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By

Doris E Lehrian, B.Sc and Gillian Smart

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Theoretical Stability Derivatives for a Symmetrically Tapered Wing at Low Supersonic Speeds

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SUMMARY

A symmetrically tapered wing of aspect ratio 4 and taper ratio $\frac{5}{11}$ with streamwise tips describes simple harmonic oscillations of low frequency in supersonic flow. Pitching and heaving derivatives are evaluated to first order in frequency, on the basis of linearized thin-wing theory, for Mach numbers in the range $1{\cdot}017 \leqslant M < 2{\cdot}50$. Comparisons are made with oscillatory strip theory and steady sonic theory.

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1. Introduction

The aerodynamic forces acting on oscillating wings of hexagonal planform are considered on the basis of linearized thin-wing theory in R. & M.3298[1]. To first order in frequency, the exact aerodynamic loading is formulated in Ref.1 for wings describing simple harmonic pitching oscillations. The formulae of Ref.1 apply to hexagonal wings having supersonic, or sonic, leading and trailing edges; an additional lower limit on Mach number is imposed by the condition that the two tip regions are non-interacting.

In this report stability derivatives are evaluated for a wing planform of aspect ratio 4 with streamwise tips and a symmetrical taper of ratio $\frac{5}{11}$. This planform is included in experimental programmes for pitching and heaving derivatives at transonic and supersonic Mach numbers up to 2·8. The present calculations provide some results by linearized theory for supersonic Mach numbers in the range 1·017 to 2·50. For the lowest Mach numbers, linearized theory does not apply to wings of practical thickness ratio and other non-linear effects will arise in the experiments; then the theoretical derivatives are not expected to agree with the measured values, but should be regarded as the first stage of a semi-empirical procedure.

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2. Evaluation of Derivatives

A symmetrically tapered wing with streamwise tips is classified in Ref.1 as case (iii); expressions for the lift and pitching-moment derivatives for low frequency pitching oscillations are formulated in Section 3 of that report. The leading and trailing edges of the wing are supersonic or sonic when $\beta \tan\lambda \geq 1$; that is, $\beta \geq \frac{3}{16}$ for the wing planform defined in Table 1. In the present report, solutions are evaluated for the eight Mach numbers corresponding to $\beta = [M^2 - 1]^{\frac{1}{2}} = \frac{3}{16}, \frac{11}{32}, \frac{1}{2}, \frac{3}{4}, \frac{9}{8}, \frac{3}{2}, \frac{15}{8}, \frac{9}{4}$. The values $\frac{11}{32}, \frac{1}{2}$ correspond to the two particular cases where the planform is divided conveniently by the Mach lines from the wing apex, as shown in Fig.1. At the lowest Mach number of 1·017 when the leading and trailing edges are sonic, the condition for non-interacting tips in equation (20) of Ref.1 is still satisfied.

The pitching derivatives ℓ_θ, $\ell_{\dot\theta}$, m_θ, $m_{\dot\theta}$ are defined according to the notation of Ref.1. Thus, for pitching oscillations of angular frequency ω and amplitude θ_0 about an axis $h_0 c_0$ downstream of the wing apex,

$$\left.\begin{aligned} \text{Lift} &= \rho_\infty U_\infty^2 S[\ell_\theta + i\nu_0 \ell_{\dot\theta}]\theta_0 e^{i\omega t} \\ &\text{Pitching moment about axis } h_0 c_0 \\ &= \rho_\infty U_\infty^2 S\, c_0[m_\theta + i\nu_0\, m_{\dot\theta}]\,\theta_0 e^{i\omega t} \end{aligned}\right\}, \qquad \ldots (1)$$

where

$$\left.\begin{aligned} c_0 &= \text{root chord} \\ S &= \text{area of wing planform} \\ \nu_0 &= \text{frequency parameter} = \omega c_0/U_\infty \end{aligned}\right\}.$$

The pitching derivatives can be expressed in terms of h_0 and the pitching and heaving derivatives corresponding to the reference axis $h_0 = 0$, i.e.,

$$\left.\begin{aligned} \ell_\theta(h_0) &= \ell_\theta(0) - h_0\ell_z(0) \\ \ell_{\dot\theta}(h_0) &= \ell_{\dot\theta}(0) - h_0\ell_{\dot z}(0) \\ m_\theta(h_0) &= m_\theta(0) + h_0[\ell_\theta(0) - m_z(0)] - h_0^2\ell_z(0) \\ m_{\dot\theta}(h_0) &= m_{\dot\theta}(0) + h_0[\ell_{\dot\theta}(0) - m_{\dot z}(0)] - h_0^2\ell_{\dot z}(0) \end{aligned}\right\}. \qquad \ldots (2)$$

Here, the heaving derivatives $\ell_z(0)$, $\ell_{\dot z}(0)$, $m_z(0)$, $m_{\dot z}(0)$ are defined by equation (1) with $h_0 = 0$ and θ_0 replaced by (z_0/c_0), the amplitude of the heaving oscillation. To first order in frequency parameter ν_0, these derivatives are given by the relations

$\ell_z(0)$/

$$\left.\begin{aligned} \ell_z(0) &= m_z(0) = 0 \\ \ell_{\dot z}(0) &= \ell_\theta(0) \\ m_{\dot z}(0) &= m_\theta(0) \end{aligned}\right\} \qquad \ldots (3)$$

The pitching derivatives $\ell_\theta(0)$, $\ell_{\dot\theta}(0)$, $m_\theta(0)$, $m_{\dot\theta}(0)$ are expressed, in equation (29) of Ref.1, as the sum of integrals of the velocity potential functions F_{mJ} (m = 0 and 1) over each region J = A, B ... H on the half-wing. The regions of integration J which occur in the present calculations are defined for four Mach numbers in Fig.1. The formulae for F_{mJ} corresponding to supersonic and sonic leading-edges are presented in respective Appendices B and C of Ref.1. In this note, the pitching derivatives for $h_0 = 0$ were evaluated for each particular Mach number by integrating analytically the appropriate functions F_{mJ} over the regions J of the half-wing. The values of these pitching derivatives are tabulated in Table 2 for the eight Mach numbers in the range $1{\cdot}017 \leqslant M \leqslant 2{\cdot}462$.

