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By

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1966

Price 3s 6d. net

C.P. No. 882

May, 1965

Bulk Compressibility Effects in the R.A.R.D.E. No.3 Hypersonic Gun Tunnel

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SUMMARY

A procedure, based on existing methods and real gas tables, for estimating flow conditions in the working section of the hypersonic gun tunnel is outlined which includes the effects of bulk compressibility. It is also shown how bulk compressibility effects will modify the performance of the tunnel.

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1. Introduction

The flow conditions in the No.3 gun tunnel may be obtained from measurements of the flow velocity and the pitot and stagnation pressures. If the flow conditions are calculated assuming that the gas behaves as an ideal gas or that the gas is thermally perfect but calorically imperfect (i.e. if the charts given in [1] are used) errors can arise from the neglect of effects of bulk compressibility. (For the sake of completeness these terms are defined in Appendix 1.) Consequently real gas tables must be used for these calculations and this note outlines the procedure and gives some examples.

In addition to modifying the flow through the nozzle further effects due to bulk compressibility will become increasingly apparent as the driving pressure is raised. These include a lower stagnation pressure for a given driving pressure, a lower isentropic stagnation temperature for a given compression ratio, a shorter running time for the high pressure portion of the 'run' and a longer total running time. These additional effects are considered briefly and the predicted trends obtained from real gas tables compared with data obtained by A. J. Cable and the author during initial tests in which the breech pressure was increased exceptionally to 13 000 p.s.i. Throughout this memorandum it is assumed that both the working gas and the driving gas are nitrogen. Some properties of nitrogen are given in Appendix 2.

2./

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2. Stagnation Temperature as a Function of Flow Velocity

The stagnation temperature may be obtained as follows:

The ideal stagnation temperature T_{ti} may be written as

$$T_{ti} = \frac{V^2\left(1 + \frac{\gamma_i - 1}{2} M_i^2\right)}{\gamma_i R M_i^2} . \qquad (1)$$

The real stagnation temperature may then be obtained by the method given by Wilson and Regan [2] by using the chart given in Fig.2 of their report since the real stagnation pressure is measured directly (Wilson and Regan use T_{EQ} where this memorandum has T_{ti}). This method assumes that the gas is in equilibrium throughout the nozzle. The ideal Mach number obtained from the ratio of the pitot and stagnation pressures may be used in equation (1) since the stagnation temperature is insensitive to small changes in Mach number provided the Mach number is high and furthermore it is shown in Section 3 that the real Mach number for normal operating conditions is the same as the ideal Mach number within the experimental accuracy.

Using the method given above the real stagnation temperature is plotted in Fig.1 as a function of flow velocity for nominal Mach numbers of 8·5, 10·4 and 12·9 and a stagnation pressure of 4015 p.s.i., and compared with the ideal temperature. It may be seen that the real gas effects are considerable, for instance at a flow velocity of 6000 ft/sec the real gas temperature is about 10% less than the ideal temperature.

3. Mach Number as a Function of Pitot Pressure/Stagnation Pressure

Wilson and Regan [2] found that, provided the Mach number is greater than 3 and the pitot pressure is less than 40 atmospheres, the ratio of total pressure behind a normal shock for real and ideal gases for the same free stream conditions is almost independent of pressure and Mach number. This enabled the correction factor for the total pressure ratio across a normal shock to be plotted as a function of temperature only (Fig.3 of [2]). Using this plot the real Mach number may be calculated, and the method is illustrated by the following example:

Consider the measured values to be:

$$\left\{\frac{p_{t2}}{p_{t1}}\right\}_r = 6{\cdot}59 \times 10^{-3}$$

$$V = 6000 \text{ ft/sec}$$

$$p_{t1r} = 4015 \text{ p.s.i. (273 atm.)}$$

From/

From Ref.[1] $M_i = 8.46$

and equation (1) then gives

$$T_{ti} \approx 1722°K = T_{EQ}.$$

From Fig.2 of Ref.[2]

$$\left(\frac{T_{EQ}}{T_{tr}}\right) = 1{\cdot}10 .$$

Hence $T_{tr} = 1560°K$.

