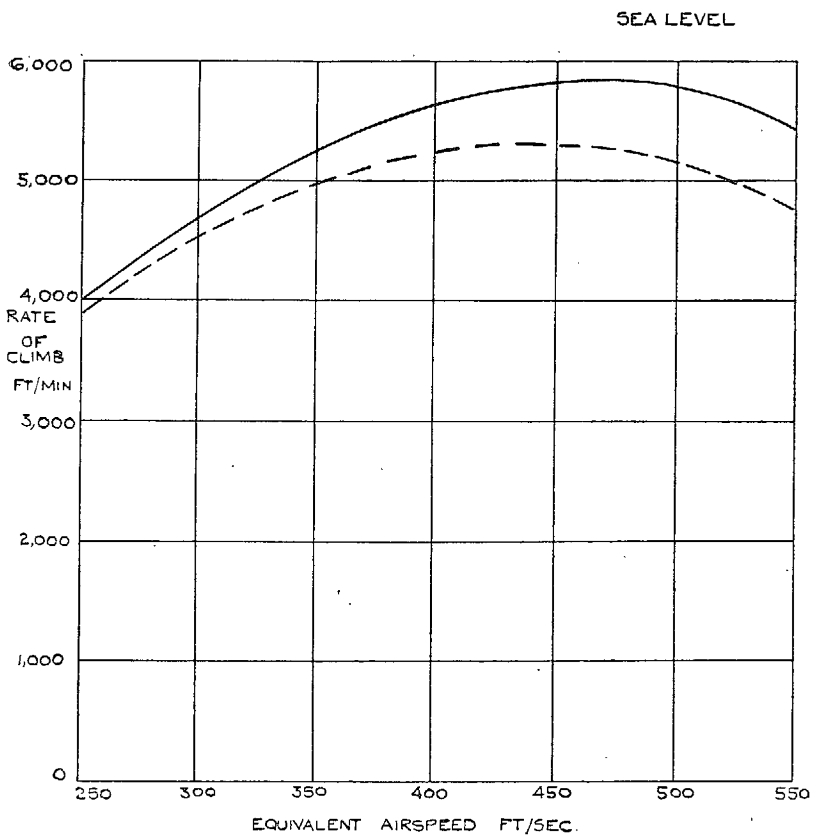


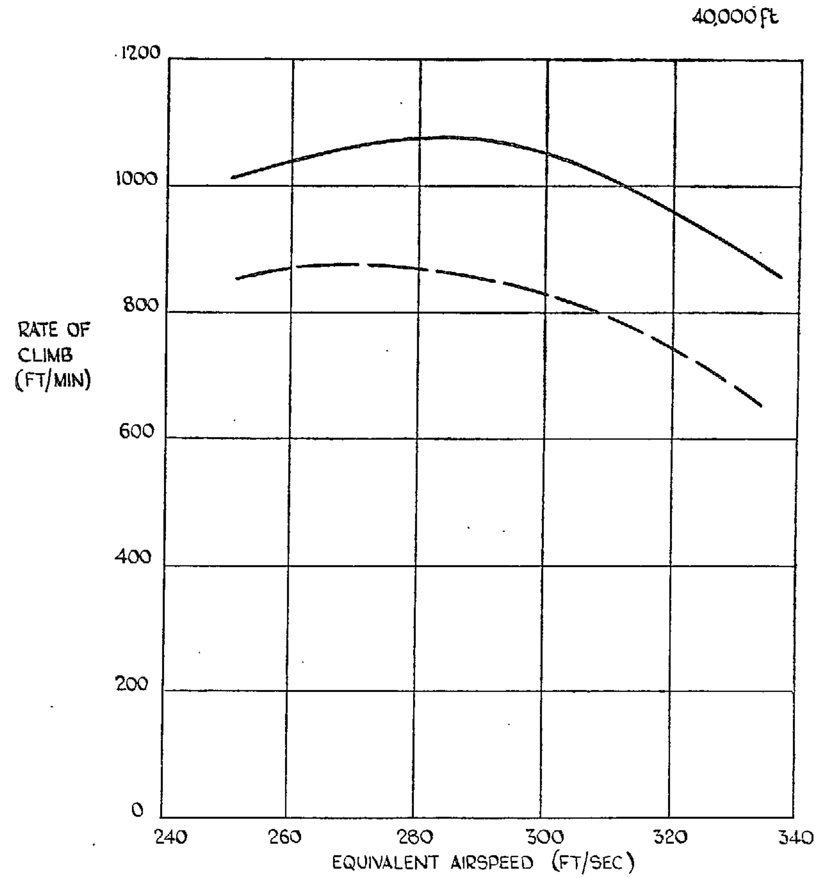
FIG. 1. Speed used on climb and height equivalent of kinetic energy—recent jet fighter.



