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# A METHOD OF CALCULATING THE EFFECT OF ONE HELICOPTER ROTOR UPON ANOTHER 

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A method of colculating the effeot of one helicopter rotor unon another
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Summary
A model of the induced flov around a lifting rotor is derived hy considering a liftung line approxumation to translational lift and a stream tube model for propelier lift. This theory is applied to a tandem rotor configuration in rectilinear flight. It is shom that the values derived are in good agreemont with the available experimental results, which relate the case of no gap or stagger. In this special case the stream tube effect is negligible and only the lifting line approximation is used.

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## 1. Introduction

Then helicopter rotors are moved into close proximity to one another, an interference flow is set up betweon thcse surfaces which results in a change of overall efficiency of the rotor configuration chosen. This change of efficiency will affoct the performanco of the rotor configuration and in cortain cases may have a marked effect on the handling qualitios. Thus any method which deals with rotor intorforonce probloms should, for completeness, treat the effect of any one rotor on any other independentiy. To do this it is necessary to determine the flow pattern around a singlc lifting rotor in free space. At least two methods (Refs. 1 and 2) have been developed to do this, but in Ref. 1 this has resulted in vory oomplicated equations which have been graphed for a limited range of rotor operating parameters, while Ref. 2 gives a series of non dimensional curves which apply to certain flight conditions. Thus both methods axo limited for practical applioation. Rof. 1 has been used by Squire (Ref. 3) to estimate the stability of a tanden rotor holicoptor, while Ref. 2 is used in this report as one of a series of mothods to be compared. Other methods have been developed to give the total intorference pover loss of a tandom rotor helicopter, those due to Stepniewski (Rers. 4 and 5) being notable for their simplicity. In Refs. 4 and 5 it is assumed that the flow line around a lifting rotor can be considered to be bounded by a stream tube, and that the interference experienced by one rotor can be found by calculating its intersection with the stroam tube of the other rotor. Whilo this muthod approximates to the total power loss of the system, it is not sufficiently detailed to give the performance of an individual rotor as is requirod for stability calculations. The mothod developed in the present roport is an attempt to compromise between these two extreme approachos, in that it can be used in all flight states, for any number of rotors or rotor ring combinations, be sufficiently simple for rapid application, and still give sufficient accuracy for use in stability colculations.

## 2. Flov pattorn in the vioinity of a lifting rotor

In forvard flight the flof in the vicinity of a lifting rotor rescmblos that of an equivalent wing. In lifting line theory the filon pattern around a tring can be represented by a bound vortex located along the quarter ohord line of the ving together with a trailing vortex from each ving tip. Such a model has been used to ropresent the flov around the rotor in the case of high formard speed flight. In hovering and flight at low forward specd the flov: around the rotor resembles that in the neighbourhood of a propeller. By suitably combining thesc two pictures it is possible to develop a model which widil apply over the whole speed range.

In lifting line theory the oirculation $K$ is given by

$$
K=\frac{I}{2 \gamma s V}
$$

Where $L$ is the lift of the wing of sami span $s, V$ the airspeed and $f^{\prime}$ the air densıty. It is proposed to use this expression to deduce the equivalent ciroulation when the lift referred to is not the total valuc but is simply that duc to the rotor acting as a wing. If this fraction of the lift $\bar{L}$ is denoted by $I_{K}$, then $I_{K}$ is obviously zero for vertical flight and zncreases with increasing airspced. It is therefore assumed that IK has the form

$$
I_{K}=W \dot{\{ } 1-\frac{v}{v_{0}} \dot{f}
$$

where $\mathbb{T}$ is the total weight of the helicopter, $v$ the induced flow and $v_{0}$ the induced flow in hovering. Thus

$$
\begin{equation*}
\mathrm{K}=\frac{\mathrm{W} Y_{1}-\frac{\mathrm{v}}{\mathrm{v}_{0}} \boldsymbol{j}}{2 \not \mathrm{RV}} \tag{1}
\end{equation*}
$$

