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The Combination of Statistical Distributions of Random Loads

By N. I. Bullen, B.Sc.

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The Combination of Statistical Distributions of Random Loads

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March, 1963

Summary.
The case of a Random Gaussian process for which the root-mean-square value is itself variable is considered. A distribution for the root-mean-square value is assumed and an expression is derived for the resulting distribution of peak values of the variable quantity.

This expression is shown to give good agreement with experimental results in many cases of aircraft gust load statistics, and its limitations are discussed.

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*Replaces R.A.E. Tech. Note No. Structures 326–A.R.C. 25 175

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1. *Introduction.*

When an aircraft flies in turbulence, the load imposed on the structure varies continuously and causes fatigue. Although hitherto, work in the fatigue laboratory has been mainly under constant amplitude conditions, workers are now turning more and more to the application of random loads. It is convenient, therefore, to have a statistical description of the load in terms of a few parameters only.

The simplest case is when the variable load behaves as a 'stationary random Gaussian process'. The usual definitions of such a time-varying quantity imply something more than that its distribution about the mean is Gaussian and invariant with time. In particular, the joint distributions of the values of the variable separated by given intervals of time are all multivariate Gaussian distributions.

The properties of a stationary random Gaussian process have been discussed extensively by Rice[1] who derives the distribution of the maxima and minima of the variable and finds an expression for their number which is related to the power spectrum.

These results are useful when the applied load behaves in such a simple fashion, but unfortunately this is not often the case. However, the load distributions met in practice can usually be described sufficiently well by assuming that they are made up of a combination of Gaussian processes, each with its own root-mean-square value.

In this paper, load distributions are derived based on the composite Gaussian case for a particular distribution of root-mean-square values, and it is shown that they give good agreement with observed load distributions.

2. *Theoretical Discussion.*

In all cases considered below the distribution of the variable x is assumed to be symmetrical about zero and the frequency distributions of the variable and of the number of crossings of a given value are all given for the range $0 \leqslant x < \infty$.

2.1. *Simple Case.*

Consider first of all the simple case in which the variable x is a stationary random Gaussian process, the distribution of which is given by:

$$\frac{\sqrt{2}}{\alpha\sqrt{\pi}} \exp\left(-\frac{x^2}{2\alpha^2}\right). \tag{1}$$

The fraction of the total time which the variable spends between x and $x + dx$ is expression (1) times dx.

Since the values of x and x' are independent (*see* Rice), it follows that the number of times the variable crosses the value x is proportional to the time spent in an elementary range dx about x, and therefore

$$N_x = N_0 \exp\left(-\frac{x^2}{2\alpha^2}\right) \tag{2}$$

where N_x is the number of x-crossings per unit time, and N_0 is the number of zero-crossings per unit time.

Press *et al*[2] have shown that for gust loads on aircraft, the crossing distribution is a good approximation to the cumulative distribution of peak values, the percentage error decreasing as x/α increases. For a relatively rigid aircraft the use of expression (2) to derive the peak distribution leads to estimates roughly 10 per cent. low at $x/\alpha = 0$, and less than 3 per cent. low at $x/\alpha = \frac{1}{2}$.

For moderately flexible aircraft such as those from which the majority of counting accelerometer and American *V-g-h* records have been obtained, the errors are somewhat larger:

$$30 \text{ per cent low at } \frac{x}{\alpha} = 0;\ 10\text{–}15 \text{ per cent low at } \frac{x}{\alpha} = 1, \text{ and 2 to 3 per cent low at } \frac{x}{\alpha} = 2.$$

These errors are not likely to be of much significance in practical cases, particularly as most of the data refer to values of x/α greater than 2.

Assuming that (2) gives the cumulative distribution of peak values, their frequency distribution can be derived from differentiating (2) and hence the probability that a given peak value falls in the range x to $x + dx$ is given by:

$$\frac{x}{\alpha^2} \exp\left(-\frac{x^2}{2\alpha^2}\right) dx. \tag{3}$$

2.2. *Composite Case.*

We now consider a family of realisations of a stationary Gaussian random process each with its own root-mean-square value. We assume the family to be large enough, – strictly speaking, infinite – to justify describing the distribution of root-mean-square values in continuous form. Now the mean-square, that is, the variance, is essentially positive and its distribution may be assumed to extend indefinitely in the positive direction, suggesting that an Eulerian (i.e. Pearson Type III) distribution might prove suitable. Accordingly, the following distribution for α^2 is assumed:

$$f(\alpha^2)\,d(\alpha^2) = \frac{1}{(n-1)!}\left(\frac{\alpha^2}{2\rho^2}\right)^{n-1} \exp\left(-\frac{\alpha^2}{2\rho^2}\right)\frac{d(\alpha^2)}{2\rho^2}. \tag{4}$$

The parameter ρ is a scale parameter, and the parameter n, which can take only positive values, determines the shape. (The factor 2 is included in (4) in order to simplify some of the subsequent expressions.) For $n < 1$, the distribution is J-shaped, and for $n > 1$ the distribution becomes hump-shaped, tending to Gaussian form for large n.

Furthermore, the coefficient of variation of the distribution is $1/\sqrt{n}$ so that as n increases the peak becomes more and more pronounced, and therefore the parameter n can be used as a measure of the heterogeneity of α.

The corresponding distribution of α is:

$$f(\alpha)\,d\alpha = \frac{1}{2^{n-1}(n-1)!}\left(\frac{\alpha}{\rho}\right)^{2n-1} \exp\left(-\frac{\alpha^2}{2\rho^2}\right)\frac{d\alpha}{\rho}. \tag{5}$$

Now providing N_0 is constant, the average number of x-crossings per unit time for the whole family (assuming all members to be of the same length) can be obtained by integrating (2) over all values of α. It must be emphasized that this is only justified providing N_0 is constant, and while this is a reasonable assumption to make, there is some evidence to suggest that in the case of aircraft loads due to turbulence, the number of zero crossings depends to some extent on turbulence intensity. This point will be taken up again in Section 2.3.