The root chord c_0 is used for the definition of the derivatives in equation (1). However, the derivatives are often required in terms of the geometric mean chord $\bar{c}$ or the aerodynamic mean chord $\bar{\bar{c}}$ which are defined in equation (49) of Ref.1. Each pitching derivative can be converted to any reference length d, say, by multiplying by the appropriate factor in the following table.

ℓ_θ	$\ell_{\dot\theta}$	m_θ	$m_{\dot\theta}$
1	c_0/d	c_0/d	$(c_0/d)^2$

The conversion factors corresponding to $d = \bar{c}$ and $\bar{\bar{c}}$ are given in Table 1.

3. Discussion of Results

Variation of the stability derivatives over the Mach number range $1{\cdot}017 \leqslant M \leqslant 2{\cdot}462$ is shown in Figs.2 to 8. The calculated lift and pitching moment derivatives for the mid-chord pitching axis $h_0 = 0{\cdot}5$ are plotted against Mach number in Figs.2 to 6. The curves drawn through the calculated values are suggested by interpolation, but may be inaccurate for the lower values of M. From Table 2 it can be seen that the variation of the stability derivatives with Mach number decreases rapidly in the range $1{\cdot}250 \leqslant M \leqslant 2{\cdot}462$. This is illustrated in Fig.3, where values of the lift derivative ℓ_θ are plotted against Mach number. The variation of the damping derivative $-m_{\dot\theta}$ with axis position is shown in Fig.7 for $M \leqslant 1{\cdot}25$ and in Fig.8 for $M \geqslant 1{\cdot}25$. The variation of $-m_{\dot\theta}$ with h_0 becomes less marked as Mach number increases.

The/

The use of the two-dimensional supersonic solution (Ref.2, Section 5.3) on a strip-theory basis leads to simple formulae for the evaluation of stability derivatives. In the case of a planform with streamwise tips and symmetrical taper, the pitching derivatives for the mid-chord axis $h_o = 0{\cdot}5$ are as follows:-

$$\left.\begin{aligned} \ell_\theta &= \frac{2}{\beta} \\ \ell_{\dot\theta} &= -\frac{1}{\beta^3}\left(\frac{\bar{\bar{c}}}{c_0}\right) = -\frac{2}{3\beta^3}\left(\frac{1+\tau+\tau^2}{1+\tau}\right) \\ m_\theta &= 0 \\ m_{\dot\theta} &= \frac{1}{12}\left(\frac{1-\beta^2}{\beta^3}\right)(1+\tau^2) \end{aligned}\right\} \qquad \ldots (4)$$

where τ = tip chord/root chord = $\frac{5}{11}$. Values of these derivatives are shown in Figs.2 to 6 for $M \geq 1{\cdot}15$. For the present planform of moderate aspect ratio and taper ratio, strip theory gives a useful rough estimate of all four derivatives at $M = 1{\cdot}25$. For $M > 2$, it is found that strip theory gives a fairly good approximation to the pitching derivatives; for $h_o = 0$ the stiffness derivatives from strip theory are within $4\frac{1}{2}$% of the exact-theory values, while the corresponding discrepancies in the damping derivatives are within $\frac{1}{2}$%. The variation of $-m_{\dot\theta}$ with axis position h_o is shown in Fig.8 to be a satisfactory approximation to the exact theoretical curves for $M \geq 1{\cdot}803$.

At $M = 1$, the stability derivatives ℓ_θ and m_θ can be determined by linearized theory for steady sonic flow[3]. For a symmetrically tapered wing with streamwise tips of aspect ratio A, the stiffness derivatives referred to mid-chord axis are obtained as

$$\left.\begin{aligned} \ell_\theta &= \tfrac{1}{4}\pi A \\ m_\theta &= \tfrac{1}{24}\pi A(1+2\tau) \end{aligned}\right\} . \qquad \ldots (5)$$

Values of ℓ_θ and m_θ for the present planform at $M = 1$, are plotted in Figs.2 and 5 and correlate satisfactorily with those for the lowest supersonic Mach number.

In Fig.7, the damping derivative $-m_{\dot\theta}$ is plotted against axis position h_o for the four values of $M = 1{\cdot}017$, $1{\cdot}057$, $1{\cdot}118$ and $1{\cdot}250$.

There/

There is considerable variation of $-m_{\dot\theta}$ with h_o, especially at the lower Mach numbers. When $M = 1{\cdot}017$, negative damping only appears for pitching axes near the wing apex. As M increases, the corresponding axis position for zero damping moves rapidly downstream to about mid-chord; for axis positions forward of this the magnitude of the negative damping decreases rapidly as M increases from 1·057. The results show a similar trend to those in Fig.10(a) of Ref.1 for another symmetrically tapered wing ($A = 4{\cdot}33$, $\tau = 0{\cdot}266$). As M increases above 1·25, in Fig.8, negative damping tends to disappear. By equations (2) to (4), strip theory gives

$$-m_{\dot\theta} = \left(\frac{0{\cdot}6006}{\beta} - \frac{0{\cdot}4812}{\beta^3}\right) - h_o\left(\frac{2}{\beta} - \frac{0{\cdot}7614}{\beta^3}\right) + h_o^2\left(\frac{2}{\beta}\right) \qquad \ldots (6)$$

which is always positive when M is greater than 1·576. Thus, from Fig.8 it is seen that exact theory gives positive damping for all axis positions when $M \geqslant 1{\cdot}505$, whereas from equation (6) strip theory predicts some negative damping in the range $1{\cdot}505 \leqslant M \leqslant 1{\cdot}576$.

