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Approximate Two-dimensional Aerofoil Theory.

Part VI. Aerofoils with Hinged Flaps

By

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Approximate Two-Dimensional Aerofoil Theory. Part VI. Aerofoils with Hinged Flaps

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Summary

Hinged flaps on aerofoils are of interest not only as control surfaces, but also as devices for introducing variable camber on low-drag wings to extend the range of lift coefficients at which low drag is obtained and to decrease the moment coefficient at high speeds.

A usual notation in the aerodynamics of control surfaces, namely $C_L = a_0 + a_1\alpha' + a_2\eta$, leads to confusion, since in ordinary aerofoil theory a_0 is generally used for the slope of the (C_L, α) curve. All confusion is avoided by writing $C_L = c_0 + c_1\alpha' + c_2\eta$; we then have $c_1 = a_0$, but there seems no objection to such a relationship.

A simple theory is developed for determining the velocity at the surface and other aerodynamic properties of an aerofoil with a hinged flap. Formulae are obtained for $\Delta\beta/\eta$, $m = -\Delta C_{M_0}/\eta$, $(\pi/a_0 + 1/2)\ \Delta C_{L_{opt}}/\eta$, $\Delta\alpha_{opt}/\eta$, c_2/c_1 (eqns.(9) – (12), (50)) and values of these quantities are tabulated in Table 1 ($\Delta\alpha_{opt}/\eta$ for $a_0 = 2\pi$ only) for E = 0·4, 0·3, 0·25, 0·2, 0·15, 0·1, 0·08; η is the downward deflection of the flap and E the flap chord as a fraction of the aerofoil chord. The reduction of C_{M_0} by the use of a flap on a low-drag wing is discussed. If the wing is originally cambered for a high design C_L, denoted by $C_{L_{opt}}$, with a centre line for which $-C_{M_0}/C_{L_{opt}} = K$ (for values of K for modern centre lines, see Ref.4, §10 and Table 1), and if, at high speeds, the flap is raised so that $C_{L_{opt}} + \Delta C_{L_{opt}}$ becomes the low-speed equivalent of the top-speed C_L, then $-(C_{M_0} + \Delta C_{M_0})$ is less than $-C_{M_0}$ would have been if the wing had been originally cambered for $C_{L_{opt}} + \Delta C_{L_{opt}}$ without a flap by $-(\frac{1}{2} - \frac{1}{2}E - K)\Delta C_{L_{opt}}$. (Note that $\Delta C_{L_{opt}}$ is negative.)

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*Of the Aerodynamics Division, N.P.L. This paper is published with the permission of the Director, N.P.L.

Of the quantities required to find the velocity at the surface of the aerofoil by the methods of Ref. 2, g_s, ψ_s, ε_s, ε_s', C_0 are unaltered by the deflection of the flap; formulae are found for $\Delta g_1/\eta$ (eqn. 31), and for r, s and t, where $\Delta\psi_c = r\eta$, $\Delta(\varepsilon_c - \beta) = s\eta$, $\Delta\varepsilon_c' = t\eta$, (eqns. (36) – (38)); these quantities are tabulated against x in Table 2 for the same values of E as before. Explicit formulae are set out for the change in the velocity due to the deflection of the flap (eqns. (40) – (46)).

A very rough calculation is given of the limiting value of the flap deflection η for which the position of maximum suction at $C_L = C_{L_{opt}} + \Delta C_{L_{opt}}$ on the upper surface of a low-drag aerofoil with the flap up (or on the lower surface with the flap down) is likely to be at the design position for $C_L = C_{L_{opt}}$, $\eta \neq 0$. If this position is at $x = X$, the limiting deflection is likely to be nearly proportional to $1 - E - X$; for a 20 per cent flap, for example, it is likely to be nearly twice as large for $X = 0{\cdot}4$ as for $X = 0{\cdot}6$. Only a gradual increase of drag at $C_L = C_{L_{opt}} + \Delta C_{L_{opt}}$ is to be anticipated when η is increased beyond the limit considered.

Formulae are found for the calculation of hinge moments. In these calculations a_0 is given its theoretical 'Kutta-Joukowski' value, since for any other value of a_0 the large values of the velocity at the trailing edge, which result from theory and are not realized in practice, completely falsify the calculation of the hinge moment. For cusped or nearly cusped aerofoils there is some reason to believe that calculations with the 'Kutta-Joukowski' value of a_0 give fairly satisfactory values of the normal loading on the flap and of the hinge moment; for aerofoils with large trailing-edge angles empirical corrections must be sought to the values so calculated; these empirical corrections will probably depend on the position of the transition to turbulence in the boundary layer, on the Reynolds number and, in wind-tunnels, on the turbulence of the stream. The present method of correlating experimental data for aerofoils which are not cusped also requires theoretical calculations with the 'Kutta-Joukowski' value of a_0.

With the hinge moment coefficient expanded in the form

$$C_H = b_0 + b_1\alpha' + b_2\eta = b_0 - \frac{b_1}{c_1} c_0 + \frac{b_1}{c_1} C_L - b\eta,$$

equations are obtained for the determination of b_0, b_1, b, b_2. Approximation I of Ref. 2 leads to the same values as the 'flat-plate' theory (Ref. 3) for b_1, b and b_2. [Eqns. (66) – (69) and Table 1]. This is equally true of the values of m and c_2/c_1, but whereas those values are also correct on more exact theories, the same is not true of b_1, b, b_2. The 'flat-plate' values of b_1, b, b_2 are, in fact, not sufficiently correct for most purposes; formulae are given for obtaining them (and also b_0) by numerical integration on Approximation III (eqns. (61) – (63), (70) – (72)); use of these formulae will usually be more correct and more convenient than the actual integration of the pressures to find the hinge moment with selected values of α'(or C_L) and η.

In/

In the simple theory in the body of the report squares of η and products of η and the aerofoil ordinates are neglected. More accurate calculations will be very rarely wanted; it is shown in the Appendix how a theory may be constructed in which the products of η and the aerofoil ordinates are not neglected.

For wings of similar thickness distribution whose thicknesses are not exceptionally large, the corrections to be applied to the values of b_0, b_1, b_2 and b found from Approximation I are all approximately proportional to the thickness; the value of b_0 according to Approximation I is proportional to the camber. The effect produced by a local modification to the thickness distribution is proportional to its scale.

For an aileron of 25 per cent chord on a certain suction aerofoil the following values have been computed.

b_0	b_1	b_2	b	Method	
0·046	−0·386	−0·725	0·490	Approx.III: eqns.(61)–(63),(70)–(72).	
0·049	−0·565	−0·944	0·599	Approx.I; eqns. (66) – (69).	
0·046	−0·383			Approx.III, $a_0=2\pi e^{C_0}$ (=7·044)	Direct integration for C_H from values of q/U at $C_L=0$ and $C_L=0{\cdot}5$ with $\eta = 0$ (Eqn. (55)).
0·050	−0·570			Approx.I, $a_0=2\pi$	
0·052	+0·029			Approx.III, $a_0=2\pi$	

For the 15 per cent thick low-drag aerofoil with a 20 per cent flap which is to be tested in the 13 ft. by 9 ft. wind-tunnel at the N.P.L., values of $\Delta\psi/\eta$, $\Delta(\varepsilon - \beta)/\eta$, $\Delta\varepsilon'/\eta$ have been computed by the theory of the Appendix for comparison with the results of the simple theory in the body of the report [Table 4; values are to be compared with those headed r, s, t under E = 0·2 in Table 2]. For the same aerofoil the following values of b_0, $-b_1$, $-b_2$ and b have been computed; the factors causing the differences between the last two lines are discussed.

b_0	$-b_1$	$-b_2$	b	Method
0	$0{\cdot}499_5$	0·923	0·648	Approximation I
0	0·349	0·739	0·547	Approximation III. Simple Theory. Eqns. (61) – (63), (70) – (72).
0	0·364	0·774	0·574	Approximation III. Theory in Appendix.

Introduction/

1. Introduction

The calculation of the velocity at the surface and of other aerodynamic properties, especially the hinge moment, of an aerofoil with a hinged flap is considered, on a simple theory, in this report; it is shown in an appendix how more accurate results may be obtained.

Hinged flaps on aerofoils are of interest not only as control surfaces, but also as devices for introducing variable camber on low-drag wings to extend the range of lift coefficients at which low drag is obtained and to decrease the moment coefficient at high speeds[1].

The theory may be expected to lead to values of the velocity in reasonable agreement with experiment except near the trailing edge and the hinge, and at large flap deflections. Boundary-layer effects near the trailing edge make predictions of hinge moments difficult; there is some reason to hope, however, that fairly satisfactory results may be obtained for cusped or nearly cusped aerofoils. It is planned to test the theory and to make a beginning of assessing its limitations by pressure-plotting experiments in the 13 ft. × 9 ft. wind tunnel at the N.P.L.

2. Notation

The chord of the aerofoil, by definition, joins the leading and trailing edges, and is normal to the aerofoil contour at the leading edge and also at a rounded trailing edge.

x, y: x is the distance from the leading edge measured along the chord and y the aerofoil ordinate, both in fractions of the chord, and both for zero flap deflection (Fig. 1).

η : the flap deflection, measured in radians and counted as positive when the flap is deflected downwards. (Fig. 2).

x_1, y_1: the coordinates with respect to the axes of x and y, for a flap deflection η, of that point on the surface of the flap which is at (x, y) for zero deflection.

x_2, y_2: the distance from the leading edge measured along the new chord, and the aerofoil ordinate normal to the new chord, respectively, both in fractions of the chord, and both for a flap deflection η. (Fig. 3).

h : the contour of the flap, when undeflected, meets the upper and lower surfaces of the aerofoil at $x = h$.

E : $E = 1 - h$, and E may be taken as the flap chord (expressed as a fraction of the original chord of the aerofoil). (Fig. 1).

θ : for zero flap deflection

$$x = \sin^2 \tfrac{1}{2}\theta = \tfrac{1}{2}(1 - \cos\theta),$$

and for a flap deflection η

$$x_2 = \sin^2 \tfrac{1}{2}\theta = \tfrac{1}{2}(1 - \cos\theta).$$

θ_1 : $\theta_1 = 2\sin^{-1}\sqrt{h} = 2\cos^{-1}\sqrt{E}.$

Figs. 1/

Fig 1.

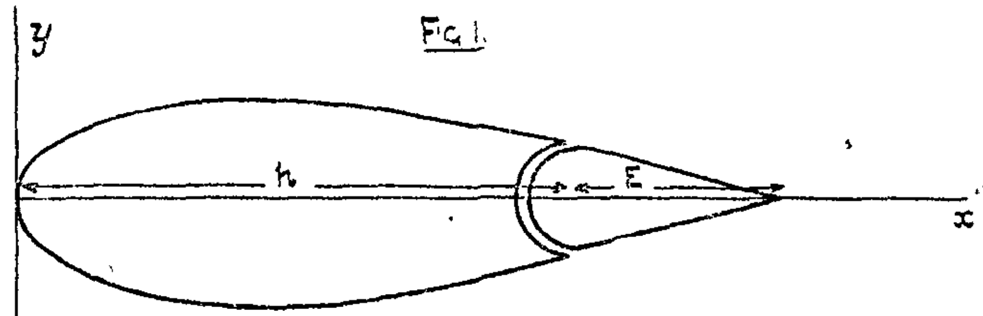


Fig 2

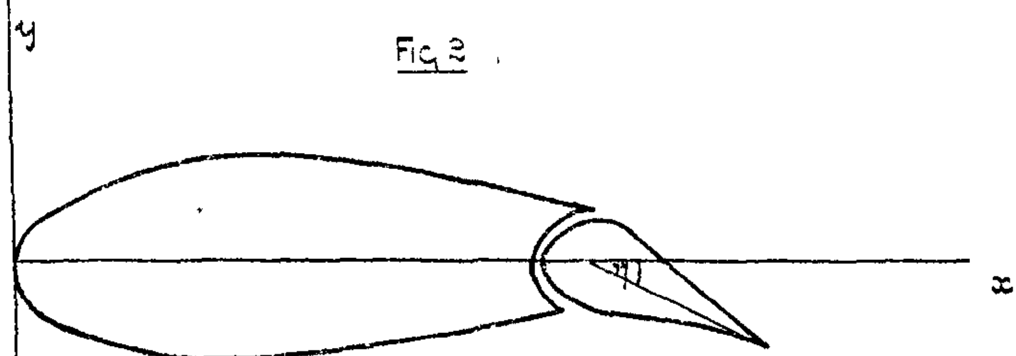
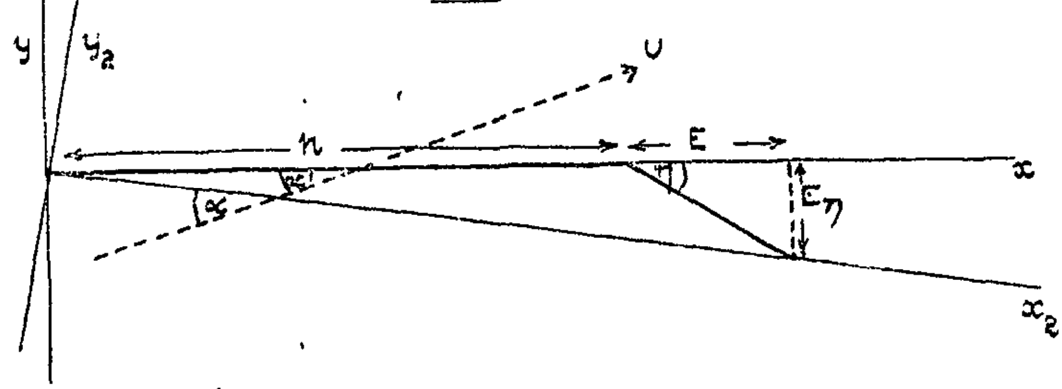


Fig 3



h_1, k_1: the coordinates of the hinge with respect to the original axes of x and y. For all but the most accurate calculations (which are very seldom required), h_1 may be taken equal to h, and the value of k_1 is largely immaterial as long as it is not greater, numerically, than each of the ordinates of the top and bottom surfaces at $x = h$.