——— $\dfrac{d\,(V^2)}{dH} = 0$

– – – $\dfrac{d\,(V_t^{\,2})}{dH} = 0 \left(\dfrac{d\,(V^2)}{dH} = +\,2{\cdot}9 \times 10^{-5}\,V^2\right)$

FIG. 2(a). Variation of rate of climb with airspeed (sea level)—hypothetical high performance fighter.

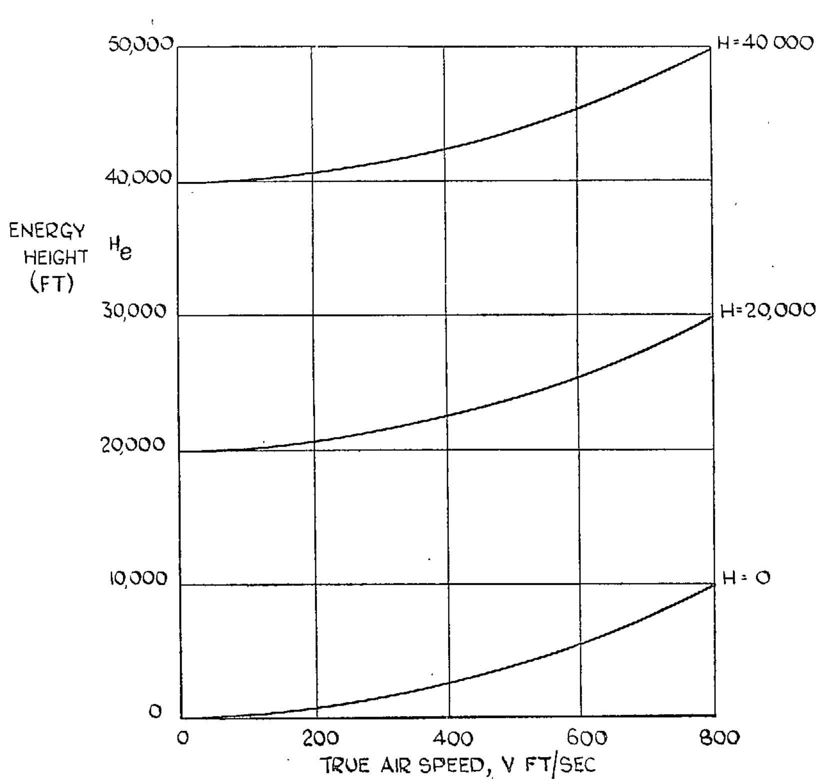


——— $\dfrac{d\,(V^2)}{dH} = 0$

– – – $\dfrac{d\,(V_t^{\,2})}{dH} = 0 \left(\dfrac{d\,(V^2)}{dH} = +\,4{\cdot}7 \times 10^{-5}\,V^2\right)$

Note.—True airspeed = 2·02 × equivalent airspeed.

FIG. 2(b). Variation of rate of climb with airspeed, 40,000 ft—hypothetical high performance fighter.