Acknowledgement

Some of the numerical results given in this report were calculated by Mrs. S. Lucas of the Aerodynamics Division, N.P.L.

References

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2	J. W. Miles	The potential theory of unsteady supersonic flow. Cambridge University Press, 1959.
3	K. W. Mangler	Calculation of the pressure distribution over a wing at sonic speeds. A.R.C. R. & M.2888. September, 1951.

Table 1 /

Table 1

Definition of Symmetrically Tapered Wing

Semi-span/root chord = $\frac{16}{11}$

Tip chord/root chord = τ = $\frac{5}{11}$

Semi-apex angle = λ = 79·38°

Aspect ratio = A = 4

$c_o/\bar{c}$ = 1·3750 , $(c_o/\bar{c})^2$ = 1·8906

$c_o/\bar{\bar{c}}$ = 1·3134 , $(c_o/\bar{\bar{c}})^2$ = 1·7251

Table 2

Stability Derivatives for Pitching Axis $h_o = 0$

M	ℓ_θ	$\ell_{\dot\theta}$	$-m_\theta$	$-m_{\dot\theta}$
1·017	3·707	-10·152	0·956	-0·513
1·057	3·718	- 6·514	1·522	-3·328
1·118	3·169	- 2·443	1·457	-1·437
1·250	2·363	- 0·235	1·139	-0·160
1·505	1·661	0·384	0·815	0·221
1·803	1·273	0·445	0·629	0·263
2·125	1·030	0·417	0·511	0·248
2·462	0·864	0·376	0·429	0·224

