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# The Time Required for High Speed Airstreams to Disintegrate Water Drops

by

D. C. Jenkins, B.Sc., A.F.R.Ae.S. and J. D. Booker, A.F.R.Ae.S.

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#### THE TIME REQUIRED FOR HIGH SPEED AIRSTREAMS TO DISINTEGRATE WATER DROPS

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#### SUMMARY

The time required for high speed airstreams to disintegrate water drops has been determined experimentally, and an empirical relation found between the time, the airstream velocity and the drop diameter. The acceleration of drops during disintegration has also been found and an empirical relationship derived. The equation of motion of a disintegrating drop has been considered and a drag coefficient determined which gives a drop motion agreeing reasonably well with that found experimentally in a particular case. Droplets produced during disintegration have been measured in a particular case and compared with the sizes that would be expected if some of the proposed mechanisms of disintegration were operative. It has not been found possible to determine conclusively what mechanism operates to cause disintegration, but the evidence favours a wavemaking mechanism.

Suggestions for further work have been made in order to establish the effect of other parameters involved in the disintegration process.

Replaces R.A.E. Tech. Note No. Mech. Eng. 401 - A.R.C. 26531

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#### 1 INTRODUCTION

In investigations of problems of erosion of the forward facing surfaces of an aircraft or missile travelling at high speed through rain, information is required on how long it takes a raindrop to be disintegrated when subject to an airblast<sup>1,2</sup> or alternatively the degree of disintegration achieved in a given time. A typical example in the case of supersonic flight through rain is the effect on the raindrop of the airflow behind a shock wave. Very little systematic experimental work appears to have been done on this problem and the various theoretical approaches which have been postulated give widely differing results<sup>1</sup>. For these reasons a study of the disintegration of water drops exposed to sudden airblasts has been undertaken using a novel experimental technique developed to overcome the difficulty experienced in determining the progress of disintegration with time. The results of the study are presented and discussed in this paper. No attempt has been made to assess the erosive effect of the small droplets resulting from disintegration.

#### 2 EXPERIMENTAL METHOD

#### 2.1 General

To determine the time taken to disintegrate a water drop in an airstream, the obvious method is to use photography and to record the state of the drop at successive intervals of time. The end of the disintegrating period can then be defined as the time at which the drop appears to have been completely reduced to a fine mist. An objection to this method is that it is subjective; different experimenters might well reach different conclusions on studying the same set of photographs. The photographic result may be sensitive to the illumination and photographic techniques used and the method does not necessarily give a measure of the droplet size reached.

To avoid these difficulties the method adopted for the tests described in this Note uses, in addition to photography, a sampling technique whereby the droplets formed and the residue of the drop remaining, are caught and measured at successive stages. Disintegration can then be said to be complete when the residue has been reduced to droplets, none of which is larger than an arbitrarily chosen small value or, alternatively, when it appears from the sampling that no further reduction in size of the residue droplets is taking place. The sampling method adopted is made possible by using a long hollow projectile closed at the rear end and open at the front, to produce the air blast.

#### 2.2 <u>Test apparatus</u>

The projectile, shown diagrammatically in Fig.1 and photographically in Fig.2, is fired from an air gun<sup>2</sup> at a water drop of given size suspended on a fine web, Fig.3, positioned such that the drop will enter the nose of the projectile, Fig.4. A column of air is carried along inside the projectile so that on entry the drop is suddenly exposed to an airstream of velocity equal to the speed of the projectile, which can be measured readily using standard timing techniques, in this case by measuring the time for the projectile to pass and break two wires placed a known distance apart. Fig.5 shows the disintegration of a 2 mm diameter water drop inside a transparent hollow projectile moving at

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560 ft/sec, the manner of disintegration being quite typical for suddenly applied airstream conditions.

To catch the residue and droplets, a sampling slide is fixed across the inside of the projectile at a known distance from the entry into the nose. If the slide is placed near to the entry so that the drop is not completely disintegrated before it reaches the slide but contains a core or residue, then very fine droplets from the disintegration process, together with several large drops from the residue of the main drop, impinge on the slide and are collected. If the slide is placed sufficiently far from the entry for disintegration to have been completed only very fine droplets will be collected. The progress of disintegration can be followed by noting the size of the droplets caught on the slide as it is placed at increasing distances from the entry in repeated tests using constant speed and constant initial drop size. In preliminary development tests of the method it was found that the residue drop size as measured in this way reduced in size to a value in the range 0.1-0.15 mm in diameter. It was, therefore, arbitrarily decided to take as the point where complete disintegration was achieved in subsequent tests the point where the residue droplets caught on the slide measured 0.1-0.15 mm. Due to the flattening of the droplets on the slide the diameter of the spherical droplets before impacting on the slide would be somewhat less than the measured diameter on the slide; no attempt was made to correlate these two diameters. The test apparatus is designed so that the projectile can be caught without damage and the sampling slide can be recovered and the droplets on it examined and measured with a measuring microscope.

Additionally, the metal version of the hollow projectile can be fitted with an internal slide running along its length, see Figs.1 and 2, called the lateral sampling slide. By this means the droplets thrown off the surface of the disintegrating drop can be caught and measured at all intervals of the disintegration period as the disintegrating drop passes along the length of the slide.

#### 2.3 Test procedure

To determine the time that has elapsed since the drop entered the projectile up to any given stage, it is necessary to determine the length  $\ell$ , Fig.1. This is made up of two components x and y. y is the penetration of the drop inside the projectile and x is the distance moved by the drop whilst being acted on by the air blast. The time that the drop has been in the airstream is then given by  $t = \ell/12U$  seconds where U is the speed of the projectile in ft/sec and  $\ell$  is in inches.

It has been assumed here that the action of the airblast on the drop starts as the drop enters the nose of the projectile. However, due to the airflow round the nose of the projectile the blast effect on the drop starts a short distance before entry and reaches its maximum value at entry. Calculations suggest that the blast effect experienced by the drop when it is at a distance in front of the entry equal to the diameter of the projectile is negligible; also photographs of drops just before entry into the projectile do not reveal any signs of premature distortion or disintegration (e.g. see Figs.4 and 9A) which would be the case if the drop experienced significant blast effects prior to entry into the projectile.

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The value of y at the point of complete disintegration is determined by repeated tests at constant speed and initial drop size using the metal hollow projectile with adjustable sampling slide position. The variation of residue maximum drop size with depth of penetration y is recorded and y is increased by adjusting the slide position until complete disintegration is obtained, as defined in para.2.2. Fig.6 shows the variation of residue drop size with penetration in a typical case where the adopted standard for complete disintegration was obtained. In a few cases, however, the combination of initial drop size, airstream velocity and projectile length was such that this standard was not reached. The particular cases where this occurred were the disintegration of 2.25 mm diameter water drops in airstreams of 250, 365 and 473 ft/sec. In these cases the drop was taken to be completely disintegrated when the largest droplets caught on the slide measured approximately 0.2 mm diameter. The reason for the larger droplets found in these cases is not known, but it is thought to be due to coalescence of droplets on the sampling slide caused by the large number of droplets caught. Fig.7 shows the variation of residue drop size with penetration in a case where the adopted standard for complete disintegration was not reached. Fig.8 shows typical droplet slide samples corresponding to the case illustrated in Fig.7.

The value of x corresponding to y at complete disintegration is obtained by determining the relationship between x and y for each speed and initial drop size. Using the transparent hollow projectile a set of photographs was taken of the progress of disintegration using flash photography with the projectile operating a trigger wire and in each photograph the movement x of the drop was measured using the web holder as a datum. Measurements were taken to the front of the drop as indicated in Fig.1. Lines engraved on the outside of the transparent projectile at regular intervals, and which appeared on the photograph, enabled corresponding values of depth of penetration y to be measured. These coincident values of x and y were recorded and used in the preparation of a set of graphs of x against y and x against t covering the various drop sizes and velocities investigated. Fig.9 shows a typical set of photographs for a 2.25 mm diameter water drop disintegrating in a projectile moving at 365 ft/sec. It is to be noted in this set of photographs that different designs of web holder have been used at various stages. This was necessary to permit correct positioning of the flash illumination trigger wire and which is operated by the projectile.

Some of the droplets formed during disintegration were collected on the lateral slide carried in the metal hollow projectile. After recovery of the projectile this slide was removed and examined for droplet size and distribution. A typical distribution of droplets from this slide is shown in Fig. 10.

#### 2.4 <u>Reliability of method</u>

#### 2.4.1 Airblast velocities

The method used in these tests for producing an air blast relies on a column of air being carried along by the moving hollow projectile. This air will be at a slightly increased pressure due to ram effect for which air must flow into the projectile through the hole at the front. During the development of the technique it had been considered that the projectile movement before reaching the drop, should be as large as possible to ensure that airflow into the

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projectile was complete before the drop entered. In the initial development stage of the method, the gun used had a barrel length of 2 ft and the drop could be placed at approximately 16 in. from the gun muzzle, the total projectile movement thus being 3 ft 4 in. A second apparatus became available after this development period with a gun having a barrel length of 4 ft and in which the drop could be placed at approximately 7 ft 6 in. from the gun muzzle, giving a total projectile movement of 11 ft 6 in. Accordingly, some experiments conducted on the first gun were repeated on the second to see if the increased projectile travel made any difference to the disintegration results. In particular, the movement of water drops of 2 mm diameter in a hollow projectile travelling at 560 ft/sec was studied photographically. The records of movement using the first gun and those using the second gun are shown in Fig.11. It is seen that both sets of results follow the same trend with the scatter of results about the mean curve being approximately the same for each set. The same values of x when plotted on a time basis are shown in Fig.12 where, again, agreement seems close between the two sets of results and one fitted curve suits both sets of results. The depth of penetration y to complete disintegration was also checked on both guns for a number of cases and the results found to be in fair agreement (para. 3.1).

The overall agreement in results using the two different test conditions is evidence that the method produces an air blast which is the same for the two test cases and, in view of the long projectile movement in the case of the second test apparatus, it is considered that the air blast velocity produced is uniform, free from wave escillations and equal to that of the projectile.

#### 2.4.2 Sampling method

It may be considered that a droplet sample showing only maximum droplets of, say, 0.1-0.15 mm diameter, which is the size arbitrarily taken to indicate complete disintegration (para.2.2), may result from larger droplets being broken to this size on impact with the sampling slide or, alternatively, any larger drops collected maybe removed by the deceleration caused by catching the projectile. However, in the case of sampling at 633 ft/see which is the highest projectile velocity recorded for sampling (Table 8) droplets of 0.1, 0.15, 0.2, 0.3 and 0.5 mm diameter were collected at different times. This is taken as evidence that maximum droplet sizes 0.1-0.15 mm diameter do not result from the removal during deceleration of any larger droplets or that they are caused by the rupture on impact of droplets larger than 0.15 mm diameter.

Near the point of complete disintegration some difficulty was experienced in distinguishing between residue droplets and droplets formed by the disintegration process. As the initial drop was positioned on the axis line of the gun residue droplets should impact on the central area of the sampling slide, but for a conservative estimate complete disintegration was taken when the largest droplets were of the size range 0.1-0.15 mm diameter appearing anywhere on the sampling slide. As mentioned in para.2.3 it was suspected that coalescence occurred in some cases due to the large number of droplets caught on the sampling slide, in these cases it may well be that the true point of complete disintegration of the residue may have occurred at a value of y somewhat less than that taken.

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#### 3 RESULTS

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#### 3.1 Disintegration times

Disintegration times were studied in test groups of (i) constant drop diameter with varying airspeed and (ii) constant airspeed with varying drop diameter. These two groups were split up into sets as follows:-

Set A Constant airspeed of 560 ft/sec with drop sizes varying from 1.0 to 2.75 mm diameter.

Set B Constant airspeed of 250 ft/sec with drop sizes varying from 1.0 to 2.25 mm diameter.

 $\underline{Set\ C}$  Constant drop size of 2.25 mm diameter with airspeeds varying from 250 to 640 ft/sec.

<u>Set D</u> Constant drop size of 1.5 mm diameter with airspeeds varying from 250 to 612 ft/sec.

The records of drop movement x for various depths of penetration y in the projectile are given in Tables 1, 2, 3, 4 and 5. In Figs.13, 14, 15 and 16 the movement x is plotted against time t. Not all the experimental points have been shown, to avoid confusion, but the least squares fitted curves of the experimental points are shown, together with some typical sets of experimental points for illustration of the degree of scatter. It is seen that a least squares curve of the form  $x = A + Bt^2$  fits the results quite well. In Figs.17, 18, 19 and 20 the movement x of the drop is plotted against the penetration y into the projectile for each set.

In fitting a polynomial curve of the form  $x = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots$ through the experimental points of x plotted against t,  $a_0$  represents an initial deflection of the drop (i.e. at time t = 0),  $a_1$  represents an initial velocity of the drop at t = 0 and  $2a_2$  represents the acceleration of the drop at t = 0. From physical considerations, if these constants have values they should be small and positive. To determine the best polynomial to fit all the experimontal results the following three polynomials were considered:-

(1)  $x = A + Bt^{2}$ (2)  $x = A + Bt + Ct^{2}$ (3)  $x = A + Bt^{2} + Ct^{3}$ 

and in each case the values of the constants were calculated so as to give a least value to the sum of the squares of the deviations of the experimental points from the curve. Equation (1) was found to give the best fit in general; the improvement in changing to equation (2) led to a significantly better fit in only one case, but gave 13 cases in which one of the constants was negative which is not tenable from physical considerations. The use of equation (3)

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gave four cases with a significantly better fit than using equation (1) but gave 17 cases with negative constants: the use of equation (1) gave only one case of a negative constant being the value of the initial displacement A in case D.3. An equation of the form  $x = A + Bt^2$  implies that the acceleration of the residue of the drop during the disintegration period (which is given by  $(2B/12) \times 10^6$ ft/sec<sup>2</sup>) is constant for all values of t. Considering the values of B found in all the test cases, it is shown in Appendix B that the acceleration of the drop varies linearly with  $U^2/D$  where U is the airstream velocity and D is the drop diameter. No regular variation in the value of A has been found.

For each case of the above sets of experiments, the depths of penetration y to the point of complete disintegration has been found by the adjustable slide method. The results are shown in Tables 6, 7, 8 and 9, where also the corresponding values of x, found from the movement data given in Tables 1, 3, 4 and 5 and plotted in Figs. 17, 18, 19 and 20 are given. From these values of x and y the time  $t_s$  of exposure to the appropriate air blast of speed U needed to cause

complete disintegration has been calculated and is also given in Tables 6 to 9. In a number of cases the values of y to the point of complete disintegration found by using the gun No.1 were checked using the gun No.2 and, where appropriate, both results are given in the Tables. The agreement is reasonably good, values being somewhat greater in general for gun No.2 than for gun No.1. In calculating t the larger value of y has been taken in each case. A time t oph

is given in the tables which is the disintegration time as judged from the photographs used for the determination of x and y. The value of  $t_{ph}$  is given as

the value at which the drop first has the appearance of being reduced to a cloud of droplets, but for the reasons explained below it is possible that, in general, the correct point of complete disintegration maybe slightly later than the times given. In the case of 2.25 mm diameter water drops in an airstream of 365 ft/sec, the disintegration progress of which is shown in Fig.9, the drop first has the appearance of a mist in Fig.9G although there is one dark area which could either be a dense portion of mist droplets or could contain a residue which has not disintegrated. Fig.9H shows a more complete stage of fine mist production but there is no absolute reason for not taking Fig.9G to represent the stage of complete disintegration and if this is accepted it is reasonable to suppose that complete disintegration cculd in fact occur at some stage between Figs.9G and 9F. The disintegration times represented by these various figures are shown in Fig.9.

During the disintegration process the drop appears to break into several separate centres which may proceed to disintegrate simultaneously with the main drop. This break-up could follow the penetration of air holes through the drop by the airstream. Such a stage in the case of 2.25 mm drops in an airstream of 365 ft/sec is shown at Fig.9E. The time at which this break-up occurs is listed as t in Tables 6-9 for the various cases considered.

#### 3.2 Droplet sizes formed during disintegration

This aspect of the disintegration process was not studied in great detail, but the method using the lateral sampling slide was checked in a few cases. Fig. 10 shows droplets thrown off the surface of a 2.75 mm diameter drop, whilst disintegrating in an airstream of 352 ft/sec: this was a special test not included in the disintegration tests. Fig.10A shows the droplet catch at the beginning of disintegration, whilst Fig.10B shows the droplet catch towards the end of the disintegration period. At the beginning of disintegration, numerous droplets were caught, the largest being about 160 microns and the smallest about 10 microns in diameter, with an average of 45 microns. Towards the end of the disintegration period, fewer droplets were caught and the range of droplet sizes was also reduced, with the largest being 90 microns and the smallest being 50 microns and the average being 68 microns.

#### 4 DISCUSSION OF RESULT'S

#### 4.1 Disintegration time

The disintegration times for sets A and B, i.e. for cases of various drop sizes and constant airspeed are plotted in Fig.21 from which it is seen that the disintegration time  $t_s$  can be taken as linearly proportional to drop size, i.e.

$$t_{s} = K_{1} D_{0}$$
(1)

where K, is a constant.

The disintegration times for sets C and D, i.e. for cases of constant drop size but varying airstream velocities, are plotted in Fig.22 on a logarithmic basis.  $\log_{10} t_s$  can be taken as linearly proportional to  $\log_{10} U$ , the slope of the curves being such that  $t_s$  can be expressed in the form

$$t_{s} = K_{2}/U^{n}$$
<sup>(2)</sup>

where  $K_2$  is a constant. Combining equations (1) and (2) an empirical relation for t can be derived of the form:-

$$t_{s} = K_{3} D_{o} / U^{n} .$$
 (3)

From the graphs it appears that a reasonable value for n is 0.72 and 20.0 for  $K_3$ . The empirical expression for  $t_s$ , within the limits of the drop size and airstream velocity cases taken, becomes:-

$$t_s = 20.0 D_o / U^{0.72} \text{ seconds}$$
 (4)

where  $D_0$  is in ft and U is in ft/sec. Figs.21 and 22 show a large amount of scatter in the values of the experimental results. This may be caused by an error in the estimate of the value of y to complete disintegration as discussed in

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para.2.4.2 or a possible error in the corresponding value of x due to the fact that it was found necessary, in some cases, to extrapolate the curve of x against y to the value of y for complete disintegration. A particularly bad case of scatter is that of a 2.25 mm diameter drop in an airstream of 365 ft/sec. This is one of the cases where coalescence of the droplets was suspected and the value of y for complete disintegration to droplets of 0.2 mm diameter was taken as 9.1 in. (Fig.7) corresponding to  $t_s = 2.69$  milliseconds. However, if a value of y = 7.7 in. is taken which is the point at which maximum size droplets of 0.2 mm were first found the corresponding value of  $t_s$  is 2.14 milliseconds and when this value is used for plotting on Fig.22 much closer agreement with the assumed curve through the experimental points is obtained. If the photographically determined value of disintegration time is used ( $t_s = 1.76$  milliseconds) (para.3.1) a value much below the assumed curve results.

