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On the Extrapolation and Scatter of Creep Data

By

K. F. A. Walles and A. Graham

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On the Extrapolation and Scatter of Creep Data

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October, 1961

SUMMARY

A method of extrapolation based on a previously given formula for creep is derived, and examples are given of its application to creep rupture and creep strain data. The accuracy of extrapolation is statistically evaluated and shown to be within the observed scatter of the experimental data. Seven other methods are shown to have errors significantly greater than that of the data.

Limitations of the method are discussed.

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1. Introduction

The problem of devising a reliable method for extrapolating the results of practicable creep tests to the long periods over which many machines are required to operate has attracted considerable attention, but the view is widely held that the changes that take place in engineering materials during creep are far too complex to be governed by laws that would be simple enough to be employed. Nevertheless a number of simple methods has been proposed. The methods differ in detail, but in principle either

(a) extrapolate a supposed law that relates time to stress with temperature constant, e.g. the well known methods of direct extrapolation on graphs of log stress versus log time or of stress versus log time

or (b) extrapolate a supposed law that relates time to temperature with stress constant, e.g. those of Bailey, Larson and Miller, Manson and Haferd, and Dorn.

In effect, the advocates of these methods advance, collectively, the opposite view that the governing laws are so simple that a change of stress, according to (a), or a change of temperature, according to (b), changes the rate of creeping of every constituent of the material by exactly the same amount. Several of the methods have shown an encouraging success, but all have been found seriously to fail on occasions in which there was no especial reason, the contrasting view apart, for expecting them to fail (c.f. Appendix). They are thus generally believed to have their uses but to be unreliable.

The method of the present paper lies between the opposed alternatives, and the results contribute evidence that the laws of creep are neither so complex as the one view would suggest nor so simple as would the other. It is based upon an extensive study of experimental data for engineering alloys[1-5] for the purpose of establishing a firm quantitative expression for the relationship during creep between the four mechanical variables concerned, namely stress, strain, time and temperature.

All interpretations of creep data for the purpose of guiding extrapolation have to contend with the extremely limited amount of data usually available relative to the considerable scatter in performance. The amount is seldom sufficient to confirm or deny any reasonable law that may be considered. The interpretation upon which the present report is based is supported by the fact that it has led to an acceptable evaluation of the scatter, and the scatter found has proved to be in quantitative agreement with that directly indicated by the available results for replicate specimens.

In the working ranges of creep, the differences of scatter between different fully-developed alloys do not appear to be large, and a scatter band of width equal to twice the standard deviation (within which only $\frac{2}{3}$ of the points may be expected to fall) is seldom narrower than $\frac{1}{5}$ cycle of log time. Thus whenever data plots are examined for evidence of quantitative trends, it is more realistic to associate with each point confidence limits of this order of magnitude than to follow the frequent practice, when a smooth curve is drawn as far as possible through the experimental points, of

accepting/

accepting all but inconvenient points at their face value. The procedure leads to a rather different weighting of previous evidence for regular trends in creep.

Almost all the data examined have been consistent with the view[2,3] that the creep strain ε comprises the sum of a number of terms of the form

$$C\sigma^{\beta}\phi^{K} \qquad \ldots \qquad (1)$$

in which σ is the stress, β and K are simple numbers, C is an arbitrary constant, and ϕ a combination of time t and temperature T of which further details are given in Section 2. The data that are unambiguously exceptional are those for which a marked change in the nature of the material appears to occur within the experimental range. A conclusion to be drawn from the research of which the present work forms part[6] is that creep closely follows the law stated in an interval between changes of this kind, but departs widely from it when they occur. They may occur during the progress of a test, as when decelerating creep follows a period of accelerating creep, or during loading, when unusually large strains may be observed. They may also be encountered within the range of stress or temperature covered by different tests, so that one part of the data applies to one condition of the material and another part to another condition.

The need for fairly extensive sets of data for study, especially for tests of extrapolation, has the effect of confining attention principally to established materials. They are materials that have been successfully developed to avoid these changes, or at least to reduce their effect to magnitudes comparable with the random irregularities in results. The need also to reject unusually scattered or ambiguous data, which is unavoidable in early studies, has the similar effect of selecting materials in which such changes are unimportant. However, this is a current limitation on the testing of any method of extrapolation.

The present method is based upon fitted curves that conform to the law given and which give uniform weighting to all the data available. The fitting of the curves is best performed by a graphical analysis of the data identical with that whereby the laws were discovered. The scatter was evaluated by comparison of experimental points with theoretical curves fitted to each set of data as a whole. Accordingly the scatter results are presented in Section 3 after a description of the method of fitting in Section 2. The method of extrapolation is described in Section 4 and results are discussed in subsequent sections.

The various features of the formula could be set out in a formal mathematical manner and the fitting and extrapolation be handled numerically. However, the procedure would be unduly cumbersome and without significant benefit.

The data considered are principally those previously analysed in detail in Ref.5 and comprise all those in the Timken Digest, complete families of creep curves for Nimonic alloys, together with data for other alloys that came to hand. Over 100 sets of data were studied relating to alloys of more than 40 compositions. The scatter of all the data was

evaluated,/

evaluated, but tests of extrapolation were necessarily confined to the rather few sets that were both without undue scatter and sufficiently extensive to be usefully divided into shorter and longer time portions with enough detail in the former to define behaviour. All sets of this category that were available when the work was in progress were used. Analyses of creep data from other sources that have since come to hand have also been made.

In order to make details available without unduly extending the present paper, these are included in Refs.6,7,8,9 and 10.

2. Analysis of Data

The following summary, which will be illustrated later by examples, will indicate both the first stages of the method of extrapolation and the experimental support and basis for the guiding theory. Rupture data involve two stages of analysis and families of creep curves three stages. Extrapolation requires a further stage.

2.1 Stage 1: Resolution of creep curves into components

Creep curves for many commercial alloys have been found to be a close fit to the equation[3]

$$\varepsilon = at^{\frac{1}{3}} + bt + ct^{3} \qquad \ldots \qquad (2)$$

in which ε is the creep strain after subtraction of the elastic strain that occurs during loading[9], t is time measured from the instant when the creep strain may be regarded as zero, and a, b and c are constants in a particular test that depend upon the constant stress σ and constant temperature T of the test. The exponents $\frac{1}{3}$ and 1 are those found by Andrade, Kennedy, and others for a variety of materials and relate to the primary and secondary stages of creep, while the term with exponent 3 represents accelerating creep. The interpretations of ε and t, and the values of the exponents have been derived and checked by detailed studies, especially of the Nimonic alloys[3,4,5,7,8,9], and there is good reason to believe that the equation is of wide applicability.

When extrapolation is based on families of creep curves, Stage 1 consists in the resolution of each curve into the three components of strain represented by the three terms on the right of Equation (1). They will be referred to as the $t_{\frac{1}{3}}$, t_1, and t_3 components and are represented by straight lines on a graph of log ε versus log t with slopes of $\frac{1}{3}$, 1 and 3 respectively. A convenient and accurate method of resolution of the curve into its component lines by the use of master curves has been devised[10].

2.2 Stage 2: Resolution of components into terms

The magnitudes of the three components of Equation (2) vary with time stress and temperature. At each temperature of testing, the components are displayed by the above method for each of the testing stresses. Stage 2

consists/

consists in plotting, for the $t_{\frac{1}{3}}$, t_1 and t_3 components at each temperature separately, the log stress of the test against the log time for the component concerned to reach an arbitrary value, say 0.1%. The curves obtained are analogous to plots of times to rupture, and indeed creep rupture data afford the simplest example.

Experience has shown that the time to rupture corresponds closely to the time for the dominant component on the right of Equation (2) to reach a critical value. Materials that rupture after a pronounced stage of accelerating creep are usually found[5] to have times to rupture that are effectively determined by only the component of Equation (2) for which the exponent is 3, while materials that display a long stage of steady-state creep and proceed to rupture without an appreciable accelerating stage have times to rupture that are effectively determined only by the component whose exponent is unity. Thus for rupture data the direct experimental graphs of log stress versus log rupture time for each of the temperatures of testing (see Figs.1, 2 and 3) are equivalent to the graphs of Stage 2, just described.

It is well known that the points in such plots of rupture data are liable to fall upon segmental curves formed by straight lines with abrupt changes of slope; indeed, the onset of the changes of slope has been one of the problems of prediction. Detailed study has shown[4] that the slopes of the segments, where the segments are clearly present, take values from the sequence

$$-1,\ -\frac{1}{2},\ -\frac{1}{4},\ -\frac{1}{8},\ -\frac{1}{16},\ -\frac{1}{32},\ \ldots. \qquad \ldots (3)$$

The clear experimental evidence for the existence of several segments indicates that each component on the right of Equation (2) is compound, i.e. each of the coefficients a, b, c consists in general of a sum of terms in stress raised to a power, so that the creep strain is to be represented by a sum of terms of the form

$$\text{const.} \times \sigma^{\beta} t^{K} \qquad \ldots (4)$$

in which K has one of the values

$$\frac{1}{3},\ 1,\ 3,$$

and the ratio K/β has one of the values at (3). The constant contains the temperature.

The qualification "where the segments are clearly present" relates to the fact that the proposed formula predicts, in agreement with experiment, that the log stress versus log time graphs are continuous curves which may be

represented/

represented adequately by straight lines with abrupt changes of slope only when K has the value 3 (c.f. Fig.1)*. When $K = \frac{1}{3}$ or 1, the theoretical curvature is more gradual, and the points must be fitted with the curve appropriate to the value of K. The curves may be regarded as being asymptotic to lines of standard slope: Fig.2 is an example.

Stage 2 thus consists of the resolution of the log stress/log time plots (either for rupture or for a constant value of the strain component concerned) into their component segments ($K = 3$) or asymptotes ($K = \frac{1}{3}$ or 1). These are conveniently referred to as the 1, $\frac{1}{2}$, $\frac{1}{4}$ etc. segments or asymptotes. Any one set of straight lines of common standard slope $-K/\beta$ for all the temperatures of testing is the graphical display of a single term of formula (4). A master-curve method[10] has been devised for determining the asymptotes.

2.3 Stage 3: Determination of the temperature dependence of the terms

The spacing in log time with respect to temperature between the lines of a given standard slope (whether segments or asymptotes) differs from slope to slope. The result is thought to indicate that the different metallurgical factors present are influenced by temperature to different extents. The time-temperature relationship may be assessed for each term by cross-plotting the Stage 2 graphs for the slope concerned for any constant value of the stress, in order to obtain a graph of log time versus temperature. Examples are inset in Figs.1, 2 and 3.

The various time-temperature parameters that have been proposed by Dorn, Larson and Miller, and others are, in effect, equations of a curve to fit cross-plots of this nature. For many purposes, but not all, the choice at this stage of a time-temperature parameter is not critical, and even a freely-drawn curve in the manner used by some authors would often be reasonable. For a number of heat-resistant alloys, however, the points derived from at least one of the standard slopes are found to fall, as in Fig.1A, upon curves that are convex towards longer times, i.e. to have both a negative slope and negative curvature. Such a result may be seen to be incompatible with all the familiar parameters, and indeed with the literal use of the usual exponential law of temperature dependence. The following parameter[2,3,11] has been found to meet requirements

$$t(T' - T)^{-A} = \text{constant, when } \sigma \text{ and } \varepsilon \text{ are constant} \quad \ldots (5)$$

in which T' is a constant temperature greater than T and A is an empirical constant. The different temperature dependence of the different factors in creep is represented by the use of different constants T' for different slopes. The exponent A is not critically determined by experiment and has been standardized at the value 20 pending more definite evidence. The parameter5 accords with a feature to be observed in creep data that extend over a sufficiently wide range of temperature that properties become anomalous near a certain temperature.

When/

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*There is evidence for some materials that an additional term component of Equation (2) with $K = 9$ is needed. This is of no consequence in the analysis of rupture data since the changes of slope are abrupt for any large value of K. The precise value of K is then of no consequence.

When the log time/temperature graphs have the opposite, i.e. positive, curvature, one or more of the alternative parameters are available. However for consistency with the above evidence and for a more specific reason given in Section 4 the requirement has been met by adapting Equation (5) to the form

$$t(T - T')^{+20} = \text{constant}, \quad T' < T \qquad \ldots (6)$$

Stage 3, which in principle completes the initial analysis of the data, consists in the determination, for each group of asymptotes of one standard slope, from a log t versus T cross-plot of a group at an arbitrary constant stress, of the various values of T'. In practice it is usually necessary to revise the analysis by one or more successive approximations in order to obtain the best overall fitting of the data.