$\ell_z = 0$ $\quad m_z = 0$

$\ell_{\dot z} = \ell_\theta$ $\quad m_{\dot z} = m_\theta$

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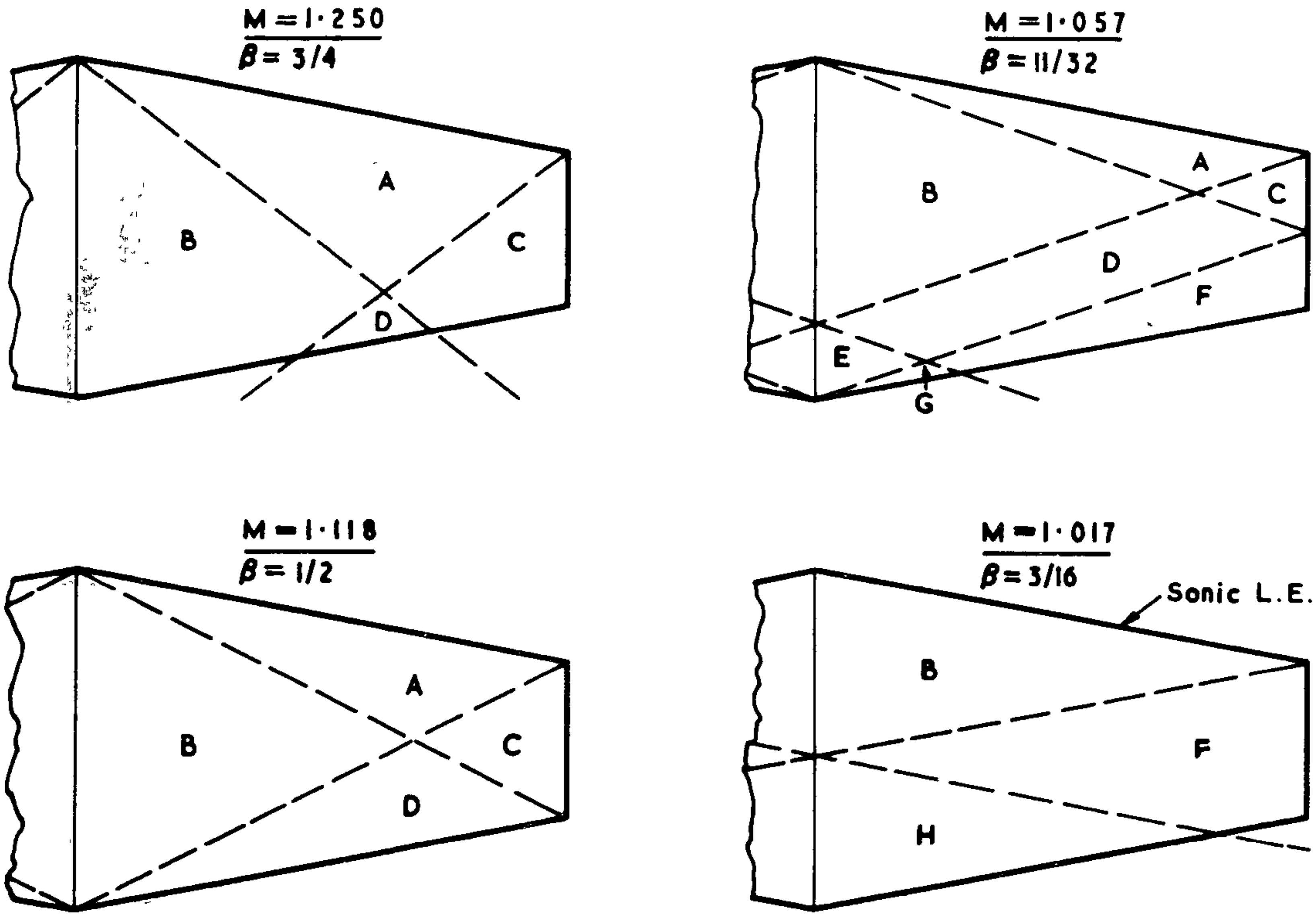
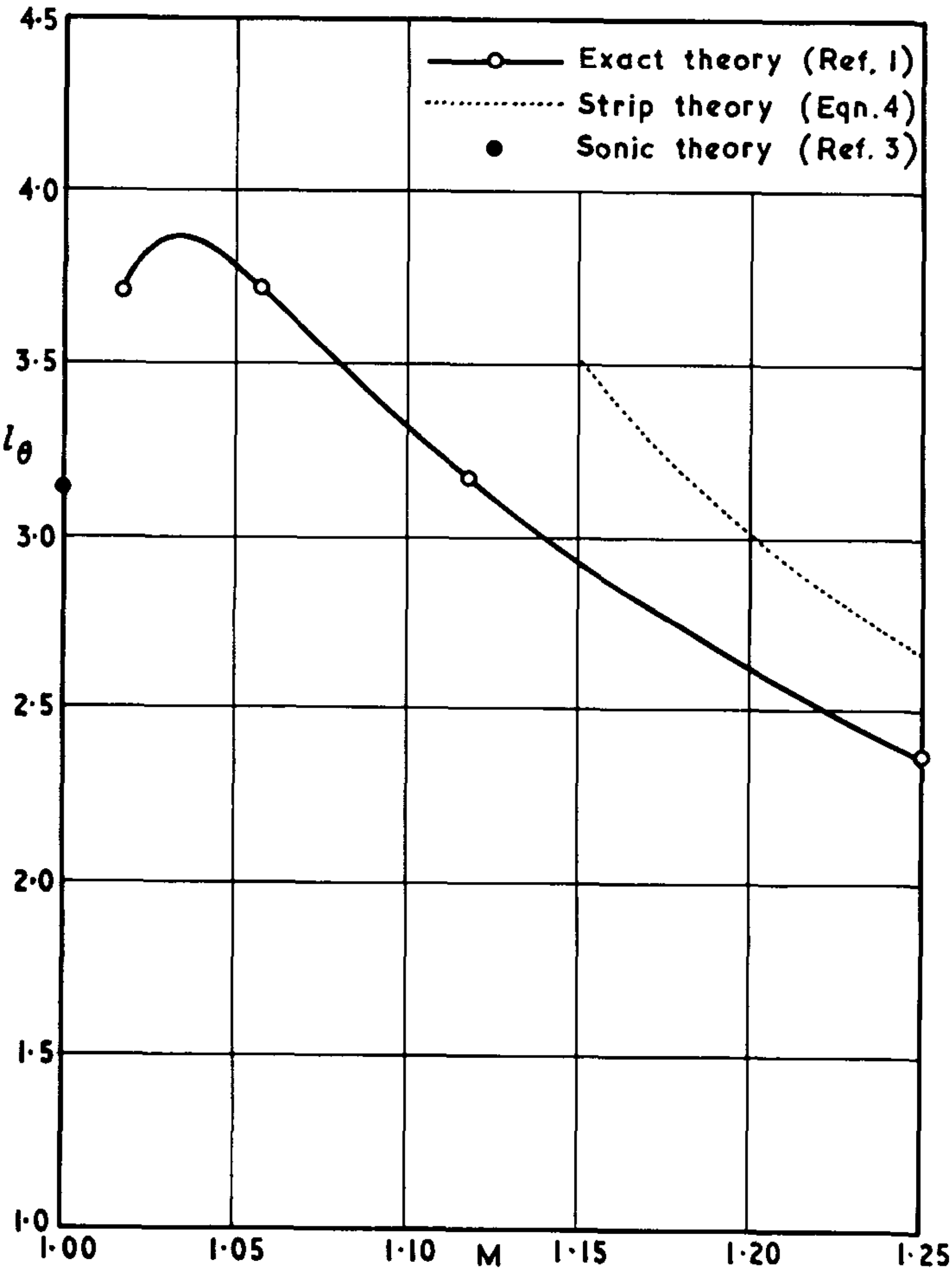


