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Influence of Variable Specific Heats on the High-Speed Flow of Air

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SUMMARY

In the calculation of compressible fluid flows it is assumed that the fluid, particularly air, has the properties of a perfect gas with constant specific heats. This assumption is adequate when the range of temperatures under consideration is not unduly wide, and the numerical results of the simplified theory are sufficiently accurate, provided that a correct value for the ratio of specific heats, γ , is selected. Flow functions are sufficiently complicated to warrant the preparation of detailed tables, and if such tables are to be of practical use, they ought to be prepared for a range of values of the ratio of specific heats, which, for example for air at low pressures varies from 1.399 at 600°F. abs. to 1.281 at 6400°F. abs., or from 1.398 at 600°F. abs. to 1.245 at 4500°F. abs. for combustion gases resulting from the combustion of hydrocarbon fuels. In actual practice most flow functions for air are calculated under the assumption of one specific value for γ , irrespective of the range of temperatures; in most cases the value

$$\gamma = 1.400$$

is used for convenience, when dealing with air.

On the other hand, with increasing Mach numbers in the study of aerodynamic phenomena, or both Mach numbers and stagnation temperatures in gas turbine problems, the error committed by stipulating constant specific heats increases and becomes significant. This paper contains an introductory analysis of this error in relation to the fundamental temperature - Mach number relationship for the steady and adiabatic flow of air. It is shown that the variation of specific heats, or their ratio, γ , with temperature, can be easily taken into account in calculations if they are based on existing tables of the thermodynamic properties of air. Skeleton tables of the temperature variation with Mach number and stagnation temperature are given and the deviation from the equation stipulating constant specific heats, namely:-

$$\frac{T}{T_o} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2}$$

is analysed.

It/

It is found that the error committed by adopting this equation, generally increases with increasing stagnation temperature T_o ; its dependence on the Mach number of the flow is more unexpected, in that it is found that it exhibits a maximum in the neighbourhood of $M = 2.0$. The maximum error in calculating T for a given M in the above equation may reach 370°F at $T_o = 5000$°F abs. with $M \hat{=} 2.2$. If the value $\gamma = 1.400$ is used, then all errors in T are negative, the more accurate value being <u>higher</u> than that calculated from the constant specific heats equation and increasing with stagnation temperature. If the value $\gamma = 1.300$ is used then, as expected, the errors are bigger for the lower range of stagnation temperatures, but smaller for the range of stagnation temperatures from 3000 to 5000°F. abs. In the lower range of stagnation temperatures the error is positive, but for higher stagnation temperature, the sign of the error depends on the Mach number. The distribution of errors is given in Fig. 5.

(1) Introduction

Present-day calculations of compressible fluid flow functions are almost exclusively based on the assumption that the working fluid is a perfect gas whose specific heats and isentropic index $\gamma = c_p/c_v$[+] are constant. The equations encountered in the calculations are sufficiently complicated to warrant the preparation of suitable tables or diagrams. Such aids to calculation have been published by Jamison and Mordell (3)[*], Keenan and Kaye (5), Emmons (2) and Dailey and Wood (1). Some functions were also calculated by Kestin and Oppenheim (7).

There is at present no general agreement as to the best value to be assigned to the isentropic index, γ , which appears in all flow functions, when the assumption of constant specific heats is made. In the case, which is most important in practical applications, namely for air, values such as

1.405 1.403 (10) 1.400 1.399 etc.

are encountered, preference being given to the rounded off value of 1.400.

In actual fact, if the working fluid is assumed to obey the perfect gas law, it can be shown that the two specific heats, c_p and c_v and their ratio, γ , are dependent on temperature[**]. The nature of this dependence and the mathematical expressions for the relevant functions are best derived from spectroscopic data. Such data are now available (5) (12) and it seems appropriate to assess with their aid the error committed in the calculation of the flow of compressible fluids, when the assumption of constant specific heats is made. It will be shown that the error is sufficiently great to justify the calculation of new tables, which would take into account the variation of the isentropic index, γ , with temperature.

It is evident that it is impossible to survey the whole field of compressible fluid flow in a single paper; on the other hand, it will be appreciated that in most cases the calculation begins with the

evaluation/

[+] A list of symbols is given on p.6

[*] Figures in brackets refer to the bibliography given on p.7

[**] See ref. 11 p. 123 or ref. 4 p.100.

evaluation of the stream temperature, T, for a given Mach number, M, and stagnation temperature, T_o. All other quantities are then easily deduced. Consequently, in this paper we shall analyse the method of calculating T for given values of T_o and M from the data for air, which have been tabulated by Keenan and Kaye (5). The results will be subsequently compared with the constant specific heat equation

$$\frac{T}{T_o} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2}, \qquad \ldots (1)$$

whose derivation can be found in a number of standard text-books[*].

(2) Assumptions and basic equations

It will be assumed that the working fluid is a perfect gas, so that its specific heats, their ratio, the enthalpy and internal energy are functions of temperature alone. These functions have been tabulated for air and several combustion gases in ref.5. Approximate methods for combustion gases were given in ref.6 and also in refs. 5 and 8. The variation of the ratio of specific heats, γ, for air is given in Fig.1.

It is well known that no simple mathematical expression can be designed to fit these data accurately, so that it is necessary to make use of tables.

From Fig.1 it is seen that the value of γ for air varies from 1.399 at 600°F. abs. to 1.281 at 6400°F. abs., moreover, the rate of change of γ with temperature, which is plotted in Fig.2, is seen to be small at the lower and higher ends of the range. It is largest in the vicinity of 1000°F. abs.

The variation of γ for gases resulting from the combustion of hydrocarbon fuels is very much the same as for air (6), and the range of values is from 1.398 at 600°F. abs. to 1.245 at 4500°F. abs. for fuels containing about 15 per cent carbon by weight and depending on the air/fuel ratio.

This type of temperature dependence suggests that the deviation from equation (1) will be largest in the middle of the temperature range and that it will tend to decrease both for very high and for very low temperatures.

In order to derive the equivalent of equation (1) for the case with variable specific heats, we can proceed as follows. The velocity of flow is given by the energy equation

$$V^2 = 2(h_o - h) \qquad \ldots (2)$$

which does not stipulate any specific thermodynamic properties of the working fluid. The velocity of sound is given by

$$a^2 = \gamma RT \qquad \ldots (3)$$

the equation being valid for perfect gases with constant or variable specific heats. In the above equations h_o, h and γ are tabulated functions of temperature.

Consequently/

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[*] For example ref. 9 p.25.

Consequently, we obtain for the Mach number:

$$M^2 = V^2/a^2$$

$$= \frac{2(h_o - h)}{\gamma RT} \;. \qquad \ldots (4)$$

If in this equation M and h_o are given, and it is desired to find T, it is necessary to proceed by trial and error, since h and γ also depend on T. Alternatively, it is possible to employ a graphical method by seeking the point of intersection of the two functions

$$Y_1 = \tfrac{1}{2}M^2\gamma RT \qquad\qquad Y_2 = h_o - h \qquad \ldots (5)$$

of which the first can easily be calculated in terms of h with M as parameter, and the second is a straight line in terms of h, with h_o as parameter. It would, therefore, be fairly simple to devise a suitable geometric construction, as e.g. in Fig.3. Neither of these methods seems to be suitable for practical calculations and it appears that it is best to proceed as outlined in the following section.

(3) Temperature - Mach number tables

It will be noticed that equation (1) represents a function of two variables, the temperature ratio T/T_o and the Mach number M, and its tabulation present no difficulty for one value of γ. It has been pointed out earlier that it is necessary to adjust the value of γ to the temperature range under consideration, so that, consequently, a set of tables for various values of γ is needed.