2.4 Discussion

The analysis corresponds to the assumption that creep strain is represented by a sum of terms of the form (1) in which ϕ is the expression given in Equations (5) or (6). Alternatively and in summary, it assumes that a family of creep curves has the following structure.

Each creep curve comprises three components represented by the three terms on the right of Equation (2). Each component is compound and comprises the sum of several similar basic terms with common K of the form (4). If each basic term is considered in isolation, the graph of log strain versus log time is a straight line of slope K with one of the values $\frac{1}{3}$, 1 or 3; the graph of log stress versus log time is a straight line of slope $-K/\beta$ with one of the values 1, $\frac{1}{2}$, $\frac{1}{4}$ ....; the graph of log time versus temperature is a standard curve (Equation (5) or (6)): the quantities not mentioned in each instance are treated as constant. The usual direct experimental graphs and cross-plots of these represent the joint contributions of several basic terms and are thus normally curves whose relation to the standard constants is not apparent until the data are analysed in terms of the present formula.

It is a feature of a sum of terms of the present form that their relative magnitudes change rapidly with changes of the variables, so that the number of terms that is important in any one set of creep curves is not unduly large.

For the purpose of prediction, the results of the graphical analysis of special interest are as illustrated in the examples of Figs.1 to 3, the set of straight lines of standard slopes on the graphs of log stress versus log time, and the associated log time versus temperature cross-plots. Before the method of prediction is discussed, however, it is desirable to consider the experimental support for the formula that has been derived and the closely associated question of the scatter of experimental data.

3. Scatter in Creep and Direct Tests of the Formula

The scatter in the sets of data referred to in Section 1 has been evaluated on the one hand by comparing experimental points with theoretical curves fitted as a family, by the procedure just outlined, to each set of

data/

data as a whole, and on the other by comparing the available results, unfortunately rather few, of replicate tests for the same condition. Results are summarized in Tables I to III. The scatter in log time was found to be substantially uniform and Gaussian over the range of stress and temperature, and has accordingly been treated on these assumptions. Evidence for the randomness of the scatter of some of the sets of data is provided by the results of Section 5. On the other hand the scatter appears to be significantly less in the tertiary stage than in primary and secondary stages of creep.

3.1 General magnitude of scatter indicated by analysis of rupture data

Table I gives the scatter, expressed as a standard deviation in log time, as evaluated for all the creep rupture data of Ref.5. The values are grouped into two supposedly random populations comprising those for materials whose times to rupture appeared to be controlled by terms in t^3 and those by terms in t (see Section 2.4). The mean s.d. for the first group is 0.14 in log time and for the second group 0.21. A check on the general correctness of these magnitudes is provided by the analysis by an independent method of the present and other data by Monkman and Grant[12]. They found that the scatter in rupture lives had s.d.'s ranging between 0.14 and 0.23 cycles of log time. For 10 sets of data, where no figure is given in the table, no scatter values were assigned. Where a dash occurs, the experimental points were too few in number effectively to define the theoretical curves, a cross denotes the presence of a characteristic anomaly referred to in Section 3.3 below, and an asterisk one of the following not immediately distinguishable possibilities: more terms should have been used in the formula to obtain satisfactory fitting (no more terms were used than for which there was clear evidence); the formula did not represent the true behaviour; or the data were unsatisfactory.

3.2 Assessment of scatter by analysis of creep data and comparison with replicate data

Table II presents in summary form the results of an analysis of the scatter in families of creep curves for the Nimonic series of alloys, an analysis set out in some detail in Ref.7. The second column of Table II(a) shows the scatter of the points that make up the experimental creep curves about the theoretical creep curves of the best fitting family. The second column of Table II(b) shows the scatter in tertiary creep alone as assessed by a comparison of the times to reach a given value of ct^3, in Equation (2), in each individual test with times to reach the same value taken from the corresponding curve of the best fitting family. In the third columns of these tables are given the corresponding s.d.'s for the available replicate tests and, in brackets, the number of pairs of tests upon which the s.d. is based. There is no clear evidence that the indicated differences of scatter between these materials are significant, and since the number of results concerned were about the same for each material, simple mean values are given. Since there were large differences between the numbers of replicate tests, however, both a simple mean of the results of these and a mean weighted for the number of tests are given. The s.d.'s estimated by fitting the formula and as directly indicated by replicate tests are in good agreement. The scatter in tertiary creep is seen to be about half the average scatter over the whole extent of the creep curves.

3.3 Assessment of scatter by analysis of rupture data and comparison with replicate data

For the materials of this table, the rupture data were analysed independently. The scatter of the rupture points about theoretical curves

fitted/

fitted to the rupture data is given in the fourth column of Table II(b). The mean in brackets at the foot of the column excludes the rather high value for Nimonic 90-III. This material showed a tendency at the highest temperature for the accelerating creep to be followed before rupture by a further period of decelerating creep instead of immediately by rupture in the usual way. The onset of this behaviour was irregular. The scatter in t_3 values is seen to be similar to that for the other materials. The scatter in rupture times is seen to be in good agreement with the scatter in the t_3 term.

Creep curves for materials that do not exhibit tertiary creep and whose rupture appears to be determined by the $t_1^{\frac{1}{3}}$ terms have not been studied in detail, but the scatter in rupture time for the two groups of materials in Table I is seen to correspond generally to the pattern of Table II.

For only four of the materials in Table I and two in Table II were results available from replicate rupture tests. The scatter of the rupture data about the fitted theoretical curves is compared in Table III(a) and (b) with the scatter in replicate tests, the two types of material being treated separately. The agreement indicated by the previous tables is here largely repeated, but there is a larger spread in some of the values. The scatter of the data about the fitted curves is indicated as being less than the scatter in replicate tests, but the result is probably due to the sampling. The opposite result indicated by S.590 may be significant. The data were particularly extensive and on this score alone would be expected to provide a stringent test; on the other hand, many of the points relate to tests of extremely short duration with the loading period comparable with the creeping period. Such tests are not then creep tests, but are combinations of creep and tensile tests. The question of the homogeneity of the data also arises.

3.4 Discussion

Differences between the scatter values assigned to different materials may in some instances represent actual differences in variability, but the possibility of sampling differences and of varying systematic departures from the formula must be recognised.

In regard to systematic effects, one that is thought to be characteristic has been mentioned in connection with Nimonic 90-III and Table II(b). Another is exhibited by the materials marked ≠ in Table I (see Ref.5). The points in the log stress/log time graph associated with a line of given standard slope fall in two distinct groups related by a parallel displacement. Two flow regimes appear to be present with either a random or systematic relation between them. The effect may well be present in some degree in other data, especially the other marked data in Table I, for it is only clearly identified by a suitable number and distribution of experimental points.

A more elaborate statistical analysis would be necessary to decide the significance of the occasional discrepancies. However, the tables show that the differences of scatter between different materials are usually little more than the difference for any one material between the scatter in its accelerating and decelerating stages. The results may therefore be simply summarized by the statement that the standard deviation in accelerating creep and associated rupture times appears to be generally rather more than $\frac{1}{10}$ cycle of log time, while the standard deviation in the earlier stages and associated

rupture/

rupture times is at least $\frac{1}{5}$ cycle. Apart from certain systematic effects upon which there is too little information for study, the formula has been shown to be in agreement with experiment to within the limmts of the least scattered data.

4. Extrapolation

Extrapolation is based upon the sets of straight lines with standard slopes $-K/\beta$ on graphs of log σ versus log t together with their associated cross-plots of log t versus T. Limiting positions can be assigned for lines of those standard slopes for which there is no direct experimental evidence, and these lead to upper and lower limits to extrapolated values.

The method is best explained by means of examples.

4.1 Rupture data, with tertiary stage

The simplest case to consider is that of a material which ruptures after a pronounced stage of accelerating creep whose life is effectively determined (c.f. Section 2.1) by only the terms for which $K = 3$ and for which the expected log stress/log rupture life graphs effectively consist of segments of standard slope joined by rather abrupt transitions*. The lower part of Fig.1A is an example of data that directly show the standard slopes $-\frac{1}{8}$ and $-\frac{1}{4}$. (The scale of log stress is four times the scale of log time, hence e.g. the line of slope $-\frac{1}{4}$ is at 45° to the horizontal.) This figure has been selected as a convenient illustration of the features and problems of extrapolation. It is not however one of the best examples of the usefulness of the method, because the test conditions, chosen for other reasons, do not establish the extrapolated values very closely.

The points relate to Stage 2 of the analysis and represent a direct plot of rupture data for Nimonic 80A. They have been arbitrarily divided into two groups: the open points are for times longer than 2000 hours and the filled points for shorter times. For a test of extrapolation, attention is first given to the filled points. The unbroken lines in this part of the figure and the whole of the upper part (which relates to Stage 3) are based on the filled points only, and represent the completed analysis of the shorter-time data. The points in the upper part are derived from the shorter-time points in the lower part by cross-plotting, for slopes $-\frac{1}{8}$ and $-\frac{1}{4}$, at stresses of 20 and 10 t.s.i. respectively. (For slope $-\frac{1}{2}$ discussed below the stress is 2 t.s.i.) Each curve shows the relation between log time and temperature for which the basic terms of the form (4) with $K/\beta = -\frac{1}{8}$ and $-\frac{1}{4}$ have each a standard value. In more detail, the points in the upper part correspond to lines of standard slope in the lower part that were first drawn in tentatively by inspection. For clarity these lines, which would be parallel to but slightly displaced from the solid lines shown, have been omitted. The lines actually shown correspond to the curves in the upper part, and both are as found by trial and adjustment to secure the best fitting of Equation (4) to the filled points. They complete the initial analysis, and extrapolation may now be considered.

Direct extension beyond 2000 hours of the lines of the steeper slope $-\frac{1}{4}$, as shown in broken line in the lower part, afford an extrapolation

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*See footnote on page 8.

that is liable to be optimistic because at any instant the trend may change abruptly on to a line of steeper slope. In this instance the next steeper slope is $-\frac{1}{2}$: the possibility of a still steeper slope is considered later. The worst possibility - still steeper slopes apart - is thus that at any of the temperatures in the figure, the result of a further experiment at a lower stress may fall on a line of slope $-\frac{1}{2}$ drawn through the last experimental (filled) point at that temperature. "Pessimistic" lines of this kind are indicated at 815°C and 750°C. They pass through points A and B respectively.

At the other temperatures, however, a similar possibility is not open because of a peculiarity of the time-temperature relationship. Equations (5) and (6) show that the spacing $\Delta \log t/\Delta T$ between lines of any one slope at different temperatures is $20/|T - T'|$. If the temperature T', at which properties are indicated by the formula to become anomalous, is to have a metallurgical significance, it must presumably lie between 0°K and the melting point; thus the spacing is indicated to be not less than a minimum value corresponding to the use of the more remote of these temperatures for T'. Apart from a circumstance that appears to be definable and will be discussed in the final section, this condition on the spacing and magnitude of T' has been found to hold for all the data examined. There has been no clear case in the study of data for more than 40 alloys in which T' was indicated to fall outside this range. Data for Nimonic 90 has also provided direct evidence to confirm the prediction of very large spacing $\Delta \log t/\Delta T$ when T approaches T'.

In view of the condition of minimum spacing, the lines of slope $-\frac{1}{2}$ at 700°C and 650°C are shown in the lower part spaced from the line of this slope through the last point B at 750°C by the minimum amount corresponding to T' = 0°K. The corresponding points and curves are shown in the upper part, the curve being chain-dashed and the points being arrowed since they refer to a minimum time rather than to a definitely established time. The condition is of no assistance for extrapolations at 815°C and 750°C because the spacing enforced by the final points A and B at these temperatures happens to be already greater than the minimum. The lines of slope $-\frac{1}{2}$ through these points thus afford a pessimistic extrapolation.

It follows from this construction, as Fig.1A shows, that the last point B at 750°C provides, with the remaining data, a sector S of unambiguous extrapolation. Outside this sector, there is a range of increasing ambiguity as shown shaded.

