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A Problem of Wing-Body Interference

By

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SUMMARY

The report considers inviscid, supersonic flow past a symmetrical wing-body combination at zero incidence. The body is a circular cylinder extending indefinitely in both the upstream and the downstream directions. The wings are two halves of a delta wing of single-wedge section with a subsonic leading edge; they are assumed to extend indefinitely in the downstream direction. Results are obtained for the pressure distribution on the body.

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1 INTRODUCTION

This section begins with a short history of work on wing-body interference in Aerodynamics Department.

In 1951 K. Stewartson (now Professor of Applied Mathematics at Durham University) spent some weeks in the Department as a vacation consultant, during which he wrote a note (unpublished) on wing-body interference. His method is applicable to wing-body combinations of which the body is approximately a circular cylinder and the wings lie approximately in a plane and are symmetrically mounted; throughout the present report the phrase "wing-body combination" is used to denote a configuration of this type. The governing partial differential equation is taken to be the linearised equation of supersonic flow. It is assumed that the velocity potential due to the wings alone is known, so that the problem reduces to the determination of an interference potential. This can be determined by Laplace transform methods, which give the pressure coefficient over the combination as a Fourier series. The coefficient of the $n^{\underline{th}}$ term of the Fourier series is an integral involving a function of n, x, and r, where x and r are cylindrical polar coordinates. Each function is an inverse of a Laplace transform that contains Bessel functions of imaginary argument; the inverse has to be obtained numerically. Even when the inverses have been computed, the numerical work involved in solving just one problem in wing-body interference is formidable. But in 1951 the inverses had not been tabulated; and the computational effort required to do this was so forbidding in the desk-machine days of 1951 that the problem was temporarily abandoned.

In 1955 the author[1] used Stewartson's method to determine the wave drag of bodies that do not depart far from circular cylinders. Reference 1 can hardly be said to have contributed to the theory of wing-body interference, since the problem solved is equivalent to determining the flow past a wing-body combination without a wing. Admittedly, some of the inverses mentioned above are tabulated in Reference 1; they are those for which the value of r is equal to the radius of the circular cylinder approximating the body. The inverses are tabulated for eleven values of n and for values of x at intervals of 0.2. In principle, they can be used to determine the pressure on the body of a wing-body combination; in practice, the coarse interval in x precludes this.

In 1957, Nielsen[2] completed his work on wing-body interference by issuing an impressively bulky set of tables of inverses of certain Laplace transforms. Nielsen's inverses are not identical with those of Stewartson, although the two sets are closely connected. The interference problem can be solved by using Nielsen's inverses in almost exactly the same way as by using Stewartson's - the pressure on the combination is given by a Fourier series whose coefficients are integrals involving the inverses. Nielsen's inverses are tabulated for eleven values of n, ten values of r, and for values of x at every 0.01. The appearance of these tables together with the arrival in the RAE of digital electronic computers meant that most of the labour required to solve wing-body interference problems had been eliminated; nevertheless, the computational effort required was still so great that simplified, but less accurate, methods were desirable.

By 1958, several members of Aero Department had produced approximate theories relating to wing-body interference: the two theories of importance here are due to J.A. Bagley[3] and D.A. Treadgold. Bagley argues that, for wings with sweptback leading edges, the flow near the initial point of a wing-body junction must be the same as that at the apex of the wing formed by joining the original wing to its reflection in the plane through the junction normal to the wing. The procedure has been applied to wing-body interference at subsonic speeds with considerable success (at least, as far as the flow over the wing is concerned). Treadgold has developed a method originally suggested by Stewartson. He considers a combination with one wing only mounted on the body, since it can be shown that, in this case, the inverses contain known functions only (exponentials, circular functions, etc.); the method gives correct answers ahead of the Mach cone from the initial point of the junction of the body and the other wing. Bagley's method is simpler than Treadgold's; on the other hand, Treadgold's method is more accurate than Bagley's.

It was decided that, to provide a test for these methods, the author should solve a particular wing-body problem exactly (that is, by using the method of Ref.2) and that the results for the pressure on the combination should be compared with those obtained by the approximate theories. The particular wing-body combination considered here consists of two halves of a symmetrical delta wing of single-wedge section mounted symmetrically on a circular cylinder; the leading edges are subsonic, and the combination is at zero incidence. The potential due to the wing alone is determined in Section 2; the apparently long-winded method for obtaining the x and r derivatives of the potential as a Fourier series is due to Stewartson (in his unpublished note); it is, in fact, a vast improvement on the obvious method. The interference potential is determined in Section 3; and the results are discussed in Section 4.

2 SUPERSONIC FLOW PAST A DELTA WING OF SINGLE-WEDGE SECTION

Consider inviscid, supersonic flow past a symmetrical delta wing at zero incidence. Let the apex of the wing be the origin of a system of right-handed rectangular cartesian coordinates, x, y, and z, and let the x axis be in the direction of the free stream. Suppose that the equation of the wing is

$$z = \delta\left(x + \frac{y}{k}\right) \qquad (-kx < y < 0),$$

$$z = \delta\left(x - \frac{y}{k}\right) \qquad (0 < y < kx);$$

the wing is assumed to extend indefinitely in the positive x direction. Let the speed of the free stream be U, its Mach number be M, its pressure be p_∞, and its density be ρ_∞; write B for $\sqrt{M^2 - 1}$.

Suppose that δ is small compared with unity and that M is neither close to unity nor large compared with unity. It is then possible[4] to introduce a disturbance velocity potential, ϕ_W (W standing for wing), such that the velocities in the x, y, and z directions are given respectively by $U\left(1 + \frac{\partial\phi_W}{\partial x}\right)$, $U\frac{\partial\phi_W}{\partial y}$, and $U\frac{\partial\phi_W}{\partial z}$; ϕ_W satisfies the linearised equation of supersonic flow,

$$B^2 \frac{\partial^2 \phi_W}{\partial x^2} = \frac{\partial^2 \phi_W}{\partial y^2} + \frac{\partial^2 \phi_W}{\partial z^2} . \quad (1)$$

The boundary conditions are that the disturbance velocity components should vanish on the Mach cone from the apex of the wing, and that, on the wing, the velocity component normal to the wing should vanish; these conditions may be written approximately

$$\frac{\partial \phi_W}{\partial x} = \frac{\partial \phi_W}{\partial y} = \frac{\partial \phi_W}{\partial z} = 0 \quad \text{when } x^2 - B^2 y^2 - B^2 z^2 = 0, \quad (2a)$$

$$\frac{\partial \phi_W}{\partial z} = \delta \quad \text{when} \quad z = 0, \quad |y| < kx. \quad (2b)$$

From the symmetry of the flow, $\phi_W(x, y, z) = \phi_W(x, y, -z)$.