Then $\dfrac{p_{t2i}}{p_{t2r}} = 0{\cdot}992$ from Fig.3 of Ref.[2] and

$\dfrac{p_{t1i}}{p_{t1r}} = 0{\cdot}93$ from Fig.1 of Ref.[2]

and since

$$\left\{\frac{p_{t2}}{p_{t1}}\right\}_i = \left\{\frac{p_{t2}}{p_{t1}}\right\}_r \left\{\frac{p_{t2i}}{p_{t2r}}\right\} \Big/ \left\{\frac{p_{t1i}}{p_{t1r}}\right\}$$

then

$$\left\{\frac{p_{t2}}{p_{t1}}\right\}_i = 6{\cdot}59 \times 10^{-3} \times 0{\cdot}992/0{\cdot}93 = 7{\cdot}10 \times 10^{-3}$$

Use of tables in Ref.[1] gives $M_i = 8{\cdot}32$.

Using this method the real Mach number is plotted in Fig.2 as a function of flow velocity for nominal Mach numbers of 8·4, 10·4 and 12·9, and a stagnation pressure of 4015 p.s.i. Taking the experimental accuracy for the determination of the Mach number to be $\pm$ 0·2 it may be seen that the difference between the real gas and ideal gas Mach number is within this value for the normal operating range. The behaviour of the real Mach number curve appears anomalous at first since it might be expected that the real curve would collapse on to the ideal curve as the velocity is reduced. However these curves are for a constant stagnation pressure, consequently as the velocity is reduced the stagnation temperature is reduced, the stagnation density is increased and the effects of bulk compressibility become more pronounced.

4./

4. Stagnation Pressure as a Function of Initial Breech Pressure

The equilibrium stagnation pressure is plotted in Fig.3 as a function of the initial driving pressure assuming an isentropic expansion from the breech to the breech plus the barrel, i.e., volume increase of 9·3%. This curve was obtained from a temperature-entropy diagram for nitrogen prepared by F. Din [3]. (A large scale version of the chart shown on p.141 is published by British Oxygen Co.). The measured pressures, Fig.3, lie slightly higher than the predicted values but significantly lower than the ideal values. The discrepancy between the measured and predicted values may be accounted for if it is assumed that the gas in the breech before the run is at a temperature of about 400°K. This temperature was not measured explicitly during the experiments but a thermocouple, placed at the outlet of the second stage compressor, recorded temperatures in excess of this value during the pump-up period.

The measured stagnation pressures obtained during the 'first wave' phase of operation are also shown on Fig.3 for general interest. For these measurements the breech/barrel pressure ratio was 133 and the piston weight was 237 gm. Under these conditions the 'first wave' stagnation pressure is roughly equal to the initial breech pressure.

5. 'Running' Time as a Function of Breech Pressure

As mentioned in Section 4 the stagnation pressure during the 'first wave' phase of operation is considerably higher than the final isentropic pressure and if this effect were to be exploited stagnation pressures significantly higher than the initial breech pressures might be obtained relatively easily. (For an ideal gas the maximum theoretical increase in pressure which might be obtained by this method is 1·84 for a steady expansion [4] but unpublished calculations by N. B. Wood show that for the "steady-unsteady" expansion process in the gun tunnel this value is reduced to 1·73.) This higher pressure phase lasts whilst the "hammer" shock from the retarded piston travels back along the barrel and the reflected rarefaction from the breech travels back to the piston. This time, Δt, is significantly reduced as the breech pressure is increased because of the greatly increased speed of sound. A rough estimate of the 'first wave' running time may be obtained by assuming that Δt is the time taken for an acoustic wave to travel up and down the barrel containing gas at the final isentropic pressure and temperature. This time Δt_a was calculated from the speed of sound data given by Lewis and Neel [5] and the final isentropic pressure and temperature in the barrel obtained from a Mollier diagram [3] (as described in Section 4). Δt_a is plotted in Fig.4 and compared with the measured value of Δt, defined in the inset. The results show that the 'first wave' running time may be fairly well represented by the acoustic wave approximation and both the approximation and the measurements show that the 'first wave' running time is significantly reduced as the breech pressure is increased. These results are in qualitative agreement with those obtained at the NPL for a reflected shock tunnel [6].