The trailing vortices of the conventional horseshoe pattern associated with the lifting line model are assumed to follor the total resultant flow
immediately boncath tho rotor and thoy aro thus inclined to the tip path plane at an angle $\sigma$ which is given by

$$
\tan P=-\frac{V+V \sin \operatorname{vec} D}{V \cos D}=\frac{-u}{V \cos \angle \alpha D}
$$

Where $二 \mathrm{D}$ is the anglo of the flight path relative to the tip path plane measured positive dormvarde from the rotor and $u$ is the total flow normal to the tip path planc.

The flon due to the rotor acting as a propeller is assumed to be bounded by a cylinder of radius $R$ and to movo with $a$ velocity $-v$; the cylinder is considercd to makc an angle $D$ to the tip path plane. It is appreciated that this does not allow for the vena-contracta and the associated increase in the value of $v$, but due to the uncertainty as to the distance below the rotor at whach this contraction takes place, it is felt that the assumption made here is reprcsentative of the actual flow in the region oonsidered.

## A diagranmatic ropresentation of the model is given in Fig. 1.

## 3. The interference betweon tro rotors arranged in tondem

3.1 General. The layout to be considered is shom in Fig. 2, together with tho flo'l pattom assoclated with cach rotor. The gap $G$ is the vortical soparation and the stagger $S$ the horizontal scparation betwoen the contros of tho rotor tip path planes moasured rolative to tho direction of the free stream. It should be notcd that bocauso the anglc betreen the fuselage and the frce stream direction deponds on the flight condition the gap and stagger as defined of any particular holicopter will not be constant but will change with flight condition. Tho interforence offect can be calculated by the sum of the effect due to the vortex pattorm and that due to the propeller slipstream. In each casc the rosult is first expressed in terms of change in induced velocity at each rotor and this is then converted into equivalent cxpressions for the change in porer and thrust.
3.2 prfect due to vortex pattorm. The horseshoc pattern induces velocitics $\vec{V}_{X}, V_{y}, \vec{V}_{z}$ at a point $X, ~ X, Z$, rclative to the axcs shown in Fig. 2, which are given by the following expressions takon from Ruf. 7 .

$$
\begin{equation*}
\left.V_{x}=\frac{K}{4}: \frac{Z}{x^{2}+z^{2}}: \frac{y+R}{\left(x^{2}+z^{2}+(y+R)^{2}\right)^{\frac{1}{2}}}-\frac{Y-R}{\left(x^{2}+z^{2}+(y-R)^{2}\right)^{\frac{1}{2}}}\right\} \tag{2}
\end{equation*}
$$

$$
\begin{align*}
V_{y}= & \frac{-K}{4^{r}}\left[\frac{-z}{z^{2}+(y-R)^{2}} i_{1}^{1}+\frac{x}{\left(x^{2}+z^{2}+(y-R)^{2}\right)^{\frac{1}{2}}}\right)+\frac{Z}{z^{2}+(y+R)^{2}} \\
& \left.\vdots 1+\frac{x}{\left(x^{2}+z^{2}+(y+R)^{2}\right)^{\frac{1}{3}}}\right] \tag{3}
\end{align*}
$$

$$
\begin{align*}
V_{z}= & \frac{-K}{4 \pi}\left\{\frac{x}{x^{2}+z^{2}}\left[\frac{y+R}{\left.i x^{2}+z^{2}+(y+R)^{2}\right\}^{\frac{1}{2}}}-\frac{y-R}{\left\{x^{2}+z^{2}+(y-R)^{2}\right\}^{2}}\right]\right. \\
& -\frac{y-R}{z^{2}+(y-R)^{2}}\left[1+\frac{x}{\left\{x^{2}+z^{2}+(y-R)^{2} \frac{1}{2}\right.}\right] \\
& \left.+\frac{y+R}{z^{2}+(y+R)^{2}}\left[1+\frac{x}{\left\{x^{2}+z^{2}+(y+R)^{2} \frac{1}{2}\right.}\right]\right) \tag{4}
\end{align*}
$$

