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A Time-Division Analogue Multiplier for Correlation Measurements and Mixing at Frequencies up to 100 Kilocycles per Second

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A Time-Division Analogue Multiplier for Correlation<br>Measurements and Mixing at Frequencies up to 100 Killocycles per Second - By -<br>R. F. Johnson, B. A., B. Sc.

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## SUMMARY

For processing noise and turbulence signals there is a need for an analogue multiplier capable of handling input frequencies from a few cycles to about 80 kilocycles per second. Previous circuits described have been unduly complex or restricted in frequency response. A circuit for a tame-division multiplier is described capable of accepting slgnals from zero frequency to $100 \mathrm{kc} / \mathrm{s}$. Preamplifiers and associated circuits are described which enable the multipller to be used for the measurement of a wide range of parameters encountered in the turbulence and noise field.

## 2. Introduction

Fluctuations of velocity or pressure in or near a turbulent flow may be converted into fluctuating electrical signals by the use of suitable transducers. For example, a hot wire anemometer may be used to produce an electrical signal proportional to the fluctuating velocity in a jet ${ }^{1}$, which will contain energy up to approximately $100 \mathrm{kc} / \mathrm{s}$ if the exit diameter is one inch and the mean velocity a few hundred feet per second.

In order to obtain information from these signals they must be passed through processing equipment, to yield such functions as the mean square and higher powers, the autocorrelation coefficient and the coefficient of correlation between two signals. In generating most of the functions of aerodynamic significance it is thus necessary to operate upon the signals at some stage with a multiplier. For example the mean square value, $\mathrm{v}^{3}(\mathrm{t})$, of a signal $\mathrm{v}(\mathrm{t})$ is the mean or zero frequency output of the multiplier when the signal is fed into both inputs. This function, or the root mean square value, can obviously be derived by the use of a true R.M.S. meter such as a thermocouple instrument, but the
measurement of correlation coefficients and mean values of higher powers requares the dernvation of instantaneous squares or products, thus making a multiplier essential.

## 3. Multiplication Methods

Since most multipliers are designed for use in analogue computers, the accuracy is good (usually better than $0.1 \%$ of full output) but the frequency response is restricted to under $1 \mathrm{kc} / \mathrm{s}$. A number of methods to effect the multiplication of two sigrals has been derised, and a few of these methods, excluding mechanical techniques, will be briefly mentioned.

### 3.1 Quarter squares

The principle of quarter squares, using the identity

$$
x y=\frac{1}{4}\left[(x+y)^{2}-(x-y)^{2}\right]
$$

reduces the problem of multiplication to one of generating the square of a function. This may be carried out by the use of a non-linear element or elemen $\ddagger$, such as a chain of silicon carbide resistors 2,3 or of biased diodes ${ }^{9}$ however both these methods are limited in frequency response, the former to about $400 \mathrm{c} / \mathrm{s}$ and the latter to about $20 \mathrm{kc} / \mathrm{s}$. An accurate squaring device with a very good frequency response can be made using the deflection of an electron beam past an effectively parabolic-shaped mask ${ }^{4}, 5$, the resulting anode current being proportional to the square of the deflecting plate potential. The accuracy of the multaplier described in Ref. 4 is about $0.5 \%$ of the maximum output and the frequency response is restricted mainly by the necessary drivang amplifiers, that of the squaring tubes themselves beang many $\mathrm{Mc} / \mathrm{s}$. A disadvantage of this system is the high cost of the tubes and the care needed with the physical layout of high impedance curcultry at the higher frequencies.

### 3.2 Logarithmic

It is also possible to multaply two functions using the relation

$$
\log x y=\log x+\log y
$$

and to recover the product by taking the antilogarithm of the sum. However, the generation of the logarithmic function, usually performed by diodes, is limited in frequency response to under $1 \mathrm{kc} / \mathrm{s}$ (Ref. 6). Some semiconductor diodes exhabit a logarithmic current-voltage characteristic over a wide range of applied voltage and it may thus be possible to reduce the number of elements required for a given accuracy and hence increase the frequency response.