When the variation of specific heats with temperature is taken into account, as in equation (4), there are still only three variables to consider; they cease, however, to be dimensionless. The variables are

$$M \quad T_o \quad \text{and} \quad T$$

since h_o is a function of T_o and γ and h are functions of T. Obviously R is still a constant. Furthermore, it will be noticed that if T and M are given, then it is easy to calculate the corresponding stagnation enthalpy h_o and to interpolate T_o from tables. This fact was utilised in preparing the set of skeleton tables given at the end of this paper. The tables were prepared in two steps. In the first step values of M and T were assumed and the stagnation enthalpy h_o was calculated. In the second step values of T_o were interpolated from ref.5 and the result was once more interpolated to obtain equally spaced values of T_o.

It is worth noting here that if the results are presented in a form resembling equation (1), that is if ratios T/T_o are calculated, then the deviation of those values from constancy, as required by equation (1), will provide a graphic description of the error committed when applying equation (1). A plot of T/T_o for various values of M for air is given in Fig.4.

The plot reveals that at the two ends of the temperature scale the value of T/T_o for a given value of M tends to become constant, as expected from the analysis of Figs. 1 and 2.

Since/

Since the variation of T/T_o is not unduly large, the results of calculations, which were performed with the aid of an electric calculating machine, were smoothed by making the second differences in

$$T/T_o = f(T_o)$$

equal to zero over fairly wide ranges of values. In this process no smoothed value differed from that calculated directly by more than 1°F., and differences were calculated for intervals of 100°F.

A direct assessment of the error is given in Fig.5 in which is plotted the difference between the value of stream temperature as calculated with the aid of equation (1), and that found in the Tables of this paper. The error was calculated for two values of γ, namely $\gamma = 1.300$ and $\gamma = 1.400$.

It is surprising to note that the error in calculating the stream temperature may reach, under certain circumstances, a value of 370°F. Generally speaking, the error increases with increasing stagnation temperature. It is always negative, that is the temperature is always calculated too low, if the value of $\gamma = 1.400$ is used. With a value of $\gamma = 1.300$ it is positive for lower values of T_o and partly positive and negative for higher values of T_o.

The most interesting feature is the dependence of the error on the Mach number. It appears that it reaches a maximum in the vicinity of $M = 2.2$ with $\gamma = 1.400$, when the temperature calculated by equation (1) is too low by some 370°F. for $T_o = 5000$°F. abs. With $\gamma = 1.300$ the error tends to have a negative maximum in the vicinity of $M = 1.4$ and a positive maximum above $M = 4.0$ for higher stagnation temperatures. With $\gamma = 1.400$ the error is negligible only about up to

$$T_o = 1000\text{°F. abs.}$$

The accurate values of T from equation (4) in terms of the interpolated stagnation temperature T_o, with the Mach number M as parameter are given in the Tables. It is believed that the accuracy of that Table is the same as in ref. 5, that is, that no appreciable additional error was introduced in the process of calculation.

(4) The temperature - Mach number chart.

The accuracy of the Tables is probably higher than that required for most purposes. In order to provide a more convenient means of rapid calculations, it was decided to present them graphically in a convenient form. The two co-ordinates of the chart are:

temperature T

and Mach number M.

The curves on the diagram represent the variation of temperature with Mach number for a given value of T_o which is indicated at $M = 0$. The values of the speed of sound, a, the ratio of specific heats, γ, and the enthalpy h, are given on the auxiliary diagram in terms of temperature. Since at each point of the main chart, the value of T, and hence, a, as well as the value of M are fixed, it follows that the velocity

$$V = M \times a \qquad \ldots (6)$$

is also fixed. Points for which these values are equal have been

connected/

connected into curves of constant velocity on the main chart. Values of $\mathcal{V}$ can be interpolated linearly along lines T i.e. a = const. The method of using the chart is explained in Fig.6.

In conclusion it might be mentioned that the T, M diagram could be used in conjunction with a temperature entropy chart for air for solving more complicated problems of flow in a graphical manner.

(5) Acknowledgements

The author wishes to acknowledge the help which he received from two of his students, Messrs. W. Konkow and S. Chmielowski, in the calculation of tables and in the preparation of diagrams and drawings.

List of symbols.

a - velocity of sound

h - enthalpy

M - Mach number

R - gas constant $\left(= 53.342 \, \frac{\text{ft. lb.}}{\text{lb.}^{*} \, {}^{\circ}\text{F}} = 0.068549 \, \frac{\text{Btu.}}{\text{lb.}^{*} \, {}^{\circ}\text{F}} \right)$ for air[*]

T - stream temperature (absolute)

$T^{(1)}$ - stream temperature calculated from constant specific heats (distinction made only when necessary).

T_o - stagnation temperature

$\mathcal{V}$ - velocity of flow

Y_1, Y_2 - temperature functions (eqn. (5))

γ - ratio of specific heats (isentropic index)

References/

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[*] See ref. 5 p. 202.

lb.^{*} denotes lb.-mass.

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TABLES/

TABLES

Stream temperature T for varying stagnation temperature T_o, with Mach number, M, as parameter.

T_o	Mach number						
	0.4	0.5	0.6	0.7	0.8	0.9	1.0
400	387.60	380.88	372.93	364.17	354.45	344.05	333.12
500	484.55	476.15	466.30	455.28	443.13	430.14	416.48
600	581.52	571.48	559.74	546.50	531.94	516.33	499.93
700	678.49	666.96	653.31	637.93	621.02	602.86	582.76
800	775.63	762.58	747.12	729.61	710.47	689.75	668.04
900	872.94	858.36	841.18	321.65	800.07	777.02	752.75
1000	970.37	954.38	935.50	914.05	890.39	865.00	838.15
1100	1067.91	1050.60	1030.10	1006.76	981.00	953.40	923.90
1200	1165.58	1146.95	1124.91	1099.75	1072.03	1042.05	1010.22
1300	1263.31	1243.44	1219.89	1193.04	1163.32	1131.26	1097.16
1400	1361.10	1340.03	1315.02	1286.52	1254.91	1220.67	1184.45
1500	1458.98	1436.73	1410.31	1380.16	1346.73	1310.48	1272.00
1600	1556.87	1533.48	1505.65	1473.92	1438.70	1400.49	1359.84
1700	1654.79	1630.23	1601.06	1567.74	1530.77	1490.65	1447.92
1800	1752.73	1727.07	1696.53	1661.68	1622.96	1580.94	1536.15
1900	1850.68	1824.00	1792.09	1755.68	1715.22	1671.32	1624.46
2000	1948.65	1920.79	1887.58	1849.67	1807.49	1761.74	1712.88
2100	2046.60	2017.64	1983.17	1943.67	1899.83	1852.18	1801.32
2200	2144.58	2114.45	2078.69	2037.71	1992.21	1942.68	1889.84
2300	2242.54	2211.29	2174.19	2131.73	2084.48	2033.16	1978.37
2400	2340.49	2308.18	2269.71	2225.74	2176.76	2123.59	2066.82
2500	2438.48	2405.00	2365.23	2319.73	2269.10	2214.07	2155.28
2600	2536.40	2501.80	2460.72	2413.67	2361.39	2304.55	2243.80
2700	2634.38	2598.74	2556.24	2507.74	2453.66	2394.55	2332.28
2800	2732.31	2695.52	2651.76	2601.65	2545.96	2485.35	2420.66
2900	2830.31	2792.41	2747.29	2695.66	2638.24	2575.79	2509.06
3000	2928.27	2889.22	2842.84	2789.73	2730.56	2666.25	2597.70

TABLES/

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Stream temperature T for varying stagnation temperature T_o, with Mach number, M, as parameter.

