

MINISTRY OF SUPPLY

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## Focussing Schlieren Systems

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The substitution of grids ior the usual knite edges in a Schlieren apparatus confers iocussing propertics on the system. Three possible systems are described and their optical lamitations discussed. Achromatio lenses are used in place of mirrors because or the excessive of feaxis abcrrations or the mirrors. Some practical suggestions for the oonstruction of the grid and the adjustment of the system are given.

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The formation of Lnages in a Schlieren system is largely limated by dirifaction which, because oi' the narrowness of the beams, loads to diffuse images and spurious iranges at the edge or any discontinuity.

Further dil'ricultios arıse in practice through shortcomings in materials; the windows need to be completely rree from inhomogeneities, mirrors have to roach an extremely high standard of workmanship and even the alr itseli' has to be kept iree i'rom turbulence.

Work recently ruported 1,2 has aimed ot overcoming these Initations by the so-called focussing systems, which gave improved derinition in the lmagc and allow optical components of lower quality. The published work, however, has not always beon specitic in outinning the optical condition, and the lamitations or size, ideld etc. ir these systems are to be successfiul.

The present note is intended to investigate these focussing systems vith particular reference to their use in wand tunnels for measuring anr densitios.

## 2. The Basic Prinoiples or the frocussing Systems

The basic optical layout of the conventional Schlieren system is shom in Fig. 1 whore the slit $S$ is at the focus of the lens $L$; from which a parallel bean or light emerges. A second lens $L_{2}$ brings this beam to a rocus and, at this position, the knire-edge $K$ is arranged to cut or'f the image of the slit. No light then appears upon the screen T. Suppose now, at a position 0 in thc parallel buam, therc is a region of d dscontinuity (a Sohliero) which deviates the incident light ray through a small angle $\varepsilon$, the ray OP becoming OP'. By the usual optical theory, all tays passing through 0 will, aiter rofraction at the lens $L_{2}$, pass through the conjugate point $I$. Tho ray Pl will no longor bo cut or 'r' by the knile-edge, and tho point I wjil appear bright vpon the screen. The lens $L_{2}$ will have produced an image ol the wohliere 0 upon the screon at I.

In order to anvestagate the quality of thas optical image, we need to consider the cxact illuminating condations at the pount 0 and the manner in which this light is presonted to the projection lens $\mathrm{L}_{2}$. Fig. 2 represents these Illumanating conditions. As the llluminating slit is at the socus or the lirst luns $I_{1}$, parallol Jight omerges corresponding to each point on the slil although beams srom various points on the slit are not all parallel to oach other. Any point 0 in the rijeld wall be illumanated by one ray from cach point of the silt, in tact by a ran of rays making an angle $\theta$ where $\theta$ is the angle subtended at the lens by the slit. For small values of $\theta$ we have

$$
\begin{equation*}
\theta=\frac{d}{I_{1}} \tag{1}
\end{equation*}
$$

whore $d$ is the slit wath
and $f_{1}^{2}$ is the tocal leng th of the lens $I_{1}$.
Fig. 3 now shows the same cone being focussed by the lens $L_{2}$ to form an image at $I$, the enture fan having been derlected sulticiently by the disturbance to prevent any out or's' at the knife edge $K$. The maximum possible light is now going into the image $I$. Should the movement at the knifeedge be insuriciciont to clear the knite-cdge completely, it can readily bo seen that thas provides an erfective reduction in the slit wadth and the angle $\theta$ will be given by

$$
\begin{equation*}
\theta=\frac{d_{2} f_{1}}{f_{2} f_{1}}=\frac{d_{2}}{f_{2}} \tag{2}
\end{equation*}
$$

where $d_{2}$ is the width of the beam transmıtted past the knıte-edge and $\quad i_{2}^{2}$ is the rocal length or the second lens.

In practice therefore the illuminating angle whll be less than the value given by equation (1) and may vary wi th the derlection $\varepsilon$.

In conventional lens theory, the angle $\theta$ determines the relative aperture or $I /$ no at which the lens $I_{2}$ is working, the approxamate relation between them bcing, lor small angles 0

$$
\begin{equation*}
\mathrm{f} / \mathrm{no}=\frac{1}{\theta} \tag{3}
\end{equation*}
$$

For a typical case, the siit width may be 0,020 " and the focal length of the lens $L_{1}$ may be $120^{\prime \prime}$. The image is then being formed by a beam of angular extent $\frac{1}{2}$ minute of arc and at an aperture equal to $\mathrm{f}^{\prime} / 6000$. The resuit is that the lens will have an enormous depth of focus in the object spare about 0. The narrow angular beam causes dirfraction er'fects to become prominent and the image, in the case when all the light passes the second knifo edge, takes on the characteristios or the duftraction pattern of the illuminating slit. When the knife edge cuts ot'f part of the light, the resultant illumination is a complicated function of the dil'fraction patterns of the slit and the knife edge.

