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# A Method for Correcting Measurements of the Heat Transfer Factor through the Skin of a Wind Tunnel Model by <br> B. A. M. Piggott, B.A. 

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# A METHOD FOR CORRECTITG MEASUREMENTS OF THE heat transfer factor through the skin of a tind iunner model 

by<br>B. A. M. Piggott, B.A.

## SUMMARY

In an experiment in which it is required to measure the heat trensfer factor at the outer surface of the skin of a wind tunnel model, the temperature of the skin is measured at various points on this surface and the heat flow rate across the skin is measured at the corresponding points on the inner surface. Caloulation of the heat flow rate across the outer surface of the skin involves the solution of Laplace's equation in a rectangular region with mixed boundary conditions. A numerical method of solution is described and illustrated by an example.

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## 1 <br> INTRODUCTION

Fig. 1 shows a longitudinal seotion of a wind tunnel model ${ }^{1}$ in the form of a cylindrical shell with a conioal forward section. The skin is heated by an airflow and heat is taken out by coolant flowing through the hollow intericr of the model. When a steady state has been reaohed, the temperature $T_{W}$ of the outer surface, and the heat flow rate $Q_{M}$ across the inner surface of the skin of the model are measured by a series of thermocouplos and heat flow meters ${ }^{2}$ at various stations along the cylindrical section of the model (see Fig.2).

The heat transfer factor $h$ across the outer surface of the skin at a given station is defined by the equation

$$
\begin{equation*}
Q=h\left(T_{R}-T_{W}\right) \tag{1}
\end{equation*}
$$

where $Q$ is the heat flow rate across the outer surface at that station and $T_{R}$ is the recovery temperature - i.e. the temperature that would be reached there if no heat were allowed to escape into the model. As a first approximation it may be assumed that $Q=Q_{M}$; this is equivalent to neglecting any flow of heat along the akin, which is thin and of low thermal conductivity. By varying the amount of coolant flowing through the interior of the model, it is possible to obtain a series of values of $Q_{M}$ and $W_{W}$ for each station. From these, using the above assumption, $h$ and $T_{R}$ for that station may be oalculated.

The method described in this paper, in which the assumption that $Q=Q_{M}$ is not made, solves the heat equation for a seotion of the skin, assuming only that there is no heat flow across the ends of the section. It thus provides an improved value of $Q$; a comparison between the calculated values of $Q$ and $Q$ for a specific case is shown in Fig. 5.

## 2 THE HEAT EQUATION AND BOUNDARY CCNDITIONS

Since the skin of the cylindrical model is thin, its curvature is neglected and it is regarded as an infinite rectangular slab - see Fig. 3 ; $A$ and $B$ are arbitrarily chosen points on the outer surface of the skin and 0 and $P$ are the corresponding points on the inner surface. Taking $O P$ and $C A$ as $x$ - and $y$-axis, respectively, with $O P=a$ and $O A=b$, the temperature $T(x, y)$ of a point in the skin in the steady state satisfies Laplace's equation:

$$
\begin{equation*}
\nabla^{2} T \equiv \frac{\partial^{2} T}{\partial x^{2}}+\frac{\partial^{2} T}{\partial y^{2}}=0 \quad(0 \leqslant x \leqslant a, 0 \leqslant y \leqslant b) \tag{2}
\end{equation*}
$$

The boundary conditions on $A B$ and $O P$ are known:

$$
\left.\begin{array}{rl}
T(x, b) & =T_{W}(x)  \tag{3}\\
\frac{\partial T}{\partial y}(x, 0) & =\frac{1}{k} Q_{M}(x)
\end{array}\right\}
$$

$(0 \leqslant x \leqslant a)$
where $k$ is the thermal conductivity of the skin, and $Q_{M}$ is taken to be positive when the heat flow is direoted towards the interior of the model. Boundary conditions are also needed along OA and PB ; since the variation of $T$ with $x$ is observed to be small at the ends of the model, the boundary conditions are taken to be

$$
\begin{equation*}
\frac{\partial T}{\partial x}(0, y)=\frac{\partial T}{\partial x}(a, y)=0 . \quad(0 \leqslant y \leqslant b) \tag{5}
\end{equation*}
$$

They are equivalent to the assumption that there is no longitudinal flow of heat across the ends of the section.

