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# Potential Flow through Cascades - A Comparison between Exact and Approximate Solutions

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# Potential Flow through Cascades - A Comparison between Exact and Approximate Solutions

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## SUMMARY

The transformation method of Merchant and Collar[1] is developed in order to obtain an exact solution to the potential flow around a cascade of derived aerofoils. This solution is then used as a check on the accuracy of an approximate method, given by Schlichting, for the prediction of the flow around the derived cascade.

## 1. INTRODUCTION

This paper is primarily concerned with the direct problem of the application of potential flow theory to cascades, i.e. that in which a solution to the flow about a cascade of given geometry is required. In general most of the solutions which have been given suffer from the need for restrictions and approximations and are of a lengthy nature.

The first solutions to the problem of potential flow in cascades were for cascades of flat plates and within the limitations of zero thickness and camber, analytical solutions for lift coefficient and outlet angle were obtained[2,3]. The more general problem of thick, cambered aerofoils in cascade, as used in axial flow compressors and turbines, has proved more difficult to solve. The solutions proposed fall into the two catagories of (a) transformation methods and (b) singularity methods.

### a) Transformation methods

Howell[4] gave a solution, based upon a conformal transformation, and by the use of suitable intermediate stages transformed the cascade of arbitrarily specified aerofoil profiles into a circle, the flow around which could be determined. This method has been extended by Carter and Hughes[5] and programmed for an electronic computer by Pollard and Wordsworth[6]. It was found that approximations arose in the transformation to an exact circle, with special difficulty at the point corresponding to the aerofoil leading edge.

Garrick[7] has also given a solution to the problem based upon the Theodorsen conformal transformations, and this method of solution has been developed by Hall[8].

b) Singularity methods

Schlichting[9], whose method has been modified by Mellor[10] and programmed for a low speed digital computer by Pollard and Wordsworth[6], distributed sources, sinks and vortices on the chord line in order to represent a given aerofoil cascade profile. This limited the application of the theory to profiles of low camber. Doubts also arise concerning the convergence of the Fourier series used for specifying the singularity distribution.

A more sophisticated approach is that due to Martenson[11], who distributed vorticity around the profile. Results from this method, which is being widely used by other workers, may be the most reliable to date, although the method seems to fail for profiles of low thickness.

In each of these methods an attempt is made to predict lift coefficient, outlet angle and distribution of pressure over a given blade profile; results have been published, based upon one or other of these methods for many different aerofoil profiles and blade configurations. However, due to the possibility of error in the lengthy computations, and to the differing assumptions made, discrepancies are noticeable when two or more of these methods are applied to the same blade profile[6].

During the years 1940-1944 Merchant and Collar[1] produced an analysis giving a transformation linking the known potential flow around a series of ovals to that around a cascade of inclined flat plates. They also gave suggestions for extension of the theory to a cascade of aerofoil profiles, in an analogous manner to the theory of isolated Joukowsky transform aerofoils. This theory has not, to the knowledge of the author, been extended prior to the work described in the present paper. The assumptions made are those of conventional potential flow theory and the accuracy of calculation is limited only by the means of computation available. Thus a standard has been provided for comparison with the approximate methods outlined above.

## 2. NOTATION AND SYMBOLS (See also Fig. 2)

$c$ chord length (distance between extremities of camber line)

$C_{L(c)}$, $C_{L(m)}$ lift coefficient (based on chord and mean line respectively.)

$C_p = \dfrac{p - p_1}{\frac{1}{2}\rho u_1^2}$ pressure coefficient

$\zeta = m + in$ complex coordinates in plane of ovals

$\zeta' = m' + in'$ centre of offset oval

$p$ local pressure at a point on the profile

$q$ local velocity on the profile

$q_\zeta = u_\zeta + iv_\zeta$ complex velocity in the $\zeta$ plane

$q_z = u_z + iv_z$ complex velocity in the z plane

$s$ space between blades ( $s = \lambda$ in this paper)

$U_1, U_2$ velocity before and after cascade

$y_s, y_t$ camber and thickness ordinate in singularity method

$z = x + iy$ complex coordinates in cascade plane

$\alpha_1$ inlet flow angle

$\alpha_2$ outlet flow angle

$\beta$ size parameter of smaller oval

$\beta'$ size parameter of larger oval

$\lambda = \ell + \sin h^2\beta \coth \ell$

$\gamma = \beta + \sin h^2\beta \coth \beta$

$\zeta = \xi + i\eta$ complex coordinates in intermediate plane

$\sigma$ aerofoil stagger angle

$\rho$ density of fluid

$\Gamma = -2\lambda W$ circulation around each aerofoil

$w = \phi + i\psi$ complex potential in plane of ovals

## 3. THE EXACT SOLUTION TO THE FLOW THROUGH A DERIVED CASCADE

The procedure for evaluation of the blade profile shape and cascade configuration follows that of Merchant and Collar[1].

i) The normal* flow past a series of ovals on the imaginary axis has been given by Lamb[12].

ii) The normal flow past a series of laminae lying along the imaginary axis is also known and a transformation can be obtained which converts the laminae into the ovals of (i) (Fig. 1a).

iii) In a similar way the general flow^ round the laminae (which is known) gives the general flow round the ovals.

iv) A particular case of the general flow round the ovals is that for which the flow at infinity is inclined to the axis but for which there is no circulation. In this ca e the ovals can immediately be transformed into a cascade of flat plates parallel to the direction of flow at infinity (Fig. 1b).

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* "Normal Flow" - flow perpendicular to the imaginary axis.

^ "General Flow" - flow with inlet angle and circulation

v) Application of this transformation to ovals which are offset from the origin produces a cascade of aerofoil shapes. This is the class of aerofoil for which the profile shape, and subsequently the aerodynamic characteristics, will be obtained.

The procedure is similar to the usual Joukowsky process for an isolated aerofoil and, if the diameter of each oval is small compared with the space, these ovals tend to become circles and the transormation used becomes the Joukowsky transformation. The full analysis is given below.

### 3.1 Derivation of aerofoils

#### 3.1.1 General Flow Past a Cascade of Ovals

The potential field due to normal flow of a uniform stream past a series of uniform doublets lying along the imaginary $\ell$ plane axis is given by Lamb[12]:-

$$w = U\left\{ \ell + \sinh^2 \beta \coth \ell \right\} \qquad .....(1)$$

This rationalises into

$$\phi = U\left\{ m + \frac{\sinh^2 \beta \sinh 2m}{\cosh 2m - \cos 2n} \right\}$$

$$\psi = U\left\{ n - \frac{\sinh^2 \beta \sin 2n}{\cosh 2m - \cos 2n} \right\}$$

where the streamline $\psi = 0$ marks part of the n = 0 axis and the closed oval

$$\cosh 2m = \cos 2n + \frac{\sinh^2 \beta \sin 2n}{n} \qquad .....(2)$$

$\beta$ is now seen to be the length of the semi-major oval axis.