α : the incidence, for a flap deflection η, with respect to the new chord (Fig. 3).

α' : the incidence with respect to the original chord — i.e. the incidence of the front part of the wing, or the incidence at zero flap deflection (Fig. 3).

U : the undisturbed velocity of the air relative to the aerofoil.

ρ : the air density.

q : the velocity of the air at the surface of the aerofoil, relative to the aerofoil

g_s, g_i, β, $C_{L_{opt}}$, α_{opt}, C_{M_0}, ψ, ε, ε', C_0, γ_0: See Ref. 2.
The symbols are used here to denote values for zero flap deflection. $-\beta$, the no-lift angle, and α_{opt}, the incidence for the optimum C_L, are both measured from the chord for zero flap deflection. ε' denotes $d\varepsilon/d\theta$.

Δg_i, $\Delta C_{L_{opt}}$, $\Delta\beta$, etc.: Values for a flap deflection η (to the first order in η) are denoted by $g_i + \Delta g_i$, $C_{L_{opt}} + \Delta C_{L_{opt}}$, $\beta + \Delta\beta$, etc. Those quantities which are functions of position, such as $g_i + \Delta g_i$, refer to a value of x_2 equal to the value of x to which g_i refers; i.e., g_i and Δg_i refer to the same value of θ. Also $-(\beta + \Delta\beta)$, $\alpha_{opt} + \Delta\alpha_{opt}$ are measured from the new chord line — i.e. they are the values of α, and not of α' (see Fig. 3) when $C_L = c$ and when $C_L = C_{L_{opt}}$, respectively.

L,M,N: When q/U is expanded in powers of C_L and η the first terms are

$$q/U = L + MC_L + N\eta.$$

Suffixes u, ℓ, s, c: the suffix u denotes values on the upper surface and the suffix ℓ values on the lower surface; the suffix s refers to values for the corresponding symmetrical aerofoil and the suffix c to values for the camber or centre line.

r,s,t,m/

r,s,t,m: $r = \Delta\psi_c/\eta,$

$s = (\Delta\varepsilon_c - \Delta\beta)/\eta,$

$t = \Delta\varepsilon_c'/\eta,$

$m = -\Delta C_{M_0}/\eta.$

a_0 : $C_L = a_0 \sin(\alpha + \beta + \Delta\beta).$

c_0, c_1, c_2: When C_L is expanded in powers of α' and η the first terms* are

$$C_L = c_0 + c_1\alpha' + c_2\eta,$$

so $c_1 = a_0$.

C_H : the hinge-moment coefficient, such that the hinge moment is $\frac{1}{2}\rho U^2 E^2 c^2 C_H$ for an aerofoil chord of length c.

b_0, b_1, b_2, b: When C_H is expanded in powers of α' and η

$$C_H = b_0 + b_1\alpha' + b_2\eta,$$

and when C_H is expanded in powers of C_L and η,

$$C_H = b_0 - \frac{b_1}{c_1} c_0 + \frac{b_1}{c_1} C_L - b\eta$$

$$\left(\text{so } b + b_2 = \frac{c_2}{c_1} b_1\right).$$

3. The Simple Theory of an Aerofoil with a Hinged Flap

We suppose that the contour of the flap in its 'zero' position meets the upper and lower surfaces of the aerofoil at $x = h$ (Fig. 1), and that the coordinates of the hinge, with respect to the axes of x and y, are (h_1, k_1). For all but the most accurate calculations (which are very seldom required) we may take $h_1 = h$, and the value of k_1 is immaterial so long as it does not exceed, numerically, the ordinate of either surface at $x = h$.

Let/

*The notation here differs from that usually adopted in the consideration of aerofoils with control flaps, according to which $C_L = a_0 + a_1\alpha' + a_2\eta$. The use of a_0 for the slope of the (C_L,α) curve, however, is now well established; in particular, it is so used by Glauert and has been so used in preceding reports of this series. Adoption of the usual notation for control flaps would therefore lead to confusion between a_0 and a_1. All confusion is avoided by the simple expedient of using c_0, c_1, c_2 for a_0, a_1, a_2 as in the text. We then have $c_1 = a_0$; there seems to be no objection to such a relationship.

Let the point on the contour of the flap whose coordinates for zero deflection are (x, y) (with polar coordinates (r, ϕ) relative to the hinge) become (x_1, y_1) for a flap deflection η. Then

$$x = h_1 + r\cos\phi, \quad y = k_1 + r\sin\phi,$$

and

$$x_1 = h_1 + r\cos(\phi - \eta) = h_1 + (x - h_1)\cos\eta + (y - k_1)\sin\eta \qquad \ldots(1)$$

$$y_1 = k_1 + r\sin(\phi - \eta) = k_1 + (y - k_1)\cos\eta - (x - h_1)\sin\eta. \qquad \ldots(2)$$

If η is small, and squares of η and products of η and the aerofoil ordinates are neglected, then approximately

$$x_1 = x, \quad y_1 = y - \eta(x - h). \qquad \ldots(3)$$

The trailing edge is at the point $(1, -\eta(1 - h))$, and to the first order in η the chord is still of unit length.

For the most part we shall be content with this simple theory*, but the axes must be changed so that the new axis of x is along the new chord, joining the leading edge to the new position of the trailing edge (Fig. 3). To the first order in η the axes must be rotated through an angle $E\eta$, where $E = 1 - h$, is the length of the chord of the flap as a fraction of the total chord of the wing. The new coordinates, with a flap deflection η, are denoted by (x_2, y_2). To the first order in η, with products of η and the aerofoil ordinates neglected,

$$x_2 = x_1 = x, \qquad \ldots(4)$$

$$\left.\begin{aligned} y_2 &= y + E\eta x \quad (0 \leqslant x \leqslant h) \\ &= y_1 + E\eta x = y + h\eta(1 - x) \quad (h \leqslant x \leqslant 1) \end{aligned}\right\} \qquad \ldots(5)$$

Since the ordinates of both the top and bottom surfaces are altered by the same amount, the half-thickness y_s is unaltered; the change in the ordinate of the centre line is

$$\left.\begin{aligned} \Delta y_c &= E\eta x \quad (0 \leqslant x \leqslant h) \\ &= h\eta(1 - x) \quad (h \leqslant x \leqslant 1) \end{aligned}\right\}. \qquad \ldots(6)$$

The incidence with respect to the new chord is denoted by α; we denote by α' the incidence with respect to the old chord — i.e., the incidence of the front part of the wing (Fig. 3). Since the new chord makes an angle $E\eta$ with the old chord

$$\alpha = \alpha' + E\eta. \qquad \ldots(7)$$

The/

*For a more accurate, and much more complicated, theory, see the Appendix.

The quantities we wish to calculate are, in addition to the hinge moment, the no-lift angle $-\beta$, the moment coefficient at zero lift C_{M_0}, the optimum lift coefficient $C_{L_{opt}}$ and incidence α_{opt}, and the velocity at the surface. In connection with the calculation of the velocity at the surface, we require g_s, ψ_s, ε_s, ε'_s, C_0, g_i, ψ_c, ε_c, ε'_c. (Ref. 2. ε' denotes $d\varepsilon/d\theta$).

On the simple theory above, y_s has the same value as for zero flap deflection, and so therefore have g_s, ψ_s, ε_s, ε'_s, C_0.

According to (6) Δy_c is represented by two straight lines, and the determination of the changes produced by the deflection of the flap in all the other quantities required (except the hinge moment) is merely a special case of the example in §9 of Ref. 2, where y_c was represented by two quartics in x. All we need to do is to make the following substitutions in the values obtained in that example.

$$a_1 = E\eta, \quad b_0 = h\eta, \quad b_1 = -h\eta, \quad c_1 = a_1 - b_1 = \eta,$$

$$a_2 = a_3 = a_4 = b_2 = b_3 = b_4 = c_2 = c_3 = c_4 = 0,$$

$$X_1 = h, \qquad \theta_1 = 2\sin^{-1}\sqrt{h} = 2\cos^{-1}\sqrt{E}. \qquad \ldots(8)$$

4. The No-Lift Angle, Moment Coefficient at Zero Lift, Optimum Lift Coefficient and Incidence

We use β, C_{M_0}, $C_{L_{opt}}$, α_{opt} to denote values at zero flap deflection, and suppose these values calculated by the theory of Ref. 2. For the changes due to a flap deflection η we find that

$$\frac{\Delta\beta}{\eta} = h - \frac{\theta_1}{\pi} + \frac{\sin\theta_1}{\pi} \qquad \ldots(9)$$

$$m = -\frac{\Delta C_{M_0}}{\eta} = h\sin\theta_1, \qquad \ldots(10)$$

$$\left(\frac{\pi}{a_0} - \frac{1}{2}\right)\frac{\Delta C_{L_{opt}}}{\eta} = 2\sin\theta_1, \qquad \ldots(11)$$

$$\frac{\Delta\alpha_{opt}}{\eta} = \frac{\theta_1}{\pi} - h + \left(\frac{2\pi - a_0}{2\pi + a_0}\right)\frac{\sin\theta_1}{\pi}, \qquad \ldots(12)$$

where

$$C_L = a_0 \sin(\alpha + \beta + \Delta\beta). \qquad \ldots(13)$$

Formulae (11) and (12) do not allow for any change in a_0 as the flap is deflected. It is easy to calculate the effect of any change whose magnitude is known or can be inferred; but such effects will usually be negligible for the purposes for which the formulae will be required.

Values/

Values of θ_1/π, $(\sin\theta_1)/\pi$, $(\Delta\beta)/\eta$, m, and $(\Delta C_{L_{opt}})/\eta$, $(\Delta\alpha_{opt})/\eta$ for $a_0 = 2\pi$, are tabulated in Table 1 for E = 0·4, 0·3, 0·25, 0·2, 0·15, 0·1, 0·08.

The formula for m is in agreement with the formula given by Glauert in Ref. 3.

The values of $\Delta\beta$, $\Delta\alpha_{opt}$ in (9) and (12) refer to the new chord: i.e., $-(\beta + \Delta\beta)$, $\alpha_{opt} + \Delta\alpha_{opt}$ are the values of α when $C_L = 0$ and when $C_L = C_{L_{opt}}$, respectively. The value of $-\alpha'$ at zero lift is

$$\beta + \eta\left[1 - \frac{\theta_1}{\pi} + \frac{\sin\theta_1}{\pi}\right]; \qquad \ldots(14)$$

the value of α' when $C_L = C_{L_{opt}}$ is

$$\alpha_{opt} - \eta\left[1 - \frac{\theta_1}{\pi} - \left(\frac{2\pi - a_0}{2\pi + a_0}\right)\frac{\sin\theta_1}{\pi}\right]. \qquad \ldots(15)$$

We note also that, for $a_0 = 2\pi$,

$$-\Delta C_{M_0} = \tfrac{1}{2}h\Delta C_{L_{opt}}. \qquad \ldots(16)$$

Consider the application to reduce the moment coefficient of a low-drag wing at high speeds. Suppose the wing is cambered for a fairly high design C_L, appropriate to the cruising speed or to the climb, and that at high speeds the flap is up, so that η and $\Delta C_{L_{opt}}$ are negative. Let $C_{L_{opt}}$ (the design C_L), for example, be 0·5; if the low-speed equivalent of the top speed C_L is 0·2, we want $C_{L_{opt}} + \Delta C_{L_{opt}} = 0{\cdot}2$, so for $a_0 = 2\pi$ we should make $\eta = -0{\cdot}3/(2\sin\theta_1)$ radians; for a 20 per cent flap, for example, we should make $-\eta = 10$ deg $44\tfrac{1}{2}$ min.

Further, if, for the particular type of centre line on the original aerofoil,

$$-C_{M_0}/C_{L_{opt}} = K,$$

then

$$-(C_{M_0} + \Delta C_{M_0}) = KC_{L_{opt}} + \tfrac{1}{2}h\Delta C_{L_{opt}},$$

and is less, with the flap up, than if the aerofoil had been originally cambered for $C_{L_{opt}} + \Delta C_{L_{opt}}$ without a flap by

$$-(\tfrac{1}{2}h - K)\,\Delta C_{L_{opt}}.$$

Suppose/

Suppose, for example, that $-\Delta C_{L_{opt}} = 0{\cdot}3$, as above; that $K = 0{\cdot}25$ (the value appropriate to a centre-line designed for constant approximate loading over the whole chord: see Ref. 4, §9); and that $h = 0{\cdot}8$, so that we have a 20 per cent flap as before. Then $-C_{M_0}$ is reduced, by the use of the flap, by $0{\cdot}3\,[0{\cdot}4 - 0{\cdot}25] = 0{\cdot}045$; in fact, with $C_{L_{opt}} = 0{\cdot}5$ as above, $-C_{M_0}$ would be $0{\cdot}125$, and with the flap up, so that $\Delta C_{L_{opt}}$ is $-0{\cdot}3$, $-\Delta C_{M_0} = -0{\cdot}4 \times 0{\cdot}3 = -0{\cdot}12$, so $-(C_{M_0} + \Delta C_{M_0}) = 0{\cdot}005$; without a flap and with a value of $C_{L_{opt}}$ equal to $0{\cdot}2$, $-C_{M_0}$ would have been $0{\cdot}05$.

5. The Velocity at the Surface and the Lift Coefficient

To find the velocity distribution at the surface we use the methods and notation of Ref. 2. Two methods may be of interest — Approximation I, based on a very rough linear theory, to give rapidly a rough idea of the changes produced by the deflection of the flap away from the nose of the aerofoil (where Approximation I fails entirely), and Approximation III for more accurate work.