Suppose that $Bk < 1$, so that the leading edge is supersonic. The potential at a point (x, y, z) can be found[4] by distributing suitable fundamental singular solutions of Equation (1) over the wing surface and integrating them over the part of the wing surface lying within the Mach fore-cone from the point being considered. The equation of the Mach cone from the point is

$$(x - x_1)^2 - B^2(y - y_1)^2 - B^2(z - z_1)^2 = 0, \quad (3)$$

where x_1, y_1, and z_1, are running coordinates. The Mach fore-cone intersects the wing in the curve

$$x_1 = x - B[(y - y_1)^2 + z^2]^{\frac{1}{2}}, \quad (4)$$

since the equation of the wing may be approximated by $z_1 = 0$. The curve given by Equation (4) intersects the leading edge, $y_1 = \pm kx_1$, in two points whose y coordinates are y_A and y_B,

where

$$y_A = -\frac{k}{(1 - B^2k^2)} \left\{(x + B^2ky) - B[(y + kx)^2 + (1 - B^2k^2)z^2]^{\frac{1}{2}}\right\}, \quad (5a)$$

and

$$y_B = \frac{k}{(1 - B^2k^2)}\left\{(x - B^2ky) - B[(y - kx)^2 + (1 - B^2k^2)z^2]^{\frac{1}{2}}\right\}. \qquad (5b)$$

It can be shown[4] that the following form for ϕ_W satisfies Equation (1) and the boundary conditions of Equations (2):

$$\phi_W = -\frac{\delta}{\pi}\int_{y_A}^{o}\int_{-y_1/k}^{x-B[(y-y_1)^2+z^2]^{\frac{1}{2}}} \frac{dx_1\ dy_1}{[(x - x_1)^2 - B^2(y - y_1)^2 - B^2(z - z_1)^2]^{\frac{1}{2}}}$$

$$-\frac{\delta}{\pi}\int_{o}^{y_B}\int_{y_1/k}^{x-B[(y-y_1)^2+z^2]^{\frac{1}{2}}} \frac{dx_1\ dy_1}{[(x - x_1)^2 - B^2(y - y_1)^2 - B^2(z - z_1)^2]^{\frac{1}{2}}}.$$

It follows that

$$\phi_W = -\frac{\delta}{\pi}\int_{y_A}^{o} \cosh^{-1}\frac{(kx + y_1)}{Bk[(y - y_1)^2 + z^2]^{\frac{1}{2}}}\ dy_1 - \frac{\delta}{\pi}\int_{o}^{y_B} \cosh^{-1}\frac{(kx - y_1)}{Bk[(y - y_1)^2 + z^2]^{\frac{1}{2}}}.$$

Hence,

$$\frac{\partial\phi_W}{\partial x} = -\frac{\delta}{\pi}\int_{y_A}^{o} \frac{k dy_1}{[(1 - B^2k^2)y_1^2 + 2k(x + B^2ky)y_1 + k^2(x^2 - B^2y^2 - B^2z^2)]^{\frac{1}{2}}}$$

$$-\frac{\delta}{\pi}\int_{o}^{y_B} \frac{k dy_1}{[(1 - B^2k^2)y_1^2 - 2k(x - B^2ky)y_1 + k^2(x^2 - B^2y^2 - B^2z^2)]^{\frac{1}{2}}}$$

$$= -\frac{\delta k}{\pi(1-B^2k^2)^{\frac{1}{2}}}\left\{\cosh^{-1}\frac{(x+B^2ky)}{B[(kx+y)^2+(1-B^2k^2)z^2]^{\frac{1}{2}}} + \cosh^{-1}\frac{(x-B^2ky)}{B[(kx-y)^2+(1-B^2k^2)z^2]^{\frac{1}{2}}}\right\}.$$

.... (6)

Introduce cylindrical polar coordinates (x, r, θ) such that

$$r^2 = y^2 + z^2, \tag{7a}$$

$$\tan\theta = z/y. \tag{7b}$$

Equation (6) becomes

$$\frac{\partial\phi_W}{\partial x} = -\frac{\delta k}{\pi(1 - B^2k^2)^{\frac{1}{2}}}\left\{\cosh^{-1}\frac{(x + B^2\,kr\cos\theta)}{B[(kx + r\cos\theta)^2 + (1 - B^2k^2)r^2\sin^2\theta]^{\frac{1}{2}}}\right.$$
$$\left. + \cosh^{-1}\frac{(x - B^2\,kr\cos\theta)}{B[(kx - r\cos\theta)^2 + (1 - B^2k^2)r^2\sin^2\theta]^{\frac{1}{2}}}\right\}.$$

It follows that

$$\frac{\partial^2\varphi_W}{\partial x\,\partial r} = \frac{\delta kx}{\pi(x^2 - B^2r^2)^{\frac{1}{2}}}\left\{\frac{(r + kx\cos\theta)}{[k^2x^2 + (1 - B^2k^2)r^2 + 2kxr\cos\theta + B^2k^2r^2\cos^2\theta]}\right.$$
$$\left. + \frac{(r - kx\cos\theta)}{[x^2 + (1 - B^2k^2)r^2 - 2kxr\cos\theta + B^2k^2r^2\cos^2\theta]}\right\}. \quad \ldots. (8)$$

Now write

$$\frac{1}{Bk} = \cosh\lambda, \tag{9a}$$

$$\frac{x}{Br} = \cosh\chi. \tag{9b}$$

Equation (8) becomes

$$\frac{\partial^2 \phi_W}{\partial x\, \partial r} = \frac{\delta \coth \chi}{\pi B r} \left\{ \frac{(\cosh \lambda + \cosh \chi \cos \theta)}{[(\cosh \lambda \cosh \chi + \cos \theta)^2 - \sinh^2 \chi \sinh^2 \lambda]} + \frac{(\cosh \lambda - \cosh \chi \cos \theta)}{[(\cosh \lambda \cosh \chi - \cos \theta)^2 - \sinh^2 \chi \sinh^2 \lambda]} \right\}$$

$$= \frac{\delta \coth \chi}{2\pi Br \sinh \lambda} \left\{ \frac{\sinh (\lambda + \chi)}{[\cosh (\lambda + \chi) + \cos \theta]} + \frac{\sinh (\lambda - \chi)}{[\cosh (\lambda - \chi) + \cos \theta]} + \frac{\sinh (\lambda + \chi)}{[\cosh (\lambda + \chi) - \cos \theta]} + \frac{\sinh (\lambda - \chi)}{[\cosh (\lambda - \chi) - \cos \theta]} \right\}$$

$$= \frac{\delta \coth \chi}{\pi Br \sinh \lambda} \left\{ \frac{\sinh 2(\lambda + \chi)}{[\cosh 2 (\lambda + \chi) - \cos 2\theta]} + \frac{\sinh 2 (\lambda - \chi)}{[\cosh 2 (\lambda - \chi) - \cos 2\theta]} \right\}. \quad \ldots.(10)$$