If we consider a series of laminae distributed along the $\eta$ axis of the $\zeta$ plane with a period of $\lambda$, the stagnation points of the flow around these laminae may be made to correspond to those in the $\ell$ plane. We can thus put $\ell = \pm \beta$, as the stagnation point, in equation (1)

For simplicity, following Merchant and Collar, $\lambda$ and $\gamma$ are defined as

$$\lambda = \ell + \sinh^2 \beta \coth \ell \qquad \ldots\ldots(3)$$

$$\gamma = \beta + \sinh^2 \beta \coth \beta \qquad \ldots\ldots(4)$$

The flow around the laminae is given by[1],

$$w = U \cosh^{-1}(\cosh \gamma \cosh \zeta) \qquad \ldots\ldots(5)$$

and since, from (1), $w = U\lambda$ for the ovals

$$\cosh \lambda = \cosh \gamma \cosh \zeta \qquad \ldots\ldots(6)$$

This is thus the required transformation connecting the $\ell$ plane ovals and the $\zeta$ plane laminae.

The general flow past the $\zeta$ plane laminae is known to be

$$\frac{dw}{d\zeta} = \frac{U \sinh \zeta + iW \cosh \zeta}{\sqrt{\sinh^2 \zeta + \tanh^2 \zeta}} - iV \qquad \ldots\ldots(7)$$

where V is the component of velocity parallel to the $\eta$ axis, and there is a circulation $2\pi W$ around each lamina.

If transformation (6) is applied to the general flow past the laminae, the following equation is obtained:-

$$u_\ell - iv_\ell = \frac{dw}{d\ell} = \left[U + i\left(\frac{W\cosh\lambda - V\sinh\lambda}{\sinh^2\lambda - \sinh^2\gamma}\right)\right]\left[1 - \frac{\sinh^2\beta}{\sinh^2\ell}\right] \qquad \ldots\ldots(8)$$

This is the general flow past the $\ell$ plane ovals.

3.1.2 Transformation of ovals into inclined flat plates and aerofoils

Considering the particular case in which $W = 0$, $V = U \tan \delta$, equation (8) becomes

$$u_\ell - iv_\ell = \frac{dw}{d\ell} = U\left[1 - \frac{i\tan\delta \sinh\lambda}{\sinh^2\lambda - \sinh^2\gamma}\right]\left[1 - \frac{\sinh^2\beta}{\sinh^2\ell}\right] \quad \ldots(9)$$

Also, considering the flow past a cascade of inclined flat plates as shown in Fig. 2,

$$\frac{dw}{ds} = U(1 - i\tan\delta) \qquad \ldots\ldots(10)$$

Elimination of w in (9) and (10) gives

$$\frac{dz}{d\zeta} = e^{i\delta}\left[\cos\delta - \frac{i\sin\delta\sinh\lambda}{\sinh^2\lambda - \sinh^2\gamma}\right]\left[1 - \frac{\sinh^2\beta}{\sinh^2\zeta}\right] \quad ..(11)$$

Hence the transformation connecting the oval and cascade planes is given by

$$z = e^{i\delta}(\lambda\cos\delta - i\sin\delta\,\operatorname{arccosh}(\operatorname{sech}\gamma\cosh\lambda)) \quad .....(12)$$

or, if the true chord is taken as abscissa,

$$Z = \lambda\cos\delta - i\sin\delta\,\operatorname{arccosh}(\operatorname{sech}\gamma\cosh\lambda) \quad .....(13)$$

which can be expressed, for ease of computer programming, as

$$Z = \lambda\cos\delta - i\sin\delta\,\ln\left[\operatorname{sech}\gamma\cosh\lambda + \sqrt{\frac{\cosh^2\lambda}{\cosh^2\gamma} - 1}\right] \quad .....(14)$$

The procedure for the derivation of a cascade of aerofoils is thus to select a suitable set of $\zeta$ plane ovals, postulate a set of larger ovals with offset centres, and apply transformation (13) to these offset ovals.

Experience enables the required type of cascade to be obtained. For example, β should be around 0.725 to give a space-chord ratio of unity and β' should be approximately 10% larger than β to give a maximum thickness of 10% of the chord. Variation of camber and position of maximum thickness is obtained by variation of n' and m' (the coordinates of the offset oval centre); the proviso is that the transformation singularities must be enclosed within the offset oval, or, for a cusped trailing edge, lie on this oval. A more detailed explanation of this procedure is given by Collar[13] and Merchant and Collar[1].

There exist two extreme particular examples of the generalised method. In the first case the transformation is applied to concentric ovals and a cascade of elliptic aerofoils is produced in the z plane. In the second example (the case under discussion in this paper) the stagnation point at the position on the ovals corresponding to the trailing edge is placed at the transformation singularity, by application of Newton's method for the determination of roots to equation (2). This case gives a cascade of cusped aerofoils. Between these two extreme cases the derivation of an infinite variety of aerofoil sections is possible. Applications of such a generalisation will be the subject of a second paper.

### 3.2.1 The Flow Around the Aerofoils

The relationship between velocities in the $\ell$ and z planes is given by

$$u_z - iv_z = \frac{u_\ell - iv_\ell}{\left|\frac{dz}{d\ell}\right|} \qquad \text{.....(15)}$$

where subscript z refers to the local velocity on the z plane cascade profile, subscript $\ell$ refers to the local velocity on the $\ell$ plane oval profile.

From equations (9) and (11) the velocity in the z plane is given by

$$u_z - iv_z = \frac{\left[U + i\left(\frac{W\cosh\lambda - V\sinh\lambda}{\sqrt{\sinh^2\lambda - \sinh^2\gamma}}\right)\right]\left[1 - \frac{\sinh^2\beta}{\sinh^2\ell}\right]}{\left|\left[\cos\delta - \frac{i\sin\delta\sinh\lambda}{\sqrt{\sinh^2\lambda - \sinh^2\gamma}}\right]\left[1 - \frac{\sinh^2\beta}{\sinh^2\ell}\right]\right|} \qquad \text{.....(16)}$$

To obtain W, the value of $\ell$ at the rear stagnation point in the $\ell$ plane is substituted into equation (9). If the trailing edge of the profile is cusped, then the Kutta condition must be satisfied. Since $\frac{dw}{dz} = \frac{dw/d\ell}{dz/d\ell}$ and both $\frac{dw}{d\ell}$ and $\frac{dz}{d\ell}$ become zero if the stagnation point is at the point of the cusp, the complex velocity in the z plane becomes finite and the Kutta condition is satisfied. If the trailing edge is rounded then the rear stagnation point is indeterminate and a suitable position must be chosen.

Thus, from equation (9), at the rear stagnation point

$$W = V\tanh\lambda_t + iU\sqrt{1 - \frac{\cosh^2\gamma}{\cosh^2\lambda_t}} \qquad \text{.....(17)}$$

where the subscript t refers to trailing edge conditions.

