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An Amplitude Probability Analyser for Use in Turbulence and Noise Measurements

By

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of the Aerodynamics Division, N.P.L.

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An Amplitude Probability Analyser for Use in Turbulence and Noise Measurements

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SUMMARY

A circuit is described for the measurement of the cumulative amplitude probability distribution of electrical signals. Details are given of the use of the apparatus in noise and aerodynamic turbulence measurements. The input level is 0.6 volts peak-to-peak (100 mV R.M.S. with a Gaussian probability distribution), and the frequency response is approximately 10 c/s to 70 kc/s. The output signal is obtained as a train of 3 volt negative-going pulses which are integrated on a galvanometer.

Introduction

The amplitude probability distribution function of a fluctuating signal u, is P(u) where P(u) du is the probability, or the fraction of total time, that the signal amplitude is between u and u + du. For instance, the probability distribution of most randomly-fluctuating signals, such as thermal noise or aerodynamic turbulence, is nearly Gaussian, with

$$P(u) \simeq \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\frac{u^2}{\overline{u^2}}}, \qquad \ldots(1)$$

where $\overline{u^2}$ is the mean square value of u.

We can also define the cumulative or integrated probability $\Pi(u)$, as the probability that the signal is less than u : thus,

$$\Pi(u) = \int_{-\infty}^{u} P(u)\,du\,. \qquad \ldots(2)$$

The probability distribution can readily be found from the cumulative probability distribution by differentiation: since $\Pi(-\infty) = 0$ and $\Pi(\infty) = 1$ it follows from (2) that,

$$d\Pi = P(u)\,du\,. \qquad \ldots(3)$$

The probability distribution is of importance in the study of noise and turbulence chiefly because of the interest in its moments of various order, given in general form by

$$\overline{u^n} = \int_{-\infty}^{\infty} u^n\,P(u)\,du \qquad \ldots(4)$$

The/

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The first order moment $(n = 1)$ is zero, and the second gives the variance, or mean square value of u. The non-dimensional mean cube moment,

$$\frac{\overline{u^3}}{(\overline{u^2})^{3/2}}$$ or skewness, represents the mean transport of turbulent energy by the turbulence itself, and the skewness of the joint probability distribution,

$$\frac{\overline{(u_1 - u_2)^3}}{\left[\overline{(u_1 - u_2)^2}\right]^{3/2}}$$, gives a more general demonstration of the non-linearity of turbulent motion. The fourth order moment and its non-dimensional form, the flatness factor or kurtosis, can be used to find the intermittency of turbulence near the free edge of a shear layer: it has also been shown by flatness factor measurements that the smallest eddies in turbulence occur intermittently because of the energy dissipating influence of viscosity.

The moments of $P(u)$ of order n can be found by squaring, cubing, etc., the signal, but it is often convenient to measure the cumulative probability distribution and to derive the moments graphically. Measurement of the cumulative probability distribution also permits plotting on 'probit' paper, the axes of which are printed to transform a Gaussian distribution into a straight line without further computation. Moments of $P(u)$ of order n can be found from the moments of $\Pi(u)$ of order $(n - 1)$ since:

$$\int_{-\infty}^{\infty} u^n P(u)du = n \left[\int_0^{\infty} u^{n-1} (1 - \Pi(u))du - \int_{-\infty}^{0} u^{n-1} \Pi(u)du \right] \quad ...(5)$$

The circuit to be described was designed for the measurement of the cumulative probability distributions of turbulence and noise signals in the frequency range 10 c/s to 70 kc/s, and uses the over-driven amplifier technique as opposed to the pulse counting method (Refs. 1 and 2). Strictly, the zero signal level should be preserved throughout by the use of D. C. coupling; however, the use of an A.C. coupled amplifier is here justified since its lower frequency response is very good (0.15 c/s 3 dB down) and is better than the response of the equipment from which the signals are derived.

Circuit Details

The circuit consists of two main sections, an A.C. coupled amplifier, and the directly-coupled output stages which drive the transistor switch.

The input amplifier (Fig. 1), has two stages of amplification, V_1 and V_2, with an intrinsic gain of approximately 2,000. Overall negative feedback is applied by the common cathode resistors of V_1 and the cathode follower output valve V_3; with 15 dB of feedback a 100 mV R.M.S. sinewave input gives 100 V peak-to-peak output.