Another method of considering the locussing problem is illustrated in Fig. 4 where the light path, because of its extremely narrow angle, is ronsidered to be merely a "light ray". The point $O_{1}$ deflects the ray by an amount $\varepsilon_{1}$ but berore striking the lens $L_{2}$, the ray meets another distur- . bance $O_{2}$ which del'lects it a further amount $\varepsilon_{2}$. On arrival at the lens the total deviation is $\varepsilon_{1}+\varepsilon_{2}$ and the lens rinds it impossible to distinguinh these derlections us separato entities. The light patch at I will merely be a function of $\Sigma \varepsilon_{1}$ taken along the entire light path and the lens has no focussing action. Bad windows, poor lenses or mirrors, convection currents: local irregularities will all shov up in the $f^{2}$ inal picture. In addition to the alteration in intensity, the position or the image $I$ of the Schliere $0_{1}$ will be af'fected by the presence on the disturbance $0_{2}$, the image shif't depending on the term $\varepsilon_{2} t$ (where $t$ is the distance $\mathrm{O}_{1} \mathrm{O}_{2}$ ) and the magnif'ication ratio between the Image planes of $O_{1}$ and $I$. This image movement is likely to be small; a window with a local deviation of 1 minute of are will move the image oi' a schliere $10^{\prime \prime}$ i'rom it and photographed at a $3: 1$ reduction only by $0.001^{\prime \prime}$.

We now consider an arrangement wh tho slits and two knife edges so arranged that the beans from both slits are cut of simultaneously. Fig. 2 will apply again and the angle between the two beams will be given by equation (1) where d is now the distance between the two slits, which can be made much greator than the individual slit width. Fig. 4 shows these two rays both being derilocted an amount $\varepsilon_{1}$ at the Schliere $O_{1}$ and coming to a focus at the print $I$, the conjugate point to $O_{1}$. The Irinal image of the Schliere is $r^{2}$ ormed by the superposition ot these two images. If a second small disturbance occurs at the point $\mathrm{O}_{2}$ in the path of one beam nnly, this beam will be deflected while the ather will be unchanged and will form the image $I$ with the sume $\perp$ intensity and at the same position. For a system wi th a large numbur of slits and corresponding knire-edges, i.e. t'or two complementary grids, there will be a large number of rays forming the image of the point $O_{1}$ and the presence of a small disturbance $O_{2}$ in one or twe of the beams will be completely swamped by tho larger number of undisturbed
beams. The systcm now has a focussing action and the image from every slit wall only coincide exactly in position and brightness for disturbances on the one plane $O_{1}$, conjugato to the image plane.

An alternative point of view uses the concept oi the $\mathrm{s} / \mathrm{no}$. With the same luns as previously considered, the single slit width of $0.020^{\prime \prime}$ may be replaced by a $12^{\prime \prime}$ grid containing 100 or more slits.

The ansle $\theta$ subtcnded by the outer beams at the projection lens $L_{2}$ as now about $6^{\circ}$, equivalent to $f / 10$ and at this relative aperture the dopth of locus rill be much reduced; small disturbances nearer or farther from the lens not being recorded. This is not exactly equivalent to the usual case of lens focussing as there is not nocessarily any phase relationsh$\perp p$ betweon the various bcams and they will therefore not interferc to form a diffraction image in all cases.

## 3 Optocal Limitations of the Focussing System

The explanations givon of the focussing action also help to bring out some of the limiting incotors in this type or optical imagery.

As the 1 'inal image is made up trom overlapping images formed by dit'lerent light rays, it as essentzal that those rays should all have the same deviation, but unless two-dimensional Schliere aro beang observed, so that otherwiso the rays are in a unirorm medium, this is not likely to happen. Each ray will pass through a dilyerent part of the throc dimenshonal incld, therc will be small din'rerencos in the deviation of each ray and the images iormed on the screen whll hive din'terent brightnosses and positions. The tinal pioture will not have the correct intensity and will bo slightly blus red $2 r^{\prime}$ these dutiterences in deviation are small, whilo ler Bohliere of small diancter a multiple mage may be Iormed. This etineot . rathor simalar to the or'sect on optical aborrations upon the fincl amage quality or a photograph.