In regard to the choice of the value $-\frac{1}{2}$, data are known that clearly exhibit the scope of -1, but steeper slopes have not been encountered. In the present example, if attention is confined to stresses greater than the lowest stress at 750°C for the short-time set, namely point B, the condition of minimum spacing may be seen to exclude the possibility of lines of slope steeper than $-\frac{1}{2}$. If lines of a steeper slope were present, the line for 815°C could reasonably pass through the point A at 6 t.s.i., but then those for 700°C and 650°C respectively are not permitted by this condition to pass to the left of positions C, D, where C and D are spaced from point B (at 10 t.s.i.) by the minimum spacing appropriate to their temperatures. At higher stresses than 10 t.s.i. (but not smaller stresses) such steeper lines would provide more optimistic extrapolations than the lines of slope $-\frac{1}{2}$ drawn in the figure. Indeed, for extrapolations towards high stresses, the smallest permissible slope is the most pessimistic. In the present instance, however, the existing points will not tolerate any slope smaller than $-\frac{1}{2}$. The rule for pessimistic extrapolation is therefore to use the most unfavourable slopes of the series that the data and minimum spacing condition

will/

will allow. The range of useful extrapolation is rather limited in this example because of the small duration of the longest test at the highest temperature (point A). A further test for a smaller stress at this temperature would have defined a wider sector of unambiguous extrapolation.

The value of such a test can be seen in the example of Fig.3, considered later.

Supplementary information that will assist extrapolation can be obtained from results from neighbouring materials of a related series. As an illustration, it appears that the value of T' for slope $-\frac{1}{2}$ for age-hardened Nimonic alloys is always greater than 100°C, also no line of slope -1 has been observed, even at the longest times. If this T' is adopted as a minimum value for the present data, the lines of slope $-\frac{1}{2}$ will fall in the positions shown by the dot-dashed lines of the slope in Fig.1B, and the unambiguous sector becomes much larger.

The results are aggregated with the other examples in the discussion of the accuracy of extrapolation in the next section. The differing choices of T' in the present instance do not affect the test of extrapolation because, whichever method is used, the longer time points deviate by the same amount or remain ambiguous.

4.2 Rupture data, no tertiary stage

Fig.2 is an example of rupture data for a material without tertiary creep and for which rupture is apparently governed by terms of form of Equation (4) with $K = 1$. It is a direct plot of the Timken data for a 4 to 6% Cr-Mo steel. The data have been arbitrarily divided into shorter and longer time groups with the division at 10 hours. The graph does not now approximate sufficiently closely to a set of segments of standard slope because, with $K = 1$, the curvature of the transitions is more gradual and they extend over much of the experimental range.

The shape of the transition curve appropriate to two terms of form (4) is dependent only upon the values of β and K for the terms and not upon the constant factors. Thus the strain contributed, at a given temperature, by two terms with common K (unity in the present instance) is

$$\varepsilon = (A_1\sigma^{\beta_1} + A_2\sigma^{\beta_2})t^K$$

in which A_1 and A_2 are constants. Choose quantities λ and σ_0 such that

$$A_1 = \lambda/\sigma_0^{\beta_1}, \quad A_2 = \lambda/\sigma_0^{\beta_2},$$

i.e.,

$$\sigma_0 = (A_1/A_2)^{\frac{1}{\beta_2 - \beta_1}}$$

Then

$$\varepsilon = \lambda\left[\left(\frac{\sigma}{\sigma_0}\right)^{\beta_1} + \left(\frac{\sigma}{\sigma_0}\right)^{\beta_2}\right]t^K$$

Now/

Now choose a quantity t_o such that

$$\lambda = t_o^{-K}$$

Then
$$\varepsilon = \left[\left(\frac{\sigma}{\sigma_o}\right)^{\beta_1} + \left(\frac{\sigma}{\sigma_o}\right)^{\beta_2}\right]\left(\frac{t}{t_o}\right)^{K}$$

This equation for the cross-plots of log σ versus log t at constant strain represents a master curve whose shape is specified by β_1, β_2, and K, and whose position is specified by log σ_0 and log t_o. Fig.4 gives examples of master curves for a few commonly-occurring combinations of K and K/β. The shapes of curves for more than two terms of form (4) depend upon the relative magnitudes of the terms, and are no longer standard; they need to be calculated as they arise.

The principles of analysis and extrapolation of rupture data for K small are exactly the same as those just described for K large, the procedure being now applied to the asymptotes rather than to the directly fitted lines of standard slopes. Each asymptote represents the independent contribution of a separate term.

The upper part of Fig.2 represents the completed analysis of the data represented by the shorter-time filled points in the lower part. The significance of lines and points is similar to that in the previous figure; thus in the lower part the broken straight lines and curves represent respectively the optimistic direct extrapolations of the asymptotes and curves fitted to the filled points, while the chain-dashed lines and curves represent the pessimistic interpretation. For extrapolation, instead of setting, as previously, a pessimistic line directly through the longest-time point at the temperature concerned, the standard curve appropriate to the indicated standard constants is now placed at the shortest time that is allowed by the trend of the points. The appropriate value of the term is defined by the asymptote. The curves of the upper part of Fig.2 relate each to the asymptote of indicated slope.

When the data are such that, as at the lower temperature in Fig.2, three or more terms are present and their transition regions overlap, the details are more complicated but the principles remain the same. It becomes necessary to estimate the positions of the asymptotes that represent the basic terms by successive approximation. The results of Fig.2 are discussed in Section 5.

Fig.3 for "killed carbon steel" from the Timken Digest, is another example. In this, the data are sufficient to define the asymptote of slope $-\frac{1}{2}$ at the highest temperature: as a result there is no ambiguity at this temperature, and the extent of ambiguity at other temperatures is reduced.

In both Fig.2 and Fig.3 the slope of -1 was found not to be more pessimistic than that of $-\frac{1}{2}$, except at the highest experimental temperature (815°C and 760°C respectively); it has not therefore been shown.

4.3 Creep curves

For the analysis and extrapolation of creep curves, the procedure outlined is applied separately to each of the components $t_{\frac{1}{3}}$, t_1, and t_3 obtained by analysis in Stage 1 of the curves. In regard to Stage 1, it is easy to show, by the method of the last paragraph, that the contribution of any pair of terms in Equation (1) to the creep curve is represented by a standard curve. Two of the three standard curves concerned are illustrated in Fig.5. For creep curves with an accelerating stage, it is convenient to fit the $(\frac{1}{3}, 1)$ standard curve to the earlier portion of the curve and obtain the t_3 term by subtraction of the ordinates of the curve from those of the experimental points. In Stage 2 the log stress/log time cross-plots of the $t_{\frac{1}{3}}$, t_1 and t_3 terms are separately fitted with the appropriate standard curves. Since K is precisely known, there is no need to use segmental curves for $K = 3$. The method leads to extrapolated values of the $t_{\frac{1}{3}}$, t_1 and t_3 terms separately: the predicted creep curve is then obtained by setting the master $(\frac{1}{3}, 1)$ curve in the predicted position and then adding on the predicted t_3 contribution.

A complication of the Stage 1 analysis of creep curves is that their shapes are appreciably affected by an uncertainty in the amount of elastic and creep strain that occurs during loading, and thus in the appropriate zeros of strain and time. Although the uncertainties are small in comparison with practical magnitudes, their effect on the analysis needs to be considered. The matter is discussed in detail in Refs.7 and 9 where the procedure of choosing the amount to suit Equation (1) is supported. Fig.6 shows the set of creep curves at 815°C from a family of creep curves for temperatures of 700°C to 940°C for Nimonic 100. The difference between the crosses, and open circles where shown, represents the difference between independent estimates of this amount and lies within the experimental latitude. The solid lines are derived from a fitting of all the data for times up to 1000 hours; where they are extended to longer times, or are interpolated as at 9 t.s.i., they represent unambiguous extensions.

Experimental points for times greater than 1000 hours and not used for the fitting are shown by filled triangles. The broken curves (to be regarded as coincident with the full lines where not drawn) refer to an independent fitting of data as a whole over the full range of time, which extends in this case to 15,000 hours.

5. Tests of Extrapolation

Owing to space limitations, the results of tests of extrapolation are mainly presented in summary form, but the examples of Figs.1, 2 and 3 are given in more detail.

Figs.1, 2 and 3 are typical of those given in Ref.5 for the seven sets of rupture data and one of fatigue data that, of the sets available, were the only ones sufficiently extensive and of small enough scatter to provide a critical test. For these three materials predicted and observed rupture times are compared in Table IV. Two sets of pessimistic values are given for Nimonic 80A, the first uses slopes of $-\frac{1}{2}$ and -1, with T' of -273°C, as in Fig.1A, the second uses only the $-\frac{1}{2}$ slope with a T' of 100°C as in Fig.1B. The confidence limits on the observed times in column 4 correspond to standard deviations in log time of ±0.07, ±0.17 and ±0.18 for the three materials respectively, as found by comparison of individual points with curves from an

independent/

independent fitting of each set of data as a whole. The results given in Section 3 show that the scatter of the data is fairly estimated in this manner. Single extrapolated times represent unambiguous values, and a range of times the extent of an ambiguity. The average ranges of extrapolation in time are by factors of 6, 104 and 11.5 respectively and the extreme extrapolations by factors of 17, 540 and 54. The errors of prediction are seen to be distributed in the same manner as the random scatter of the data, with 3 out of 38 results deviating by more than twice the appropriate standard deviation. The Gaussian figure would be 2 in 38. Results for the remaining 5 sets, (treated individually in Ref.10) are treated collectively below.

For the family of creep curves for Nimonic 100, of which Fig.6 is a sample, predicted and observed values are compared in Table V. The bracketed values in the extrapolation column are given only to complete the table: they are for times less than 1000 hours and are not therefore extrapolations: differences from observed times represent the scatter in direct fitting. For the long-time comparisons that these data afford there is no ambiguous range of extrapolation. Where two figures are given in the extrapolation column, the raw experimental data exhibited one of the anomalous effects noted in Section 3 and was double valued. The average range of extrapolation in time is by a factor of 4.4 and the extreme factor is 10.6. For this material, the standard deviation of points about fitted curves, averaged over the data as a whole, is 0.21 in log time. It may be seen that 31 out of 36 extrapolated values fall within this range. The number to be expected for random errors is 25. This better-than-random result is probably because the value 0.21 relates to the whole range of creep, while the scatter is appreciably smaller in the tertiary stage than in the primary and secondary stages. To distinguish between the three stages would involve consideration of the errors of distributing the observed creep amongst the three terms of Equation (2) and was not thought worth while.

5.1 Statistical presentation of scatter in rupture times

The results for the three sets of rupture data in Tables II and III are joined with those for the remaining five sets of data and reduced to a common basis in Fig.7. The solid stepped curve represents the distribution of scatter in log time as obtained by direct fitting of the theoretical curves, while the stepped curve in broken line represents the observed distribution of errors in prediction. The distribution to be expected from the normal curve of errors is shown by the continuous curve. To obtain the broken curve, the scatter of the points for each material about the fitted curves for that material have been scaled to correspond to a common s.d. in log time for all eight materials. For example, the individual differences in log time for 4 to 6% Cr-Mo steel in column 4 of Table IV were divided by 0.18, the value of the s.d. for the data. Those for the other materials were similarly divided by their s.d. and all values so obtained were treated as a single population. To obtain the broken curve, the errors of extrapolation for each material were similarly divided by the appropriate s.d. and aggregated together. The close agreement between the three curves suggests that the distributions are Gaussian and that predictions are in error mainly because of the uncertainty of the data. The average extrapolation was by a factor of 26 in time.

5.2 Statistical presentation of scatter in creep

The above method of aggregating the rupture results for different materials may conceal irregularities in the data for a single material. A more penetrating test is offered by a set of creep curves that is adequate to define the scatter ogive in detail. Fig.8 for Nimonic 100 and Fig.9 for Nimonic 90 show the ogives for two sets of this kind.

In/

In both figures, A gives the observed scatter of short-time points about creep curves fitted to these points, B the scatter of long-time points about curves extrapolated from the short-time data, and C the scatter of all the points about curves fitted to all the points. The observed ogives are the stepped curves: each is to be compared with the continuous Gaussian ogive which in both figures is drawn for a standard deviation of 0.2 in log time.

In regard to Fig.8 for Nimonic 100, the central sections of 8A and 8C fit closely to the calculated curve, suggesting that the scatter is essentially Gaussian with a standard deviation of 0.2; 8B is of similar form but steeper, suggesting that for extrapolated points the scatter is less than 0.2.