According to Approximation I, for zero flap deflection

$$q/U = 1 + g_s \pm \left[g_i + \frac{1}{2}\left(\frac{1}{a_0} + \frac{1}{2\pi}\right)(C_L - C_{L_{opt}}) \cot \tfrac{1}{2}\theta - \frac{1}{2}\left(\frac{1}{a_0} - \frac{1}{2\pi}\right) C_L \tan \tfrac{1}{2}\theta\right]. \qquad ...(17)$$

(the upper sign applying to the upper surface and the lower sign to the lower surface), where

$$x = \tfrac{1}{2}(1 - \cos\theta) = \sin^2 \tfrac{1}{2}\theta, \quad 0 \leqslant \theta \leqslant \pi, \qquad ...(18)$$

and

$$C_L = a_0(\alpha + \beta). \qquad ...(19)$$

The theoretical value of a_0, if the Kutta-Joukowski relation is satisfied, is 2π on this theory.

According to Approximation III, for the upper surface

$$\frac{q_u}{U} = \left|\left(1 - \frac{C_L^2}{a_0^2}\right)^{\frac{1}{2}} F_u \sin(\theta + \varepsilon_u - \beta) + \frac{C_L}{a_0} F_u \cos(\theta + \varepsilon_u - \beta) + \frac{C_L}{2\pi e^{C_0}} F_u\right|, \qquad ...(20)$$

where/

where

$$F_u = \frac{e^{\gamma_0}\,(1 + \varepsilon_u')}{(\psi_u^2 + \sin^2\theta)^{\frac{1}{2}}}; \qquad \ldots(21)$$

and for the lower surface

$$\frac{q_\ell}{U} = \left|\left(1 - \frac{C_L^2}{a_0^2}\right)^{\frac{1}{2}} F_\ell \sin(\theta - \varepsilon_\ell + \beta) - \frac{C_L}{a_0} F_\ell \cos(\theta - \varepsilon_\ell + \beta)\right.$$

$$\left. - \frac{C_L}{2\pi e^{C_0}} F_\ell \right|, \qquad \ldots(22)$$

where

$$F_\ell = \frac{e^{\gamma_0}\,(1 + \varepsilon_\ell')}{(\psi_\ell^2 + \sin^2\theta)^{\frac{1}{2}}}. \qquad \ldots(23)$$

ε' denotes $d\varepsilon/d\theta$. In both (20) and (22)

$$C_L = a_0 \sin(\alpha + \beta); \qquad \ldots(24)$$

in (20) – (23), θ is defined as in (18);

$$\gamma_0 = C_0 + [\varepsilon_s'\,\psi_s] + [\varepsilon_c'\,\psi_c], \qquad \ldots(25)$$

where

$$[\varepsilon_s'\,\psi_s] = \frac{1}{\pi}\int_0^\pi \varepsilon_s'(\theta)\,\psi_s(\theta)\,d\theta, \qquad \ldots(26)$$

and similarly with the suffix c; also

$$\left.\begin{aligned} \psi_u &= \psi_s + \psi_c, & \psi_\ell &= \psi_s - \psi_c, \\ \varepsilon_u &= \varepsilon_c + \varepsilon_s, & \varepsilon_\ell &= \varepsilon_c - \varepsilon_s, \\ \varepsilon_u' &= \varepsilon_s' + \varepsilon_c', & \varepsilon_\ell' &= \varepsilon_s' - \varepsilon_c' \end{aligned}\right\}. \qquad \ldots(27)$$

The theoretical value of a_0, when the Kutta-Joukowski condition is satisfied, is $2\pi e^{C_0}$. To find the velocity for a flap deflection η, in Approximation I we change g_s, g_i, $C_{L_{opt}}$ into $g_s + \Delta g_s$, $g_i + \Delta g_i$, $C_{L_{opt}} + \Delta C_{L_{opt}}$; in Approximation III we change ψ, ε, β, ε' into $\psi + \Delta\psi$, $\varepsilon + \Delta\varepsilon$, $\beta + \Delta\beta$, $\varepsilon' + \Delta\varepsilon'$. (In addition we should make any change in a_0 which we can infer from experiment or which experience has led us to expect; for small flap deflections we should not expect any appreciable change.) The resulting value of q/U then applies to a value of x_2 equal to the value of x at which the original value of q/U (for $\eta = 0$) applied, – i.e. if we write

x_2/

$$x_2 = \tfrac{1}{2}(1 - \cos\theta) = \sin^2 \tfrac{1}{2}\theta \qquad \ldots(28)$$

for a flap deflection of η, the calculated values of q/U apply to the same values of θ as before.

$\Delta\beta$ and $\Delta C_{L_{opt}}$ were found in §4; and, as explained at the end of §3, since $\Delta y_s = 0$,

$$\Delta g_s = \Delta\psi_s = \Delta\varepsilon_s = \Delta\varepsilon_s' = \Delta C_0 = 0. \qquad \ldots(29)$$

Consequently

$$\left.\begin{aligned} \Delta\psi_u &= -\Delta\psi_\ell = \Delta\psi_c, \\ \Delta\varepsilon_u &= \Delta\varepsilon_\ell = \Delta\varepsilon_c, \\ \Delta\varepsilon_u' &= -\Delta\varepsilon_\ell' = \Delta\varepsilon_c' \end{aligned}\right\}. \qquad \ldots(30)$$

Δg_i, $\Delta\psi_c$, $\Delta\varepsilon_c$, $\Delta\varepsilon_c'$ are then immediately found from §9 of Ref. 2, as explained at the end of §3. We find that*

$$\Delta g_i = \frac{\eta}{\pi}\,\ell n\,\frac{\sin\frac{1}{2}(\theta + \theta_1)}{\sin\frac{1}{2}|\theta - \theta_1|}, \qquad \ldots(31)$$

$$\left.\begin{aligned} \Delta\psi_c &= \frac{2\Delta y_c}{\sin\theta} = \frac{2E\eta\sin^2\frac{1}{2}\theta}{\sin\theta} = E\eta\tan\tfrac{1}{2}\theta \;(= \Delta\psi_1,\text{ say}) \text{ for } 0<x<h, \\ &= \frac{2h\eta\cos^2\frac{1}{2}\theta}{\sin\theta} = h\eta\cot\tfrac{1}{2}\theta \;(= \Delta\psi_2,\text{ say}) \text{ for } h<x<1, \end{aligned}\right\} \qquad \ldots(32)$$

$$\Delta\varepsilon_c = \frac{1}{\pi}[\Delta\psi_1 - \Delta\psi_2]\,\ell n\,\frac{\sin\frac{1}{2}(\theta + \theta_1)}{\sin\frac{1}{2}|\theta - \theta_1|} + \eta\left(h - \frac{\theta_1}{\pi}\right), \qquad \ldots(33)$$

and

$$\left.\begin{aligned} \Delta\varepsilon_c' &= \frac{1}{\pi}[\Delta\psi_1' - \Delta\psi_2']\,\ell n\,\frac{\sin\frac{1}{2}(\theta + \theta_1)}{\sin\frac{1}{2}|\theta - \theta_1|} - \eta\,\frac{\sin\theta_1}{\pi}\,\mathrm{cosec}\,\theta, \\ &\text{where} \\ \Delta\psi_1' &= \tfrac{1}{2}E\eta\sec^2\tfrac{1}{2}\theta, \qquad \Delta\psi_2' = -\tfrac{1}{2}h\eta\,\mathrm{cosec}^2\tfrac{1}{2}\theta \end{aligned}\right\} \qquad \ldots(34)$$

Since we are concerned only with a theory linear in η, $\Delta\psi_c$, $\Delta\varepsilon_c$, $\Delta\varepsilon_c'$ are proportional to η. In the expression for q/U, ε_c occurs in the combination $\varepsilon_c - \beta$ (and $\Delta\beta$, of course, is also proportional to η). It is therefore convenient to write

$$\Delta\psi_c = r\eta, \quad \Delta\varepsilon_c - \Delta\beta = s\eta, \quad \Delta\varepsilon_c' = t\eta. \qquad \ldots(35)$$

From/

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*ℓn is used for $\log_e$.

From (32), (33) and (34) we then find (substituting for E and h in terms of θ_1 in s and t) that

$$\left.\begin{aligned} r &= E \tan \tfrac{1}{2}\theta \quad \text{for} \quad 0 \leqslant x \leqslant h \\ &= h \cot \tfrac{1}{2}\theta \quad \text{for} \quad h \leqslant x \leqslant 1 \end{aligned}\right\}, \qquad \ldots(36)$$

$$s = \frac{1}{\pi}\left(\frac{\cos\theta_1 - \cos\theta}{\sin\theta}\right) \ell n \frac{\sin\frac{1}{2}(\theta+\theta_1)}{\sin\frac{1}{2}|\theta-\theta_1|} - \frac{\sin\theta_1}{\pi}, \qquad \ldots(37)$$

$$t = \frac{1}{\pi}\left(\frac{1-\cos\theta_1\cos\theta}{\sin^2\theta}\right) \ell n \frac{\sin\frac{1}{2}(\theta+\theta_1)}{\sin\frac{1}{2}|\theta-\theta_1|} - \frac{\sin\theta_1}{\pi} \operatorname{cosec}\theta, \qquad \ldots(38)$$

where

$$\cos\theta_1 = 2E - 1. \qquad \ldots(39)$$

t vanishes at $\theta = 0$ and $\theta = \pi$; s vanishes at $\theta = \pi$, and at $\theta = 0$

$$s = -(2/\pi)\sin\theta_1.$$

At $\theta = \theta_1$, r is continuous and equal to $\frac{1}{2}\sin\theta_1$; s is continuous and equal to $-(\sin\theta_1)/\pi$; but t is logarithmically infinite. Consequently the velocity is logarithmically infinite at $x = h$ according to this simple theory. If this infinity is to be avoided, the details of the way in which the surface is rounded off near $x = h$ must be studied, and some such theory used as that expounded in the Appendix. This logarithmic infinity is a defect of our simple theory, but it is not to be considered a very serious defect. For the effect is purely local; it is correct that the velocity will vary rapidly near $x = h$; and the exact values reached will depend critically on the way in which the surface is rounded off, details of manufacture included.

Tables of $(\Delta g_i)/\eta$, r, s and t against x for E = 0·4, 0·3, 0·25, 0·2, 0·15, 0·1, 0·008 are to be found in Table 2.

If, in Approximation III, we wish to include the effect of the change in γ_0 due to the flap deflection, we note that

$$(\Delta\gamma_0)/\eta = [t\psi_c]^* + [r\,\varepsilon_c'];$$

this change is, in any case, very small.

It/

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*Although t is logarithmically infinite at $\theta = \theta_1$, $\int_0^{\pi} t\,\psi_c(\theta)\,d\theta$ converges, and for small values of δ

$$\int_{\theta_1-\delta}^{\theta_1+\delta} t\,\psi_c(\theta)\,d\theta = \frac{2\delta}{\pi}\psi_c(\theta_1)\,\ell n \frac{\delta}{2\sin\theta_1} + O(\delta^3\,\ell n\,\delta).$$

It may be convenient to have available explicit formulae for the change in q due to the flap deflection. On Approximation I

$$q/U = 1 + g_s \pm \left\{ g_i + \frac{1}{2}\left(\frac{1}{a_0} + \frac{1}{2\pi}\right) (C_L - C_{L_{opt}}) \cot \tfrac{1}{2}\theta - \frac{1}{2}\left(\frac{1}{a_0} - \frac{1}{2\pi}\right) C_L \tan \tfrac{1}{2}\theta + \frac{\eta}{\pi}\left[\ell n \frac{\sin \frac{1}{2}(\theta + \theta_1)}{\sin \frac{1}{2}|\theta - \theta_1|} - \sin\theta_1 \cot \tfrac{1}{2}\theta \right] \right\} \quad \ldots(40)$$

For Approximation III we write

$$G = \frac{\Delta F}{\eta F}, \quad \ldots(41)$$

so that

$$\left.\begin{aligned} G_u &= \frac{\Delta\gamma_0}{\eta} + \frac{t}{1 + \varepsilon_u'} - \frac{r\,\psi_u}{\psi_u^2 + \sin^2\theta}, \\ G_\ell &= \frac{\Delta\gamma_0}{\eta} + \frac{t}{1 + \varepsilon_\ell'} + \frac{r\,\psi_\ell}{\psi_\ell^2 + \sin^2\theta} \end{aligned}\right\}. \quad \ldots(42)$$