FIG. 1

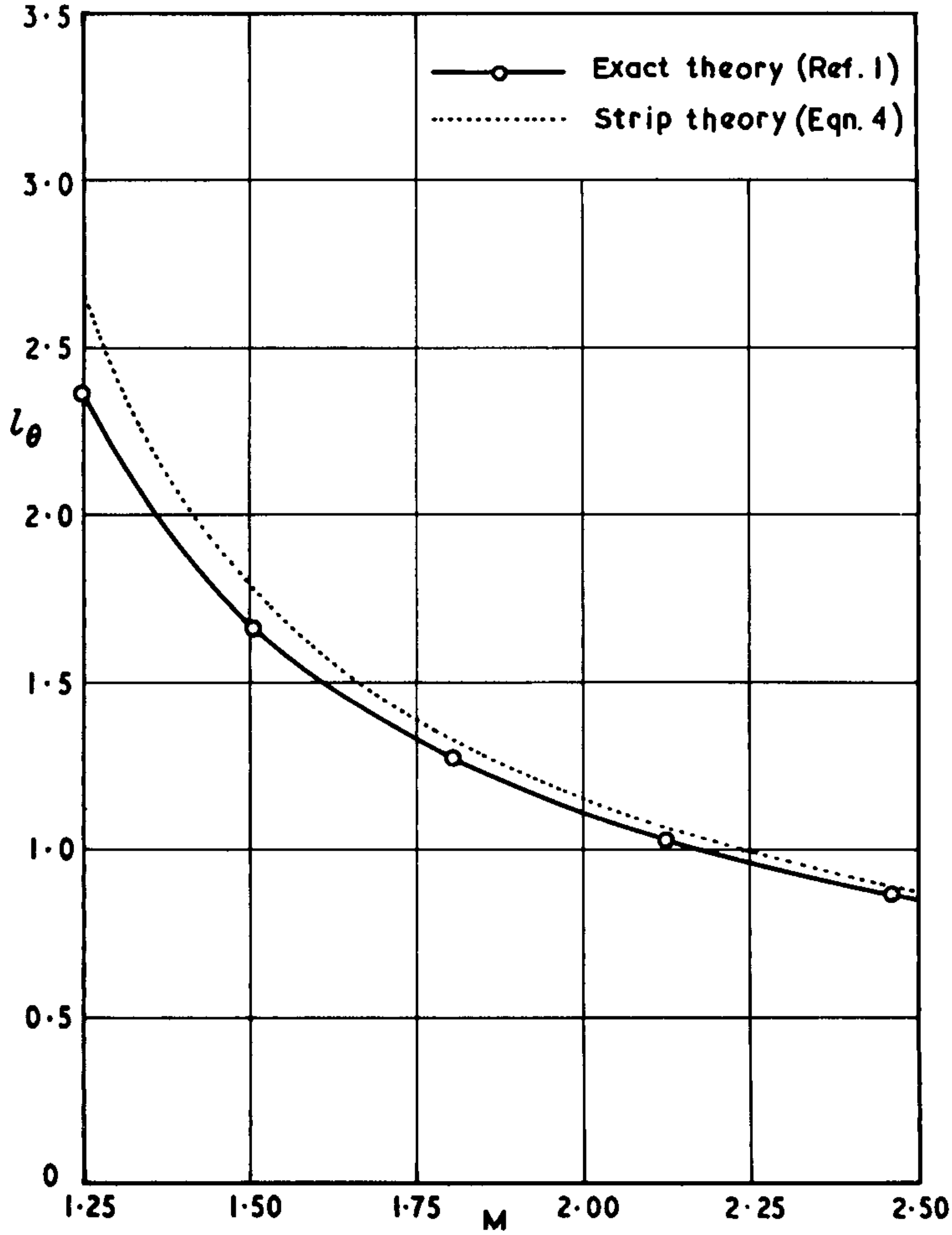
Division of half-wing by Mach lines into regions A, B ---- H