Now, it is known that[5]

$$\frac{1}{\pi} \int_0^{\pi} \frac{\cos n\theta \, d\theta}{(\cosh \eta - \cos \theta)} = \frac{e^{-n|\eta|}}{\sinh |\eta|} .$$

Therefore, Equation (10) can be expanded as

$$\frac{\partial^2 \phi}{\partial x\, \partial r} = \frac{4\delta \coth \chi}{\pi Br \sinh \lambda} \left[\frac{1}{2} + \sum_{n=1}^{\infty} e^{-2n\lambda} \cosh 2n\chi \cos 2n\theta \right] \quad (\chi < \lambda), \qquad (11a)$$

$$\frac{\partial^2 \phi}{\partial x\, \partial r} = - \frac{4\delta \coth \chi}{\pi Br \sinh \lambda} \sum_{n=1}^{\infty} \sinh 2n\lambda \, e^{-2n\chi} \cos 2n\theta \quad (\chi > \lambda). \qquad (11b)$$

From Equation (9b), the Mach cone from the origin is given by $\chi = 0$; from Equation (2a), $\frac{\partial\phi_W}{\partial r} = 0$ on the Mach cone from the origin; hence, from Equation (11a),

$$\frac{\partial\phi_W}{\partial r} = \frac{2\delta}{\pi \sinh\lambda}\left\{\sinh\chi + \sum_{n=1}^{\infty} e^{-2n\lambda}\left[\frac{\sinh(2n+1)\chi}{(2n+1)} + \frac{\sinh(2n-1)\chi}{(2n-1)}\right]\cos 2n\theta\right\} \quad (\chi < \lambda). \qquad \ldots. (12)$$

Equation (11b) may be integrated with respect to x to give

$$\frac{\partial\phi_W}{\partial r} = \frac{2\delta}{\pi} + \frac{2\delta}{\pi \sinh\lambda}\sum_{n=1}^{\infty}\left\{\sinh 2n\lambda\left[\frac{e^{-(2n+1)\chi}}{(2n+1)} + \frac{e^{-(2n-1)\chi}}{(2n-1)}\right] - \frac{2\sinh\lambda}{(4n^2-1)}\right\}\cos 2n\theta \quad (\chi > \lambda), \qquad \ldots. (13)$$

where the constant of integration has been chosen to make the values of $\frac{\partial\phi_W}{\partial r}$ as given by Equations (12) and (13) equal when $\chi = \lambda$. From Equation (2a), $\frac{\partial\phi_W}{\partial x} = 0$ on the Mach cone from the origin, which corresponds to $\chi = 0$; hence from Equation (11a),

$$\frac{\partial\phi_W}{\partial x} = -\frac{4\delta}{\pi B \sinh\lambda}\left[\frac{1}{2}\chi + \sum_{n=1}^{\infty}\frac{e^{-2n\lambda}\sinh 2n\chi}{2n}\cos 2n\theta\right] \quad (\chi < \lambda). \qquad (14)$$

Equation (11b) may be integrated with respect to r to give

$$\frac{\partial\phi_W}{\partial x} = -\frac{4\delta}{\pi B \sinh\lambda}\left[\frac{1}{2}\lambda + \sum_{n=1}^{\infty}\frac{\sinh 2n\lambda\ e^{-2n\chi}}{2n}\cos 2n\theta\right] \quad (\chi > \lambda), \qquad (15)$$

where the constant of integration has been chosen to make the values of $\frac{\partial\phi_W}{\partial x}$ given by Equations (14) and (15) equal when $\chi = \lambda$.

Equations (12) to (15) inclusive may be written as

$$\frac{\partial\phi_W}{\partial r} = \frac{2\delta}{\pi \sinh\lambda} \sum_{n=0}^{\infty} a_{2n}(x, r) \cos 2n\theta, \tag{16a}$$

$$\frac{\partial\phi_W}{\partial x} = -\frac{2\delta}{\pi B \sinh\lambda} \sum_{n=0}^{\infty} b_{2n}(x, r) \cos 2n\theta; \tag{16b}$$

here,

$$a_o = \sinh\chi, \qquad a_{2n} = e^{-2n\lambda}\left[\frac{\sinh(2n+1)\chi}{(2n+1)} + \frac{\sinh(2n-1)\chi}{(2n-1)}\right] \qquad (n > 0), \quad .... \tag{17a}$$

$$b_o = \chi, \qquad b_{2n} = \frac{e^{-2n\lambda}\sinh 2n\chi}{n} \qquad (n > 0), \tag{17b}$$

when $\chi < \lambda$; and

$$a_o = \sinh\lambda, \qquad a_{2n} = \sinh 2n\lambda\left[\frac{e^{-(2n+1)\chi}}{(2n+1)} + \frac{e^{-(2n-1)\chi}}{(2n-1)}\right] - \frac{2\sinh\lambda}{(4n^2-1)} \qquad (n > 0), \quad .... \tag{18a}$$

$$b_o = \lambda, \qquad b_{2n} = \frac{\sinh 2n\lambda\, e^{-2n\chi}}{n} \qquad (n > 0), \tag{18b}$$

when $\chi > \lambda$.

3 SUPERSONIC FLOW PAST A CERTAIN WING-BODY COMBINATION

Consider inviscid, supersonic flow past a wing-body combination consisting of two halves of a symmetrical delta wing of single-wedge section mounted symmetrically on a circular cylinder; suppose the configuration is at zero incidence. It is convenient to imagine the two halves of the wing to be continued through the wall of the circular cylinder until they meet inside; the wing thus formed is assumed to be the same as the one considered in Section 2. Let x, y, z, r, and θ be defined as before; and suppose that the free stream direction and the flow quantities in the free stream are the same as in Section 2. Let the radius of the circular cylinder be r_0. The wing is assumed to extend indefinitely in the positive x direction, and the circular cylinder to extend indefinitely in the negative and positive x directions.

A disturbance velocity potential, ϕ, is introduced; it is written as the sum of two terms,

$$\phi = \phi_W + \phi_I . \tag{19}$$

ϕ_W is the potential due to the wing alone, and has been determined in Section 2; ϕ_I is the interference potential. ϕ must satisfy the linearised equation of supersonic flow, and the first derivatives of ϕ (the velocity components) must vanish on the Mach cone from the apex of the wing (in fact, they must vanish in a region behind this cone, since, in reality, the flow is undisturbed up to the point where the wing appears through the cylinder). ϕ_W satisfies these requirements, and so ϕ_I must also satisfy them.