The mean voltage of the cathode of V_3 is stabilised by V_4 at approximately -200 volts. The signal from V_3 is fed directly to the anode of the slipping diode V_5 (Fig.2). The potential of the cathode of this valve may be varied over the range -100 to -200 volts by the 50 kΩ bias-setting potentiometer, and its magnitude found using a measuring potentiometer

connected/

connected to the 100 : 1 attenuator across the bias voltage. V_5 will only conduct when the instantaneous voltage at the cathode of V_3 is greater than this bias potential. The resulting potential change across the 47 KΩ resistor is fed to the differential amplifier V_7 and V_8, and the ensuing fall in the anode potential of V_7 causes the transistor Tr1 to conduct. The potential of the collector, which in the non-conducting state is governed by the voltage across the Zener diode D_1, falls for the duration of the switch-on period. The switching signal is very much greater than that required for saturation, and causes square output pulses to appear at the collector of the transistor: the rise-time of these pulses is 0.3 μs.

The output pulses are clipped by D_2 to prevent a small amount of high frequency breakthrough, which would distort the pulses when the input frequency was in excess of 50 kc/s.

The 50Ω, set-zero potentiometer is used to reduce the output to zero when the bias voltage is zero. The output can either be fed to a galvanometer via a suitable series resistor or to an electronic integrator. The former is satisfactory when dealing with near Gaussian signals having no very low frequency components. However, the longer averaging time afforded by the latter method is essential if accurate measurements are to be made on signals having low frequency components or when these signals are intermittent. An electronic integrator with timing circuit is described in Ref.3.

When it is desired to explore the regions of the probability distribution close to the negative input peaks, the sensitivity of the galvanometer may be increased. For distributions close to the positive peaks a phase inverter and cathode follower are inserted immediately before the analyser (Fig.3). The cathode follower was included to permit a convenient length of cable to be used between the units.

Performance

The input amplifier has a frequency response from 0.15 c/s to 220 kc/s (3 dB down); a rise of approximately 1 dB occurs between 0.5 c/s and 5 c/s. The total hum and noise at the cathode of V_3 is less than 50 mV peak-to-peak when the input is fed from a source impedance of 100 KΩ, and is thus less than 0.05% of the total signal at this point.

Fig.4 shows a comparison with theory of the results obtained for the cumulative probability distribution of a sinewave signal. The theoretical curve is given by

$$\Pi = 0.5 - \frac{1}{\pi} \sin^{-1} \left(\frac{V_M - V_B}{A} \right)$$

where:- V_M is the mean potential of the cathode of V_3

V_B is the bias potential of the cathode of V_5

A is the peak amplitude of the sinewave at V_3

The agreement is seen to be very good over a large range of values of $\Pi(u)$, and excepting extreme values, the measured probability is within about 1% of the theoretical figure. The disagreement is a result of the output pulses no longer having a rectangular shape, when the input voltage to the differential amplifier is small.

Fig.5 shows the variation in output for a fixed input amplitude sinewave as the frequency is varied, for three fixed bias potentials.

Fig.6/

Figure 6 shows a plot of the cumulative probability distribution for the turbulence signal near to the edge of a subsonic jet. These results are plotted on 'probit' paper in Figure 7 for comparison, and show a departure from the Gaussian distribution (dotted line) at the higher values of $\Pi(u)$.

Conclusions

The instrument has proved satisfactory for the measurement of the probability distribution of electrical signals in the frequency range 10 c/s to 70 kc/s. The reliability has been good and the circuit has the merit of being relatively simple in construction and use.

Acknowledgement

Much helpful advice was given by Mr. P. Bradshaw of the Aerodynamics Division, N.P.L.