Assuming then that N_0 is constant, the average number of x-crossings per unit time is given by

$$N_x = \frac{N_0}{2^{n-1}(n-1)!\,\rho^{2n}}\int_0^\infty \alpha^{2n-1}\exp\left(-\frac{x^2}{2\alpha^2}-\frac{\alpha^2}{2\rho^2}\right)d\alpha$$

i.e.

$$N_x = \frac{N_0}{2^{n-1}(n-1)!}\left(\frac{x}{\rho}\right)^n K_n\left(\frac{x}{\rho}\right). \tag{6}$$

where K_n is a modified Bessel function of the second kind as defined and tabulated in Ref. 3.

Fig. 1 shows the distributions of α/ρ given by (5) for a range of values of n, and indicates the progressive change of shape. Fig. 2 gives the corresponding numbers of x-crossings given by (6), (for $N_0 = 1$). As explained above, these are approximately the cumulative distributions of peak values.

Differentiating (6) and dividing out the factor N_0 to obtain the approximate frequency distribution of peak values gives:

$$f(x)\,dx = \frac{1}{2^{n-1}(n-1)!}\left(\frac{x}{\rho}\right)^n K_{n-1}\left(\frac{x}{\rho}\right)\frac{dx}{\rho}. \tag{7}$$

The frequency distribution of the original variable, x itself, is obtained by integrating (1) over all values of α, giving

$$f(x)\,dx = \frac{1}{2^{n-3/2}\sqrt{\pi}\,(n-1)!}\left(\frac{x}{\rho}\right)^{n-\frac{1}{2}} K_{n-\frac{1}{2}}\left(\frac{x}{\rho}\right)\frac{dx}{\rho}. \tag{8}$$

The value of σ^2, the mean square value of x, can be found either by multiplying (8) by x^2 and integrating*; or by finding the average value of α^2 from (4), giving:—

$$\sigma^2 = 2n\rho^2 . \tag{9}$$

The mean-square value of the peak distribution is $4n\rho^2$, twice that of the distribution of x itself. A convenient way to determine the parameters n and ρ for a given peak distribution is by finding the second and fourth moments and the coefficient of Kurtosis (Pearson's β_2). We have

$$\mu_2 = 4n\rho^2, \tag{10}$$

$$\mu_4 = 32n(n+1)\rho^4 \tag{11}$$

and

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{2(n+1)}{n} \tag{12}$$

giving

$$n = \frac{2}{\beta_2 - 2} \tag{13}$$

When $\beta_2 = 2$ we have a Rayleigh distribution of peaks, corresponding to an infinite value of n and a constant value of α, that is, the homogenous case. As α becomes more and more variable, β_2 increases.

Although the use of the second and fourth moments is a convenient way of determining the parameters of the distribution it is not the most accurate. It is more accurate to use the mean deviation and the second moment, as the higher moments have larger sampling errors. However, using this method, it is necessary to solve a transcendental equation involving Gamma functions, and it is by no means so convenient.

An important particular case arises when $\beta_2 = 6$ and $n = \frac{1}{2}$. For this value of n, expressions (6) and (7) degenerate into exponential distributions giving

$$N_x = N_0 \exp\left(-\frac{x}{\rho}\right).$$

The distribution of α^2 is J-shaped, and substituting $n = \frac{1}{2}$ in (5) shows that the distribution of α is the positive half of a Gaussian distribution. The distribution of x from (8) is

$$\frac{2}{\pi\rho} K_0\left(\frac{x}{\rho}\right) dx$$

and (9) gives

$$\sigma = \rho .$$

It is thus a simple matter to determine the root-mean-square value of x from the slope of the peak distribution, or cumulative peak distribution, plotted on a logarithmic scale. The increase in x necessary to decrease the number of peaks by a factor 'e' is its root-mean-square value (but not, of course, the root-mean-square of the peak distribution, which is $\sqrt{2}\,\rho$).

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*In determining the moments of the distribution of x and of the distribution of peak values, the following relation is useful:

$$\int_0^\infty x^r K_s(x)\, dx = 2^{r-1}\left(\frac{r-s-1}{2}\right)!\left(\frac{r+s-1}{2}\right)!$$

2.3. *The Case when N_0 Varies.*

It was emphasised earlier that the results obtained in the above analysis are only true providing N_0 is constant. In general, when N_0 varies the problem becomes more intractable, but in the particular case when N_0 is a power of α the solution follows similar lines to that above.

As before, let the distribution of x be given by (1) and the distribution of α^2 by (4), but in this case let N_0 be dependent on α, the relation being given by

$$N_0 = A\,\alpha^{2m},$$

where A and m are constants. Writing M_x for the number of x-crossings, the integral expression for M_x then becomes

$$M_x = \frac{A}{2^{n-1}(n-1)!\,\rho^{2n}} \int_0^\infty \alpha^{2n+2m-1} \exp\left(-\frac{x^2}{2\alpha^2} - \frac{\alpha^2}{2\rho^2}\right) d\alpha$$

i.e.

$$M_x = \frac{A\rho^{2m}}{2^{n-1}(n-1)!}\left(\frac{x}{\rho}\right)^{n+m} K_{n+m}\left(\frac{x}{\rho}\right). \tag{14}$$

Thus the expression for M_x is of the same form as that previously obtained for N_x; it has the same scale parameter ρ, but the shape parameter is changed from n to $n+m$. The value of M_0 is given by

$$M_0 = 2^m A\,\rho^{2m}\frac{(n+m-1)!}{(n-1)!}. \tag{15}$$

In the case of aircraft normal acceleration response to atmospheric turbulence, there are indications that as the turbulence intensity increases, the number of zero crossings decreases slightly. Expression (14) shows that if the variation can be approximated over the range of interest by a power law, it may still be possible to fit a distribution of the same form. However, unless the power law is known, it will not be possible to infer the parameter n in the distribution of α, or the value of A, although it will be seen that the parameter ρ again determines the scale. If the root-mean-square value of x is inferred in usual way, the spuriously low value of $\sqrt{2(n+m)}\rho$ instead of $\sqrt{2n}\,\rho$ is obtained (m negative), although this may be quite a good approximation if n is numerically large compared with m.