Then

$$\frac{q_u}{U} = \left| \left(1 - \frac{C_L^2}{a_0^2}\right)^{\frac{1}{2}} F_u \sin(\theta + \varepsilon_u - \beta) + \frac{C_L}{a_0} F_u \cos(\theta + \varepsilon_u - \beta) + \frac{C_L}{2\pi\, e^{C_0}} F_u + \frac{\Delta q_u}{U} \right| \quad \ldots(43)$$

and

$$\frac{q_\ell}{U} = \left| \left(1 - \frac{C_L^2}{a_0^2}\right)^{\frac{1}{2}} F_\ell \sin(\theta - \varepsilon_\ell + \beta) - \frac{C_L}{a_0} F_\ell \cos(\theta - \varepsilon_\ell + \beta) - \frac{C_L}{2\pi\, e^{C_0}} F_\ell + \frac{\Delta q_\ell}{U} \right|, \quad \ldots(44)$$

where

$\frac{1}{\eta}$/

$$\frac{1}{\eta}\frac{\Delta q_u}{U} = \left\{\left(1 - \frac{C_L^2}{a_0^2}\right)^{\frac{1}{2}} G_u - \frac{C_L}{a_0} s\right\} F_u \sin(\theta + \varepsilon_u - \beta)$$

$$+ \left\{\left(1 - \frac{C_L^2}{a_0^2}\right)^{\frac{1}{2}} s + \frac{C_L}{a_0} G_u\right\} F_u \cos(\theta + \varepsilon_u - \beta)$$

$$+ \frac{C_L}{2\pi\ e^{C_0}} G_u F_u \qquad \ldots(45)$$

and

$$\frac{1}{\eta}\frac{\Delta q_\ell}{U} = \left\{\left(1 - \frac{C_L^2}{a_0^2}\right)^{\frac{1}{2}} G_\ell - \frac{C_L}{a_0} s\right\} F_\ell \sin(\theta - \varepsilon_\ell + \beta)$$

$$- \left\{\left(1 - \frac{C_L^2}{a_0^2}\right)^{\frac{1}{2}} s + \frac{C_L}{a_0} G_\ell\right\} F_\ell \cos(\theta - \varepsilon_\ell + \beta)$$

$$- \frac{C_L}{2\pi\ e^{C_0}} G_\ell F_\ell . \qquad \ldots(46)$$

The value of $\Delta g_i(\theta)$ in (31) enables us to determine if, in any particular case, the position of maximum suction for $C_L = C_{L_{opt}} + \Delta C_{L_{opt}}$ on the upper surface of a low-drag aerofoil with the flap up (or on the lower surface with the flap down) is at the design position for $C_L = C_{L_{opt}}$, $\eta \gtrless 0$. If this position is at $x = X$, and if, as is usual, $g_i'(\theta) = 0$ for $0 \leqslant x \leqslant X$ (where the dash denotes differentiation with respect to θ), then very roughly, we require[5]

$$|\Delta g_i'(\theta)| = \frac{|\eta|}{2\pi}\frac{\sin\theta_1}{h - x} < g_s'(\theta) + 2\rho_L \frac{\cos\theta}{\sin^3\theta} \quad \text{for } 0 \leqslant x \leqslant X.$$

Normally, but not necessarily always, the least value of the right-hand side occurs at $x = X$. Then we require, roughly,

$$\frac{|\eta|}{2\pi}\sin\theta_1 < (h - X)\, g_s'\,(2\sin^{-1} X^{\frac{1}{2}})^*.$$

For example, if $h = 0{\cdot}8$, $\sin\theta_1 = 0{\cdot}8$; and if dg_s/dx is constant and equal to s for $0 \leqslant x \leqslant X$, with $X = 0{\cdot}6$, $s = 0{\cdot}12$, then $|\eta|$ must not exceed about $5\frac{1}{2}$ deg.

If/

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* $g_s'(2\sin^{-1} X^{\frac{1}{2}})$ is the value of $g_s'(\theta)$ at $\theta = 2\sin^{-1} X^{\frac{1}{2}}$.

If e is kept constant and X varies between, say, 0·6 and 0·4, then $g_s'(2\sin^{-1} X^{\frac{1}{2}})$ varies very little, so the limiting value of $|\eta|$ is nearly proportional to $h - X$. With $h = 0{\cdot}8$, this limiting value will therefore be about half as large again for $X = 0{\cdot}5$ as for $X = 0{\cdot}6$; for $X = 0{\cdot}4$ it will be twice as large. In any case when η exceeds the values considered here, the position of maximum suction will move forward gradually, and only a gradual rise of drag at $C_L = C_{L_{opt}}$ (rather faster for flap up than for flap down) is to be anticipated.

Finally we note that

$$C_L = a_0 \sin(\alpha + \beta + \Delta\beta)$$

$$= a_0 \sin\left[\alpha' + \beta + \eta\left(1 - \frac{\theta_1}{\pi} + \frac{\sin\theta_1}{\pi}\right)\right],$$

so if, when C_L is expanded in powers of α' and η the first terms are

$$C_L = c_0 + c_1\alpha' + c_2\eta, \qquad \ldots(47)$$

we have

$$c_0 = a_0 \sin\beta, \qquad \ldots(48)$$

$$c_1 = a_0 \cos\beta \doteqdot a_0, \qquad \ldots(49)$$

$$\frac{c_2}{c_1} = 1 - \frac{\theta_1}{\pi} + \frac{\sin\theta_1}{\pi}. \qquad \ldots(50)$$

The theoretical value of a_0, for the Kutta-Joukowski condition to be satisfied, is $2\pi e^{C_0}$, but the value of c_2/c_1 is independent of the value of a_0. Values of c_2/c_1 are tabulated in Table 1 for the same values of E as before. It is clear that the formula for c_2/c_1 is the same as on the rough linear theory leading to Approximation I, and is in agreement with the formula given by Glauert in Ref. 3.

On the simple theory here presented, in the calculation of the effect of the flap deflection on the velocity distribution products of η and the aerofoil ordinates are neglected. It is shown in the Appendix how, by complicated calculations, these products may be taken into account; and it is worth remarking that if, for example, products of η and the square of the thickness are neglected, the corrections to the simple theory will, for geometrically similar thickness distributions, be proportional to the thickness. Except for very thick aerofoils such an approximation is always justified (except perhaps at the nose of the aerofoil). In fact, away from the nose of the aerofoil, the velocity on the surface at a given incidence and flap deflection is nearly a linear function of the thickness for aerofoils having a given centre line and geometrically similar thickness distributions, so long as the thickness is not exceptionally large.

Hinge/

6. Hinge Moments

If C_H is the hinge-moment coefficient, such that, for unit chord, $\frac{1}{2}\rho U^2E^2C_H$ is the moment about the hinge of the normal pressure p on unit width of the flap, then

$$\tfrac{1}{2}\rho U^2E^2C_H = \oint p\left[(x_2 - h_2)dx_2 + (y_2 - k_2)\,dy_2\right], \quad \ldots(51)$$

the axes of x_2 and y_2 being used, (h_2, k_2) being the coordinates of the hinge relative to those axes, and the integral being taken round the contour of the flap in the negative sense. Now if H is the total head, assumed constant,

$$p = H - \tfrac{1}{2}\rho q^2, \quad \ldots(52)$$

and

$$\oint H\,[(x_2 - h_2)dx_2 + (y_2 - k_2)dy_2] = 0, \quad \ldots(53)$$

so

$$E^2C_H = -\oint (q/U)^2\,[(x_2 - h_2)dx_2 + (y_2 - k_2)dy_2]. \quad \ldots(54)$$

It is usual to neglect the contribution to the integral in (54) from the part of the contour of the flap forward of $x = h$. If this part of the contour is a circle with its centre on the hinge, so that the lines of action of the normal pressures on the surface forward of $x = h$ all intersect the hinge, this contribution is identically zero; otherwise it should be specifically noted that the effects of the difference from the total head of the normal pressures on this part of the surface of the flap have been neglected; moreover these effects could not be theoretically calculated in many cases when there is no seal; ideal fluid theory would provide no guide.

It should also be specifically stated that, since this investigation is a theoretical one on the basis of potential flow, all effects of skin friction are neglected. Moreover, the pressures are assumed to have their theoretical values. Now boundary-layer effects are greatest at the trailing edge, and the contribution to the hinge moment of the pressures near the trailing edge is considerable, so boundary-layer effects may be expected to have a considerable influence on the hinge moment. There is some evidence, however, that for cusped or nearly cusped aerofoils, hinge moments calculated by potential theory will provide a good guide, especially at large Reynolds numbers in streams of low turbulence or in flight, provided that a_0 is given its theoretical 'Kutta-Joukowski' value. This statement does not imply that the actual pressures on the surface near the trailing edge will be at all accurately found in this manner, but it does seem that the difference between the pressures on the upper and lower surfaces - the normal loading - may be fairly well reproduced. For aerofoils with large trailing-edge angles the hinge moment appears to be more dependent on the position of the transition to turbulence in the boundary layer, on Reynolds number and on turbulence in the stream than for cusped aerofoils; at the present time our only hope of being able to forecast hinge moments on aerofoils with large tailing-edge angles seems to be to find empirical corrections to values calculated in the manner described above. In

any/

any case it seems valueless to calculate hinge moments with any value of a_0 other than the theore tical 'Kutta-Joukowski' values, since even if the trailing edge is rounded, it will be only slightly rounded, and with any such value of a_0 the velocity q will, according to potential theory, reach large values which will not be obtained in experiment or in practice; these large values will entirely falsify the calculation of the hing moment. We shall therefore give a_0 its 'Kutta-Joukowski' value.*

If we now neglect the second term in (54), and use the simple approximation (5), we may write

$$E^2 C_H = \int_h^1 \left[\frac{q_\ell^2 - q_u^2}{U^2}\right] [x - h]\, dx. \qquad \ldots(55)$$

Now suppose that, when q/U is expanded in powers of C_L and η, the first terms are

$$q/U = L + MC_L + N\eta. \qquad \ldots(56)$$

When the hinge-moment coefficient is expanded in powers of α' and η it is usual to write, for the first terms,

$$C_H = b_0 + b_1\alpha' + b_2\eta; \qquad \ldots(57)$$

If the first terms in the expansion of C_L in powers of α' and η are, as before,

$$C_L = c_0 + c_1\alpha' + c_2\eta, \qquad \ldots(58)$$

then the first terms in the expansion of C_H in powers of C_L and η are

C_H/

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*It is found experimentally[8] that, for cusped or nearly cusped aerofoils, b_1 and b_2 (see eqn.(57)) are very close to their theoretical values based on the Kutta-Joukowski circulation. Thus b (see eqns. (59) and (60)) will be near to its theoretical value only if c_2/c_1 (see eqn.(58)) has its theoretical value. At present the experimental evidence is that c_2/c_1 may be slightly less than its theoretical value, but this small departure may perhaps be lessened by an increase of Reynolds number and a decrease of free-stream turbulence, and by ensuring smoothness of the aerofoil surface with the flap deflected.

For aerofoils with large trailing-edge angles the most successful method at present of correlating experimental data is to plot the ratio of the measured value of b_1 to its theoretical value against the ratio of the measured value of b_2 to its theoretical value, and similarly for c_1 and c_2. For any given value of E it appears that the points thus plotted lie on straight lines independent of shape or of the position of the transition to turbulence in the boundary layer. Hence b_2 and c_2 may be estimated from a knowledge of b_1 and c_1 and the theoretical values of b_1, c_1, b_2, c_2. These theoretical values are those found with the Kutta-Joukowski circulation; we therefore require to be able to calculate these theoretical values even for aerofoils which are not cusped. It seems well within the bounds of possibility that it will prove possible to calculate c_1 with sufficient accuracy by allowing for boundary layers, with scale and transition effects included; and it is hoped ultimately to be able to compute b_1 as well.

$$C_H = b_0 - \frac{b_1}{c_1} c_0 + \frac{b_1}{c_1} C_L - b\eta, \quad \ldots(59)$$

where

$$b = \frac{c_2}{c_1} b_1 - b_2. \quad \ldots(60)$$

Hence, from (55),

$$E^2 \left(b_0 - \frac{b_1}{c_1} c_0 \right) = \int_h^1 (L_\ell^2 - L_u^2)(x - h)dx, \quad \ldots(61)$$

$$E^2 \frac{b_1}{c_1} = \int_h^1 (2\, L_\ell M_\ell - 2L_u M_u)(x - h)\, dx, \quad \ldots(62)$$

$$-E^2 b = \int_h^1 (2L_\ell N_\ell - 2L_u N_u)(x - h)dx. \quad \ldots(63)$$

On Approximation I it follows from (40) (with $a_0 = 2\pi$) that

$$\left.\begin{aligned} L_u &= 1 + g_s + g_i - \frac{1}{2\pi} C_{L_{opt}} \cot \tfrac{1}{2}\theta, \\ L_\ell &= 1 + g_s - g_i + \frac{1}{2\pi} C_{L_{opt}} \cot \tfrac{1}{2}\theta, \\ M_u &= -M_\ell = \frac{1}{2\pi} \cot \tfrac{1}{2}\theta \\ N_u &= -N_\ell = \frac{1}{\pi} \left[\ell n \frac{\sin \frac{1}{2}(\theta + \theta_1)}{\sin \frac{1}{2}|\theta - \theta_1|} - \sin \theta_1 \cot \tfrac{1}{2}\theta \right] \end{aligned}\right\} \quad \ldots(64)$$

Hence, if we keep only the most important terms,

$$\left.\begin{aligned} E^2 \left(b_0 - \frac{b_1}{c_1} c_0 \right) &= -4 \int_h^1 \left(g_i - \frac{1}{2\pi} C_{L_{opt}} \cot \tfrac{1}{2}\theta \right) (x - h)dx, \\ E^2 \frac{b_1}{c_1} &= -\frac{2}{\pi} \int_h^1 (x - h) \cot \tfrac{1}{2}\theta\, dx, \\ E^2 b &= \frac{4}{\pi} \int \left(\ell n \frac{\sin \frac{1}{2}(\theta + \theta_1)}{\sin \frac{1}{2}|\theta - \theta_1|} - \sin \theta_1 \cot \tfrac{1}{2}\theta \right) (x-h)dx \end{aligned}\right\} \quad \ldots(65)$$