The upper and lower sections of Figs.8A and 8C - but more particularly the upper sections - depart significantly from the Gaussian curve. The points observed at three to four times the standard deviation (whose number is considerably in excess of those predicted by the Gaussian curve) are attributed to a double-valued behaviour in the $t^{\frac{1}{3}}$ term which was clearly observed at 870°C and 940°C (c.f. Ref.7, Section 4.3). The effect is not observed in Fig.8B, presumably because very few of the extrapolated points fell in the primary range. It will be observed from Fig.8B that the double-valued behaviour of the short-time data has not biased the extrapolation, suggesting that the method of analysis has been successful in distinguishing regular behaviour at larger strains in the presence of irregularities at smaller strains.

Fig.9 for Nimonic 90 shows generally similar features, again being affected by double values in the $t^{\frac{1}{3}}$ group of terms. Fig.9B indicates that the extrapolation is slightly pessimistic overall, to the extent of 0.05 in log time or about 12% in time, an error which is much less than the uncertainty of a single point. This overall pessimism is probably due to a tendency to caution in the analysis. There is a suggestion of the influence of double values in the extrapolation ogives (see Fig.9B), probably because, for this material, as many as one third of the extrapolated values were in the $t_{\frac{1}{3}}$ region.

6. General Discussion

The feature of the formula that has been least critically tested is the precise form of the time-temperature parameter. One of the more usual exponential forms was not used because these predict, in particular instances, the wrong curvature of the log time-temperature cross-plots. The form proposed meets the experimental requirement that the plots may have either positive or negative curvature. The condition of minimum spacing to which it leads, between lines for different temperatures on the log stress/log time cross-plots, has enabled the most unfavourable slopes of the lines to be selected for pessimistic extrapolation. The condition is part and parcel of the result that T' lies between the absolute zero and the melting point which was yielded by direct analysis of all the sets of data as a whole, and is not independently checked by the examples of extrapolation given, which relate to the same data.

The method evidently provides means of predicting the onset of the metallurgical changes that are responsible for a steepening at longer times of the graphs of log stress versus log life. However, metallurgical changes, presumably of another kind, are known to occur, of which the onset is not directly predicted by the method. In some, though apparently not all instances, these appear to be associated with a flattening rather than a steepening of the graphs, and the present method will then be unduly pessimistic. Further research

along/

along the present lines is necessary however, both to define the circumstances in detail and to assess the significance of the anomalies that have been mentioned.

The operations of fitting the formula have been carried out without difficulty by junior staff; a typical time for fitting a set of creep rupture data, including plotting of primary data, is 4 to 6 hours, and for a set of creep curves, 3 to 5 days. These times are appreciably reduced when suitable copying equipment is available. The saving in machine time on the 3 sets of creep curves discussed in Ref.7 amounts to 48 machine years.

A formula whose agreement with the data was achieved by a purely arbitrary flexibility would be most unlikely to provide satisfactory predictions, and the result found for the present formula that the errors of prediction are in substantial agreement with both the errors of direct fitting and the directly-measured scatter in replicate tests is thus evidence for the validity, within the circumstances concerned, of both the formula and the method of prediction.

REFERENCES /

REFERENCES

No.	Author(s)	Title, etc.
1	A. Graham	The Engineer, Vols.193, 198 and 234. 1952.
2	A. Graham	The phenomenological method in rheology. Research Vol.6, p.92. 1953.
3	A. Graham K. F. A. Walles	Relationships between long and short-time creep and tensile properties of a commercial alloy. J. Iron and Steel Inst. Vol.179, p.105. 1955.
4	A. Graham K. F. A. Walles	Regularities in creep and hot-fatigue data. Part I. A.R.C. C.P.379. December, 1956.
5	K. F. A. Walles A. Graham	Regularities in creep and hot-fatigue data. Part II. A.R.C. C.P.380. December, 1956.
6	A. Graham	Unpublished M.o.A. Report.
7	K. F. A. Walles	A quantitative presentation of the creep of Nimonic alloys. A.R.C.20,935. March, 1959.
8	A. Graham G. J. Bates	An analysis of the scatter in creep of an alloy of Nimonic 90 type. A.R.C.21,108. March, 1959.
9	A. Graham G. J. Bates	On the shape of the creep curve particularly of Nimonic alloys. A.R.C.21,109. April, 1959.
10	K. F. A. Walles	The extrapolation of creep rupture data. A.R.C.21,253. October, 1957.
11	A. Graham	Numerical regularities in uniaxial deformation. IXe Congrès International de Mechanique Appliquee, Vol.8, 227. 1957.
12	F. C. Monkman N. J. Grant	An empirical relationship between rupture life and minimum creep-rate in creep-rupture tests. Amer. Soc. for Testing Materials. Preprint 72. 1956.

TABLE I/

TABLE I

Scatter of Rupture Data about Fitted Curves for Materials of Reference 5

Standard deviations in log time

Materials that rupture on t^3 term	Nimonic 80-I 0.13	Nimonic 80-II 0.05	Nimonic 80A-I 0.07	Nimonic 80A-II 0.14	Nimonic 80A-III 0.04
	Nimonic 90-I 0.10	Nimonic 90-II 0.11	Nimonic 90-III 0.23	Nimonic 95-I -	Nimonic 95-II 0.14
	Nimonic 100-I 0.09	G.32 0.14	Inconel X 0.25	G.34 0.18	35-15 0.11
	H.46 -	18-8+Cb 0.18	Red Fox 36 0.12	FCB(T)-I -	FCB(T)-II -
	DM.2 0.10	25-20 0.24	G.19 -	25-12 0.17	16-13-3 0.22
	Rex 337A 0.17	Rex 448 /	Red Fox 31 /	$2\frac{1}{4}$ Cr-1Mo 0.11	CML-I 0.20
	CML-II 0.11	CRM-I 0.16	CRM-II 0.15	CRM-III 0.18	Killed carbon 0.18
	Silmo 0.11	Mean of t^3 rupture 0.14_4			
Materials that rupture on t^1 term	C-Mo 0.26	2% Cr-Mo *	Sicromo 2 0.13	Sicromo 3 *	4-6% Cr-Mo+ 0.17
	4-6% Cr-Mo+Ti 0.13	Sicromo 5S *	Sicromo 5MS 0.24	Sicromo 7 0.14	Sicromo 9M 0.28
	18-8 0.16	S.816 0.22	S.590 0.32	Mean of t' rupture 0.20_6	

\- data were too sparse to define scatter

* systematic errors of fitting curves to data were present, so that scatter figures would be unreliable

/ characteristic anomaly

TABLE II/

TABLE II

Comparison of Scatter in Creep

Standard deviations in log time

(a) Creep data		
Material	Data about fitted curves	Replicate data
Nimonic 80A-I	0.14	0.05 (2)
Nimonic 80A-III	0.27	0.18 (2)
Nimonic 90-I	0.18	0.15 (8)
Nimonic 90-II	0.15	0.27 (6)
Nimonic 90-III	0.25	0.19 (2)
Nimonic 100-II	0.21	-
Mean	0.20	0.18* 0.17/

(b) Creep and rupture data			
Material	Creep data		Rupture data about fitted curves
	t_3 values about fitted curves	Replicate t_3 values	
Nimonic 80A-I	0.07	0.04 (1)	0.07
Nimonic 80A-III	0.06	0.07 (2)	0.04
Nimonic 90-I	0.11	0.18 (7)	0.10
Nimonic 90-II	0.08	0.04 (3)	0.11
Nimonic 90-III	0.09	-	0.23
Nimonic 100-II	0.11	-	-
Mean	0.09	0.12* 0.08/	0.11 (0.08 without N90-III)

* Mean weighted by number of tests

/ unweighted mean

bracketed figures indicate number of pairs of tests

TABLE III /

TABLE III

Comparisons of Scatter in Rupture Data

Standard deviations in log time

Material		Data about fitted curves	Replicate data
(a) t^s type	Nimonic 80A-III Nimonic 90-II Red Fox 36	0.04 0.11 0.12	0.04 (4) 0.36 (1) 0.06 (1)
	Mean	0.09	0.15 plain 0.10 weighted
(b) t' type	18-8 stainless S.816 S.590	0.16 0.22 0.32	0.40 (1) 0.45 (3) 0.05 (2)
	Mean	0.23	0.30 plain 0.31 weighted

TABLE IV /

TABLE IV

Extrapolated Creep Rupture Times compared with Observed Times

Nimonic 80A						
T (°C)	σ t.s.i.	Observed time (hours)	Range for ± standard error	Range for ±2 × standard error	Extrapolated time (hours)	
					T' = -273°C	T' = 100°C
650	22	2650	2250-3130		3160	3160
	20	6200	5300-7300	4400-8600		
	20	4800	4100-5700		3900-4200 (rows 2–4)	4200 (rows 2–4)
	20	5300	4500-6200	3700-7200		
	18	8200	7000-9700	5900-11400	4700-6400	6400
	16	13400	11400-15900	9700-18800	6100-10300	10300
700	13	4800	4100-5700		3600-4200	4200
	10	11000	9400-13000		6200-12100	9500-12100
	7	34000	29000-40000		8500-49000	19000-49000
750	8	4500	3800-5300		2400-4600	3100-4600
	6	13100	11100-15500		3100-14500	5600-14500
	4	22700	19300-26700		4600-73000	11500-73000

4-6% Cr-Mo steel					
T (°C)	p.s.i. × 1000	Observed time (hours)	Range for ± standard error	Range for ±2 × standard error	Extrapolated time (hours)
538	24	104	70-154		114-125
	20	1600	1100-2400		420-740
593	13	310	210-460		200-260
	11	1400	930-2100	640-3100	450-730
649	9.1	140	93-210		105-116
	6	2800	1900-4200	1280-6100	420-1300
704	6	86	57-129		72-86
	5	250	170-370		140-225
	4	1100	730-1650	500-2400	290-620
	3.5	1200	800-1800		420-1160
	2.5	5300	3500-8000		1000-5000
815	2.0	90	60-135		114-180
	1.5	160	110-240	73-350	280-670
	1.1	480	320-720		600-2500
	0.9	670	450-1000		970-6000
	0.7	890	600-1340	410-1950	1780-16900

Killed carbon steel					
538	12	1550	1030-2340	680-3500	570-660
	10	3600	2400-5400	1580-8200	1020-1550
	9	4800	3200-7200	2100-11000	1430-2500
	6	13400	8900-20000		4600-13500
649	3	620	410-930		430-680
	2	2100	1400-3200		1300-2750
704	2	290	190-440		230-320
	1.8	450	300-680		320-450
	1.5	850	560-1280		540-840
760	0.75	900	600-1360		870

TABLE V /

TABLE V

Extrapolated Times compared with Observed Times for Creep of Nimonic 100

T °C	σ t.s.i.	Times for indicated strains, hours								
		0.1%			0.2%			0.5%		
		Observed	Range for ± standard error	Extrapolation	Observed	Range for ± standard error	Extrapolation	Observed	Range for ± standard error	Extrapolation
700	23	420	260-680	(730)	1400	870-2300	1850	4800	3000-7800	5000
	20	3000	1900-4800	1850	-	- -	4300	-	- -	8800
	17	4000	2500-6500	4500	10000	6200-16200	8800	-	- -	15800
750	17	300	190-480	(670)	920	570-1500	1500	2900	1800-4700	2900
	15	950	590-1540	1400	2800	1700-4500	2850	5200	3200-8400	6800
	12	6000	3700-9700	3700	-	- -	7400	-	- -	13100
815	11	335	210-540	(310)	790	490-1280	(720)	1800	1100-2900	1420
	9	800	500-1300	(880)	1750	1080-2800	1820	3850	2400-6200	3550
	7	970	600-1570	1870	3900	2400-6300	4500	7800	4800-12600	8900
	6	3700	2300-6000	3050	9800	6100-15800	7500	-	- -	16200
	4	-	- -	8600	-	- -	21500	-	- -	57000
870	5	200	120-320	(730)	520	320-840	1590	1190	730-1920	3100
	4	1250	770-2000	1500	3200	2000-5200	3500	6600	4100-10700	7200
	3	2500	1500-4000	2900	6600	4100-10300	7000	-	- -	17000
	2.5	1650	1000-2700	1300-4500	5000	3100-8100	4900-10800	-	- -	18700-29000
	2	3750	2300-6000	3750-8600	10600	6600-17100	11500-17400	-	- -	37000-49000
940	2	280	170-450	(560)	780	480-1260	1420	2450	1500-4000	3500
	1.5	940	580-1520	1130	2400	1500-3900	2550-2850	6800	4200-11000	5500-7800
	1	1400	870-2300	1200-2800	5400	3300-8700	4000-6500	-	- -	11000-12700

TABLE VI

Scatter about Extrapolated Curves compared with Scatter about Fitted Curves

Material	Standard deviation in log time		
	(i)	(ii)	(iii)
Nimonic 80A-II Nimonic 90-III Nimonic 100	0.22 0.21 0.17	0.27 0.16 0.21	0.22 0.17 0.13
Mean weighted for numbers of points	0.19	0.21	0.15

(i) Longer-time points about extrapolated curves (e.g., triangles about solid curves in Fig.6).