The total running time is also plotted in Fig.4 and it may be seen that for a constant breech/barrel pressure ratio the net effect of the real gases is significantly to increase the running time.

6./

6. Isentropic Stagnation Temperature

The stagnation temperature obtained by isentropic compression alone is plotted in Fig.5 as a function of compression ratio and stagnation pressure. These curves were obtained from the Mollier diagram prepared by Wilson and Regan [7]. The compression ratio C is defined here as:

$$C = \frac{\text{measured stagnation pressure}}{\text{initial barrel pressure}}$$

The ideal temperature is also shown in Fig.5 and it may be seen that the real temperature is considerably lower than the ideal temperature. Estimates of stagnation temperature from flow measurements made by J. E. Bowman [8] show that for a breech pressure of 4000 p.s.i. the gas is heated non-isentropically so these real gas isentropic curves may only be used to indicate the lower limit to the stagnation temperature.

List of Symbols

C	Compression ratio $= \frac{\text{stagnation pressure}}{\text{initial barrel pressure}}$
M	Mach number
p	Pressure
R	Gas constant
T	Temperature
t	Run time
Δt	Run time for 'first wave' phase
Δt_a	Time for an acoustic wave to travel up and down the barrel
V	Velocity
γ	Ratio of specific heats

Subscripts

EQ	Equivalent
f	First wave
i	Ideal
r	Real
t	Stagnation
1	Upstream of a normal shock
2	Downstream of a normal shock

References/

References

No.	Author(s)	Title, etc.
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7	J. L. Wilson and J. D. Regan	Calculations of the thermodynamic properties of nitrogen at high pressures. A.R.C. C.P. 771. January, 1964.
8	J. E. Bowman	Determination of stagnation temperatures in the RARDE No.3 hypersonic gun tunnel from streak camera measurements of flow velocity. A.R.C. 27 768. February, 1966.

Definitions

A gas is thermally perfect when it obeys the equation of state given by

$$P = \rho RT$$

A gas is calorically perfect when its specific heats are constant.

The equation of state of a real gas may be written as

$$P = Z\rho RT$$

where Z is the bulk compressibility factor.

Properties of Nitrogen

This data is reproduced from Ref.[5].

Conditions at one atmosphere pressure (1 atm.) and 273·15°K

1 atm. $= 14{\cdot}70$ lbf/in^2 $=$ 2116 lbf/ft^2 $1{\cdot}013 \times 10^6$ dynes/cm^2

$=$ 760 mm Hg $= 1{\cdot}013 \times 10^5$ Newtons/m^2

Speed of sound $=$ 1106 ft/sec $=$ 0·3370 km/sec

Density $= 2{\cdot}424 \times 10^{-3}$ lbf sec^2/ft^4 or slugs/ft^3

$= 7{\cdot}794 \times 10^{-2}$ lbm/ft^3

$= 1{\cdot}249$ kg/m^3 $= 1{\cdot}249 \times 10^{-3}$ gm/cm^3

R $= 3197$ ft^2/sec^2 °K $= 1{\cdot}987$ cal/mole °K

$= 7{\cdot}094 \times 10^{-2}$ cal/gm °K or Btu/lbm°R

Molecular weight $=$ 28·01 gm/mole

FC

D76946/1/125875 K4 11/66 XL.