## 3 THE FINITE DIFFERENCE EOUATIONS

The rectangle $O A B P$ is covered by a uniform rectangular grid of points ( $x_{i}, y_{j}$ ) with the mesh lengths $\delta x$ and $\delta y$ respectively in the $x$ - and $y$-direotions (see Fig.4) and grid-points are added along $x=-\delta x$ and $x=a+\delta x$. The equations of the previous section are then approximated by the finitedifference equations:

$$
\left.\begin{array}{c}
\frac{T\left(x_{i-1}, y_{j}\right)-2 T\left(x_{i}, y_{j}\right)+T\left(x_{i+1}, y_{j}\right)}{(\delta x)^{2}}+\frac{T\left(x_{i}, y_{j-1}\right)-2 T\left(x_{i}, y_{j}\right)+T\left(x_{i}, y_{j+1}\right)}{(\delta y)^{2}}=0 \\
(1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n-1) \\
T\left(x_{i}, b\right)=T T_{W}\left(x_{i}\right)  \tag{7}\\
T\left(x_{i}, \delta y\right)-T\left(x_{i}, 0\right)=\frac{\delta y_{i}}{k} Q_{M}\left(x_{i}\right) \\
T\left(-\delta x, y_{j}\right)=T\left(0, y_{j}\right) \\
T\left(a+\delta x, y_{j}\right)=T\left(a, y_{j}\right)
\end{array}\right\} \begin{gathered}
(1 \leqslant i \leqslant m) \\
(1 \leqslant j \leqslant n-1)
\end{gathered}
$$

The heat flow rate across $A B$ is approximated by the equations

$$
\begin{equation*}
Q\left(x_{i}\right)=\frac{k}{\delta y}\left\{T_{W}\left(x_{i}\right)-T\left(x_{i}, y_{n-1}\right)\right\} . \quad(1 \leqslant i \leqslant m) \tag{11}
\end{equation*}
$$

It is convenient at this stage to combine equations (6), (7), (9) and (10) in order to eliminate $T\left(x_{i}, b\right), T\left(-\delta x, y_{j}\right)$, and $T\left(a+\delta x, y_{j}\right)$; the resulting equations, together with (8), provide mn linear equations for the unknowns $T\left(x_{1}, y_{j}\right),(1 \leqslant i \leqslant m, 0 \leqslant j \leqslant n-1)$. These unknowns may be written as a column vector $t$, where $T\left(x_{i}, y_{j}\right)=t_{r}(1 \leqslant r \leqslant m n)$ when $r=(i-m)+m(n-j)$, which corresponds to a numbering of the grid-points by rows, starting with the first row inside the outer surface. The equations then take the form

$$
\begin{equation*}
\mathrm{Mt}=\underline{\underline{u}} \tag{12}
\end{equation*}
$$

$$
u_{r}=\left\{\begin{array}{cl}
-\left(\frac{\delta x}{\delta y}\right)^{2} T_{W}\left(x_{r}\right) & 1 \leqslant r \leqslant m  \tag{13}\\
0 & m+1 \leqslant r \leqslant m(n-1) \\
-\frac{1}{k} \delta y Q_{M}\left(x_{r-m(n-1)}\right) & m(n-1)+1 \leqslant r \leqslant m n
\end{array}\right\}
$$

and $M$ is the $\mathrm{mn} \times \mathrm{mn}$ block tridiagonal matrix

$I_{m}$ is the $m \times m$ unit matrix and $N$ the $m \times m$ triaiagonal matrix

4 THE METHCD OF SUCCESSIVE OVER-REL,AXATION
Equation (12) corresponds to the mn scalar equations

$$
\begin{equation*}
\sum_{s=1}^{m n} m_{r s} t_{s}=u_{r} \quad(r=1,2, \ldots, m n) \tag{16}
\end{equation*}
$$

Given any approximate solution $t_{r}^{(p)}$, the method of (point) successive overrelaxation replaces it by a new approximation $t_{r}^{(p+1)}$ given by

$$
\begin{equation*}
t_{r}^{(p+1)}=t_{r}^{(p)}+\frac{\omega}{m_{r r}}\left\{u_{r}-\sum_{s=1}^{r-1} m_{r s} t_{s}^{(p+1)}-\sum_{s=r}^{m n} m_{r s} t_{s}^{(p)}\right\} \quad(r=1,2, \ldots, m n) \tag{17}
\end{equation*}
$$

where $\omega$ is a suitably chosen "relaxation factor".