Since $m+n$ may be negative and since $K_n = K_{(-n)}$, expression (14) suggests that in some cases it may be useful to fit distributions of the form:

$$\text{constant} \times \left(\frac{\rho}{x}\right)^p K_p\left(\frac{x}{\rho}\right) \tag{16}$$

where $p = -(n+m)$. This expression becomes infinite when $x = 0$ but may give a good fit over the observed range of values.

3. *Fitting the Distribution to Observed Values.*

A convenient body of data for testing the validity of expression (6) in describing gust load statistics is provided by the results of an investigation, using Canberra aircraft, of low level atmospheric turbulence in North Africa. The full scope of this investigation, known as 'Operation Swifter', is described in Ref. 4.

In the first instance, expression (6) will be fitted to observational gust load material extending over fairly short periods of time during which conditions remain fairly constant, and some of the single runs over the desert will be examined. These runs were of about 100 to 150 miles in length and were made throughout the year under a variety of meteorological conditions and intensities of turbulence. Counting accelerometer records for each run were converted to equivalent gust velocities by the standard discrete gust method. The resulting distributions may be assumed to represent reasonably well the peak load distribution, to some arbitrary scale, that would be produced by the gusts on the aircraft, if its weight and speed remained constant throughout the run.

Tables 1 to 4 give a selection of these cumulative distributions ranging from an exponential to one approaching a Rayleigh, together with the calculated distributions based on expression (6). The comparisons are made diagrammatically in Figs. 3 to 6. (In order to use published tables*, cases where n is close to an integer have been selected, but in other respects these distributions are quite typical.)

It will be seen that the agreement is very good, and confirms that the suggested theoretical expression adequately describes these distributions.

It is also to be expected that the distribution will give a good fit to data representing more extensive flying, providing it is done under fairly uniform conditions. The next example is for the gust distribution obtained from all flights made in the middle of the day over the flat desert during the month of June. Table 5 gives the observed and calculated distributions and they are shown plotted in Fig. 7. The agreement is not as good as for the single legs, but is probably sufficient for most purposes. The most serious discrepancy, from the practical point of view, is for the gusts above 25 ft/sec, where 4 occur, compared with the expected mean of 1.2. However, the actual numbers involved are small, and such a distortion of the 'tail' of the distribution could easily be produced by a few manoeuvre loads inadvertently applied, probably in combination with an already moderately large gust load.

When much longer periods, involving a variety of meteorological conditions, are considered however, expression (6) will not usually give a satisfactory representation of the gust distribution. For example, most of the gust distributions for the year's flying shown in Ref. 4 are obviously different in character from those considered in this paper, having a point of inflection and a marked 'tail'. This is not surprising, since the distribution of α for the whole year might be expected to be far flatter than that implied by expression (5).

An exception is in the case of flying over the sea, where the turbulence intensity varied throughout the year between much narrower limits. As a final example, illustrating the use of the distribution as modified in (16), a curve will be fitted to these data.

It is found that reasonable agreement is given by $p = \frac{1}{2}$ and since

$$\sqrt{\frac{2}{\pi}}x^{-\frac{1}{2}} K_{\frac{1}{2}}(x) = \frac{1}{x} e^{-x}$$

tables of logarithms can be used in the calculation. Table 6 gives the observed and calculated distributions, and they are plotted in Fig. 8.

Generally speaking, the agreement is good but again the tendency is observed for a few additional high loads to occur. The discrepancy, however, is only about 4 occurrences in 44,000 miles and, as before, might easily be due to a few manoeuvre loads.

4. *Conclusions.*

In the case of a random Gaussian process for which the mean square itself varies, when the mean square distribution is of Eulerian (Person Type III) form, the cumulative distribution of peak values of the variable is given approximately by:

$$\frac{1}{2^{n-1}(n-1)!}\left(\frac{x}{\rho}\right)^n K_n\left(\frac{x}{\rho}\right)$$

---

*Tables of the function and its logarithm have now been prepared[5] for the values:

$$x = 0{\cdot}1\,(0{\cdot}1)\,20$$

$$n = \tfrac{1}{2}\,(\tfrac{1}{2})\,6\,(1)\,10$$

This expression gives a good fit to many observed distributions, and it is thus confirmed that the suggested mathematical model is a useful one.

In deriving the above expression, n is necessarily positive, but in some cases a function of the form

$$C\left(\frac{\rho}{x}\right)^{p} K_{p}\left(\frac{x}{\rho}\right)$$

where C is a constant, and p positive, may give good agreement over the observed range of values.