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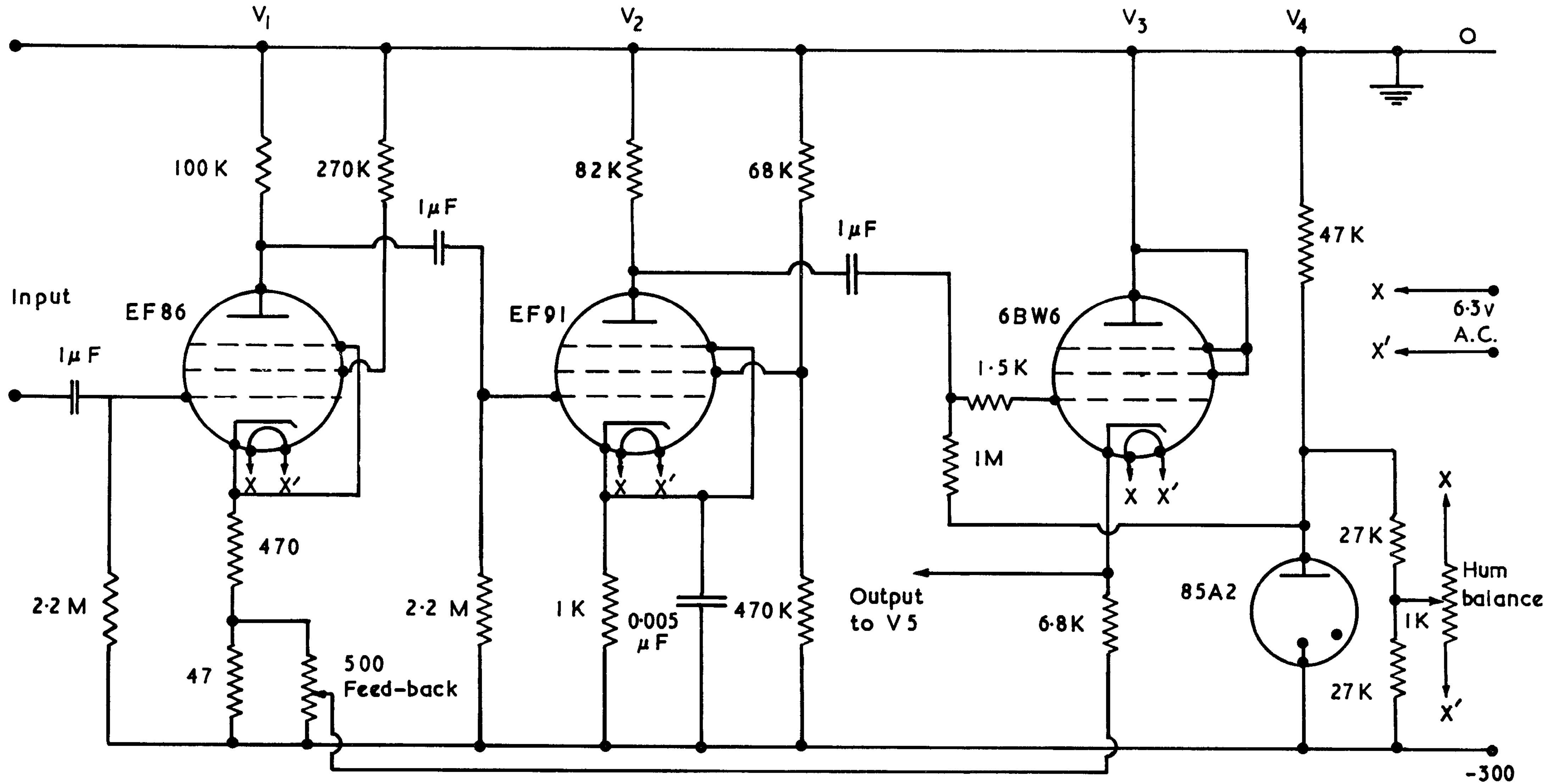


FIG.I.

Probability analyser (Input Amplifier)