FIG. I

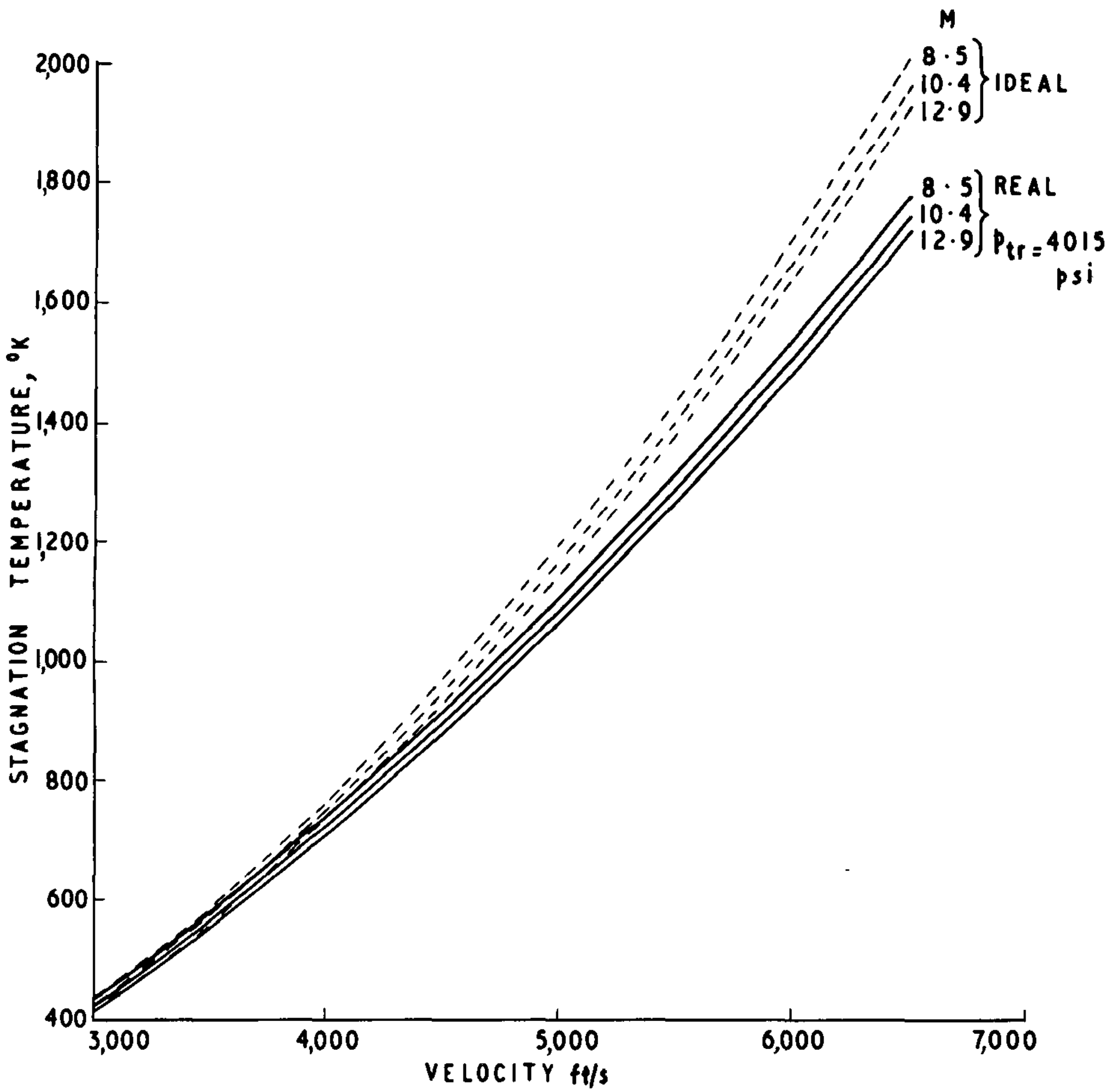

FIG I STAGNATION TEMPERATURE AS A FUNCTION OF FLOW VELOCITY

FIG. 2