A lurther diriticulty arisos l'rom the fact that the beams straking the schliure, do not do so all at the samo angle, and wall theretore bo deviatcd by unoqual amounts. Since the angle subtended by the boam is small, this dirference is untikely to be large unless the mean angle betweon the boam and the dunity grodient is large. This erroct dlways exists however and must lead to an uncortain rolationship butwcen the density gradient and the intonsity in the Schlieren mage.

The use of the torm "depth or focus" suggests that, outside a cortain range, duviating objucte vill not show in the inal picture. This is only partly trve, depending largely upon the slze of the disturbance. For a very small disturbance, woll outside focus, only one beam may be aftectod and the loss to the fimal 1 mage may be small. But 1 the disturbance affects a large proportion of the total beams, it will be very notinedulu in the tinal picture $2 s$ in intensity dirt'erence and a very small shit't in position. Should ine disturbance cover completely the required Schliero then the deviaticns will be additive, the position of the image on the screen will be altercd and the intensity of the image will correspond to the combincd dcviation and will, therefore, not be a true indication of the deviation oi the Schlicre in pocus.

While it is true that the eircecu oi an out of focus disturbance on the foarsed Schliere 15 much reduced, the opposite $1 s$ also true and an out oi locus disturbanco artects the image over a much larger area. In Fig. 5 a iisturbance $0_{2}$, not in focus, would in a single slit system merely at'rect on aca $A B$ equal to 2 ts own area in the plane oi focus. Using a multi-slit system with an illuminating cone or angle $\theta$ an increased area $A^{\prime} B^{\prime}$ will be arituctod by the disturbance even though the in'luence of the
disturbance will be much reduced in the original area $A B$. The er'fect oir the out of l'ocus disturbance is spread out and weakened.

The tocussing er'rect is in many ways very similar to that seen in normal pictorial photography where objects not in focus still appear in the rinnd picture unless they are very small. They appear to be larger than when in focus, are very difiuse and when subtending a small angle at the camera, allow other objects to be seen through them. This is the appearance of out or rocus disturbances in a Schlieren fireld and ir these disturbances are large in area or possess large deflections they will always be apparent in the final picture although in a dir'ruse manner. In the Schlieren case moreover there may be an increase in illumination and a slight change of the position of a Schliere in the plane of focus, due to an out of focus disturbance.

In spite of these dificiculties, in a great many cases information about a three-dimensional system can be obtolned by taking a series of photographs through iocus. The position of any Schliere can then be estrmated by finding the plane of sharpest magery. When sharp discontinuities are present, this car be done wi th an accuracy comparable to that or focussing in normal photography. In general, however, a change of focus will merely alter tho intunsity distribution in the image, there will be nothing definite to focus upon and the new intensity distribution will not be a reliable guide to the actual density gradients in the plane of focus.

## 4 Derects in Optical Components

In f'ocussing systems it may be possible to reduce the quality required in the vindows, mirrors ctc. oi the system. The vind tunnel is normally closed by windows of thick glass and the problem of obtaining glass of surfircient strongth and homogenelty is very dirficult to solve except at groat expense and 'rith greai waste or' glass in selection. Fig. 6 shows a focussing system for a wind tunnel. Each Schlıere 0 in the plane of cocus is illuminated by a cone oi angle $\theta$ given by equation (1), and this cone will intersoct tho window in a patch of size $h$ given by

$$
\begin{equation*}
h=l \theta=\frac{\ell d}{\mathrm{~m}_{1}}=\frac{\ell}{\mathrm{f} / \mathrm{no}} \tag{4}
\end{equation*}
$$

where $l \quad$ is the distance from the Sohliere to the window
d $1 s$ the size or the grid containing the slits
$f_{1}$ is the tocal length of the rirst lens or mirror
$I$ /no is the relative aperture of the illuminating beam.
For a beam of local ratio $\mathrm{f} / 10$, the wandow patch size fer a wind tunncl or total length $20^{\prime \prime}$ ( $\ell \approx 10^{\prime \prime}$ ) will be about $1^{\prime \prime}$. The window will not affoct the Schliuren images $2 \hat{f}$ any patoh $1^{\prime \prime}$ ir diameter contains a large proportion of undeviating glass. By far the most common derect or glass is the presence of ream (or veins) in the form of long threads or filaments which; in a single slit system, are registered directly upon the screen. They possuss. however, only a small area and would disappear in many cases when a focussing systom is used. Ordınary twin ground and polıshed plate glass, seleoted to be frec from major dercots, should be satisiactory for a system irith the dimensions quoted.

The mirrors or lenses are illuminated by each point in the Schlieren field over an area instead of there being a point to point correspondence. The sizo or the illuminated patch will be governed by equation (4) where $c$ is now the distance irom the plane of focus to the mirror. The mirror can then have local der'ects, polishing marks, small pits or narrow grinding
zones without at'recting the amage, providing these never cover a high proportion os this illumnated area.