5. *Summary of Results.*

(a) *Simple case.*

Frequency distribution of x

$$f(x)\,dx = \frac{\sqrt{2}}{\alpha\sqrt{\pi}} \exp\left(-\frac{x^2}{2\alpha^2}\right) dx\,.$$

The number of x-crossings per unit time

$$N_x = N_0 \exp\left(-\frac{x^2}{2\alpha^2}\right).$$

Approximate frequency distribution of peak values

$$f(x)\,dx = \frac{x}{\alpha^2} \exp\left(-\frac{x^2}{2\alpha^2}\right) dx\,.$$

(b) *Composite case.*

Let the variance of x, that is, α^2, have the distribution given by:

$$f(\alpha)^2\,d(\alpha^2) = \frac{1}{(n-1)!}\left(\frac{\alpha^2}{2\rho^2}\right)^{n-1} \exp\left(-\frac{\alpha^2}{2\rho^2}\right)\frac{d(\alpha^2)}{2\rho^2}\,.$$

The frequency distribution of α is then:

$$f(\alpha)\,d\alpha = \frac{1}{2^{n-1}(n-1)!}\left(\frac{\alpha}{\rho}\right)^{2n-1} \exp\left(-\frac{\alpha^2}{2\rho^2}\right)\frac{d\alpha}{\rho}\,.$$

Frequency distribution of x

$$f(x)\,dx = \frac{1}{2^{n-3/2}\sqrt{\pi}\,(n-1)!}\left(\frac{x}{\rho}\right)^{n-\frac{1}{2}} K_{n-\frac{1}{2}}\left(\frac{x}{\rho}\right)\frac{dx}{\rho}\,.$$

The number of x-crossings per unit time

$$N_x = \frac{N_0}{2^{n-1}(n-1)!}\left(\frac{x}{\rho}\right)^{n} K_n\left(\frac{x}{\rho}\right).$$

Aproximate frequency distribution of peak values

$$f(x)\,dx = \frac{1}{2^{n-1}(n-1)!}\left(\frac{x}{\rho}\right)^n K_{n-1}\left(\frac{x}{\rho}\right)\frac{dx}{\rho}.$$

Moments and kurtosis of the frequency distribution of x

$$\mu_2 = 2n\rho^2 = \sigma^2$$

$$\mu_4 = 12n\,(n+1)\,\rho^4$$

$$\beta_2 = \frac{3\,(n+1)}{n}.$$

Moments and kurtosis of the frequency distribution of peak values

$$\mu_2 = 4n\rho^2,$$

$$\mu_4 = 32n\,(n+1)\,\rho^4,$$

$$\beta_2 = \frac{2\,(n+1)}{n}.$$

LIST OF SYMBOLS

A	A quantity in the expression defining N_0, when N_0 depends on α
K_r	Modified Bessel function of the second kind as defined and tabulated in Ref. 3
M_x	The number of times the variable crosses the value x per unit time in the case when N_0 depends on α
m	A quantity in the expression defining N_0, when N_0 depends on α
N_x	The number of times the variable crosses the value x per unit time
n	A parameter in the distributions of α and x determining their shape
p	A parameter in the distribution of x given by expression (16)
v	Gust velocity in ft/sec
x	A quantity varying randomly with time
α	The root-mean-square value of x over an elementary time interval
β_2	The coefficient of kurtosis, defined as $\frac{\mu_4}{\mu_2^2}$
μ_r	the 'r'th moment of a distribution
ρ	A scale parameter in the distributions of α and x
σ	The root-mean-square value of x over a long period of time

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3	—	British Association Mathematical Tables Vol. X. Bessel Functions Part II.
4	N. I. Bullen	Gusts at low altitude in North Africa. A.R.C. C.P. No. 581. September, 1961.
5	N. I. Bullen, ... Elizabeth Busby	Tables of function $\frac{x^n K_n(x)}{2^{n-1}(n-1)!}$ for use as Cumulative Frequency Distributions. A.R.C. C.P. No. 765. February, 1964.

TABLE 1

Comparison of observed and calculated gust distributions for 'Swifter' leg 399 (80 *miles*)

Gust velocity v ft/sec	Number of gusts exceeding v ft/sec	
	Observed	Calculated
5	236	236
7·5	73	72·2
10	22	22·1
15	2	2·1

Calculated values given by $2522 \exp\left(-\frac{v}{2{\cdot}056}\right)$

TABLE 2

Comparison of observed and calculated gust distributions for 'Swifter' leg 289 (155 *miles*)

Gust velocity v ft/sec	Number of gusts exceeding v ft/sec	
	Observed	Calculated
5	711	711
7·5	237	235·8
10	67	67·3
15	5	4·2

Calculated values given by $297{\cdot}7\left(\frac{v}{1{\cdot}385}\right)^3 K_3\left(\frac{v}{1{\cdot}385}\right)$

TABLE 3

Comparison of observed and calculated gust distributions for 'Swifter' leg 1735 (140 miles)

Gust velocity v ft/sec	Number of gusts exceeding v ft/sec	
	Observed	Calculated
5	919	919
7·5	414	414·6
10	164	160·3
15	15	17·7
20	1	1·5

Calculated values given by $41{\cdot}63\left(\frac{v}{1{\cdot}524}\right)^4 K_4\left(\frac{v}{1{\cdot}524}\right)$

TABLE 4

Comparison of observed and calculated gust distributions for 'Swifter' leg 119 (152 miles)

Gust velocity v ft/sec	Number of gusts exceeding v ft/sec	
	Observed	Calculated
5	1209	1209
7·5	559	562·3
10	217	216·5
15	21	21·7
20	1	1·5

Calculated values given by $0{\cdot}6324\left(\frac{v}{1{\cdot}289}\right)^6 K_6\left(\frac{v}{1{\cdot}289}\right)$

TABLE 5

Comparison of observed and calculated gust distributions for flying over flat desert at 200 ft at midday during the June period of 'Swifter' trials (2100 miles)

Gust velocity v ft/sec	Number of gusts exceeding v ft/sec	
	Observed	Calculated
5	17,173	17,173
7·5	7,813	7,661
10	2,803	2,863
15	262	279·3
20	26	20·2
25	4	1·2

Calculated values given by $95{\cdot}49\left(\frac{v}{1{\cdot}362}\right)^5 K_5\left(\frac{v}{1{\cdot}362}\right)$