(ii) Shorter-time points about directly fitted curves (crosses about solid curves).

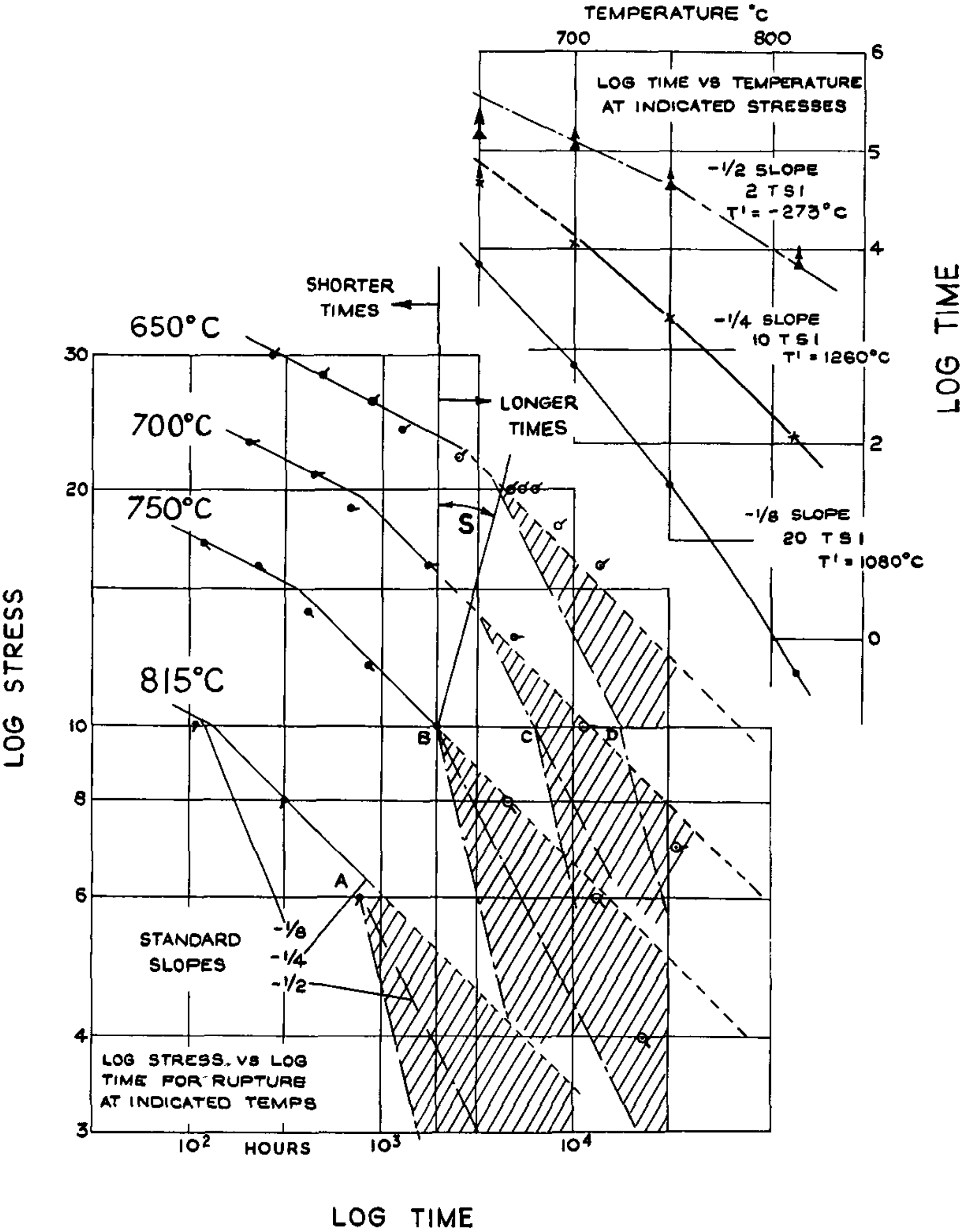
(iii) Longer-time points about directly fitted curves (triangles about broken curves).

APPENDIX I

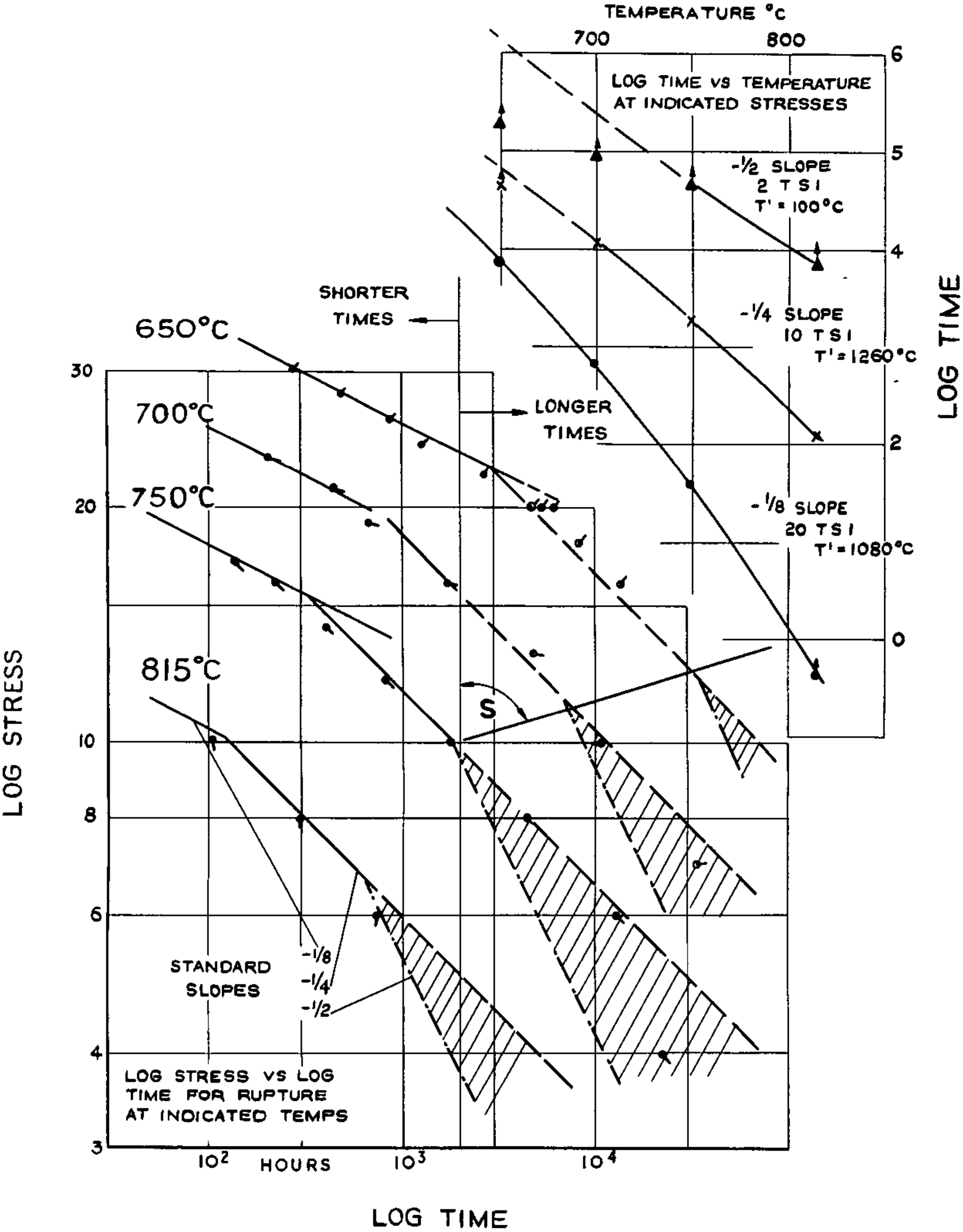
Test of Other Methods of Extrapolation

Seven other methods described in the literature (see Section 1) have been tested upon the two sets of rupture data of Figs.1 and 2 and Table IV. Significant errors were found, and in view of their magnitude, it was not considered necessary to extend the testing to other materials. The methods generally require comparisons of results at common stresses or temperatures, and consequently involve the use, unless the experimental conditions are specially selected in advance, of a supplementary and unspecified method of interpolation or even extrapolation. The procedure is subject to personal errors, and was therefore avoided; thus the number of experimental points that could be used was rather limited. They were sufficient to provide the cumulative distribution diagrams of errors shown in Figs.10 and 11. The methods considered are indicated in the figures.

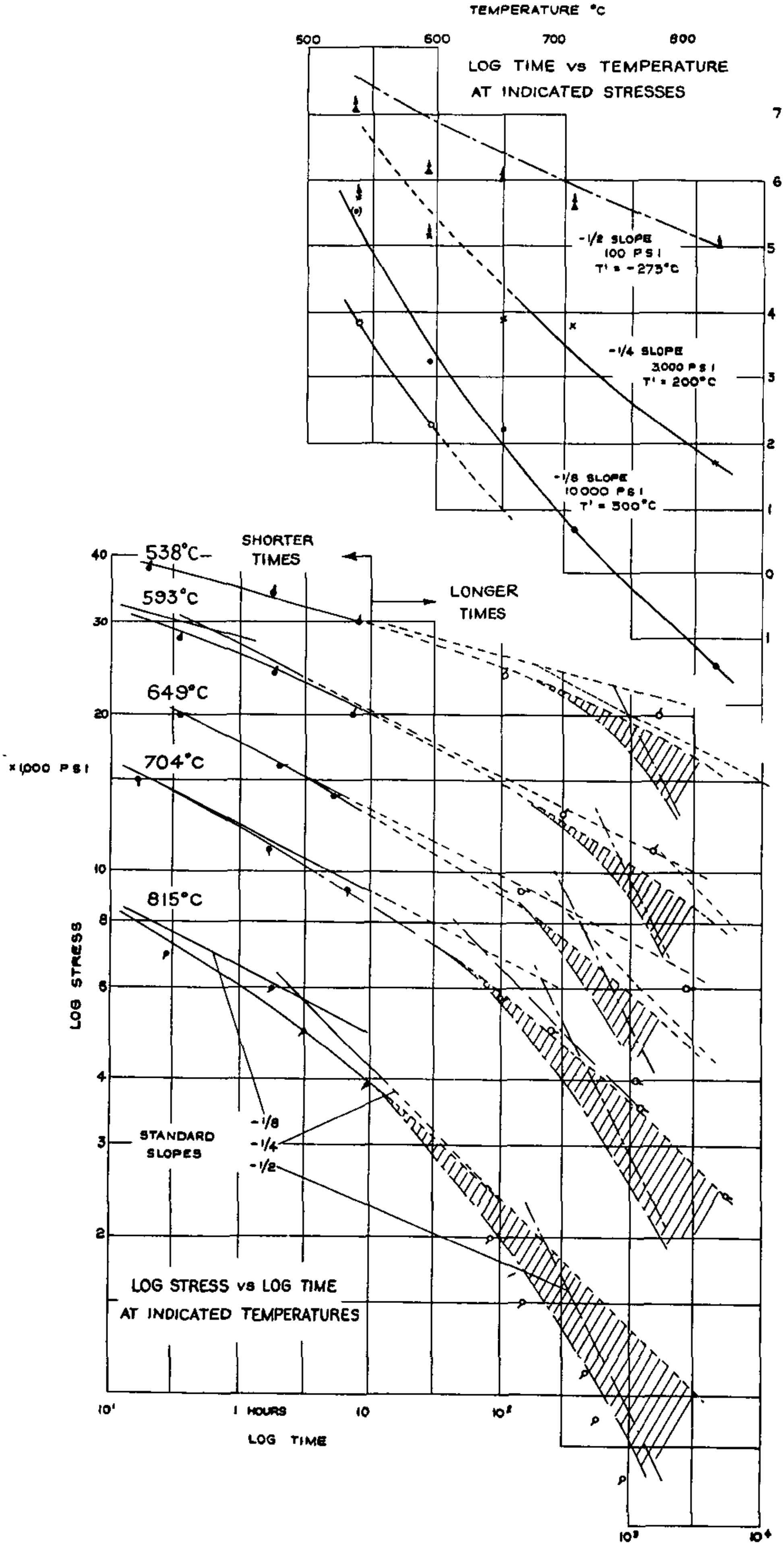
Each method gives a roughly Gaussian distribution of errors whose standard deviation is between two and five times that of the curve of data scatter shown by the continuous curve. Thus they can all be significantly in error. Although the comparison is limited to two sets of data, the form of the Gaussian distribution is such that even if a very large number of predictions with small error were added, the observed errors, which include some in excess of five times the standard deviation, would remain significant. The general view that these methods are unreliable is thus confirmed.