The empirical formula for disintegration time [equation (4)] does not appear at first sight to be dimensionally consistent. This is because the constant term includes the effect of other parameters such as viscosity, density and possibly surface tension which in a complete expression would appear separately such as in equations (21) and (24) which give disintegration times in terms of all the parameters.

 $Engel^2$  quotes disintegration times t<sub>se</sub> for 1.4 mm diameter water drops in airstreams of various velocities. The disintegration time t is that required to reduce the drop to a fine mist as seen photographically. The airstreams are those produced by plane shock waves of various Mach numbers  $N_{\rm m}$  giving an air velocity U ft/sec behind the shock wave. The results are tabulated below, together with the corresponding disintegration times t found by using the empirical formula  $(l_{+})$ . It is seen that quite close agreement is reached despite the fact that considerable extrapolation of the range of validity of the empirical formula has been used. The discrepancy between calculated and test results can be due to a number of causes. Firstly, the airstream conditions used in the shock wave tests differ considerably from those used in deriving the empirical formula in that the relative density  $\sigma$  and the temperature  $\phi$  are higher. Secondly, the times used in deriving the empirical formula are based on disintegration to a measured residual drop size of the order 0.1-0.15 nm, whereas the times quoted by Engel are based on the rather less precise method of determining photographically when the drop has been reduced to a fine mist. The relative density  $\sigma$  and relative kinematic viscosity  $\beta$  of the air in the shock wave tests are given in the table, but it is not immediately obvious what arrangement or power of these parameters is required to bring the calculated results to closer agreement with the observed results. It is noticed that times based on the formula are all slightly larger than those based on photography; this will be discussed further later. It is suggested that, in fact, the effect of these parameters is of second order importance and that the discrepancies can be accounted for in differences of test techniques and extrapolation of empirical results.

As discussed in para.3.1 the determination of disintegration time by photography may not always be straightforward or precise and in the series of experiments reported here disintegration times determined from photographs have generally been found to be shorter than times found by the sampling method (Tables 6-9). This is also reflected in the comparison above of the photographic results of Engel and the corresponding values extrapolated from the empirical formula.

N <sub>m</sub>	¢ °C	σ	β	t <sub>se</sub> secs	t sc secs	U ft/sec
1.3	84	1.38	1.12	$0.90 \times 10^{-3}$	1.05 × 10-3	498
1.5	123	1.69	1.27	$0.70 \times 10^{-3}$	$0.76 \times 10^{-3}$	777
1.7	165	1.99	1.38	$0.58 \times 10^{-3}$	$0.62 \times 10^{-3}$	1036

### 4.2 Disintegration rate

1

The volume of the disintegrated drop =  $V = \pi D_0^3/6 \text{ ft}^3$ .

Hence the average rate of disintegration =  $V/t_s$  (= -dV/dt)

$$= (\pi D_{o}^{3}/6) \times (U^{0.72}/20.0 D_{o}) \text{ ft}^{3}/\text{sec}$$
(5)

i.e. 
$$dV/dt = -(\pi D_0^2 U^{0.72})/(6 \times 20.0) ft^3/sec$$
 (6)

This suggests that the instantaneous rate of disintegration could reasonably be expressed in the form

 $dV/dt = -KD^2 (U - w)^{0.72}$  (7)

where D is the instantaneous drop diameter and (U - w) is the instantaneous relative velocity.

Also 
$$dV/dt = (\pi D^2/2) \times (dD/dt)$$
 (8)

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Hence in (7)

$$dD/dt = 2 dR/dt = (-2K/\pi) \times (U-w)^{0.72} = (-2K/\pi) \times U^{0.72} \times (1-w/U)^{0.72} . (9)$$

By using the equations of the fitted curves and the values of the disintegration times t to determine w at the end of disintegration it is found that  $(w/U)_s$  has an average value of 0.27. The average final value of  $(1 - w/U)^{0.72}$  is thus approximately 0.8. The initial value of  $(1 - w/U)^{0.72} = 1$  when w = 0. Thus the average value of  $(1 - w/U)^{0.72}$  during the disintegration interval = 0.9. i.e. equation (9) thus becomes

$$dD/dt = -1.8 \text{ KU}^{0.72}/\pi$$
 (10a)

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integrating (10a) gives

$$D = -1.8 \, \text{KU}^{0.72} \, \text{t/}\pi + A \qquad (10b)$$

D = D<sub>o</sub> when t = 0 hence A = D<sub>o</sub>.  
D = 0 when t = t<sub>s</sub>. Hence in (10b) 0 = - 1.8 KU<sup>0.72</sup> t<sub>s</sub>/
$$\pi$$
 + D<sub>o</sub>.

$$t_s = \pi D_0 / 1.8 KU^{0.72}$$
 (11)

Comparing (11) with (4) gives  $K = \pi/36$ . Hence in (9)

$$dR/dt = - (U - w)^{0.72}/36.0$$
 (12)

#### 4.3 Premature break-up

The drop break-up discussed in para.3.1 could be due to perferation or penetration of the drop by air pressure. Gordon<sup>4</sup> has discussed the possibility of penetration of water drops by airstreams, a simplified form of his theory in which the effect of surface tension and viscosity of the fluid are neglected is given in Appendix A, where theoretical results are compared with experimental values of t. The simplified Gordon method gives a time t for penetration of the drop by air pressure where:-

$$t_{p} = 2(D/U)(\rho_{\ell}/\rho_{a})^{\frac{1}{2}}$$
 (13)

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Taking standard values for density, this leads to

$$t_{\rm p} = 57({\rm D/U})(1/\sigma^2)$$
 (14)

From the discussion of Appendix A, the time at which breakaway of portions of the drop occurs is given empirically as:-

$$t_p = 50 \times (D/U)(1/\sigma^{\frac{1}{2}})$$
 (15)

The good agreement between the simple Gordon theory and the experimental breakaway time t suggests that breakaway could follow penetration of the drop in the manner discussed by Gordon.

# 4.4 Droplet sizes formed during disintegration

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Taylor<sup>5,6,7</sup> has suggested that when air blows over a drop unstable waves may be formed, the crests of which become detached and blow away in the form of droplets, the diameters of which may be of the same order as the wave length. The expression developed by Taylor for the wave length of the most unstable wave is:-

$$\lambda = 2\pi x_{\rm m} T/\rho_{\rm a} U^2$$
 (16)

where  $x_m$  is a function of another parameter  $\theta$  given by:-

$$\theta = (\rho_{\ell} / \rho_{a}) (\mathbb{T}^{2} / \mu^{2} \mathbb{U}^{2})$$
 (17)

 $\mu$  is the coefficient of viscosity of the fluid, and T the surface tension. Values of x for a range of values of  $\theta$  have been computed. Using these values in the case of a drop of diameter 2.75 mm in an airstream of velocity 352 ft/sec (see para.3.2) gives a value of  $\lambda = 50$  microns at the start of disintegration. This is to be compared with the range of drop sizes found at this stage of 10-160 microns with an arithmetic mean value of 45 microns (para.3.2). This mean size is in reasonable agreement with the predicted value.

At the end of the disintegration period in the case under consideration, the velocity of the airstream relative to the drop is (U - w) where w is the velocity of the drop and is given empirically by  $w = g_{\ell} \times t_s$  where  $g_{\ell} = 2.08 \times 10^{-3} \times U^2/D_o$  (see Appendix B) and  $t_s = 20 D_o/U^{0.72}$  (para.4.1) with U = 352 ft/sec this gives w = 73 ft/sec. Hence relative velocity over the drop at the end of disintegration = 279 ft/sec. In (17)  $\theta = 580$  and  $x_m = 1.52$ . Hence in (16)  $\lambda = 75$  microns. This is an increase in droplet size over the

- 15 -

predicted value of 49 microns at the start of disintegration and is to be compared with the droplet size range caught at the end of disintegration of 50-90 microns with an arithmetic mean of 68 microns (para.3.2).

It is seen from equations (16) and (17) that the wave length and hence the droplet diameter is independent of the main drop diameter and dependent only on the airstream velocity. This could be checked in a number of cases of constant airstream velocity, but with varying drop diameter.

An alternative theory also due to Taylor<sup>5</sup>, to describe the disintegration of a drop in an airstream is to imagine that the airflow continually strips layers off the drop equal in thickness to the boundary layer generated in the fluid. This layer then breaks up into droplets which it is assumed here will have a diameter of the same order as the thickness of the layer. The boundary layer thickness calculated by Taylor is given by:-

$$\delta = 8.6(X/U)^{\frac{1}{2}}$$
(18)

where X is the distance from the stagnation point to the rim of the flattened drop in om and can be approximately taken = 2R. Considering the above case, at the start of disintegration U = 352 ft/sec (= 10,700 cm/sec) and X = 0.275 cm. Hence  $\delta = 8.6(0.275/10.7 \times 10^3)^{\frac{1}{2}} = 0.0435$  cm = 435 microns.

Towards the end of the disintegration process X approaches zero. Hence droplets should be vanishingly small at this stage whereas they were measured to be in the range 50-90 microns with a mean value of 68 microns.

Thus neither of the predictions made above using the boundary layer concept of disintegration give droplet sizes similar to those caught in the case of disintegration of a drop of diameter 2.75 mm in an airstream of 352 ft/sec. (10,700 cm/sec) being larger than found at the start of disintegration and smaller than found at the end of disintegration.

#### 4.5 Acceleration of a disintegrating drop

As recorded in para.3.1, the movement in an airstream of a water drop which is both accelerating and disintegrating, has generally been found to be expressible in the form  $x = A + Bt^2$  where x is the movement of the drop in inches and t the time in the airstream in milliseconds, A and B are constants. This type of relationship implies a constant acceleration  $g_{\ell}$  of the drop (= 2B × 10<sup>6</sup>/12 ft/sec<sup>2</sup>).  $g_{\ell}$  has been calculated for the cases considered and in Appendix B it is shown that there is an empirical relationship between  $g_{\ell}$  and  $U^2/D_0$  of the form:-

$$g_{\ell} = 2K_6 U^2/D_o$$
(19)

where  $K_6$  is a constant, U is the airstream velocity and D<sub>o</sub> is the initial diameter of the drop. The analysis of Appendix B shows that the constant acceleration of the disintegrating drop is the same as the constant acceleration of a drop of constant radius R<sub>o</sub> acted on by an airstream of constant relative velocity U with a constant drag coefficient  $C_D = 2.26$ . This compares with a drag coefficient  $C_D = 3.26$  which it is found in Appendix C to satisfy the equation of motion of a disintegrating drop in the case when U = 24.7 ft/sec and  $D_c = 1.0$  mm.

By combining the empirical relations for acceleration  $g_{\ell}$  and disintegration time  $t_s$ , it is shown in Appendix B that an empirical relation for the speed w of the residue of the disintegrating drop at the end of the disintegration period when d = 0.1-0.15 mm can be deduced in the form:-

$$(w/U)_{s} \alpha U^{0.28}$$
 (20)

This is independent of R and depends only on the airstream velocity U.

## 5 DISCUSSION OF SOME SUGGESTED DISINTEGRATION MECHANISMS

#### 5.1 Boundary layer stripping

Taylor<sup>5</sup> has proposed a boundary layer stripping mechanism to explain the disintegration of water drops by an airstream. His method leads to the following equation for the time of disintegration:-

$$\mathbf{t}_{s} = \left(\frac{O_{\bullet} 685}{2\pi}\right) \left(\frac{\rho_{\ell}}{\rho_{a}}\right)^{1/3} \left(\frac{\nu_{a}}{\nu_{\ell}}\right)^{1/6} \left(\frac{r_{o}^{3}}{U\nu_{\ell}}\right)^{\frac{1}{2}} \left[1 + \frac{C_{1}}{4}\right]$$
(21a)

$$C_{1} = \frac{3}{8} C_{D} \left(\frac{\rho_{a}}{\rho_{\ell}}\right)^{2/3} \left(\frac{\nu_{a}}{\nu_{\ell}}\right)^{1/6} \left(\frac{r_{o}U}{\nu_{\ell}}\right)^{\frac{1}{2}}$$
(22a)

where

r is the radius of the lenticular shape assumed by the flattened drop and is related to the radius R of the spherical drop of the same volume by r = 1.83R. Expressing equations (21a) and (22a) in terms of R gives:-

$$\mathbf{t}_{s} = \left(\frac{O \cdot \partial L_{t} \hat{G}}{\pi}\right) \left(\frac{\rho_{\ell}}{\rho_{\alpha}}\right)^{1/3} \left(\frac{\nu_{a}}{\nu_{\ell}}\right)^{1/6} \left(\frac{R_{o}^{2}}{U \nu_{\ell}}\right)^{\frac{1}{2}} \left[1 + \frac{C_{1}}{L_{t}}\right]$$
(21b)

$$C_{1} = 0.507 \left(\frac{\rho_{a}}{\rho_{\ell}}\right)^{2/3} \left(\frac{\nu_{a}}{\nu_{\ell}}\right)^{1/6} \left(\frac{R_{o}U}{\nu_{\ell}}\right)^{\frac{1}{2}} C_{D}$$
(22b)

where

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$$t_s = \lambda_{23} \left(\frac{D_c^3}{U}\right)^{\frac{1}{2}} \left[1 + 0.495 (D_c U)^{\frac{1}{2}}\right]$$
 (23)

where D is the initial diameter of the drop.

This is not of the same form as that found experimentally (equation (4)). Further, the times found by (23) are considerably greater than those found in tests. A comparison of droplet sizes formed during disintegration with what would be expected from Taylor's theory also shows disagreement (para.4.4), the possible theoretical size being greater than that found at the start of disintegration and smaller than found at the end of disintegration. This does not afford a sure basis for establishing why the discrepancy in disintegration rates exists. On the other hand promature break-up of the drop (para.4.3) would enable disintegration to proceed from a larger surface area and might be an explanation for the more rapid disintegration which occurs in practice compared with prediction.

### 5.2 Bursting due to curved flow

Dodd<sup>8</sup> has suggested a mechanism for the disintegration of drops in an airstream by which viscous flow round the drop sets up a flow of the curved surface of the drop. The contripetal force needed for this curved flow is taken to exceed the centripetal force provided by surface tension, so that fluid escapes from the drop. The equation for the time of disintegration deduced by Dodd is:-

$$t_{s} = (D/U^{3/2}) \times (1/0.332) (T \rho_{\ell} F/\mu \rho_{a} f)^{\frac{1}{2}}$$
(24)

where F and f are constants.

No values are given for F and f, so that direct comparisons between times found experimentally and those based on equation (24) cannot be made. The form of equation (24) agrees with the empirical relation (4) as regards the power of D, but differs in regard to the power of U. The method does not provide an estimate of droplet sizes that would be formed so that a comparison with droplets found in a test cannot be made.

### 5.3 Wave making

Priestley<sup>10</sup> considers the loss of liquid from a large surface which, under the action of a wind, is breaking up into droplets of uniform size. He assumes that the diameter d of the droplets is propertional to the wavelength  $\lambda$  of the waves formed on the surface of the liquid. Also if  $\tau$  is the time taken for the complete process of formation of a droplet, then

where  $\psi$  is the rate of growth of the waves.

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 $\tau$  represents the time taken for the formation of a complete layer of droplets of diameter  $d(\alpha \lambda)$ . Thus in the time  $\tau$  a layer of thickness proportional to  $\lambda$  is torn away from the liquid, so that the rate of loss of liquid from the surface is proportional to

$$\lambda/\tau$$
 or  $\lambda \times \psi$ . (26)

The wave length  $\lambda^*$  of the most rapidly growing wave is given as

$$\lambda^* = 2\pi (\ell_{4}/3)^{1/3} \rho_{\ell}^{-1/3} \rho_{a}^{-2/3} T^{1/3} \mu^{2/3} U^{-4/3}$$
(27)

where U is the velocity of the wind. The maximum value of  $\psi$  is:-

$$\psi^* = \rho_a U^2 / 2\mu \quad . \tag{28}$$

Hence in (26) the rate of less of liquid due to unstable waves is proportional to  $\lambda^* \psi^*$  where  $\lambda^*$  and  $\psi^*$  are given by equations (27) and (28) respectively.

If it is assumed that the same formulae can be applied to the surface of a liquid drop of radius R, then the rate of loss of liquid becomes:..

$$-dV/dt = -A dR/dt \alpha(\rho_a U^2/2\mu) \left[ 2\pi(4/3)^{1/3} \rho_e^{-1/3} \rho_a^{-2/3} T^{1/3} \mu^{2/3} U^{-4/3} \right] A \quad (29)$$

where A is the surface area of the drop from which droplets are being formed by wave action.

From (29)

$$dR/dt \alpha - \pi (4/3)^{1/3} (\rho_a/\rho_\ell)^{1/3} (\mathbb{T}^{1/3}/\mu^{1/3}) \mathbb{U}^{2/3}$$
(30)

integrating (30) and using the boundary conditions  $R = D_0/2$  when t = 0 and R = 0 when  $t = t_s$  gives

$$t_{s} \alpha(D_{0}/U^{2/3}) \times (1/2\pi) \times [(3/4)(\mu/T)(\rho_{\ell}/\rho_{a})]^{1/3} .$$
(31)

Equations (30) and (31) agree in form with the corresponding empirical formulae (12) and (4). The difference occurs in the power of U, being 2/3 for the theoretical wave making mechanism and 0.72 for the empirical result. Both disintegration rate expressions are independent of D<sub>o</sub>, the drop diameter.