### 3.3 Hall effect

The Hall effect ${ }^{7}$, whereby in some materials a potential is developed which is proportional to the product of the transverse current
flowing and the magnetic field in which the material is placed, can be used to generate the product of two signals. This device is capable of working up to high frequencies, the main difficulty being to provide the magnetic field without phase shift and stray coupling to the output carcuit. By careful design of the colls and by the use of correction fields the Hall effect has been used at radio frequencies $8,9,10$. However, commercially available models do not appear to have frequency responses in excess of $10 \mathrm{kc} / \mathrm{s}$; accuracies at these frequencies are about $0.5 \%$ of maximum output.

### 3.4 Time division

used 11-18 The principle of time division in multiplication has been widely be switched on for a time proportional to the instantaneous value of $x$. If the rate of sampling is much higher than the highest frequency component in $x(t)$ or $y(t)$, the output may be passed through a low-pass filter to remove the carrier frequency gaving a filtered output proportional to both $x(t)$ and $y(t)$ and hence to the product $x y$. In the carcuits described in Refs. 11 to 15 the highest sampling rate is a few kilocycles per second and hence the maxamum frequency of input is restricted to an order less than this, or about $1 \mathrm{kc} / \mathrm{s}$. The accuracy in these carcuits is better than $0.5 \%$ of full output.

## 4. General Description of the Multiplier

In view of the lack of frequency response of existing designs, a new multiplier has been developed for the N.P.L. noise and turbulence research programme. A time division multıplier, (U.K. Pat. App. 8103/61; D.S.I.R. Pat. $24 / 1540 / 1$ ) working up to $20 \mathrm{kc} / \mathrm{s}$ has been described by Barber ${ }^{16}$ : the basic circuit to be described is a development from this.

A block diagram showing the principles of the multaplier is given in Fig. 1. The $X$ signal controls the Mark/Space Modulator which generates a train of rectangular pulses. The departure from unity mark/space ratio of these pulses is proportional to the instantaneous value of the input signal $X$, so that if $\alpha$ is the fraction of time that the output of the Modulator is in the "switch on" state, we have:-

$$
\begin{equation*}
\alpha=\frac{1+A X}{2} \tag{1}
\end{equation*}
$$

The Modulator also generates an antiphase switching signal such that

$$
\begin{equation*}
a^{\prime}=\frac{1-A X}{2} \tag{2}
\end{equation*}
$$

where $A$ is the modulation constant.
These trains of pulses are used to operate switches to gate the $Y$ signal. When the swatching signal is in the "switch on" state, switch $S_{1}$
is open and the potential at $A$ rises to $+Y$. When the switching signal is in the "off" state $S_{1}$ is closed and the porential at $A$ falls to zero. Thus the signal at $A$ is a train of pulses having the same width and repetition rate as the Modulator pulse train, but with pulse height + Y. Similarly the signal at $B$ is the inverted pulse train having height $-Y$. The signal at A now passes through a Phase Inverting Amplifier and is added to the $B$ signal. The resultant at $C$ is a rectangular wave signal between $+Y / 2$ and $-Y / 2$. The fraction of time spent in the $+Y / 2$ state is $\frac{1+A X}{2}$, and in the $-Y / 2$ state, $\frac{1-A X}{2}$. If the mean value of this signal is obtained by passing it through a filter that removes the carrier slgnal but passes the signal frequency without appreciable phase-shift, the output $V_{0}$ Is given by:-

$$
\begin{align*}
\mathrm{V}_{\mathrm{O}} & =\left(\frac{\mathrm{Y}}{2}\right) \cdot\left(\frac{1+\mathrm{AX}}{2}\right)+\left(-\frac{Y}{2}\right) \cdot\left(\frac{1-\mathrm{AX}}{2}\right) \\
& =\frac{\mathrm{AXY}}{2}=\mathrm{KXY} . \tag{3}
\end{align*}
$$

Thus the output is proportional to the product of $X$ and $Y$.