## 5 Pocussing System No. 1

This, the moit armple system, consists of a single lens and as shown in Fig. 7. Here J , 15 the lens, $A B$ the grid, $C D$ the focussing ' $p$ iane containing the Schlıere, EF tho plane of the complementary or knire-edge grid upon which the lens forms an 2mage or the primary grid. The plane gri, conjugato to the sichliero plane CD, contans the soreen or photographin rilm upon phich the image is recorded.

With the notation shown in the clagram, we can derive quite simply the iollowing rulationships. The maxumm diamcter $h$ or the sohliere obscrvale is

$$
h=\frac{d \ell}{L}+\frac{D(L-\ell)}{L} .
$$

$$
\cong
$$

In this system, then, a sizeablc field $a\left(\frac{l}{L}\right)$ can be obtained with a lens ot zero aporture $D$. Using the fiull field given by equation (5) the extreme points suffor I'rom consisderable vignetting and the focussing ertoct is accordingly much roduced. The largest ijield that can be used without vagnetting is given by

$$
\begin{equation*}
h=\frac{\ell a}{L}-\frac{D(L-e)}{L} \tag{6a}
\end{equation*}
$$

If the Schliere is nearer the grid and a large grid is used or by

$$
\begin{equation*}
h=\frac{D(T-l)}{I}-\frac{l d}{L} \tag{6b}
\end{equation*}
$$

If the Schliure is near the lens and a large lens is used. The ettect of vignetting is to make the light intonsity less in the outer parts of the rield. As this upsets completely the relationship between light intensity and deflection, and may lead to the masking of Schliere in the outer parts of the fileld, vignetting is a bad forit in these systems. It should be investigated in the undisturbed field and only the central unvignetted area used in subsequent work.

The total illuminating cone is given lor small angles by the smallor value or the two exprossions

$$
\begin{equation*}
\theta=\frac{d}{\ell} \tag{7a}
\end{equation*}
$$

or

$$
\begin{equation*}
\theta=\frac{D}{\mathrm{~L}-\ell} \tag{7b}
\end{equation*}
$$

These are for corresponding conditions to those ot equations (6a) and (6b) above.

The size' of 'tho' complomontary grid must be

$$
\begin{equation*}
d_{1}^{\prime}=d^{\prime}\left(\frac{L^{r}}{L}\right) \tag{8}
\end{equation*}
$$

The size or the Image on the screen 2 s

$$
\begin{equation*}
h^{\prime}=h\left(\frac{e^{\prime}}{l}\right) \tag{9}
\end{equation*}
$$

This image size can always be made more convenient by the use of an auxiliary lens between the complementary grid and the screen.

The necessary focussing conditions are

$$
\begin{equation*}
\frac{1}{F}=\frac{1}{L}+\frac{1}{L^{\prime}}=\frac{1}{\ell}+\frac{1}{l^{\prime}} \tag{10}
\end{equation*}
$$

In this equation all the terms are regarded as positive in Fig.7.
The 1mage movement $\Delta s^{\prime}$ at the knife-edge for a deviation $\varepsilon$ at the plane $O_{1}$ is given by

$$
\begin{equation*}
\Delta s^{\prime}=\varepsilon \frac{(L-\ell) L^{\prime}}{L} \tag{11}
\end{equation*}
$$

The total deviation to bring the mage from zero to maximum intensity is given by

$$
\begin{equation*}
\varepsilon_{\max }=\frac{s}{(L-\ell)} \tag{12}
\end{equation*}
$$

where $s$ is the width of the individual slits in the primary grid.
The black spaces in the primary grid should be large enough to prevent interference between the individual beams at large deflections. The thickness of the black spaces is given by

$$
\begin{equation*}
t=E(J-l) \tag{13}
\end{equation*}
$$

wherc $E$ is the maximum deflection possible in the Schlieren system.
Equations (5) to (13) suifizce to design or to calculate the performance of this type of system. As an example wo take a system with a $3: 1$ reduction of the grid and a rendering or the Schlieren rield at $1: 1$ magnification. From an application of equations (8), (9) and (10) the dimensions become $L=4 F, \quad l=2 F$ and if the illuminating cone is to work at $F / 10, \quad D=2 F / 10$ or $=F / 5$. The lens should have a minimum aperture of $\mathrm{f}^{\prime} / 5$. I ${ }^{2}$ the system is to work with a $20^{\prime \prime}$ wind tunnel, the Schliere being approximately central, then $I$ must be above $20^{\prime \prime}$ i.e. $4 \mathrm{~F}=20^{\prime \prime}$ or $\mathrm{F}=5^{\prime \prime}$ and $D=1^{\prime \prime}$. The system can use a $5^{\prime \prime} f / 5$ lens.