T_o	Mach number						
	0.4	0.5	0.6	0.7	0.8	0.9	1.0
3100	3026.21	2986.10	2938.36	2883.70	2822.90	2756.77	2686.03
3200	3124.17	3082.93	3033.89	2977.69	2915.09	2847.22	2774.58
3300	3222.12	3179.75	3129.37	3071.70	3007.48	2937.63	2863.01
3400	3320.13	3276.67	3224.93	3165.71	3099.75	3028.11	2951.20
3500	3418.11	3373.57	3320.52	3259.71	3192.13	3118.56	3039.94
3600	3516.04	3470.37	3416.04	3353.80	3284.48	3209.05	3128.50
3700	3613.99	3567.20	3511.51	3447.79	3376.84	3299.55	3217.15
3800	3711.95	3664.07	3607.07	3541.77	3469.15	3390.10	3305.49
3900	3809.89	3760.91	3702.61	3635.80	3561.45	3480.54	3394.08
4000	3907.85	3857.80	3798.10	3729.82	3653.77	3571.00	3482.80
4100	4005.90	3954.70	3893.72	3823.84	3746.10	3661.48	3571.02
4200	4103.85	4051.55	3989.17	3917.90	3838.44	3751.96	3659.51
4300	4201.80	4148.64	4084.80	4011.92	3930.81	3842.45	3748.00
4400	4299.77	4245.56	4180.35	4105.97	4023.15	3932.97	3836.53
4500	4397.72	4342.50	4275.87	4200.03	4115.50	4023.50	3925.07
4600	4495.68	4439.00	4371.36	4294.01	4207.62	4114.02	4013.65
4700	4593.66	4535.97	4466.90	4388.01	4300.18	4204.58	4102.20
4800	4691.59	4632.62	4562.38	4481.98	4392.37	4294.95	4190.74
4900	4789.65	4729.53	4657.92	4575.94	4484.76	4385.40	4279.20
5000	4887.58	4826.41	4753.56	4669.98	4577.00	4475.82	4367.64
5100	4985.55	4923.21	4849.16	4764.14	4669.40	4566.27	4456.10
5200	5083.52	5020.16	4944.69	4858.22	4761.88	4656.82	4544.58
5300	5181.52	5117.02	5040.17	4952.24	4854.27	4747.47	4633.06
5400	5279.49	5213.90	5135.70	5046.28	4946.59	4838.06	4721.79
5500	5377.46	5310.75	5231.34	5140.32	5038.94	4928.53	4810.41
5600	5475.42	5407.60	5326.86	5234.37	5131.29	5019.03	4898.32

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Stream temperature T for varying stagnation temperature T_o, with Mach number, M, as parameter.

T_o	Mach number						
	1.1	1.2	1.3	1.4	1.5	1.6	1.7
400	321.81	310.32	298.63	287.03			
500	402.32	387.94	373.38	358.86	344.30	330.32	
600	483.05	465.76	448.33	430.86	413.31	396.52	379.98
700	564.05	543.91	523.55	503.10	482.44	463.15	443.73
800	645.50	622.46	599.12	575.96	552.62	530.17	507.84
900	727.48	701.53	675.47	649.38	623.36	597.64	572.72
1000	810.03	781.46	752.51	723.41	694.48	666.14	638.28
1100	893.30	861.99	830.20	798.17	766.54	735.24	704.41
1200	977.15	943.16	908.52	873.85	839.30	804.96	771.58
1300	1061.57	1024.97	987.67	950.23	912.73	875.76	839.46
1400	1146.54	1107.35	1067.53	1027.27	987.00	947.27	907.99
1500	1231.77	1190.16	1147.79	1104.89	1062.06	1019.41	977.44
1600	1317.28	1273.29	1228.39	1182.94	1137.50	1092.23	1047.56
1700	1403.07	1356.75	1309.31	1261.41	1213.27	1165.45	1118.14
1800	1489.19	1440.49	1390.63	1340.19	1289.49	1239.03	1189.12
1900	1575.33	1524.41	1472.24	1419.25	1366.07	1312.96	1260.51
2000	1661.61	1608.38	1553.91	1498.61	1442.91	1387.24	1332.19
2100	1747.96	1692.57	1635.69	1578.00	1519.92	1461.86	1404.16
2200	1834.32	1776.68	1717.60	1657.55	1596.97	1536.55	1476.48
2300	1920.72	1860.87	1799.48	1737.13	1674.27	1611.29	1548.81
2400	2007.15	1945.13	1881.44	1816.67	1751.49	1686.21	1621.22
2500	2093.54	2029.35	1963.47	1896.34	1828.70	1761.04	1693.81
2600	2179.90	2113.48	2045.44	1976.08	1905.99	1835.93	1766.26
2700	2266.32	2197.72	2127.31	2055.69	1983.44	1910.90	1838.79
2800	2352.65	2281.98	2209.31	2135.29	2060.72	1986.01	1911.42
2900	2438.94	2366.11	2291.33	2214.98	2138.00	2060.92	1984.19
3000	2525.28	2450.23	2373.15	2294.72	2215.36	2135.84	2056.78

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Stream temperature T for varying stagnation temperature T_o, with Mach number, M, as parameter.

T_o	Mach number						
	1.1	1.2	1.3	1.4	1.5	1.6	1.7
3100	2611.64	2534.39	2455.01	2374.27	2292.78	2210.86	2129.35
3200	2698.04	2618.58	2536.91	2453.80	2369.95	2285.95	2202.00
3300	2784.49	2702.76	2618.82	2533.41	2447.15	2360.79	2274.11
3400	2870.84	2787.08	2700.77	2613.00	2524.40	2435.62	2347.27
3500	2957.20	2871.24	2782.84	2692.68	2601.68	2510.50	2419.75
3600	3043.57	2955.39	2864.74	2772.40	2679.00	2585.42	2492.24
3700	3129.97	3039.57	2946.65	2852.04	2756.40	2660.40	2564.80
3800	3216.40	3123.78	3028.60	2931.64	2833.75	2735.44	2637.42
3900	3302.87	3208.04	3110.55	3011.29	2911.00	2810.51	2710.09
4000	3389.37	3292.35	3192.56	3090.92	2988.32	2885.40	2782.84
4100	3475.80	3376.72	3274.66	3170.69	3065.67	2960.37	2855.45
4200	3562.14	3460.93	3356.76	3250.46	3143.04	3035.35	2928.03
4300	3648.53	3545.10	3438.77	3330.25	3220.49	3110.33	3000.65
4400	3734.95	3629.31	3520.70	3410.03	3297.80	3185.40	3073.97
4500	3821.40	3713.56	3602.66	3489.66	3375.21	3260.53	3146.03
4600	3907.86	3797.82	3684.65	3569.33	3452.81	3335.71	3218.78
4700	3994.38	3882.12	3766.66	3649.01	3530.10	3410.87	3291.60
4800	4080.80	3966.40	3848.69	3728.70	3607.46	3485.79	3364.43
4900	4167.28	4050.70	3930.74	3808.41	3684.87	3560.79	3437.15
5000	4253.67	4135.04	4012.81	3888.18	3762.20	3635.80	3509.75
5100	4340.03	4219.35	4094.91	3967.99	3839.63	3710.83	3582.42
5200	4426.41	4303.50	4177.04	4047.80	3917.10	3785.91	3655.13
5300	4512.76	4387.68	4259.00	4127.56	3994.58	3861.02	3727.84
5400	4599.14	4471.85	4340.92	4207.44	4072.08	3936.17	3800.54
5500	4685.78	4556.05	4422.85	4287.04	4149.61	4011.32	3873.35
5600	4772.41	4640.37	4504.74	4366.64	4227.05	4086.47	3946.17

TABLES/

TABLES

Stream temperature T for varying stagnation temperature T_o , with Mach number, M , as parameter.