FIG. 2



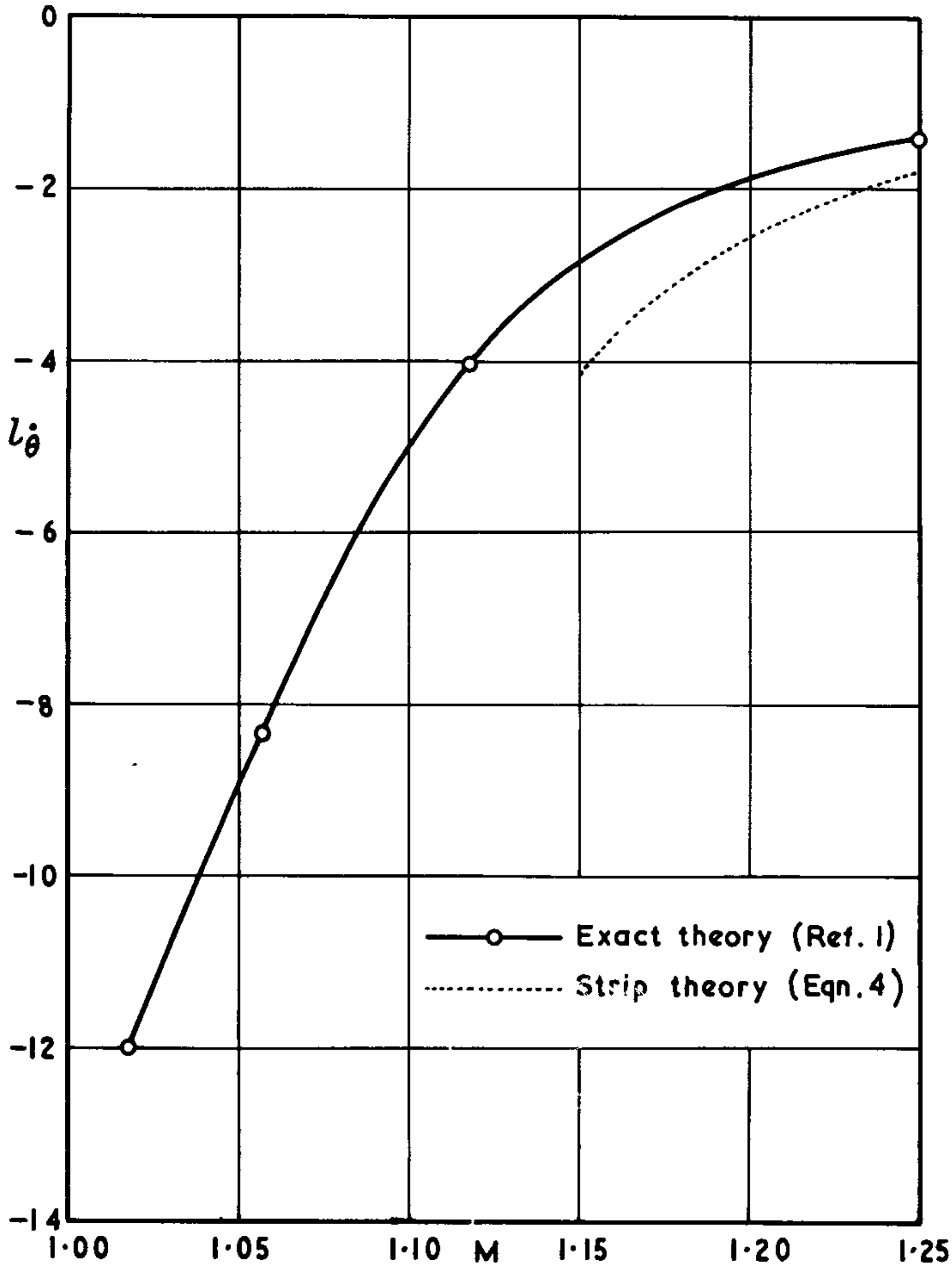
Lift derivative l_θ against Mach number

FIG. 3



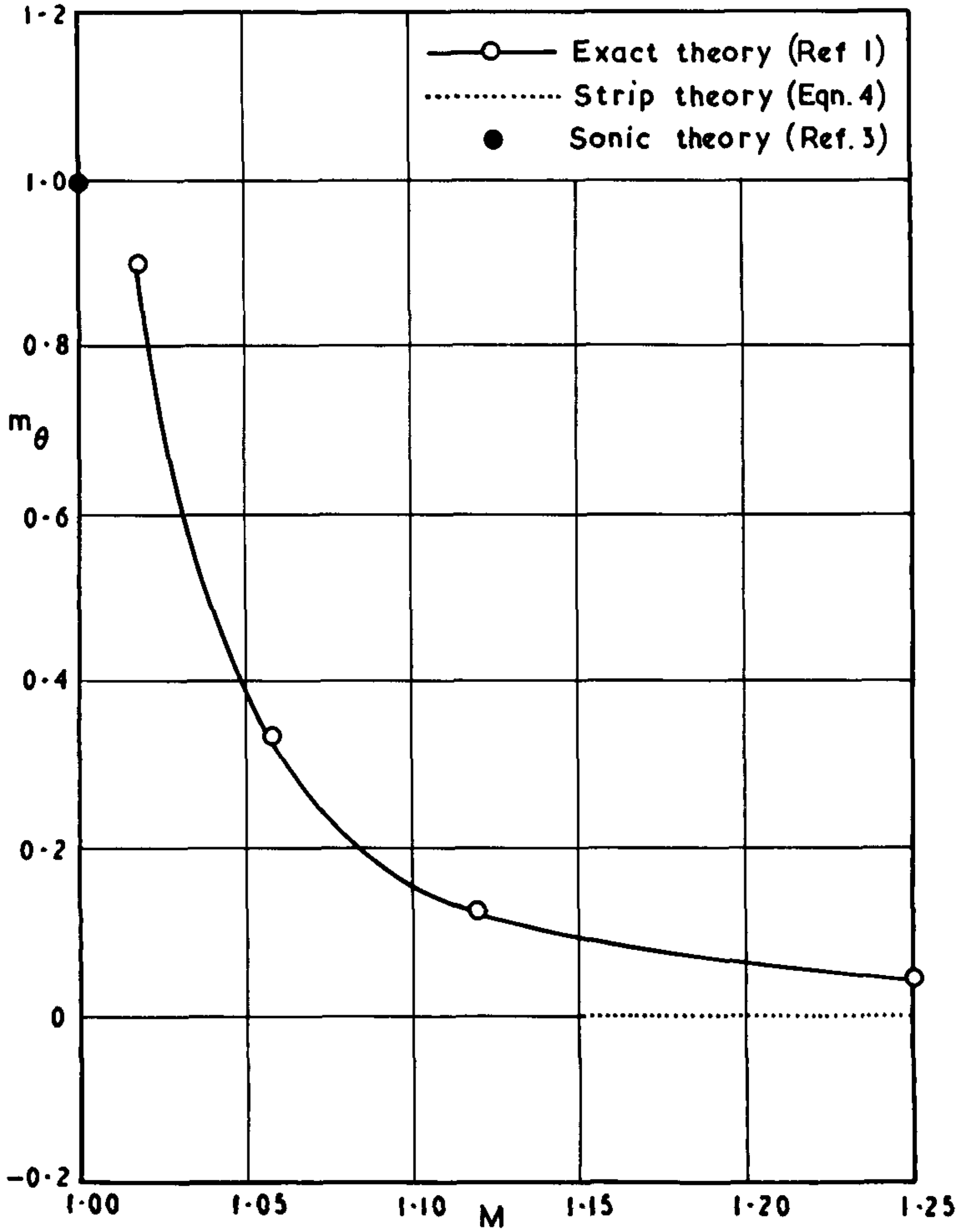
Lift derivative l_θ against Mach number

FIG.4



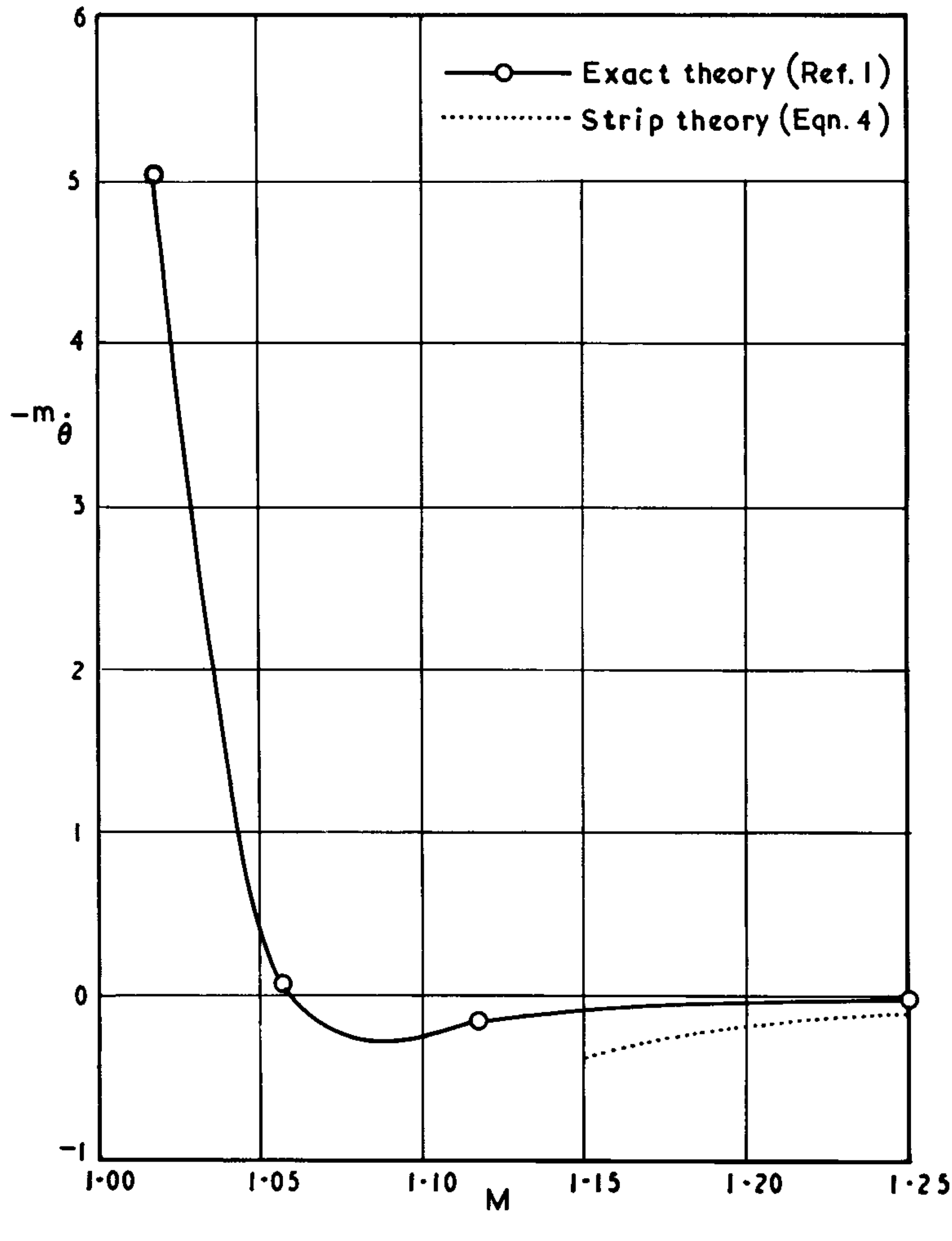