Now the circulation is taken as $\Gamma = -2\pi W$, giving the following values for air angles $\alpha_1$ and $\alpha_2$

$$\tan\alpha_1 = \frac{V - W}{U} \qquad \tan\alpha_2 = \frac{V + W}{U}$$

Hence

$$V = U\tan\alpha_1 + W$$

and from equation (17)

$$W = U\left[\frac{\tan\alpha_1\tanh\lambda_t + \sqrt{\dfrac{\cosh^2\gamma}{\cosh^2\lambda_t} - 1}}{1 - \tanh\lambda_t}\right]$$

$$V = U\left[\frac{\tan\alpha_1 + \sqrt{\dfrac{\cosh^2\gamma}{\cosh^2\lambda_t} - 1}}{1 - \tanh\lambda_t}\right]$$

Substituting these values for W and V into equation (16) we obtain,

$$\frac{u_z - iv_z}{U} = \frac{(1 + iA)\left(1 - \dfrac{\sinh^2\beta}{\sinh^2\rho}\right)}{\left|\left[\cos\sigma - \dfrac{i\sin\sigma\sinh\lambda}{\sinh^2\lambda - \sinh^2\gamma}\right]\left[1 - \dfrac{\sinh^2\beta}{\sinh^2\rho}\right]\right|} \quad \ldots(18)$$

where $A = \dfrac{\left[\tan\alpha_1\tanh\lambda_t + \sqrt{\dfrac{\cosh^2\gamma}{\cosh^2\lambda_t} - 1}\right]\cosh\lambda - \left(\tan\alpha_1 + \sqrt{\dfrac{\cosh^2\gamma}{\cosh^2\lambda_t} - 1}\right)\sinh\lambda}{(1 - \tanh\lambda)\sqrt{\sinh^2\lambda_t - \sinh^2\gamma}}$

Also

$$\frac{q}{U} = \sqrt{\frac{u_z^2 + v_z^2}{U}}$$

and we define the pressure coefficient

$$Cp = \frac{p - p_1}{\frac{1}{2}\rho U_1^2}$$

From Bernoulli's equation

$$p + \tfrac{1}{2}\rho q^2 = p_1 + \tfrac{1}{2}\rho U_1^2$$

therefore

$$\frac{p - p_1}{\frac{1}{2}\rho U_1^2} = 1 - \frac{q^2}{U_1^2}$$

Now

$$U = U_1\cos\alpha_1$$

therefore

$$Cp = \frac{p - p_1}{\frac{1}{2}\rho U_1^2} = 1 - \frac{(u_z^2 + v_z^2)}{U^2}\cos^2\alpha_1 \quad \ldots\ldots(19)$$

For the potential flow around the β' ovals to be determined, these ovals must be relocated with their centres at the points (0,0), (0, π), (0,2π),..in the ℓ plane.

If $\ell_{t_2} = \ell_t - \ell'$ , where $\ell' = m' + in'$

then

$$\lambda_{t_2} = \ell_{t_2} + \sinh^2\beta' \coth \ell_{t_2}$$

Now

$$\tan \alpha_1 = \frac{V - W}{U} \quad \text{and} \quad \tan \alpha_2 = \frac{V + W}{U}$$

thus

$$\tan \alpha_2 = \frac{\tan \alpha_1(1 + \tanh\lambda_{t_2}) + 2\sqrt{\dfrac{\cosh^2\gamma_2}{\cosh^2\lambda_{t_2}} - 1}}{1 - \tanh\lambda_{t_2}} \quad ..(20) *$$

The complete flow around the relocated β' ovals will be needed, for use in obtaining the cascade profile pressure distribution; from (8)

$$\frac{u_\ell - iv_\ell}{U} = \left[1 + i\left(\frac{\left(\tan\alpha_1 \tanh\lambda_t + \sqrt{\dfrac{\cosh^2\gamma}{\cosh^2\lambda_t} - 1}\right)\cosh\lambda - \left(\tan\alpha_1 + \sqrt{\dfrac{\cosh^2\gamma}{\cosh^2\lambda_t}} - 1\right)\sinh\lambda}{(1 - \tanh\lambda_t)\sqrt{\sinh^2\lambda - \sinh^2\gamma}}\right)\right]$$

$$\times\left[1 - \frac{\sinh^2\beta'}{\sinh^2\ell'}\right] \quad .....(22)$$

In equation (22) all λ's and ℓ's refer to conditions around the β' oval relocated and centred at the origin.

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* This corresponds to the equation

$$\tan \alpha_2 = \frac{\tan \alpha_1\, y + 2 \tan \phi}{y + 2} \quad .....(21)$$

of ref. 15 and it can easily be demonstrated that

$$\frac{y}{y + 2} = \frac{1 + \tan h\lambda_{t_2}}{1 - \tan h\lambda_{t_2}}$$

and

$$\frac{\tan \phi}{y + 2} = \frac{\sqrt{\dfrac{\cosh^2\gamma_2}{\cosh^2\lambda_{t_2}} - 1}}{1 - \tanh \lambda_{t_2}}$$

The next stage is the evaluation of the scale factor $\left|\frac{dz}{d\ell}\right|$

Making use of equation (11) we have

$$\left|\frac{dz}{d\ell}\right| = \left|\left(\cos\delta - \frac{i\sin\delta\sinh\lambda}{\sqrt{\sinh^2\lambda - \sinh^2\gamma}}\right)\left(1 - \frac{\sinh^2\beta}{\sinh^2\ell}\right)\right| \qquad .....(23)$$

Here, as in the basic transformation used to determine the aerofoil profile, the $\beta$ of the smaller oval is employed.

It is now possible to evaluate the $\frac{u_z - iv_z}{U}$ of equation (18) using the right hand side of equation (22) as the numerator, and that of (23) as the denominator. The value of the pressure coefficient for the corresponding point on the aerofoil surface is now given by equation (19).

The only remaining aerodynamic parameter which can be calculated from potential flow theory is the lift coefficient. This is defined and calculated in two different ways below, both of which are in common usage.

3.3 Lift Coefficients

Firstly it is possible to base the lift coefficient on the chord line of the profile. The advantage of this definition is that the resulting value of lift coefficient can be compared with the value obtained by integration of the pressure distribution as is shown in Fig. 6.

$$C_{L(c)} = \int_{x/c = 0}^{x/c = 1} Cp \; d\left(\frac{x}{c}\right)$$

Thus the lift coefficient perpendicular to the chord line is defined as

$$C_{L(c)} = \frac{L(\perp\text{'r to chord})}{\frac{1}{2}\rho U_{\alpha_1}^2 \; C} \qquad .....(24)$$

It may be shown that

$$C_{L(c)} = \frac{s}{c}\cos^2\alpha_1(\tan\alpha_1 - \tan\alpha_2)\cdot\left((\tan\alpha_1 + \tan\alpha_2)\sin\zeta + 2\cos\zeta\right) \qquad .....(25)$$

An alternative definition of the lift coefficient is obtained from a consideration of the lift perpendicular to the mean flow direction.

$$C_{L(m)} = \frac{\rho c_m \Gamma}{\frac{1}{2}\rho c_m^2 c} \qquad .....(26)$$

It follows that

$$C_{L(m)} = \frac{2 \cos\delta \, C_{Lc}}{\sqrt{4 + (\tan\alpha_1 + \tan\alpha_2)^2}} \qquad .....(27)$$

4. APPROXIMATE SOLUTIONS TO THE FLOW THROUGH A DERIVED CASCADE

Of the potential flow solutions mentioned in the introduction the author was only able to use the singularity method of Schlichting. However, due to the cooperation of Dr. Hall of Southampton, who used his extended Garrick method and the use by a team at Rolls-Royce of a modified Martensen-Isay method, a more complete comparison was possible.

These methods for determinations of the potential flow were applied to the cascade of blades with the profile shown in graph 2c, having the given stagger, space/chord ratio and inlet angle, the object being to determine the outlet angle at downstream infinity, the lift coefficient, and the distribution of pressure around the blade profile.

This process was carried out by the author using the Schlichting singularity method and a brief description of the procedure is given below. Results of the comparison between the analysis and the application of the above mentioned methods are given in Fig. 4.