T_o	Mach number						
	1.8	1.9	2.0	2.5	3.0	3.5	4.0
400							
500							
600	363.78	348.12	333.18				
700	424.84	406.46	389.06	310.95			
800	486.26	465.38	445.17	355.92			
900	548.42	524.74	501.73	401.13	322.56		
1000	611.01	584.71	559.43	447.00	359.40		
1100	674.57	645.60	617.49	493.50	396.44	321.83	
1200	738.89	706.95	676.41	540.60	434.28	352.57	
1300	803.78	769.43	736.12	588.36	472.55	383.31	314.86
1400	869.80	832.56	796.46	636.89	511.14	414.64	340.86
1500	936.42	896.30	857.76	685.76	550.20	446.66	366.75
1600	1003.58	960.80	919.57	735.36	590.16	478.68	392.96
1700	1071.70	1026.26	982.12	785.63	630.58	511.19	419.69
1800	1140.11	1092.10	1045.42	836.28	671.29	544.23	446.81
1900	1208.80	1158.60	1109.08	887.77	712.31	577.39	473.94
2000	1277.96	1224.75	1173.14	939.79	754.09	611.10	501.20
2100	1347.43	1291.32	1237.51	992.05	795.87	644.99	529.13
2200	1417.13	1358.97	1302.11	1045.09	838.48	679.12	557.04
2300	1487.10	1426.41	1367.12	1098.26	881.18	713.69	585.20
2400	1557.10	1494.06	1432.32	1151.75	924.37	748.50	613.58
2500	1627.18	1561.71	1497.70	1205.31	967.95	783.44	642.36
2600	1697.39	1629.48	1563.07	1259.30	1011.92	818.78	671.14
2700	1767.52	1697.42	1628.56	1313.39	1055.94	854.28	700.11
2800	1837.46	1765.22	1694.20	1367.79	1100.18	890.14	729.32
2900	1907.96	1833.09	1759.74	1422.33	1144.76	926.32	758.73
3000	1977.87	1901.07	1825.32	1477.09	1189.35	962.68	788.14

TABLES/

TABLES

Stream temperature T for varying stagnation temperature T_o, with Mach number, M, as parameter.

T_o	Mach number						
	1.8	1.9	2.0	2.5	3.0	3.5	4.0
3100	2048.34	1969.19	1891.02	1531.83	1234.24	999.04	817.91
3200	2118.94	2037.17	1956.83	1586.55	1279.22	1036.05	846.68
3300	2189.22	2105.03	2022.58	1641.42	1324.41	1073.09	877.94
3400	2259.61	2173.05	2088.18	1696.47	1369.76	1110.17	908.10
3500	2329.89	2241.11	2153.87	1751.33	1415.22	1147.40	938.67
3600	2399.98	2309.16	2219.62	1806.21	1460.93	1184.62	969.23
3700	2470.15	2376.95	2285.45	1861.24	1506.63	1222.06	999.80
3800	2540.35	2444.78	2351.02	1916.32	1552.68	1259.66	1030.94
3900	2610.64	2512.66	2416.54	1971.52	1598.22	1297.25	1062.09
4000	2680.97	2580.61	2482.13	2026.61	1643.78	1335.13	1093.23
4100	2751.39	2648.61	2547.79	2081.58	1689.66	1373.06	1124.51
4200	2821.76	2716.66	2613.50	2136.62	1735.44	1411.06	1155.81
4300	2891.96	2784.79	2679.24	2191.71	1781.20	1449.27	1187.12
4400	2962.24	2852.74	2745.03	2246.91	1826.95	1487.49	1218.60
4500	3032.55	2920.67	2810.82	2302.13	1872.69	1525.69	1250.22
4600	3102.89	2988.65	2876.50	2357.08	1919.05	1563.87	1281.83
4700	3173.30	3056.68	2942.20	2411.86	1965.18	1602.07	1313.56
4800	3243.76	3124.75	3007.91	2466.72	2011.24	1640.48	1345.46
4900	3314.24	3192.87	3073.67	2521.82	2057.12	1678.89	1377.35
5000	3384.77	3261.09	3139.70	2576.87	2103.00	1717.25	1409.33
5100	3455.11	3329.30	3205.35	2631.98	2149.02	1755.53	1441.50
5200	3525.42	3397.48	3271.28	2687.14	2195.04	1793.82	1473.67
5300	3595.76	3465.48	3337.24	2742.37	2241.18	1832.26	1505.84
5400	3666.28	3533.48	3403.21	2797.63	2287.35	1870.73	1537.97
5500	3736.70	3601.52	3468.27	2852.67	2333.24	1909.24	1570.11
5600	3807.03	3669.51	3534.16	2908.02	2379.02	1947.89	1602.19

TABLES/

TABLES

Stream temperature T for varying stagnation temperature T_o, with Mach number, M, as parameter.

T_o	Mach number						
	4.5	5.0	6.0	7.0	8.0	9.0	10.0
400							
500							
600							
700							
800							
900							
1000							
1100							
1200							
1300							
1400							
1500	305.55						
1600	327.36						
1700	349.29						
1800	371.52	312.66					
1900	394.06	331.71					
2000	416.73	351.00					
2100	439.74	370.23					
2200	462.86	389.46					
2300	486.24	409.01					
2400	509.69	428.94	313.60				
2500	533.56	448.87	328.39				
2600	557.43	468.81	343.03				
2700	581.29	488.74	357.48				
2800	605.28	508.91	372.12				
2900	629.73	529.39	386.95				
3000	654.17	549.86	401.64	304.78			