Hence,

$$B^2 \frac{\partial^2 \phi_I}{\partial x^2} = \frac{\partial^2 \phi_I}{\partial y^2} + \frac{\partial^2 \phi_I}{\partial z^2} = \frac{\partial^2 \phi_I}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_I}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi_I}{\partial \theta^2} = 0; \tag{20}$$

and

$$\frac{\partial \phi_I}{\partial x} = \frac{\partial \phi_I}{\partial y} = \frac{\partial \phi_I}{\partial z} = 0 \quad \text{when } x^2 - B^2y^2 - B^2z^2 = 0. \tag{21}$$

The boundary condition on the wing is satisfied by ϕ_W on its own; hence,

$$\left(\frac{\partial \phi_I}{\partial z}\right)_{z=0} = 0. \tag{22}$$

The boundary condition on the circular cylinder is that the velocity component normal to the cylinder should vanish; this means that

$$\left(\frac{\partial \phi}{\partial r}\right)_{r=r_0} = 0;$$

hence, from Equations (19) and (16a),

$$\left(\frac{\partial\phi_I}{\partial r}\right)_{r=r_0} = -\left(\frac{\partial\phi_W}{\partial r}\right)_{r=r_0} = -\frac{2\delta}{\pi \sinh\lambda} \sum_{n=0}^{\infty} a_{2n,0}(x) \cos 2n\theta, \tag{23}$$

where

$$a_{2n,0}(x) = a_{2n}(x, r_0). \tag{24}$$

Equation (20), with the boundary conditions given by Equations (21), (22), and (23), can be solved[1] by Laplace transform methods. The Laplace transform of $f(x)$ is $\bar{f}(p)$, where

$$\bar{f}(p) = \int_0^\infty e^{-px} f(x)\, dx.$$

Equation (20) transforms to

$$B^2 p^2 \bar{\phi} = \frac{\partial^2 \bar{\phi}_I}{\partial r^2} + \frac{1}{r}\frac{\partial\bar{\phi}_I}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \bar{\phi}_I}{\partial\theta^2}; \tag{25}$$

Equation (22) transforms to

$$\left(\frac{\partial\bar{\phi}_I}{\partial z}\right)_{z=0} = 0; \tag{26a}$$

Equation (23) transforms to

$$\left(\frac{\partial\bar{\phi}_I}{\partial r}\right)_{r=r_0} = -\frac{2\delta}{\pi \sinh\lambda} \sum_{n=0} \bar{a}_{2n,0} \cos 2n\theta; \tag{26b}$$

from Equation (21),

$$\bar{\phi}_I \to 0 \quad \text{as} \quad r \to \infty. \tag{26c}$$

By the method of separation of variables it can be shown that the solution of Equation (25) is

$$\bar{\phi}_I = \sum_{n=0}^{\infty} \{[A_n K_n(Bpr) + C_n I_n(Bpr)] \cos n\theta + [B_n K_n(Bpr) + D_n I_n(Bpr)] \sin n\theta\},$$

where A_n, B_n, C_n, and D_n are arbitrary functions of p, and K_n and I_n are Bessel functions of imaginary argument[1]. From Equations (26) and the fact that $I_n(Bpr) \to \infty$ as $r \to \infty$, it follows that $\bar{\phi}_I$ can be written as

$$\bar{\phi}_I = \sum_{n=0}^{\infty} A_{2n}(p) \; K_{2n}(Bpr) \; \cos 2n\theta. \tag{27}$$

$A_{2n}(p)$ is to be determined from Equation (26b), which gives

$$Bp \sum_{n=0}^{\infty} A_{2n}(p) \; K'_{2n}(Bpr_0) \; \cos 2n\theta = -\frac{2\delta}{\pi \sinh \lambda} \sum_{n=0}^{\infty} \bar{a}_{2n,0} \cos 2n\theta.$$

It follows that

$$A_{2n}(p) = -\frac{2\delta \; \bar{a}_{2n,0}}{\pi B p \sinh \lambda \; K'_n(Bpr_0)} .$$

From Equation (27),

$$\bar{\phi}_I = -\frac{2\delta}{\pi \; Bp \; \sinh \lambda} \sum_{n=0}^{\infty} \frac{\bar{a}_{2n,0} \; K_{2n}(Bpr)}{K'_{2n}(Bpr_0)} \cos 2n\theta.$$

Hence,

$$\overline{\frac{\partial \phi_I}{\partial x}} = -\frac{2\delta}{\pi \; B \; \sinh \lambda} \sum_{n=0}^{\infty} \frac{\bar{a}_{2n,0} \; K_{2n}(Bpr)}{K'_{2n}(Bpr_0)} \cos 2n\theta. \tag{28}$$

Now define a function $W_{2n}(\zeta, \nu)$ by the equation

$$e^{p(\nu-1)} \frac{K_{2n}(p\nu)}{K'_{2n}(p)} + \frac{1}{\sqrt{\nu}} = \int_0^\infty e^{-p\xi} W_{2n}(\xi, \nu)\, d\xi; \qquad (29)$$

W_{2n} is tabulated in Ref.2.

It follows that

$$e^{Bp(r-r_0)} \frac{K_{2n}(Bpr)}{K'_{2n}(Bpr_0)} + \sqrt{\frac{r_0}{r}} = \int_0^\infty \frac{1}{Br_0} W_{2n}\left(\frac{x}{Br_0}, \frac{r}{r_0}\right) e^{-px}\, dx. \qquad (30)$$

Now Equation (28) may be written

$$\frac{\overline{\partial \phi_I}}{\partial x} = -\frac{2\delta}{\pi B \sinh\lambda} \sum_{n=0}^{\infty} \left\{\left[e^{Bp(r-r_0)} \frac{K_{2n}(Bpr)}{K'_{2n}(Bpr_0)} + \sqrt{\frac{r_0}{r}}\right] - \sqrt{\frac{r_0}{r}}\right\} e^{-Bp(r-r_0)}\, \bar{a}_{2n,0} \cos 2n\theta.$$