TABLE 6

Comparison of observed and calculated gust distributions for all flying over sea at 200 ft during 'Swifter' trials (43,660 miles)

Gust velocity v ft/sec	Number of gusts exceeding v ft/sec	
	Observed	Calculated
5	32,932	32,932
7·5	4,503	4,541
10	714	705
15	21	20·1
20	5	0·6
25	2	—

Calculated values given by $2{\cdot}423 \times 10^6 \times \frac{1{\cdot}587}{v} \exp\left(-\frac{v}{1{\cdot}587}\right)$

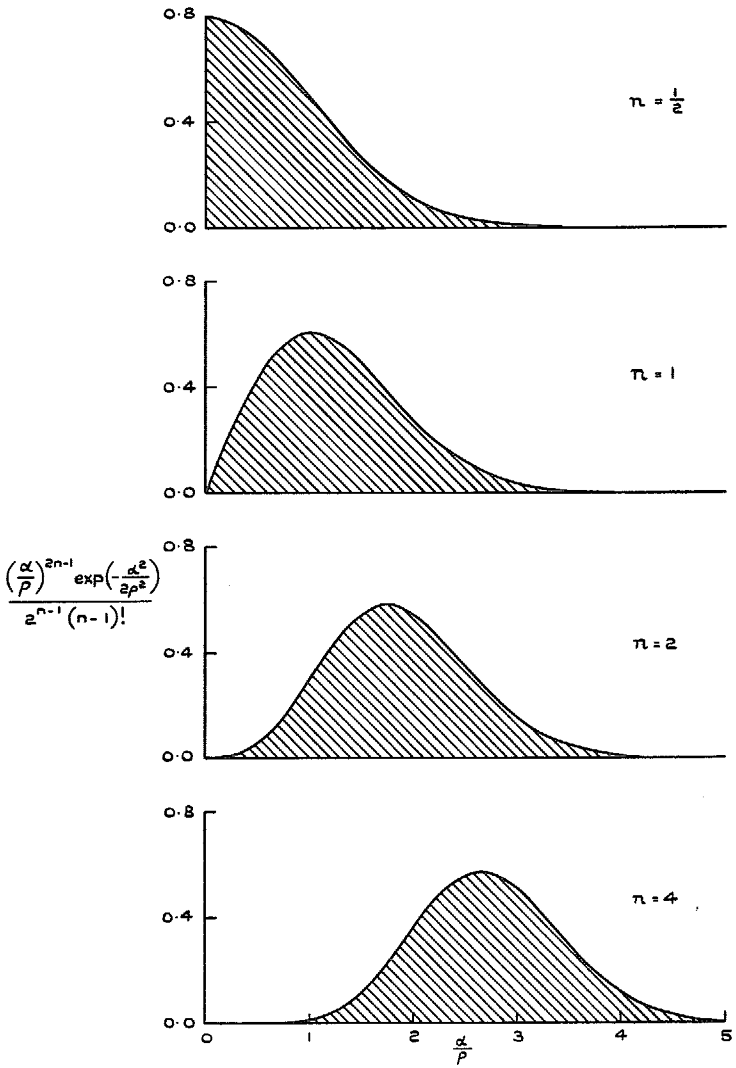

FIG. 1. Distributions of root-mean-square values for a range of values of n.

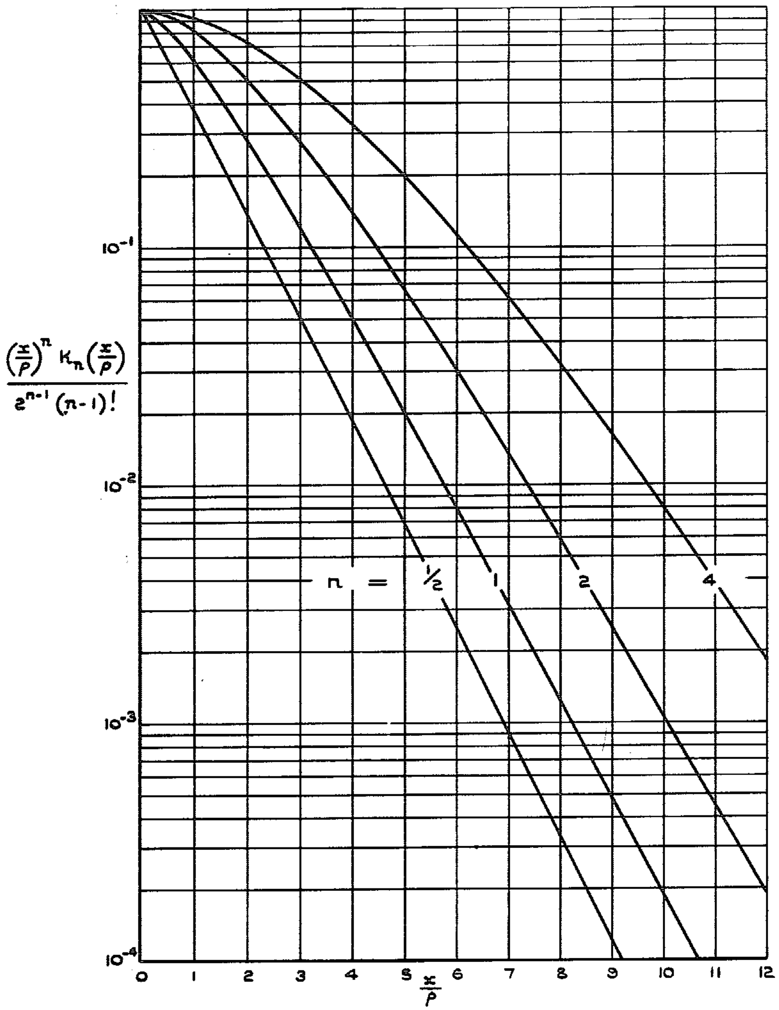

FIG. 2. Numbers of x-crossings for a range of values of n.