In the Schlichting method, sources, sinks and vortices are distributed along the true chord of the blade and the velocity induced by the sum of these singularities is calculated throughout the flow regime and added to the free stream velocity. The magnitude of the singularities is chosen so that a fluid streamline corresponds to each blade profile.

The main assumptions and approximations are as follows:-

i) a distribution of singularities is used to match the profile at a finite number of points.

ii) this number of matching points is restricted by the stability of the Fourier series which is used to represent the singularity distribution[6].

iii) the blade profile is split into a camber line and thickness distribution; these are considered separately.

iv) the singularities are distributed along the chord line. Hence the induced velocities are calculated on the chord line and corrected to give the velocity on the profile, utilising a factor

$$\frac{V_\iota}{V_{mx}} = \frac{V_x}{V_{mx}} \quad \frac{1}{\sqrt{1 + (y_s' \pm y_t')^2}}$$

given by Riegels[14].

v) the blade profile shape is not introduced in the form of (x,y) coordinates but in the form $(x, \frac{dy}{dx})$ and since the profile gradients of an arbitrary profile are difficult to measure or compute with good accuracy it is difficult to avoid small errors in profile specification.

The calculations were carried out on the Deuce computer for the given cascade profile of Fig. 2, matching camber and thickness gradients at seventeen stations along the chord. The lift coefficient, outlet angle and pressure distribution were obtained. Provision had been made, in the work of Pollard and Wordsworth, for integrating the expressions for camber line and thickness gradient as finally obtained, to give the actual "integrated" profile around which the flow had been found. This integrated profile proved to be slightly different from the given profile, as shown in Fig. 3a.

The entire calculation using the singularity method was carried out independently several times in attempts to improve the profile matching. The final pressure distribution was found to vary only slightly with change in integrated profile. The curves shown in Figs. 3a and 4 are for the integrated profile nearest to the required one.

5. CONCLUSIONS

The analysis of Merchant and Collar[1] has been programmed for an électronic computer in order to obtain a cascade of aerofoil profiles; this analysis has been extended in order to calculate fully the potential flow around these profiles. It was also found possible to determine the variation of outlet angle, theoretical lift coefficient and pressure distribution over a wide range of inlet angles. As a check on the accuracy of the calculations the theoretical lift coefficient was compared with the value of lift coefficient obtained by planimeter integration of pressure distribution, the results being shown in Fig. 6. Good agreement was obtained, as was to be expected since no assumptions other than those of potential flow theory were made and the only limitations on the accuracy were those of the computing equipment (viz. 7 decimal places, allowance having been made for rounding off errors). The results of the calculations are presented both graphically and in the form of tables for x/c, y/c and Cp, thus facilitating a check on the accuracy of other, more general, potential flow solutions.

Comparisons have been made with the singularity method of prediction of potential flow in cascades, as developed by Pollard and Wordsworth. Difficulties and limitations of this method have been discussed and graphs are presented showing the difficulties of matching the profile exactly. The outlet angle, as predicted by the singularity method, is seen to be in error by 0.7° and the pressure distribution is seen to be in reasonable general agreement, although discrepancies occur near the suction peak.

The results which Dr. Hall has provided, based on the Garrick method, show an accuracy in outlet angle of almost four decimal places and excellent agreement in pressure distribution.

A generalisation of the preceding potential flow solution is to be presented in a further paper in which the possibilities and limitations of the solution will be explored.

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APPENDIX A.

PRELIMINARY CALCULATIONS FOR THE GIVEN EXAMPLE.

A profile with a stagger $\sigma$ of $37.5^{\circ}$ and a space/chord ratio of 0.9901573 has been computed and the results are given below.

As mentioned in paragraph 3.1.2 a value of $\beta = 0.725$ was chosen as the parameter for the basic oval. From equation (4) we obtain

$$\cosh \gamma = 2.91481083.$$

Examining equation (11) it will be seen that the zeroes of this equation are given by

$$\sinh\lambda = \pm \cos\sigma \sinh \gamma.$$

Since $\gamma$ and $\sigma$ are known, the values of $\lambda$ at the positions of the zeros are determined. Since $\lambda$ is a function only of $\ell$ for constant $\beta$ the two values of $\ell$ can be obtained.

For the example of the text

$$\ell_1 = +0.632248112 - 0.351257149i$$

$$\ell_2 = -0.632248112 - 0.351257149i$$

The negative value, $\ell_2$, is taken to be the position of the rear stagnation point in the $\ell$ plane.

Considering the larger ovals, given by $\beta' = 0.8$, these ovals can be placed anywhere in the $\ell'$ plane so long as they include all zeros and infinities. To obtain the limiting case of a finally transformed aerofoil which has a cusped extremity, the $\beta'$ oval is positioned so as to actually pass through the previously determined zero. In this example the $\beta'$ oval was displaced so that its centre was at the point

$$\ell' = m' + in' = +0.112512215 - 0.0632i.$$

The procedure was then purely a computation of the aerofoil profile from equation (14) and subsequently computation of the pressure distribution and aerodynamic parameters from equations (18) and (19).

## APPENDIX B

## MERCHANT AND COLLAR CASCADE BLADE PROFILE AND PRESSURE DISTRIBUTION

Calculated correct to 7 d.p. on the Liverpool University Deuce electronic computer.

Arbitrarily selected parameters:-

$n' = -0.0632$

$\alpha_1 = 53.5^\circ$

Stagger angle= $37.5^\circ$ (Compressor)

$\beta' = 0.8$

$\beta = 0.725$

Derived parameters

$s/c = 0.9901573$

$m' = +0.112512215$

$n'$ and $m'$ are coordinates of $\beta' = 0.8$ oval centre in $l = m + in$ plane. Also $\tan \alpha_2 = +0.57793012$

| Reference Number N | Profile coordinates Based upon unit chord X | + iY | Cp $Cp = \dfrac{p - p_1}{\frac{1}{2}\rho u_1^2}$ |
|---|---|---|---|
| 1 | +0.1840367 | +0.0949930 | -0.7329363 |
| 2 | +0.6500792 | +0.0889340 | +0.0219360 |
| 3 | +0.5685042 | +0.1010401 | -0.0941639 |
| 4 | +0.5211449 | +0.1062824 | -0.1659479 |
| 5 | +0.4030553 | +0.1130286 | -0.3597584 |
| 6 | +0.3231776 | +0.1117387 | -0.4992400 |
| 7 | +0.2599128 | +0.1067143 | -0.6094902 |
| 8 | +0.2069867 | +0.0992816 | -0.6973123 |
| 9 | +0.1834664 | +0.0948774 | -0.7337957 |
| 10 | +0.1616129 | +0.0900909 | -0.7656136 |
| 11 | +0.1223507 | +0.0795404 | -0.8156043 |
| 12 | +0.0884371 | +0.0679148 | -0.8470333 |
| 13 | +0.0595318 | +0.0554474 | -0.8571795 |
| 14 | +0.0356275 | +0.0423583 | -0.8368069 |
| 15 | +0.0170584 | +0.0288868 | -0.7538457 |
| 16 | +0.0046149 | +0.0153348 | -0.4766543 |
| 17 | -0.0001414 | +0.0021437 | +0.4363445 |
| 18 | +0.0000631 | -0.0003936 | +0.6827467 |
| 19 | +0.0059857 | -0.0099254 | +0.9211494 |
| 20 | +0.0308941 | -0.0192226 | +0.4121105 |
| 21 | +0.1109821 | -0.0193545 | +0.2977494 |
| 22 | +0.1451326 | -0.0159400 | +0.3248899 |
| 23 | +0.1464393 | -0.0157915 | +0.3260522 |
| 24 | +0.1653660 | -0.0135341 | +0.3432965 |
| 25 | +0.2472125 | -0.0025069 | +0.4167019 |
| 26 | +0.2708527 | +0.0007947 | +0.4359517 |
| 27 | +0.2725677 | +0.0010332 | +0.4373066 |