In the present case, the process can be started by choosing $\omega=1$ and assuming that the temperature gradient through the skin is $\frac{1}{k} Q_{M}\left(x_{i}\right)$, depending only on $x$, so that

$$
\begin{align*}
t_{r}^{(0)} & =T^{(0)}\left(x_{i}, y_{i}\right) \\
& =T^{(0)}\left(x_{i}, y_{j+1}\right)-\frac{Q_{M}\left(x_{i}\right)}{k} \delta y \\
& =T_{W}\left(x_{i}\right)-\frac{M-1}{K} Q_{M}\left(x_{i}\right) \delta y \tag{18}
\end{align*}
$$

where $r=(i-m)+m(n-j) \quad(1 \leqslant i \leqslant m, 0 \leqslant j \leqslant n-1)$. If $\mu=\lim _{p \rightarrow \infty} \mu_{p}$ where

$$
\mu_{p}=\frac{\sum_{r=1}^{m n}\left(t_{r}^{(p)}-t_{r}^{(p-1)}\right)^{2}}{\sum_{r=1}^{m n}\left(t_{r}^{(p+1)}-t_{r}^{(p)}\right)^{2}}
$$

then the best value of $\omega$ for use in (17) is given ${ }^{3}$ by

$$
\begin{equation*}
\omega(\mu)=1+\mu^{\frac{1}{2}} /\left(1+\sqrt{1-\mu^{\frac{1}{2}}}\right)^{2} \tag{20}
\end{equation*}
$$

The iterative process described by equation (17) is repeated with $\omega=1$ until $\left|\mu_{p}-\mu_{p-1}\right|<f$ where $f$ is a small positive number. An approximation $\mu^{\prime}$ to $\mu$ is then calculated by Aitken's $\delta^{2}$-process ${ }^{4}$ :

$$
\begin{equation*}
\mu^{\prime}=\mu_{p}-\frac{\left(\mu_{p}-\mu_{p-1}\right)^{2}}{\mu_{p}-2 \mu_{p-1}+\mu_{p-2}} . \tag{21}
\end{equation*}
$$

Repetition of (17) is resumed with $\omega=\omega\left(\mu^{\prime}\right)$ and continued until

$$
\begin{equation*}
\sum^{m n}\left(t_{r}^{(p+1)}-t_{r}^{(p)}\right)^{2}<e \tag{22}
\end{equation*}
$$

where $e$ is a small positive number. Then $t^{(p+1)}$ is taken as the solution of (12) and substituted into (11) to give the required flow rate across the outer surface.

## 5 ACCURACY CHECKS

### 5.1 Convergence of the iterative process

Let $T\left(x_{i}, y_{j}\right)=t_{r}^{(p+1)}$ denote the final values of the temperatures computed as described above. Owing to the truncation of the process, equation (6) will not be satisfied exactly and, from it, new values $T_{1}\left(x_{i}, y_{j}\right)$ can be calculated:

$$
\begin{align*}
& T_{1}\left(x_{i}, y_{j}\right)=\frac{1}{2}\left\{\frac{1}{(\delta x)^{2}}+\frac{1}{(\delta y)^{2}}\right\}^{-1}\left[\frac{T\left(x_{i-1}, y_{j}\right)+T\left(x_{i+1}, y_{j}\right)}{(\delta x)^{2}}+\frac{T\left(x_{i}, y_{j-1}\right)+T\left(x_{i}, y_{j+1}\right)}{(\delta y)^{2}}\right\} \\
&  \tag{23}\\
& =T\left(x_{i}, y_{j}\right)+\frac{1}{2}\left\{\frac{1}{(\delta x)^{2}}+\frac{1}{(\delta y)^{2}}\right\}^{-1}\left\{\frac{x^{2} T\left(x_{i}, y_{j}\right)}{(\delta x)^{2}}+\frac{\delta_{j}^{2} T\left(x_{j}, y_{j}\right)}{(\delta y)^{2}}\right\}  \tag{24}\\
& \\
& =T\left(x_{i}, y_{j}\right)+\frac{1}{2}\left\{\frac{1}{(\delta x)^{2}}+\frac{1}{(\delta y)^{2}}\right\}^{-1} \nabla^{2} T\left(x_{i}, y_{j}\right)  \tag{25}\\
& \text { where } \quad \\
& \quad \delta_{x}^{2} T\left(x_{i}, y_{j}\right)=T\left(x_{i-1}, y_{j}\right)+T\left(x_{i+1}, y_{j}\right)-2 T\left(x_{i}, y_{j}\right) \\
& \text { and } \quad \delta_{y}^{2} T\left(x_{i}, y_{j}\right)=T\left(x_{i}, y_{j-1}\right)+T\left(x_{i}, y_{j+1}\right)-2 T\left(x_{i}, y_{j}\right)
\end{align*}
$$