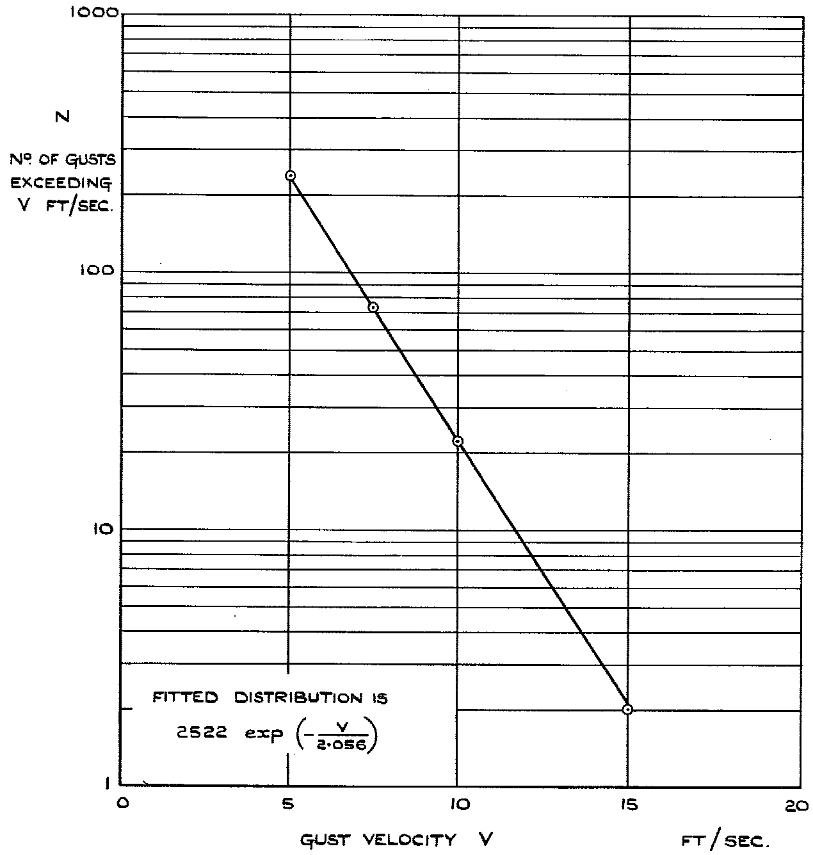

FIG. 3. Gust distribution for 'Swifter' leg 339.

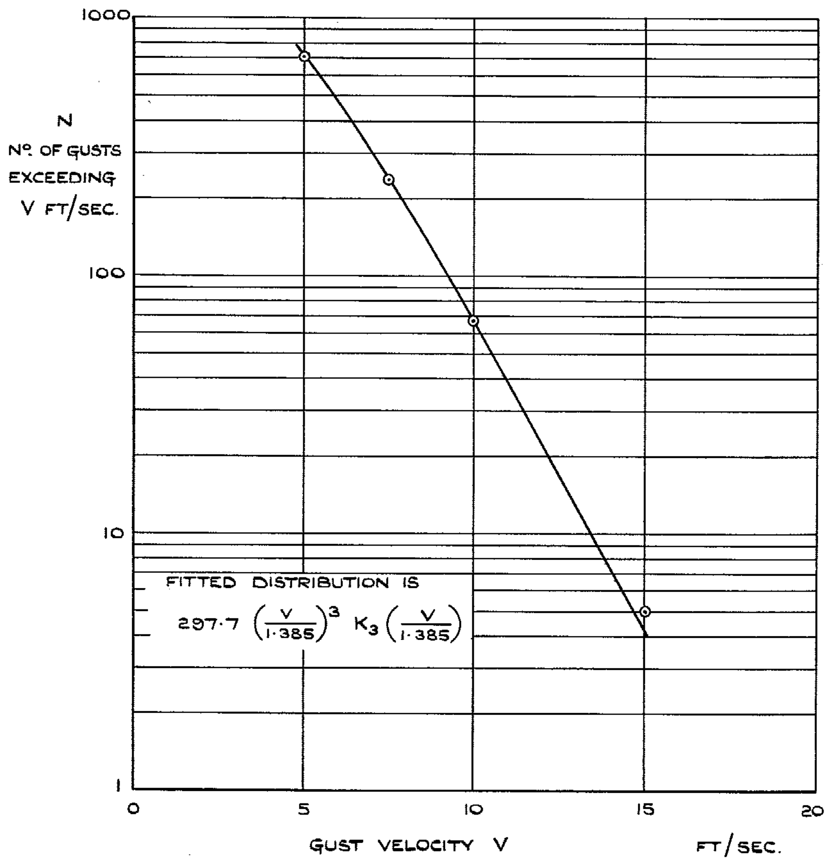

FIG. 4. Gust distribution for 'Swifter' leg 289.