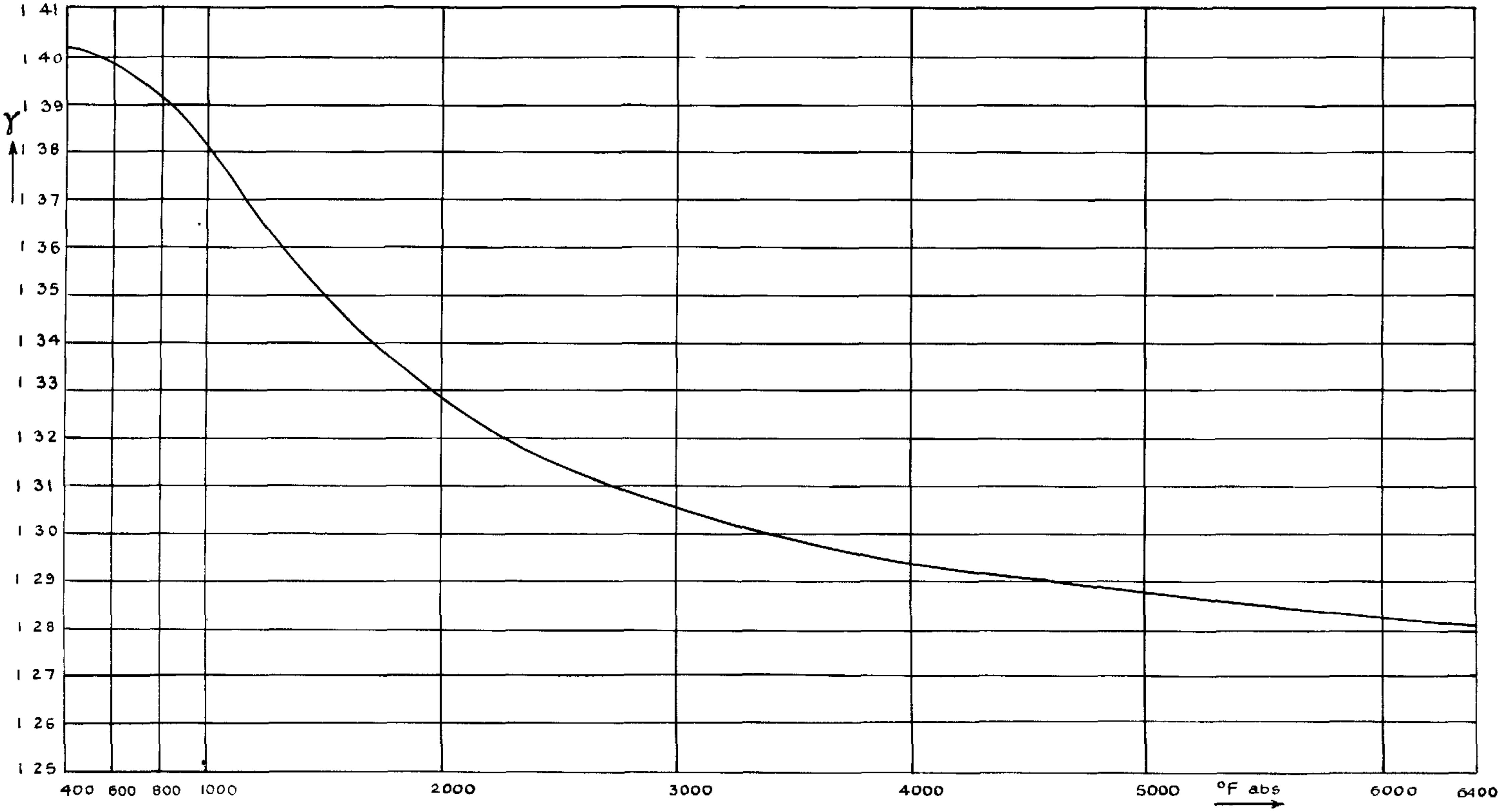
TABLES/

TABLES

Stream temperature T for varying stagnation temperature T_o, with Mach number, M, as parameter.

T_o	Mach number						
	4.5	5.0	6.0	7.0	8.0	9.0	10.0
3100	678.62	570.34	416.72	316.41			
3200	703.12	590.81	431.81	327.85			
3300	728.07	611.54	446.89	339.29			
3400	753.03	632.47	461.97	350.72			
3500	777.98	653.40	477.05	362.16			
3600	803.16	674.34	492.13	373.60			
3700	828.43	695.27	507.26	385.03	301.20		
3800	853.83	716.52	522.45	396.47	310.34		
3900	879.30	737.86	537.64	408.11	319.48		
4000	904.83	759.21	552.84	419.83	328.62		
4100	930.74	780.56	568.03	431.56	337.76		
4200	956.65	801.94	583.22	443.28	346.90		
4300	982.57	823.69	598.41	455.01	356.04		
4400	1008.64	845.45	615.65	466.74	365.18		
4500	1035.05	867.21	631.38	478.46	374.32	300.16	
4600	1061.45	888.96	647.10	490.19	383.46	307.61	
4700	1087.86	910.91	662.83	501.95	392.60	315.06	
4800	1114.33	933.06	678.56	513.91	401.78	322.51	
4900	1140.86	955.21	694.21	525.87	411.12	329.97	
5000	1167.49	977.36	710.20	537.83	420.47	337.42	
5100	1193.93	999.51	726.21	549.79	429.81	344.87	
5200	1220.67	1022.08	742.23	561.75	439.15	352.32	
5300	1247.47	1044.65	758.25	573.71	448.49	359.78	
5400	1274.27	1067.23	774.26	585.67	457.83	367.23	300.24
5500	1301.08	1089.81	790.28	597.63	467.17	374.68	
5600	1328.14	1112.45	806.41	609.77	476.52	382.13	

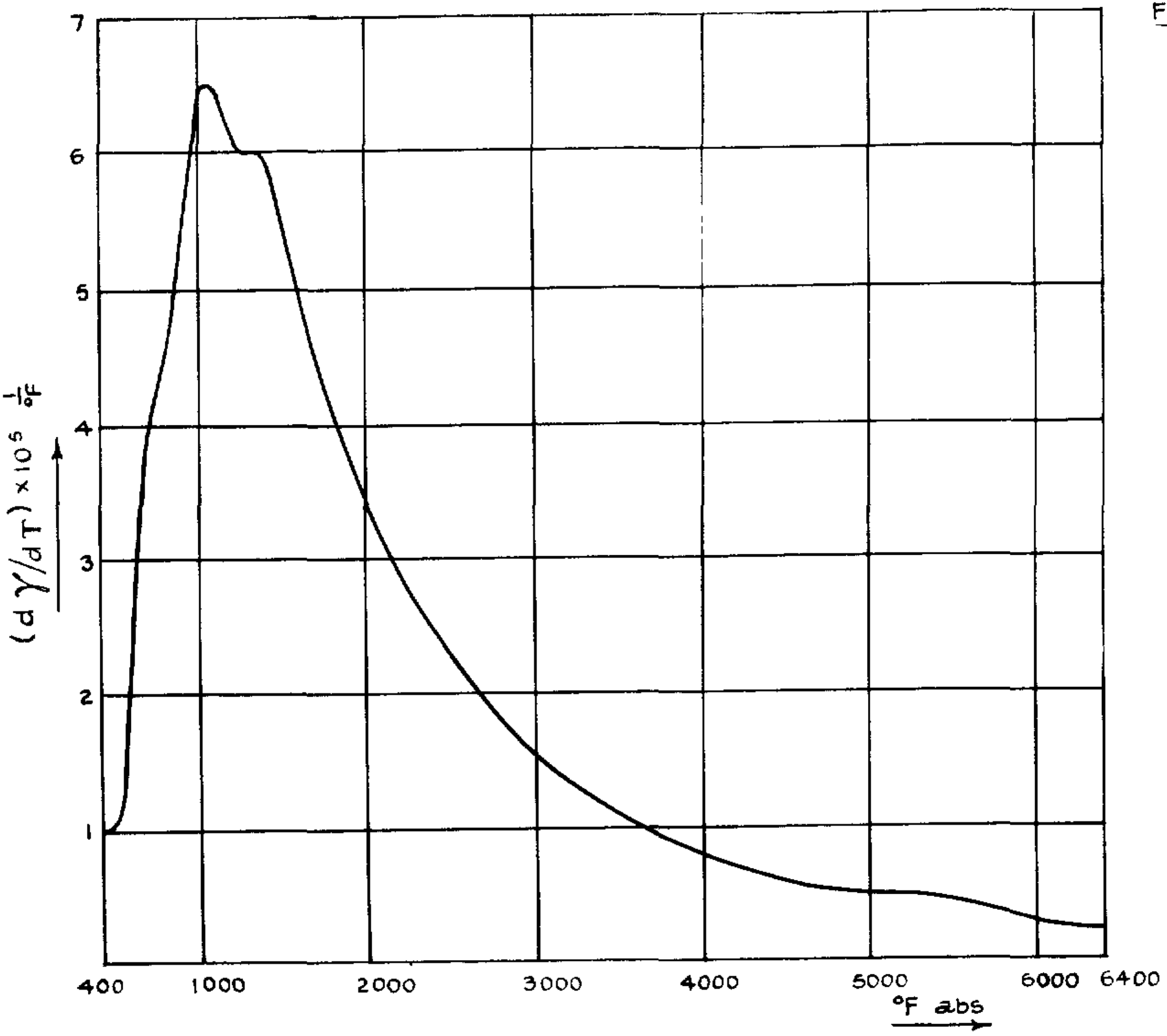
JW.