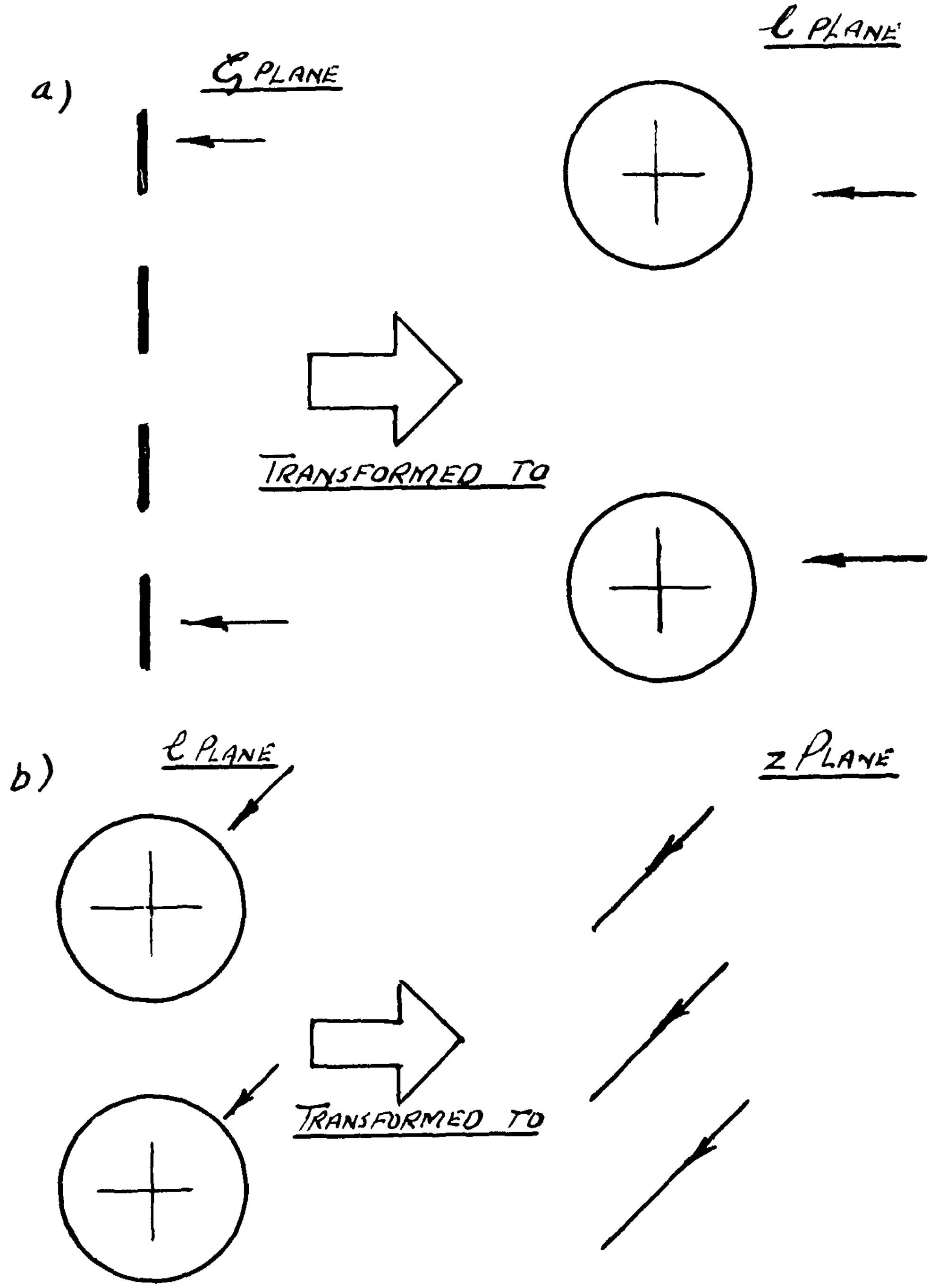
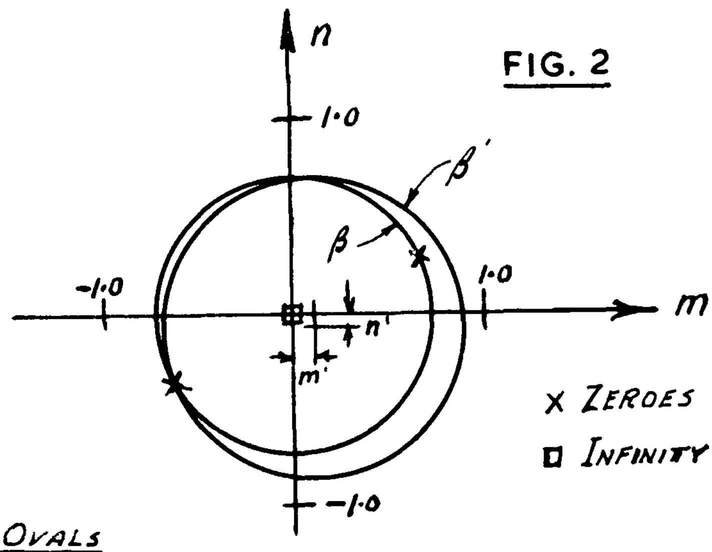
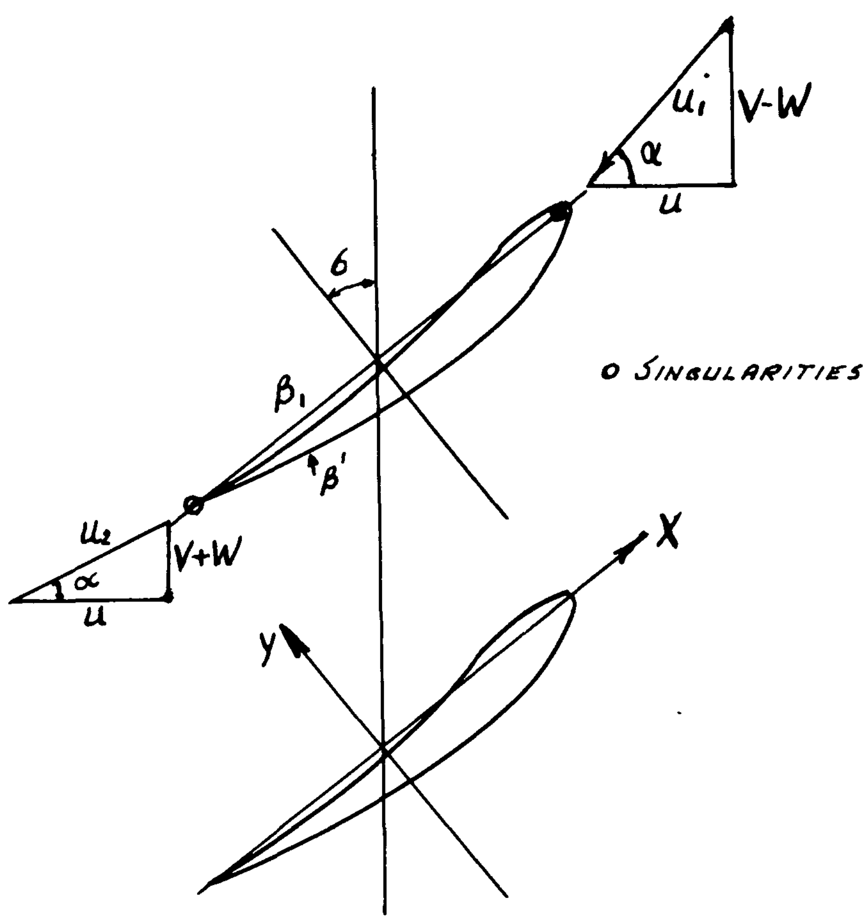