For practical reasons, the largest grid would appear to be about $12^{\prime \prime}$ square 1.e. $d=12^{\prime \prime}$. Application ot equations (5) and (6) gives

$$
h=\frac{12 \times 2}{4}+\frac{1 \times 2}{4}=6 \frac{1}{2}{ }^{11} \quad 1 x^{2} \text { vignetting is permitted }
$$

or $\quad h=6^{\prime \prime}-\frac{1}{2}^{\prime \prime} \quad=5 \frac{1}{2}$ " $\quad$ if no vignetting is permitted.
The complementary grid at 3 : 1 reduction will be 4" square and at this size the wiath or the In obtainable with suriticient detimition is about $0.003^{\prime \prime}$ equivalent to $0.009^{\prime \prime}$ slit width in the primary grid. With
this slit width and the grid-Schlieren separation of $10^{\prime \prime}$, the total deviation from zero to maximum intensity will be $\frac{0.009}{10}=3$ minutes of arc. Writh the system set to mean antensaty a range of $\pm 1 \frac{1}{2}$ minutes of deviation oan be measured.

This calculated system is relatively insensitive, at mean setting a $10 \%$ change in brightness boing given by a 9 sec deviation. The field cover is high for the diameter on the lens, an unvignetted field of $5 \frac{1}{2} "$ being given with a lens of only $1^{\prime \prime}$ diameter. It has therefore veen called a wide-field system. To achieve this wide field, however, requires the use of large grids with rine slits and these are not easy to make and set up.

The lons requirement should be easily met, the system calculated requiring a $5^{\prime \prime}$ lens at $f / 5$ covering a fiold angle of $30^{\circ}$ total. The main disadvantage $0 \underset{\sim}{\text { en }}$ the mod $2 s$ that the principle rays are inclined at an angle to the axis of the vind tunnel, and that thererore, unless the Schliere aro approximatoly two-dimensional, the intensities recorded upon the soreen will not be truly representative of the density gradients of the Schliere in fcous.

## 6 Focussing Systom No. 2

The previous systom requires a largo diameter grid unlike the usual systom which requires large diameter lenses. A moditication to the first system makus usc or' a large grid produced optically. The arrongement is shown in fig. 8 in which an image of the redl grid $S$ is formed by a lens $I_{1}$ at a position $S^{\prime}$. A condensing or ficld lens $L_{2}$ must be used to form an image of the lens $I_{1}$ upon the thard projuction $1 \mathrm{cns} L_{3}$ to ensure an adoquate illuminating beam. The Iunction of this third lons is sumilar to that of tho lons in tho first fooussing systom, namely to produce an mage of tho grid S' upon a complementary grid St and an Image of tho Schliere o upon a screen at I. Thu equations of the provious section govorn the performanou of this part of tho system. A complotely symmetrical arrangement can be used with a supplumontary lens, if required, between the complementary grid and the soroen.

This system may be more simple in use, as it will be easier to make a large field lons $L_{2}$ than a large grid, partioularly as the lons need not have great accuracy of $\mathrm{I}^{2} \mathrm{~g} \boldsymbol{g r e}$. The main disadvantage of this system is its length for it wall bo at least twioc as long as the previous system, and if the lens $L_{2}$ is to havo a reasonable rocal length may be moro than twice as long. It also suifers irom the same difficulty as the previous system in the Schliere aro not all in one plane, for the principal rays are inclined to the axis.

## 7 Focussing System No. 3

This is the counterpart of the nommon parallel beam Schlieren system, with grids replacing the slit and knife-adge. The system 2 s shown in Fig. 9 , in which the grid $S$ is placed at the rocus of the lens $L_{2}$, so that a parallel beam passes through the wind tunnel to the projection lens $L_{2}$. At the foous of this lens, an image of the primary grid is formed upon the complementary grid $\mathrm{S}^{\prime}$. An imago of the Schliere 0 is formed upon the screen at $I$.

The introduction oi- a finite grid of size $d$, illuminates the Schliero with a cone of angle $\theta$ given, for small values of $\theta$, by

$$
\begin{equation*}
\theta=\frac{\mathrm{d}}{\mathrm{~F}}=\frac{1}{\mathrm{I}^{2} / \mathrm{no}} \tag{13}
\end{equation*}
$$

where $F$ is the focal length oi the two ienses $f /$ no is the relative aperture at which the Illuminating beam works.