### 4.1 The use of feedback

The basic principle of Fig. 1 is used as shown in Fig. 2. The Mark/Space Modulator, 2, is controlled by the X Input Amplifier, 1. The two trains of switching pulses are passed through a pair of Switch Amplifiers, 3. The output of one of these Switch Amplifiers is used to apply negative feedback to the Modulator; the pulse train at this point has a constant peak-to-peak amplitude with mark/space ratio given by

$$
a^{\prime}=\frac{1-A X}{2}
$$

If the mean potential of this pulse train for unity mark/space ratio is zero, and the pulse height is $2 \mathrm{~K}^{\prime}$, then the mean value, $\mathrm{V}_{\mathrm{f}}$. is given by

$$
\begin{equation*}
V_{f}=-K^{\prime} A X \tag{4}
\end{equation*}
$$

Hence it is possible to apply negative feedback in the usual way from the output of the Switch Amplifier stage to the X Input Amplifier, and thus linearize the departure from unity mark/space ratio with the applied input signal $X$.

The Switches, 4, and Phase Inverting Amplifier, 5, are used as described in Section 4 above. An Output Amplificer, 6, is used to give a low output impedance and isolate the summing junction, $C$ from load effects.

This amplifier also acts as a low pass filter by the inclusion of a capacitor, $C^{\prime}$ in its feedback path.

## 5. Circuit Details

5.1 The X Input Amplifier (Fig. 3) is a long-tailed pair of silicon transistors, $\operatorname{Tr} 3$ and $\operatorname{Tr} 4$, mounted on a common heat sink to minimise effects of temperature difference and hence reduce drift. The base of $\operatorname{Tr} 3$ is taken to an adjustable potential to allow the currents flowing in the collectors of these input transistors to be set differentially as described below. For convenience 'coarse' and 'fine' balance controls are provided, the latter being on the front panel of the instrument. The $X$ input is applied to the base of Tr 4 via an 8.2 K resistor, giving an input sensitivity of about $\pm 4 \mathrm{~V}$ for maximum output; the value of this resistor may be changed to give different input sensitivities.
5.2 The Modulator, consisting of $\operatorname{Tr} 5$ and $\operatorname{Tr} 6$, is a symmetrical, emitter-coupled multivibrator carcuit. The deviation from unity mark/space ratio is proportional to the ratio of the currents in these two transistors. This ratio is controlled by the currents in $\operatorname{Tr} 3$ and $\operatorname{Tr} 4$ and hence by their base potentials. For equal currents the repetition frequency is approximately $3 \mathrm{Mc} / \mathrm{s}$; at the maximum excursion of the X input signal this repetition rate falls to about $800 \mathrm{Kc} / \mathrm{s}$ when the mark/space ratio is about 10:1. Thus at the maximum excursions of the $X$ input signal the sampling rate decreases; the high-frequency response of the multiplier is thereby reduced. The balance controls VR1 and VR2 are used to set the mark/space ratio to unity when $X=0$. A variable collector load, VR4 is used to equalise the outputs of $\operatorname{Tr} 5$ and $\operatorname{Tr} 6$, to overcome variations in component values.
5.3 The Switch Amplifiers $\operatorname{Tr} 2$ and $\operatorname{Tr} 7$ are directly coupled to the collectors of Tr5 and Tr6. Negative feedback is taken from a tapping in the collector load of $\operatorname{Tr} 7$ at such a point that the mean potential for unity mark/space ratio is zero. The negative feedback is applied by returning the 470 ohm resistor in the base circuit of $\operatorname{Tr} 4$ to the output of the feedback filter, $2.7 \mathrm{~K} \Omega$ and 100 pF . The amplitude of the pulse trains at the collectors of $\operatorname{Tr} 2$ and $\operatorname{Tr} 7$ is approximately $\pm 4$ volts.
5.4 The Transistor Switches $\operatorname{Tr} 1$ and $\operatorname{Tr} 8$ operate in the grominded emitter configuration and are driven from the collectors of $\operatorname{Tr} 2$ and $\operatorname{Tr} 7$ via the diodes D1 and D2. These act as catchers which prevent the reverse base-emitter voltage exceeding the limit of 3 volts f'or the 2 N 706 . (Types 2N706A and 2N706B may be used here and elsewhere in the circuit with some advantage. Both have a higher reverse emitter-base voltage limit of 5 V , and the latter type a lower emitter-collector saturation voltage.) When the base potential of the swatching transistor is positive the impedance between emitter and collector falls, and the transistor "turns on" thus switching off the signal at the collector. The mean potential of the collectors can be set by VR5 to minimise the component of X signal in the output when $Y=0$. VR3 compensates for differences in the transistor switches and is set for minimum output when $Y$ is at maximum a.c. signal and $X=0$. The outputs from the collectors of $\operatorname{Tr} 1$ and $\operatorname{Tr} 8$ are fed respectively to the Phase Inverter and Output Amplifier, Fig. 4.
5.5 The Phase Inverting Amplifier consists of a long-tailed pair $\operatorname{Tr} 9$ and $\operatorname{Tr} 10$ directly coupled to the output transistor $\operatorname{Tr} 11$. Overall feedback is applied and the gain is set to unity by VR6; the 4.7pF capacitor across the feedback resistor prevents positive feedback at high frequencies. VR7 sets the d.c. output level to zero for zero input signal. The signal at the collector of $\operatorname{Tr} 11$ is added to that at the collector of $\operatorname{Tr} 8$ at the junction of the two 27 K ohm resistors. Across one of these is a small variable capacitor to prevent unbalance occurring at- the higher frequencies. This unbalance, and also the loading effect previously mentioned in 4.1, would occur because of the unequal output impedances at the collectors of $\operatorname{Tr} 8$ and $\operatorname{Tr} 11$.
5. 6 The Output Amplifier is similar to the Phase Inverting Amplifier except that lower frequency transistors are used, $\operatorname{Tr} 12$ and $\operatorname{Tr} 13$ being mounted on a common heat sink to minimise temperature effects. Coarse and fine "set zero" controls are provided, the latter being mounted on the front panel of the instrument.