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The effect of using values of n derived in the foregoing theoretical treatments instead of the experimentally determined value n = 0.72 in an equation of the form  $t_s = K/U^n$  for the disintegration time of the drop in an airstream U is shown in Fig.22 where it is seen that the value n = 2/3 (using the Priestley wave making theory) gives the best fit of the theoretical values of n to the results.

The use of the Taylor equations ((16), (17)) gives calculated wave lengths for unstable waves which agree quite closely with the average drop sizes measured in a test. This supports Taylor's hypothesis<sup>5</sup> that droplets formed might be comparable with the wave length of the unstable wave which increases most rapidly in amplitude. Although the use of the Priestley equations ((27), (28)) for the most rapidly growing wave leads to an expression for the rate of removal of fluid (30) which is similar in form to that derived empirically (12) in respect of the parameter air velocity, these equations yield smaller calculated wave lengths. The form given by Priestley for the most rapidly growing wave (equation (27)) differs from that given by Taylor (16) due to a difference in neglected terms in the mathematical development.

In calculating the wave lengths for waves generated with flow over a drop, the operative velocity is assumed to be the velocity of the airstream U. In the flow over a drop the local velocity will vary so that a range of wave lengths and hence of drop sizes could be expected. In the case of a sphere in a uniform stream of velocity U, the local velocity varies from 0 at the stagnation point to 1.5U at the periphery. Assuming that the wave length formulae hold for flow over a sphere and that the case considered, of a drop of diameter 2.75 mm in an airstream of 352 ft/sec can be regarded as a sphere in a uniform airstream, then the minimum drop sizes that would be expected at the start and finish of disintegration are, using equation (16)  $50/1.5^2 = 22$  and  $75/1.5^2 = 33$  microns compared with minimum drop sizes actually caught at these two times of 10 and 50 microns respectively (para.3.2).

Of the suggested disintegration mechanisms considered, the wave making mechanism appears the most likely. A relationship for the time required for disintegration which is approximately of the right form as regards the parameters drop size and airstream velocity, can be derived using Priestley's formula although it does not provide a numerical value to the constant of proportionality. It may be necessary to find this constant experimentally for any mechanism due to the complication of possible premature break-up of the drop and the increase of surface area that this could give. The drop sizes that the wave making mechanism predict using Taylor's formula seem in reasonable agreement with those actually found in the one case in which droplets were caught and measured. It would be worthwhile continuing the study of droplets formed during disintegration using the present sampling methods.

### 6 <u>CONCLUSIONS</u>

The time required for airstreams of various velocities to disintegrate water drops of different diameters down to droplets of size order 100-150 microns diameter, has been determined experimentally. The acceleration and velocity of the drops whilst being disintegrated was also determined. The general conclusions derived from these results are given below:-

(1) An empirical formula for the time  $t_s$  needed to disintegrate water drops in airstreams has been deduced and is given by:-

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$$t_s = 20.0 D_o / U^{0.72} secs$$

where  $D_0$  is the diameter of the drop in ft and U the velocity of the airstream in ft/sec.

(2) From the above equation it has been deduced that the rate of disintegration can be expressed in the form:-

$$dR/dt = -(U - w)^{0.72}/36.0 \text{ ft/sec}$$

where R is the radius of drop in ft and w is the velocity of the drop in ft/sec.

(3) Premature break-up of the drop appears to occur at a time t after the start of acceleration and disintegration given by the empirical relation:-

$$t_p = 50 D_0 / U \sigma^{\frac{1}{2}}$$

where  $\sigma$  is the relative density of the airstream.

(4) During the disintegration period the drop accelerates with approximately constant acceleration  $g_\ell$  which is found to be given empircally by the relation:-

$$g_{\ell} = 2 K (U^2 / D_0) ft / sec^2$$

where  $K = 1.04 \times 10^{-3}$ .

(5) The equation of motion of a drop which is both accelerating and disintegrating in an airstream can be expressed as:-

$$(dw/dR) - (3w/R) \times (\alpha - 1) = \left[\frac{3}{8}C_F(\rho_a/\rho_\ell)(U - w)^2/R\right] \times (1/dR/dt)$$

where  $\alpha$  and  $C_F$  are constants which for the particular case of a 1.0 mm diameter water drop being accelerated and disintegrated by an airstream of velocity 247 ft/sec have the values  $C_F = 3.26$  and  $\alpha = 1.14$ .

(6) Droplets formed during disintegration have been caught and measured in the particular case of a 2.75 mm diameter drop being disintegrated in an airstream of 352 ft/sec and compared with values that would be expected if certain suggested mechanisms of disintegration were operating. In the case referred to the droplets caught at the start of disintegration ranged in size from 10-160 microns with an average of  $l_{15}$  microns. At the end of disintegration fewer droplets were caught, having a size range of 50-90 microns with an average of 68 microns. These values can be compared with droplets produced by an unstable wave making mechanism which would be 50 microns at the start and 75 microns at the end of disintegration. The agreement between the means and predicted values is quite good, although some very much larger drops were also present.

Using Taylor's theory of disintegration by boundary layer stripping and taking expected droplet diameters to be of the same order as the boundary layer thickness, then the above droplet sizes found are to be compared with droplet sizes of the order 435 microns at the start of disintegration and vanishingly small at the end of disintegration.

(7) It has not been found possible to establish conclusively what mechanism operates to cause disintegration, but the evidence tends to favour the wave making mechanism. It appears from photographs taken of disintegrating drops and other evidence, that break-up of the drop into smaller drops occurs during disintegration enabling the disintegration process to operate on a larger surface area than if the drop had remained whole. This could account for the observed times for complete disintegration being shorter than predicted using Taylor's boundary layer theory.

#### 7 FURTHER WORK

In order to determine the effect of parameters that might be operating in the disintegration process other than airspeed and drop size, the present techniques could be used to find disintegration times of other fluids, so that the effect of surface tension, fluid density and viscosity can be studied. Also by doing tests in conditions of low pressure the effect of air density could be investigated.

A more extensive study of the droplets produced during disintegration might also shed light on the mechanism of disintegration operating. Also by dispensing with the single lateral sampling slide and using a slide extending right round the inside of the projectile, and catching all the droplets shed in a given interval, it should be possible to estimate the rate of disintegration for various cases. This, again, would throw light on the disintegration process in operation.

It would be worthwhile studying the break-up of water drops disintegrating in an airstream caused by the penetration of the drop by air holes, so as to form an idea of the increase in surface area that occurs when break-up takes place. This could be done by viewing the drop photographically from the front.

#### 8 ACKNOWLEDGMENT

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# SYMBOLS

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R	liquid drop radius	ft
r	liquid cylinder radius	ft
D = 2R	liquid drop diameter	ft
đ	liquid droplet diameter	ft
L	length of liquid cylinder	ft
Z	distance moved by liquid cylinder	ft
λ	wave length	ft
х	distance measured on surface of liquid drop	ft
x	distance moved by liquid drop	in.
У	penetration into projectile	in.
l	= x + y	in.
δ	boundary layer thickness	cm
v	volume of liquid drop	ft <sup>3</sup>
m	mass of liquid drop	slugs
U	airspeed	ft/sec
v	velocity of liquid cylinder	ft/sec
w	velocity of liquid drop	ft/sec
М	mass of liquid	slugs
t	time	sec
τ	time	sec
ψ	rate of growth of waves	sec <sup>-1</sup>
σ	rclative density of air	
$\phi$	temperature of air	°C
β	relative kinematic viscosity of air	
ρ	density	slugs/ft3
v	kinematic viscosity	ft <sup>2</sup> /sec

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# SYMBOLS (CONTD)

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	SYMBOLS (CONTD)			Units
μ	viscosity	1b	or	slugs/ft/sec
Т	surface tension			lb/ft
g	acceleration			ft/sec <sup>2</sup>
C <sub>D</sub>	drag coefficient			
$c_{\rm F}$	force coefficient			
C <sub>p</sub>	pressure coefficient			
А	constant			in.
В	constant			in./millisecond <sup>2</sup>
α	fractional increase in liquid velocity			
٢	fractional increase in liquid drop diameter due flattening	to		
π	= w/U			
Σ	$= R/R_{o}$			
Г	$= t/t_{g}$			
k	fractional drop diameter			
к <sub>1</sub> к <sub>2</sub> к <sub>3</sub>	constants			
К.	constant = L/D			
к <sub>5</sub>	constant = 2 $\begin{pmatrix} K_{l_{4}}/C_{p} \end{pmatrix} (\rho_{a}/\rho_{\ell})^{\frac{1}{2}}$			
к <sub>6</sub>	constant = $\frac{3}{8}(C_D/k)(\rho_a/\rho_\ell)$			
<sup>к</sup> 7	constant = 0.507 $C_D (\rho_a / \rho_\ell)^{2/3} (\nu_a / \nu_\ell)^{1/6} (1/\nu_\ell)^{\frac{1}{2}}$			
° <sub>1</sub>	constant = 0.507 $C_D (\rho_a / \rho_\ell)^{2/3} (\nu_a / \nu_\ell)^{1/6} (R_0 U / \nu_\ell)^{1/6}$	e) <sup>1</sup> 2		
Suffixes				
0	initial conditions			
8	complete disintegration conditions			
с	calculated values			

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# SYMBOLS (CONTD)

Suffixes	(Contd)	
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- experimental values
- a air

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- l liquid
- p penetration
- ph photographic

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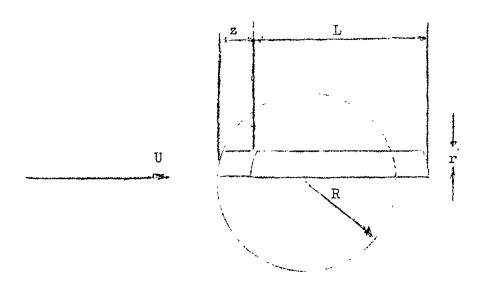
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### APPENDIX 'A'

#### PENETRATION OF WATER DROPS BY AN AIRSTREAM



## Fig.A.1

Gordon<sup>4</sup> has considered the case of a water drop of radius R in an airstream of velocity U and has calculated the time required for the airstream to cause a cylinder of water to be extruded through the drop (Fig.A.1). The full treatment includes surface tension and viscous forces as well as inertia and pressure forces. Reproduced below is the case of negligible surface tension and viscous forces.

The air pressure is considered as extruding through the drop a cylinder of water of length L and radius r. The pressure difference over the two ends of the cylinder is proportional to  $\frac{1}{2} \rho_a U^2$ .

Thus the force on the cylinder = 
$$C_p \times \frac{1}{2} \rho_a U^2(\pi r^2)$$
 (32)

where C is a pressure coefficient.

The inertia force = 
$$g_{\ell}(\pi \times r^2 L) \rho_{\ell}$$
 (33)

where  $g_{\ell}$  is the acceleration of the cylinder = dv/dt and v is the velocity of the cylinder. It is assumed that the rest of the drop is stationary. Hence, equating inertia force (33) to applied force (32) gives:-

$$dv/dt = (C_p/2)(\rho_a/\rho_\ell)(U^2/L)$$
 (32)

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Integrating gives

$$v = (C_{p}/2)(\rho_{a}/\rho_{\ell})(U^{2} t/L) + A$$

if U is constant during the interval. When t = 0 v = 0. Hence A = 0,

therefore 
$$v = dz/dt = (C_p/2)(\rho_a/\rho_\ell)(U^2 t/L)$$
 (35)

$$z = (C_{p}/4)(\rho_{a}/\rho_{\ell})(U^{2} t^{2}/L) + B$$
(36)

when t = 0 z = 0. Therefore B = 0.

Therefore 
$$z = (C_{p}/4)(\rho_{a}/\rho_{\ell})(U^{2} t^{2}/L)$$
 (37)

When  $t = t_p z = L$ 

### therefore

$$t_{p} = (2/C_{p}^{\frac{1}{2}})(L/U)(\rho_{\ell}/\rho_{a})^{\frac{1}{2}} .$$
 (38)

But  $\rho_a = \sigma \rho_{ao}$  and taking  $L = K_4 D$  gives

$$t_{p} = K_{5}(D/U)(1/\sigma^{\frac{1}{2}})$$
(39)

$$K_{5} = (2 K_{1} / C_{p}^{\frac{1}{2}}) (\rho_{\ell} / \rho_{ao})^{\frac{1}{2}} . \qquad (40)$$

The approximate solution given by Gordon<sup>4</sup> for the case when viscosity and surface tension are included is:-

$$t_{p} = (2D \rho_{\ell}^{\frac{1}{2}}) / (\rho_{a} U^{2} - 16 T/D)^{\frac{1}{2}} + 32\mu / (\rho_{a} U^{2} - 16 T/D)$$
(41)

This reduces to equation (39) when the surface tension and viscous force terms are ignored, and taking  $C_p$  and  $K_{\mu} = 1$ .

From photographs of drops disintegrating in an airstream, it appears that portions of the drop break away during disintegration. This could follow penetration of the drop by an air hole. On the assumption that this, in fact, does happen, and that breakaway occurs at the end of penetration, penetration times t have been found from various photographs of disintegrating drops and are given p in Table A.1 below, together with a reference to the photograph from which they have been taken. (The negatives quoted are held in Mech Eng Department, R.A.E.)

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TABLE A.1	

.

Case	Diameter d cm	Diameter D ft	Velocity U ft/sec	D/U sec x 10 <sup>5</sup>	σ	$D/U\sigma^{\frac{1}{2}} \sec \times 10^{5}$	$t_p \sec \times 10^3$	Figure or Negative
C1	0.225	$7.38 \times 10^{-3}$	248	2.98	1.03	2.93	1.25	652
C2	0.225	$7.38 \times 10^{-3}$	365	2.02	1.05	1.97	1.04	890 <b>(9</b> E)
C3	0.225	7.38 × 10 <sup>-3</sup>	474	1.56	1.09	1.49	0.78	766
C4	0.225	$7.38 \times 10^{-3}$	560	1.32	1.13	1.24	0.69	706
C5	0.225	$7.38 \times 10^{-3}$	633	1.16	1.17	1.07	0.67	777
A1	0.100	$3.28 \times 10^{-3}$	563	0.58	1.13	0.55	0. 38	116
A2	0.125	$4.10 \times 10^{-3}$	554	0.74	1.13	0.70	0. 31	692
A3	0.150	$4.92 \times 10^{-3}$	567	0.87	1.13	0.82	0.37	162
A4	0.175	$5.74 \times 10^{-3}$	559	1.03	1.13	0•97	0.54	700
A5	0.200	$6.56 \times 10^{-3}$	566	1.16	1.13	1.09	0.64	35
A8	0.275	$9.02 \times 10^{-3}$	553	1.63	1.13	1.53	0.87	722
D4	0.150	$4.92 \times 10^{-3}$	477	1.03	1.09	0.99	0.74	5 <b>96</b>
D7	0.150	$4.92 \times 10^{-3}$	<b>7</b> 02	0.70	1.21	0.64	0.34	788
	0.270	$8.85 \times 10^{-3}$	498	1.78	1.51	1.45	0.742	4(6)*
	0.270	$8.85 \times 10^{-3}$	777	1.14	1.85	0.84	0.667	10(5)*
	0.140	$4.59 \times 10^{-3}$	777	0.59	1.85	0.43	0.411	11(4)*
	0.270	$8.85 \times 10^{-3}$	1036	0.85	2.18	0.58	0.428	12(5)*

Penetration times for water drops

\*These figure numbers refer to Ref.2 - The fragmentation of water drops in the zone behind an air shock

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Appendix "A"

Appendix 'A'

The above results of t and  $D/(U\sigma^2)$  have been plotted on Fig.A.2 from which it is seen that the linear relationship suggested by equation (39) is followed. From Fig.A.2 the empirical value of K<sub>5</sub> is found to be 50 with t and  $D/(U\sigma^2)$  both in seconds. This compares with a value of K<sub>5</sub> = 57 which is obtained from equation (40) by taking C<sub>p</sub> = 1, K<sub>4</sub> = 1,  $\rho_{ao} = 0.002378$  slugs/ft<sup>3</sup> and  $\rho_{\ell} = 1.94$  slugs/ft<sup>3</sup>. Engel<sup>2</sup> has considered the acceleration of a water drop in an airstream and concludes that it would have to be perforated to have a drag coefficient that would yield the correct acceleration. It has been found<sup>3</sup> that when a spherical drop of water strikes a piece of aluminium at high speed (800 ft/sec) the damage mark produced is a smooth indentation similar to that made in a Brinell hardness test. If, however, the water drop instead of being spherical has the ragged shape of a drop that is being disintegrated, then the damage marks formed on impact with the aluminium surface at the same speed consists of a number of separate indentations which could be consistent with the impact of a number of separate small drops. This lends support to the idea of water drops being split up into separate pertices whilst disintegrating.