For the tandam rotor configuration in rectilinear flight the induced flow due to one rotor on the other is symnetrical about the line of flight so that the mean value of $V_{y}$ across the rotor will be zoro. Due to the nearmess of the two rotors, and to the simplifying assumption that the translational lift can be represented by a lifting line approxination, there is nothing to be gainod by attempting to calculate a mean induced effect over the complete rotor disc; the usual biplane approximation of calculating the cffect along the position of the lifting lino :ijll be used in this work.

The value of $x$ and $z$ in equations 2 and 4 is therefore that of the contro of the scoond rotor reierred to the axes of co-ordinates based on the other rotor; thesc values may be denoted by $x_{0}$ and $z_{0}$ where

$$
\begin{aligned}
& x_{0}=S \cos \left(Q_{1}+\cdots D_{1}\right)-G \sin \left(P+\left(D_{1}\right)\right. \\
& \left.z_{0}=G \cos (P)+W D_{1}\right)-S \sin \left(P+G D_{1}\right)
\end{aligned}
$$

Thus mean valucs of $V_{x}, V_{y}$ and $V_{z}$ denoted by $\bar{V}_{X}, \overline{\mathrm{~V}}_{y}$ and $\vec{V}_{z}$ are given by
$\widetilde{V}_{y}=0$.
$\vec{V}_{z}=-\frac{1}{2 R} \int_{-R}^{R} \frac{R}{4 i}\left\{\frac{x_{0}}{x_{0}^{2}+Z_{0}^{2}}\left\{\frac{y+R}{\left\{x_{0}^{2}+Z_{0}^{2}+(y+R)^{2} ;^{2}\right.}-\frac{y-R}{\left.x_{0}^{2}+Z_{0}^{2}+(y-R)^{2}\right\}^{\frac{1}{2}}}\right\}\right.$
$-\frac{y-R}{z_{0}^{2}+(y-R)^{2}}\left[1+\frac{x_{0}}{\left\{x_{0}^{2}+z_{0}^{2}+(y-R)^{2}\right]^{\frac{1}{2}}}\right)$
$+\frac{y+R}{z_{0}^{2}+(y+R)^{2}}\left[1+\frac{x_{0}}{\left\{x_{0}^{2}+z_{0}^{2}+(y+R)^{2}\right\}^{\frac{1}{2}}}\right]_{1}^{\prime} d y$
Evaluating the integrals by simple substitutions it is found that

$$
\begin{align*}
\bar{V}_{x}= & \left.\frac{K}{4 \pi R} \frac{z_{0}}{x_{0}^{2}+z_{0}^{2}} ;\left[x_{0}^{2}+z_{0}^{2}+4 R^{2}\right\}^{\frac{1}{2}}-\left[x_{0}^{2}+z_{0}^{2}\right\}^{\frac{1}{2}}\right\}  \tag{8}\\
\bar{V}_{z}= & \frac{-K}{4 \pi R}\left\{\frac{x_{0}}{x_{0}^{2}+z_{0}^{2}}\left[\left[x_{0}^{2}+z_{0}^{2}+4 R^{2}\right]^{\frac{1}{2}}-\left[x_{0}^{2}+z_{0}^{2}\right)^{\frac{1}{2}}\right\}\right. \\
& \left.+I_{n}\left[\sqrt{x_{0}^{2}+z_{0}^{2}+x_{0}} \quad x^{z_{0}^{2}+4 R^{2}}\right]\right\}
\end{align*}
$$

