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A Note on the Estimation of the Effect of
Wind Tunnel Walls on the Forces on Slowly Oscillating Slender Wings

By<br>W.EA. Acum

# A Note on the Estimation of the Effect of Wind Tunnel Walls on the Forces on Slowly Oscillating Slender Wings <br> - By - <br> W. E. A. Acum 

$\qquad$

April, 1963

## 1. Introduction

The formulae for tunnel interference upwash on small wings performing slow oscillations in closed rectangular or circular wand tunnels were derived some years ago (Refs.1 and 2). A "small wing" is one whose cross-stream and streamwise dimensions are all small compared wath those of the tunnel cross section; such a "small wing" is in fact merely an element of area wath its associated lift, and, within the assumptions of linearised theory, any wing may be regarded as made up of such lifting elements. In particular a slender wing on the tunnel axis is equivalent to a streamwise row of small wings, and this leads to a very simple method for calculating the interference upwash on a slender wing. As given the theory is for incompressible flow, but subsonic compressibility effects may be introduced by a minor alteration.

It must be observed that, besides the fact that the wing is slender, it is assumed that the frequency parameter of the oscillation is small and that the streamwise extent of the wing is small enough for the interference upwash to be regarded as varying linearly over the length of the wing. Moreover, the mean position of the wing is assumed to lie on the tunnel axis. Although the theory thus appears to be severely restricted these conditions are lakely to be satisfied in a signaficant number of experiments.

## 2. Notation

| A | aspect ratio of wing |
| :---: | :---: |
| b | breadth of rectangular tunnel |
| $\mathrm{C}_{L}$ | theoretical lift coefficient, Lift/( $\left.\frac{1}{2} \mathrm{pU}{ }^{2} \mathrm{~S}\right)$ |
| $\mathrm{C}_{\mathrm{L}}^{\prime}$ | measured value of $C_{L}$ |
| $\delta C_{L}$ | tunnel induced increment in $\mathrm{C}_{\mathrm{L}}$ |
| $C_{m}$ | theoretical pitching moment coefficient, (nose up pitching moment) $/\left(\frac{1}{2} \rho U^{2} \mathrm{~S}_{\mathrm{c}}^{-}\right)$ |


| $c_{\text {m }}^{\prime}$ | measured value of $C_{m}$ |
| :---: | :---: |
| $\delta C_{m}$ | tunnel induced increment to $\mathrm{C}_{\text {II }}$ |
| ${ }^{c}$ r | root chord of wing |
| $\overline{0}$ | mean chord of wing |
| h | height of rectangular tunnel |
| L | lift force per unit streamwise distance |
| $\ell$ | local lift coefficient, (local lift per unit area)/( $\frac{1}{2} \mathrm{pJ}^{2}$ ) |
| M | Mach number |
| S | area of planform |
| s(x) | local semi-span |
| $t$ | time |
| U | undisturbed wind velocity |
| w | component of flow velocity in the $z$ direction ("upwash") |
| $w_{i}$ | tunnel-induced upwash |
| $\mathrm{x}, \mathrm{y}, \mathrm{z}$ | rectangular co-ordinates |
| $\mathrm{x}_{0}$ | value of $x$ at pitching axis |
| $\beta$ | $\left(1-M^{2}\right)^{\frac{1}{2}}$ |
| $\theta=\theta_{0} e^{i \omega t}$ | angle of incidence |
| $\bar{\nu}$ | frequency parameter, $\omega \bar{c} / \mathrm{J}$ |
| $\rho$ | density |
| $\omega$ | angular frequency of oscillation |

## 3. Interference Upwash due to a Slender Wing in a Rectangular Tunnel

Consider a wind tunnel of closed rectangular section of height $h$, and breadth $b$, composed of solid surfaces in the planes $y= \pm \frac{1}{2} h$, and $z= \pm \frac{1}{2} \mathrm{~b}$ and extending from $x=-\infty$ to $x=+\infty$, where $x, y, z$ is a rectangular co-ordinate system. Now consider a wing performing harmonic oscillations but lying at all times near to the plane $\mathbf{z}=0$. According to linearised theory the total interference upwash is the sum of the interference upwashes due to each of the lifting elements into which the wing may be divided. In particular, if the wing is slender, it may be divided into a large number of
small elements by planes $x=\xi$; the element lying between $x=\xi$ and $x=\xi+\delta \xi$ will, since the span is small, have d工mensions small compared with the cross-section of the tunnel, and may therefore be treated by the theory for small wings given in Ref. 1 or the appendix to Ref. 2. Thus the interference upwash caused by a slowly oscillating small wing at the origin is