EXTRAPOLATION OF RUPTURE DATA
NIMONIC 80A

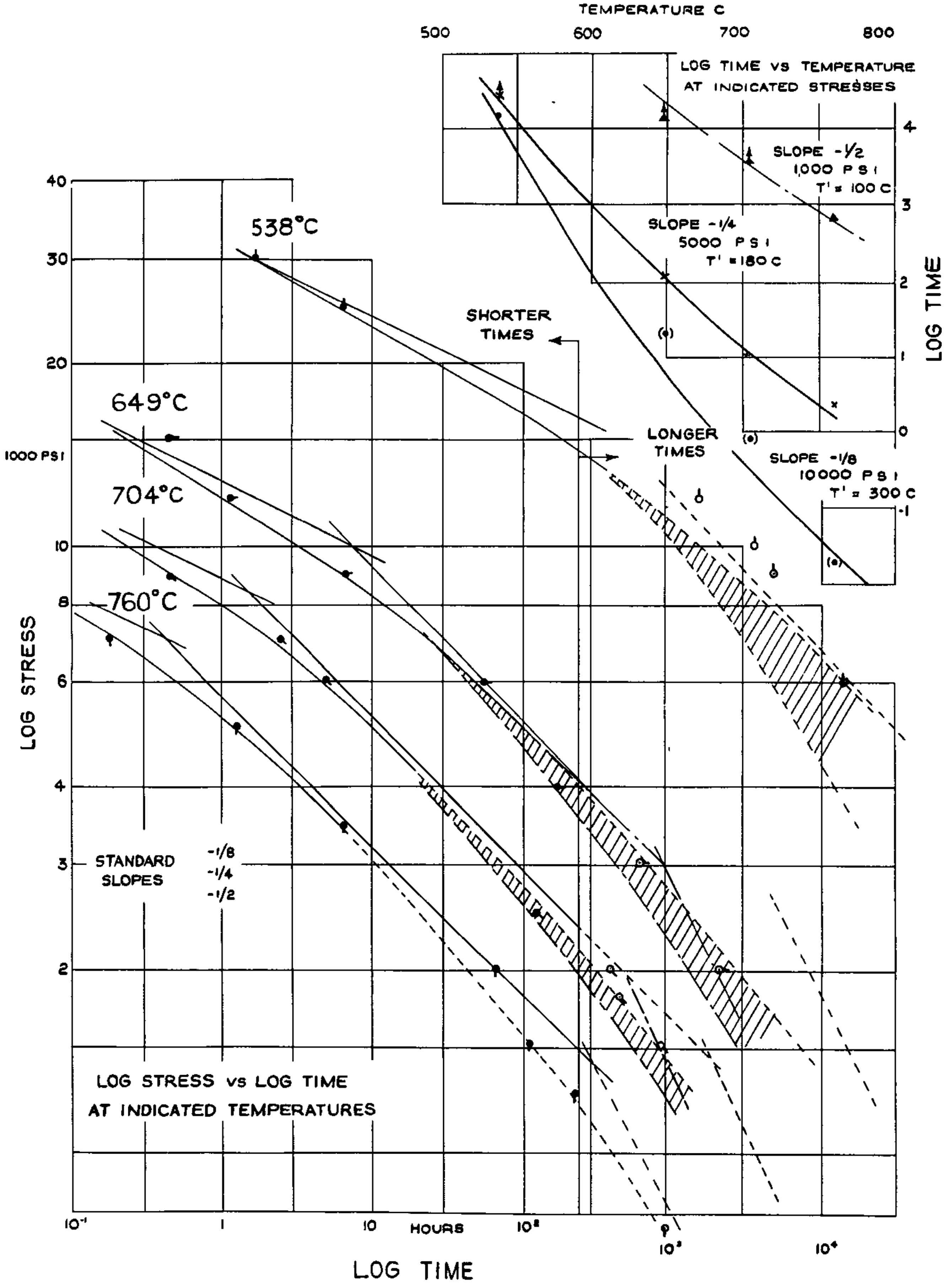


ALTERNATIVE EXTRAPOLATION OF FIGURE 1A.



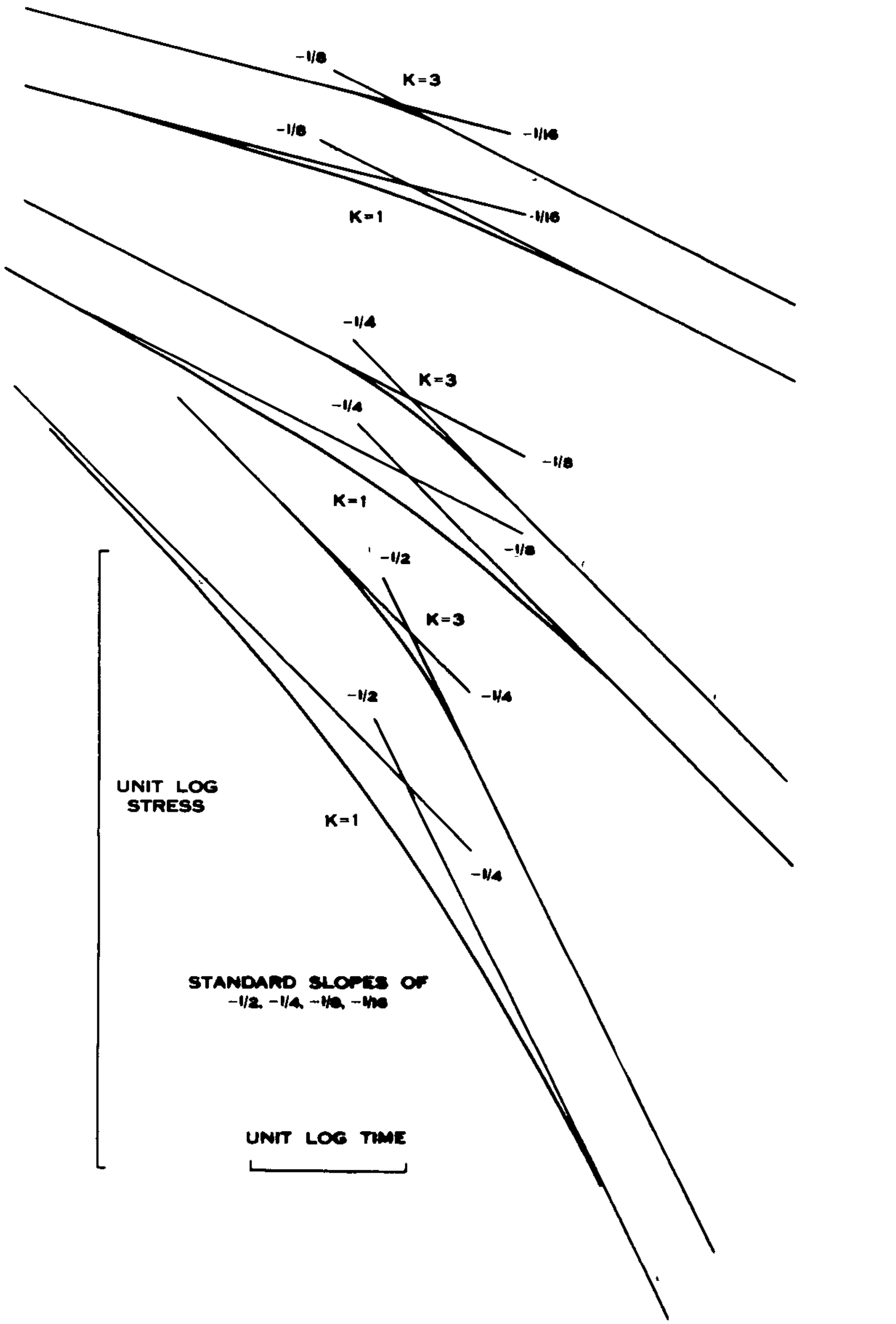
EXTRAPOLATION OF RUPTURE DATA FOR 4-6% CHROME-MOLYBDENUM STEEL

FIG 3



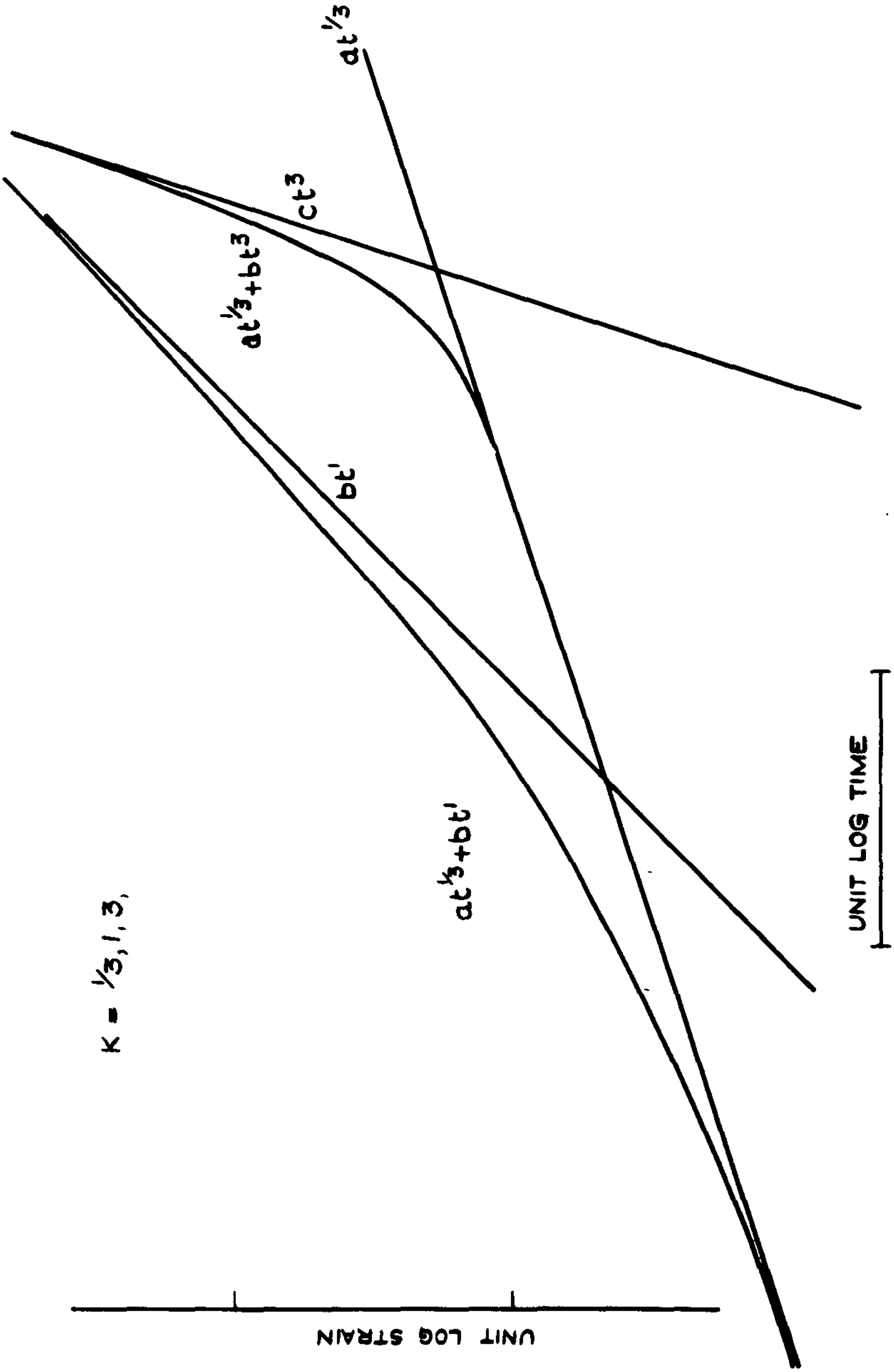
EXTRAPOLATION OF RUPTURE DATA FOR KILLED CARBON STEEL

FIG. 4.