Lift derivative $l_{\dot{\theta}}$ against Mach number for mid-chord pitching axis ($h_o = 0{\cdot}5$)

FIG.5



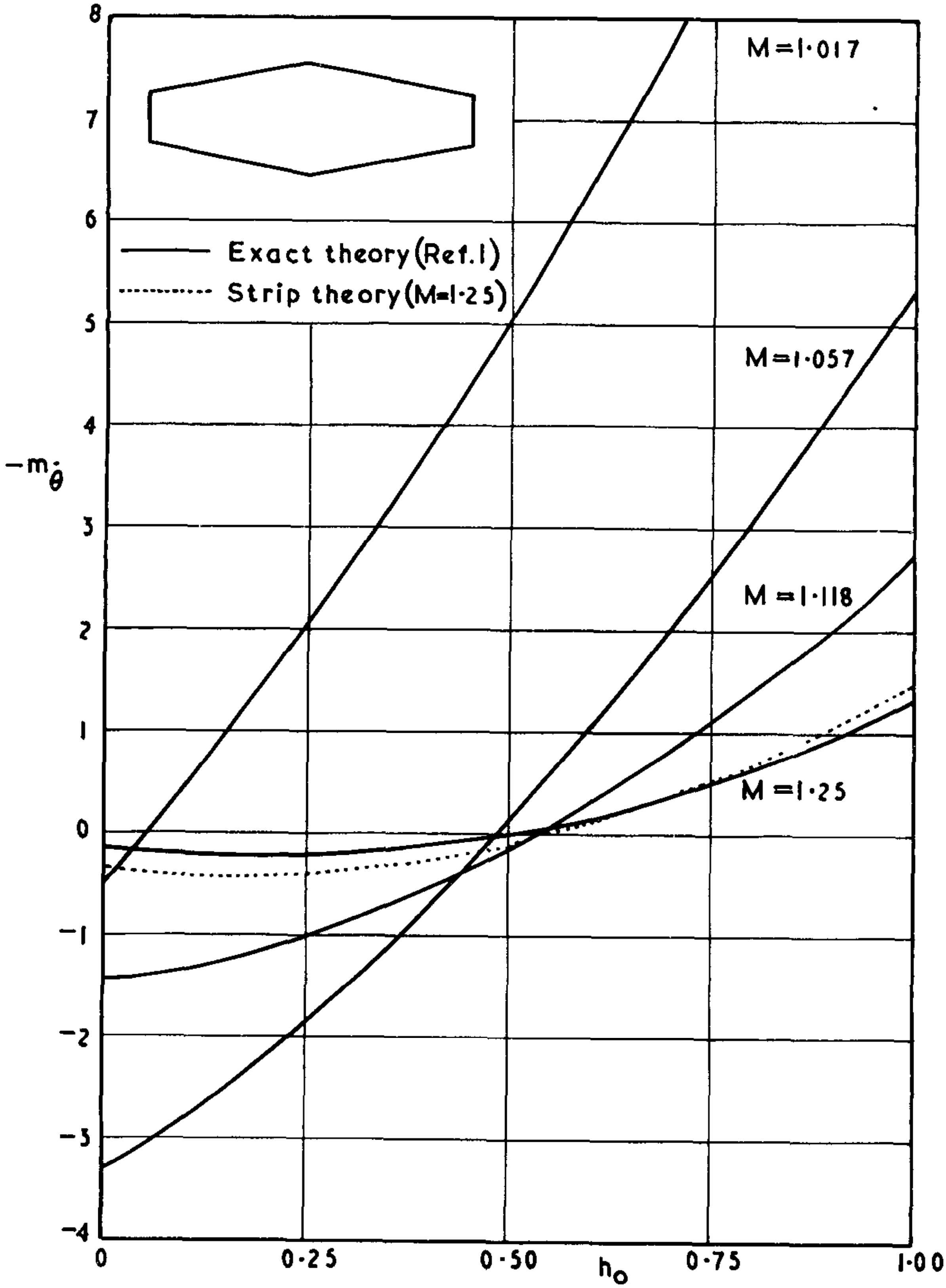
Direct pitching derivative m_θ against Mach number for mid-chord pitching axis ($h_o = 0·5$)