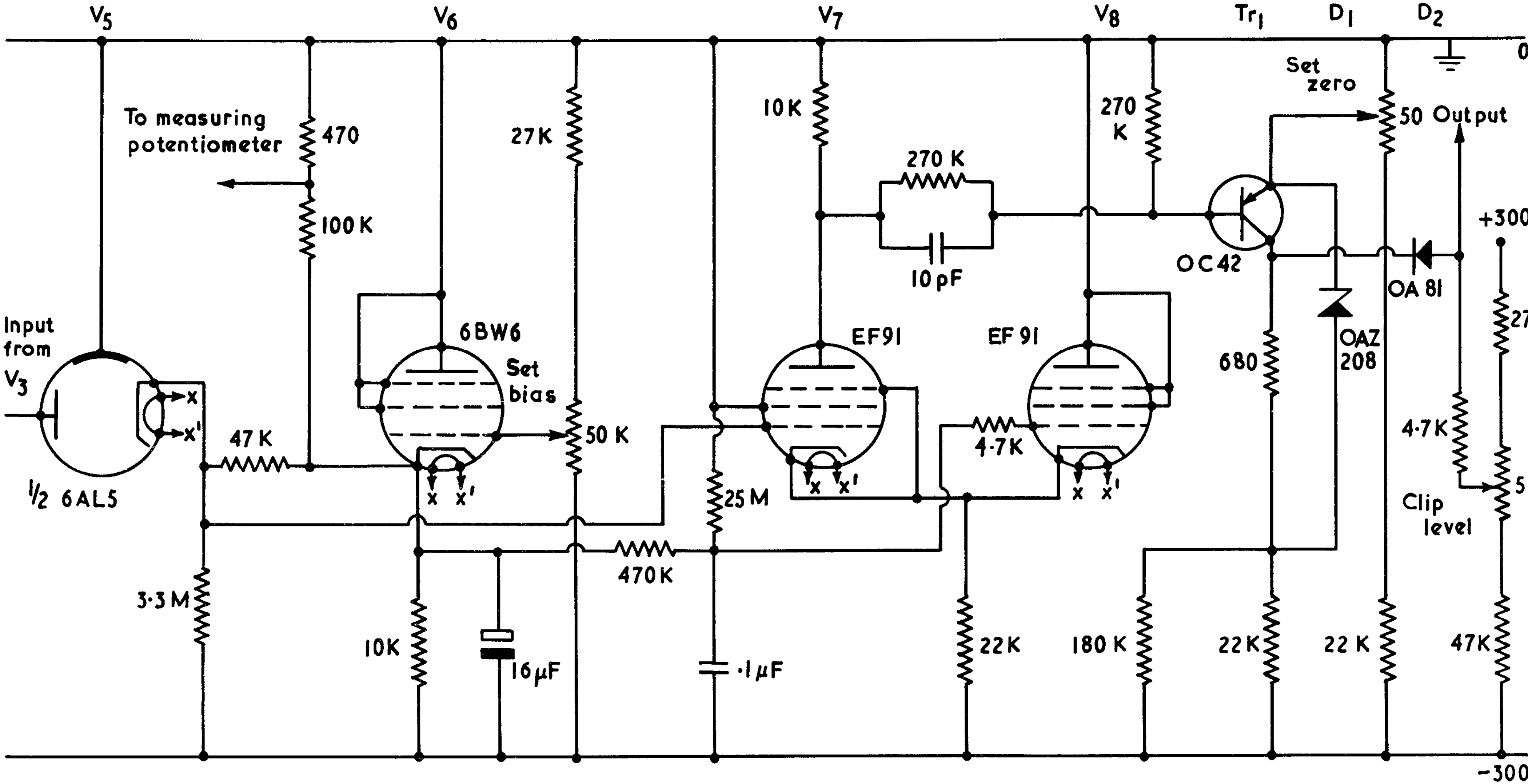
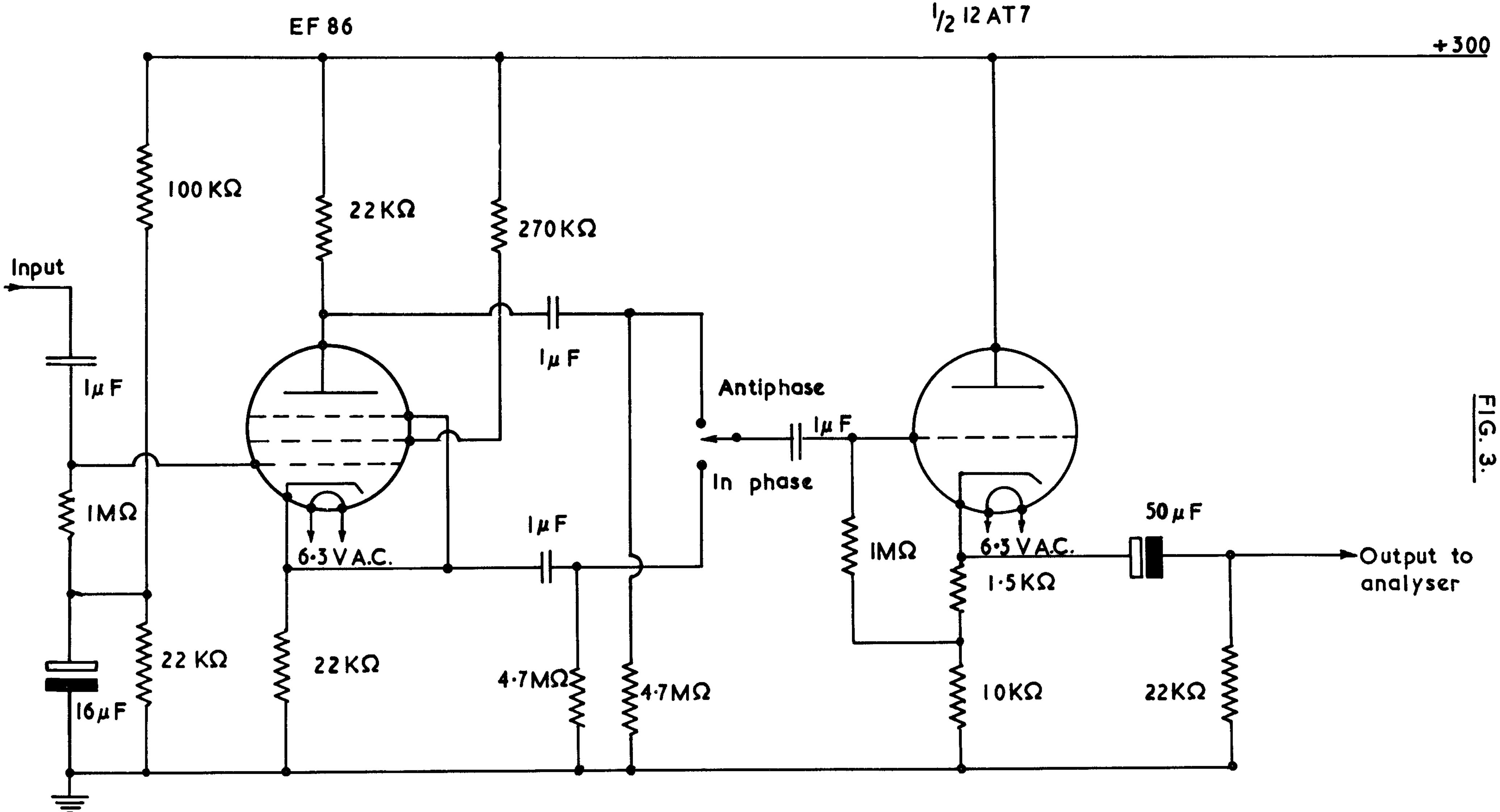