$$
\begin{equation*}
\frac{w_{i}(x, 0,0)}{U}=\frac{S C^{I}}{b h}\left\{\delta_{0}+\frac{x \delta_{1}}{h}+\frac{i \omega h}{U}\left[\delta_{0}^{\prime}+\frac{x \delta_{i}^{\prime}}{h}\right]\right\}+0\left(\frac{x}{h}\right)^{a} \tag{1}
\end{equation*}
$$

where $\delta_{0}, \delta_{1}, \delta_{0}^{\prime}$ and $\delta_{1}^{\prime}$ are constants depending on the shape of the tunnel, $b / h$. Tables of $\delta_{0}, \ldots \delta_{1}^{\prime}$ may be found in Ref.2. It is assumed. that $\omega h / \sigma$ and $x / h$ are sma11. Equation (1) applies to incompressible flow, but compressibility may be accounted for by replacing $\delta_{f}$ in equation (1) by $\delta_{1} / \beta$ and $\delta_{0}^{\prime}$ by $\delta_{0}^{\prime} / \beta$, where $\beta=\left(1-\mathbb{N}^{2}\right)^{\frac{1}{2}}$.

When applied to that part of the slender wing between $x=\xi$
and $x=\xi+\delta \xi$ equation (1) becomes
$\frac{\delta w_{1}(x, 0,0)}{U}=\frac{1}{b h}\left[\int_{-s(\xi)}^{s(\xi)} \ell(\xi, y) d y\right] \cdot\left[\delta_{0}+\frac{(x-\xi) \delta_{1}}{h}+\frac{i \omega h}{U}\left(\delta_{0}^{\prime}+\frac{\left(x-\zeta_{0}\right) \delta_{1}^{\prime}}{h}\right)\right] \delta \xi \cdot$

The total interference upwash follows by integration

$$
\begin{align*}
& \frac{w_{i}(x, 0,0)}{U}=\frac{S}{b h}\left[C_{I_{i}}\left\{\delta_{0}+\frac{x \delta_{1}}{h}+\frac{i \omega h}{U}\left(\delta_{0}^{\prime}+\frac{x \delta_{1}^{\prime}}{h}\right)\right\}\right. \\
&\left.+C_{m}^{\bar{c}}\left\{\delta_{1}+\frac{i \omega h \delta_{1}^{\prime}}{U}\right\}\right] \tag{3}
\end{align*}
$$

In equation (3) $C_{m}$ refers to the pitohing moment about $x=0$.
The behaviour of $W_{i} / J$ when $x$ is not small is considered briefly in the Appendix.

When the flow is steady it is possible to use the more elaborate formula derived by Berndt ${ }^{3}$ for slender wings in rectangular tunnels. In Berndt's expression corresponding to equation (1) $\omega$ is, of course, zero, and it is not assumed that $w_{i}$ is linear in $x_{\text {. }}$
4. Calculation of Interference Forces

In equation (3) $w_{i}$ is the additional upwash caused by the presence of the tunnel walls. In fact the upwash at the wing is dictated by its mode of oscillation, and we may therefore regard the tunnel walls as supplying $w_{i}$ towards the total upwash, thus reducing by $w_{i}$ the part to be supplied by the lift distribution over the wing. The incremental forces due to the turnel interference are therefore obtained from linearised theory by taking the prescribed upwash at the wing as $-w_{i}$.

From the formulae of slender wing theary (Ref.4) the lift per unit length in the x direction is given by

$$
L(x)=-\pi \rho U^{2}\left\{s^{2}\left[\frac{\partial}{\partial x}+\frac{i \omega}{U}\right]^{W} \frac{W}{U}+2 s \frac{d s}{d x} \frac{w}{U}\right\}, \quad \ldots \text { (4) }
$$

where $w$ is the upwash prescribed by the motion of the wing. Equation (4) may be used to calculate the incremental lift due to tunnel interference by taking $w / J$ equal to $-w_{i} / J$ as given by equation (3).

In the absence of direct experimental values, or more accurate theoretical values, the complex quantities $C_{L}$ and $C_{\text {II }}$ an equation (3) may also be estimated by (4).