From (39) the average velocity of penetration  $v_p$  is given by:-

$$v_{p} = D/t_{p} = U\sigma^{\frac{1}{2}}/K_{5}$$
 (42)

For these cases where  $\sigma \div 1$ ,

$$v_p \neq U/K_5$$
 (43)

From Lewis's<sup>9</sup> tests on penetration of liquid layers under acceleration

$$v_{p} = 1.11 \left[ (g_{\ell} - g)r \right]^{\frac{1}{2}}$$
 (44)

where r is the radius of curvature of the air finger which penetrates the liquid layer, v is the velocity of penetration of the finger and g is the acceleration due to gravity. In Appendix B it is shown that for a disintegrating drop an empirical expression for  $g_{\ell}$  is given by  $g_{\ell} = K_6(U^2/R)$  where  $K_6 = 1.04 \times 10^{-3}$ . g is small compared with  $g_{\ell}$ . If it can be taken that Lewis's formula also applies to the acceleration of a spherical water drop, then substituting the values derived for v and  $g_{\ell}$  in (44) gives:-

$$U/K_5 = 1.11 [K_6 U_2 r/R]^{\frac{1}{2}} = 1.11 K_6^{\frac{1}{2}} U \times (r/R)^{\frac{1}{2}}$$
 (45)

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Appendix 'A'

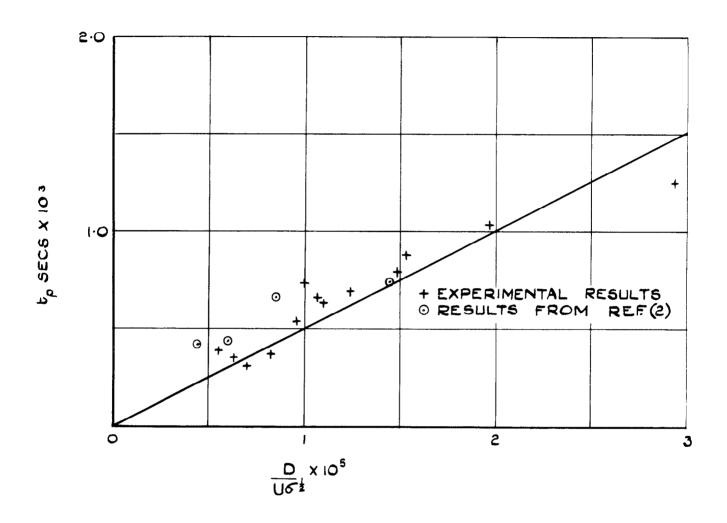
Taking  $K_5 = 50$  and  $K_6 = 1.04 \times 10^{-3}$  gives:-

$$(r/R) = 1/(1.11^2 \times K_5^2 \times K_6) = 0.313$$
 (46)

Thus using empirically found values for the average velocity of penetration  $v_p$ , in the break-up of water drops and of the acceleration  $g_\ell$  of the drop in Lewis's formula for the rate of penetration of air fingers into a layer of water under acceleration, suggests that fingers, if so produced in a drop, might have a radius of curvature of the order 0.313R where R is the radius of the drop.

It would be worthwhile to study disintegrating drops in some detail, viewing them photographically from the front so as to arrive at a better understanding of the penetration process and to establish if penetration does in fact occur and if it is regular or erratic. A drop which has been penetrated and split up into a number of separate smaller drops presents a larger area than the original drop to the airstream so that disintegration would proceed at a higher rate. Some knowledge of the increase in surface area would be helpful in establishing what mechanism operates to disintegrate a drop in an airstream. \_ ŧ ٠

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FIG. A.2. TIME  $(t_p)$  NEEDED TO PENETRATE WATER DROPS BY AN AIRSTREAM.

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#### APPENDIX 'B'

#### THE ACCELERATION BY AN AIRSTREAM OF WATER DROPS OF REDUCING SIZE (EMPIRICAL METHOD)

It is found experimentally that the motion in an airstream of water drops which are being both disintegrated and accelerated by the airstream approximates to one of constant acceleration. This suggests that the equation of motion of the reducing drop can be represented by the acceleration of a drop of constant radius by a constant accelerating force produced by the flow of an airstream of constant relative velocity past a drop giving rise to a drag coefficient  $C_D$  of constant value. The equation of motion of the equivalent drop then becomes:-

$$C_{\rm D} \times \frac{1}{2} \rho_{\rm a} U^2 \pi (kR_{\rm o})^2 = \rho_{\ell} 4/3\pi (kR_{\rm o})^3 g_{\ell}$$
 (47)

where  $g_{\ell}$  is the acceleration of the drop and  $(kR_{o})$  is the radius of a drop of constant value which is assumed to be equivalent to the reducing drop which starts with a radius  $R_{o}$  and finishes with a radius vanishingly small at the end of the accelerating and disintegrating period. The velocity acquired by the drop is assumed to be small compared with U, so that the airspeed relative to the drop can be taken as constant and equal to U.

From equation (47)

$$g_{\ell} = \frac{3}{8} (C_{\rm D}/k) (\rho_{\rm a}/\rho_{\ell}) U^2/R_{\rm o}$$
(48)

$$= \kappa_{6}(U^{2}/R_{o}) = 2\kappa_{6}(U^{2}/D_{o})$$
(49)

$$K_{6} = \frac{3}{2} (C_{D}/k) (\rho_{a}/\rho_{\ell})$$
(50)

This suggests that for the equivalent drop system, the acceleration  $g_\ell$  is proportional to  $U^2/R_o$  if  $(C_D/k)$  is constant. From the measured values of drop movement in airstreams, the values of  $g_\ell$  have been calculated for a number of cases. These are given in Table B.1 and plotted in Fig.B.1 against  $(U^2/R_o)$ . It is seen that there is a linear relationship between  $g_\ell$  and  $U^2/R_o$  as suggested by equation (48) indicating a constant value of  $K_6$ . There is a moderate amount of scatter in the results, but this is to be expected in view of the scatter of the basic experimental data on which the values of  $g_\ell$  have been based. From Fig.B.1  $K_6 = 1.04 \times 10^{-3}$ . Thus  $g_\ell = 2.08 \times 10^{-3} \times U^2/D_o = 1.04 \times 10^{-3} \times (U^2/R_o)$  (equation (49)).

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Hence in equation (50)

$$1.04 \times 10^{-3} = \frac{3}{8} \times (C_{\rm D}/k) \times (0.002378/1.938)$$
 (51)

$$(C_{\rm D}/{\rm k}) = 2.26$$
 (52)

 $(C_D/k)$  can be regarded as an equivalent drag coefficient. Thus, within the range of cases considered, the drop of initial radius  $R_o$  which is accelerated and disintegrated by an airstream of velocity U has the same acceleration as a drop of constant radius  $R_o$  acted on by an airstream of constant relative velocity U with a constant equivalent drag coefficient = 2.26.

#### TABLE B.1

#### Acceleration of water drops

Case	В	$g_{\ell} \times 10^{-4}$ ft/sec <sup>2</sup>	U ft/sec	$R_{o} \times 10^{3}$ ft	
A1 A2 A3 A4 A5 A6 A7 A8	1.085 1.088 0.695 0.785 0.448 0.758 0.322 0.666	18.10 18.15 11.58 13.08 7.47 12.63 5.37 11.10	563 554 567 559 566 560 561 553	1.64 2.05 2.46 2.87 3.28 3.69 4.09 1.51	19.30 14.95 13.05 10.87 9.76 8.50 7.69 6.78
B1 B2 B3	0.335 0.243 0.161	5•59 4•05 2•69	247 251 248	1.64 2.46 3.69	3.72 2.56 1.67
C1 C2 C3 C4 C5	0.334 0.397 0.724	5.58 6.62 12.06	Same as 6 365 474 Same as 6 633	case B3 3.69 5.69 case A6 3.69	3.61 6.08 10.87
D1 D2 D3 D4 D5 D6 D7	0.281 0.513 0.402 0.984 1.074	4.69 8.56 6.69 16.40 17.90	322 389 477	case B2 2.1.6 2.1.6 2.46 2.46 case A3 2.46 2.46	4.21 6.14 9.24 15.20 20.05

The velocity w of the residue of the reducing drop is given by:-

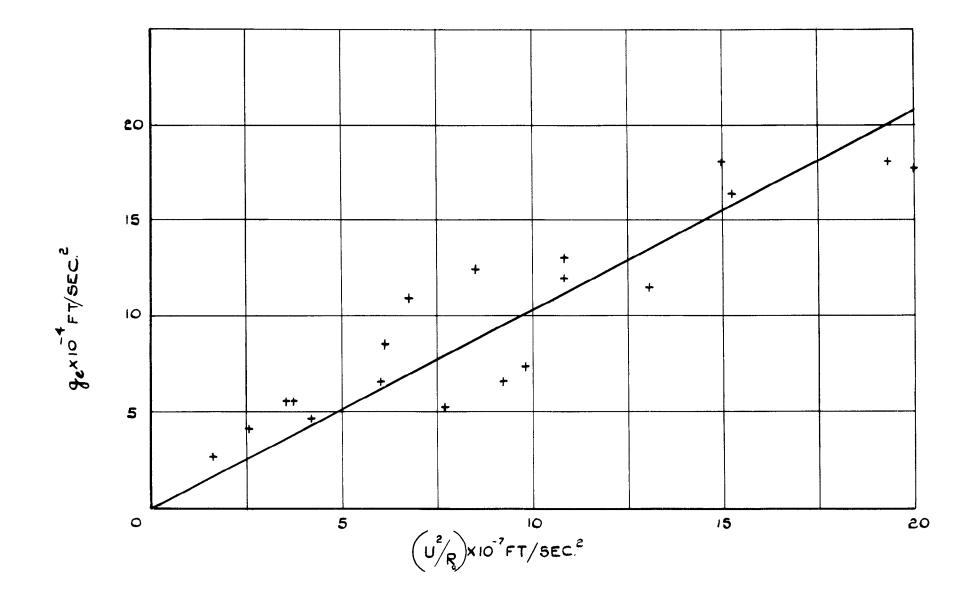
$$w = g_{\ell} t \quad . \tag{53}$$

At the end of the disintegrating period t = t<sub>s</sub> which is found experimentally to be given by t<sub>s</sub> = 20 D<sub>o</sub>/U<sup>0.72</sup> and the empirically found value of  $g_{\ell} = 2.08 \times 10^{-3} \times U^2/D_{o}$ . Hence w, at the end of disintegration = 20 D<sub>o</sub>/U<sup>0.72</sup> × 2.08 × 10<sup>-3</sup> × U<sup>2</sup>/D<sub>o</sub>.

Hence  $(w/U)_{s} = l_{1.6} \times 10^{-3} \times U^{0.28}$  (54)

which is independent of  ${\rm D}_{_{\rm O}}$  the initial drop diameter.

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FIG. B.I. ACCELERATION OF DISINTEGRATING WATER DROPS.

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#### APPENDIX 'C'

#### THE ACCELERATION BY AN AIRSTREAM OF WATER DROPS OF REDUCING SIZE (ANALYTICAL METHOD)

A water drop placed in an airstream is accelerated and, if the difference of speed between the drop and the airstream is higher than a critical value, the airstream also reduces the size of the drop by stripping surface layers off the drop. This change of mass as well as the change of velocity must be allowed for in the equation of motion of the drop. This problem is considered below by estimating the change of momentum in an interval of time  $\delta t$ .

#### Momentum at time t

A drop of water of mass  $m_1$  is considered to have a velocity  $w_1$ .

Momentum of water system = 
$$m_1 w_1$$
 (55)

#### Momentum at time t + $\delta t$

The mass of the drop is now assumed to have changed to  $m_2$  and its speed to  $w_2$ . In the interval  $\delta t$  a mass of water M is assumed to have been shed at a speed slightly greater than that of the drop, i.e.  $\alpha w$  where  $\alpha$  is a constant to be found.

Momentum of water system = 
$$m_2 w_2 + \alpha M(w_1 + w_2) \times \frac{1}{2}$$
 (56)

hence,

Increase in momentum of the system = 
$$m_2 w_2 + \alpha M(w_1 + w_2) \times \frac{1}{2} - m_1 w_1$$
 (57)

but 
$$m_2 = m_1 + (dm/dt) \times \delta t$$
  $w_2 = w_1 + (dw/dt) \times \delta t$   $M = (dM/dt) \times \delta t$ 

hence in (57)

Increase in momentum = 
$$m(dw/dt)\delta t + w(dm/dt)\delta t + \alpha w(dM/dt)\delta t + terms in (\delta t)^2$$
.... (58)

but from conservation of mass(dn/dt) = -dh/dt. Hence from (58)

Rate of increase of momentum = 
$$m^{dw}/dt - (\alpha - 1)w^{dm}/dt$$
. (59)

The applied aerodynamic force which causes the increase in momentum can be written in the form:-

$$\mathbf{F} = C_{\mathbf{F}} \times \frac{1}{2} \rho_{\mathbf{a}} \pi R^2 (\mathbf{U} - \mathbf{w})^2 = \mathbf{m} \, d\mathbf{w}/d\mathbf{t} - (\alpha - 1) \, \mathbf{w} \, d\mathbf{m}/d\mathbf{t}$$
(60)

but  $m = 4 \pi R^3 \rho_{\ell}/3$  and hence

$$dm/dt = 4 \pi R^2 \rho_{\ell} dR/dt$$
 (61)

hence in (60)

$$(\rho_{a} C_{F} \pi R^{2}/2)(U - w)^{2} = (4/3)\pi R^{3} \rho_{\ell} dw/dt - w(\alpha - 1) 4 \pi R^{2} \rho_{\ell} dR/dt$$
(62)

dividing both sides by  $(4/3)\pi R^3~\rho_{\ell}$  and re-arranging gives

$$dw/dt - 3(w/R)(\alpha - 1) dR/dt = \left[\frac{3}{3}C_{F}(U - w)^{2}/R\right](\rho_{a}/\rho_{\ell})$$
(63)

dividing now by dR/dt gives

$$dw/dR - (3w/R)(\alpha - 1) = \begin{bmatrix} \frac{3}{8} C_{F}(\rho_{a}/\rho_{c})(U - w)^{2}/R \end{bmatrix} (1/dR/dt) . (64)$$

The integration of this equation is now considered for two cases where dR/dt is known.

An empirical value of dR/dt has been derived from tests and is given by (12):-

$$dR/dt = - (U - w)^{0.72}/36.0$$
 (65)

hence in (64)

$$dw/dR - (3w/R)(\alpha - 1) = -\frac{3}{8} \times 36.0 C_F \times (\rho_a/\rho_\ell)(U - w)^{1.28}/R$$
. (66)

The force coefficient  ${\rm C}_{\rm F}$  allows for the increase in frontal area of the drop due to flattening such that:-

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$$C_{\rm F} \times \pi R^2 = C_{\rm D} \times \pi (\gamma R)^2$$
(67)

i.e. 
$$C_{\rm F} = \gamma^2 C_{\rm D}$$
 (68)

where 
$$\gamma$$
 is the increase in radius of the drop due to flattening.  
Taylor<sup>5</sup> calculates

$$\gamma = 1.83$$
 and takes  $C_D = 1.0$ , to give  $C_F = 3.35$ . (69)

Taking  $\rho_a$  = 0.002378 and  $\rho_\ell$  = 1.937 equation (66) becomes

$$dw/dR - (3w/R)(\alpha - 1) = -K_8(U - w)^{1.28}/R$$
(70)

$$K_8 = 0.01656 C_F$$
 (71)

To solve equation (70) it is necessary to assume values of  $\alpha$  and  $C_{F^{\bullet}}$ . For the case that  $\alpha = 1$  equation (70) becomes:-

$$dw/dR = -K_8 (U - w)^{1.28}/R$$
 (72)

integrating gives

$$w/U = 1 - [1 - 0.28 U^{0.28} K_8 \log_e(R/R_0)]^{-3.58}$$
 (73)

when  $R/R_o = 1 \quad w/U = 0$ , which agrees with the test conditions corresponding to the start of acceleration and disintegration. When  $(R/R_o) \rightarrow 0 \quad w/U \rightarrow 1$ .

This does not agree with experimentally found values of (w/U). It would thus appear that  $\alpha = 1$  is not a suitable value.

Putting (w/U) =  $\pi$ ; (R/R<sub>o</sub>) =  $\Sigma$  and (t/t<sub>s</sub>) =  $\Gamma$  equations (63) and (65) give:-

$$d\pi/d\Gamma = -\left[\frac{3\pi}{\Sigma}(\alpha-1) - \left\{\frac{3}{8} \times 36 \times (\rho_{a}/\rho_{\ell})C_{f}\right\} \frac{U^{0.28}(1-\pi)^{1.28}}{\Sigma}\right] \frac{U^{0.72} \cdot t_{s} \cdot (1-\pi)^{0.72}}{36R_{o}}$$
.... (74)

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Tests show the acceleration of the drop to be constant during the disintegrating period, i.e.  $d\pi/d\Gamma$  is constant (=  $\pi_{\alpha}$ ) during the period  $0 \leq \Gamma \leq 1$ ,

also  $\pi = \pi_0 = 0$ ;  $\Sigma = \Sigma_0 = 1$  when  $\Gamma = 0$  (initial value)

 $\pi = \pi_{s}$ ;  $\Sigma = \Sigma_{s}$  when  $\Gamma = 1$  (final value).

Substituting the above values in equation (74) gives two equations from which  $\alpha$  and  $C_{\rho}$  can be found. This leads to

$$C_{f} = \frac{\pi_{s} R_{o}}{U t_{s}} \left( \rho_{\ell} / \rho_{a} \right) \frac{8}{3} .$$
 (75)

Putting  $\pi_s = w_s/U$  and  $w_s = g_\ell t_s$ , where  $g_\ell$  is the acceleration of the disintegrating drop gives

$$C_{f} = \frac{g_{\ell} R_{o}}{u^{2}} \left( \rho_{\ell} / \rho_{a} \right) \frac{8}{3} .$$
 (76)

Equation (76) is the same as the equation for the equivalent drag coefficient  $(C_D/k)$  derived in Appendix B (equation (48)) using an empirical method,

also 
$$\alpha = 1 + \frac{12 R_o}{U^{0.72} t_s} \left[ \frac{-(1 - \pi_s)^2 - \Sigma_s}{(1 - \pi_s)^{0.72}} \right].$$
 (77)

Using equations (75) and (77) the values of  $\alpha$  and  $C_{f}$  have been calculated for each of the test cases for which the appropriate constants are known. The results are given in Table C.1 from which it is seen that there is a moderate amount of scatter in the derived values of  $C_{f}$ . The average value is 2.60 and lies in the range  $1.52 \leq C_{f} \leq 3.56$ . This value can be compared with the empirically found average value of equivalent drag coefficient  $(C_{f}/k) = 2.26$ (Appendix B) and the Taylor value of 3.35 (equation (69)). The average value of  $\alpha$  is 1.17 and lies in the range  $1.10 \leq \alpha \leq 1.24$ . Neither  $C_{f}$  nor  $\alpha$  appear to vary in a regular manner. The large values of  $C_{f}$  approximate to the Taylor value of 3.35 (equation (69)) whereas the small values of  $C_{f}$  possibly correspond with drops that have been perforated (Appendix A).

Using equation (70) the case of a water drop of initial diameter 1.0 mm disintegrating and accelerating in an airstream of 247 ft/sec has been examined in some detail. It is known from the test data that when the drop has been reduced to a diameter equal to 0.1 times its initial diameter its velocity is given approximately by w/U = 0.287 (Table C.1). Tests also show that the drop accelerates to this velocity with constant acceleration in a time of 1.27 milliseconds (Table 7).