FIG. 2.

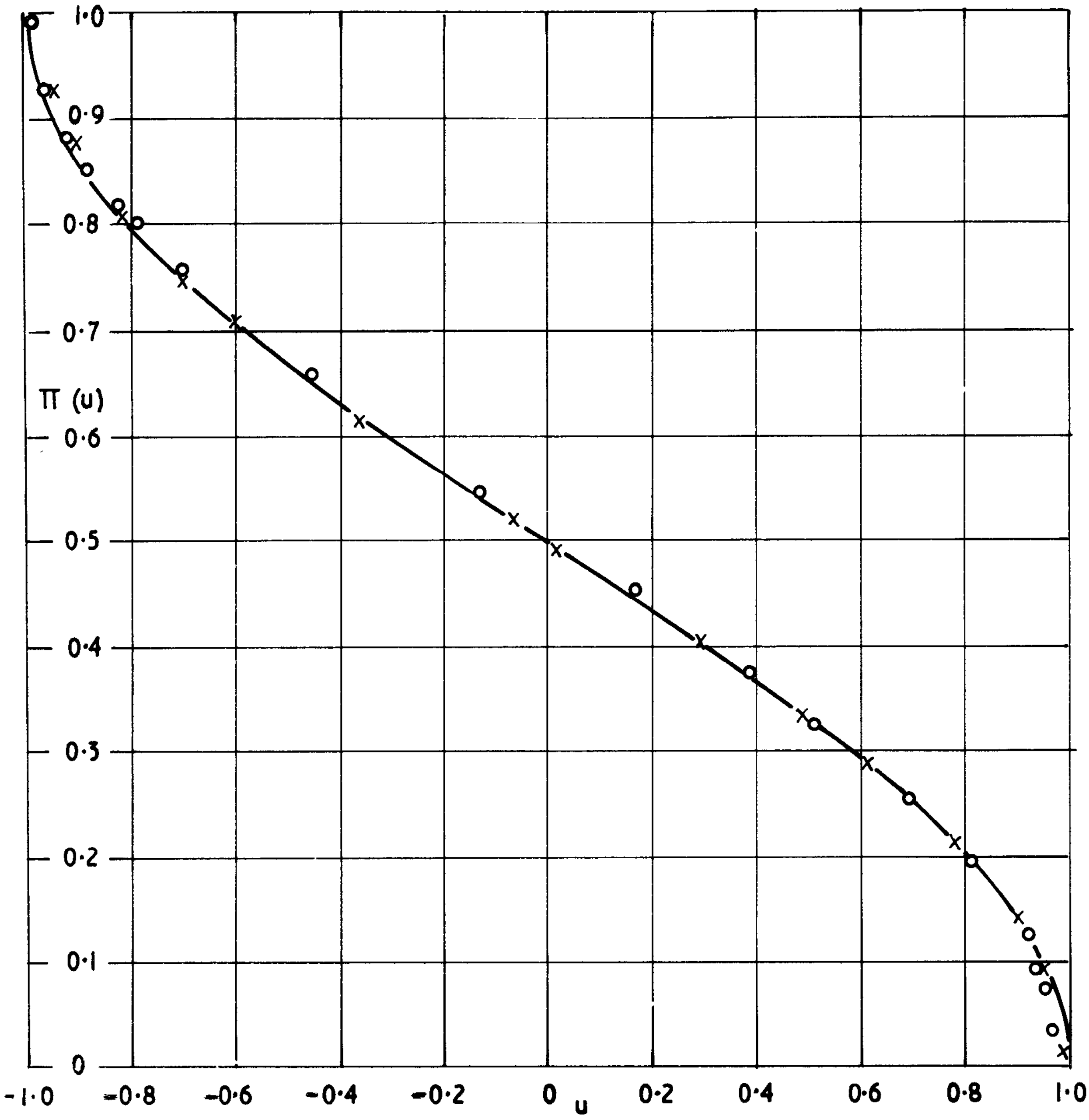
Probability analyser (Gate and switch)