Since/

Since

$$x = \tfrac{1}{2}(1 - \cos\theta),$$
$$h = \tfrac{1}{2}(1 - \cos\theta_1),$$

it follows that

$$4\int_h^1 (x - h)\cot\tfrac{1}{2}\theta\,dx = \int_{\theta_1}^{\pi} (\cos\theta_1 - \cos\theta)\sin\theta\cot\tfrac{1}{2}\theta\,d\theta$$
$$= \int_{\theta_1}^{\pi} (\cos\theta_1 - \cos\theta)(1 + \cos\theta)\,d\theta$$
$$= (\pi - \theta_1)(\cos\theta_1 - \tfrac{1}{2}) + (1 - \tfrac{1}{2}\cos\theta_1)\sin\theta_1,$$

and

$$4\int_h^1 \ell n\frac{\sin\frac{1}{2}(\theta+\theta_1)}{\sin\frac{1}{2}|\theta-\theta_1|}(x-h)dx = \int_{\theta_1}^{\pi} \ell n\frac{\sin\frac{1}{2}(\theta+\theta_1)}{\sin\frac{1}{2}|\theta-\theta_1|}(\cos\theta_1 - \cos\theta)\sin\theta\,d\theta$$
$$= \Big[\tfrac{1}{2}(\cos\theta_1 - \cos\theta)^2\,\ell n\frac{\sin\frac{1}{2}(\theta+\theta_1)}{\sin\frac{1}{2}|\theta-\theta_1|} + \tfrac{1}{2}\sin\theta_1(\theta\cos\theta_1 - \sin\theta)\Big]_{\theta_1}^{\pi}$$

$$= \frac{\pi - \theta_1}{4}\sin 2\theta_1 + \frac{\sin^2\theta_1}{2}.$$

Hence

$$-\frac{b_1}{c_1} = \frac{1}{2\pi E^2}\{(1 - \tfrac{1}{2}\cos\theta_1)\sin\theta_1 + (\pi - \theta_1)(\cos\theta_1 - \tfrac{1}{2})\}$$
$$= \frac{1}{2\pi E^2}\{(\tfrac{3}{2} - E)\sin\theta_1 - (\tfrac{3}{2} - 2E)(\pi - \theta_1)\}, \qquad \ldots(66)$$

$$b_0 = -\frac{4}{E^2}\int_h^1 g_1(x - h)dx + \frac{\alpha_{opt}}{E^2}\{(\tfrac{3}{2} - E)\sin\theta_1 - (\tfrac{3}{2} - 2E)(\pi - \theta_1)\}$$
$$\ldots(67)$$

(since on Approximation I with $a_0 = 2\pi$,

$$c_0 = 2\pi\beta$$

and

$$C_{L_{opt}} - c_0 = 2\pi(\alpha_{opt} + \beta) - 2\pi\beta = 2\pi\alpha_{opt}),$$

and/

and

$$b = \frac{h \sin \theta_1}{\pi E^2} (\pi - \theta_1 - \sin \theta_1). \qquad \ldots(68)$$

It follows that

$$- b_2 = b - \frac{c_2}{c_1} b_1$$

$$= \frac{1}{\pi E^2} \{2Eh + (\pi - \theta_1) \sin \theta_1 - (\tfrac{3}{2} - 2E)(\pi - \theta_1)^2\}.$$

$$\ldots(69)$$

These values agree with those found by Glauert, by 'flat-plate' theory and summation of infinite series, in Ref. 3. The same is true of the values previously found in this report for m and c_2/c_1 (eqns. (10) and (50)), but whereas those values are also correct on more exact non-linear theories, eqns. (66) - (69) are not so correct. In fact, the values found from these equations are not sufficiently accurate for most purposes, and values should be calculated on Approximation III. Nevertheless the values of $-b_1/c_1$, b and $-b_2$ from the formulae above are tabulated, for what they are worth, in Table 1 for $E = 0{\cdot}4, 0{\cdot}3, 0{\cdot}25, 0{\cdot}2, 0{\cdot}15, 0{\cdot}1, 0{\cdot}08$.

On Approximation III it follows from eqns. (43) - (46), with $a_0 = 2\pi e^{C_0}$, that

$$\left.\begin{aligned} L_u &= F_u \sin (\theta + \varepsilon_u - \beta), \\ L_\ell &= F_\ell \sin (\theta - \varepsilon_\ell + \beta), \end{aligned}\right\} \qquad \ldots(70)$$

$$\left.\begin{aligned} M_u &= \frac{F_u}{2\pi e^{C_0}} [1 + \cos (\theta + \varepsilon_u - \beta)], \\ M_\ell &= -\frac{F_\ell}{2\pi e^{C_0}} [1 + \cos (\theta - \varepsilon_\ell + \beta)], \end{aligned}\right\} \qquad \ldots(71)$$

$$\left.\begin{aligned} N_u &= G_u F_u \sin (\theta + \varepsilon_u - \beta) + sF_u \cos (\theta + \varepsilon_u - \beta), \\ N_\ell &= G_\ell F_\ell \sin (\theta - \varepsilon_\ell + \beta) - sF_\ell \cos (\theta - \varepsilon_\ell + \beta), \end{aligned}\right\} \qquad \ldots(72)$$

where G_u, G_ℓ are given by (42), and r, s, t by (36), (37), (38). (See also Table 2).

More accurate values of b_0, b_1 and b are to be found from eqns. (61) - (63) by numerical integration, with the above values of L, M, N.

For more accurate calculations still, in which the second term in (54) is included, see §3 of the Appendix.

We/

We may remark also that, for wings of similar thickness distribution whose thicknesses are not exceptionally large, b_1, b_2 and b are approximately linear functions of the thickness, and the corrections to the results found by Approximation I (eqns. (66), (68), (69)) are approximately proportional to the thickness. Similarly if a local modification is made to the thickness distribution, e.g. by 'convexing' a control surface to change the balance, then the effect produced is proportional to the scale of the local modification.

The correction to be applied to the value of b_0 given by (67) is also approximately proportional to the thickness; the value given by (67) is itself proportional to the camber.

7. Numerical Illustrations.

Our first numerical illustration relates to a control surface on a suction wing designed for a flight test. The fairing was designed from Approximation I with g_s as shewn in Fig. 4[7,8]

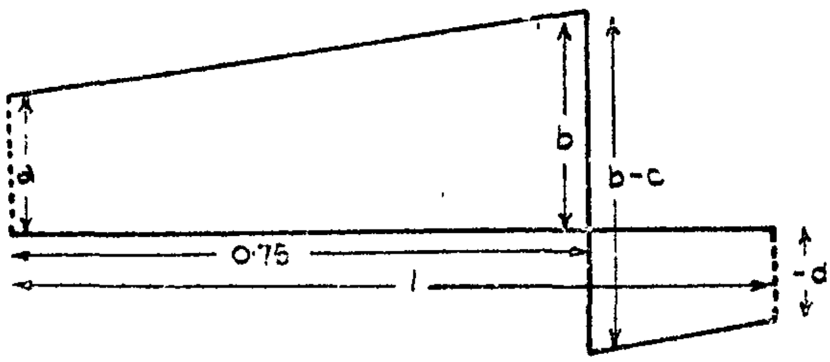

Fig. 4.

$a = 0{\cdot}1448964$, $b = 0{\cdot}2321126$, $d = -0{\cdot}0937799$, $b - c = 0{\cdot}3550309$ and $x = 0{\cdot}75$. The centre line was designed from Approximation I with g_i as shewn in Fig. 5 (see Ref. 4 §12), $k = 0{\cdot}073$, $k' = 0{\cdot}02$, $X = 0{\cdot}75$. The design C_L is $0{\cdot}2$, C_{M_0} is $-0{\cdot}015$ and the aerofoil is 16 per cent thick; a sketch of it is shewn in Fig. 6. The aileron is a 25 per cent flap — i.e. it consists of the whole of the wing aft of the suction slot.

Fig. 5/

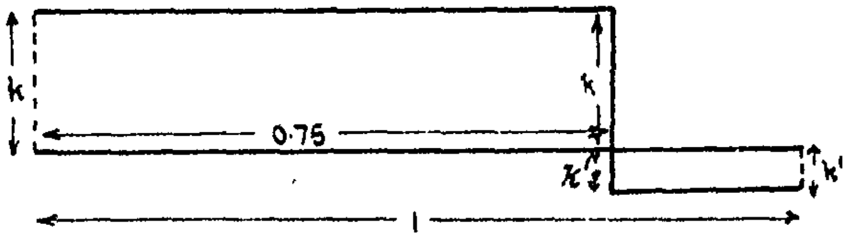

Fig. 5.

Values of b_0, b_1, b, b_2 have been worked out from equations (61) – (63), with L, M, N, given by (70), (71), and (72), with the following results:

$b_0 = 0{\cdot}046, \quad b_1 = -0{\cdot}386, \quad b_2 = -0{\cdot}725, \quad b = 0{\cdot}490.$

These are probably the most satisfactory values it is possible to obtain without empirical corrections. The values according to Approximation I are given below for comparison.

$b_0 = 0{\cdot}049, \quad b_1 = -0{\cdot}565, \quad b_2 = -0{\cdot}944, \quad b = 0{\cdot}599.$

These values of b_1, b_2, b are the same as those obtained on a 'flat-plate' theory[3]; the differences from the more accurate values given above are considerable.

Average values of b_0 and b_1 from $C_L = 0$ to $C_L = 0{\cdot}5$ were also found by computing C_H for these two C_L-values (with $\eta = 0$) directly by integration of the right-hand side of (55). On Approximation III, with $a_0 = 2\pi e^{C_0}$ (which for this aerofoil is 7·0440), it was thus found that $b_0 = 0.046$ and $b_1 = -0{\cdot}383$; on Approximation I, with $a_0 = 2\pi$, $b_0 = 0{\cdot}050$, $b_1 = -0{\cdot}570$. These values compare quite satisfactorily with those given above, especially as this last computation is not very accurate. In order to exhibit the large change in b_1 produced by a small change in a_0, average values of b_0 and b_1 between $C_L = 0$ and $C_L = 0{\cdot}5$ were also computed on Approximation III with $a_0 = 2\pi$; the values obtained were $b_0 = 0{\cdot}052$, $b_1 = +0{\cdot}029$. The change in the value of b_1, from $-0{\cdot}383$ to $+0{\cdot}029$ when a_0 is changed from $2\pi e^{C_0}$ to 2π, is quite striking.

Our second numerical illustration concerns certain computations which have already been carried out for the aerofoil on which pressure-plotting experiments are planned in the 13 ft. × 9 ft. wind tunnel at the N.P.L. The aerofoil is a symmetrical low-drag aerofoil of the so-called 'roof-top' variety designed from Approx. I with g_s as shown in Fig. 7 (see Ref. 7 §6),

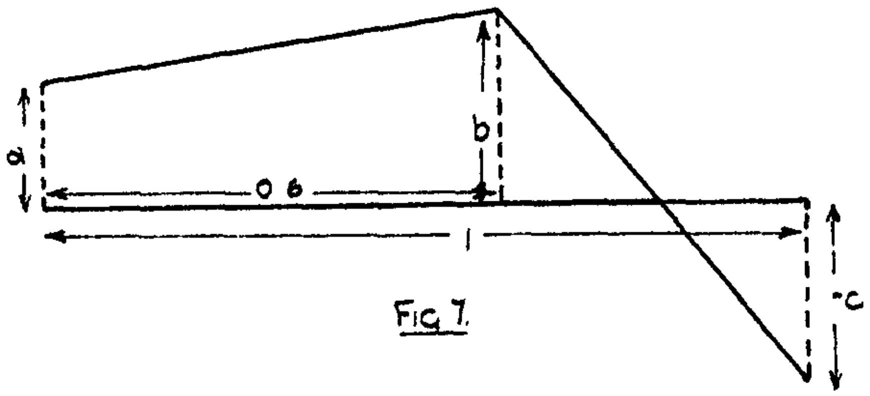

Fig 7.

$a = 0{\cdot}1336624$, $b = 0{\cdot}206$[illegible]19, $c = -0{\cdot}1817961$ and $x = 0{\cdot}6$; it is 15 per cent thick, with a cusp, and a sketch of it is shown in Fig. 8. The flap is to be a 20 per cent flap, with the hinge at the intersection of the normals to the upper and lower surfaces at $x = 0{\cdot}8$, i.e. on the chord line at $x = h_1 = 0{\cdot}793534$.

In the first place, values of the aerofoil ordinates are given in Table 3, and of ψ_s, ε_s, ε_s' in Table 4. C_0 is $0{\cdot}10696$, and the theoretical value of $a_0 (= 2\pi e^{C_0})$ is $6{\cdot}9925$.

It was considered that this aerofoil would be a suitable one for which to compute the difference between the simple theory of the preceding paragraphs and the more accurate theory of the Appendix. Since the original aerofoil is symmetrical, $\Delta\psi_s$, $\Delta\varepsilon_s$, $\Delta\varepsilon_s'$ are zero, even on the theory in the Appendix; values of $(\Delta\psi_c)/\eta$, $\Delta(\varepsilon_c - \beta)/\eta$, $\Delta\varepsilon_c'/\eta$, calculated by the theory in the Appendix, are reproduced in Table 4; these values should be compared with those headed r, s, t, respectively under $E = 0{\cdot}2$ in Table 2. The value of $\Delta\varepsilon_c'/\eta$ at $x = 0{\cdot}8$ has not been computed, since it depends critically on the amount of rounding-off on the surface near that value of x. Apart from values at $x = 0{\cdot}8$, the biggest difference between the entries in the two tables for $\Delta\varepsilon_c'/\eta$ is $0{\cdot}0166$ (at $x = 0{\cdot}85$); it appears from the calculations that this difference may rise to $0{\cdot}038$ near $x = 0{\cdot}8$. For $\Delta\psi_c/\eta$, the biggest difference is $0{\cdot}0084$ (at $x = 0{\cdot}8$); for $\Delta(\varepsilon_c - \beta)/\eta$ it is $0{\cdot}0063$ at $x = 0$. The more accurate value of $\Delta\beta/\eta$; however, is $0{\cdot}3560$, compared with $0{\cdot}3498$ on the simple theory; when allowance is made for the change in $\Delta\beta/\eta$, it appears that for $\Delta\varepsilon_c/\eta$ the biggest difference would be $0{\cdot}0099$ (at $x = 0{\cdot}9$).

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At this stage it seems best to leave detailed sample calculations of velocity distributions until the experiments are performed, so that the calculations may be made for the same circumstances as the experiments. We may say in a general way, however, that whereas the correction terms introduced in the Appendix may make a difference of as much as 7 in the third decimal place in q/U near $x = 0{\cdot}8$ for a flap deflection of 10 deg., they are unlikely to make a bigger difference than about 5 or 6 in the fourth decimal place near $x = 0{\cdot}5$, for example.