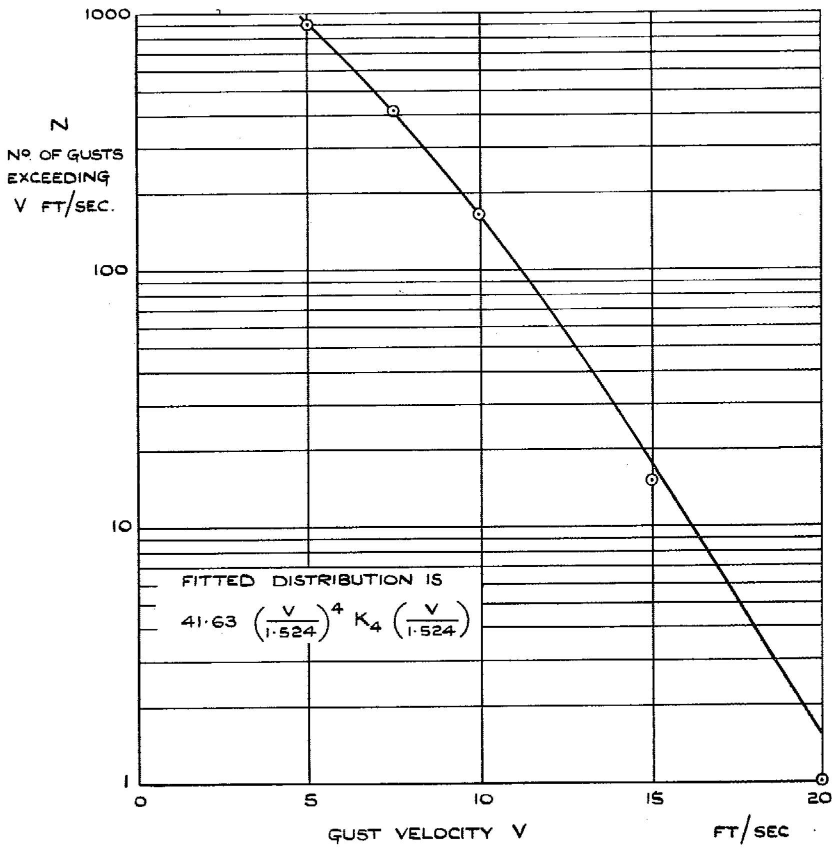

FIG. 5. Gust distribution for 'Swifter' leg 1735.

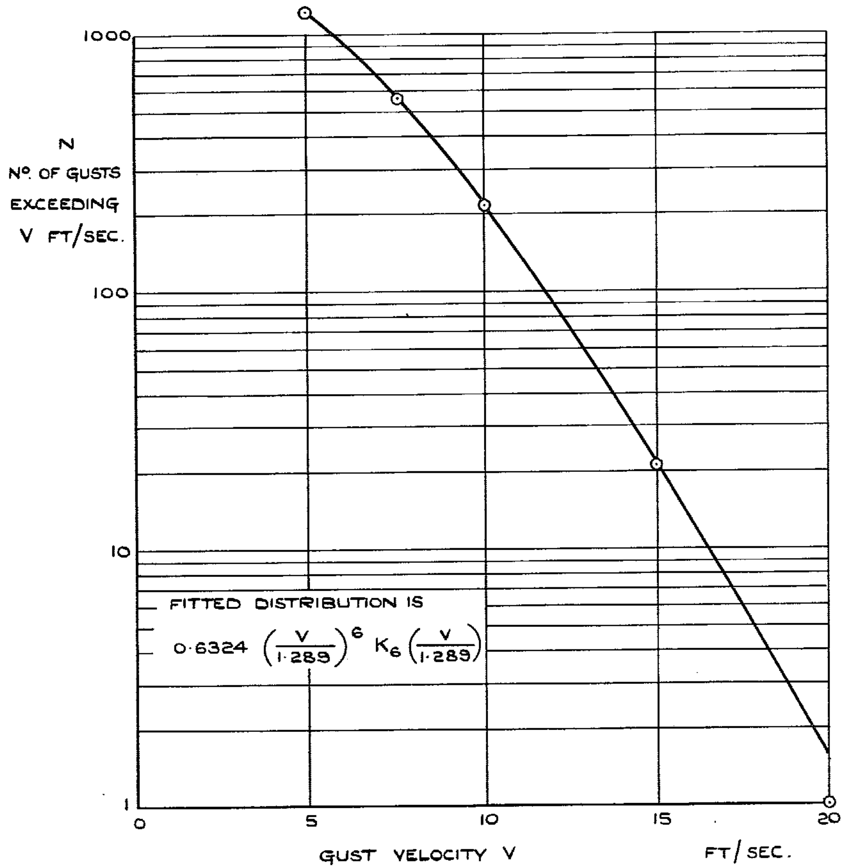

FIG. 6. Gust distribution for 'Swifter' leg 119.