Phase splitter

FIG. 3.

FIG. 4.



Comparison with theory for sine-wave signals.

—— Theory
o 30 c/s
x 30 Kc/s

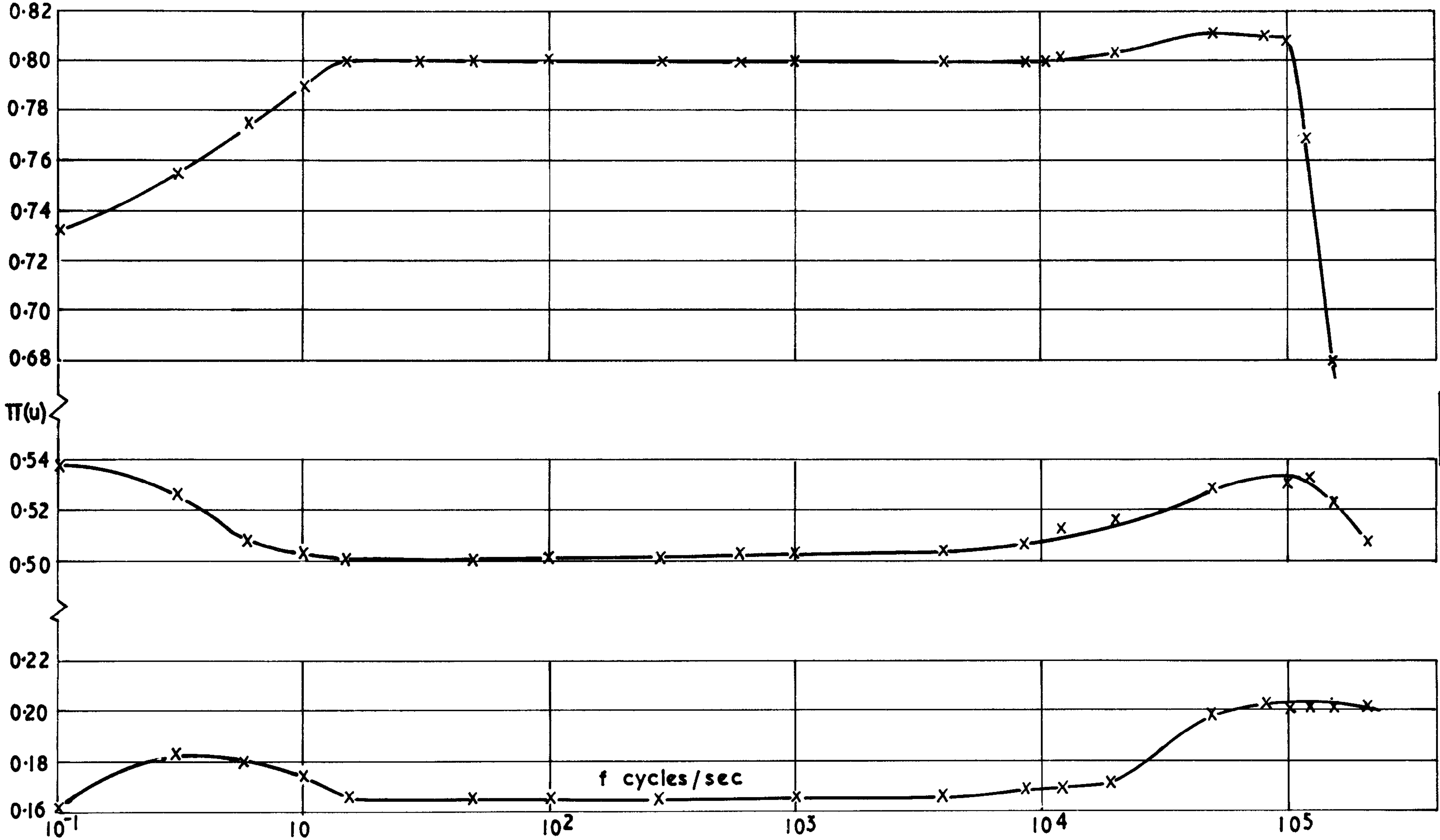
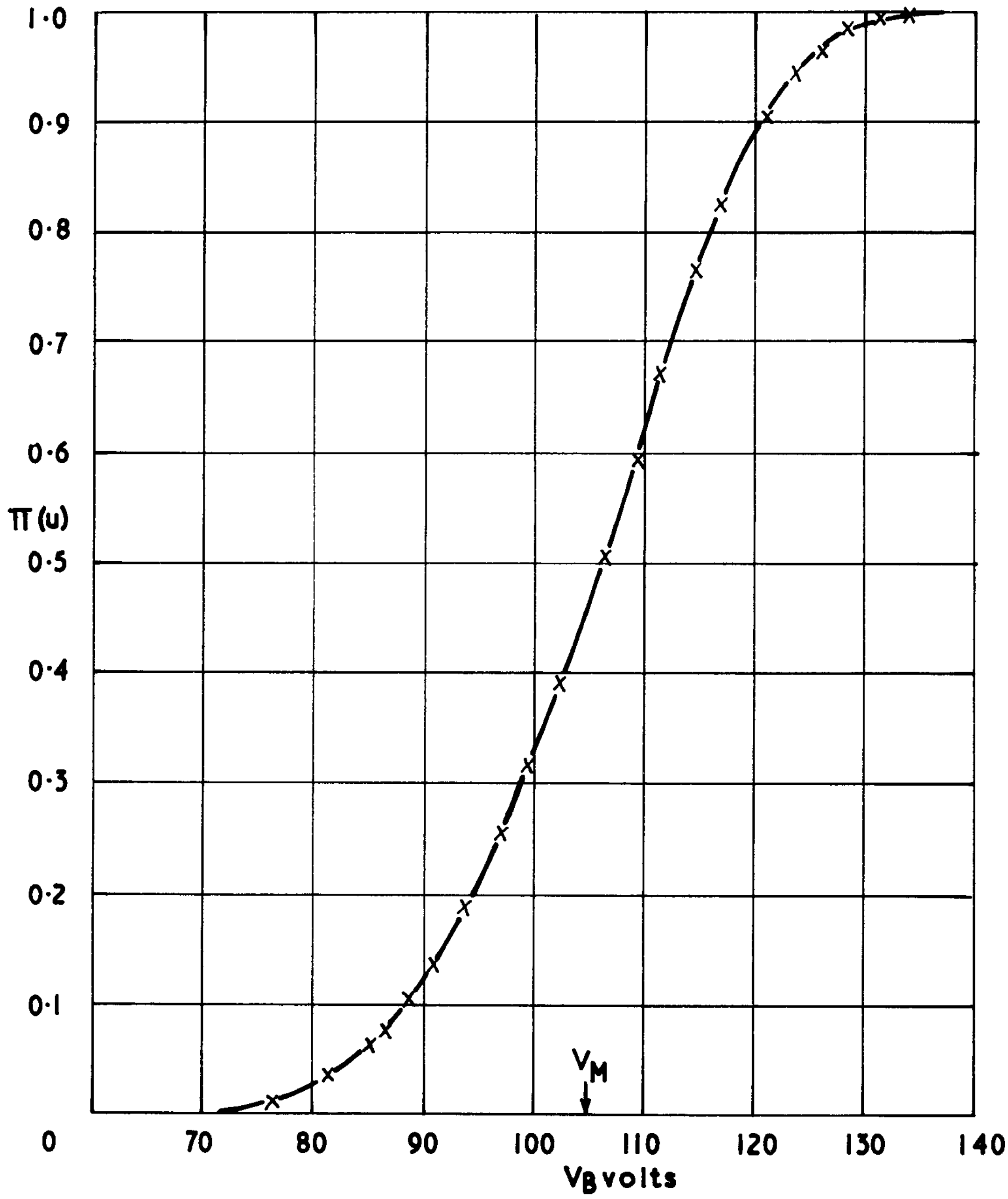


FIG. 5.