The inverse of $e^{-Bp(r-r_0)}\bar{a}_{2n,0}$ is $H(x - B\overline{r - r_0})\, a_{2n,0}(x - B\overline{r - r_0})$, where H is the unit function (zero for negative values of its argument, unity for positive values); hence, from Equation (30) and the convolution theorem[1],

$$\frac{\partial \phi_I}{\partial x} = -\frac{2\delta}{\pi B \sinh\lambda} \sum_{n=0}^{\infty} \left\{ \int_{B(r-r_0)}^{x} a_{2n,0}(x_1 - B\overline{r - r_0}) \frac{1}{Br_0} W_{2n}\left(\frac{x - x_1}{Br_0}, \frac{r}{r_0}\right) dx_1 \right.$$

$$\left. - \sqrt{\frac{r_0}{r}}\, H(x - B\overline{r - r_0})\, a_{2n,0}(x - B\overline{r - r_0}) \right\} \cos 2n\theta$$

$$= -\frac{2\delta}{\pi B \sinh\lambda} \sum_{n=0}^{\infty} \left\{ \int_{\left(\frac{r}{r_0} - 1\right)}^{x\, Br_0} C_{2n}\left(\xi - \frac{r}{r_0} + 1\right) W_{2n}\left(\frac{x}{Br_0} - \xi, \frac{r}{r_0}\right) d\xi \right.$$

$$\left. - \sqrt{\frac{r}{r_0}}\, H\left(\frac{x}{Br_0} - \frac{r}{r_0} + 1\right) C_{2n}\left(\frac{x}{Br_0} - \frac{r}{r_0} + 1\right)\right\} \cos 2n\theta, \qquad \ldots. (31)$$

where ξ has been written for $\frac{x_1}{Br_0}$. From Equation (24), C_{2n} is defined by

$$C_{2n}(\vartheta) = a_{2n,0}(Br_0\,\vartheta) = a_{2n}(Br_0\,\vartheta,\ r_0);$$

hence, from Equations (17a) and (18a),

$$C_0(\vartheta) = \sinh\chi_0, \quad C_{2n}(\vartheta) = e^{-2n\lambda}\left[\frac{\sinh(2n+1)\chi_0}{(2n+1)} + \frac{\sinh(2n-1)\chi_0}{(2n-1)}\right] \quad (n > 0), \qquad \ldots.(32a)$$

when $\chi_0 < \lambda$, and

$$C_0(\vartheta) = \sinh\lambda, \quad C_{2n}(\vartheta) = \sinh 2n\lambda\left[\frac{e^{-(2n+1)\chi_0}}{2n+1} + \frac{e^{-(2n-1)\chi_0}}{(2n-1)}\right] - \frac{2\sinh\lambda}{(4n^2-1)} \quad (n>0) \qquad \ldots.\ (32b)$$

when $\chi_0 > \lambda_1$.

λ is given by Equation (9a); from Equation (9b), χ_0 is defined by $\cosh\chi_0 = \vartheta$.

The linearised approximation for C_p, the pressure coefficient (equal to $\frac{p - p_\infty}{\frac{1}{2}\rho_\infty V^2}$, where p is the local pressure), is given by

$$C_p = -2\frac{\partial\phi}{\partial x}.$$

From Equations (19), (31), and (16b), it follows that

$$C_p = \frac{4\delta}{\pi B \sinh\lambda}\sum_{n=0}^{\infty}\left\{\int_{\left(\frac{r}{r_0}-1\right)}^{x/Br_0} C_{2n}\left(\xi - \frac{r}{r_0} + 1\right) W_{2n}\left(\frac{x}{Br_0} - \xi,\ \frac{r}{r_0}\right) d\xi\right.$$

$$\left. - \sqrt{\frac{r_0}{r}}\, H\left(\frac{x}{Br_0} - \frac{r}{r_0} + 1\right) C_{2n}\left(\frac{x}{Br_0} - \frac{r}{r_0} + 1\right) + b_{2n}(x,\ r)\right\} \cos 2n\theta; \qquad (33)$$

The b_{2n} are given by Equations (17b) and (18b).

C_p is required on both the wing and the body. On the wing $\theta = 0$ (or π), and Equation (33) becomes

$$(C_p)_{wing} = \frac{4\delta}{\pi B \sinh\lambda} \sum_{n=0}^{\infty} \left\{ \int_{\left(\frac{r}{r_0}-1\right)}^{r/Br_0} C_{2n}\left(\xi - \frac{r}{r_0} + 1\right) W_{2n}\left(\frac{x}{Br} - \xi, \frac{r}{r_0}\right) d\xi \right.$$

$$\left. - \sqrt{\frac{r_0}{r}}\, H\left(\frac{x}{Br_0} - \frac{r}{r_0} + 1\right) C_{2n}\left(\frac{x}{Br_0} - \frac{r}{r_0} + 1\right) + b_{2n}(x, r) \right\} \quad .\qquad \ldots\ldots (34)$$

On the body $r = r_0$, and Equation (33) becomes

$$(C_p)_{body} = \frac{4\delta}{\pi B \sinh\lambda} \sum_{n=0}^{\infty} \left\{ \int_0^{x/Br_0} C_{2n}(\xi)\, W_{2n}\left(\frac{x}{Br_0} - \xi, 1\right) d\xi \right.$$

$$\left. - H\left(\frac{x}{Br_0}\right) C_{2n}\left(\frac{x}{Br_0}\right) + b_{2n}(x, r_0) \right\} \cos 2n\theta. \qquad \ldots\ldots (35)$$

The pressure due to the wing alone has been written in the form $\frac{4\delta}{\pi B \sinh\lambda} \sum b_{2n}(x, r) \cos 2n\theta$ rather than in the closed form derived in Section 2, since experience shows that convergence of the Fourier series for the interference pressure alone is less satisfactory than convergence for the combined pressure.

4 RESULTS AND DISCUSSION

Equations (34) and (35) apply to inviscid supersonic flow past a wing-body combination consisting of two halves of a symmetrical delta wing of single-wedge section mounted symmetrically on a circular cylinder; the leading edges are subsonic, and the combination is at zero incidence. δ and r_0 appear as scaling factors only; 2δ is the angle of the wedge section, and r_0 is the radius of the circular cylinder. The remaining parameters are k and B; k is the tangent of the angle between the leading edge and the wing junction, and $B = \sqrt{M^2-1}$, where M is the free stream Mach number. Results have been obtained for B equal to $\sqrt{2}$ and k equal to 0.5.

In Figs. 1 to 4 inclusive $B(C_p)_{body}/\delta$ is plotted against x/Br_0 for four values of θ - 0, $\pi/6$, $\pi/3$, and $\pi/2$. The flow is undisturbed ahead of the Mach cones from the tips of the two wing-body junctions. These cones intersect the circular cylinder in helices, and it is easily shown that the flow should be undisturbed ahead of a value of x/Br_0 equal to 2 when $\theta = 0$, equal to $2 + \pi/6$ when $\theta = \pi/6$, equal to $2 + \pi/3$ when $\theta = \pi/3$, and equal to $2 + \pi/2$ when $\theta = \pi/2$; the effect of the other half wing ($\theta = \pi$) is first felt at a value of x/Br_0 equal to $2 + \pi$ when $\theta = 0$, equal to $2 + 5\pi/6$ when $\theta = \pi/6$, equal to $2 + 2\pi/3$ when $\theta = \pi/3$, and equal to $2 + \pi/2$ when $\theta = \pi/2$.