The hinge-moment coefficients of this aerofoil and flap have been worked out on Approximation I, and according to Approximation III both for the simple theory in §6 (eqns. (61), (62), (63), (70), (71), (72) etc.) and from the complicated theory of the Appendix. The results are set out below.

b_0	$-b_1$	$-b_2$	b	Method
0	$0{\cdot}499_5$	0·923	0·648	Approximation I.
0	0·349	0·739	0·547	Approximation III. Simple Theory. Eqns. (61)–(63), (70)–(72).
0	0·364	0·774	0·574	Approximation III. Theory in Appendix.

There is a fair difference between the values in the last two lines, and something may be said about the various effects, and their magnitude, which are taken into account in the third line and not in the second. The coefficients computed by integration are $-b_1$ and b. In the computation of $-b_1$ account is taken, in the more exact theory, (i) of the exact location of the hinge, (ii) of the effect of the thickness on the moment in so far as the moment of the pressure component parallel to the chord is included (the second term in eqn. (51)). In the computation of $-b_1$ these are the only extra effects. In the computation of b these effects also enter, together with (iii) the effect of the thickness on the velocity change produced by a deflection of the flap, and (iv) the effect of the product of the thickness and the flap deflection on the geometry of the system, as expressed by the terms in X_1 in eqn. (25A) in the Appendix.

No exact computation of these separate effects has been made; a judgment of their signs and relative magnitudes has been attempted from an inspection of the work necessary to obtain the finished results set out in the table above. In the computation of $-b_1$, the largest effect is due to (i) (the location of the hinge); the effect of (ii) is of the opposite sign and about one-third as big. In the computation of b, (iii) and (iv) separately are quite large – about 2 and $2\frac{1}{2}$ times the effect of (i), respectively. (i) is again positive, (ii) negative and about one-third as large, (iii) negative and (iv) positive, and the combined effect of (iii) and (iv) is positive and just over half as big as the effect of (i). It will therefore be seen that if accurate results are required, the location of the hinge is of considerable importance.

Appendix/

Appendix

1. Additional Notation

Y : the aerofoil ordinate at $x = h$.

Y' : the slope of the aerofoil surface, — i.e. the values of dy/dx — at $x = h$.

$f(x)$, $F(x)$: $y = f(x)$ is the equation of the contour of the aerofoil for $x < h$; $y = F(x)$ is the equation of the contour of the flap for zero deflection.

ρ_L, ρ_T, $\Delta\rho_L$, $\Delta\rho_T$; ρ_L and ρ_T are the radii of curvature at the original leading and trailing edges (zero deflection), respectively; $\rho_L + \Delta\rho_L$, ρ_T, $+ \Delta\rho_T$ the radii of curvature at the new leading and trailing edges for a flap deflection η.

h_2, k_2 : the coordinates of the hinge with respect to the axes of x_2 and y_2.

X, X_1 : See eqns. 23A and 24A.

ℓ, ℓ' : Near $x = 0$

$$f(x) = \sqrt{(2\rho_L)}x^{\frac{1}{2}} + \ell x + \dots ;$$

near $x = 1$

$$F(x) = \sqrt{(2\rho_T)}(1 - x)^{\frac{1}{2}} + \ell'(1 - x) + \dots .$$

The values of ℓ and ℓ' are those appropriate to the upper or the lower surface according as the new leading and trailing edges, respectively lie on the original upper or lower surface.

2. Geometrical Considerations. The Determination of ψ, and of the Velocity at the Surface.

If more accurate calculations are desired than those of the simple theory in the body of the report, the position of the hinge and the contour of the flap near and forward of $x = h$ must be given exactly. Calculations may then be made for any flap deflection, but the questions at issue are sufficiently illustrated if we restrict our attention to small values of η, neglecting its square, but retaining its products with the aerofoil ordinates. Even for such calculations, the exact position of the hinge is of importance. In the very rare cases when such calculations are necessary and justified, it will probably be legitimate to assume that care has been exercised in designing the flap so as to preserve, at any rate for small values of η, as fair an external surface to the aerofoil as possible: one way of achieving this aim is to locate the hinge at the intersection of the normals to the upper and lower surfaces at $x = h$, and to make the portions of the contour of the flap near $x = h$ on both surfaces circular arcs with their centres on the hinge. Let (h_1, k_1) be the coordinates of the hinge, as thus determined, relative to the original axes of x and y; denote by Y and Y' the ordinate and slope of the aerofoil surface — i.e. the values of y and dy/dx, respectively — at $x = h$, and use subscripts u and ℓ for

values/

values on the upper and lower surfaces, respectively. (The subscripts will be omitted when the analysis is the same for both surfaces.) Then since (h_1, k_1) lies on both normals

$$\left.\begin{aligned} h_1 - h &= -Y_u'(k_1 - Y_u), \\ h_1 - h &= -Y_\ell'(k_1 - Y_\ell), \end{aligned}\right\} \qquad \ldots(1A)$$

whence

$$\left.\begin{aligned} h_1 &= h - \frac{Y_u' \, Y_\ell'(Y_u - Y_\ell)}{Y_u' - Y_\ell'}, \\ k_1 &= \frac{Y_u' Y_u - Y_\ell' \, Y_\ell}{Y_u' - Y_\ell'} \end{aligned}\right\}. \qquad \ldots(2A)$$

Since

$$Y_u = Y_c + Y_s, \quad Y_\ell = Y_c - Y_s, \qquad \ldots(3A)$$

and similarly for Y', where Y_c is the ordinate of the centre line and Y_s the half-thickness at $x = h$,

$$\left.\begin{aligned} h_1 &= h - Y_u' \, Y_\ell' \, Y_s/Y_s' \\ k_1 &= Y_c + Y_c' \, Y_s/Y_s' \end{aligned}\right\}. \qquad \ldots(4A)$$

If now in equations (1) and (2) of §3 we neglect squares of η, we have

$$x_1 = x + \eta(y - k_1), \quad y_1 = y - \eta(x - h_1). \quad \ldots(5A)$$

The point which was at the trailing edge for $\eta = 0$ is now at $(1 - k_1\eta, \; -\eta(1 - h_1))$.

If the equation of the contour of the flap for zero deflection* is $y = F(x)$, then

$$\begin{aligned} y_1 &= F(x) - \eta(x - h_1) \\ &= F(x_1) - \eta\{x_1 - h_1 + F'(x_1)[F(x_1) - k_1]\} \quad \ldots(6A) \end{aligned}$$

to the first order in η (except perhaps very near the trailing edge). Eqn. (6A) is the equation of the contour of the flap for a deflection η, with x_1 and y_1 as current coordinates.

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*This use of F is not to be confused with the use of F_u and F_ℓ in (20) and (22) <u>et seq</u>.

By definition the leading and trailing edges are at the ends of the chord, and the chord is normal to the aerofoil contour at the leading edge, and also at the trailing edge if the trailing edge is rounded. Consequently, when the flap is deflected, the position of the leading edge is slightly altered, and if the trailing edge is rounded, the position of the trailing edge is slightly different from the new position of the old trailing edge. A short analysis shows that the new leading edge is at

$$x = o(\eta^2), \quad y = \eta \frac{(1 - h_1 - \rho_T)\rho_L}{1 - \rho_L - \rho_T}, \qquad \ldots(7A)$$

and the new trailing edge at

$$x = 1 - k_1\eta, \quad y = -\eta(1 - h_1) + \eta \frac{(h_1 - \rho_L)\rho_T}{1 - \rho_L - \rho_T}, \qquad \ldots(8A)$$

to the first order in η, where ρ_L and ρ_T are the radii of curvature at the leading and trailing edges, respectively.

The new chord is of length $1 - k_1\eta$, to the first order in η.

We now take the new leading edge as a new origin, and the new chord as the new axis of x. The axes are therefore rotated through an angle

$$-\eta \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T}; \qquad \ldots(9A)$$

we also divide the coordinates by $1 - k_1\eta$, so that the new chord may be of unit length. We again denote the new coordinates, with a flap deflection η, by (x_2, y_2). Then to the first order in η

$$\left.\begin{aligned} x_2 &= \left(x - \eta y \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T}\right) \Big/ [1 - k_1\eta] \\ &= x + \eta \left\{k_1 x - \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T} y\right\}, \\ y_2 &= \left[y + \eta(x - \rho_L) \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T}\right] \Big/ [1 - k_1\eta] \\ &= y + \eta \left\{k_1 y + \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T} (x - \rho_L)\right\} \end{aligned}\right\} \qquad \ldots(10A)$$

Hence/

Hence if $y = f(x)$ is the equation, for $0 < x < h$, of the contour of the aerofoil in the original coordinates (x, y), its equation in the new coordinates is, to the first order in η,

$$y_2 = f(x) + \eta \left[k_1 y_2 + \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T} (x_2 - \rho_L) \right]$$

$$= f(x_2) + \eta \left\{ \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T} [x_2 - \rho_L + f'(x_2) f(x_2)] + k_1 [f(x_2) - x_2 f'(x_2)] \right\}$$

$$\text{for} \quad x_2 \leqslant h + \eta \left[k_1 h - \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T} Y \right] \qquad \ldots(11A)$$

except very near the leading edge.

Referred to the old axes the equation of the contour of the flap, for a deflection η, is given by (6A) with x_1 and y_1 as current coordinates. We must therefore write x_1 and y_1 for x and y in (10A) (or x and y for x_1 and y_1 in (6A)) in order to find the equation of the contour of the flap in the new coordinates. We find that the new equation is

$$y_2 = F(x_2) + \eta \left\{ \frac{h_1 - \rho_L}{1 - \rho_L - \rho_T} [1 - x_2 - F'(x_2) F(x_2) - \rho_T] \right.$$

$$\left. + k_1 [F(x_2) + (1 - x_2) F'(x_2)] \right\}$$

$$\text{for } x_2 \geqslant h + \eta \left[k_1 h - \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T} Y \right], \qquad \ldots(12A)$$

except very near the trailing edge.

It is now not difficult to verify that y_2 and y_2' are continuous. It is necessary only to remark that $f(x) = F(x)$ and $f'(x) = F'(x)$ at $x = h$, and that, since $y = F(x)$ is, near $x = h$, part of a circle with its centre at (h_1, k_1),

$$(x - h_1)^2 + [F(x) - k_1]^2 = \text{constant}$$

and

$$x - h_1 + F'(x)[F(x) - k_1] = 0,$$

in the neighbourhood of $x = h$ on both surfaces. Since y_2 and y_2' are now both continuous, the logarithmic infinity in ε', found on the simple theory in the body of the report, does not now occur.

Equations (11A) and (12A) are valid for both the upper and lower surfaces, and provide equations for the contour of the

aerofoil/

aerofoil with the flap deflected referred to axes suitable for the calculation of the velocity distribution. With $x_2 = \frac{1}{2}(1 - \cos\theta)$, as before, ψ is determined from the equation

$$\psi = 2y_2 \operatorname{cosec}\theta, \qquad \ldots(13A)$$

where y_2 is given by (11A) and (12A). We may compare (11A) and (12A) with the equations (5), §3, of the simple theory, which, in the present notation, are

$$y_2 = f(x_2) + \eta(1 - h)x_2 \quad \text{for} \quad 0 < x < h \ \ldots(14A)$$

and

$$y_2 = F(x_2) + \eta h(1 - x_2) \quad \text{for} \quad h < x < 1. \ \ldots(15A)$$

The simplest method of carrying out the calculations would appear to be to find ψ, ε, ε' on the simple theory, as explained in §5, and then, from (13A), to find the alteration necessary to ψ to allow for the difference between eqns. (11A) and (12A) on the one hand and eqns. (14A) and (15A) on the other; from these values of the necessary changes in ψ the necessary changes in ε and ε' would then be found numerically, except that the value of ε' at $x = h$, if required, would need a special computation; its value depends critically on the amount of fairing-off near $x = h$.

Equation (11A) does not hold at the leading edge ($\theta = 0$), nor equation (12A) at the trailing edge ($\theta = \pi$). Special methods are necessary, therefore, to find the new values of ψ, with the flap deflected, at $\theta = 0$ and $\theta = \pi$. The appropriate approximation to $\psi(0)$ is $2\sqrt{(\rho_L + \Delta\rho_L)}$, where ρ_L is the radius of curvature at the original leading edge, and $\rho_L + \Delta\rho_L$ the radius of curvature at the new leading edge. If, near $x = 0$,

$$f(x) = \sqrt{(2\rho_L)}\, x^{\frac{1}{2}} + \ell x + \ldots, \qquad \ldots(16A)$$

we find, making due allowance for the change of the length of the chord, that the value of $\psi(0)$ is now

$$\psi(0) = \sqrt{(2\rho_L)} \left\{1 + \eta\left[\tfrac{1}{2}k_1 + \frac{3}{2}\,\ell\,\frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T}\right]\right\} \ldots(17A)$$

to the first order in η. Similarly if, near $x = 1$,

$$F(x) = \sqrt{(2\rho_T)}(1 - x)^{\frac{1}{2}} + \ell'(1 - x) + \ldots, \qquad \ldots(18A)$$

then the new value of $\psi(\pi)$ is

$$\psi(\pi) = \sqrt{(2\rho_T)} \left\{1 + \eta\left[\tfrac{1}{2}k_1 + \frac{3}{2}\,\ell'\,\frac{h_1 - \rho_L}{1 - \rho_L - \rho_T}\right]\right\}. \qquad \ldots(19A)$$

The/

The values of ℓ and ℓ' are those appropriate to the upper or the lower surface according as the new leading and trailing edges, respectively, lie on the original upper or lower surface.