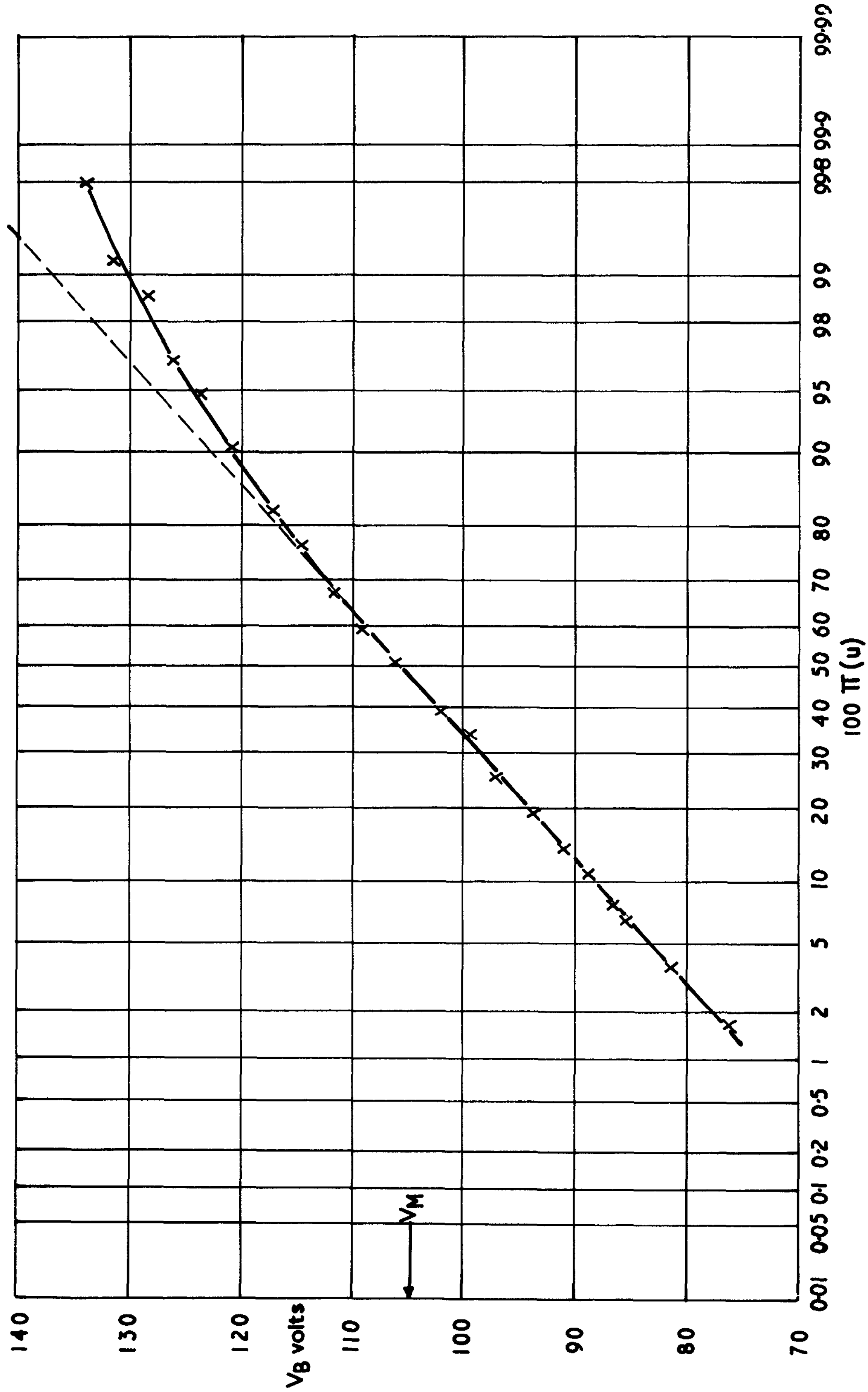
Frequency response of system for sine-wave signal.

FIG. 6.



Probability distribution for turbulence signal near to edge of sub-sonic jet.

FIG. 7.



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