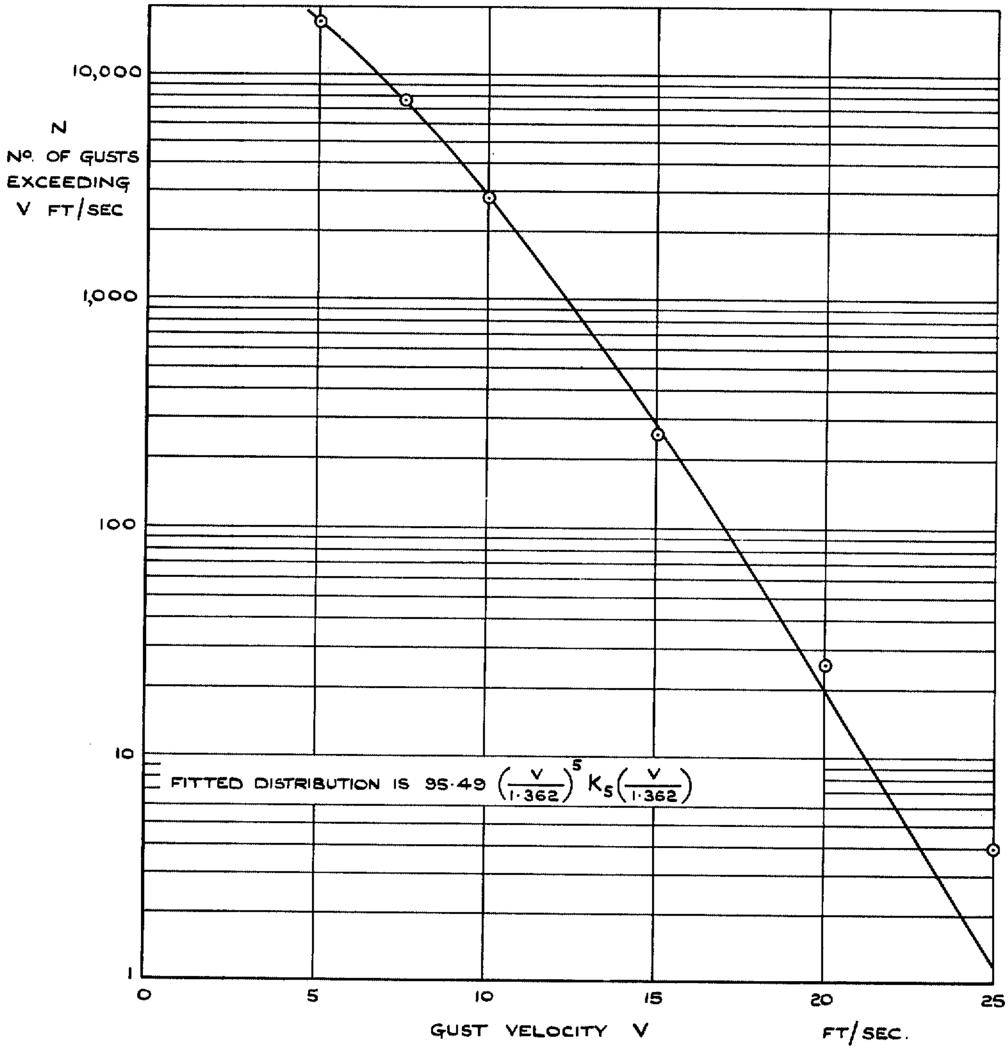

FIG. 7. Gust distribution for flying over flat desert at 200 ft. at midday during the June period of 'Swifter' trials.

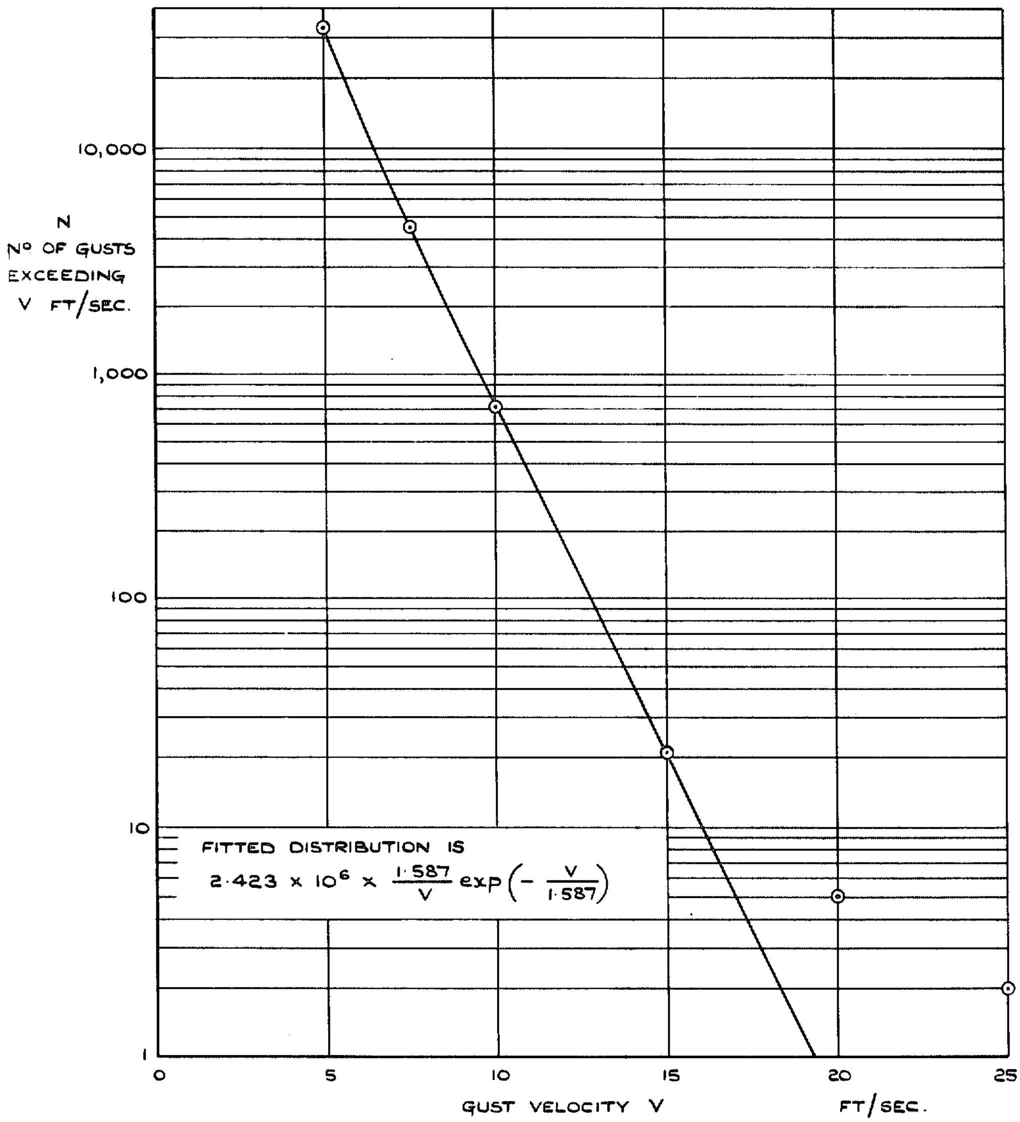

FIG. 8. Gust distribution for all flying over the sea at 200 ft. during 'Swifter' trials.

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