FIG 1. STAGES IN THE TRANSFORMATION

FIG. 2




THE L PLANE OVALS




Z PLANE CASCADE AND NOTATION

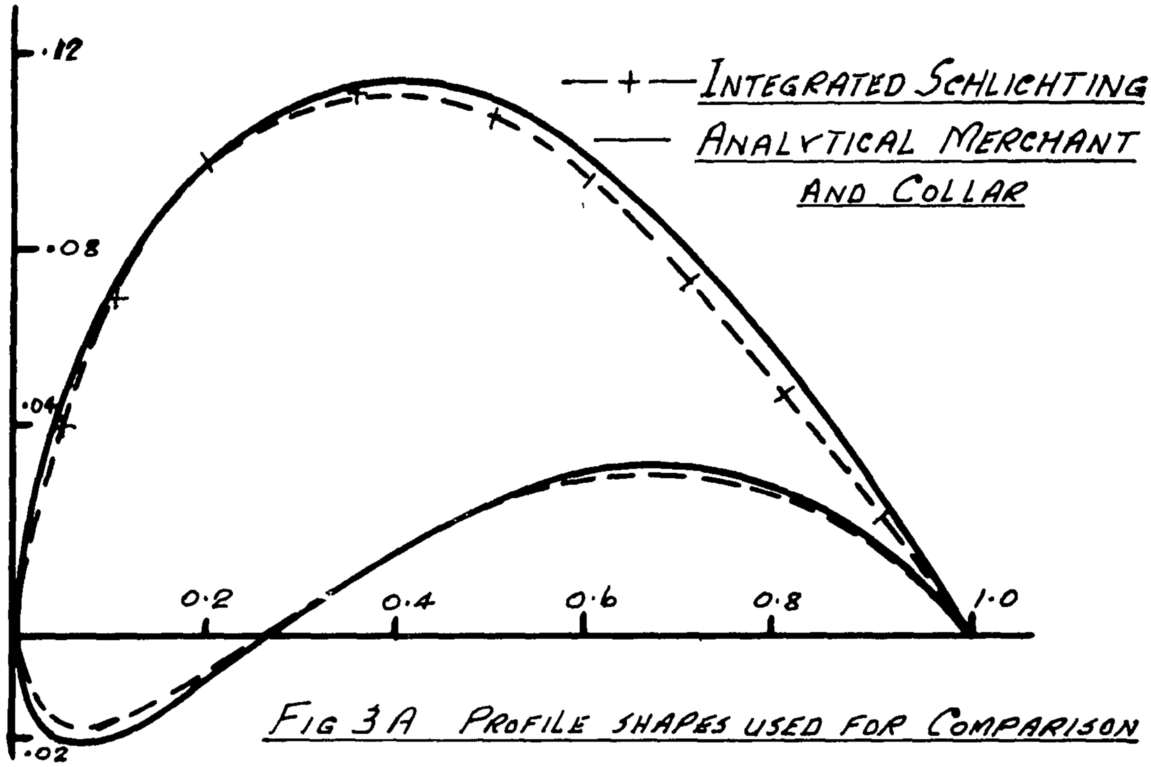



FIG 3A PROFILE SHAPES USED FOR COMPARISON

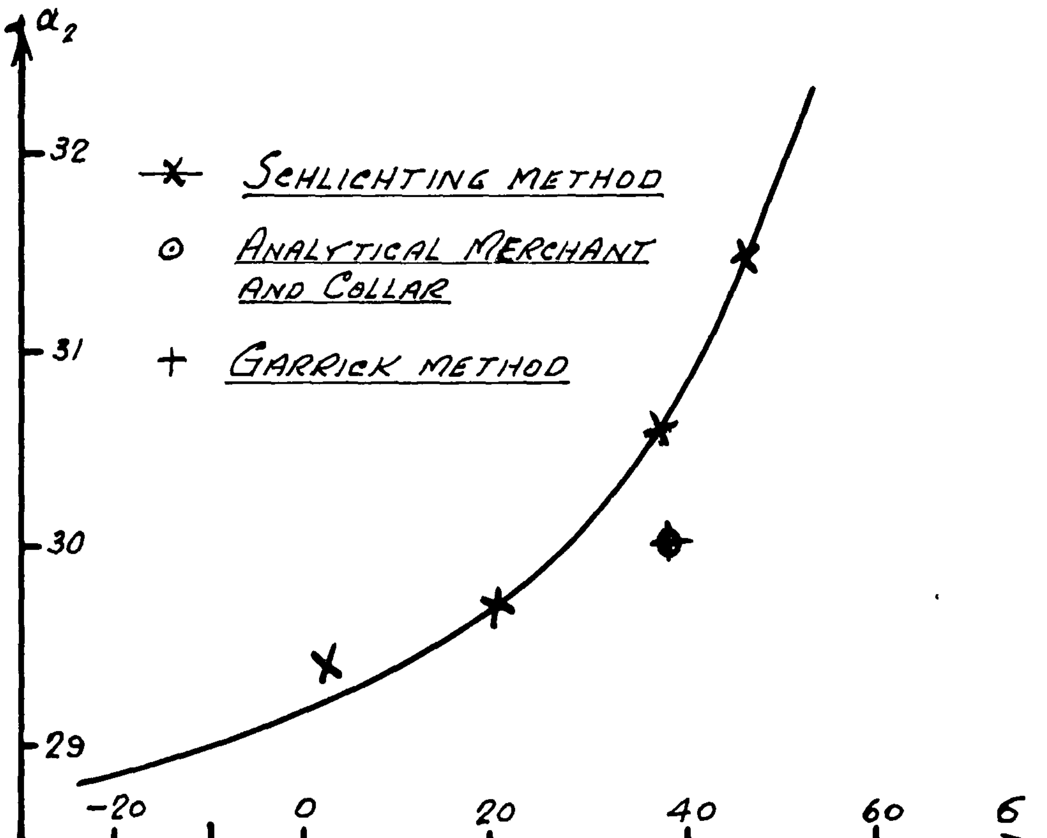



FIG 3B. OUTLET ANGLE AS A FUNCTION OF STAGGER

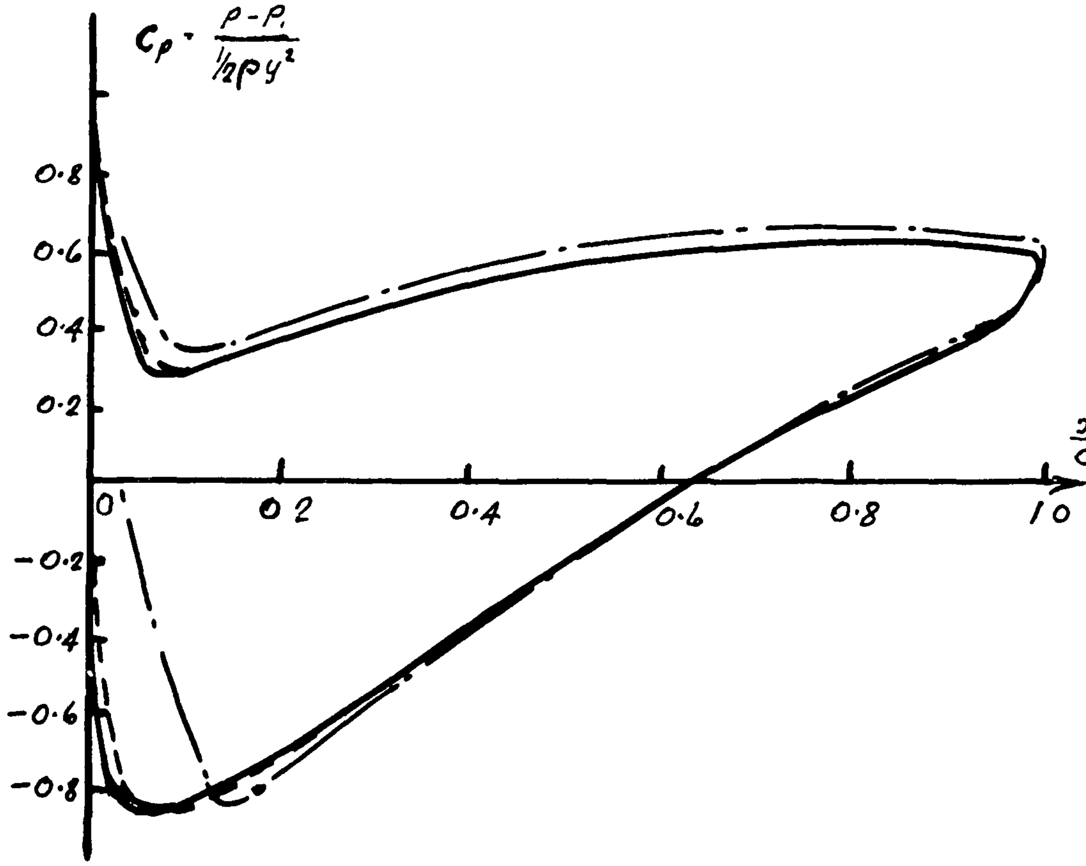



FIG 4. PRESSURE DISTRIBUTION FOR MERCHANT AND COLLAR PROFILE

STAGGER $37\frac{1}{2}°$ $s/c$ 0·9901573 $\alpha_1 = 53·5°$

KEY

———— ANALYSIS

—— - —— SCHLICHTING METHOD (POLLARD & WORDSWORTH)

- - - - GARRICK METHOD (HALL)