TYPICAL TWO-TERM LOG STRESS vs LOG TIME MASTER CURVES

FIG. 5



TYPICAL TWO-TERM LOG STRAIN

v. LOG TIME MASTER CURVES.

EXTRAPOLATED CREEP CURVES FOR NIMONIC 100 AT 815°C

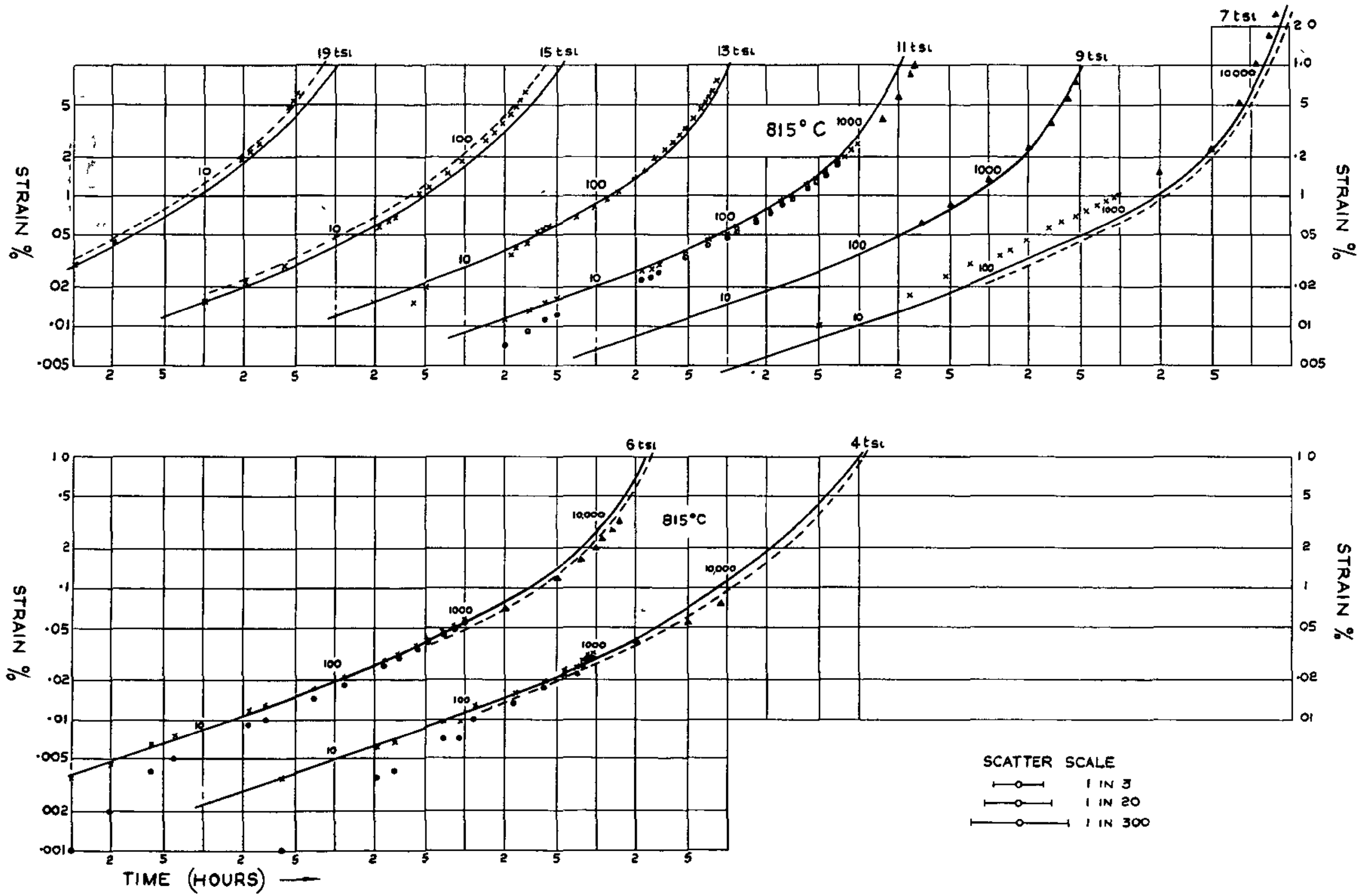
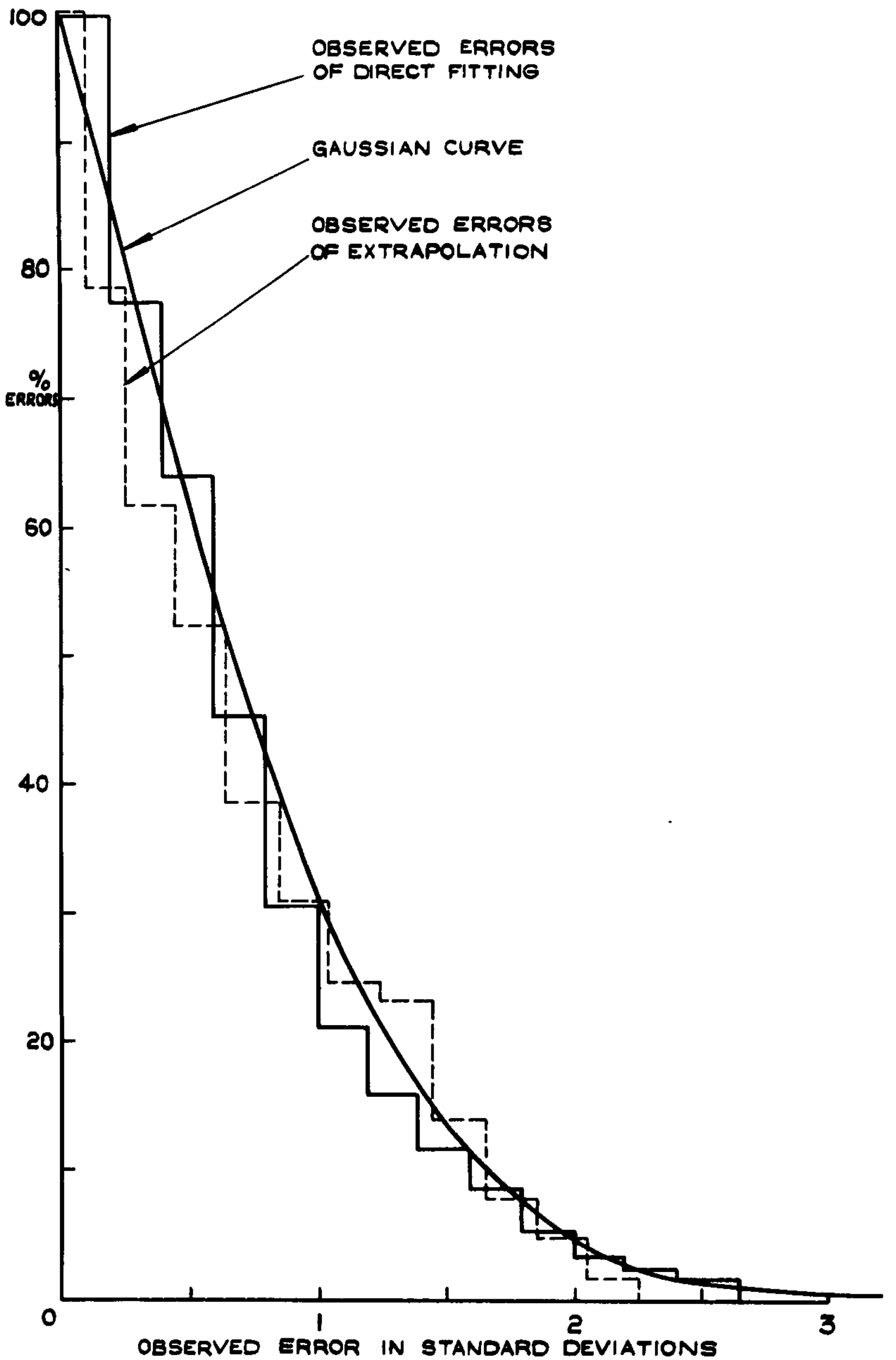
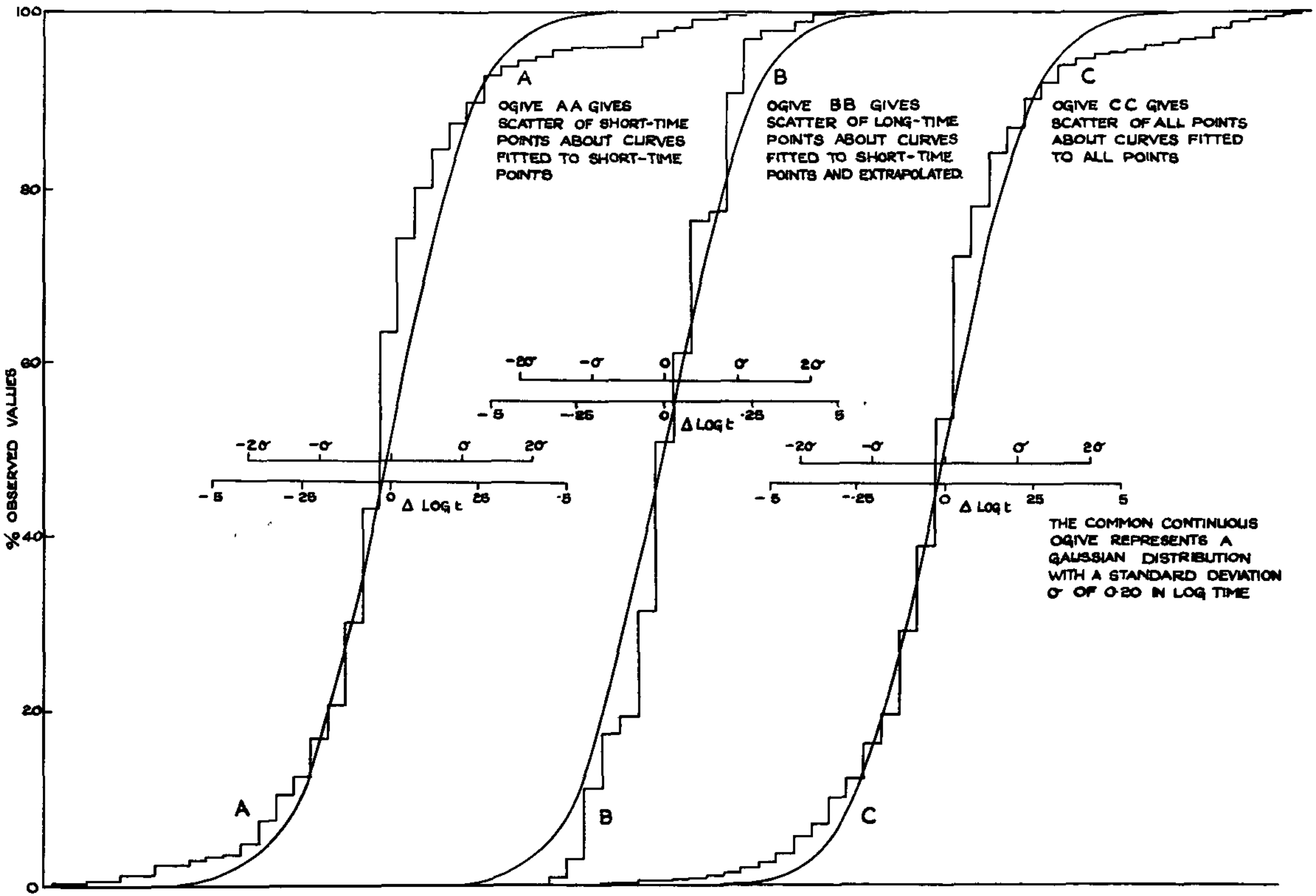