Variation of the ratio of specific heats, γ, for air, with temperature

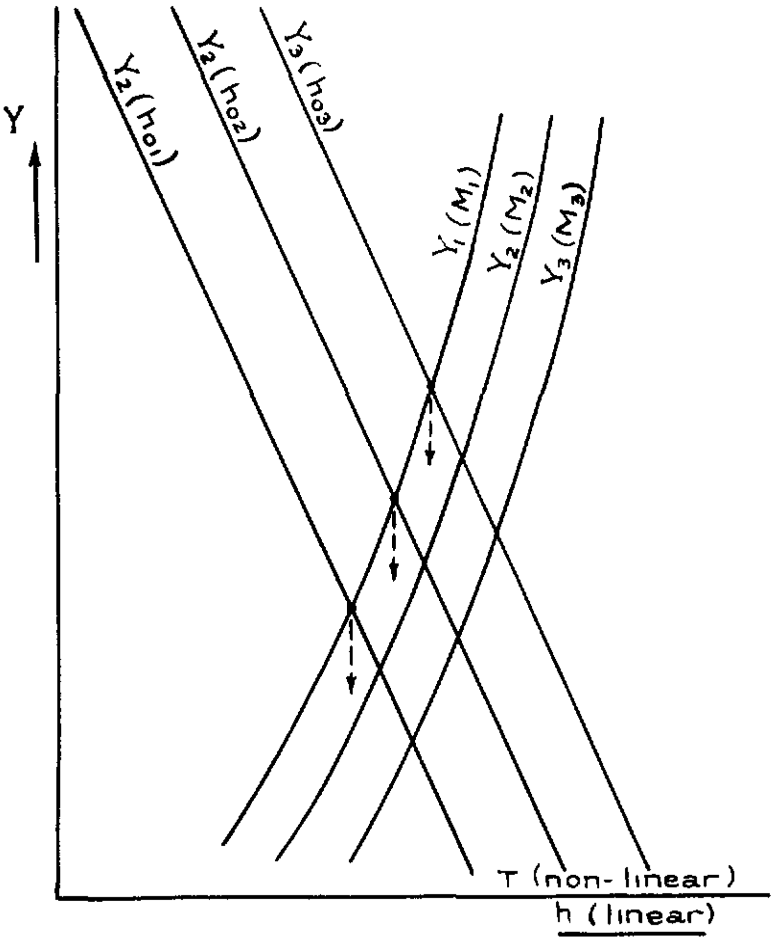
13,176
FIG 1

FIG 2

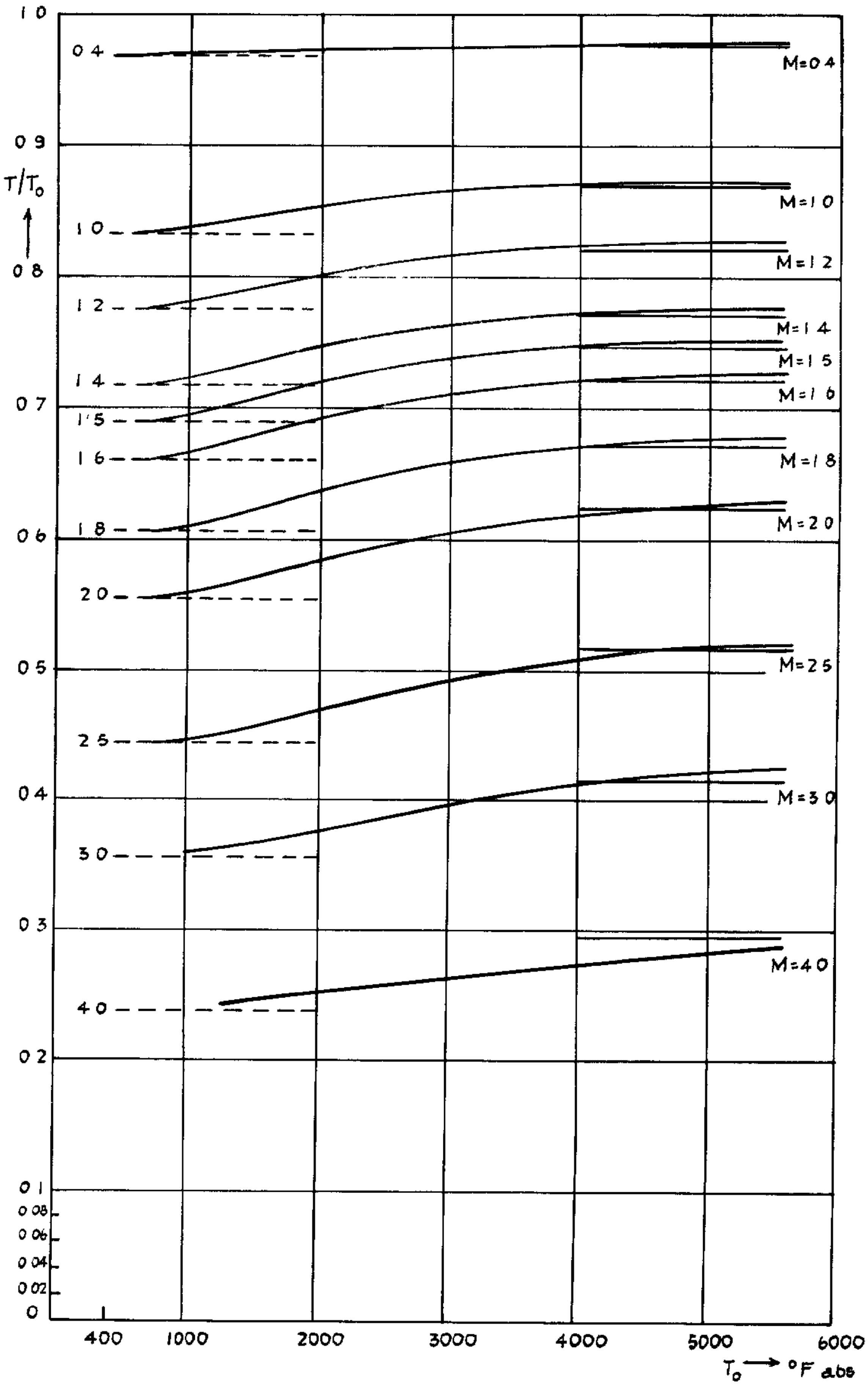


Rate of Change of γ with temperature for air

FIG 3



Geometrical construction for evaluating T from equation (4)