The diagram shows that Schlıere in the plane 0 will not all be illuminated by this full cone of light, that vignatting will occur and that the to tal dzameter of the fleld without vignetting will be glven by

$$
\begin{equation*}
h=D-C \theta=D-\frac{\ell \alpha}{F} \tag{14}
\end{equation*}
$$

where $\varepsilon$ is the distance of the Schliere from eqther lens if placed centrally or is the longer of the distances $L_{1} \mathrm{O} \mathrm{I}_{2} \mathrm{O}$ if unevenly spaced. The field is therefore restricted comparod m the usual single slit systom in which a ficld equal to the lens diameter $D$ is obtained. The size of the Schlicren.ficld upon the soreen I is given by

$$
h^{\prime}=n \frac{l^{\prime}}{l}
$$

and $l^{\prime}$ and $l$ arc conncoted by the equation

$$
\begin{equation*}
\frac{1}{F}=\frac{1}{\ell^{\prime}}+\frac{1}{\ell} \tag{16}
\end{equation*}
$$

with both distances being regarded as positive in the diagram.
The change in ray position $\Delta s^{\prime}$ at the complementary grid for a deviation $\varepsilon$ is given by

$$
\begin{equation*}
\Delta s^{\prime}=\varepsilon F \tag{17}
\end{equation*}
$$

and so is independent of the separation ot the two lenses and the Schliere position. The maximum measurable deflection from zero to maximum intensity is then given by

$$
\begin{equation*}
\varepsilon_{\max }=\frac{s}{\mathrm{~F}} \tag{18}
\end{equation*}
$$

where $s$ is the wath of the individual sints in the grid.
These equations (13) to (18) allow the complete system to be designed or the performance of a given system to be calculated.

As a numerical example we wll assume twn $36^{\prime \prime} f / 6$ lenses are used for the lenses $L_{1}$ and $L_{2}$. For an illuminating cone working at $f / 10$ the grid size from equation (13) is $d=3.6 "$. From equation (16), in order to have a real image of the Schliere 0 formed upon a screen, $\ell$ would necl to be larger than F i.e. $36^{\prime \prime}$ becomes the least value for $\ell$. Under suoh circumstances we find

$$
h=\frac{36}{6.3}-\frac{36}{10}=5.7^{\prime \prime}-3.6^{\prime \prime}=2.1^{\prime \prime}
$$

The fizeld without vignetting is much reduced and would be reduned to zero if with this arrangement the lenses had been separated by $144^{\prime \prime}$ to produce a 1:1 image of the Schliere upon the screen at I. However the use of an auxiliary lens placed beyona the knife-edge allows the two lenses to be brought oloser together, and we therefore take the case of Fig. 9 with the lenses separated by $20^{\prime \prime}$ and an auxiliary-lens placed at $I_{3}$ to focus the Schlieren Image.

The unvignetted diameter then becomes

$$
h=5.7^{\prime \prime}-1^{\prime \prime}=4.7^{\prime \prime}
$$

By application of equation (16) to $I_{2}$ and $L_{3}$ successively, the necessary 'ocal length of the lens $\mathrm{L}_{3}$ can be fround. 'For sli't wadhs o1' 0.005 " in the primary grid, the maximum deviation becomes (equation 18) equal to 28 seconds or arc or at a mean intensity setting $\pm 14$ seconds oi arc. The $10 \%$ illumanation change in sensitivity is then $1 \frac{1}{2}$ secs of arc. The system may be made less senstive by having whiter slits on the grid.

This system has gratiter sensithvity than focussing system No.1, has rather less ticud and ruquiros three large lenses and two small grids. Compared with the usual non-focussing type, it achzovos its focussing action at the cxponse or a roduced tield and the necossity for an auxiliary lens.

## 8 Application to Mirror Systoms

A common wand tunnel Schlicren system uses concave mirrors in place or a lens, the systom being shown in Fig.10., The slit and knife-edge are not on the axis or the concave mirrors but are offset to prevent interference with the main parallel beam. They are oris'set by equal angles on opposite siden oll the axis, the coma orrors introduced by the two mirrors are then oqual and opposite, the ilnal image at tho knife-edge being comalrue but sux'rering ircm astigmatisg thich does not cancel.

A typical systum uscis mirrors $10^{\prime \prime}$ in diameter, working at $\mathrm{f}^{\circ} / 10$ or $f^{\prime} / 15$, with a mirror suparation o1 $120^{\prime \prime}$. This may be converted into a arooussing syston by the substitution ol grids for the ilit and knifo-edege The sizo of the grids required for a givon illuminating angle can be found from equation (13). For a 2ocal longth $0 A^{\prime} 120^{\prime \prime}$ and a diametcr of 10", an illuminating conu of $\mathrm{r}^{2} / 10$ roquires a $12^{\prime \prime}$ grid and with the mirror soparation oI $120^{\prime \prime}$ and tho schlicre mid-way botween the mirrors, this gives an unvignetted ticld of only 4 ".