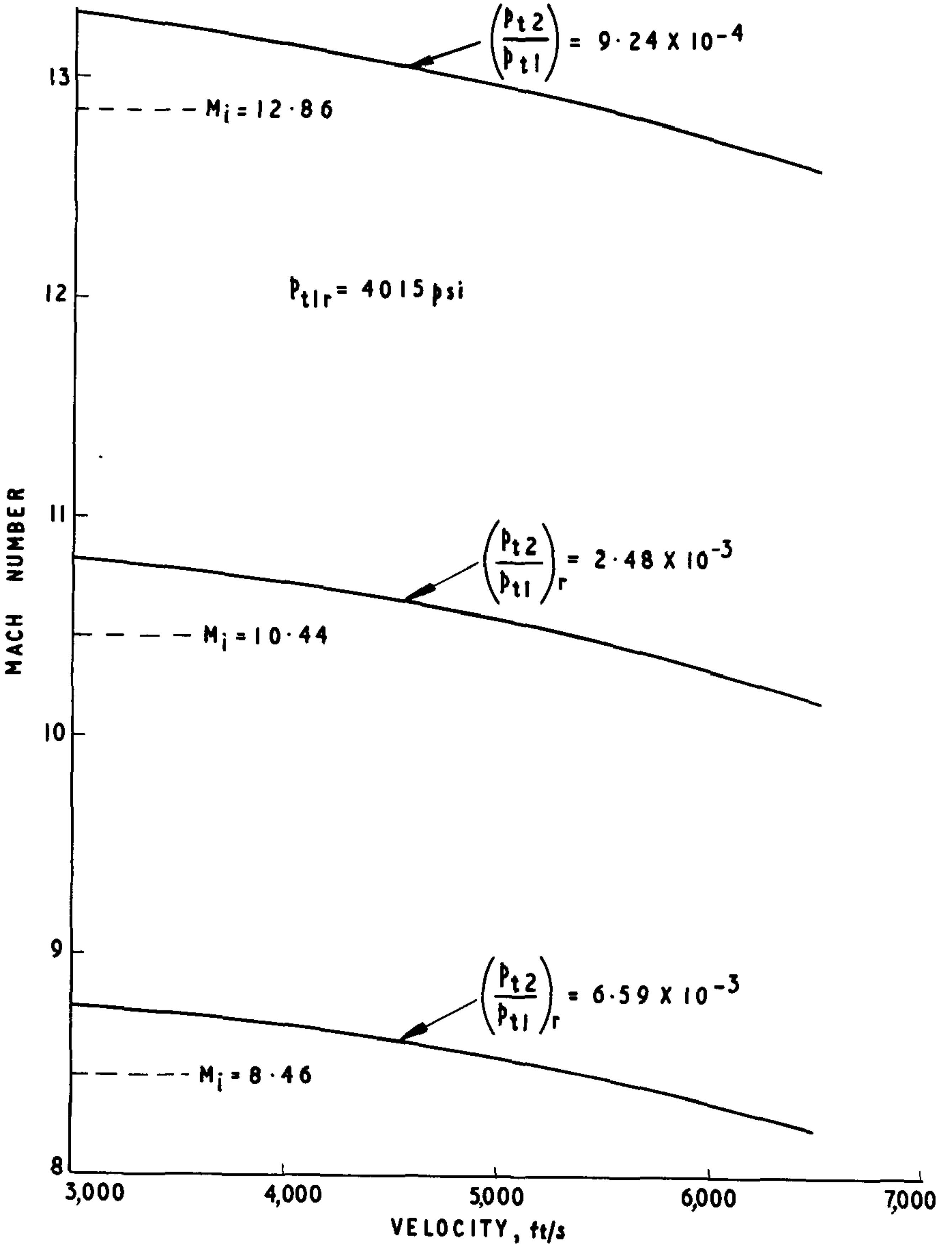

FIG. 2 MACH NUMBER AS A FUNCTION OF FLOW VELOCITY

FIG. 3

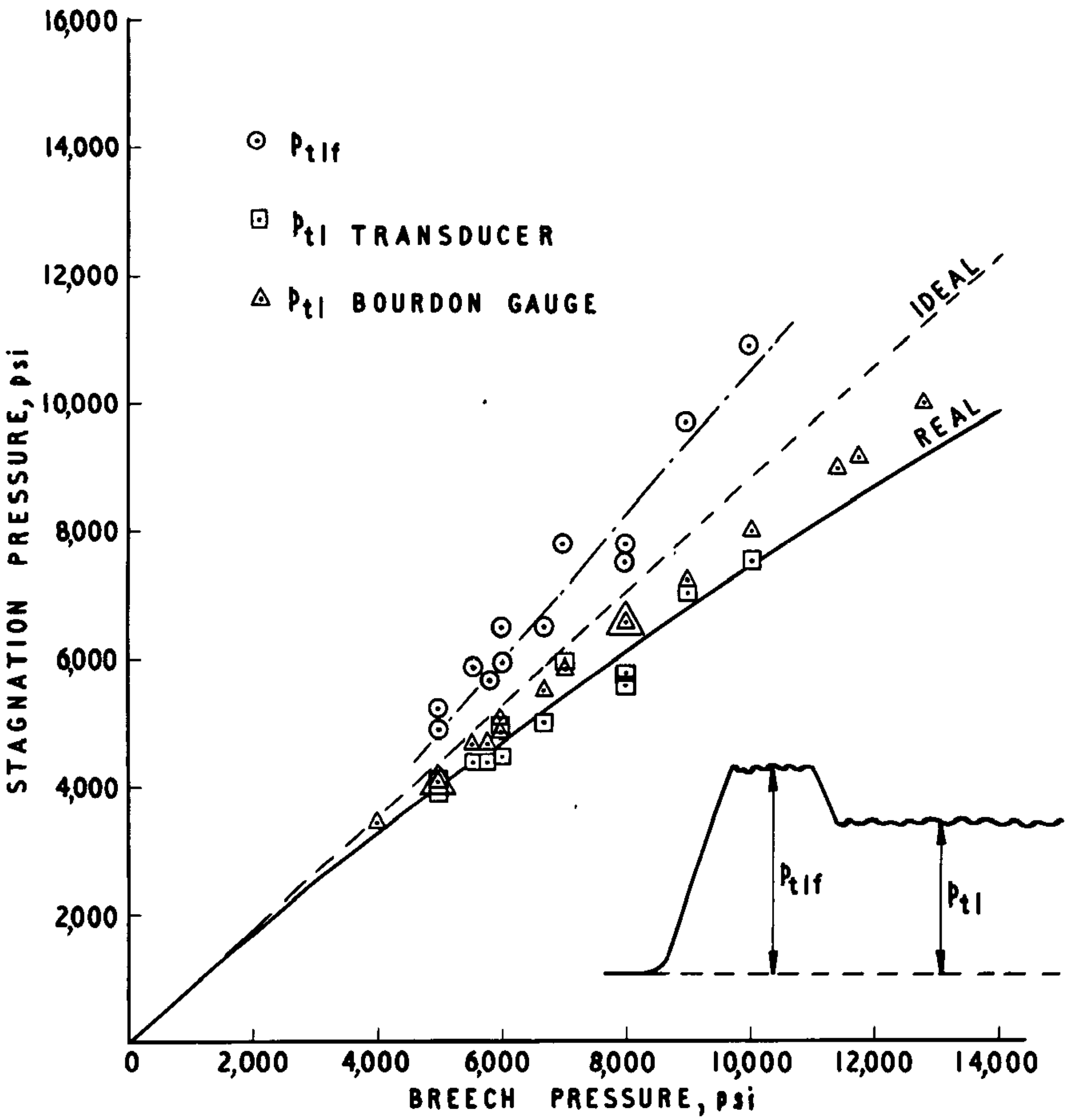