Finally, we may note that since the angle between the old and new chords is given by (9A),

$$\alpha = \alpha' + \eta \frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T} . \qquad ...(20A)$$

It should be mentioned that, on an aerofoil specially designed for suction, with the velocity at the surface decreasing discontinuously at one chordwise position on each surface, where a slot is cut and suction applied, the theory of this appendix will not be applicable if the suction slot is at $x = h$ – i.e. if the wing aft of the suction slot is to be used as a flap. The hinge cannot then be located at the intersection of the normals to the upper and lower surfaces at $x = h$; theoretically there is a singularity in the equation of the aerofoil contour there and practically the position of the hinge is largely determined by mechanical and other geometrical considerations, arising from the necessity of leaving the slot free for all values of η without cutting away too much of the surface at and near the slot.

3. Hinge Moments

We return to equation (54) of §6 and retain the second term, using eqn. (12A) for y_2. Note first that it follows from eqns.(10A) that

$$\left.\begin{aligned} h_2 &= h_1 - \eta k_1 \left\{\frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T} - h_1\right\} , \\ k_2 &= k_1 + \eta \left\{\frac{1 - h_1 - \rho_T}{1 - \rho_L - \rho_T}(h_1 - \rho_L) + k_1^2\right\} \end{aligned}\right\} \qquad ...(21A)$$

From (12A) and (21A) it follows, after some reduction, that, to the first order in η

$$x_2 - h_2 + (y_2 - k_2)\frac{dy_2}{dx_2} = \frac{1}{2}\frac{d}{dx_2}\left\{(x_2 - h_2)^2 + (y_2 - k_2)^2\right\}$$

$$= X + \eta(X_1 + 2k_1X), \qquad ...(22A)$$

where

$$X = \frac{1}{2}\frac{d}{dx_2}\{[x_2 - h_1]^2 + [F(x_2) - k_1]^2\}$$

$$= x_2 - h_1 + F'(x_2)\,[F(x_2) - k_1], \qquad ...(23A)$$

X_1/

$$X_1 = \frac{d}{dx_2}\left\{X\left[k_1(1-x_2) - \frac{h_1-\rho_L}{1-\rho_L-\rho_T}F(x_2)\right]\right\}$$

$$= -X\left[k_1 + \frac{h_1-\rho_L}{1-\rho_L-\rho_T}F'(x_2)\right] + \left\{k_1(1-x_2) - \frac{h_1-\rho_L}{1-\rho_L-\rho_T}F(x_2)\right\}$$

$$\times \{1 + F''(x_2)[F(x_2) - k_1] + [F'(x_2)]^2\}. \qquad ...(24A)$$

Since $y = F(x)$ is, near $x = h$, part of a circle with its centre at (h_1, k_1), it follows that X and X_1 both vanish identically in the neighbourhood of $x_2 = h$.

E^2C_H is given by eqn. (54) of §6, but that equation was obtained with the length of the new chord as unity, whereas, strictly speaking, C_H should be defined so that the hinge moment is $\frac{1}{2}\rho U^2E^2c^2C_H$, where c is the original aerofoil chord. With the new chord of unit length, the length of the orignal chord, to the first order in η, is $1 + k_1\eta$. Consequently the right-hand side of (54) should be multiplied by $1 - 2k_1\eta$, and

$$E^2C_H = \int_h^1\left\{\frac{q_\ell^2}{U^2}(X_\ell + \eta X_{1\ell}) - \frac{q_u^2}{U^2}(X_u + \eta X_{1u})\right\} dx_2. \qquad ...(25A)$$

We have taken the lower limit of integration as h because, although the flap extends from $x_2 = h + \eta[k_1h - (1 - h_1)Y]$, the integrand vanishes identically in the neighbourhood of $x_2 = h$.

With L, M, N as in (70), (71) and (72) (but with r, s, t replaced, on the upper surface, by the more accurate values of $\Delta\psi_u/\eta$, $\Delta(\varepsilon_u - \beta)/\eta$, $\Delta\varepsilon_u'/\eta$ derived from eqns. (11A) and (12A), and on the lower surface by the more accurate values of $-\Delta\psi_\ell/\eta$, $\Delta(\varepsilon_\ell - \beta)/\eta$, $-\Delta\varepsilon_\ell'/\eta$ similarly derived), the following equations are obtained from (25A) for b_0, b_1, b, b_2.

$$E^2\left(b_0 - \frac{b_1}{c_1}c_0\right) = \int_h^1 (L_\ell^2X_\ell - L_u^2X_u)\, dx_2, \qquad ...(26A)$$

$$E^2\frac{b_1}{c_1} = \int_h^1 (2L_\ell M_\ell X_\ell - 2L_uM_uX_u)\, dx_2, \qquad ...(27A)$$

$$-E^2b = E^2\left(b_2 - \frac{c_2}{c_1}b_1\right) = \int_h^1 (2L_\ell N_\ell X_\ell - 2L_uN_uX_u + L_\ell^2X_{1\ell} - L_u^2X_{1u})dx_2.$$

$$...(28A)$$

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Table 1

measured from new chord line

E	h	θ_1/π	$\sin\theta_1/\pi$	$\Delta\beta/\eta$	m
0·4	0·6	0·564094	0·311879	0·3478	0·5879
0·3	0·7	0·630990	0·291736	0·3607	0·6416
0·25	$0{\cdot}7_5$	0·666667	0·275664	0·3590	0·6495
0·2	0·8	0·704833	0·254648	550- 48. 0·3498	0·6400
0·15	0·85	0·746817	0·227319	0·3405	0·6070
0·10	0·9	0·795167	0·190986	0·2958	0·5400
0·08	0·92	0·817445	0·172711	0·2753	0·4992

E	$\Delta C_{L_{opt}}/\eta$	$\Delta\alpha_{opt}/\eta$	c_2/c_1	$-b_1/c_1$	b	$-b_2$
0·4	1·9596	—0·0359	0·7478	0·1185	0·4557	1·0127
0·3	1·8330	—0·0690	$0{\cdot}6607_5$	0·0999	0·5508	0·9654
0·25	1·7321	—0·0833	0·6090	0·0900	0·5993	0·9436
0·2	1·6000	—0·0952	0·5498	0·0795	0·6483	0·9229
0·15	1·4283	—0·1032	0·4805	0·0680	0·6978	0·9031
0·10	1·2	—0·1048	0·3958	0·0549	0·7477	0·8842
0·08	1·0852	—0·1026	0·3553	0·0489	0·7678	0·8769

Table 2/

Table 2.

x	E = 0·4				E = 0·3			
	$\Delta g_i/\eta$	r	s	t	$\Delta g_i/\eta$	r	s	t
0	0	0	−0·6238	0	0	0	−0·5835	0
0·005	0·0369	0·0284	−0·6231	0·0099	0·0296	0·0213	−0·5830$_5$	0·0059
0·0075	0·0453	0·0348	−0·6227	0·0122	0·0363	0·0261	−0·5828	0·0073
0·0125	0·0586$_5$	0·0450	−0·6220	0·0158	0·0470	0·0337$_5$	−0·5824	0·0095
0·025	0·0837	0·0640$_5$	−0·6202	0·0229	0·0670	0·0480	−0·5813	0·0137
0·05	0·1207	0·0918	−0·6164	0·0340	0·0963	0·0688	−0·5791	0·0203
0·075	0·1508	0·1139	−0·6124	0·0436	0·1201	0·0854	−0·5767	0·0260
0·1	0·1777$_5$	0·1333	−0·6081	0·0529	0·1412	0·1000	−0·5741	0·0315
0·15	0·2276	0·1680	−0·5987	0·0720	0·1797	0·1260	−0·5685	0·0425
0·2	0·2760	0·2000	−0·5879	0·0931	0·2163	0·1500	−0·5622	0·0545
0·25	0·3259	0·2309	−0·5753	0·1178	0·2532	0·1732	−0·5548	0·0682
0·3	0·3797	0·2619	−0·5605	0·1479	0·2917	0·1964	−0·5463	0·0845
0·35	0·4404	0·2935	−0·5427	0·1861	0·3333	0·2201	−0·5363	0·1043
0·4	0·5123	0·3266	−0·5210	0·2367	0·3797	0·2449$_5$	−0·5243	0·1294
0·45	0·6031	0·3618	−0·4937	0·3079	0·4335	0·2714	−0·5096	0·1622
0·5	0·7297	0·4000	−0·4578	0·4178	0·4987	0·3000	−0·4912	0·2070
0·55	0·9463	0·4422	−0·4070	0·6233	0·5828	0·3317	−0·4675	0·2719
0·6	∞	0·4899	−0·3119	∞	0·7026	0·3674	−0·4351$_5$	0·3755
0·65	0·9330	0·4403	−0·2141	0·6368	0·9118$_5$	0·4088	−0·3873	0·5760
0·7	0·7026	0·3928	−0·1586	0·4292	∞	0·4583	−0·2917	∞
0·75	0·5611	0·3464	−0·1175	0·3132	0·8813	0·4041$_5$	−0·1900	0·6032
0·8	0·4537	0·3000	−0·0850	0·2340	0·6399	0·3500	−0·1318	0·3952
0·85	0·3621	0·2520$_5$	−0·0583	0·1739	0·4845	0·2941	−0·0882	0·2755
0·9	0·2760	0·2000	−0·0359	0·1242	0·3575	0·2333	−0·0534	0·1891
0·925	0·2317	0·1708$_5$	−0·0259	0·1011	0·2967	0·1993	−0·0383	0·1518
0·95	0·1838	0·1376$_5$	−0·0167	0·0778	0·2330	0·1606	−0·0245	0·1154
0·975	0·1265	0·0961	−0·0081	0·0520	0·1589	0·1121	−0·0118	0·0764
0·9875	0·0883	0·0675	−0·0040	0·0358	0·1105	0·0788	−0·0058	0·0523
1	0	0	0	0	0	0	0	0

Table 2 Continued/

Table 2 Contd.

x	E = 0·25				E = 0·2			
	$\Delta g_i/\eta$	r	s	t	$\Delta g_i/\eta$	r	s	t
0	0	0 0	−0·5513	0	0	0	−0·5093	0
0·005	0·0261	0·0177	−0·5510	0·0044	0·0226	0·0142	−0·5091	0·0030
0·0075	0·0320	0·0217	−0·5509	0·0054	0·0277	0·0174	−0·5090	0·0037
0·0125	0·0414	0·0281	−0·5505$_5$	0·0070	0·0358$_5$	0·0225	−0·5088	0·0048
0·025	0·0590	0·0400	−0·5498^5	0·0101	0·0511^5	0·0320	−0·5082	0·007[illegible]50
0·05	0·0848	0·0573$_5$	−0·5481	0·0149	0·0733$_5$	0·0459	−0·5071	0·0103
0·075	0·1056	0·0712^5	−0·5463	0·0191	0·0913^5	0·0569$_5$	−0·5058	0·0132
0·1	0·1241	0·0833	−0·5445	0·0230	0·1071	0·0667^5	−0·5045$_5$	0·0159
0·15	0·2875	0·1050	−0·5404	0·0310	0·1357	0·0840	−0·5017^5	0·0214
0·2	0·1891$_5$	0·1250	−0·5347$_5$	0·0396	0·1626	0·1000	−0·4985$_5$	0·0272
0·25	0·2206^5	0·1443	−0·5304^5	0·0494	0·1891$_5$	0·1155	−0·4949^5	0·0338
0·3	0·2532	0·1637	−0·5243	0·0609	0·2163^5	0·1309	−0·4907	0·0415
0·35	0·2878	0·1834$_5$	−0·5171	0·0748	0·2450	0·1468	−0·4858	0·0507
0·4	0·3259	0·2041^5	−0·5085	0·0920	0·2760	0·1633	−0·4800	0·0621
0·45	0·3689	0·2261	−0·4981	0·1142	0·3104	0·1809	−0·4730	0·0764
0·5	0·4192	0·2500	−0·4853	0·1435	0·3497	0·2000	−0·4645	0·0951
0·55	0·4808	0·2764	−0·4690	0·1843	0·3962	0·2211	−0·4537$_5$	0·1203
0·6	0·5611	0·3062	−0·4475	0·2447	0·4537	0·2449$_5$	−0·4399^5	0·1560
0·65	0·6767	0·3407	−0·4175	0·3431	0·5295	0·2725$^5_5$	−0·4212	0·2102
0·7	0·8813	0·3819	−0·3718	0·5386	0·6399	0·3055^5	−0·3943	0·3011
0·75	∞	0·4330	−0·2757	∞	0·8384	0·3464	−0·3515	0·4885
0·8	0·8384	0·3750	−0·1709	0·5724	∞	0·4000	−0·2546$_5$	∞
0·85	0·5880	0·3151	−0·1110	0·3634	0·7778	0·3361	−0·1457^5	0·5280
0·9	0·4192	0·2500	−0·0661	0·2392	0·5123	0·2667	−0·0839	0·3156
0·925	0·3439$_5$	0·2136	−0·0471	0·1894	0·4117$_5$	0·2278	−0·0592	0·2437
0·95	0·2677^5	0·1721	−0·0300	0·1425	0·3157^5	0·1835	−0·0374	0·1800
0·975	0·1813	0·1201	−0·0144	0·0935	0·2113	0·1281	−0·0178	0·1164
0·9875	0·1257	0·0844	−0·0070	0·0638	0·1457	0·0900	−0·0087	0·0790
1	0	0	0	0	0	0	0	0