For a point upstroam of the lifting rotor the expression for $\overline{\mathrm{V}}_{\mathrm{x}}$ ramains unchanged; $x_{0}$ is nov nogative in $V_{z}$ and the $I_{n}$ term will be small since the distance from the trailing vortuces to the point in question vill in general be large. It will be notod that

$$
x_{0}^{2}+z_{0}^{2}=s^{2}+G^{2}
$$

and this is a characteristzo of the helicoptur, beang equal to the distance between the rotor hubs squared.
$\overrightarrow{\mathrm{V}}_{\mathrm{x}}$ and $\overrightarrow{\mathrm{V}}_{\mathrm{z}}$ are rosolved into two components normal and parallel to the rotor tip path plane, the normal velocity component.s $L$ R is givon by
and the component parallel to the disc, $-1-\mathrm{R} \dot{\rho} \mu_{\mathrm{D}}$ by.

$$
\begin{equation*}
s-\mathrm{R} \dot{\dot{Y}}, \mu_{\mathrm{D}}=\overrightarrow{\mathrm{V}}_{\mathrm{x}} \cos \left(Q-Y_{\mathrm{D} 2}+W_{\mathrm{D} 1}\right)-\overrightarrow{\mathrm{V}}_{\mathrm{z}} \sin \left(Q-\alpha_{\mathrm{D} 2}+\omega_{\mathrm{D} 1}\right) \tag{11}
\end{equation*}
$$

where $\mathcal{S}_{\text {- is }}$ the rotational speed or the rotor.
3.3 Effoct due to propeller slipstream. For the tandem rotor configuration illustratcd in Fig. 2, it can be seen that part of the front rotor will exporience a loss of lift due to the downash from the rear rotor. The rear rotor will expcrience a loss in lift duc to the flow into the front rotor, but since the ohange induced in the flow above and upstream of a rotor in the power on flight condition is much smaller than the change induced in the flor' pattern bencath the same rotor, the loss in lif't by the rear rotor vill be small by comparison vith the loss of lift of the front rotor and vill be noglected. The increase i, $u$ in the mean value of the induced flow $u$ over the whole disc oan be exprossed as

$$
\begin{equation*}
\hat{\partial u}=u \frac{\varrho_{A}}{A} \tag{12}
\end{equation*}
$$

where $\int_{A}$ is the masked area of the rotor disc of area $A$, and assuming as in para. 2 that the inoroase in velocity belor the rotor diso to the vona contracta is negligiblo in the region considered. Since the mean downvash from the rotor inclinca backwards at an angle $P$, $\delta A$ will be the arca of interscotion between two circles of radius F with oentres separated by $\mathrm{S}^{\prime}$ where

$$
S^{\prime}=S-G \frac{V}{u}
$$

Thus $6 A / A$ is expressed by

$$
\begin{equation*}
\frac{\lambda_{A}}{A}=\frac{2}{11} \int_{-}^{\cos ^{-1}}\left(\frac{S^{\prime}}{2 R}\right)-\frac{S^{\prime}}{2 R}\left(1-\frac{S^{1}}{4 R^{2}}\right)^{\frac{1}{2}} j \tag{13}
\end{equation*}
$$

provided that $S^{\prime} \leqslant 2 R$. Hence cambining 12 and 13 and dividang by $\operatorname{li}^{\prime}-R$ it is

$$
\begin{equation*}
\therefore \lambda=\frac{2 \lambda}{\pi}\left\{\cos ^{-1}\left(\frac{S^{i}}{2 R}\right)-\frac{S^{1}}{2 R}\left(1-\frac{S^{2}}{4 R^{2}}\right)^{\frac{1}{2}}\right\} \tag{14}
\end{equation*}
$$

There will also be a small contribution to ${ }^{\mu}$ ad but for the usual valuos of $G$ and $S$ this is negligible.

Combining equations (10) and (14) together gives the change in $\lambda$ due to slipstroam and vortex effects, so that the combined equation together with equation (11) will enable the performance of one rotor in the field of another to bo calculated.