Equation (23) gives an indication of the error in $T\left(x_{i}, y_{j}\right)$ and, from equation (11), since $T_{W}\left(x_{i}\right)$ is fixed, the corresponding error in $Q\left(x_{i}\right)$ is

$$
\begin{equation*}
E_{1}\left[Q\left(x_{i}\right)\right]=\frac{k}{\delta y}\left\{T_{1}\left(x_{i}, y_{n-1}\right)-T\left(x_{i}, y_{n-1}\right)\right\} \cdot \quad(1 \leqslant i \leqslant m) \tag{26}
\end{equation*}
$$

### 5.2 Aocuracy of the finite-difference approximation

A more accurate representation of the Laplacian operator $\nabla^{2}=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}$ then has been used ebove is given by

$$
\begin{align*}
& \nabla^{2} T\left(x_{i}, y_{j}\right) \div \frac{1}{(\delta x)^{2}}\left\{\delta_{x}{ }^{2} T\left(x_{i}, y_{j}\right)-\frac{1}{12} \delta_{x}{ }^{4} T\left(x_{i}, y_{j}\right)\right\} \\
& \quad+\frac{1}{(\delta y)^{2}}\left\{\delta_{y}{ }^{2} T\left(x_{i}, y_{j}\right)-\frac{1}{12} \delta_{y}{ }^{4} T\left(x_{i}, y_{j}\right)\right\} \tag{27}
\end{align*}
$$

where

and
$\delta_{y}^{4} T\left(x_{i}, y_{j}\right)=T\left(x_{i}, y_{j-2}\right)+T\left(x_{i}, y_{j+2}\right)-4\left\{T\left(x_{i}, y_{j-1}\right)+T\left(x_{i}, y_{j+1}\right)\right\}+6 T\left(x_{i}, y_{j}\right)$

An improved value for $T\left(x_{i}, y_{j}\right)$ may be obtained by substituting (27) into (24); but to show separately the effects of the choice of step lengths in the two coordinate directions, it is convenient to calculate the following quantities:

$$
\begin{align*}
& T_{2}\left(x_{i}, y_{j}\right)=T\left(x_{i}, y_{j}\right)-\frac{1}{24}\left\{\frac{1}{(\delta x)^{2}}+\frac{1}{(\delta y)^{2}}\right\}^{-1} \delta_{x}^{4} T\left(x_{i}, y_{j}\right) \\
& T_{3}\left(x_{i}, y_{j}\right)=T\left(x_{i}, y_{j}\right)-\frac{1}{24}\left\{\frac{1}{(\delta x)^{2}}+\frac{1}{(\delta y)^{2}}\right\}^{-1} \delta_{y}^{4} T\left(x_{i}, y_{j}\right) . \tag{29}
\end{align*}
$$

When $j=n-1$, these formulae involve velues of $T$ at points outside the region ABPO; therefore it is assumed that the errors in $T\left(x_{i}, y_{n-1}\right)$ have the same orders of magnitude as those in $T\left(x_{i}, y_{n-2}\right)$ and the corresponding errors in $Q\left(x_{i}\right)$ are estimated by

$$
\begin{align*}
& E_{2}\left[Q\left(x_{i}\right)\right]=\frac{k}{\delta y}\left\{T_{2}\left(x_{i}, y_{n-2}\right)-T\left(x_{i}, y_{n-2}\right)\right\} \\
& E_{3}\left[Q\left(x_{i}\right)\right]=\frac{k}{\delta y}\left\{T_{3}\left(x_{i}, y_{n-2}\right)-T\left(x_{i} ; y_{n-2}\right)\right\} \quad(3 \leqslant i \leqslant m-2)
\end{align*}
$$