## 5. Tunnels of Other Cross Sections

Values of $\delta_{0}, \delta_{1}, \delta_{0}^{\prime}$ and $\delta_{1}^{\prime}$ for a small wing on the axis of a closed circular tunnel are given in Ref.2. Equation (1) is unchanged except that $h$, taken as the typical length in (1), must be replaced by the diameter, and bh replaced by the area of cross section of the circular tumel.

For the circular tumnel Goodman ${ }^{5}$ has extended equation (1) to cover all frequenoy parameters and streanwise positions, but the equation for $w_{i}$ is naturally more complicated and the integration leading to equation (3) would no longer be so simple.

## 6. Example

Consider a slender triangular wing pitching about an axis through its in-phase centre of pressure, in a low-speed tunnel for which $b / h=977$. The length of the root-chord, $c_{r}$, will be assumed to be equal to the tunnel height, $h$. The aspect ratio, A, will be assumed small but otherwise left unspecified. Denote the pitching axis by $x=x_{0}$. Then the equation of the wing surface is

$$
\begin{equation*}
z=-\theta_{0} e^{i \omega t}\left(x-x_{0}\right), \tag{5}
\end{equation*}
$$

and the upwash angle at the wing is therefore

$$
\begin{equation*}
\frac{w}{U}=-\theta_{0} e^{i \omega t}\left[1+\frac{i \omega\left(x-x_{0}\right)}{U}\right] . \tag{6}
\end{equation*}
$$

For a delta wing take the origin of co-ordinates at the apex so that $s(x)=A x / 4$. Then substitution in equation (4) yields the following approximate theoretical lift and pitching moment,

$$
\left.\begin{array}{l}
c_{L}=\frac{\pi A}{2} \theta_{0} e^{i \omega t}\left[1+\frac{i \omega o_{r}}{0}\left(\frac{4}{3}-\frac{x_{0}}{c_{r}}\right)\right], \\
c_{m}=-\pi A \theta_{0} e^{i \omega t}\left[\frac{2}{3}+\frac{i \omega c_{r}}{U}\left(1-\frac{2 x_{0}}{3} \frac{c_{r}}{U}\right)\right], \tag{7}
\end{array}\right\}
$$

where $C_{m}$ is referred to $x=0$.
It follows that the in-phase centre of pressure is at $x=4 \bar{c} / 3$, so that $x_{0}=4 \bar{c} / 3=2 c_{r} / 3$, and equations (7) become

$$
\left.\begin{array}{l}
C_{L}=\pi A \theta_{0} e^{i \omega t}\left[\frac{1}{2}+\frac{1}{3} \frac{i \omega c_{r}}{U}\right],  \tag{8}\\
C_{m}=-\pi A \theta_{0} e^{i \omega t}\left[\frac{2}{3}+\frac{5 i \omega c_{r}}{9}\right] \cdot
\end{array}\right\}
$$

Now from Table AII of Ref.2, for $b / h=9 / 7$

$$
\left.\begin{array}{l}
\delta_{0}=0.120390,  \tag{9}\\
\delta_{1}= \\
\delta_{0}^{\prime}=-0.228247, \\
\delta_{1}^{\prime}=--0.1202224, \\
\hline
\end{array}\right\}
$$

It has been assumed that $h=2 \bar{c}=c_{r}$, so that $b=9 \mathrm{~h} / 7=18 \bar{c} / 7$, and for delta wings $S=A \overline{A C}^{2}$. Then from equations (3) and (9)
$\frac{W_{i}}{U}=\frac{7 A}{36}\left[C_{L}\left\{\left(0.120390+0.114124 \frac{x}{\bar{c}}\right)-i \bar{\nu}\left(0.040448+0.120390 \frac{x}{\bar{c}}\right)\right\}\right.$

$$
\begin{equation*}
\left.+C_{m}\{0.114124-i \bar{\nu} 0.120390\}\right], \tag{10}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{\nu}=\frac{\omega \bar{\omega}}{\bar{u}} \tag{11}
\end{equation*}
$$

and it has been assumed that $\bar{\nu}$ is small.
From equation (4) it may be deduced that the incremental forces due to tunnel interference are

$$
\begin{equation*}
\delta C_{L}=\frac{\pi A}{2}\left(\frac{w_{1}}{U}\right)_{x=c_{r}}+\frac{i \omega}{U} \frac{\pi A}{2} \frac{1}{c_{r}^{2}} \int_{0}^{c_{r}} x^{2} \frac{w_{i}}{U} d x, \tag{12}
\end{equation*}
$$

and $\delta c_{m}=-\pi A\left(\frac{w_{i}}{U}\right)_{x=0_{r}}+\pi A \frac{1}{c_{r}^{3}} \int_{0}^{c_{r}} x^{2} \frac{w_{i}}{U} d x-\frac{i \omega}{U} \pi A \frac{1}{c_{r}^{3}} \int_{0}^{c_{r}} x^{3} \frac{w_{i}}{U} d x$.