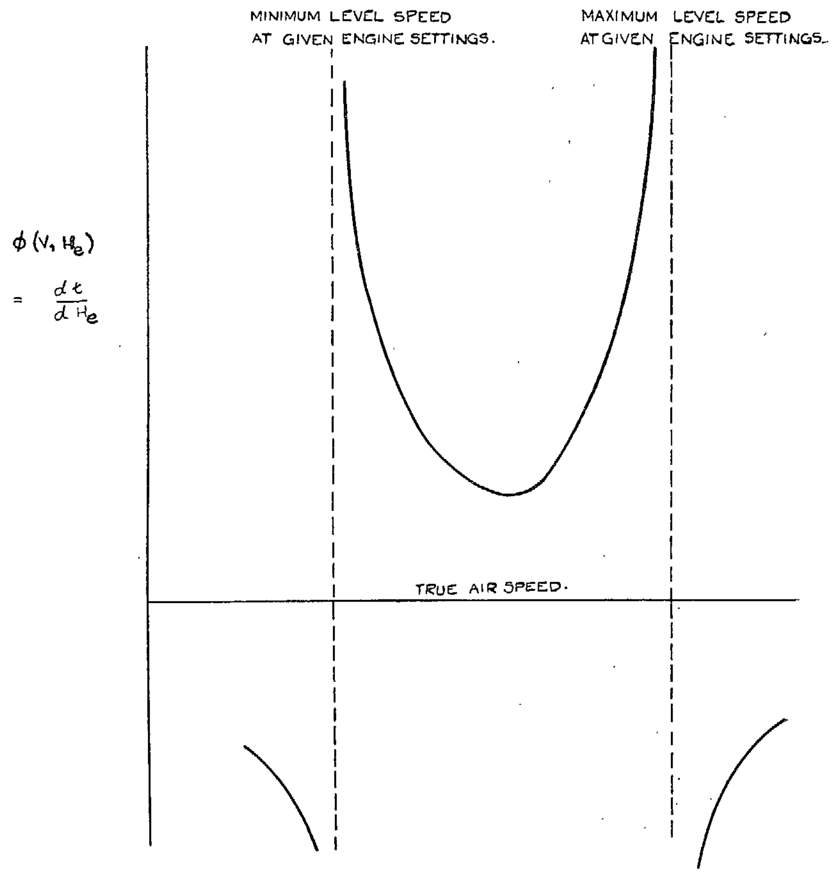


Notes :—

$$H_e = H + \tfrac{1}{2}\frac{V^2}{g}$$

Where H is the true height $H_e - H$ is a measure of the kinetic energy of the aircraft.

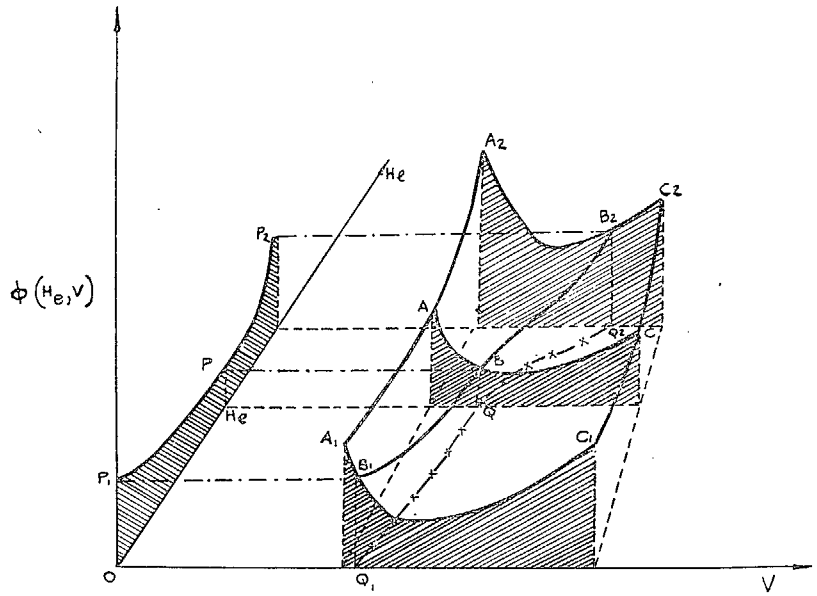
FIG. 3. The relation between " energy height," true height and airspeed.



Note :—

Lower end of curve is cut off by loss of control due to stalling.

FIG. 4. Sketch of variation of ϕ (V, H_e) with airspeed.



Notes.—1. $H_e = H + \frac{1}{2} \frac{V^2}{g}$

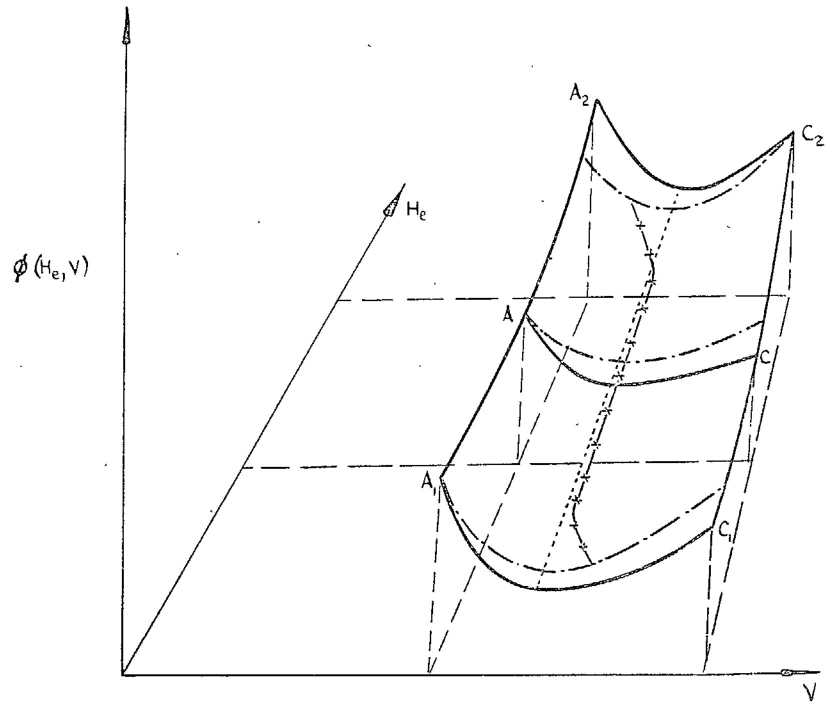
2. $\phi\ (H_e,\ V) = \dfrac{dt}{dH_e}$

3. The surface $A_1\ B_1\ C_1\ CC_2\ B_2\ A_2$ A represents the variation of ϕ $(H_e,\ V)$ with V and H_e. The curves $A_1\ B_1\ C_1$, ABC, $A_2\ B_2\ C_2$ on this surface represent the variation of $\phi\ (H_e,\ V)$ with V at constant H_e.