Fig.C.1 shows the variation of (w/U) with  $(R/R_0)$  for various values of  $\alpha$ and C<sub>f</sub> taking R<sub>0</sub> = 0.05 cm and U = 247 ft/sec and using a step by step integration of equation (70). This figure illustrates the previously found condition that  $\alpha = 1$  is not suitable in that it produces a very sharp rise in the value of w at small values of R (i.e. see curves B and E of Fig.C.1). By the use of equation (65) the curves of Fig.C.1 have been replotted in Fig.C.2 on the basis of variation of (w/U) with  $(t/t_s)$  where t<sub>s</sub> is the time for complete disintegration. From an examination of these curves it can be seen that the required linear acceleration is obtained with a value of C<sub>f</sub> = 3.2 and a value of  $\alpha$  lying between 1.1 and 1.2 (compared with calculated values of C<sub>f</sub> and  $\alpha$  of 3.26 and 1.14 respectively. (Table C.1.))

It is noted that the analysis of Appendix B indicates that a drag coefficient of constant value can be found, which, associated with a drop of assumed constant radius R and an airstream of constant relative velocity U

ft/sec will yield a calculated value of acceleration equal to that measured for a drop of initial radius R<sub>o</sub> disintegrating and accelerating in an airstream of velocity U whereas the analysis of Appendix C shows that using a drag coefficient

cf constant value for evaluating the motion of a disintegrating drop of initial radius R in an airstream of velocity U and allowing for changes in radius of

the drop will yield a calculated value of acceleration which is much higher than measured but which can be corrected by including in the calculations a term with a positive constant  $\alpha$  of value greater than 1.0. This term corrects for changes of momentum due to changes of mass.

Case	t <sub>s</sub> sccs	U ft/sec	R C mm	π S	Σs	α	°f
A1 A2 A3 A4 A5 A6 A7 A8	0.58 × 10 <sup>-3</sup> 0.79 " 0.92 " 1.15 " 1.26 " 1.79 " 1.69 " 2.18 "	563 554 567 559 566 560 561 553	0.500 0.625 0.750 0.875 1.000 1.125 1.250 1.375	0.187 0.258 0.187 0.270 0.166 0.404 0.162 0.438	0.10 0.08 0.067 0.057 0.050 0.024 0.024 0.0240 0.036	1.23 1.18 1.23 1.17 1.22 1.11 1.21 1.10	2.04 2.62 1.91 2.62 1.66 3.23 1.52 3.56
B1 B2 B3	1.27 " 2.13 " 2.88 "	247 251 248	0.500 0.750 1.125	0.287 0.344 0.312	0.100 0.067 0.02µ4	1.14 1.12 1.15	3.26 3.44 3.49
C1 C2 C3 C4 C5	2.69 " 1.96 " 1.45 "	365 474 S	ame as c 1.125 1.125 ame as c 1.125	0•411 0•274	0.044 0.044 0.044	1.10 1.15 1.17	3•35 2•36 2•41
D1 D2 D3 D4 D5 D6	1.35 " 1.26 " 1.02 " 0.90 "	322 389 477	ame as c 0.750 0.750 0.750 ame as c 0.750	0.197 0.277 0.143	0.067 0.067 0.067 0.067	1.24 1.17 1.24 1.24	2•43 3•05 1•57 2•35

TABLE C.1

Calculated values of  $\alpha$  and  $C_{\mathbf{f}}$ 

# Case 2 - dR/dt has a value calculated by Taylor

The expression for the rate of removal of fluid given by Taylor<sup>5</sup> for the case where disintegration is assumed to proceed by boundary layer thicknesses of fluid being stripped off the surface of the drop is given by

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\mathbf{t}} = -\left(\frac{4\pi}{3\times0.685}\right) \times \left(\frac{\rho_{\mathrm{a}}}{\rho_{\mathrm{e}}}\right)^{1/3} \times \left(\frac{\nu_{\mathrm{e}}}{\nu_{\mathrm{a}}}\right)^{1/6} \times \nu_{\mathrm{e}}^{\frac{1}{2}} \frac{(\mathrm{U}-\mathrm{w})^{\frac{1}{2}}}{\mathbf{r}^{\frac{1}{2}}}$$
(78a)

here r is the radius of the lenticular shape assumed by the flattened drop. r is related to the radius R of a spherical drop of the same volume by r = 1.83R. Expressing equation (78a) in terms of R gives:-

$$\frac{\mathrm{dR}}{\mathrm{dt}} = -2.48 \times \left(\frac{\rho_{\mathrm{a}}}{\rho_{\ell}}\right)^{1/3} \times \left(\frac{\nu_{\ell}}{\nu_{\mathrm{a}}}\right)^{1/6} \times \left[\frac{(\nu_{\ell}) \times (\mathrm{U} - \mathrm{w})}{\mathrm{R}}\right]^{\frac{1}{2}}$$
(78b)

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substituting in (64). gives, with  $\alpha = 1:-$ 

where

$$\frac{dw}{dR} = -\kappa_{\gamma} \frac{(U - w)^{3/2}}{R^{\frac{1}{2}}}$$
(79)

$$K_{\gamma} = 0.507 \frac{C_{\rm D}}{\nu_{\ell}^{\frac{1}{2}}} \cdot \left(\frac{\rho_{\rm a}}{\rho_{\ell}}\right)^{2/3} \cdot \left(\frac{\nu_{\rm a}}{\nu_{\ell}}\right)^{1/6} .$$
(80)

Integrating (79) with the boundary condition that  $R = R_0$  when w = 0 gives:-

$$\frac{\mathbf{w}}{\mathbf{U}} = 1 - \left\{1 + C_1 \left[1 - \left(\frac{\mathbf{R}}{\mathbf{R}_0}\right)^{\frac{1}{2}}\right]\right\}^{-2}$$
(81)

where 
$$C_1 = 0.507 C_D \cdot \left(\frac{\rho_a}{\rho_\ell}\right)^{2/3} \left(\frac{\nu_a}{\nu_\ell}\right)^{1/6} \left[\frac{R}{\nu_\ell}U_{-\frac{1}{2}}\right]^{\frac{1}{2}}$$
 (82)

when  $(R/R_0) = 1$  (w/U) = 0, in (81). This agrees with test conditions

and when  $\left(\frac{R}{R_0}\right) \rightarrow 0 \quad \frac{W}{U} \rightarrow 1 - \left[1 + C_1\right]^{-2}$ , in (81). (83)

Considering the case where U = 247 ft/sec,  $R_{_{O}}$  = 0.05 cm and taking the following values for  $\rho$  and  $\nu:-$ 

$$\rho_{\ell} = 1.937 \qquad \nu_{\ell} = 1.075 \times 10^{-5}$$
  
 $\rho_{a} = 0.002378 \qquad \nu_{a} = 1.56 \times 10^{-4}$ 

gives a value of  $C_1 = 1.76$  from equation (82). Hence in equation (83) (w/U) = 0.868. This is greatly in excess of the value of w/U = 0.287 found experimentally for this case. Taking  $R/R_0 = 0.1$  at the end of disintegration then (81) gives a value for (w/U) = 0.793 which is still far larger than the experimentally determined value for (w/U). The large calculated values of (w/U) may be caused by taking  $\alpha = 1$  as was similarly found for case 1 where an empirical value of dR/dt was used.

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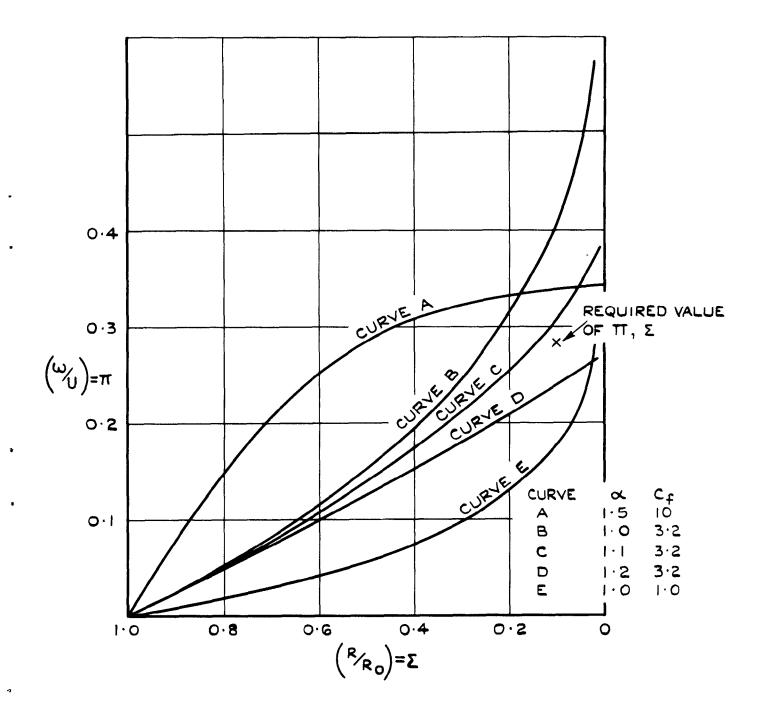


FIG.C.I. VARIATION OF  $\binom{\omega_{u}}{u}$  with  $\binom{R_{R_{o}}}{r_{o}}$  for a water drop initially of 1.0 millimetres diameter accelerating and disintegrating in an airstream of velocity 248 ft/sec.

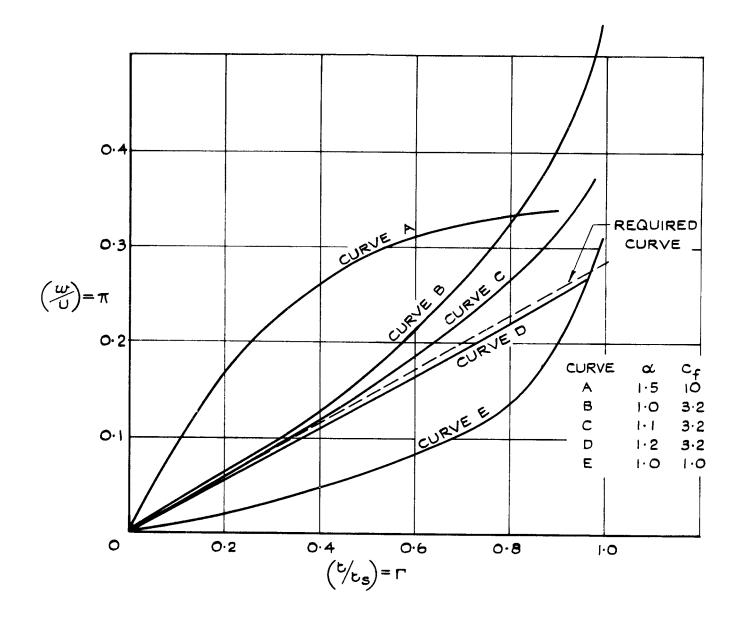


FIG.C.2. VARIATION OF  $\binom{\omega}{u}$  WITH  $\binom{t}{ts}$  FOR A WATER DROP INITIALLY OF I-O MILLIMETRES DIAMETER ACCELERATING AND DISINTEGRATING IN AN AIRSTREAM OF VELOCITY 248 FT/SEC.

# Movement of drops of various sizes in an airstream of velocity 560 ft/sec

SET A

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Remarks	Negative number	Drop movement x - in.	Drop penetration y - in.	Total movement (x+y)in.	Speed U ft/sec	Time t millisecs
<u>Case A.1</u> Average airspeed = 563 ft/sec Drop size = 1 mm diameter	114 167 119 115 116 117 118	0.03 0.08 0.12 0.14 0.21 0.27 0.48	0.90 1.20 1.1.6 2.03 2.39 2.78 3.75	0.17 1.28 1.58 2.17 2.60 3.05 4.23	546 566 566 562 566 566 566	0.03 0.19 0.23 0.32 0.38 0.45 0.62
Case A.2 Drop size = 1.25 mm diameter Average airspeed = 554 ft/see	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		1.20 1.23 1.47 1.86 3.03 4.30	1.33 1.47 1.61 2.06 3.51 5.01	552 569 551 547 562 545	0.20 0.22 0.24 0.31 0.52 0.77
<u>Case A.3</u> Drop size = 1.50 mm diameter Average airspeed = 567 ft/sec	79 57 162 80 81 82 1013 1019 164	0.12 0.10 0.14 0.24 0.41 0.54 0.42 0.59 0.07	1.87 1.80 2.30 3.34 3.93 5.63 4.00 4.30 1.01	1.99 1.90 2.44 5.58 4.34 6.17 4.42 4.89 1.08	562 570 558 565 562 578 569 578	0.30 0.28 0.37 0.53 0.65 0.92 0.64 0.72 0.16
<u>Case A.4</u> Drop size = 1.75 mm diameter Average airspeed = 559 ft/sec	685 696 700 707 711 712 717 1020 1021 1029	0.07 0.13 0.29 0.53 0.47 0.65 1.00 0.59 0.55 0.66	0.97 2.15 3.31 4.06 1.60 5.07 6.56 4.75 4.17 4.75	1.04 2.28 3.60 4.59 5.07 5.72 7.56 5.34 4.72 5.41	556 568 556 552 552 556 567 569 562	0.16 0.34 0.54 0.71 0.77 0.86 1.11 0.79 0.69 0.80

TABLE 1 (CONTD)

SET A (CONTD)

Remarks	Negative number	Drop movement x - in.	Drop penetration y - in.	Total movement (x+y)in.	Speed U ft/sec	Time t millisecs		
Case A.5								
Drop size = 2.0 mm diameter Average airspeed = 566 ft/sec	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		0.38 0.96 1.39 2.00 2.80 2.23 2.64 3.00 4.03 3.07 3.64 5.45 8.17 8.76	0.46 1.01 1.47 2.12 2.92 2.41 2.77 3.23 4.27 3.23 3.96 6.00 8.88 9.89	582 562 562 569 558 566 560 566 569 563 - *	0.07 0.15 0.22 0.31 0.43 0.36 0.41 0.48 0.64 0.48 0.64 0.48 0.58 0.58 0.59 1.32 1.47		
Case A.6								
Drop size = 2.25 mm diameter		Thi	s is the same	as case C	•4			
<u>Case A.7</u> Drop size = 2.50 mm diameter Average airspeed = 561 ft/sec	166 83 34 87 88 89 90	0.06 0.11 0.13 0.16 0.42 0.68 0.71	1.01 1.93 2.47 3.66 7.31 7.66 9.05	1.07 2.04 2.60 3.82 7.73 8.34 9.76	566 558 558 565 562 558 <b>-</b> *	0.16 0.31 0.39 0.56 1.15 1.25 1.45		
Case A.8	(0)							
Drop size = 2.75 mm diameter Avcrage airspeed = 553 ft/sec	684 698 702 708 715 722 726 725 731	0.11 0.25 0.31 0.56 0.62 1.10 0.90 1.29	1.25 2.40 3.40 4.48 5.43 6.03 7.30 6.63 8.00	1.36 2.51 3.65 4.79 5.99 6.65 8.40 7.53 9.29	555 550 563 555 544 558 558 550 550 562	0.20 0.37 0.54 0.72 0.92 0.87 1.24 1.13 1.38		

\*Where speeds have not been recorded the average speed for the rest of the case has been used in calculating times.

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# Movement of 2.0 mm diameter water drops in an airstream of average velocity 569 ft/sec (results with Gun 2)

<u>SET A</u>

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Remarks	Negative number	Drop movement x - in.	Drop penetration y - in.	Tctal movement (x+y)in.	Speed U ft/sec	Time t millisecs
Case A.5 Average airspeed = 569 ft/sec	X732 X733 X734 X739 X735 X740 X744 X752	0.11 0.19 0.27 0.36 0.49 0.40 0.56 0.75	2.50 3.55 4.45 4.30 5.30 5.30 6.25 6.75	2.61 3.74 4.72 4.66 5.79 5.70 6.81 7.50	557 577 595 580 595 550 530	0.39 0.54 0.66 0.67 0.78 0.80 0.88 1.18

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#### Movement of water drops of various sizes in an airstream of velocity 250 ft/sec

SET B

Remarks	Negative number	Drop movement x - in.	Drop penetration y - in.	Total movement (x+y)in.	Speed U ft/sec	Time t millisecs
Case B.1 Drop size = 1.0 mm diameter Average airspeed = 247 ft/sec	643 647 650 655	0.09 0.27 0.36 0.90	1.20 2.18 3.22 3.70	1.29 2.47 3.58 4.60	250 2149 2146 2143	0.4.3 0.83 1.21 1.58
<u>Case B.2</u> Drop size = 1.5 mm diameter Average airspeed = 251 ft/sec	$\frac{.2}{\text{diameter}} = 641  0.10$ $\frac{.10}{.17}$ $\frac{.11}{.17}$ $\frac{.11}{.17}$ $\frac{.11}{.17}$ $\frac{.11}{.17}$ $\frac{.11}{.17}$ $\frac{.11}{.17}$		1.42 2.11 3.35 3.84 4.94 5.56 5.63 7.05	1.52 2.28 3.50 4.42 6.01 6.89 7.07 9.24	249 251 251 245 256 254 249 255	0.51 0.76 1.20 1.51 1.96 2.26 2.36 3.04
Case B.3 Drop size = 2.25 mm diameter Average airspeed = 248 ft/sec	642 644 648 652 653 661 656 659 663 667 670	0.09 0.13 0.23 0.29 0.35 0.66 0.71 0.84 1.06 1.20 1.97	1.24 2.24 3.00 3.39 4.26 5.20 5.33 6.01 6.70 7.60 8.02	1.33 3.37 3.23 3.68 4.55 5.86 6.04 6.85 7.76 8.80 9.99	244 251 251 246 248 248 250 245 251 251 251	0.45 1.12 1.07 1.25 1.55 1.97 2.01 2.33 2.57 2.99 3.32

#### Movement of water drops of 2.25 mm diameter in airstreams of various velocities

SET C

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Remarks	Negative number	Drop movement x - in.	Drop penetration y - in.	Total movement (x+y)in.	Speed U ft/sec	Time t millisecs
Case C.1 Average airspeed = 24.8 ft/sec		Thi	s case is the	same as c	ase B.3	
Case C.2 Average airspeed = 365 ft/sec	755 754 756 890 757 893 892 758 894 895 759 910 763 911 891	0.07 0.09 0.14 0.14 0.33 0.66 0.68 0.40 0.71 0.56 0.73 1.08 0.89 1.36 0.41	1.27 2.16 3.04 4.06 4.59 4.92 4.94 5.30 5.04 5.19 6.16 6.57 6.80 7.14 4.02	1.34 2.25 3.18 4.48 4.92 5.58 5.62 5.74 5.75 6.89 7.65 7.69 8.50 4.43	- * - * 360 - * 320 372 396 372 376 360 364 360 364 361 - * 371	0.31 0.51 0.73 1.04 1.12 1.45 1.26 1.18 1.29 1.28 1.28 1.58 1.73 1.76 1.92 1.00
<u>Case C.3</u> Average airspeed = 474 ft/sec	verage airspeed 765 0.1		2.02 3.09 4.07 1.75 6.09 6.76 7.56 7.97	2.16 3.29 4.41 5.02 6.51 7.30 8.25 9.25	476 465 468 476 476 476 471 485	0.38 0.58 0.78 0.88 1.15 1.29 1.46 1.59

# TABLE 4 (CONTD)

## SET C (CONTD)

Remarks	Negative number	Drop movement x - in.	Drop penetration y - in.	Total movement (x+y)in.	Speed U ft/sec	Time t millisecs
Case C.4						
Average airspeed = 560 ft/sec	683 682 697 701 706 714 719 723	0.07 0.09 0.14 0.23 0.31 0.52 0.70 0.98	1.30 1.40 2.36 3.28 4.28 5.32 5.68 6.30	1.37 1.49 2.50 3.50 4.59 5.84 6.33 7.28	559 554 564 556 556 554 559 566	0.21 0.22 0.37 0.52 0.69 0.88 0.96 1.07
Case C.5 Average airspeed = 633 ft/sec	77! <sub>+</sub> 775 776 777 778 779 780(R)	0.09 0.15 0.32 0.36 0.56 0.86 0.91	1.20 2.25 3.82 4.74 5.92 6.23 7.30	1.29 2.40 4.14 5.10 6.48 7.09 8.21	641 636 636 636 631 636 637	0.17 0.32 0.54 0.67 0.86 0.93 1.11

\*Where speeds have not been recorded the average speed for the rest of the case has been used in calculating times.