Then $\delta \mathrm{C}_{\mathrm{L}}$ and $\delta \mathrm{C}_{\text {m }}$ are obtained numerically by substituting from equations (8) into equation (10), and then from (10) into (12) and (13). After some calculation we find
and

$$
\left.\begin{array}{l}
\delta C_{L}=\pi A^{3} \theta_{0} e^{i \omega t}\{0.030004+i \bar{\nu} 0.028053\}  \tag{14}\\
\delta C_{m}=-\pi A^{3} \theta_{0} e^{i \omega t}\{0.045815+i \bar{\nu} 0.043130\}
\end{array}\right\}
$$

where $\delta C_{m}$ is referred to $x=0$.

Now $\delta C_{L}$ and $\delta C_{m}$ are those parts of the measured lift and moment due to tunnel interference. The obvious way of applying tunnel corrections is therefore to subtract $\delta C_{L}$ and $\delta C_{m}$ from the measured $C_{L}$ and $C_{m}$ respectively; if this is done the measured values of $C_{L}$ and $C_{m}$ should be used in equation (3) to obtain $w_{i}$.

Alternatively the correction may be regarded primarily as one to incidence with a resldual correction to pitching moment. Let the measured values of $C_{L}$ and $C_{m}$ be $C_{L}^{\prime}$ and $C_{m}^{\prime}$ while undashed symbols represent theoretical estimates. Thus from the method described in Ref. 1 the correction to be added to the incidence is

$$
\begin{equation*}
\Delta \theta=C_{L}^{\prime} \frac{\partial \delta C_{L}}{\partial \theta} /\left(\frac{\partial C_{L}}{\partial \theta}\right)^{2}, \tag{15}
\end{equation*}
$$

with a residual correction to pitching moment

$$
\begin{equation*}
\Delta C_{m}=\left(C_{m}^{\prime}\right)_{\text {corrected }}-\left(C_{m}^{\prime}\right)_{\text {measured }}=\frac{C_{L}^{\prime}}{\left(\frac{\partial C_{L}}{\partial \theta}\right)^{2}}\left\{\frac{\partial C_{m}}{\partial \theta} \frac{\partial \delta C_{L}}{\partial \theta}-\frac{\partial C_{L}}{\partial \theta} \frac{\partial \delta C_{m}}{\partial \theta}\right\} \tag{16}
\end{equation*}
$$

The suffices $I$ and $m$ may be interchanged throughout equations (15)
and (16), that is the correction to incidence may be made to depend on pitching moment. This has to be done if only $\mathrm{C}_{\mathrm{m}}^{\prime}$ is measured.

Up to this point it has been assumed that the pitching moments are taken about $\mathbf{x}=0$; it is more usual to refer them to the pitching axis. This does not affect the form of equation (15), nor, in fact, does a change of axis which is the same for both $C_{m}$ and $\delta C_{m}$, alter the value of $\Delta C_{m}$. In the present example, a triangular wing, it was assumed that $x_{0}=4 \overline{0} / 3$, and the theoretical tunnel induced and free-stream pitching moments referred to this axis are

$$
\begin{align*}
\delta C_{m} & =-\pi A^{3} \theta_{0} e^{i \omega t}[0.005810+i \bar{\nu} 0.005726]  \tag{17}\\
C_{m} & =-\pi A \theta_{0} e^{i \omega t} \frac{2}{9} i \bar{\nu} \tag{18}
\end{align*}
$$

while $\delta C_{L}$ and $C_{L}$ are, of course, unchanged. Then, from the first of equations (8), the first of equations (14), and equation (15), it follows that

$$
\begin{equation*}
\Delta \theta=C_{L}^{\prime} A(0.038202-i \bar{\nu} 0.066155), \tag{19}
\end{equation*}
$$

and from equations (17), (18) and (16) that

$$
\begin{equation*}
\Delta C_{\mathrm{m}}=\mathrm{C}_{\mathrm{L}}^{\prime} \mathrm{A}(0.01162-i \bar{\nu} 0.01909) . \tag{20}
\end{equation*}
$$