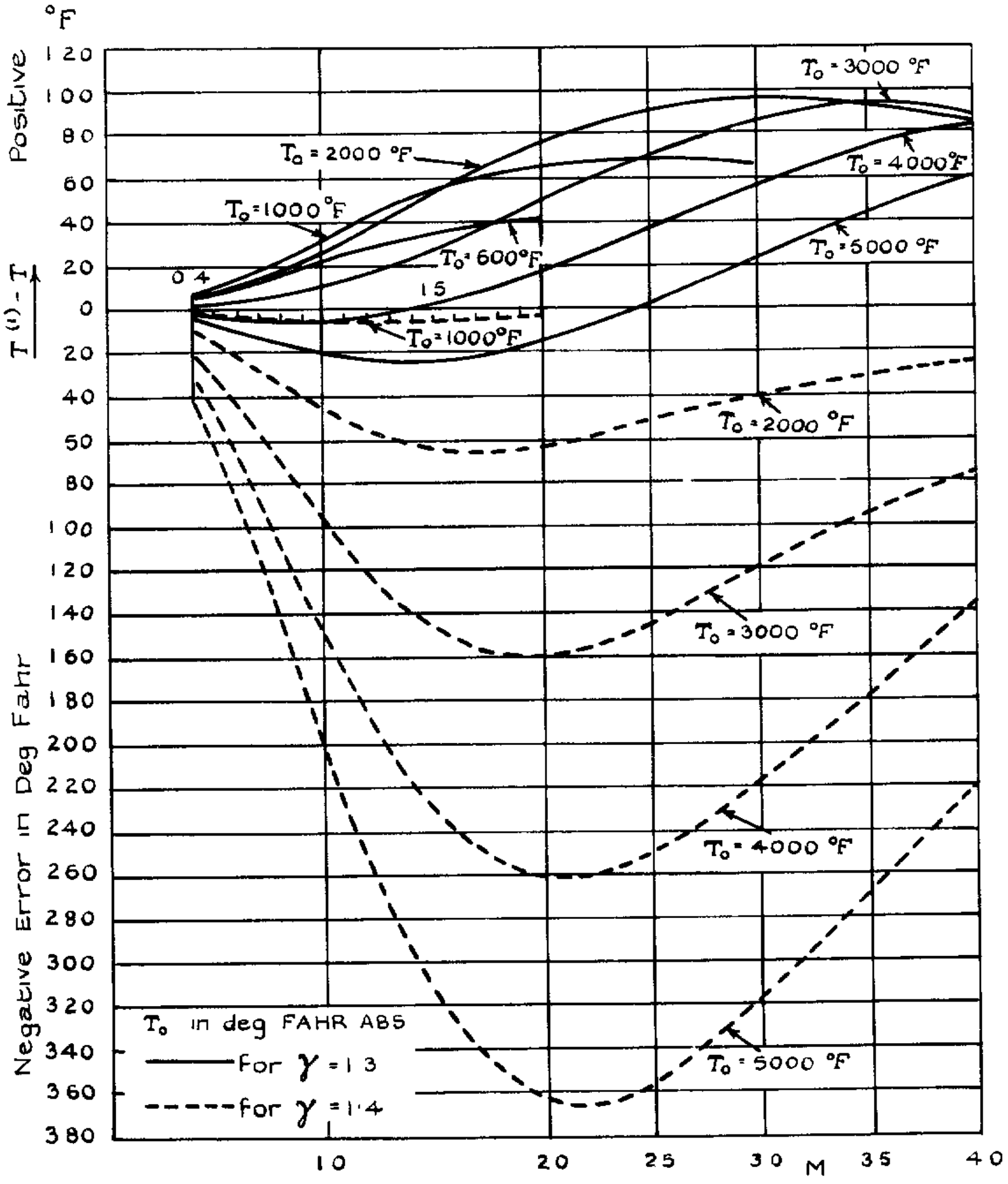


Values of the ratio of T/T_0 in terms of stagnation temperature T_0 with Mach number, M, as parameter

——— constant values for $\gamma = 1\ 300$

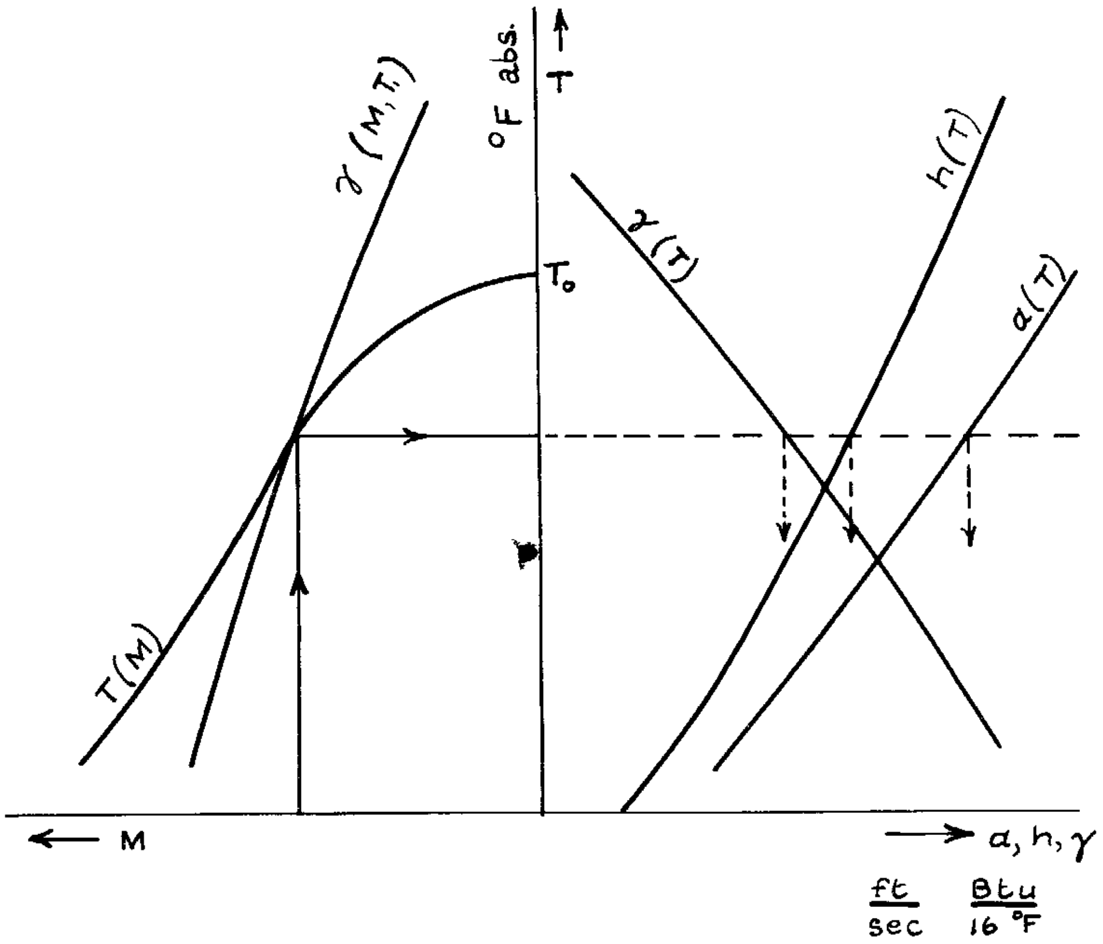
– – – – constant values for $\gamma = 1\ 400$