Only the first six terms of the Fourier series in Equation (35) have been used to obtain the curves of Figs. 1 to 4. This is why, in the figures, $(C_p)_{body}$ does not vanish ahead of the Mach cone from the junction tip. For example, for $\theta = \pi/2$, $(C_p)_{body}$ should be zero ahead of a value of x/Br_0 equal to roughly 3.57; instead, it oscillates about zero; fortunately, the amplitude of oscillation is small, which suggests that truncation of the Fourier series at the sixth term produces small errors only. Equation (35) apparently gives values for $(C_p)_{body}$ ahead of the section given by $\frac{x}{Br_0} = 2$; this is because the wing is assumed to be continued inside the cylinder. In reality, no disturbances can exist ahead of this section; and, in fact, Equation (35) with the Fourier series truncated at the sixth term does give values extremely close to zero for values of x/Br_0 less than 2.

At the wing-body junction itself the value of $(C_p)_{body}$ is known, since the flow locally is the same as that at the apex of a symmetrical delta wing of wedge section; from Equation (66) it is found that $B(C_p)_{body}/\delta = 0.968$ there. Bagley's approximate theory, therefore, gives the straight line $B(C_p)_{body}/\delta = 0.968$, it is shown in Fig. 1 as a dashed line. On the line $\theta = 0$, $r = r_0$, ahead of the point given by $\frac{x}{Br_0} = 2$, $(C_p)_{body}$ vanishes, so that there is a discontinuity in $(C_p)_{body}$ at this point. From the figure it is seen that Equation (35) with the Fourier series truncated at the sixth term does try to reproduce the discontinuity. For values of $\frac{x}{Br_0}$ other than zero there is no discontinuity in $(C_p)_{body}$.

These results do not call for discussion, and it seems appropriate to close with a few remarks on future work in the field of wing-body interference. The main difficulty in solving interference problems of this kind is still the large amount of computation required. The arrival of large digital electronic computers has made the solution of particular problems feasible; but it is unlikely that the effort could be spared to determine pressure distributions on a whole range of combinations. Indeed, the reason why pressure distributions on only the body have been calculated in this report is a reluctance to spend any more computational effort on the problem at the moment; to have used Equation (34) to find $(C_p)_{wing}$ would have put the publication of this report back by a considerable length of time.

The most promising way of reducing the amount of computational work is probably to develop Luke's technique[6]. He shows how to replace the functions by very simple approximations that contain exponential and circular inverses only. It seems possible that use of these approximations might lead to considerable simplifications in the formulas for the pressure, Equations (34) and (35).

ACKNOWLEDGMENT

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LIST OF SYMBOLS

$A_{2n}(p)$	arbitrary function of p
$a_{2n}(x, r)$	defined by Equations (17a) and (18a)
$a_{2n,0}(r)$	$a_{2n}(x, r_0)$
B	$(M^2-1)^{\frac{1}{2}}$
$b_{2n}(x, r)$	defined by Equations (17b) and (18b)
C_p	pressure coefficient
C_{2n}	$C_{2n}(\vartheta) = a_{2n}(B\ r_0\ \vartheta)$
H	unit function; $H(\xi) = 0$ if $\xi < 0$, $H(\xi) = 1$ if $\xi > 0$
k	tangent of half the apex angle of the wing
K_n	Bessel function of imaginary argument
M	Mach number of free stream
p	local pressure
p_∞	pressure in free stream
r	cylindrical polar coordinate, $(y^2 + z^2)^{\frac{1}{2}}$
r_0	radius of body
U	speed of free stream

LIST OF SYMBOLS (CONTD)

x, y, z	cartesian coordinates defined in Section 2
y_A, y_B	defined by Equations (5)
δ	2δ is the angle of the wedge section of the wing
θ	cylindrical polar coordinate, $\tan^{-1}(z/y)$
λ	$\cosh^{-1}(1/Bk)$
ρ_∞	density in free stream
ϕ	disturbance velocity potential of wing-body combination
ϕ_I	interference velocity potential
ϕ_W	disturbance velocity potential of wing alone
χ	$\cosh^{-1}(x/Br)$

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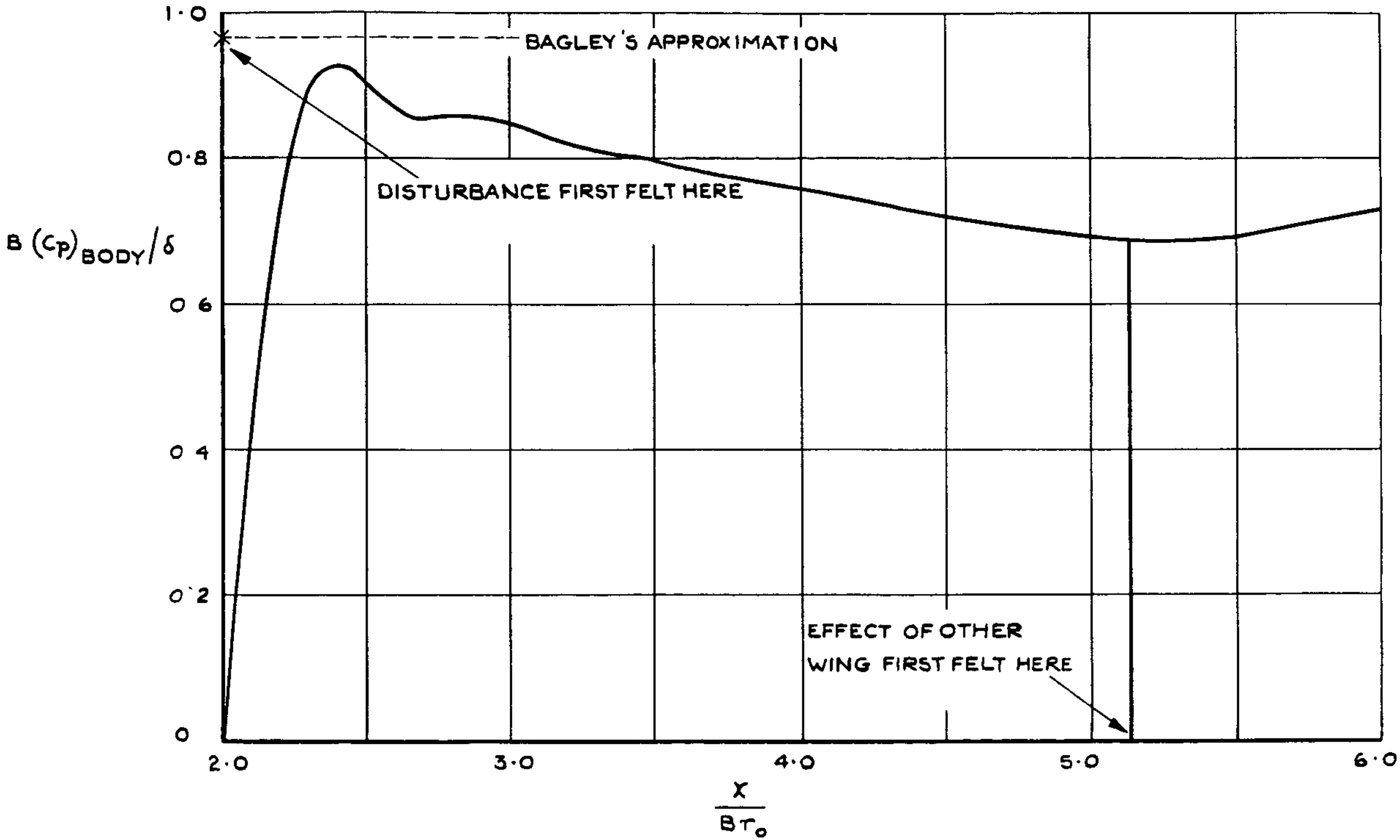