#### Movement of water drops of 1.5 mm diameter in airstreams of various velocities

<u>SET D</u>

Remarks	Negative number	Drop movement x - in.	Drop penetration y - in.	Total movement (x+y)in.	Speed U ft/sec	Time t millisecs						
Case D.1												
Average airspeed = 251 ft/sec		This case is the same as case B.2										
Case D,2												
Average airspeed = 322 ft/sec	68 70 71 72 73 74	0.06 0.09 0.15 0.26 0.70 1.00	1.28 1.87 2.91 4.08 5.30 5.93	1.34 1.96 3.07 1.34 6.00 6.93	312 310 333 318 354 303	0.36 0.53 0.77 1.14 1.41 1.91						
Case D.3												
Average airspeed = 389 ft/sec	1 30 1 31 1 32 1 33 1 51	0.08 0.20 0.30 0.62 0.87	1.55 2.83 3.63 4.69 5.23	1.64. 3.04 3.93 5.31 6.00	392 - 388 383 394	0.35 0.65 0.84 1.16 1.27						
Case D.l.												
Averagc airspeed = 4.77 ft/sec	593 121 123 594 124 595 125 596 127 598 597 126 599 128 129 600	0.09 0.08 0.10 0.13 0.17 0.15 0.25 0.22 0.37 0.29 0.36 0.61 0.43 0.50 0.93	1.36 1.68 2.00 2.22 2.92 3.00 3.98 4.00 4.60 5.00 5.00 5.00 5.00 5.30 5.77 5.95 6.60 6.74	1.45 1.76 2.10 2.32 3.04 3.17 4.13 4.24 4.82 5.37 5.29 5.65 6.38 6.38 6.38 7.10 7.58	477 475 481 475 483 475 480 475 492 470 475 470 477 474 473 475	0.25 0.31 0.37 0.40 0.53 0.55 0.73 0.74 0.85 0.91 0.924 0.99 1.12 1.12 1.12 1.24 1.24 1.33						

# TABLE 5 (CONTD)

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#### SET D (CONTD)

Remarks	Negative number	Drop movement x - in.	Drop penetration y - in.	Total movement (x+y)in.	Speed U ft/sec	Time t millisecs			
Case D.5 Average airspeed = 567 ft/sec		This case is the same as case A.3							
Case D.6 Average airspeed = 612 ft/sec	168 153 146 143 144 147 148 152	0.06 0.06 0.12 0.13 0.15 0.30 0.33 0.35	0.91 1.00 1.97 2.36 2.68 3.41 3.69 3.98	0.97 1.06 2.09 2.49 2.87 3.71 4.02 133	626 540 627 616 616 630 621 621	0.13 0.16 0.28 0.34 0.38 0.49 0.54 0.58			
<u>Case D.7</u> Average airspeed = 702 ft/sec	785 797 788 800 789 791 794 793 792	0.16 0.30 0.37 0.14 0.50 0.22 0.68 0.47 0.73	1.76 2.17 2.72 3.78 3.25 1.23 1.41 1.98 1.98 1.87	1.92 2.47 3.10 3.92 3.81 4.45 5.12 5.45 5.56	688 700 761 694 694 694 694 700 694	0.23 0.30 0.34 0.47 0.53 0.54 0.61 0.65 0.67			

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#### SUMMARY OF DISINTEGRATION TIMES

## TABLE 6

$\operatorname{Set}$	А	 Constant	speed	of	560	ft/sec	with	various	drop	sizes

Dro Case dia	Drop	Penetration y in.		Max y	Drop movement	(x+y)	$t_s \times 10^3$	$t_{\rm ph} \times 10^3$	$t_p \times 10^3$
Case	min	Gun 1	Gun 2	in.	x in.	in.	secs	secs	Secs
A1 A2 A3 A4 A5 A6 A7 A8	1.00 1.25 1.50 1.75 2.00 2.25 2.50 2.75	2.9 4.5 4.8 6.7 7.8 9.0	3.5 5.7 - 7.7 9.6 10.5 11.8	3.5 7.5 5.7 6.7 7.8 9.6 10.5 11.8	0.4 0.8 0.48 1.0 0.68 2.4 0.90 2.82	3.9 5.3 6.18 7.7 8.48 12.0 11.14 14.62	0.58 0.79 0.92 1.15 1.26 1.79 1.69 2.18	0.45 0.52 0.64 0.69 1.32 0.88 1.25 1.13	0.38 0.31 0.37 0.54 0.64 0.69 -

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## TABLE 7

# Set B - Constant speed of 250 ft/sec with various drop sizes

Case	Drop dia mm	Penetration y in,		Max y	Drop	(x + y)	$t_s \times 10^3$	$t_{ph} \times 10^3$	$t_p \times 10^3$
		Gun 1	Gun 2	in.	movement x in.	in.	Secs	secs	secs
B1 B2 B3	1.00 1.50 2.25	3•3 5•2 7•3	-	3.3 5.2 7.3	0.52 1.17 1.34	3.82 6.37 8.64	1.27 2.13 2.88	1.58 2.26 2.33	- 1.25

Case	Air- speed ft/sec	Penetration y in.		Max y	•	(x + y)	$t_s \times 10^3$		$t_p \times 10^3$
		Gun 1	Gun 2	in.	ment x in.	in.	Secs	secs	Secs
C1 C2 C3 C1 <sub>4</sub> C5	24,8 365 474 560 633	7.3 9.1 9.3 9.0 9.3	- 9.6 -	7.3 9.1 9.3 9.6 9.3	1.26 2.68 1.80 2.1+0 1.70	8.56 11.78 11.10 12.00 11.00	2.87 2.69 1.96 1.79 1.45	2.33 1.76 1.15 0.88 0.86	1.25 1.0½ 0.78 0.69 0.67

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TABLE 8

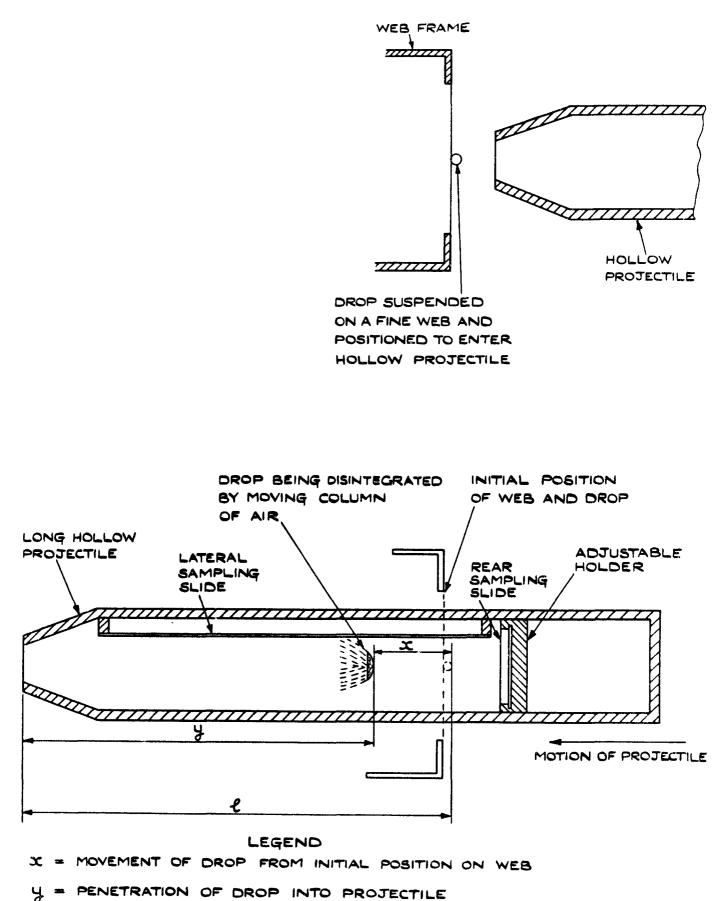
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<u>Set C - Constant</u>	drop size	of 2.25	mn diameter	with various	airspeeds

TABLE 9

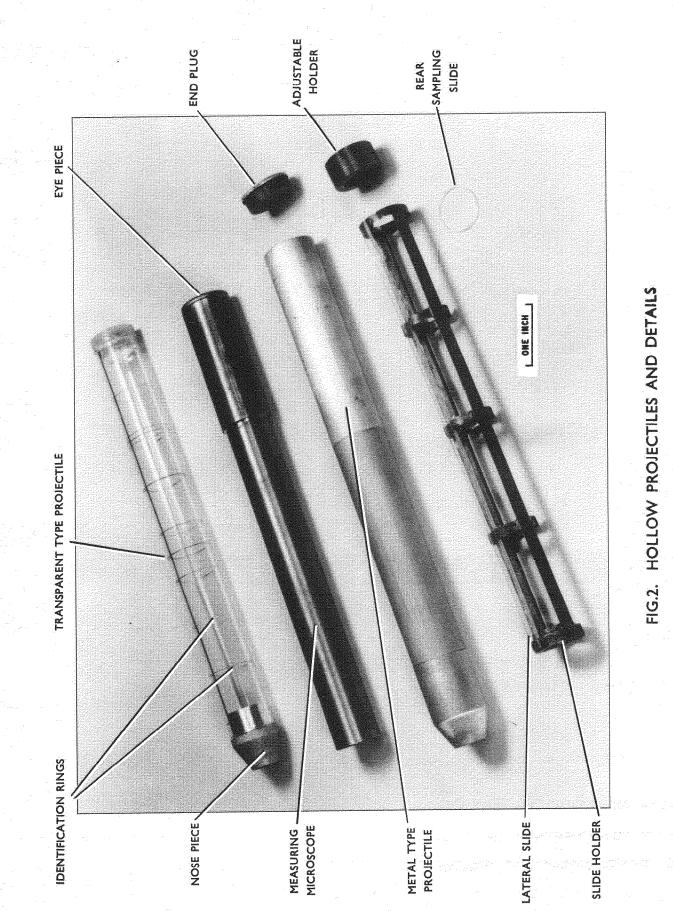
Set D - Constant drop size of 1.50 mm diameter with various airspeeds

Case	Air- speed ft/sec	Penetration y in.		Max y	1	(x + y)	$t_s \times 10^3$	$t_{\rm ph} \times 10^3$	
		Gun 1	Gun 2	in.	ment x in.	in.	Secs	secs	secs
D1 D2 D3 D4 D5 D6 D7	251 322 389 477 567 612 702	5.2 4.8 5.1 5.4 4.8 5.0	- - 5.7 5.9	5.2 4.8 5.1 5.4 5.7 5.9	1.17 0.46 0.81 0.40 0.56 0.69	6.37 5.26 5.91 5.80 6.26 6.59	2.11 1.35 1.26 1.02 0.91+ 0.90	2.26 1.41 1.16 0.99 0.64 0.52 0.1.7	- 0.74 0.140 - 0.34



- CILERATION OF DROF INTO FROJECTILE
- $\ell$  = MOVEMENT OF PROJECTILE WHILST DROP IS INSIDE = x + y

# FIG. I. DIAGRAMMATIC REPRESENTATION OF THE HOLLOW PROJECTILE METHOD,



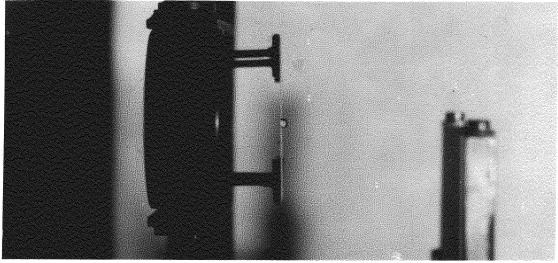


FIG.3. 2mm. DIAMETER WATER DROP SUSPENDED ON A FINE WEB

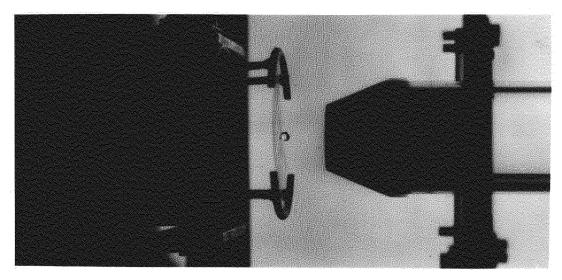


FIG.4. 2mm. DIAMETER WATER DROP ABOUT TO ENTER HOLLOW PROJECTILE MOVING AT 560ft./sec.

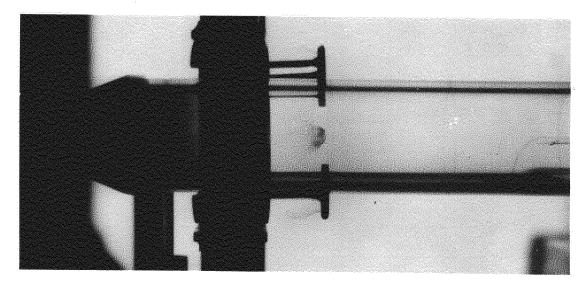


FIG.5. DISINTEGRATION OF A 2mm. DIAMETER WATER DROP INSIDE A HOLLOW PROJECTILE MOVING AT 560ft./sec.

R.A.E: 166348 64

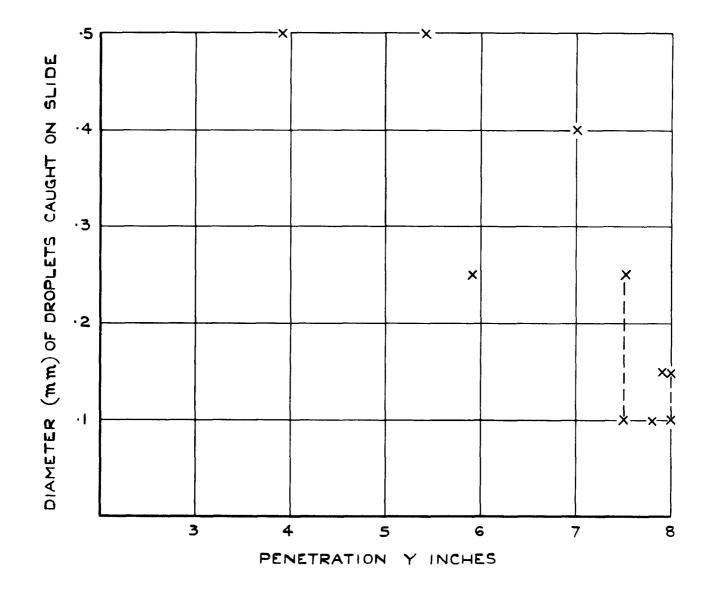
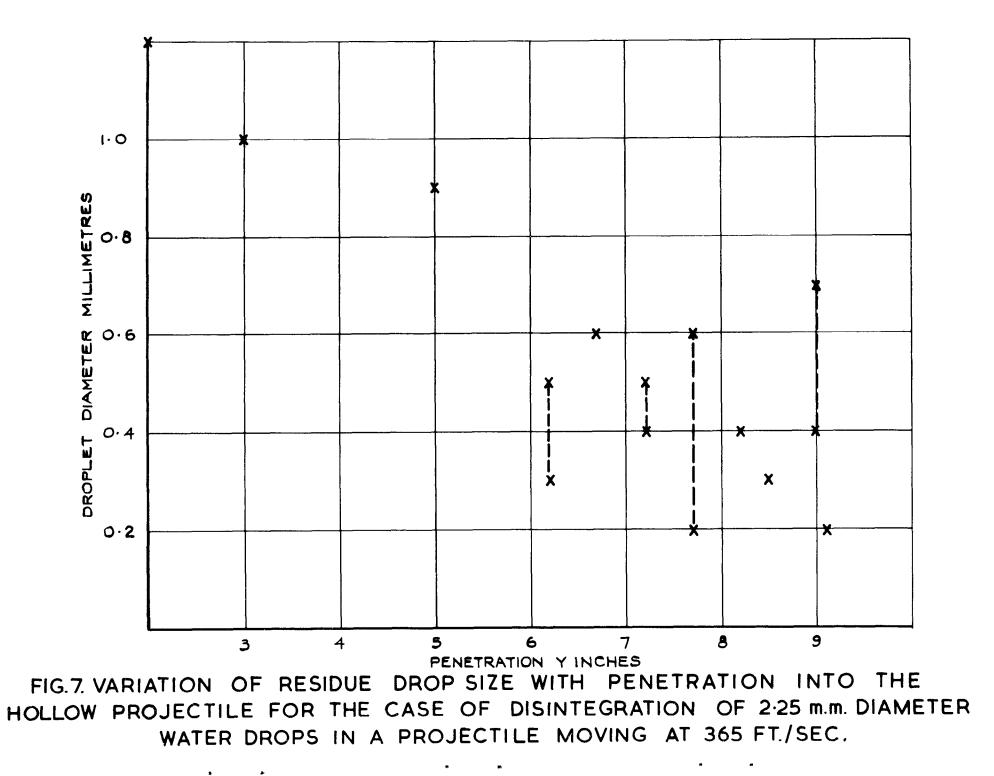
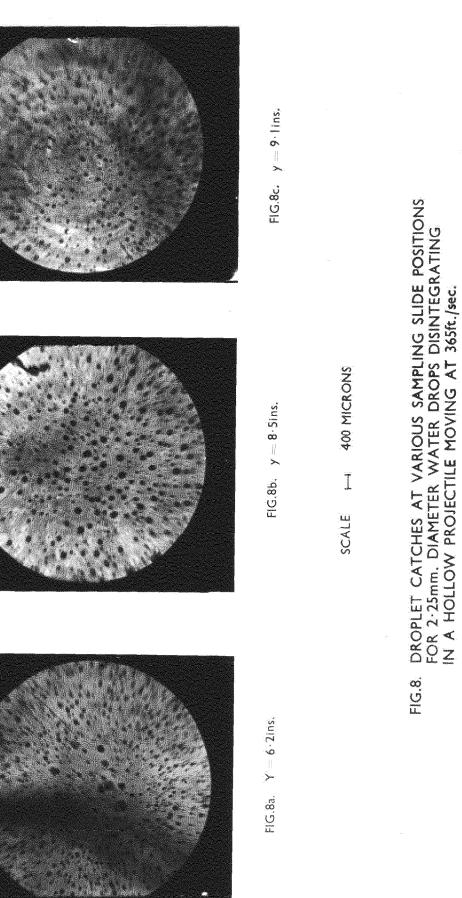


FIG. 6. VARIATION OF RESIDUE DROP SIZE WITH PENETRATION INTO HOLLOW PROJECTILE FOR THE CASE OF DISINTEGRATION OF 2.0 m.m DIAMETER WATER DROPS IN A PROJECTILE MOVING AT 560 FT./SEC.