## 4. The performance of trio rotors armanged in tandem

4.1 General. The performance of a rotor is given in blade clement theory by the expression for thrust $T$ and torque 0 ,

$$
\begin{align*}
& T=\frac{1}{2} f^{\prime a b c J} \operatorname{la}^{2}\left\{\frac{\theta}{3}+\frac{\lambda}{2}+\frac{1}{2} \mu_{d}^{2} \theta^{j}\right\}  \tag{15}\\
& Q=\frac{1}{2} \mu_{b c s}{ }^{2} R^{4} \hat{\lambda} \frac{\hat{i}}{4}\left(1+\mu \alpha^{2}\right)-a \therefore \frac{\theta}{3} \hat{j} \tag{16}
\end{align*}
$$

when $a$ is the laft curve slope of the blade section, $b$ the number of blades in the rotor, 0 the mean chord, $\theta$ the goomotrical collective pitoh, $\mu \lambda$ the tip spocd ratio, $\lambda$ the inflor ratio and $l s$ the mean drag of the blade at a mean lift cocfficiont $C_{L}$. When this rotor is moved anto the vicinity of anothor rotor assuming that the controls of the rotors arc fixed, the thrust $T$, and torque 8 , are given by

$$
\begin{equation*}
\left.T_{1}=\frac{1}{2} \mu_{a b o L^{2} R^{3}\left\{\frac{\theta}{3}+\frac{\lambda}{2}+\frac{\xi_{i} \lambda}{2}+\frac{1}{2}\left(\mu_{d}+\xi \mu_{d}\right)^{2} \theta\right\}}^{?}\right\} \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.e_{1}=\frac{1}{2} f-b \cos { }^{2} R^{4} \hat{\{ } \frac{\Delta}{4}\left(1+\left[\mu_{a}+\delta \mu_{a}\right]^{2}\right)-a(\lambda+\ell \lambda) \frac{\theta}{3}\right\} \tag{18}
\end{equation*}
$$

Hence, to the first order of small quantities,

$$
\begin{equation*}
\frac{T_{1}}{T}=1+\frac{1}{2}\left\{\frac{\hat{A}_{1} \lambda+2 / \mu_{d} E / \mu_{a^{\theta}}}{\frac{\theta}{3}+\frac{\lambda}{2}+\frac{\mu_{d}^{2}}{2} \theta}\right\} \tag{19}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{Q_{1}}{Q}=1+\frac{\frac{1}{2} \Delta \mu_{\mathrm{a}} \delta \mu_{\mathrm{a}}-\mathrm{a} \delta \lambda \theta / 3}{\frac{\Delta}{4}\left(1+/ \mu_{\mathrm{d}}^{2}\right)-a \lambda \frac{\theta}{3}} \tag{20}
\end{equation*}
$$

$\delta \lambda$ is very much largor than $i \mu_{d}$ for normal values of $G$ and $S$ and hence it can be scon from equations (19) and (20) that the effect of one rotor on another is to decreaso the thrust and increasc the torque vhen $\lambda \lambda$ is negative i.e. when the rotor being considered is above the datum rotor.

### 4.2 Calculation of the interference power loss for a tandem rotor

 configuration operating at twice the woight of an equivalent single rotor helioopter. If in any flight condition a rotor operating at particular values of $\mu, \lambda, \int L, \theta$ and producing a thrust $T$, is brought intg the vicinity of anothor rotor it will experience a decrease in thrust if $S$ and $\theta$ are kropt constant. To maintain constant thrust $\theta$ is incroased to $\theta+\mathcal{L} \theta$, where$$
\begin{equation*}
\left\{, \theta\left\{\frac{1}{3}+\frac{\mu_{a}^{2}}{2}\right\}=-\left\{\frac{\hat{i} \lambda^{\prime}}{2}+\mu^{\prime}+\hat{0} \mu_{\mathrm{a}} \theta\right\}\right. \tag{21}
\end{equation*}
$$