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## APPENDIX

The Variation of Interference Upwash with Streamwise Distance for Small Wings in Rectangular Tunnels

From the treatment of the small wing given in Ref. 1 it follows that the interference upwash for such a wing in a rectangular tunnel is

$$
\begin{gather*}
\frac{w_{i}(x, 0,0)}{U}=\frac{S C^{L}}{2 b^{2}}\left[F_{2}(\xi, 0)-\frac{i \omega b}{U} \exp \left(\frac{-i \omega x}{U}\right) \int_{-\infty}^{x / b} \exp \left(\frac{i \omega b \theta}{U}\right) F_{2}(\theta, 0) d \theta\right], \\
\text { where } F_{a}(\xi, 0)=\frac{-b^{2}}{4 \pi h^{2}} \sum_{m=-\infty}^{m=+\infty} \sum_{n=-\infty}^{n=+\infty}(-1)^{n}\left[\frac{n^{2}-Y^{2}}{\left(n^{2}+Y^{2}\right)^{2}}\left(1+\frac{X}{\sqrt{X^{2}+Y^{2}+n^{2}}}\right)\right. \\
\left.+\frac{n^{2}}{Y^{2}+n^{2}} \frac{\left.X^{2}+Y^{2}+n^{2}\right)^{3 / 2}}{\left(X^{2}\right.}\right],
\end{gather*}
$$

where $X=\mathrm{b} \xi / \mathrm{h}, \mathrm{Y}=\mathrm{mb} / \mathrm{h}$, and the term in $\mathrm{m}=\mathrm{n}=0$ is omitted from the double summation.

$$
\text { For small } x \text { expansion of equation (A.2) as a power series in }
$$ x gives

$$
\begin{align*}
F_{2}(\xi, 0)= & \frac{-b^{2}}{4 \pi h^{2}} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty}(-1)^{n}\left[\frac{n^{2}-Y^{2}}{\left(n^{2}+Y^{2}\right)^{2}}+\frac{X}{h} \frac{2 n^{2}-Y^{2}}{\left(n^{2}+Y^{2}\right)^{5 / 2}}\right. \\
& \left.-\frac{x^{3}}{h^{3}} \frac{2 n^{2}-\frac{1}{2} Y^{2}}{\left(n^{2}+Y^{2}\right)^{5 / 2}}+\cdots\right], \tag{A.3}
\end{align*}
$$

provided $x$ is less than the smaller of $b$ or $h$. The first two terms lead to $\delta_{0}$ and $\delta_{1}$, which are already known, but obviously higher terms could be included. If $\omega \mathrm{b} / \mathrm{J}$ is small the second term in the right-hand side of equation (A.1) may be replaced by

$$
-\frac{\dot{i} \omega \mathrm{~b}}{\mathrm{U}} /
$$

$$
\begin{align*}
& -\frac{i \omega b}{U} \int_{-\infty}^{x / b} F_{a}(\theta, 0) d \theta \\
= & \frac{i \omega b}{U} \frac{b}{4 \pi h} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty}(-1)^{n}\left[\frac{n^{2}-Y^{2}}{\left(n^{2}+Y^{2}\right)^{2}}\left(\frac{x}{h}+\sqrt{\left(\frac{X}{h}\right)^{2}+Y^{2}+n^{2}}\right)\right. \\
& \left.-\frac{n^{2}}{n^{2}+Y^{2}} \sqrt{\left(\frac{X}{h}\right)^{2}+Y^{2}+n^{2}}\right] \\
= & \frac{1 \omega h}{U} \frac{b^{2}}{4 \pi h^{2}} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty}(-1)^{n}\left[\frac{-Y^{2}}{\left(n^{2}+Y^{2}\right)^{3 / 2}}+\frac{x}{h} \frac{n^{2}-Y^{2}}{\left(n^{2}+Y^{2}\right)^{2}}+0\left(\frac{x^{3}}{h^{3}}\right)\right] . \tag{A.4}
\end{align*}
$$