FIG 5

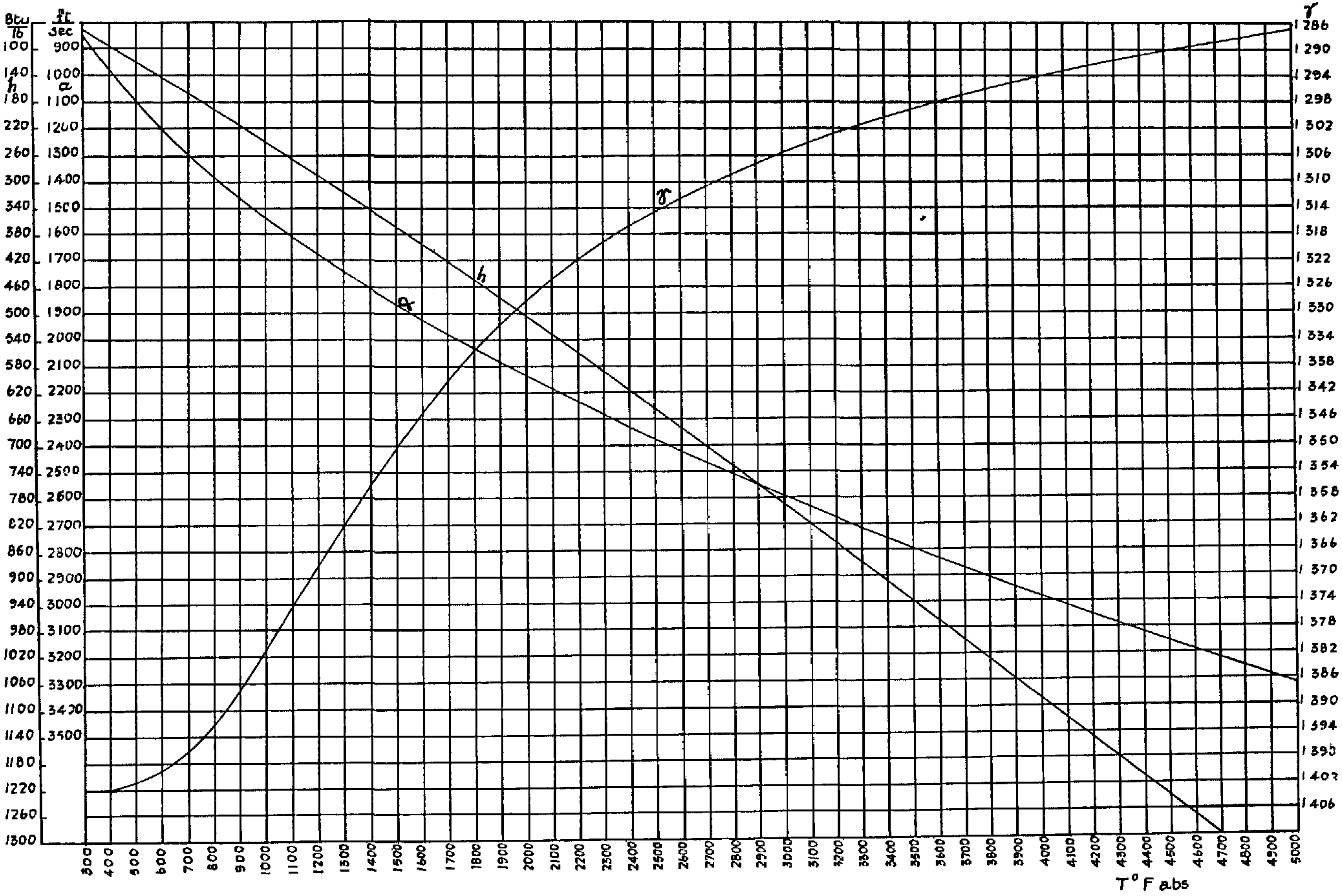


Error in using constant specific heats theory $T = T^{(1)} - T$ where $T^{(1)}$ is calculated from equation (1) and T is taken from the Tables on pages 8-15

13,176
Fig 6.



The T, M chart


Btu/lb
h
100
140
180
220
260
300
340
380
420
460
500
540
580
620
660
700
740
780
820
860
900
940
980
1020
1060
1100
1140
1180
1220
1260
1300
ft/sec
a
900
1000
1100
1200
1300
1400
1500
1600
1700
1800
1900
2000
2100
2200
2300
2400
2500
2600
2700
2800
2900
3000
3100
3200
3300
3400
3500
γ
1.286
1.290
1.294
1.298
1.302
1.306
1.310
1.314
1.318
1.322
1.326
1.330
1.334
1.338
1.342
1.346
1.350
1.354
1.358
1.362
1.366
1.370
1.374
1.378
1.382
1.386
1.390
1.394
1.398
1.402
1.406
γ
h
a
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600
1700
1800
1900
2000
2100
2200
2300
2400
2500
2600
2700
2800
2900
3000
3100
3200
3300
3400
3500
3600
3700
3800
3900
4000
4100
4200
4300
4400
4500
4600
4700
4800
4900
5000
T° F abs

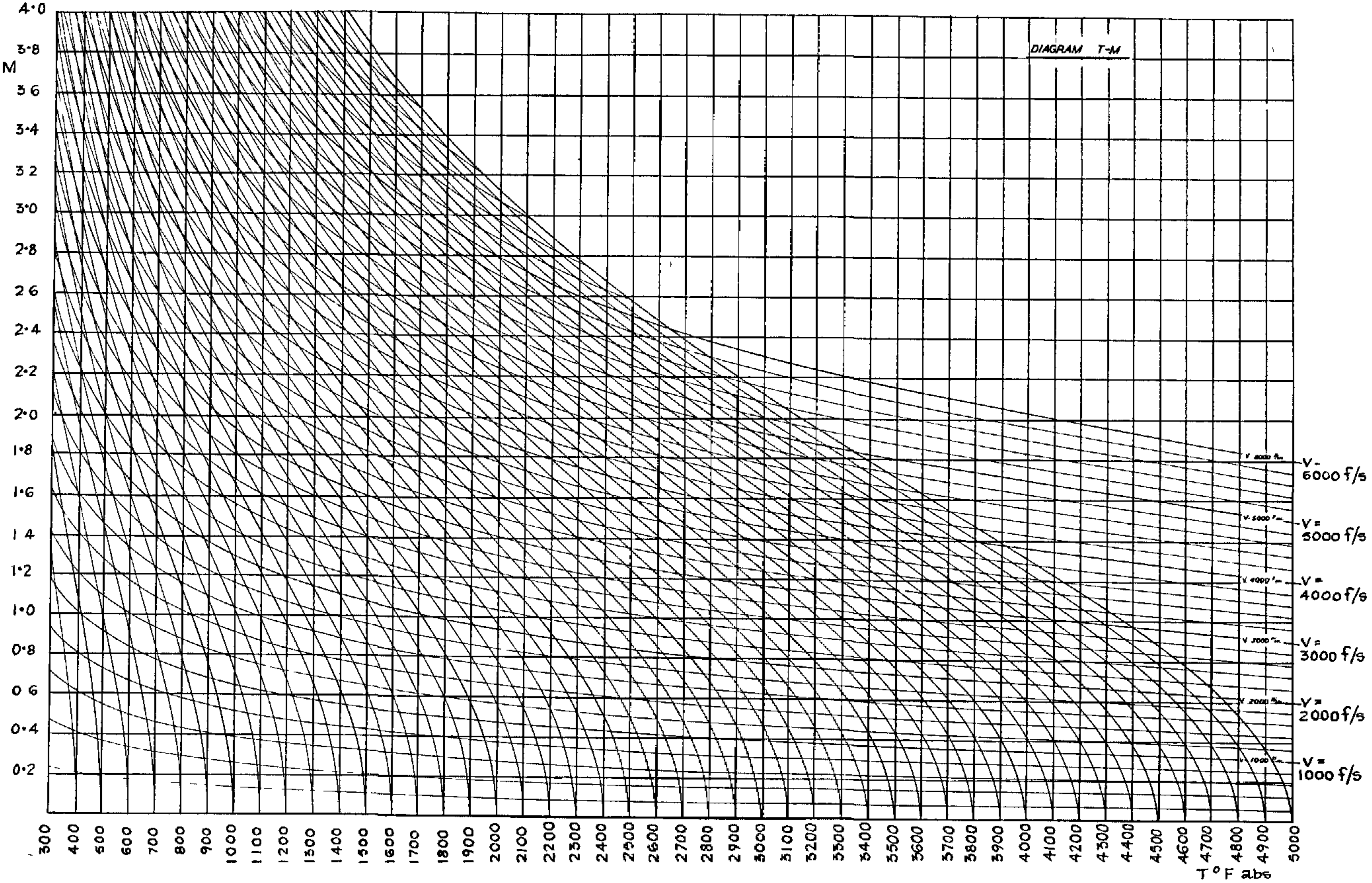


DIAGRAM T-M
M
4·0
3·8
3·6
3·4
3·2
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600
1700
1800
1900
2000
2100
2200
2300
2400
2500
2600
2700
2800
2900
3000
3100
3200
3300
3400
3500
3600
3700
3800
3900
4000
4100
4200
4300
4400
4500
4600
4700
4800
4900
5000
T °F abs
V = 6000 f/s
V = 5000 f/s
V = 4000 f/s
V = 3000 f/s
V = 2000 f/s
V = 1000 f/s

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