to the first order of small quantities provided that $f$ is kept constant. In keoping 51 oonstant the torque 2 vili increase to $Q_{2}$ given by

$$
\begin{align*}
& Q_{2}=\frac{1}{2} f^{\prime} b 0 \ln ^{2} R^{4}\left\{\frac{a}{4}\left(1+\left(\mu^{1}+\delta \mu_{a}\right)^{2}\right)-\frac{a}{3}(\lambda+\delta \lambda)(\theta+\delta \theta)\right\} \\
& =Q+\frac{1}{2} \rho_{b} c-N^{2} R^{4}\left\{\frac{\Delta}{2} /{ }^{2} \delta_{3} \mu \mathrm{a}-\frac{a}{3}(\theta \delta \lambda+\lambda \delta \theta)\right\} \tag{22}
\end{align*}
$$

Substituting from equation (21) for $\hat{\imath} \theta$, equation (22) becomes $Q_{2}=Q+\frac{1}{2} \% b c+2 R^{4} \int \mu \mathrm{a} \delta / \mu_{\mathrm{d}}\left[\frac{\theta}{2}+a \frac{\lambda}{3} \frac{\theta}{\left(\frac{1}{3}+\mu \mu_{d}^{2 / 2}\right)}\right]$

$$
\begin{equation*}
-\frac{a^{6} \lambda}{3}\left[\frac{\lambda}{2\left(\frac{1}{3}+\mu_{a}^{2 / 2}\right)}-\theta\right]^{\prime} / \tag{23}
\end{equation*}
$$

Hence the increase in power $\{P$ is given by

$$
\begin{align*}
& -\mathrm{a} \frac{\lambda}{3}\left[\frac{\lambda}{2\left(\frac{1}{3}+\mu a^{2 / 2}\right)}-\theta\right] \tag{24}
\end{align*}
$$

## 5. Comparison of theory and exvoriment

The theory developed here has been compared with some of the theories discussod in para. 1 for the case of a tondcm rotor helicopter with rotors comparable to those of the Bristol type 173 inks. 1 and 2, and in a flight condition there $G=0$ and $S=2 R$. It is soen from Fig. 3 that the cerliest theory of Stcpnnowski (Ref. 4) is in closest agreement with the prosent mothod. The thoory of Rof. 4 vas designed for this configuration only and was used in the devolopment or the Piascoki H. 21 helioopter which has $G=0$ and $S=2 R$ and the agreement with it of the prosent theory is oncouraging. Although it must be noted that the propeller slipstream effect is negligiblo when there is nu gap, and stagger is equal to twice the rotor radius and that only the "lifting line",part of the tineory is confinued by this argunent. The more advanced method of Stepniewski (Rer.5), which zas deszgned to allow for variation in $S$, shows poorer agreement with the presont theory. Comparing with the more elegant method of Castles and de Leeurl (Ref. 2) reasonable agreoment is found particularly at speeds where $\delta(P$ is a maximum. The present theory is therefore found to bc in reasonable agreement with the established methods of References 2 and 4 for the case $G=0$ and $S=2 R$; which uses only the "lifting line" part of the theory; its advantages over these methods have boen discussed in para. 1.

In Reference 8 Dingldein has mado an experimental study of a model tandan rotor configuration again with no gap and $S=2 R$ and mas compared the results obtained with the theory of Castles and do Leeum. In this comparison It was show that reasonable agreemont existed at low forward speeds but that the theory was optimistio at high speed. A comparison of the experimental rosults of Reference 8 and the present theory is show in Fig. 4 from which it can bo scen that the agreement is good over the whole specd range.