## 6 METHOD OF USE

A Mercury Autocode programe has been written which performs the calculations described in this papers it requires the following quantities as data, which must be punched on a data tape in the order given:
a length of the skin section, $A B$
b skin thickness, OA
$m$ number of grid-points on $A B$
$1 / k \quad$ reciprccal of the thermal conductivity of the skin
$Q_{M}\left(x_{i}\right)$ for $1 \leqslant i \leqslant m$ measured heat flow rate at grid-points on op
$T_{W}\left(x_{i}\right)$ for $1 \leqslant i \leqslant m$ measured temperature at grid-points on $A B$
$n$ number of grid-points on $O A$, excluding 0
f, e see section 4
a distance of $A$ from the beginning of the cylindrical section of the model

The units in which $a, b, k, Q_{M}, T_{W}$, and $a$ are given must be consistent; if $k$ and $Q_{M}$ have been measured in units not consistent with those of the remaining quantities, the simplest procedure is to apply a suitable scaling factor to k , leaving $Q_{M}$ unchanged; the pragramme will then calculate $Q$ in the units in which $Q_{M}$ is given. Thus if, as in the example mentioned below, $a$ and $b$ are given in inches, $k$ in cog.s. units, $T_{W}$ in degrees $k$, and $Q_{M}$ in $\mathrm{CHO} / \mathrm{rt}^{2} / \mathrm{hr}$, the data should include, in place of $1 / \mathrm{k}$, the value of
$\frac{9 \times 1055}{5 \times(12 \times 60)^{2} \times 2.54 \mathrm{k}}$ 。

The appendix shows a specimen data sheet and the corresponding print-out of results, the computation of which occupied 95 seconds on the RAE Mercury computer. The first line of results gives the dimensions ( $A B$ by $O A$ ) of the
section of skin considered, the relaxation faotor used, and the number of iterations made in each stage of the calculation; the suoceeding table gives, for $1=1,2, \ldots ., m$, the velues of $\left(a+x_{i}\right), Q\left(x_{i}\right), T\left(x_{i}, 0\right), E_{1}\left[Q\left(x_{i}\right)\right]$, $E_{2}\left[Q\left(x_{i}\right)\right]$, and $E_{3}\left[Q\left(x_{i}\right)\right]$.

Fig. 5 shows $Q$ and $Q_{M}$ plotted against $x_{i}$ for this example.

## SYMBOLS

| A, B | initial and final points on outer surfaoe of section of skin considered |
| :---: | :---: |
| a | length of skin section $A B$ |
| b | skin thickness, OA |
| $e, f$ | see section 4 |
| h | heat transfer factor across outer surface of skin |
| k | thermal conductivity of skin |
| m | number of grid-points on $A B$ |
| $M=\left(m_{r s}\right)$ | matrix of coefficients in finite-difference equations, see equations (14) and (15) |
| n | number of grid-points on 0 , excluding 0 |
| 0, P | initial and final points on section of skin considered |
| $Q\left(\mathrm{x}_{\mathrm{i}}\right)$ | computed heat flow rate across outer surface of skin at ith gridmpoint |
| $Q_{M}\left(x_{1}\right)$ | measured heat flow rate aoross inner surface of skin at ith grid-point |
| $\underline{t}=\left(t_{r}\right)$ | vector of temperatures at grid-points, see section 3 |
| $\underline{t}^{(p)}$ | pth approximation to $\underline{t}$ according to equation (17) |
| $T\left(x_{i}, y_{j}\right)$ | temperature of skin at ( $i, j$ ) th grid-point |
| $\mathrm{T}_{\mathrm{R}}$ | recovery temperature at outer surface of skin |
| $T_{W}\left(x_{i}\right)$ | measured temperature at outer surface of skin at ith grid-point |

## SYMBOLS (CONMD)

$\underline{u}=\left(u_{r}\right) \quad \begin{gathered}\text { vector of right-hand-sides in finite-differenoe equations, } \\ \text { see equation (13) }\end{gathered}$
$x, y \quad$ coordinates measured along and into the skin from 0, see Fig. 3
$\delta x, \delta y \quad m e s h-l e n g t h s$ of finite-difference grid
a distance of A from beginning of cylindrical section of model
(3) relaxation factor

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## Title, etc.