FIG. 1. PRESSURE COEFFICIENT ON THE BODY AT $\theta = 0$

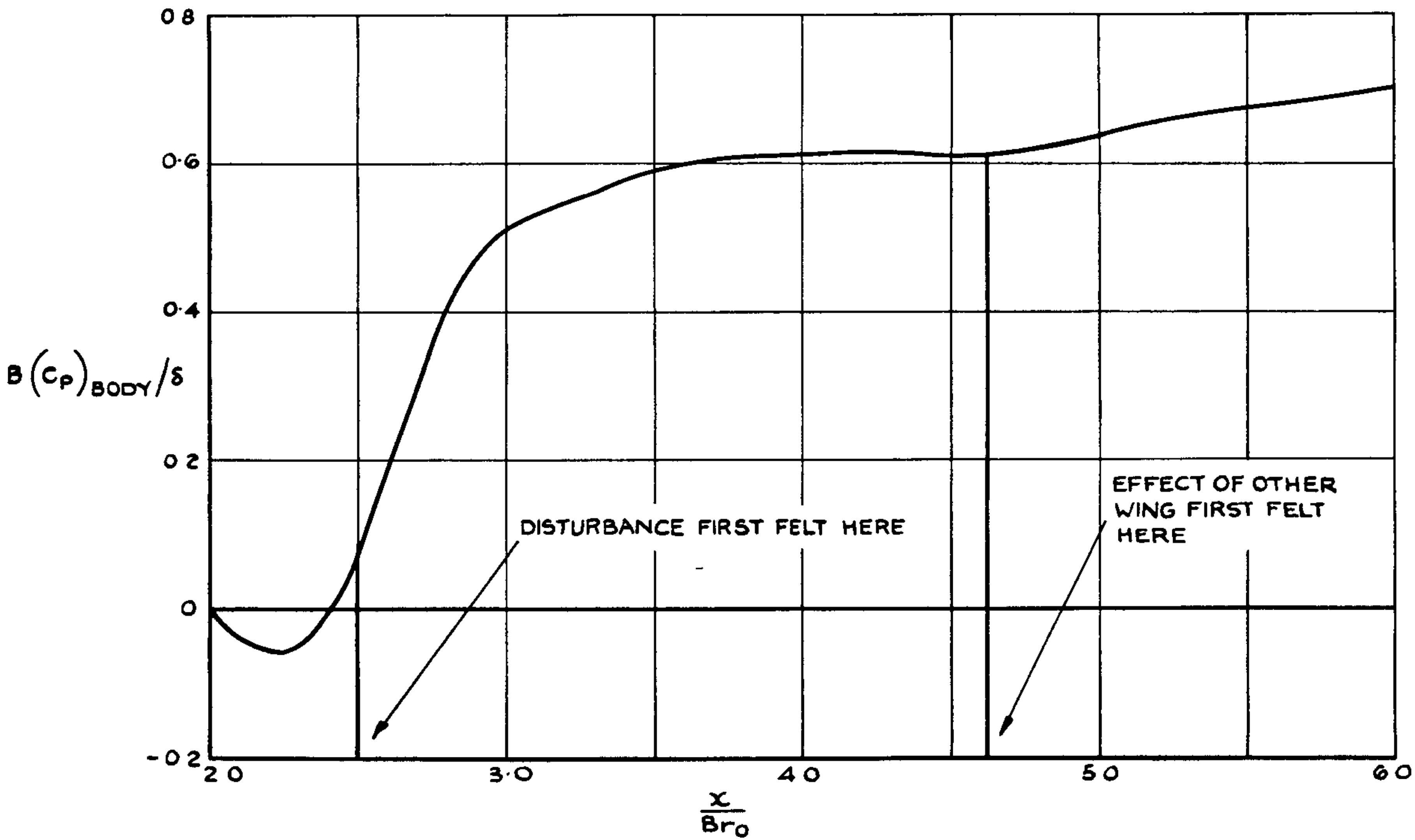

FIG. 2. PRESSURE COEFFICIENT ON THE BODY AT $\theta = \pi/6$.