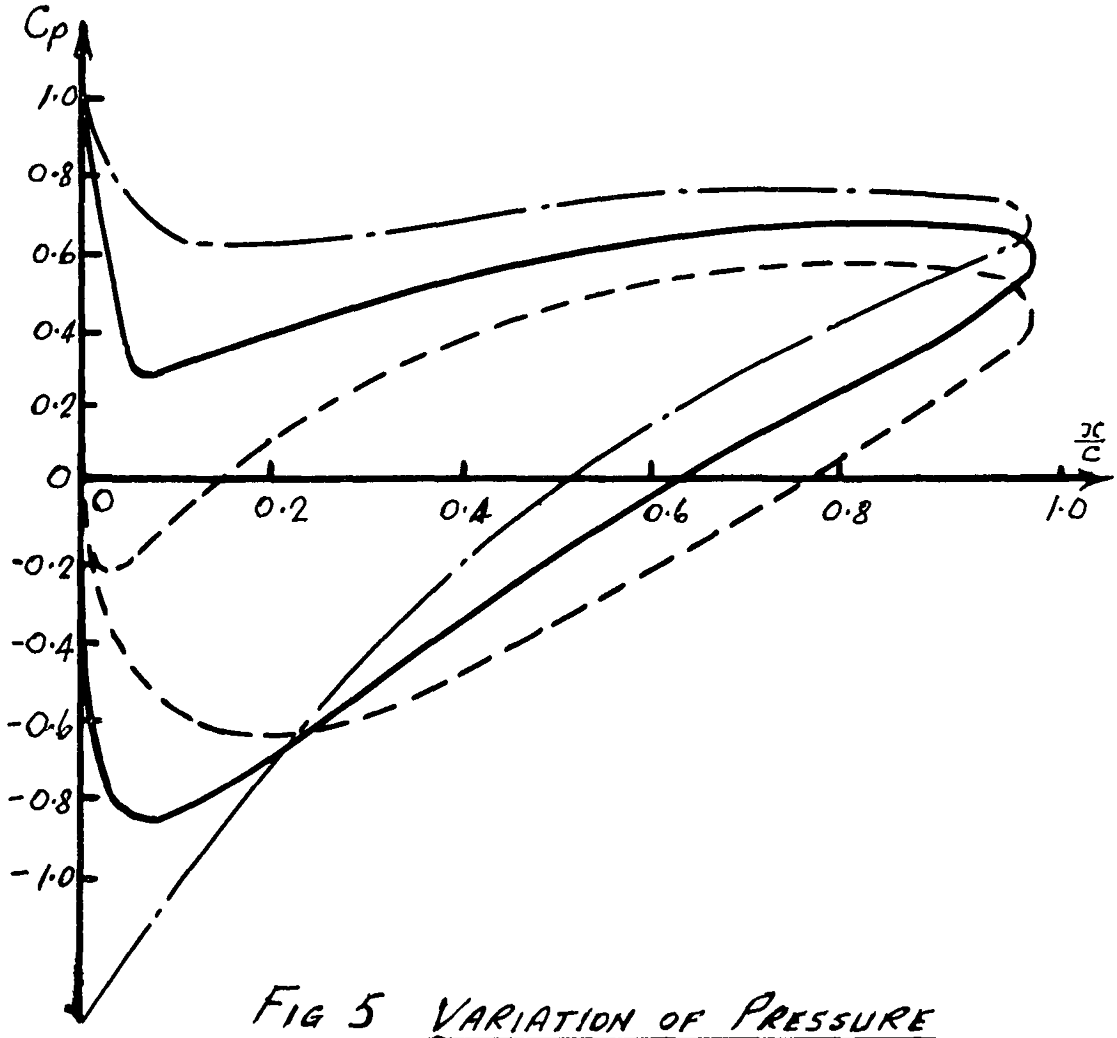



FIG 5 VARIATION OF PRESSURE DISTRIBUTION WITH INFLOW ANGLE

ANALYTICAL METHOD

STAGGER 37½°  S/C = 0·9901573

| KEY | α° |
|---|---|
| - - - - - - | 47·5 |
| ———— | 53·5 |
| ——·—— | 59·5 |

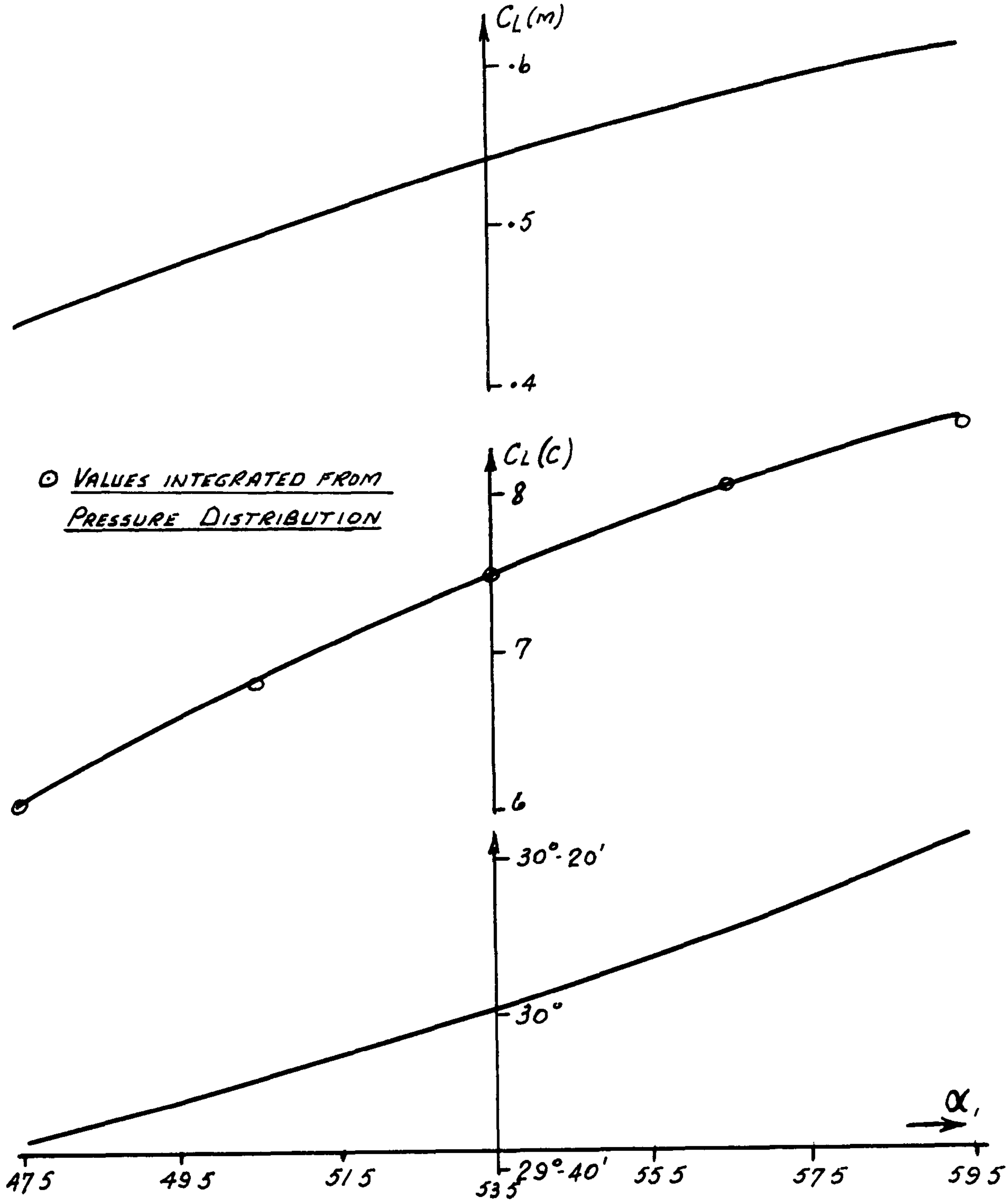
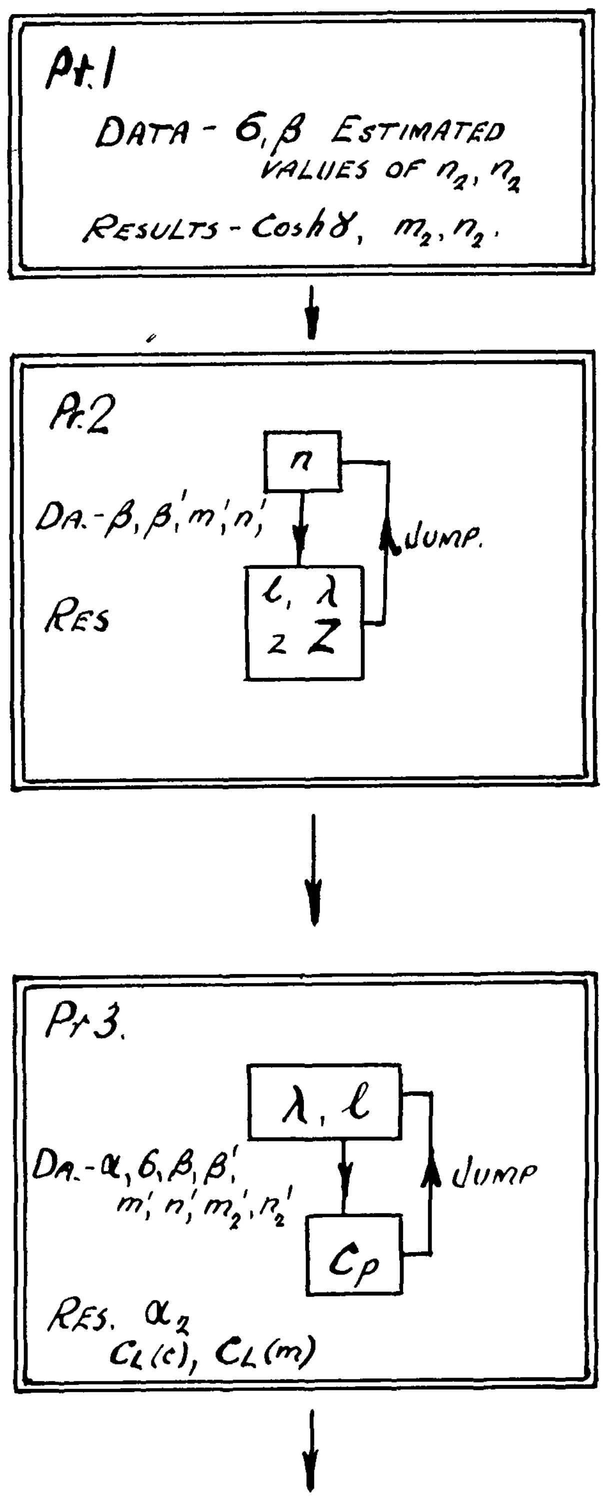



FIG 6 VARIATION OF LIFT COEFFICIENT AND OUTLET ANGLE WITH INLET ANGLE $\alpha_1$




APPENDIX C. COMPUTER PROGRAM BLOCK DIAGRAM.

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April, 1964
J. P. Gostelow

POTENTIAL FLOW THROUGH CASCADES
A COMPARISON BETWEEN EXACT AND APPROXIMATE SOLUTIONS

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