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## APPENDIX

## SPECIMEN DATA AND RESULTS

(a) Data


|  | 4 |
| :--- | :--- |
| 0.05 | $n$ |
| $\because 0.0001$ | $e$ |
| $\because 0.1$ | $a$ |

(b) Results
$T$ DATE 14.4 .64
B.A.M. PIGGOTT JOB NO. 1655


| $4 \cdot 00$ | $46 \cdot 35$ | $265 \cdot 43$ |
| :--- | ---: | ---: |
| $4 \cdot 10$ | $53 \cdot 92$ | $265 \cdot 52$ |
| $4 \cdot 20$ | $55 \cdot 27$ | $265 \cdot 52$ |
| $4 \cdot 30$ | $63 \cdot 43$ | $265 \cdot 43$ |
| $4 \cdot 40$ | $78 \cdot 19$ | $265 \cdot 28$ |
| $4 \cdot 50$ | $86 \cdot 39$ | $265 \cdot 01$ |
| $4 \cdot 60$ | $91 \cdot 25$ | $264 \cdot 74$ |
| $4 \cdot 70$ | $105 \cdot 78$ | $264 \cdot 48$ |
| $4 \cdot 80$ | $108 \cdot 96$ | $264 \cdot 27$ |
| $4 \cdot 90$ | $120 \cdot 66$ | $264 \cdot 20$ |
| $5 \cdot 00$ | $122 \cdot 15$ | $264 \cdot 15$ |
| $5 \cdot 10$ | $122 \cdot 96$ | $264 \cdot 16$ |
| $5 \cdot 20$ | $126 \cdot 89$ | $264 \cdot 23$ |
| $5 \cdot 30$ | $129 \cdot 58$ | $264 \cdot 32$ |
| $5 \cdot 40$ | $131 \cdot 92$ | $264 \cdot 42$ |
| $5 \cdot 50$ | $134 \cdot 68$ | $264 \cdot 54$ |
| $5 \cdot 60$ | $133 \cdot 75$ | $264 \cdot 65$ |
| $5 \cdot 70$ | $133 \cdot 79$ | $264 \cdot 73$ |
| $5 \cdot 80$ | $134 \cdot 10$ | $264 \cdot 82$ |
| $5 \cdot 90$ | $134 \cdot 52$ | $264 \cdot 89$ |
| $6 \cdot 00$ | $134 \cdot 98$ | $264 \cdot 97$ |
| $6 \cdot 10$ | $135 \cdot 51$ | $265 \cdot 06$ |
| $6 \cdot 20$ | $136 \cdot 77$ | $265 \cdot 14$ |
| $6 \cdot 30$ | $133 \cdot 66$ | $265 \cdot 20$ |
| $6 \cdot 40$ | $135 \cdot 35$ | $265 \cdot 26$ |
| $6 \cdot 50$ | $137 \cdot 38$ | $265 \cdot 32$ |
| $6 \cdot 60$ | $135 \cdot 84$ | $265 \cdot 35$ |
| $6 \cdot 70$ | $135 \cdot 34$ | $255 \cdot 36$ |
| $6 \cdot 80$ | $135 \cdot 18$ | $255 \cdot 37$ |


| $2 \cdot 7$, | -4 |
| ---: | ---: |
| $9 \cdot 0$, | -6 |
| $-1 \cdot 8$, | -4 |
| $1 \cdot 1$, | -4 |
| $1 \cdot 8$, | -5 |
| $-3 \cdot 5$, | -4 |
| $6 \cdot 7$, | -5 |
| $-4 \cdot 0$, | -4 |
| $-6 \cdot 9$, | -4 |
| $-5 \cdot 6$, | -4 |
| $-3 \cdot 1$, | -4 |
| $7 \cdot 1$, | -5 |
| $-4 \cdot 7$, | -4 |
| $-2 \cdot 1$, | -4 |
| $-2 \cdot 0$, | -4 |
| $-1 \cdot 7$, | -4 |
| $-8 \cdot 1$, | -5 |
| $-3 \cdot 7$, | -4 |
| $-3 \cdot 0$, | -4 |
| $-2 \cdot 4$, | -4 |
| $-2 \cdot 9$, | -4 |
| $-3 \cdot 4$, | -5 |
| $-8 \cdot 8$, | -5 |
| $-1 \cdot 1$, | -5 |
| $-1 \cdot 5$, | -4 |
| $-1 \cdot 4$, | -4 |
| $3 \cdot 7$, | -5 |