FIG. 3 STAGNATION PRESSURE AS A FUNCTION OF BREECH PRESSURE

FIG. 4

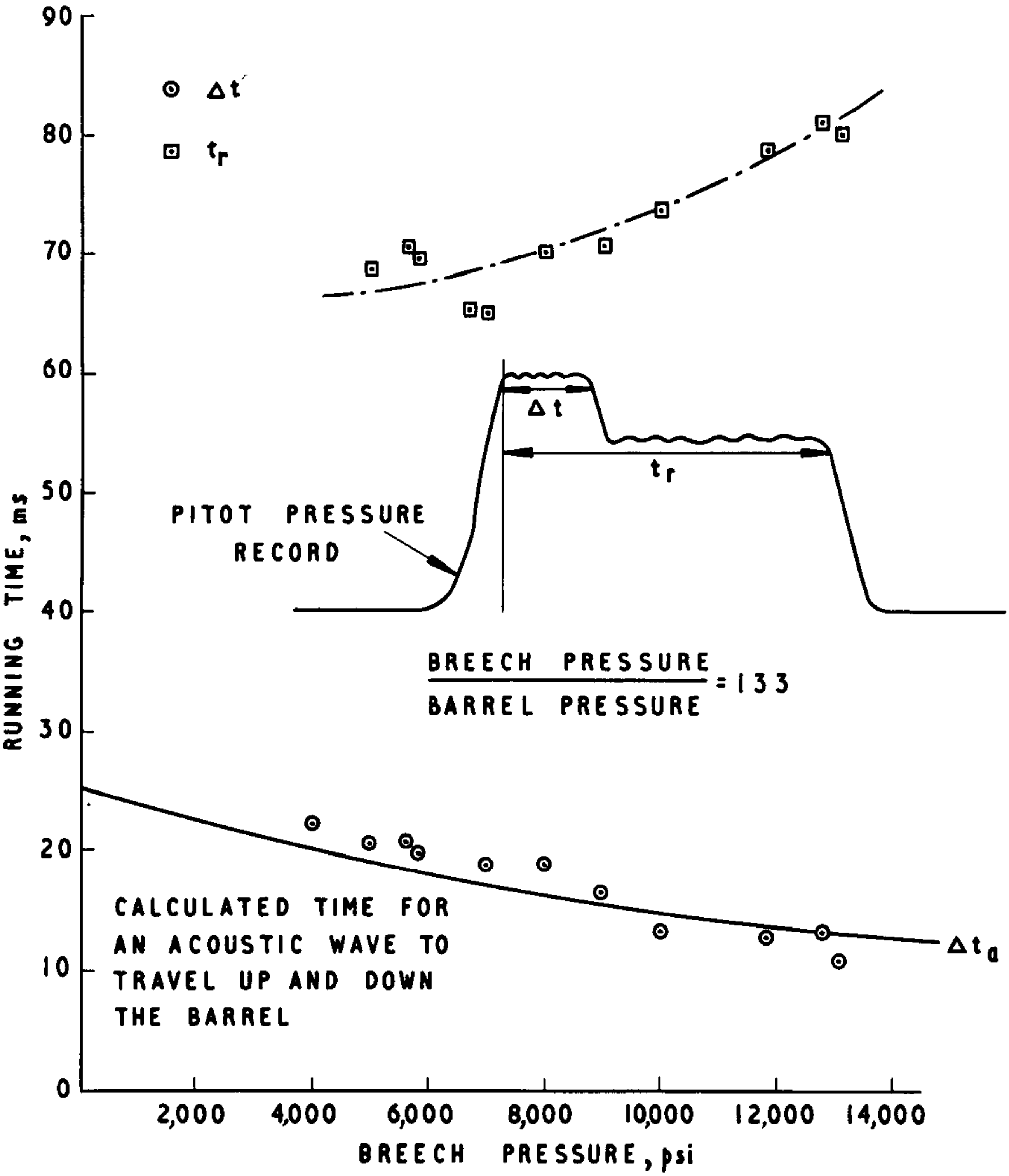