4. The curve B_1BB_2 on the surface corresponds to a particular climb technique. The projection $Q_1\ QQ_2$ represents the variation of V with H_e on this climb and the projection P_1PP_2 the corresponding variation of $\phi\ (H_e,\ V)$ with H_e for the particular climb technique.

5. The area under the curve $P_1\ PP_2$ is the "time to height" corresponding to the particular climb technique.

FIG. 5. A geometrical representation of the climb performance characteristics of an aircraft.



— · — · — · — ·	Lines on the surface for which true height (H) is constant.
. . . .	The bottom of the "valley" in the surface $A_1\ C_1\ CC_2\ A_2$ A.
—×—×—×—×	Line illustrating technique probably giving quickest change of true height.

FIG. 6. Geometrical illustration of the technique for quickest increase of true height.

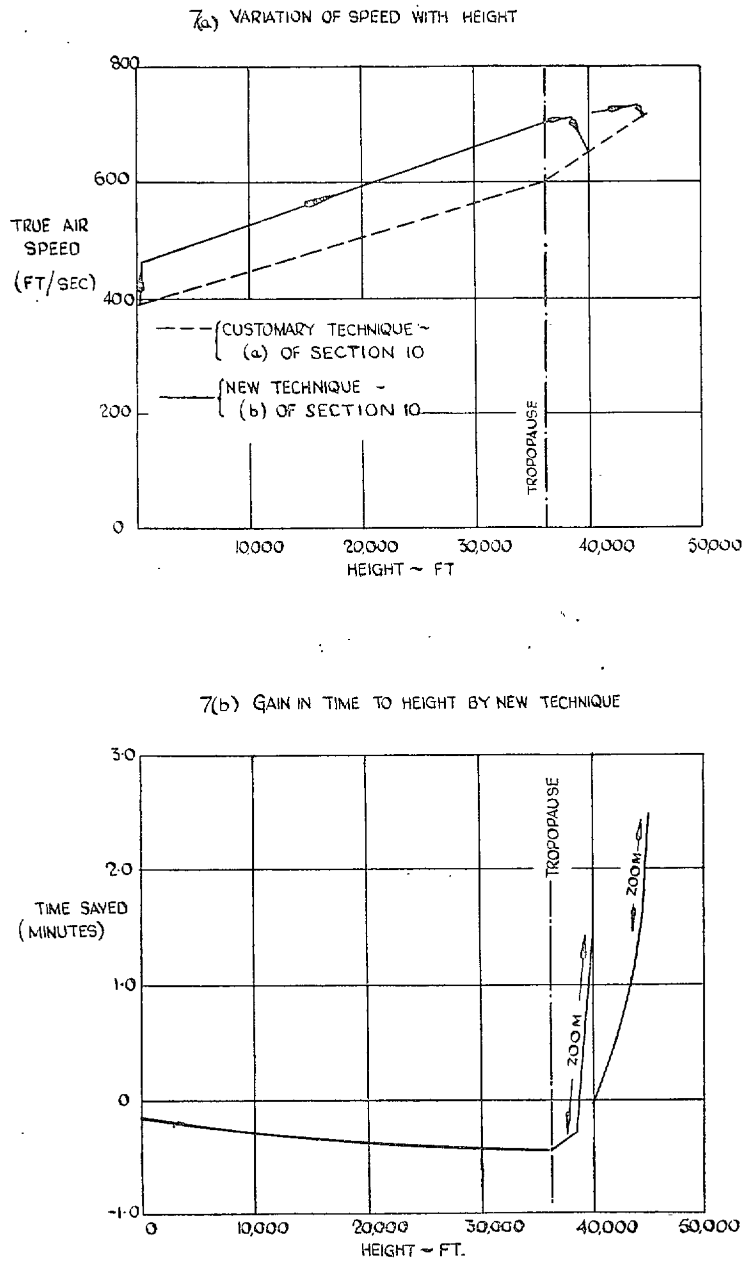


FIG. 7. Climbs to 40,000 ft and 45,000 ft—comparison of new with customary technique.

J4253 Wt.13/806 K9 7/51 D&Co. 34/263

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