R.A.E. 166349 64

FIG.8

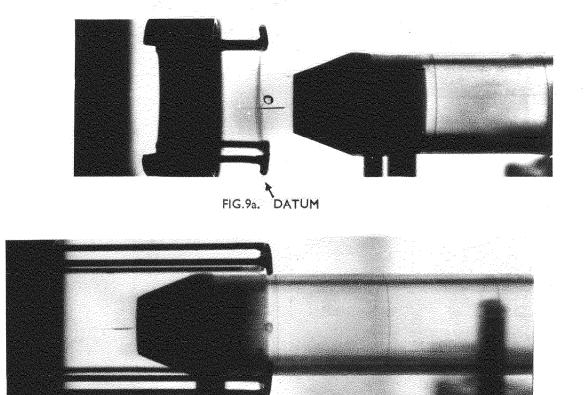


FIG.9

FIG.9b. y = 1.27 ins. x = 0.07 ins. t = 0.31 millisecs

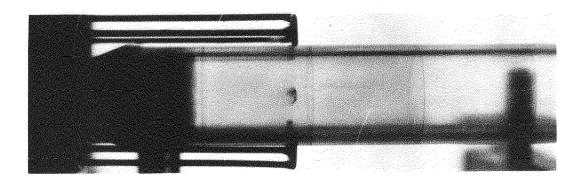
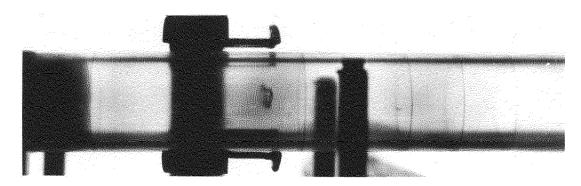


FIG.9c.  $y = 2 \cdot 16$  ins.  $x = 0 \cdot 09$  ins.  $t = 0 \cdot 51$  millisecs.



 $FIG.9d. \quad y=3\cdot 04 ins. \quad x=0\cdot 14 ins. \ t=0\cdot 73 millisecs.$ 

FIG.9. PHOTOGRAPHIC RECORD OF THE PROGRESS OF DISINTEGRATION OF 2.25mm. DIAMETER WATER DROPS IN A HOLLOW PROJECTILE MOVING AT 365ft./sec.

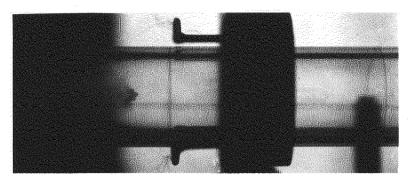


FIG.9e. y = 4.06 ins. x = 0.42 ins. t = 1.04 millisecs.

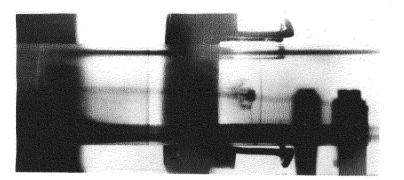


FIG.9f. y = 5.30 ins. x = 0.40 ins. t = 1.18 millisecs.

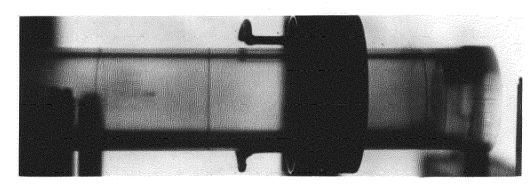


FIG.9g. y = 6.80 ins. x = 0.89 ins. t = 1.76 millisecs.

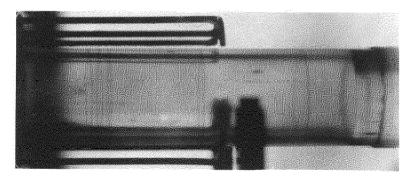
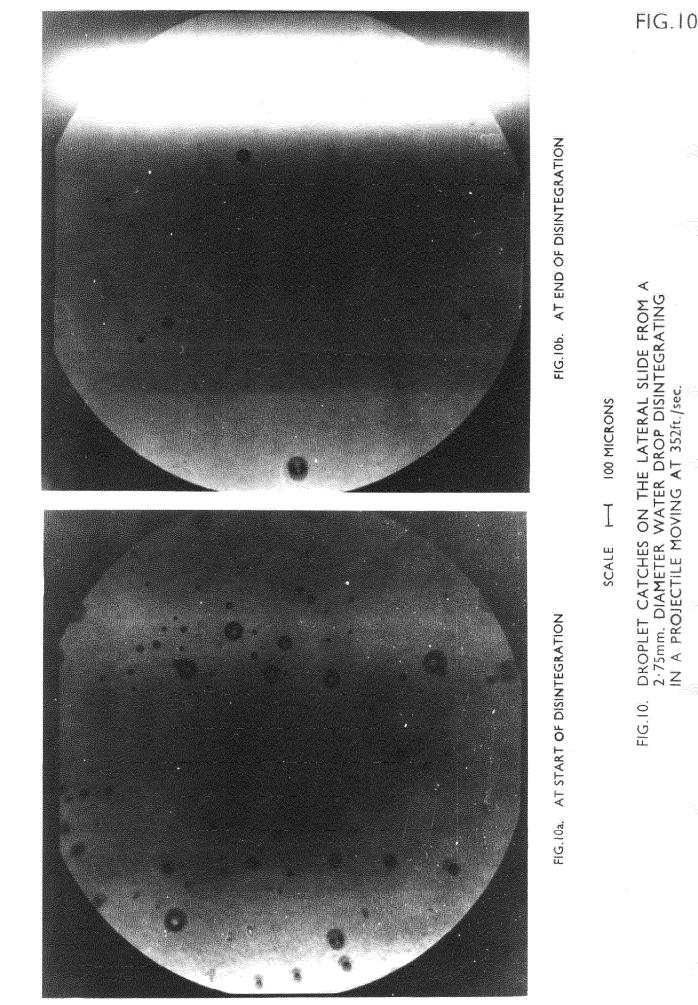


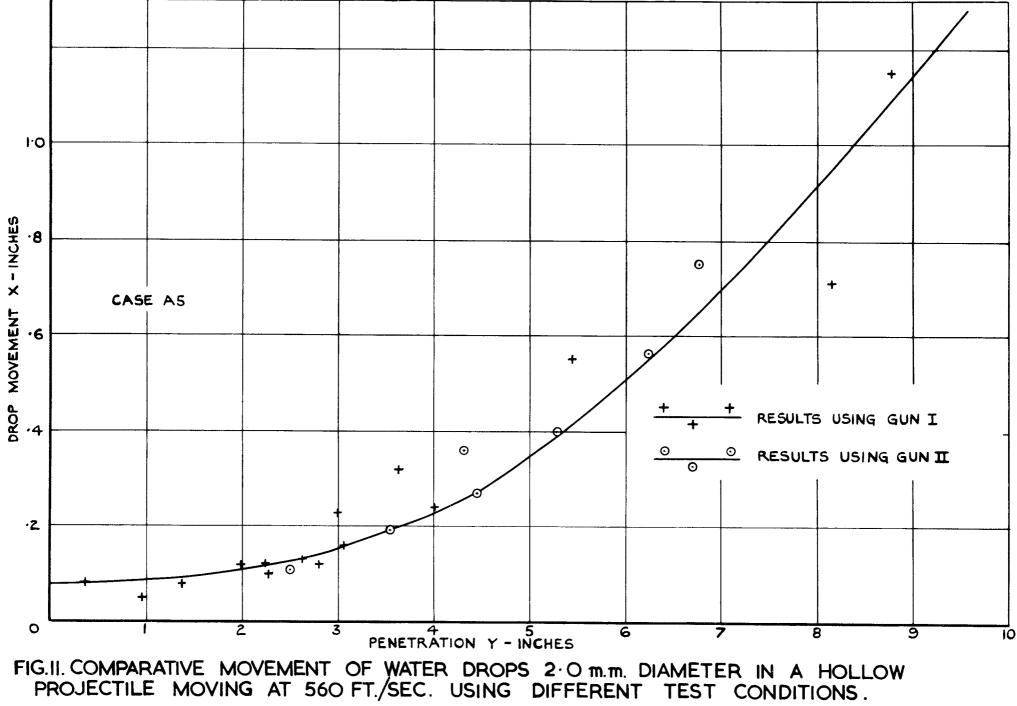
FIG.9h.  $y = 7 \cdot 14$  ins.  $x = 1 \cdot 36$  ins.  $t = 1 \cdot 92$  millisecs.

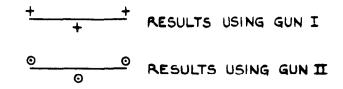
FIG.9(cont'd.) PHOTOGRAPHIC RECORD OF THE PROGRESS OF DISINTEGRATION OF 2.25mm. DIAMETER WATER DROPS IN A HOLLOW PROJECTILE MOVING AT 365ft./sec.

R.A.E. 166351 64



R.A.E: 166352 64





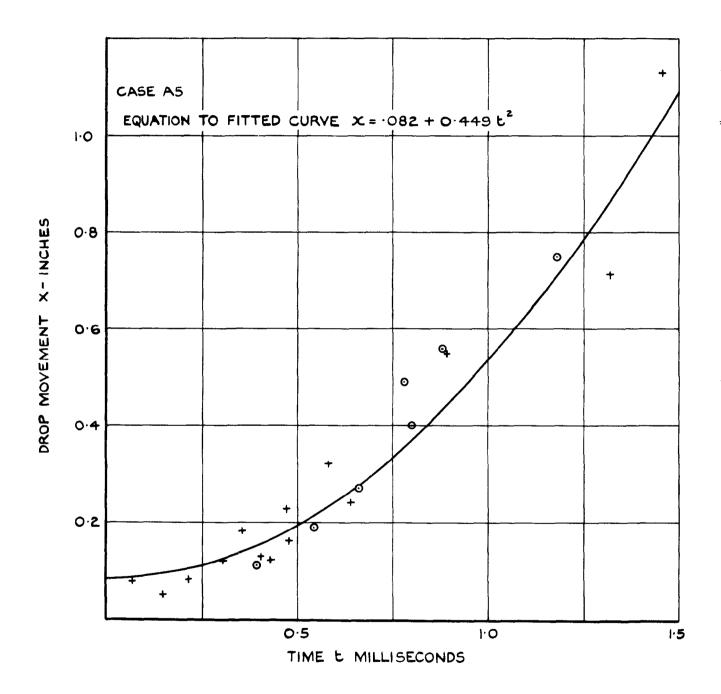


FIG.12. MOVEMENT OF 2.0 m.m. DIAMETER WATER DROPS IN A HOLLOW PROJECTILE MOVING AT 560 FT./SEC. UNDER DIFFERENT TEST CONDITIONS.

EQUATIONS TO FITTED CURVES

1.0mm DROPS	X = · 0539 + 1·085 t <sup>2</sup>	(CASE AI )
1.25mm DROPS	X = · 0996 + 1·088 t <sup>2</sup>	(CASE A2 )
1.50mm DROPS	X ≖·0753 +0695 t²	(CASE A3 )
1.75mm DROPS	X = 9823 + 0.785 t <sup>2</sup>	(CASE AA)
2.0mm DROPS	$X = 0828 + 0.448 t^2$	(CASEA5)
2.25mm DROPS	X = 0186 + 0.758 t <sup>2</sup>	(CASE A6 )
2.50mm DROPS	$X = 0679 + 0.322 t^2$	(CASE A7)
2.75mmDROPS	$X = 0.43 + 0.666 t^2$	(CASE A8)

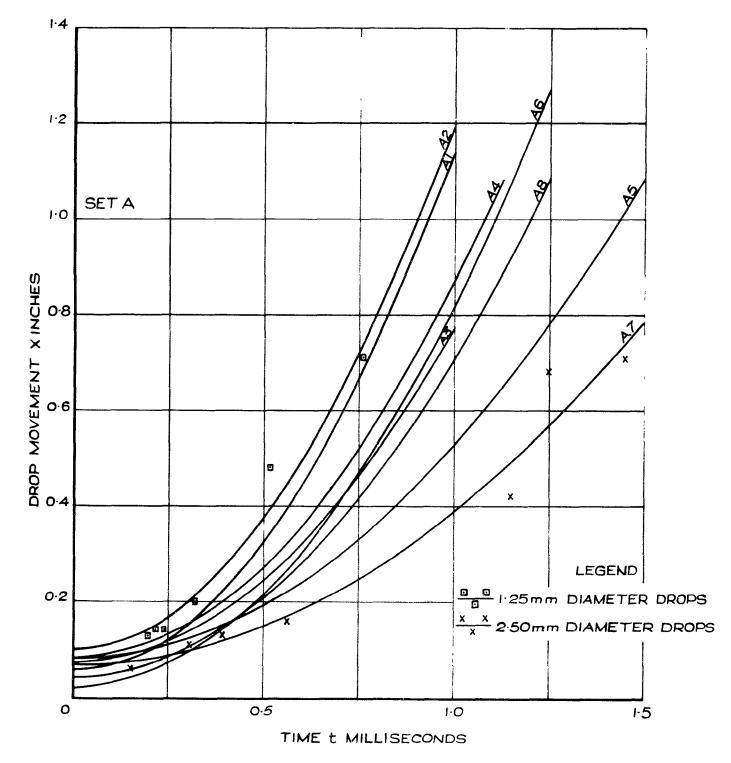


FIG.13. MOVEMENT OF DROPS OF VARIOUS DIAMETERS IN AN AIRSTREAM OF VELOCITY 560 FT/SEC.

(1.0mm DROPS)	$X = .0003 + .335 t^2$	(CASE BI)
	$x = .0307 + .243 t^2$	
(2.25mm DROPS)	$x = .0065 + .1605 t^{2}$	(CASE B3)

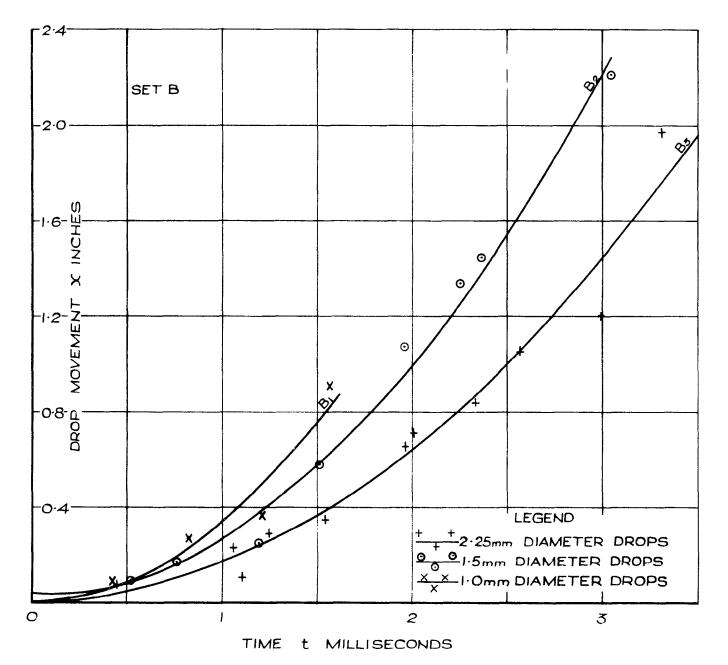
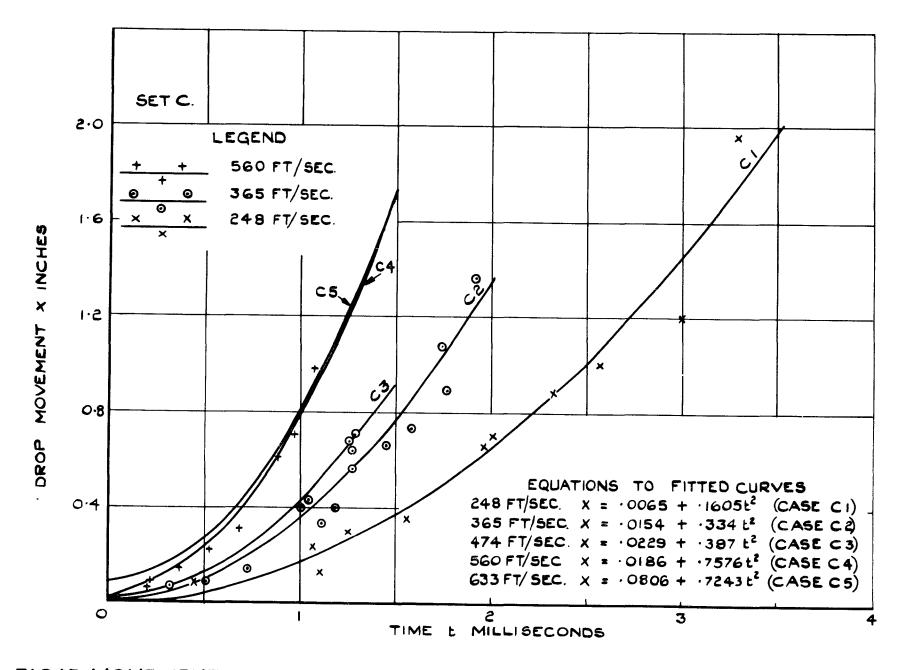


FIG. 14. MOVEMENT OF DROPS OF VARIOUS DIAMETERS IN AN AIRSTREAM OF VELOCITY 250 FT/SEC.