FIG. 4 RUNNING TIME AS A FUNCTION OF BREECH PRESSURE

FIG. 5

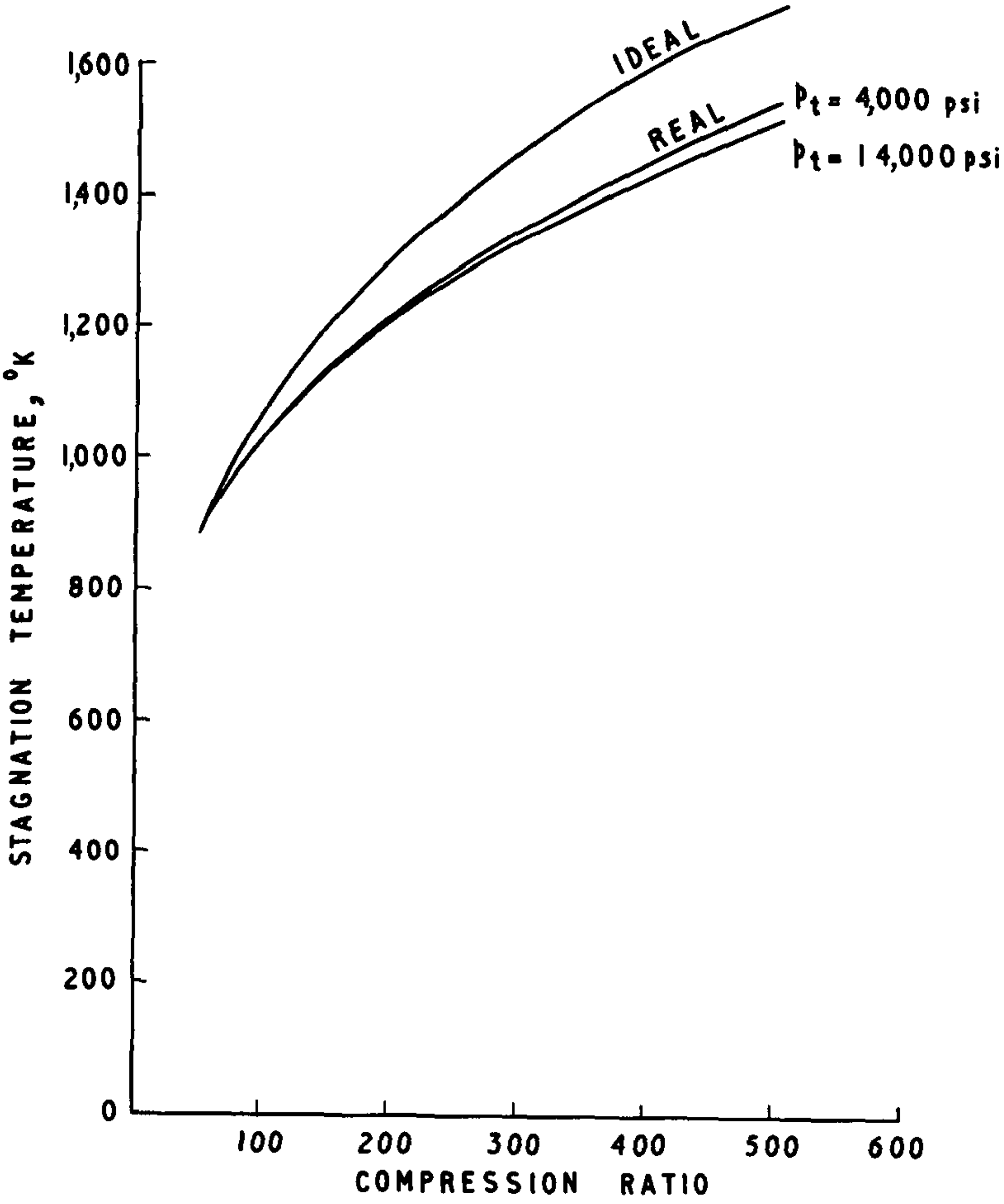

FIG. 5 STAGNATION TEMPERATURE AS A FUNCTION OF COMPRESSION RATIO

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S O Code No 23-9016-82