As the Gchliore is within tho focus of the second mirror an auxiliary lens must be used to tocus the Schliere upon the dinal soroen and this lens must have a diameter as large as the second grid, in thas case of $12^{\prime \prime}$ or over in diameter.

These figures aoamittedly reicer to an $f / 10$ system, this being a purely arbitrary choice. Using in illuminating cone or $f / 20$ corresponding values berome $6^{\prime \prime}$ for the grid and auxiliary lens diameters and the unvignetted $f^{\prime i} e l d$ becomes $7^{\prime \prime}$. These values are morc acceptable in many respects but the focussing ericot will bo much reduced by this change. The illuminatoa patch size on the window or a wind tunnel $20^{\prime \prime}$ in length will be only $\frac{1}{2}{ }^{\prime \prime}$, or ror a $5^{\prime \prime}$ wind tunncl only $\frac{1}{8}$ ". As disturbances have to be small compared with this patch size, it is, clear that there will be very little focussing etirect 1.3 the $5^{\prime \prime}$ tunnel although some advantage might be obtained in the $20^{\prime \prime}$ tunnel.

Another dueliculby in using a conventional mirror system becomos apparent in Fig.11. The optical path is halved by using an auxiliary plane mirror to iold up the light path and thus allow the main mirrors to be brought closer together. When a grid is used, the necessary diameter* of the plane mirror is increasod, $r^{\prime} r o m 5^{\prime \prime}$ to $8^{\prime \prime}$ r $^{\prime}$ or an $\mathrm{I}^{\prime} / 20$ systom or to $11^{\prime \prime}{ }^{2}$ or an $f / 10$ systom. The ort axis angle will have to be increased considerably to prevent the interference of this, much enlarged mirror with the main beam. This incruase in angle brings increased astigmatism in the lmage, this astigmatism growing rapidly with angle. It is also apparent
that all the slits do not subtend the same angle with the axis and for those at one edge of the grid on extra $3^{\circ}$ angle is necessary for an $f / 10$ system. The incrensed astigmatism present in some ot the slit images W1ll er'rectively reduce the focussing action of the system and cause a general loss of definition, this being accentuated by the loss due to the increase oi the overall angle whth the axis and consequent astigmatism.

The sensitivaty of the mirror system should not be affected by the substitution of the grid providing the slit dimensions have been correctly chosen. It seems unlikcly, however, that the full benefit of a focussing system can be obtalned with the usual concave mirrors owng to the large $\mathrm{f} / \mathrm{no}^{\prime} \mathrm{s}$ and because of the neaessity of forming zmages of good quality at large angles off-axis. lifrors are known to be poorly corrected for oriaxis aberrations and, from this point of view, the use of well corrected flat-field lenses of much shorter focal length seems to be necessary ir the I'ull advantage of the focussing system is to be obtained.

## 9 Practical Details

It is not easy to suggest the best values for all the variables of these systems, and It may vell be that the Schlieren user will require a number or systems having difforent sensitivities and field diameters, these belng the main variables. For fields of $4-5$ inches diameter with a $10 \%$ sensitivity of $2-3$ secs deviation, the system of section 7 using $36^{\prime \prime}$ $f / 6.3$ lenses should give good results. Shorter focal lengths could be used with a loss of field ard sensituvaty.

The two main lenses could be photographic lenses or even doublet lenses could be uscd as they have to handle a total angular field of only 60. They should be well corrocted tor spherical aborration, coma and chromatic aberration if the illumination in the Schlieren field is to be uniform, free from colour, and the full rocussing extect is to be obtainud. The auxiliary lens roquired cannot be the usual single lens as tho imageforming light is no longer confincd to a single ray but has a cone working at around $\mathrm{P} / 10$ in the object space and according to the final image saze may work at even smaller apcrture ratios in the image space. The spherical aberration present in a single lens may then be sufficient to destroy the definition in the Schlieren amage. For an image suze of $4^{\prime \prime}$ in the $36^{\prime \prime}$ system, the auxiliary lens has an aperture of $3 \frac{1}{2}^{\prime \prime}$, a focal length of $18^{\prime \prime}$ and has to cover an image $5^{\prime \prime}$ in diametor. A flat field photographic lons is necosscxy here.