FIG.6



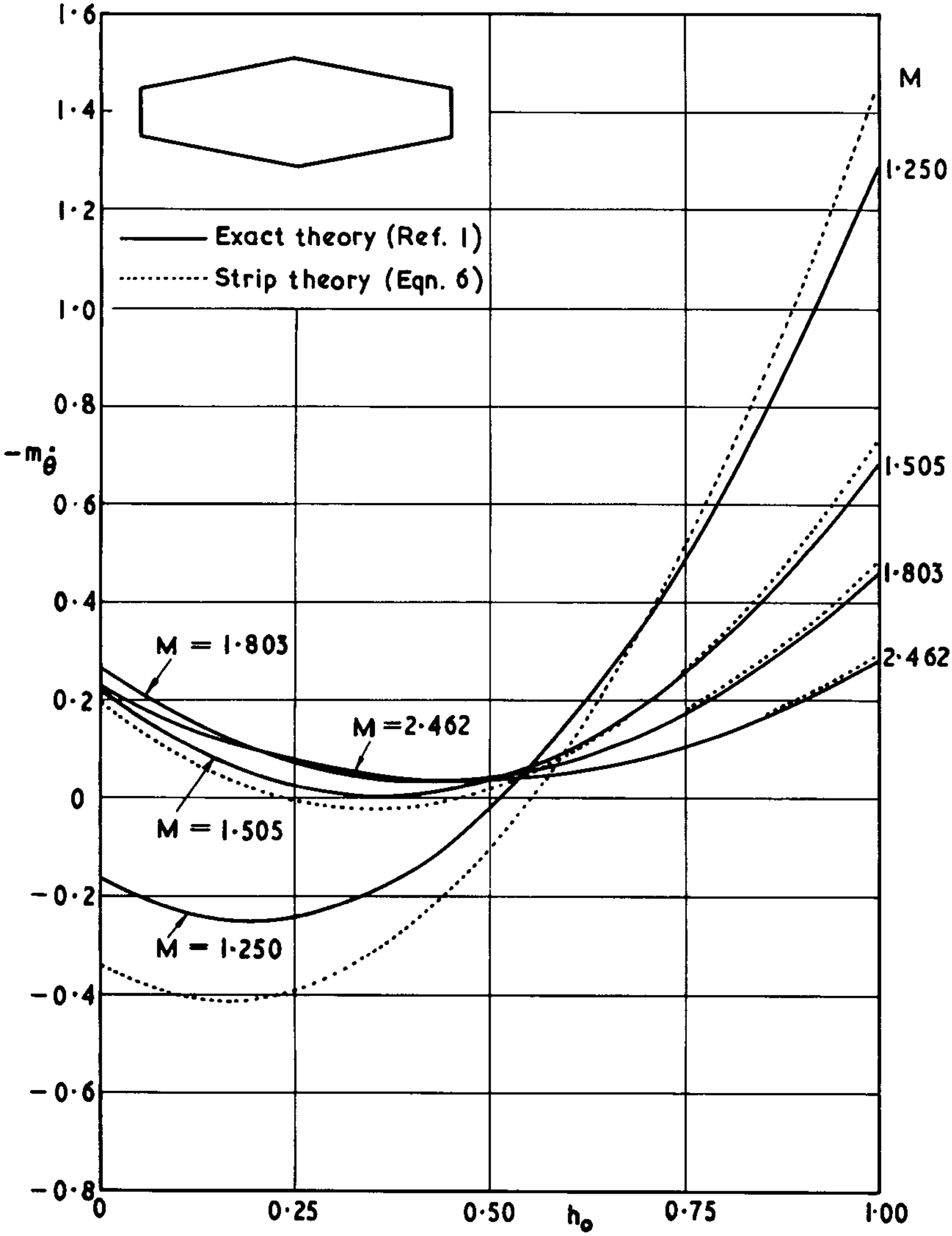
Direct pitching derivative $-m_{\dot{\theta}}$ against Mach number for mid-chord pitching axis ($h_o = 0·5$)

FIG.7



Effect of Mach number on the pitching damping $-m_{\dot{\theta}}$ against h_o

FIG. 8



Effect of Mach number on the pitching damping $-m_{\dot\theta}$ against h_o

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