Zener diodes D3 and D4, mounted on a common heat sink, provide $\pm 5.2$ volt supplies internally from the $\pm 12$ volt lines. The current required for both the 12 volt rails is approximately 40 mA , and this is supplied by two comnercial transistor-regulated power supply units.

The Multiplier unit described can be used on its own provided that inputs of 8 volts peak to peak are avallable with a source impedance of less than about 1 K ohm. For convenience of use in making measurements on turbulence and noise signals, a circuit will be described which incorporates the necessary attenuators, amplifiers and switching facilities, for use with fluctuating input signals down to 60 mV peak to peak.

## 6. The Complete Instrument

A block diagram is shown in Fig. 5. Cathode followers provide a high input impedance to the instrument and the necessary low output impedance to feed the attenuators which are calibrated in $\sqrt{2}(3 a B)$ steps. A function switch enables the multiplier to read the products $X_{0} Y, X^{2}$ or $Y^{2}$. This is of convenience in the measurement of r.m.s. values, $\left(\overline{X^{2}}\right)^{\frac{1}{2}}$, and of $\overline{\mathrm{X} . \mathrm{Y}}$ correlation coefficients, e.g., $\overline{\left(\overline{\mathrm{X}^{2}}\right)^{\frac{1}{2}} \cdot\left(\overline{\mathrm{Y}^{\overline{2}}}\right)^{\frac{1}{2}}}$, since these operations may be performed without making external changes in wiring or attenuator settings. Variable gain feedback amplifiers follow the function switch. The inputs to the multiplier unit can be selected either from the internal amplifiers, ("Preamp") or from an external input ("Direct"). A selector switch and cathode follower enable an oscilloscope to monitor the inputs and the output of the multiplier unit. Attenuators are connected between the inputs to the unit and the monitor switch to set the gain from $X$ or $Y$ input to "C.R.O." output on $X$ or $Y$ to unity.

### 6.1 Gircuit Details

The circuits are shown in Figs. 6 to 10 and follow conventional design. $V 1$ and $V 2$ are the $X$ and $Y$ input cathode followers and V3
the C.R.O. cathode follower. $V 4-V 6$ and V7-V9 are the preamplifiers with overall negative feedback applied via the cathode circuit of the first atages. The outputs of these amplifiers have two stages of high pass RC filtering $(100 \mu \mathrm{~F}, 8.2 \mathrm{~K} \Omega$ and $100 \mu \mathrm{~F}, 18 \mathrm{~K} \Omega)$ to prevent any spurious d. c. reaching the input of the multiplier unit. Catcher diodes D3-D6 are also included at this point to prevent any transients in excess of 12 volts causing damage to the transistors. For this purpose also, diodes D7 and D8 (Fig. 9) remove the high voltage pulse generated when the relay is de-energised.