| $-3 \cdot 2$, | -2 |
| ---: | ---: |
| $-6 \cdot 1$, | -2 |
| $4 \cdot 2$, | -2 |
| $4 \cdot 8$, | -2 |
| $-7 \cdot 8$, | -2 |
| $-1 \cdot 8$, | -2 |
| $8 \cdot 0$, | -2 |
| $-8 \cdot 0$, | -2 |
| $1 \cdot 1$, | -1 |
| $-8 \cdot 9$, | -2 |
| $9 \cdot 8$, | -3 |
| $3 \cdot 6$, | -2 |
| $-1 \cdot 0$, | -2 |
| $-3 \cdot 0$, | -3 |
| $1 \cdot 7$, | -2 |
| $-3 \cdot 5$, | -2 |
| $7 \cdot 5$, | -3 |
| $3 \cdot 4$, | -3 |
| $1 \cdot 3$, | -3 |
| $8 \cdot 2$, | -4 |
| $6 \cdot 1$, | -3 |
| $5 \cdot 0$, | -3 |
| $-4 \cdot 0$, | -2 |
| $4 \cdot 5$, | -2 |
| $4 \cdot 1$, | -3 |
| $-3 \cdot 5$, | -2 |
| $8 \cdot 3$, | -3 |


| $-7 \cdot 0$, | -3 |
| ---: | ---: |
| $-1 \cdot 3$, | -2 |
| $9 \cdot 2$, | -3 |
| $1 \cdot 1$, | -2 |
| $-1 \cdot 7$, | -2 |
| $-4 \cdot 1$, | -3 |
| $1 \cdot 8$, | -2 |
| $-1 \cdot 8$, | -2 |
| $2 \cdot 5$, | -2 |
| $-2 \cdot 0$, | -2 |
| $2 \cdot 1$, | -3 |
| $8 \cdot 0$, | -3 |
| $-2 \cdot 3$, | -3 |
| $-6 \cdot 8$, | -4 |
| $3 \cdot 7$, | -3 |
| $-7 \cdot 7$, | -3 |
| $1 \cdot 6$, | -3 |
| $7 \cdot 5$, | -4 |
| $1 \cdot 7$, | -4 |
| $1 \cdot 2$, | -4 |
| $1 \cdot 4$, | -3 |
| $1 \cdot 1$, | -3 |
| $-8 \cdot 8$, | -3 |
| $1 \cdot 0$, | -2 |
| $9 \cdot 7$, | -4 |
| $-7 \cdot 7$, | -3 |
| $1 \cdot 9$, | -3 |



FIG. I. LONGITUDINAL SECTION OF WIND-TUNNEL MODEL.


FIG. 2. MEASUREMENT STATION IN SKIN OF MODEL.


FIG. 3. HEAT EQUATION AND BOUNDARY CONDITIONS FOR THE SKIN.


FIG. 4. THE FINITE-DIFFERENCE MESH.


FIG. 5. COMPARISON OF THE ObSERVED AND CALCULATED heat flow rates in a special case.
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In an experimant in mich it is required to masure the beat transfer factor at the cuter surface of the esin of a wind tunnel model, the temperature of the akin is measared at varions points on this surface and the heat fiow rate scross the skin is measured at the corresponding points on the inner surface. Calculation of the heat for rate across the outer gurface of the skin involves the salution of Laplacet 8 equation in a rectangular ragion with wixed boundary conditions. A mamerical rethod of solution is deseribed and illostrated by an example.
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In an experiment in mich it is required to measure the hear rransfer factor at the outer surface of the skin of a wind tunnel model, the temperature of the skin is measured at various points on this surface and the heat fiow rate across the skin is measured at the corresponding points on the imer surface. Calculation of the heat flom rate across the auter surface of the skin involves the solution of Laplace's equation in a rectangilar region with mixed boundary conditions. A namerical eethod of solution is described and illustrated by an example.

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