Figure 5 shows the interfcrence povicr curve calculated for the Brastol 173 which has $G,=0.20, S=1.7 \mathrm{R}$. Comparison with the ourvo for $S=0$ and $G=2 R$ copicd from $\mathrm{Fi}_{\mathrm{j}} \mathrm{G} 3$ shows that the offect of the change of gap and stagger is only notzceable at iow speeds.
6. Conclusion

The thoory which has beon developed in this report is applicable to any multi-rotor configuration and to any flight condition other than when a rotor is working in the vortex ring statc. Comparison of the theory and the only roliable set of experamental results has shown good agrecment over the whole of the speed range tested, the experimental conditions being such that only the "lifting line" part of the present theory could be confirmed.

This method can obviously be oxtended to oover the case of lifting wing-rotor configurations. It may also be modificd to cover the onse of yarred flight of a tandem rotor helicopter vhich is important for stability considorations.

## 7. Acknowledgoments

I wish to record my approciation of the assistance given me by G. F. Langdon in his helpful criticisms and suggestions (in partzcular the form of the cquation for $I_{K}$ ), and by $S$. Thite who carried out much of the computation work.
8. Reforences
(1) Mangler \& Squiro
(2) Castios and de Lecuv
(3) Squire
(4) Stepniewski
(5) Stepnicwski
(6) Oliver
(7) Glauert
(8) Dingledein

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## List of Symbols

a
A.
$\therefore \mathrm{A}$
b
c

G
K
工
$x, y, z$
$x_{a}$
$x_{b}$
$O \cdot D$
lift curve slope of rotor blade section
area of rotor
arca of front rotor masked by rear motor
number of blados per rotor
rotor blade mean chord
gap between rotors - see Fig. 1.
circulation over acrofoil
lift of wing
translational lift of helicopter rotor.
rotor power
change in rotor powor due to intcrferenco effects
rotor torque
rotor torque defincd by equation 20.
rotor torque defined by equation 24 .
rotor radius
wing semi span
stagger between rotors - sce Fig. 1.
$S-G V / u$
thrust of single rotor
rotor thrust dofined by equation 19.
total flow normal to the rotor, negative for flow domnard
forvard airspecd
induced flow of rotor at a forward speed $V$, negative for flow downvard
induced flow of rotor in hovering, negative for flow dommard. $\left(i \pi / 2 \pi / \rho_{0} R^{2}\right)^{\frac{1}{2}}$, negativc for flow downard velocitics at point $x, y, z$, inđuced by horseshoe vortex pattern. all up veight of holicoptor co-ordinates of point relative to axes shown in Fig. 2 $S \cos \left(Q-\alpha_{D 1}\right)-G \sin \left(Q-\alpha_{D 1}\right)-R \cos \left(\varphi+\infty \alpha_{D 2}-\alpha_{D 1}\right)$ $S \cos \left(P-\alpha_{D 1}\right)-G \sin \left(P-\alpha_{D 1}\right)+R \cos \left(P+\alpha_{D 2}-\alpha_{D 1}\right)$
angle of flight path relative to tip path plane - negative upwards

| U | mean drag of rotor blade oporating at a mean $\mathrm{C}_{\mathrm{L}}$ |
| :---: | :---: |
| $\theta$ | geanctrical collcotivo pitch |
| 入 | u/S S |
| $\mu \mathrm{a}$ | $V \cos x_{D} / 1 . \mathrm{R}$ |
| - | air density |
| $p$ | $\tan ^{-1}\left[-u / v \cos \chi_{D}\right]$, |

FIG.I.



## DIAGRAMMATIC REPRESENTATION OF FLOW AROUND A LIFTING ROTOR.

FIG. 2.

'Y'AXE ARQANGED TO
EOXM DEHT HANCEO
SET NTH XAZ AXES

DIÄGRAMMATIC REPRESENTATION OF FLOW AROUND A TANDEM ROTOR CONFIGURATION.


$$
a^{-1} 0
$$

COMPARISON OF THEORETICAL METHODS.

FG. 4.


FIG 5.


THEORETICAL POWER LOSS FOR BRISTOL TYPE 173.

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