From a comparison of equations (A.3) and (A.4) with equation (1), it follows that

$$
\left.\begin{array}{rl}
\delta_{0}=-\delta_{1}^{\prime} & =-\frac{1}{8 \pi} \frac{b}{h} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty}(-1)^{n} \frac{n^{2}-m^{2} b^{2} / h^{2}}{\left(n^{8}+m^{2} b^{2} / h^{2}\right)^{2}}, \\
\delta_{1} & =-\frac{1}{8 \pi} \frac{b}{h} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty}(-1)^{n} \frac{2 n^{2}-m^{2} b^{2} / h^{2}}{\left(n^{2}+m^{2} b^{2} / h^{2}\right)^{5 / 2}},  \tag{A.5}\\
\delta_{0}^{\prime} & =-\frac{1}{8 \pi} \frac{-}{h} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty}(-1)^{n} \frac{m^{2} b^{3} / h^{2}}{\left(n^{2}+m^{2} b^{2} / h^{2}\right)^{3 / 2}},
\end{array}\right\}
$$

In the double summations the term $\mathrm{m}=\mathrm{n}=0$ is to be omitted.
The series for $\delta_{0}$ and $\delta_{1}$ are the well-known sums for a small
wing in steady flow. The double series for $\delta_{0}^{\prime}$ is not convergent; it does give a sum if summed first with respect to $n$ and then with respect to $m$,
and this sum is the same as that obtained by a numerical integration of $\int_{-\infty}^{0} \mathrm{~F}_{-\infty}(\theta, 0) \mathrm{d} \theta$, and is therefore, as it happens, correct.

The ooefficients of higher powers of $x / h$ depend on sums of the type

$$
\begin{equation*}
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty}(-1)^{n} \frac{c_{1} n^{2}+c_{2} n^{2} b^{3} / h^{2}}{\left[n^{2}+m^{2} b^{2} / h^{2}\right]^{p+\frac{1}{2}}} \tag{A.6}
\end{equation*}
$$

where $p$ is an integer and $c_{1}$ and $c_{2}$ are constants. Now consider

$$
\begin{equation*}
S_{p}(x)=\sum_{-\infty}^{\infty}(-1)^{n} \frac{x}{\left(n^{2}+x^{n}\right)^{p+\frac{1}{2}}} \tag{A.7}
\end{equation*}
$$

By Poisson's formula (Ref.6) if $f(x)$ is continuous and of bounded variation in $0 \leqslant x<\infty, f$ tends to zero as $x$ tends to infinity, and $\int_{0}^{\infty} f(t) d t$ exists then

$$
\frac{1}{2} f(0)+\sum_{n=1}^{\infty} f(n)=\int_{0}^{\infty} f(t) d t+2 \sum_{n=1}^{\infty} \int_{0}^{\infty} f(t) \cos 2 \pi n t d t
$$ ... (A.8)

To apply this to $S_{p}$ take

$$
\begin{equation*}
f(n)=\frac{\cos \pi n_{0} x}{\left(n^{2}+x^{2}\right)^{p+\frac{1}{2}}} \tag{A.9}
\end{equation*}
$$

so that

$$
\int_{0}^{\infty} /
$$

$$
\int_{0}^{\infty} f(t) \cos 2 \pi n t d t=\int_{0}^{\infty} \frac{\cos 2 \pi n t \cos \pi t_{0} x}{\left(t^{2}+x^{2}\right)^{p+\frac{1}{2}}} d t, \quad \ldots(\text { A. 10) }
$$

and it follows that

$$
\begin{equation*}
S_{p}(x)=4 x \sum_{n=1}^{\infty} \int_{0}^{\infty} \frac{\cos [(2 n-1) \pi t]}{\left(t^{2}+x^{2}\right)^{p+\frac{1}{2}}} d t \tag{A.11}
\end{equation*}
$$

The integrals in this summation may be expressed as Bessel functions (Ref.7) and it follows that

$$
S_{p}(x)=\frac{2 \pi^{p}}{(2 x)^{p-1}} \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(p+\frac{1}{2}\right)} \sum_{n=1}^{\infty}(2 n-1)^{p} K_{p}((2 n-1) \pi x) \ldots(A \cdot 12)
$$

This transformed series is rapidly convergent unless $x$ is small. If $x$ is small and positive,

$$
\begin{aligned}
s_{p}(x)= & \frac{1}{x^{n}}+2 \sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^{2 p+1}} x \\
& -(2 p+1) \sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^{2 p+3}} x^{3}+\frac{(2 p+1)(2 p-1)}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^{2 p+5}} x^{5}
\end{aligned}
$$

Thus $S_{p}(x)$ is easily calculated for all $x$, and equation ( $A_{0} 12$ ) shows that it tends to zero exponentially as $x$ tends to infinity. Thus sums of the form (A.6) are also easily evaluated. The treatment when the factor $(-1)^{n}$ is omitted from equation (A.7) is analogous.

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