The primary and complementary grids are required to match each othor with great accuracy, and the optical system should form an image of the primary grid with the highest possible definution. The complementary grid may well be made by photography through the system using a photographic plate in the position at which the primary grid is focussed. The photographic plate, atter processing, becomes the complementary grid and has to be replaced in the sane position. Auxiliary marks should be placed on the edges of the primary grid, so that the plate after processing can be replaced and re-alignod with the optical image of the primary grid. This method of making tho complomontor; grid has the advantage that any distortion in the lens svstem will be componsatod and even though the siit imago may be curved slightly, the complomentary grid will still match it. If the complementary grid is made by contract printing from the primary grid, this matching vill not occur in the presonce of lens distortions and intonsity orrors will arlso.
!
It should also be noted that it is essential for the complementary grid to be replaced in the correct plane with great care. If this is not done, the mage or the primary grid will not be in focus on the complementary grid and intensity differences will bo blurred over. In addition
the scale of the mage will be wrong, it will not be possible to have all the slits cutting off together, the intensity due to each slit will be difererent, the focussing crifect will be reauced and the relationship between intensity and anr denizty gradient $11 y$ vary over the ficld. It will therofore be necossary to ray more attention to the correct focussing and aligrament of the complementary grid when sotting up tho apparatus, than is usval in the casc of a single knifomodge.

As the grid systum requires the optical compononts to give an image over un extended ficld with a finnte aperture, lenses aro more appropriato than mirrors. They have however appreciable chromatic aberration and it may be necossary to work with monochromatic light porhaps from a sodium lamp or green mercury light filtcred from a high pressure mercury lamp. Unless approximately monochromatıc light is used the image formed on the complemontary grid may havo dir'fuss coloured edges and the visual Schlioren field will have colourod patches.

Illumination of the primary grid may present a problem as the aroa'to bo illuminated is lurge (up to $12^{\prime \prime}$ square) and each slit in the grid must illuminato the whole of the unvignotted field. A condensing lens of large diameter can be used if the source has a large enough aroa to illuminato the contire ficld, a condition not hound in the compact source type ol high pressure mercury lamp. The compact source lamp is so desirable for its nearly monochromatis, highly intense light output that it will largely be used for focussing systems whth an opal glass diffuser closo to the grad. This method of zlluminating loses a groat deal of light in the opal glass and the final image braghtness of the Schliere upon the screen may bo lower than the brightnoss normally mot in single slit systems.

## 10 Conclusions

(1) The focussing effoct dopends upon the incroased alluminating anglo obtainod when grads aro substituted for the usual kniferedges.
(2) The effect upon the final image plane of an out-oi-1 ${ }^{2}$ ocus Schliurc depends largely upon the amount of deviation caused by the Schlicro and its area.
(3) Arcas wh th Jarge deviations mil always be recorded in the final image even though much out of focus.
(4) Tho location ol discontinuaties can bo found from their plano of best focus.
(5) The untensity distribution in the plane in focus as not a reliable guide to the density gradients in that plane.
(6) Focussing systoms have by normal wind tunnel standards a small working field.
(7) For parallcl beam systems tho unvignetted fleld is considerably less than the marror or lens diamoter although the grids can be small in size.
(8) For inclined buam systoms the field can be larger than the lens diameter although very largo grids may be required and these are difficult to make and Illuminato.
(9) The inclined bean systems are at a disadvantage with threedimensional schlicre.
(10) Nirror systems are not as well suited as lens systems because of the large off-axis aberrations likely to be introduced. Present wind tunnel mirrors cannot effectively be adanted to focussing systems.
(11) These focussing systems require very great care in setting up and adjustment.
(12) There are likely to be more sources of intensity errors in a focussing system and quantitative work is not likely to be accurate.
(13) The focussing systems are not suitable for general wind tunnel work because of their small fields but may be of use as special tools for investigating problems in three dimensions.
(14) The use or lower quality windovs and optical components is possible in focussing systoms.

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FIG.I.2.3\& 4


FIG.I CONVENTIONAL SCHLIEREN SYSTEM.


FIG. 2 THE ILLUMINATING CONE .


FIG. 4 EFFECT OF AN OUT - OF - FOCUS DISTURBANCE.

FIG. 5,6.7\& 8


FIG. 5 AREA AFFECTED BY A DISTURBANCE


FIG. 6 WIND TUNNEL WINDOWS IN SCHLIEREN SYSTEM.


FIG. 7 SINGLE LENS FOCUSSING SYSTEM


FIG. 8 SYSTEM USING OPTICAL ENLARGEMENT OF GRID


FIG. 9 PARALLEL BEAM FOCUSSING SYSTEM.


FIG. IO CONVENTIONAL MIRROR SCHLIEREN SYSTEM.


FIG. II FOCUSSING MIRROR SYSTEM.

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