Table 2 Continued/

Table 2 Continued

x	E = 0·15				E = 0·1			
	$\Delta g_i/\eta$	r	s	t	$\Delta g_i/\eta$	r	s	t
0	0	0	−0·4546	0	0	0	−0·3820	0
0·005	0·0190	0·0106	−0·4545	0·0019	0·0150$_5$	0·0071	−0·3819	0·0010
0·0075	0·0233	0·0130	−0·4544	0·0023	0·0184$_5$	0·0087	−0·3819	0·0012
0·0125	0·0301	0·0169	−0·4543	0·0030	0·0239	0·0112$_5$	−0·3818	0·0016
0·025	0·0429	0·0240	−0·4539$_5$	0·0044	0·0340	0·0160	−0·3816	0·0023
0·05	0·0615	0·0344	−0·4532	0·0065	0·0488	0·0229	−0·3812	0·0034
0·075	0·0765	0·0427	−0·4525	0·0083	0·0606	0·0285	−0·3808	0·0044
0·1	0·0897	0·0500	−0·4516	0·0100	0·0710	0·0333	−0·3804	0·0053
0·15	0·1135	0·0630	−0·4499	0·0134	0·0897	0·0420	−0·3795	0·0070
0·2	0·1357	0·0750	−0·4479	0·0170	0·1071	0·0500	−0·3784	0·0089
0·25	0·1575	0·0866	−0·4456	0·0211	0·1241	0·0577	−0·3772	0·0111
0·3	0·1797	0·0982	−0·4430	0·0258	0·1412	0·0655	−0·3758$_5$	0·0135
0·35	0·2028	0·1101	−0·4399$_5$	0·0314	0·1589	0·0734	−0·3743	0·0164
0·4	0·2276	0·1225	−0·4364	0·0382	0·1777	0·0816$_5$	−0·3724	0·0199
0·45	0·2547	0·1357	−0·4321	0·0468	0·1981	0·0904$_5$	−0·3702	0·0242
0·5	0·2851	0·1500	−0·4269	0·0578	0·2206	0·1000	−0·3675	0·0296
0·55	0·3202	0·1658	−0·4204	0·0723	0·2462	0·1105$_5$	−0·3642	0·0368
0·6	0·3621	0·1837	−0·4121	0·0924	0·2760	0·1225	−0·3600	0·0466
0·65	0·4146	0·2044	−0·4011$_5$	0·1216	0·3120	0·1363	−0·3545	0·0603
0·7	0·4845	0·2291	−0·3850	0·1673	0·3575	0·1527$_5$	−0·3470	0·0811
0·75	0·5880	0·2598	−0·3631	0·2471	0·4192	0·1732	−0·3362	0·1148
0·8	0·7778	0·3000	−0·3245	0·4207	0·5123	0·2000	−0·3191	0·1775
0·85	∞	0·3571	−0·2273	∞	0·6880$_5$	0·2380$_5$	−0·2873	0·3262
0·9	0·6880$_5$	0·2833	−0·1126	0·4621	∞	0·3000	−0·1910	∞
0·925	0·5253	0·2420	−0·0777$_5$	0·3351	0·8095$_5$	0·2563	−0·1141$_5$	0·5710
0·95	0·3901	0·1950	−0·0483	0·2382	0·5377	0·2065	−0·0676	0·3542
0·975	0·2556	0·1361	−0·0227	0·1501	0·3333	0·1441	−0·0309	0·2087
0·9875	0·1748	0·0956	−0·0110	0·1008	0·2236	0·1013	−0·0148$_5$	0·1370
1	0	0	0	0	0	0	0	0

Table 2 Continued/

Table 2 Continued

x	E = 0·08			
	$\Delta g_i/\eta$	r	s	t
0	0	0	—0·3454	0
0·005	0·0133	0·0057	—0·3454	0·0007
0·0075	0·0163	0·0069$_5$	—0·3453$_5$	0·0009
0·125	0·0211	0·0090	—0·3453	0·0011
0·025	0·0301	0·0128	—0·3452	0·0016
0·05	0·0431	0·0183$_5$	—0·3449	0·0024
0·075	0·0536	0·0228	—0·3446	0·0031
0·1	0·0628	0·0267	—0·3443	0·0037
0·15	0·0793	0·0336	—0·3436$_5$	0·0050
0·2	0·0945$_5$	0·0400	—0·3429	0·0063
0·25	0·1094$_5$	0·0462	—0·3421	0·0078
0·3	0·1245	0·0524	—0·3411	0·0095
0·35	0·1400	0·0587	—0·3400	0·0115
0·4	0·1563$_5$	0·0653	—0·3387	0·0140
0·45	0·1740	0·0724	—0·3371	0·0170
0·5	0·1935	0·0809	—0·3352	0·0208
0·55	0·2154	0·0884	—0·3329	0·0257
0·6	0·2408	0·0980	—0·3300	0·0324
0·65	0·2711	0·1090	—0·3262	0·0418
0·7	0·3089	0·1222	—0·3210	0·0558
0·75	0·3589	0·1386	—0·3136	0·0781
0·8	0·4312	0·1600	—0·3021	0·1183
0·85	0·5546	0·1904	—0·2814	0·2062
0·9	0·8892	0·2400	—0·2320	0·5223
0·925	1·2879	0·2620	—0·1483	0·9995
0·95	0·6622	0·2111	—0·0815$_5$	0·4542
0·975	0·3873	0·1473	—0·0363	0·2494
0·9875	0·2558	0·1035	—0·0173	0·1606
1	0	0	0	0

Table 3/

Table 3

Ordinates for 15 per cent Thick Aerofoil

X = 0·6

a = 0·1336624 b = 0·2064419 c = —0·1817961

x	y	x	y
0		0·35	0·0724356
0·001	0·0045298	0·36	0·0729000
0·002	0·0064046	0·38	0·0736830
0·003	0·0078422	0·4	0·0743030
0·004	0·0090532	0·42	0·0747260
0·005	0·0101194	0·44	0·0749564
0·006	0·0110826	0·45	0·0749971
0·007	0·0119678	0·46	0·0749866
0·0075	0·0123863	0·48	0·0748065
0·008	0·0127910	0·5	0·0744030
0·009	0·0135637	0·52	0·0737593
0·01	0·0142939	0·54	0·0728527
0·012	0·0156506	0·55	0·0722912
0·0125	0·0159714	0·56	0·0716511
0·014	0·0168963	0·58	0·0701054
0·016	0·0180540	0·6	0·0681156
0·018	0·0191396	0·62	0·0654984
0·02	0·0201649	0·64	0·0624562
0·025	0·0225165	0·65	0·0608156
0·03	0·0246338	0·66	0·0591083
0·035	0·0265726	0·68	0·0555237
0·04	0·0283692	0·7	0·0517543
0·05	0·0316306	0·72	0·0478436
0·06	0·0345508	0·74	0·0438300
0·07	0·0372090	0·75	0·0417959
0·075	0·0384565	0·76	0·0397491
0·08	0·0396565	0·78	0·0356351
0·09	0·0419289	0·8	0·0315212
0·1	0·0440521	0·82	0·0274412
0·12	0·0479245	0·84	0·0234297
0·14	0·0513836	0·85	0·0214610
0·15	0·0529812	0·86	0·0195235
0·16	0·0544999	0·88	0·0157625
0·18	0·0573215	0·9	0·0121920
0·2	0·0598821	0·92	0·0088651
0·22	0·0622064	0·925	0·0080788
0·24	0·0643125	0·94	0·0058481
0·25	0·0652881	0·95	0·0044827
0·26	0·0662138	0·96	0·0032316
0·28	0·0679200	0·975	0·0016143
0·3	0·0694380	0·98	0·0011593
0·32	0·0707726	0·9875	0·0005759
0·34	0·0719263	1	0

Table 4/

Table 4

$$\frac{\Delta\beta}{\eta} = 0{\cdot}3560$$

x	$\frac{\Delta\psi_c}{\eta}$	$\frac{\Delta\varepsilon_c - \Delta\beta}{\eta}$	$\frac{\Delta\varepsilon'_c}{\eta}$	ψ_s	ε_s	ε'_s
0	0	−0·5156	0	0·143278	0	0·0134
0·005	0·0146	−0·5153	0·0034	0·143469	0·0019	0·0139
0·0075	0·0179	−0·5152	0·0042	0·143564	0·0024	0·0141
0·0125	0·0232	−0·5149	0·0054	0·143754	0·0031	0·0147
0·025	0·0331	−0·5143	0·0079	0·144221	0·0045	0·0160
0·05	0·0473	−0·5130	0·0116	0·145131	0·0068	0·0187
0·075	0·0587	−0·5116	0·0149	0·146005	0·0089	0·0214
0·1	0·0687	−0·5102	0·0176	0·146840	0·0109	0·0243
0·15	0·0865	−0·5071	0·0236	0·148377	0·0150	0·0302
0·2	0·1028	−0·5036	0·0296	0·149705	0·0194	0·0364
0·25	0·1185	−0·4996	0·0365	0·150776	0·0242	0·0431
0·3	0·1341	−0·4952	0·0446	0·151526	0·0294	0·0503
0·35	0·1501	−0·4898	0·0538	0·151866	0·0352	0·0581
0·4	0·1668	−0·4837	0·0655	0·151670	0·0416	0·0667
0·45	0·1845	−0·4764	0·0798	0·150750	0·0488	0·0764
0·5	0·2036	−0·4675	0·0984	0·148806	0·0570	0·0874
0·55	0·2245	−0·4564	0·1228	0·145311	0·0664	0·1001
0·6	0·2478	−0·4426	0·1516	0·139040	0·0773	0·1153
0·65	0·2754	−0·4247	0·2008	0·127504	0·0873	0·0784
0·7	0·3095	−0·3984	0·2989	0·112937	0·0937	0·0433
0·75	0·3523	−0·3555	0·4966	0·096523	0·0967	+0·0097
0·8	0·4084	−0·2556	*	0·078803	0·0958	−0·0228
0·85	0·3420	−0·1423	0·5446	0·060103	0·0907	−0·0542
0·9	0·2694	−0·0802	0·3138	0·040640	0·0801	−0·0849
0·925	0·2292	−0·0557	0·2384	0·030672	0·0719	−0·1000
0·95	0·1837	−0·0346	0·1720	0·020568	0·0607	−0·1149
0·975	0·1276	−0·0162	0·1082	0·010340	0·0443	−0·1297
0·9875	0·0804	−0·0077	0·0724	0·005184	0·0318	−0·1371
1	0	0	0	0	0	−0·1444

*See remarks in §7.

AH.

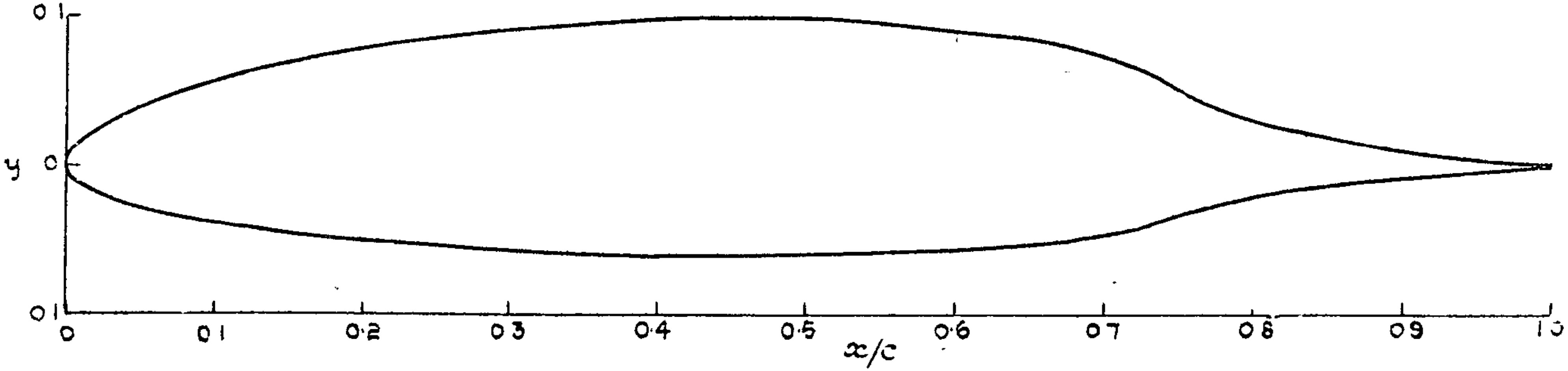

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FIG. 6.

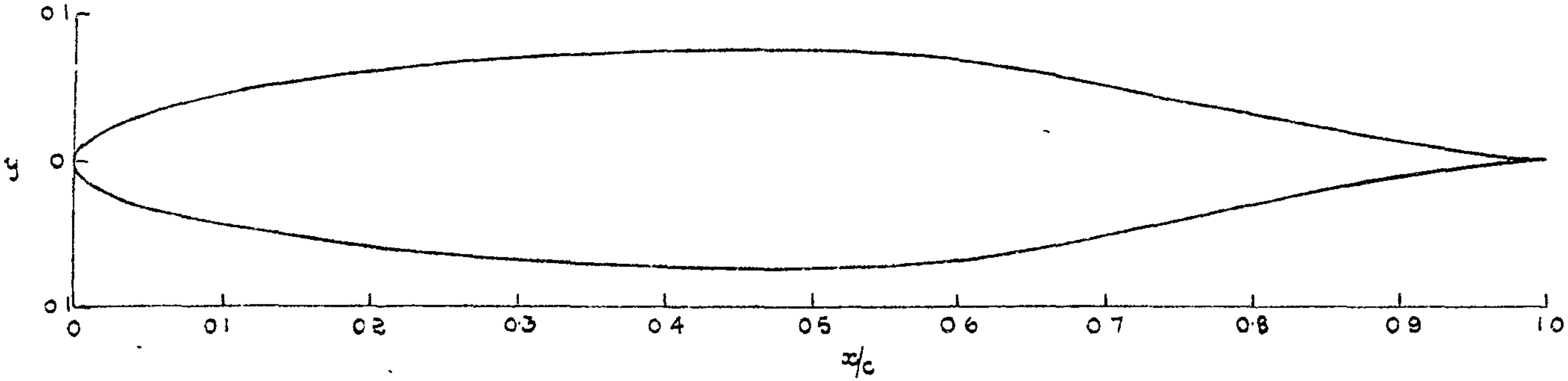

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Fig 8

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