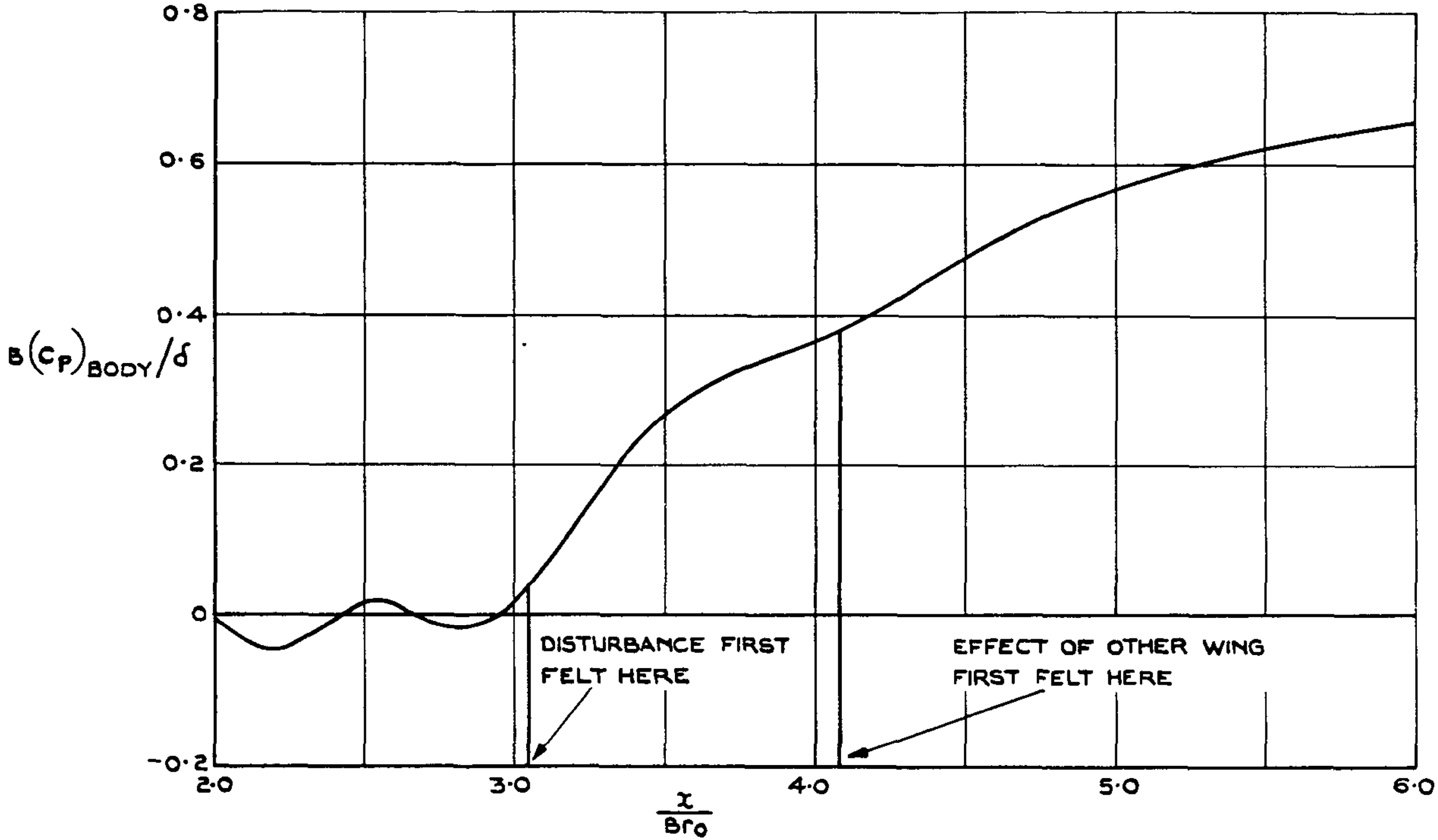

FIG. 3. PRESSURE COEFFICIENT ON THE BODY AT $\theta = \pi/3$

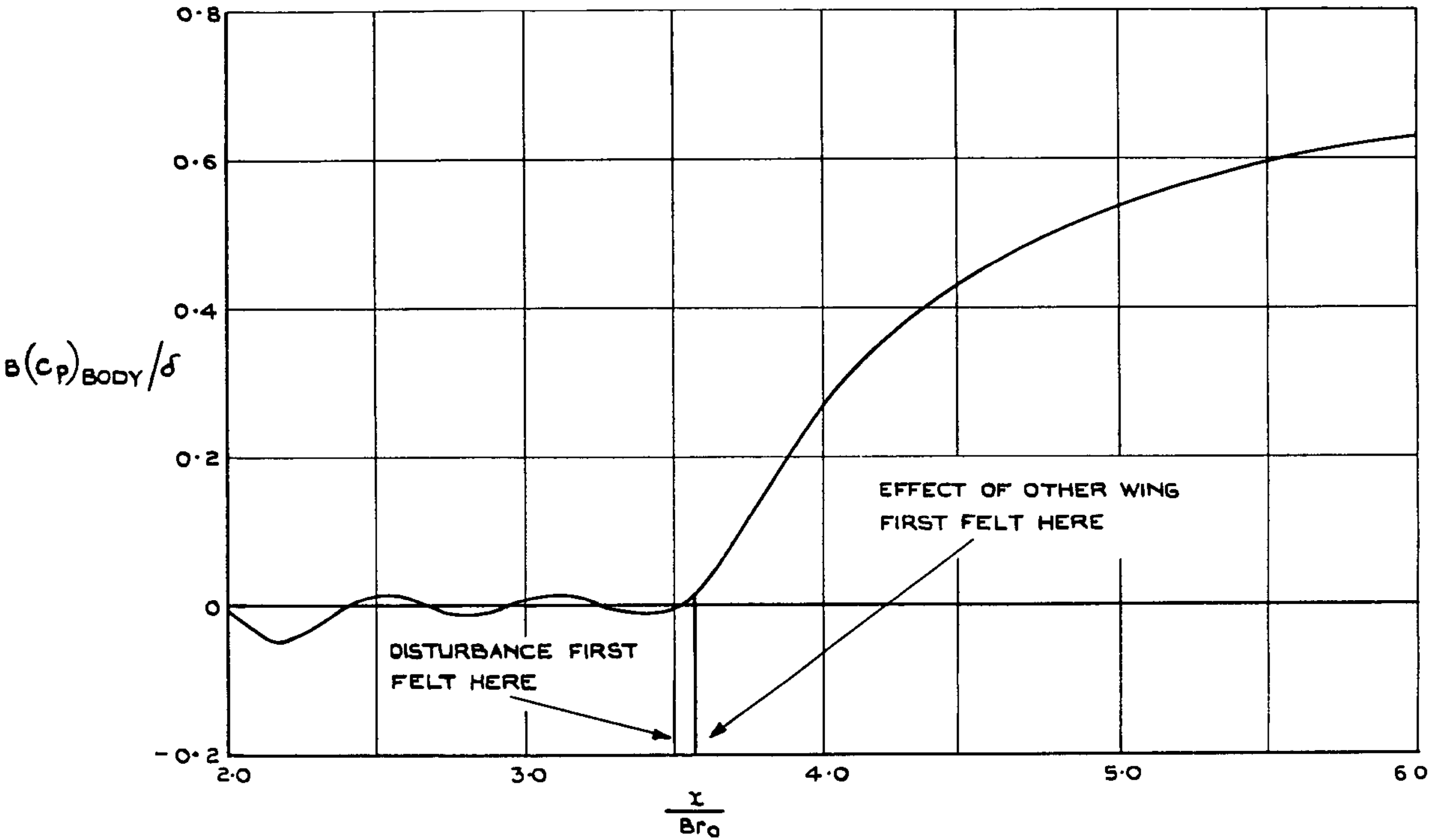

FIG. 4. PRESSURE COEFFICIENT ON THE BODY AT $\theta = \pi/2$

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533.6.011.5 :

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April 1963.

The report considers inviscid, supersonic flow past a symmetrical wing-body combination at zero incidence. The body is a circular cylinder extending indefinitely in both the upstream and the downstream directions. The wings are two halves of a delta wing of single-wedge section with a subsonic leading edge, they are assumed to extend indefinitely in the downstream direction. Results are obtained for the pressure distribution on the body.

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