Details of the $\sqrt{2}$ attenuator are shown in Fig. 10. Most of the values are made up from two resistors in series, selected from $\frac{1}{4}$ watt high stability carbon resistors to an accuracy of $\pm 0.1 \%$. The switch is an 11 way "make before break" wafer switch.

### 6.2 Operation

In operation it is necessary only to adjust VR9 for zero output with both input attenuators at " O ", and to adjust VR1 for minimum output for $X=0$ and $Y=$ maximum signal. The input level of the signals are set by observing the C.R.O. output and setting the attenuators to give less than 60 mV peak to peak when the monitor switch is at "X" or "Y". The maximum input to the instrument is 1.3 volts peak to peak when the input attenuators are on $\frac{" 1 "}{16 \sqrt{2}}$.

The output of the multiplier may be used to drive a galvanometer or other meter with an impedance greater than $1 \mathrm{~K} \Omega$, when measuring the mean value of the output signal. For greater accuracy when dealing with low frequencies the output may be fed to an integrator as described in Ref. 1.

Details of the initial adjustments to the instrument are given in the Setting-up Procedure in Appendix I.

Appendix II gives details of the components used in the instrument.

## 7. Performance of Multiplier

The variation of percentage output error with input voltage in one quadrant is shown in Fig. 11. The draft of the output for zero input, which amounted to $0.3 \%$ of the output at 3 volts input during the course of the observations, has been subtracted from these results.

Dynamic tests for the complete instrument and attenuators are shown in Figs. 12 and 13. The effect of slight overload is shown when the attenuators are in the "1" setting.

Fig. 14 demonstrates the dynamic range of the instrument when used as an operational squaring device, with $1 \mathrm{kc} / \mathrm{s}$ sinewave inputs. The outputs were within 0.1 dB of the correct value over a dynamic output range of 60 dB , the results at $80 \mathrm{kc} / \mathrm{s}$ input frequency being within 0.2 dB over the same range.

The frequency response of the Multiplier unit $1 s$ shown in Fig. 15 where one input is supplied from d.c. and the other from a variable frequency source. The outputs are $5 \%$ down at $120 \mathrm{kc} / \mathrm{s}$ and $175 \mathrm{kc} / \mathrm{s}$, for alternating signals on $X$ and $Y$ respectively.

The variation of rejection ratio with frequency 18 shown in Fig. 16. The rejection ratio as taken as the ratio of the output when one input is zero and the other is maximum to the output when both signals are maximum. This ratio is seen to be better than 1:100 (i.e., -40 dB ) at frequencles below $100 \mathrm{kc} / \mathrm{s}$.

### 7.1 Limitations on accuracy

The static multiplying error is small, less than $\pm 0.5 \%$ of absolute value. The major limitation is drift of output due to temperature changes of the components, particularly transistors. Heating or cooling the complete assembly gave a temperature coefficient of less than 0.1 mV per degree Centigrade, that is less than $0.1 \%$ of maximum output. However, drafts of an order of magnitude greater than this can occur if the apparatus is exposed to strong currents of air, as is liable to happen in aerodynamic experiments. This is attributed to differential cooling of certain elements in the carcuit and it has been found possable to reduce these effects by the inclusion of a resistance of about 10 K ohms in the base to earth carcuit of $\operatorname{Tr} 13$, although the temperature coefficient is thereby raised to $-0.4 \mathrm{mV} /{ }^{\circ} \mathrm{C}$.

The dynamic accuracy is whth $5 \%$ of the absolute value from $\mathrm{DC}-110 \mathrm{kc} / \mathrm{s}$ when used on "Direct" or from $12 \mathrm{c} / \mathrm{s}$ to $110 \mathrm{kc} / \mathrm{s}$ on "Preamp". The corresponding frequencies at which the output voltage is 3 dB down are $2 \mathrm{c} / \mathrm{s}$ and $180 \mathrm{kc} / \mathrm{s}$. The limitations on the high frequency performance are imposed by the maxamum swatching rate possible wath the transistors, and hence the number of sampling cycles during each signal cycle. This can only be amproved by the use of higher speed transistors.
7.2 Performance summary