FIG 6

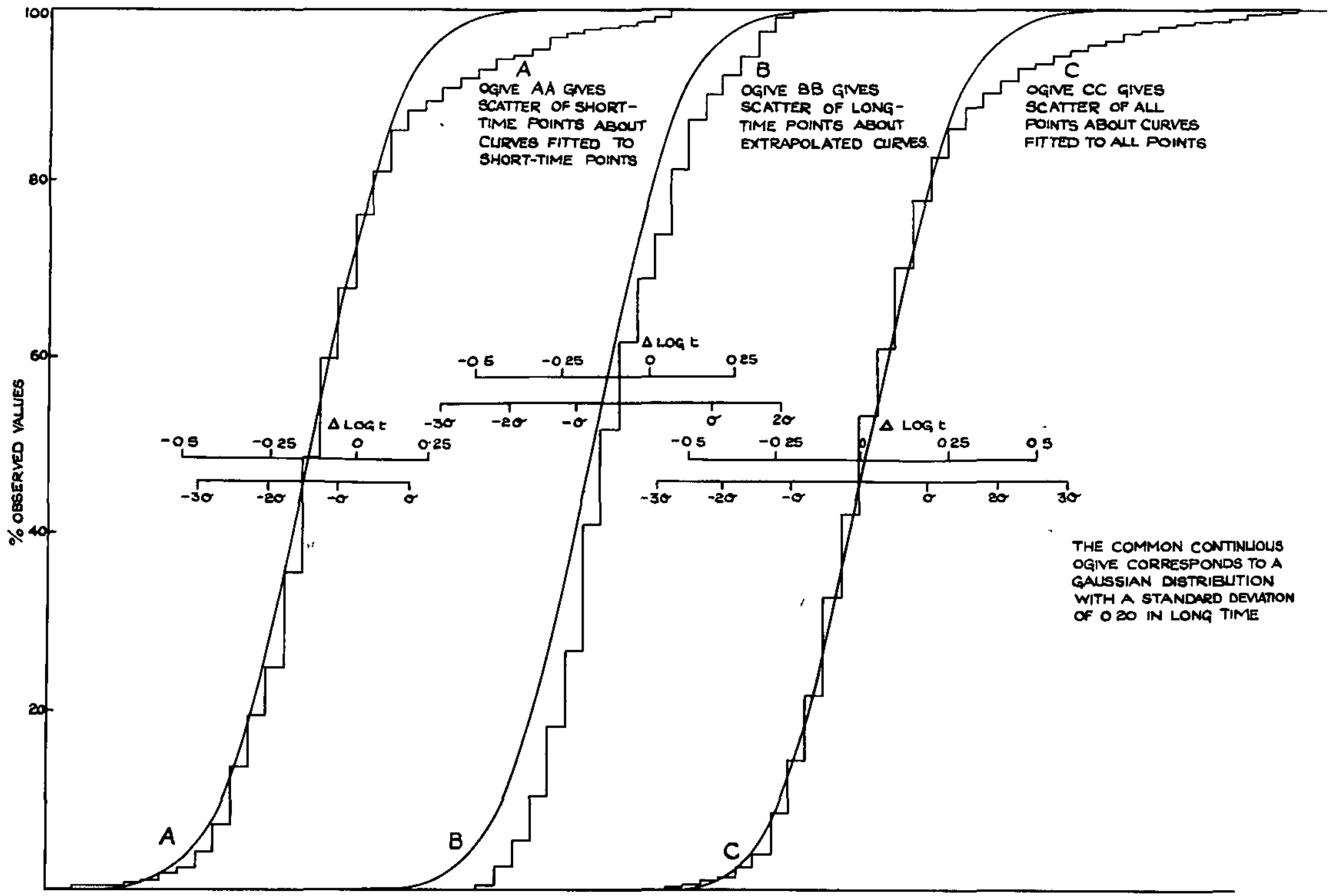
FIG. 7



OGIVE OF ERRORS IN FITTING CREEP-RUPTURE DATA BY PRESENT FORMULA.

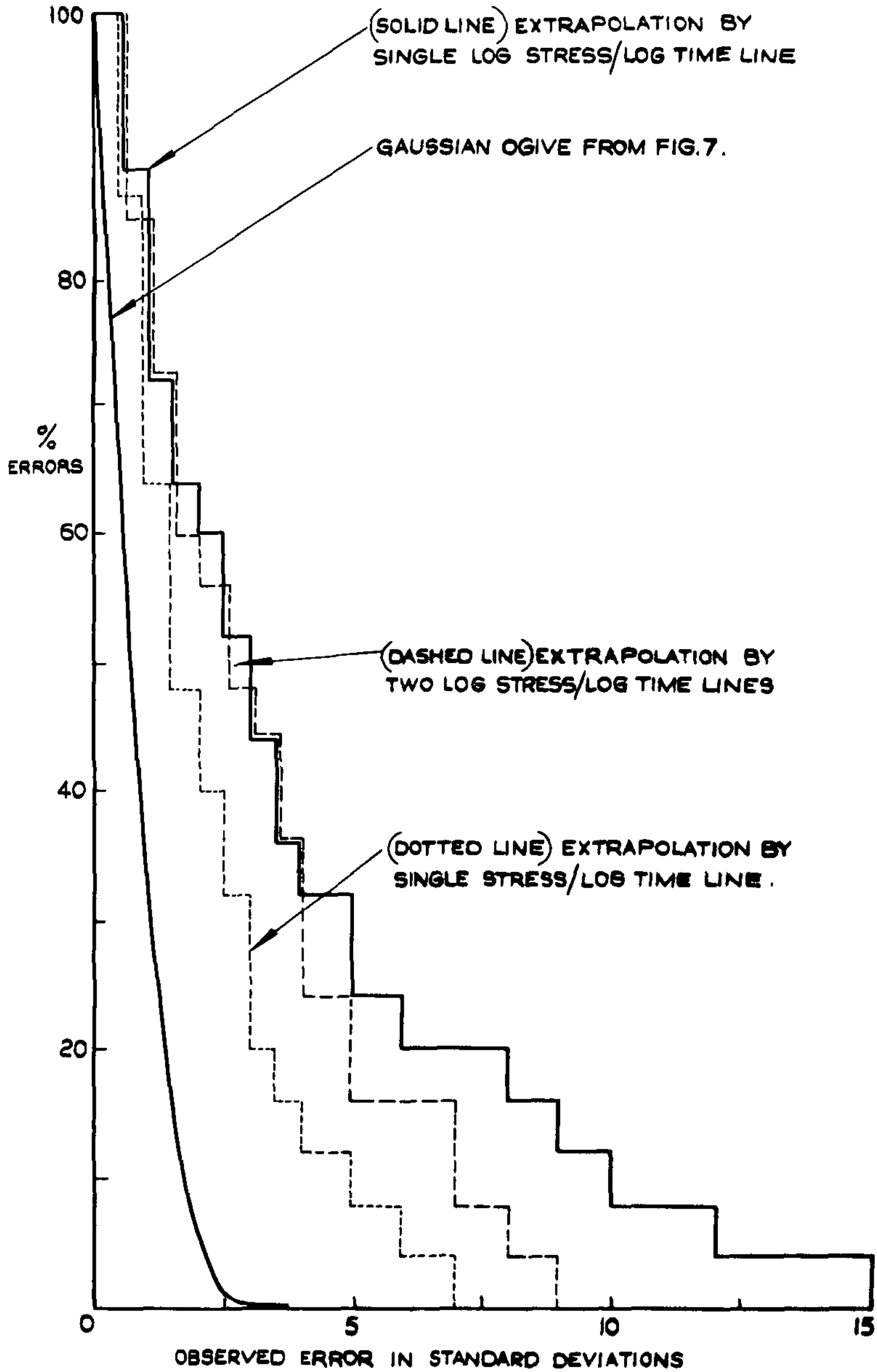


OGIVES OF ERRORS IN FITTING CREEP CURVES FOR NIMONIC 100 BY PRESENT METHOD.



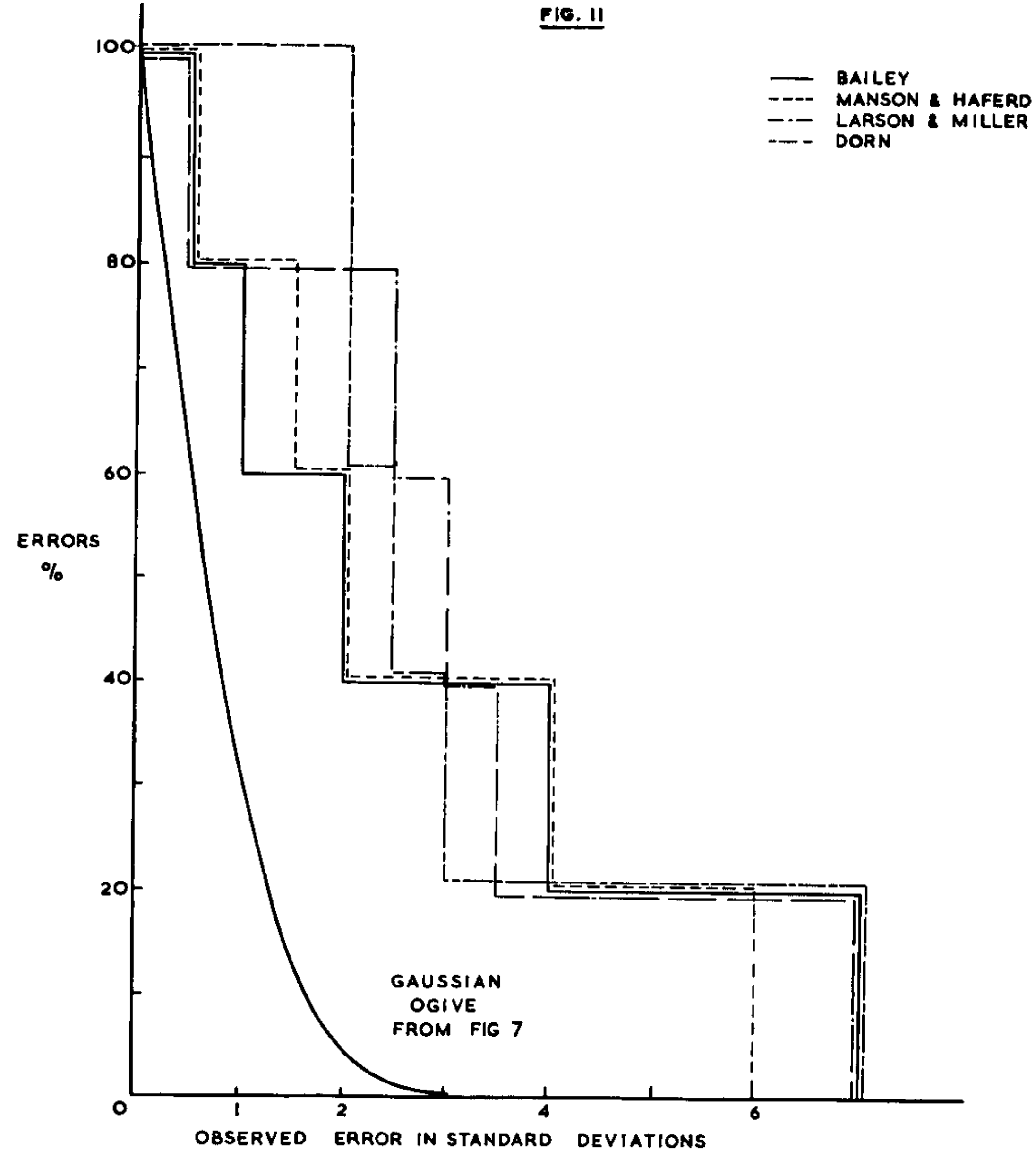
OGIVES OF ERRORS IN FITTING CREEP CURVES FOR NIMONIC 90 BY PRESENT METHOD.

FIG. 10



OGIVE OF ERRORS IN STRESS-TIME EXTRAPOLATIONS AT EACH TEMPERATURE SEPARATELY.

FIG. 11



OGIVE OF ERRORS IN TIME – TEMPERATURE EXTRAPOLATIONS AT EACH STRESS SEPARATELY

A.R.C. C.P.No.680. October, 1961 539.434:620.172.251

Walles, K. F. A. and Graham, A.

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