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FIG.15. MOVEMENT OF 2.25 m.m. DIAMETER WATER DROPS IN AIRSTREAMS OF VARIOUS VELOCITIES.

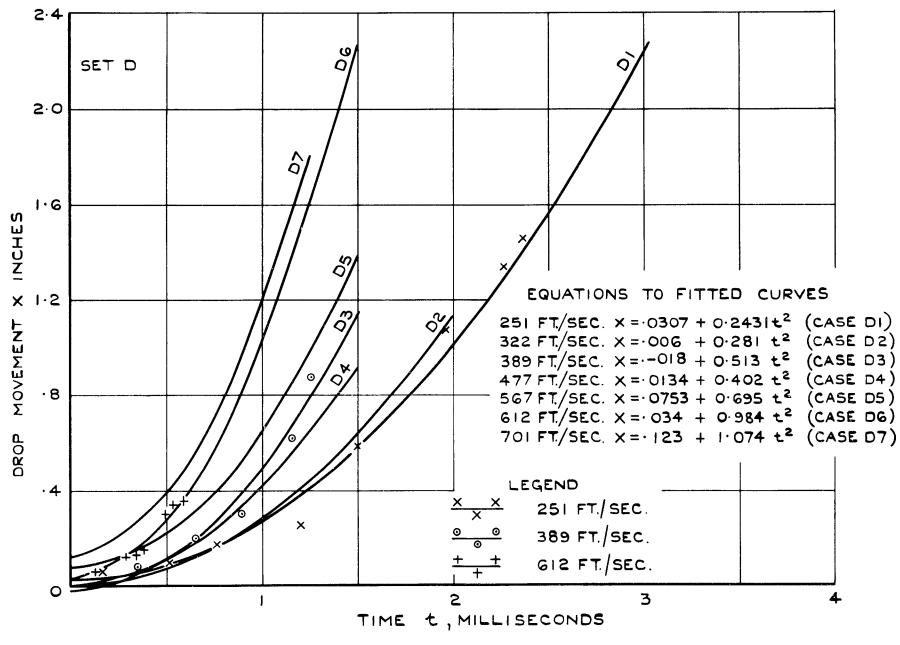
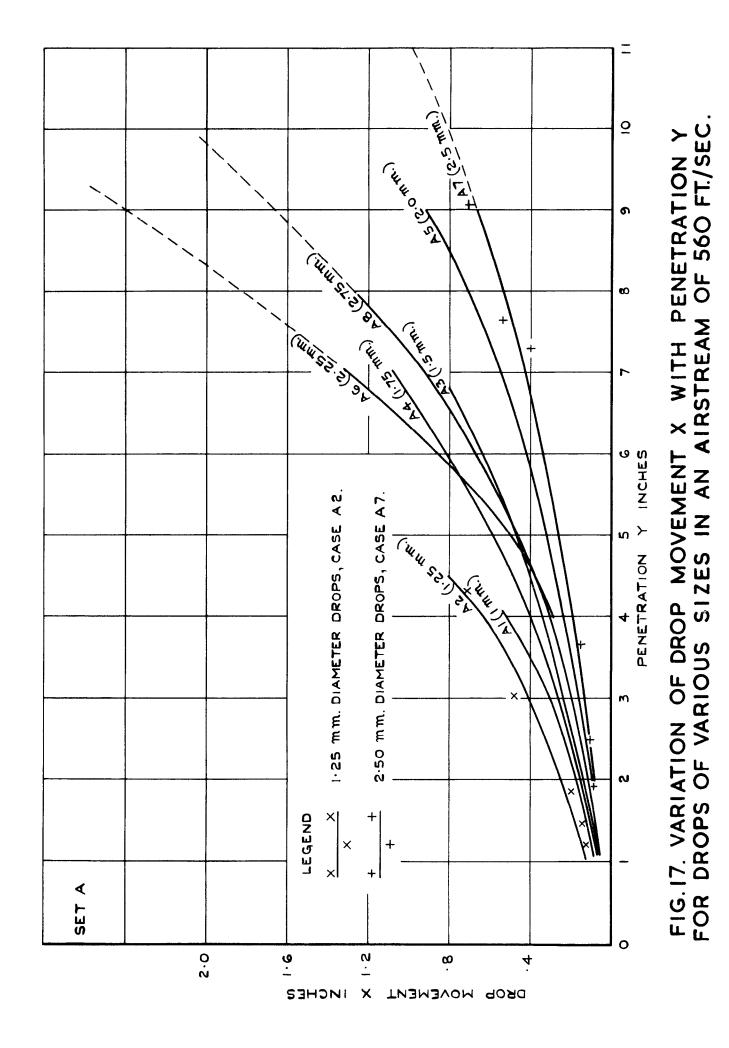


FIG. 16. MOVEMENT OF WATER DROPS OF 1.5 mm. DIAMETER IN AIRSTREAMS OF VARIOUS VELOCITIES.

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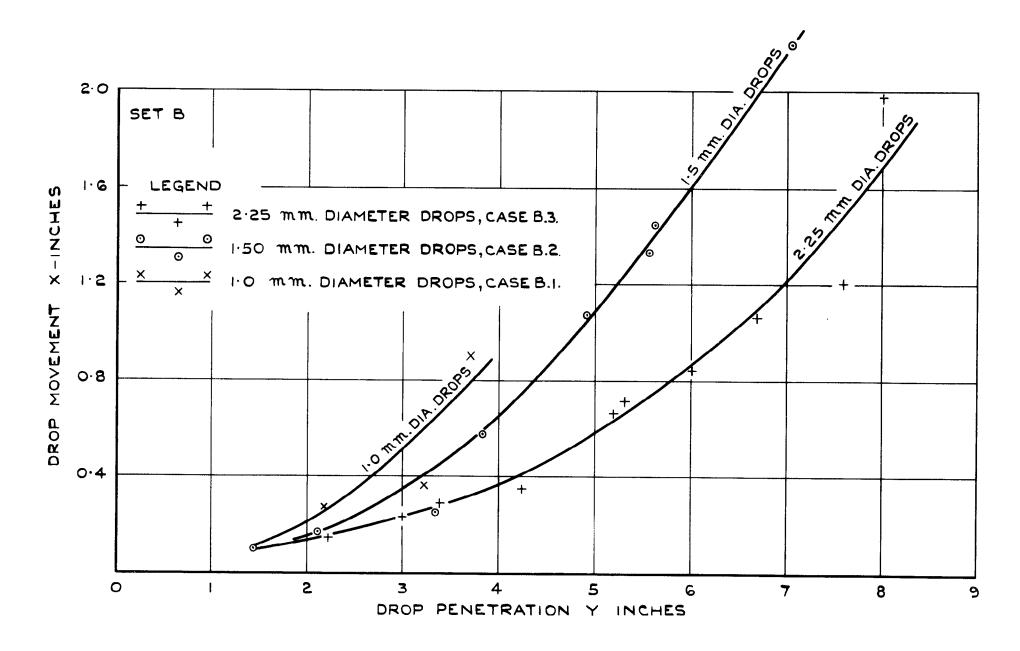


FIG.18. VARIATION OF DROP MOVEMENT X WITH PENETRATION Y FOR DROPS OF VARIOUS SIZES IN AN AIRSTREAM OF 250 FT/SEC.

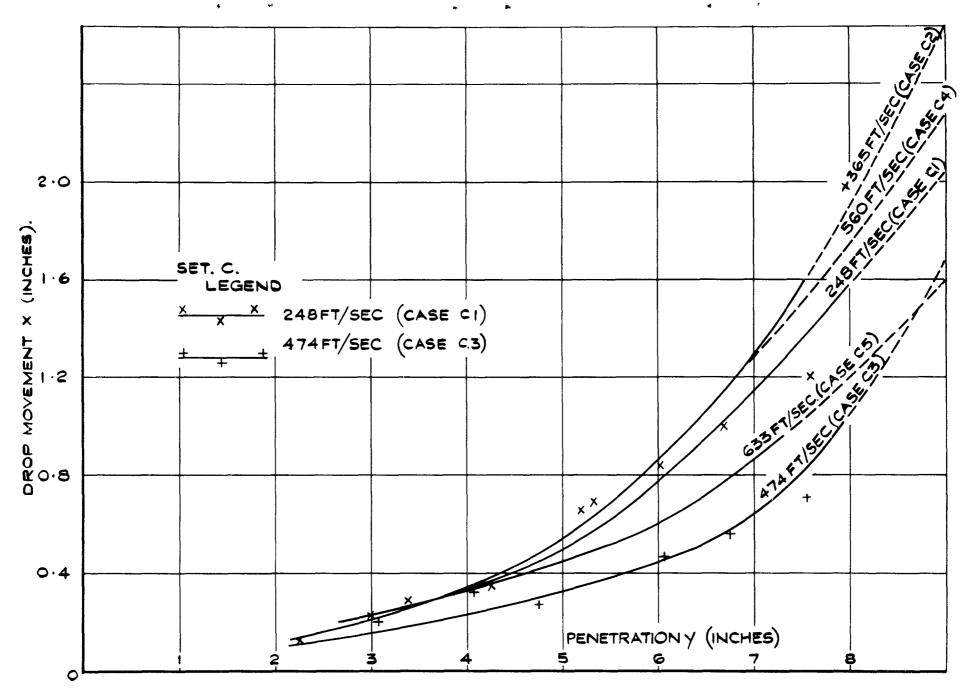


FIG. 19. VARIATION OF DROP MOVEMENT X WITH PENETRATION Y FOR WATER DROPS OF DIAMETER 2.25 mm IN AIRSTREAMS OF VARIOUS VELOCITIES.

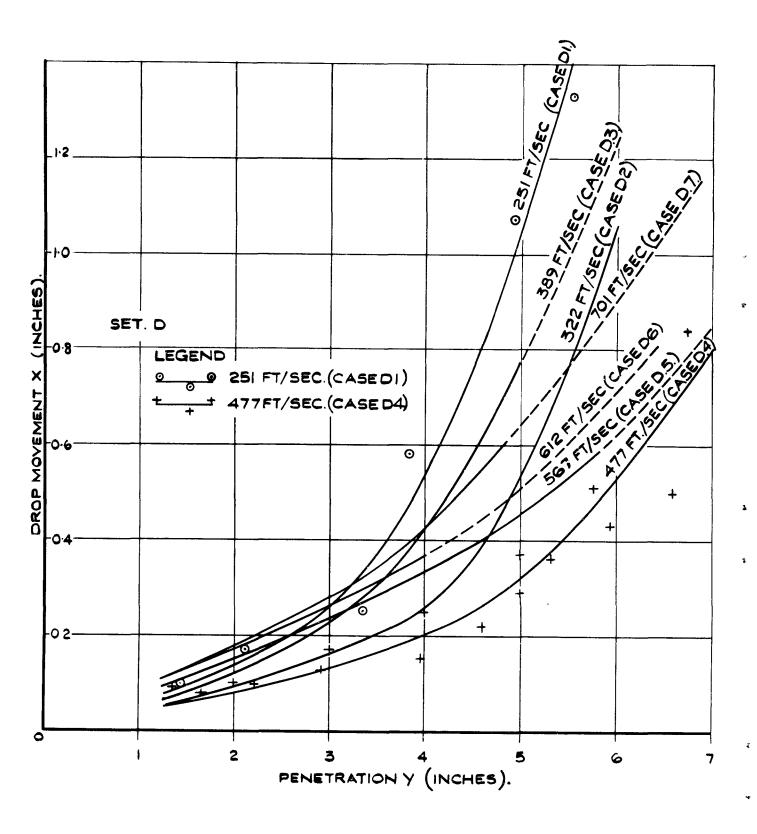


FIG. 20. VARIATION OF DROP MOVEMENT X WITH PENETRATION Y FOR WATER DROPS OF DIAMETER 1.5 mm IN AIRSTREAMS OF VARIOUS VELOCITIES.

3 ts= 200/250.72 DISINTEGRATION TIME MILLISECONDS + 2 ts= 200/ 560.72 0 2 3 DROP DIAMETER MILLIMETRES.

FIG.21. VARIATION OF DISINTEGRATION TIME WITH DROP SIZE IN AIRSTREAMS OF VELOCITY 250 AND 560 FT/SEC.

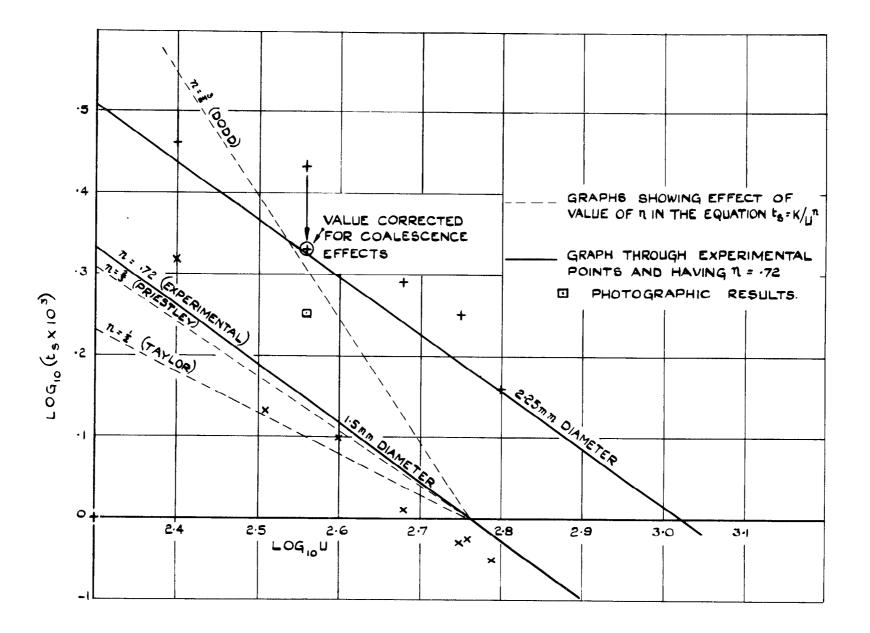


FIG. 22. VARIATION OF  $LOG_0$  (DISINTEGRATION TIME  $t_s \times 10^3$ ) WITH  $LOG_{10}$  (AIRSTREAM VELOCITY U) FOR CONSTANT DROP SIZES OF 1.50 AND 2.25 MILLIMETRES DIAMETER.

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A.R.C. C.P. No.827	532.6 : 533.6.011	A.R.C. C.P. No.827	532.6 : 533.6.011	
THE TIME REQUIRED FOR HIGH SPEED AIRSTREAMS TO DISINTEGRAT Jenkins, D. C. and Booker, J. D. May 1964.	TE WATER DROPS.	THE TIME REQUIRED FOR HIGH SPEED AIRSTREAMS TO DISINTEGRATE WATER Jenkins, D. C. and Booker, J. D. May 1964.	DROPS.	
The time required for high speed airstreams to disin drops has been determined experimentally, and an empirical between the time, the airstream velocity and the drop dian acceleration of drops during disintegration has also been empirical relationship derived. The equation of motion of drop has been considered and a drag coefficient determined drop motion agreeing reasonably well with that found exper particular case. Droplets produced during disintegration measured in a particular case and compared with the sizes	and an empirical relation found and the drop diameter. The on has also been found and an tion of motion of a disintegrating icient determined which gives a t that found experimentally in a g disintegration have been determined experiment between the time, the airstream velo acceleration of drops during disinte empirical relationship derived. The drop has been considered and a drag drop motion agreeing reasonably well particular case. Droplets produced		ity and the drop diameter. The ration has also been found and an equation of motion of a disintegrating oefficient determined which gives a with that found experimentally in a uring disintegration have been	
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		A.R.C. C.P. No.827 THE TIME REQUIRED FOR HIGH SPEED AIRSTREAMS TO DISINTEGRATE WATER Jenkins, D. C. and Booker, J. D. May 1964. The time required for high speed airstreams to disintegrate drops has been determined experimentally, and an empirical relatible between the time, the airstream velocity and the drop diameter. acceleration of drops during disintegration has also been found a empirical relationship derived. The equation of motion of a disider drop has been considered and a drag coefficient determined which drop motion agreeing reasonably well with that found experimental particular case. Droplets produced during disintegration have be measured in a particular case and compared with the sizes that we (Compared the sizes that we compared the sizes that we compare the size	e water Ion found The and an integrating gives a Lly in a een	

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expected if some of the proposed mechanisms of disintegration were operative. It has not been found possible to determine conclusively what mechanism operates to cause disintegration, but the evidence favours a wave-making mechanism.

Suggestions for further work have been made in order to establish the effect of other parameters involved in the disintegration process.

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