|  | Direct input | Preamplifier input |
| :---: | :---: | :---: |
| X input impedance <br> $Y$ input impedance <br> Input for full output <br> Frequency response, $-5 \%$ on output <br> Frequency response, -3 dB on output | $\begin{gathered} 8.2 \mathrm{~K} \Omega 40 \mathrm{pF} \\ 2.8 \mathrm{~K} \Omega 20 \mathrm{pF} \\ \pm \end{gathered}$ | $\begin{gathered} >1 \mathrm{M} \Omega 20 \mathrm{pF} \\ >1 \mathrm{M} \Omega 20 \mathrm{pF} \\ \pm 30 \mathrm{mV} \\ 12 \mathrm{c} / \mathrm{s}-110 \mathrm{kc} / \mathrm{s} \\ 2 \mathrm{c} / \mathrm{s}-180 \mathrm{kc} / \mathrm{s} \end{gathered}$ |
| Output <br> Mean sampling rate Multiplication error <br> Power requirements of complete instrument | $\begin{array}{r}  \pm 4 \\ 3 \\ <1 \% \text { of } f \\ +12 \\ -12 \\ +300 \\ 6.3 \mathrm{~V} \mathrm{A.} \end{array}$ | $\begin{aligned} & 00 \mathrm{mV} \\ & \mathrm{Mc} / \mathrm{s} \\ & \text { ull output } \\ & \text { V } \quad 50 \mathrm{~mA} \\ & \mathrm{~V} \\ & \mathrm{~V} \\ & \mathrm{C} . \\ & \mathrm{C} . \\ & \hline 0.70 \mathrm{~mA} \\ & \hline \end{aligned}$ |

## 8. Applications of the Multiplier

The application of the multiplier to the measurement of mean powers and of correlation coefficients has been mentioned in Section 2, and the block diagrams for such measurements are shown in Figs. 17, 18 and 19.

The multiplier may also be used as a mixer in a constant bandwidth frequency analyser as shown in Fig. 20. The frequency of analysis is set by the sinewave oscillator at $f_{1}$ and the bandwidth, $2 f_{2}$, by the cutoff frequency of the low pass filter which passes sygnals having frequencies from zero to $f_{a}$.

A further use of the multiplier as a frequency changer is the generation of a sweep frequency, demonstrated in the block diagram of Fig. 21. A function generator provides a voltage $\phi$ varying with time. This potential controls the frequency of oscillation of the frequency modulation oscillator, which may be a square wave generator, giving a fundamental output frequency of $f_{1}+f_{2}(\phi)$. If this output is multiplied by a signal with frequency $f_{1}$ in the multiplier and the output passed through a low pass filter to pass $f_{2}$ but not $f_{1}$ or higher frequencies, the resultant will be a frequency $f_{2}(\phi)$. Thus if the function generator, (which may conveniently be a slow oscilloscope time-base), provides a saw-tooth waveform the resultant will be a frequency varying linearly with time. The lowest frequencies obtanable in this way are limited solely by the rate of drift of the two oscillator frequencies, which may eassly be held to within a few cycles per second per minute when $f_{1}$ is about $100 \mathrm{kc} / \mathrm{s}$. The upper frequency obtainable is limated by the low pass filter characterıstics for a carrier frequency below $100 \mathrm{kc} / \mathrm{s}$. Thus this system can provide a suitable sweep frequency for the response testing of apparatus encountered in the turbulence and noise field of research.

The Integrator used in conjunction with the circuats mentioned above is the elapsed time integrator described in Ref. 1, using an operational d.c. amplifier as the computing element and a dekatron chain as the timing circuit.

## 9. Conclusions

The variable mark/space Multiplier has proved very satisfactory for the measurement of a wide range of parameters in the turbulence and noise field, the accuracy and frequency response being amply adequate. The cost and sumplicity are an advdntage over any comparable multiplier. The reliability has been excellent over an extended period of use.

## 10. Acknowledgement

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## APPENDIX I

## Setting-up Procedure

1. Set all variable controls to their mid position, $X$ and $Y$ input attenuators to zero.
2. Observe the waveform at the collector of $\operatorname{Tr} 5 \mathrm{wi}$ th a high impedance probe and oscilloscope. Adjust VR2 for unity mark/space ratio.
3. Observe the waveform at the collector of $\operatorname{Tr} 6$. Adjust VR4 for the same output as at the collector of $\operatorname{Tr} 5$.
4. Cornect a sinewave oscillator to the $X$ and $Y$ preamplifier inputs in parallel and set for $1 \mathrm{kc} / \mathrm{s}, 60 \mathrm{mV}$ peak to peak. Set function switch to "XY" and both input attenuators to "1". Set "Direct-Preamp" switches to "Preamp". Observe output of preamplifiers and adjust $X$ and $Y$ gains for 8 V peak to peak.
5. Observe "C.R.O. Output" with monitor switch at "X", adjust
"X C.R.O. Attn." for 60 mV peak to peak. Repeat for Y.
6. Set $X$ and $Y$ input attenuators to " $O$ ", function switch at "XY". Observe the output of multiplier with a d.c. instrument having a sensitivity of 1 mV . Earth the junction of the two $27 \mathrm{~K} \Omega$ resistors (Fig. 4) and adjust VR10 for zero potential.
7. Adjust VR7 so that the potential at $U$ (Fig. 4) is the same as at the collector of Tr11. Remove the shorting link.
8. Adjust VR8 and VR9 for zero potential at the output of the multiplier.
9. Set the $Y$ attenuator to "1" and adjust firstly VR6 and then VR3 for minimum output.
10. Set the $Y$ attenuator to " 0 " and the $X$ attenuator to "1". Adjust VR5 for minimum output.
11. Set the $X$ attenuator to " 0 " and the $Y$ attenuator to "1". Adjust VR3 and VR2 for minimum output.
12. Check that the r.m.s. value of the output signal when one input attenuator is at " 0 " and the other is at "1" is less than $1 \%$ of the signal when both attenuators are on "1". If not, repeat operations 9 to 11.
13. Set the $X$ input attenuator to " $O$ " and the $Y$ input attenuator to "1". Set the frequency of the input signal to $80 \mathrm{kc} / \mathrm{s}$. Adjust the 8 pF capacitor across one of the $27 \mathrm{~K} \Omega$ resistors (Fig. 4) to give minimum output.
14. The gains of preamplifiers can be set for other input sensitivities, or to adjust the input to the multiplier unit to a voltage different from that quoted in instruction 4 above. This latter operation may be necessary to achieve the maximum range of linear operation of the unit and can be determined only by trial. Operation 5 should be repeated to achieve the desired overall gain.

## APPENDIX II/

## APPENDIX II

## Constructional Details

'The values of the resistors are given in ohms, the suffices $K$ and M denoting $10^{3}$ and $10^{8}$ multipliers. High stability cracked carbon resistors with a power rating of $\frac{1}{4}$ watt and a tolerance of $\pm 5 \%$ are used unless otherwise specified. Resistors with a power dissipation above 1 watt are vitreous wire-wound types.

Capacitor values are given in farads, the suffices $\mu$ and $p$ denoting $10^{-6}$ and $10^{-12}$ multipliers. For values less than $0.01 \mu \mathrm{~F}, 500 \mathrm{~V}$ polystyrene components are used; above this capacity the non-electrolytic types are of paper construction with a manimum working voltage of 350 . Electrolytic capacitors also have a minimum working voltage of 350 unless specyfied to the contrary.

The preset potentiometers associated with the transistor circuits are $\frac{1}{10}$ th watt subminiature components, others are $1 \frac{1}{2}$ watt wire-wound types.

The layout of the circuits is not critical, but that of the modulator and switches should be maintained as symmetrical as possible to equalise wiring capacitance in the two halves of the circuit.


Multiplier principles.
Idealized waveforms for d.c. inputs are shown


Multiplier - block diagram



Phase inverter and output amplifier.



See Fig. 10 for $\sqrt{2}$ attenuator detalls.

$x$ input amplifier


Y input amplifier


Power input details


D.C. Calibration of multiplier



FIG. 14


Multiplier and preamplifiers used as an operational squarer at $f=1 \mathrm{kc} / \mathrm{s}$ and $80 \mathrm{kc} / \mathrm{s}$

$\left.\right|_{\frac{\pi}{0}} ^{\square}$

Frequency response of multiplier


FIG. 16

Rejection ratios


FIG. IB
Cross-correlation


FIG. 20


FIG. 21


Applications of the multiplier
A.R.C. C.P. No. 685

August, 1962
Johns on, R. F.
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